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Luca Tagliacozzo

Tensor Networks and Quantum Simulator for dynamical LGT

Many enlihtening discussions with:
M.C. Banuls, I. Cirac (MPQ), P. Corboz (ETH),
G. Evenbly (Caltech), F. Gliozzi (Torino),R. Orus (Mainz)
D. Perez Garcia(UCM), D. Poilblanc (Toulouse),
G. Sierra (Madrid), N. Schuch (Aachen),
F. Verstraete (Vienna), G. Vidal (PI)

Alessio Celi
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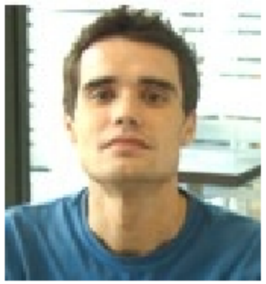
Peter Orland
CUNY

Maciej Lewenstein
ICFO

Morgan Mitchell
ICFO



Alejandro Zamora
ICFO

Motivation

Tensor Networks

Quantum simulators (QS)

Lattice Gauge Theories (LGT)

Tensor Networks 4 LGT

QS of GT with Rydberg atoms

Conclusions and outlook

Motivation

Tensor Networks

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Tensor Networks states

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$$\{A_i\} \longrightarrow |\psi(\{A\})\rangle$$

Tensors

Variational state

Tensor Networks states

i j

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Tensors

Variational state

Example:

out of equilibrium
dynamics of spin chain

$$H = \sum_{i,j} h_{ij}$$

$$h_{ij} = J_{ij}\vec{S}_i\vec{S}_j$$

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Tensor Networks states

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P. Hauke LT
PRL 111, 207202 (2013)

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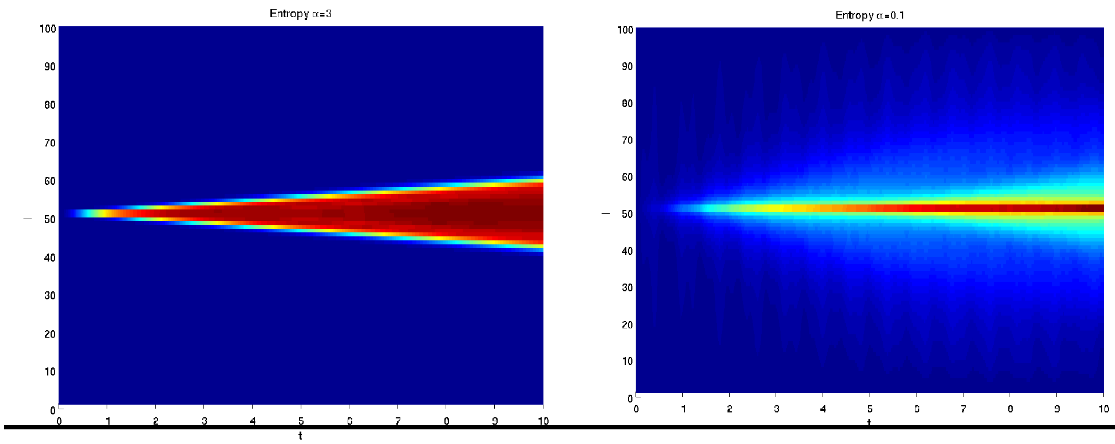

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Long Times, cold-atoms experiments

nature physics ARTICLES
PUBLISHED ONLINE: 19 FEBRUARY 2012 | DOI: 10.1038/NPHYS2232

Probing the relaxation towards equilibrium in an isolated strongly correlated one-dimensional Bose gas

S. Trotzky[1,2,3]*, Y-A. Chen[1,2,3], A. Flesch[4]*, I. P. McCulloch[5], U. Schollwöck[1,6], J. Eisert[6,7,8] and I. Bloch[1,2,3]

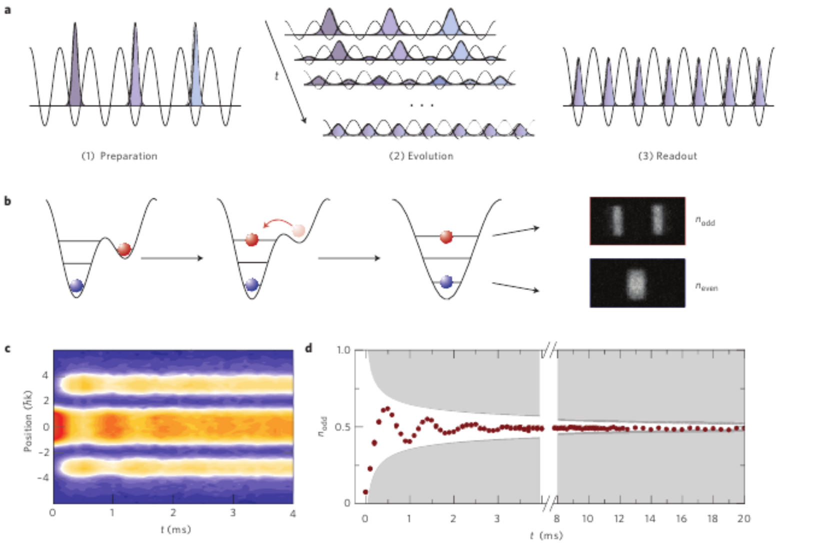

Figure 1 | Relaxation of the density pattern. a, Concept of the experiment: after having prepared the density wave $|\psi(t=0)\rangle$ (1), the lattice depth was rapidly reduced to enable tunnelling (2). Finally, the properties of the evolved state were read out after all tunnelling was again suppressed (3). **b**, Even–odd resolved detection: particles on sites with odd index were brought to a higher Bloch band. A subsequent band-mapping sequence was used to reveal the odd- and even-site populations[13,14]. **c**, Integrated band-mapping profiles versus relaxation time t for $h/(4J) \simeq 0.9$ ms, $U/J = 5.16(7)$ and $K/J \simeq 1.7\times10^{-2}$. **d**, Odd-site density extracted from the raw data shown in **c**. The shaded area marks the envelope for free bosons (light grey) and including inhomogeneities of the Hubbard parameters in the experimental system (dark grey stripe near border).

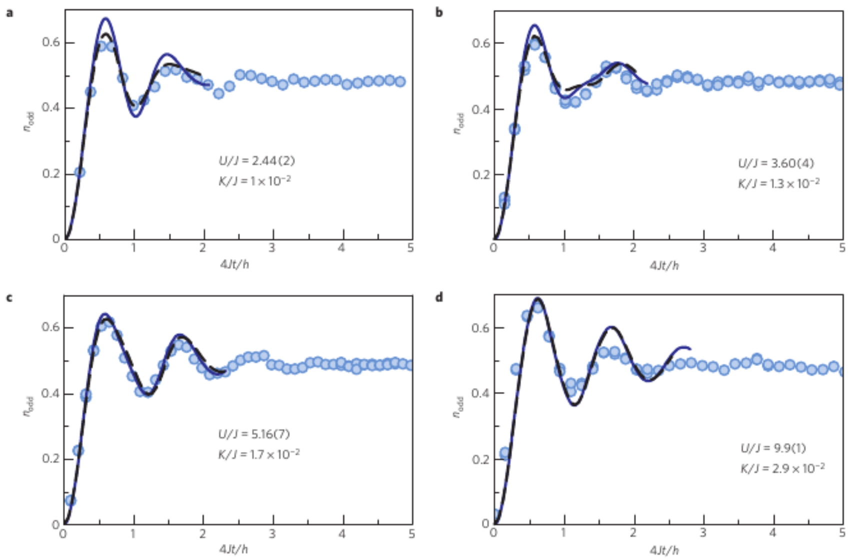

Figure 2 | Relaxation of the local density for different interaction strengths. We plot the measured traces of the odd-site population $n_{odd}(t)$ for four different interaction strengths U/J (circles). The solid lines are ensemble-averaged results from t-DMRG simulations without free parameters. The dashed lines represent simulations including next-nearest neighbour hopping with a coupling matrix element $J_{NNN}/J \simeq 0.12$ (**a**), 0.08 (**b**), 0.05 (**c**) and 0.03 (**d**) calculated from the single-particle band structure.

Directions

Directions

Improve tensor networks

Higher d

Longer Times

Exact Symmetries

Control on systematics

Directions

Higher d

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Exact Symmetries

Control on systematics

Propose interesting experiments for cold atoms

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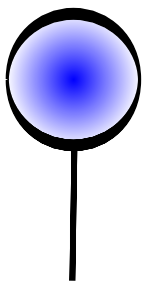
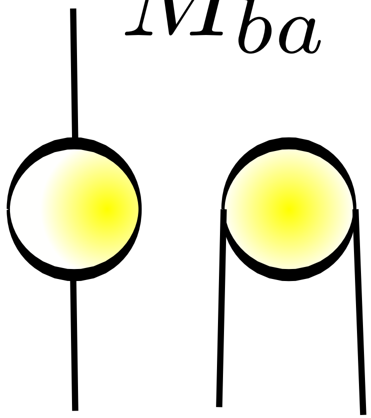
Notation

$$\vec{v} = v_a |i_a\rangle$$



$$M_{ba}$$



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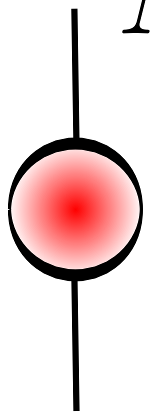
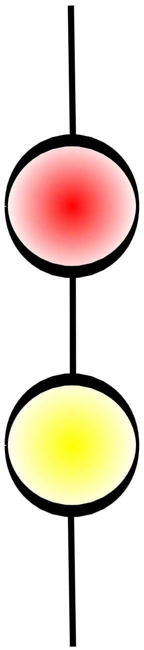
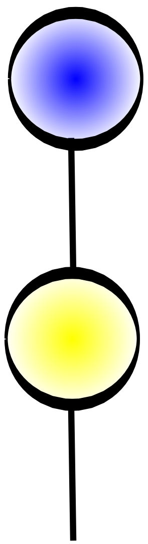
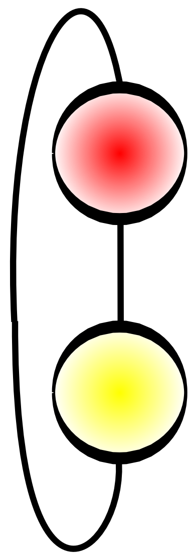


$$MN$$



$$M\vec{v}$$





Many Body quantum system

Statement of the problem

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$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \cdots \otimes \mathcal{H}_N$$

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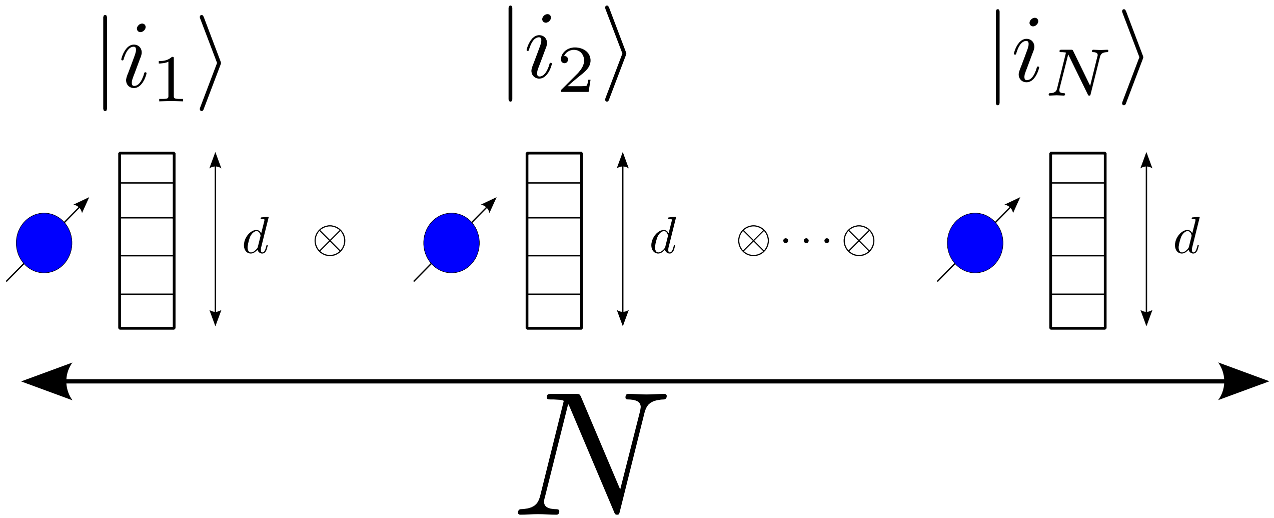

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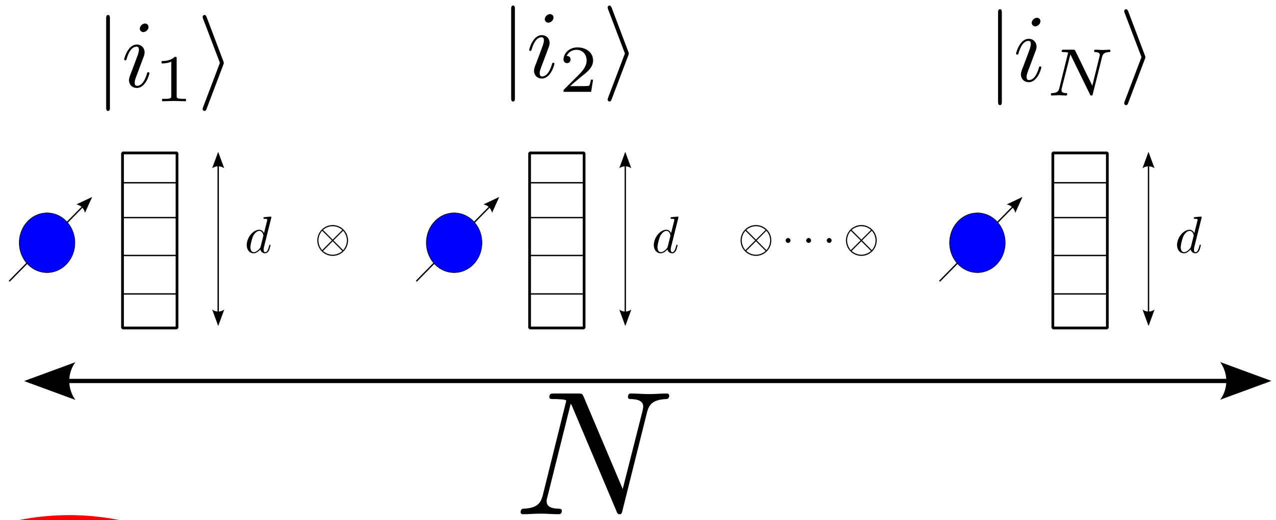

$$|\psi\rangle = c_{i_1 i_2 \cdots i_N} |i_1\rangle \otimes |i_2\rangle \cdots \otimes |i_N\rangle$$

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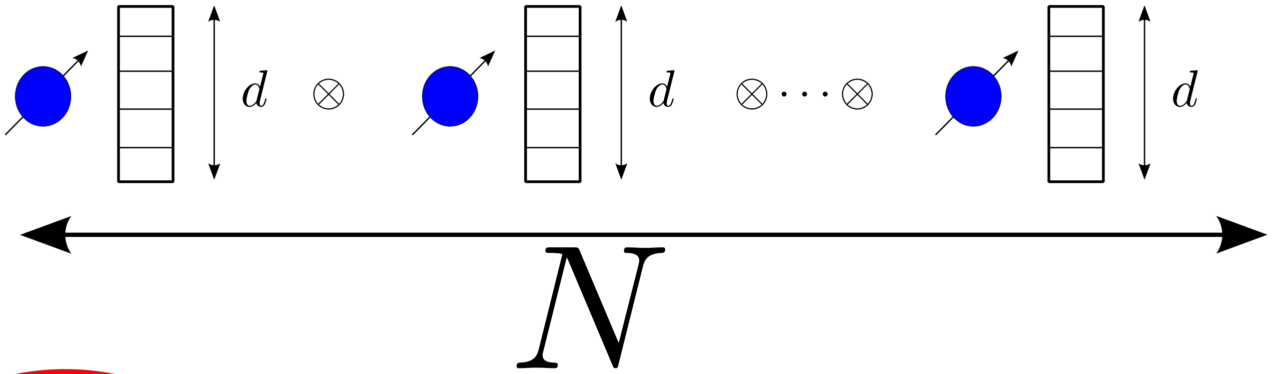
$$|i_1\rangle \qquad |i_2\rangle \qquad |i_N\rangle$$



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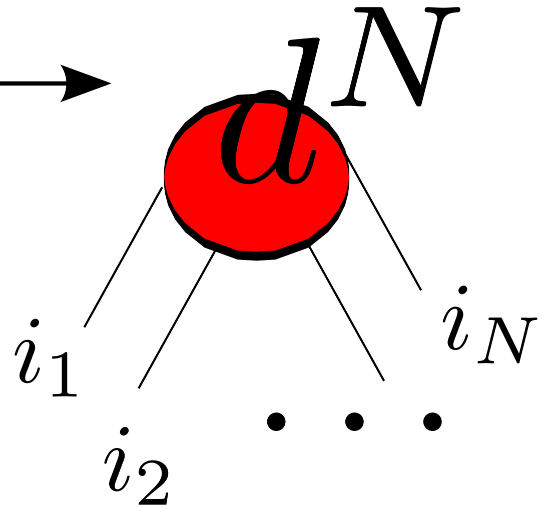
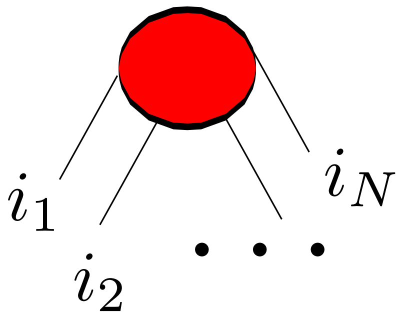


Tensor Network states



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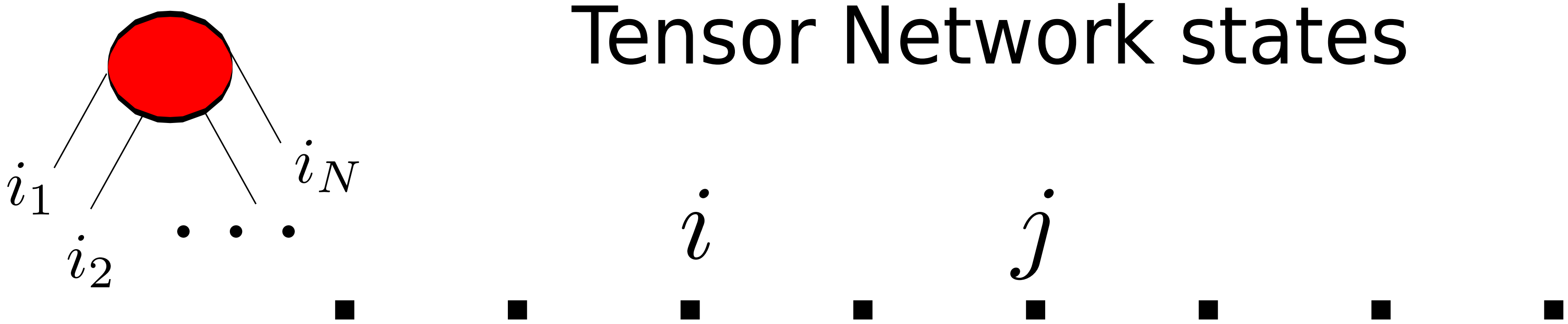

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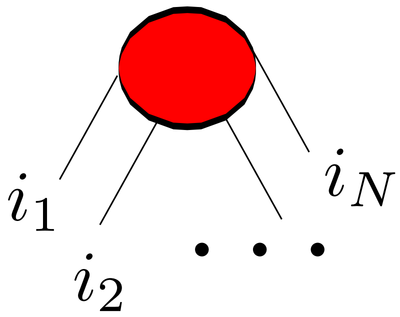

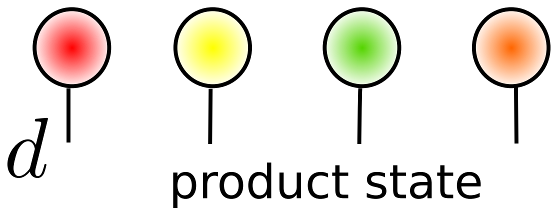

$$|\psi\rangle = |v_1\rangle|v_2\rangle|v_3\rangle|v_4\rangle$$
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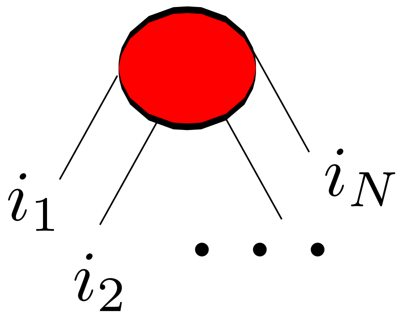
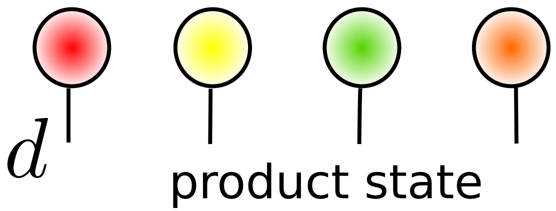
Tensor Network states





$$d^N \to dN$$

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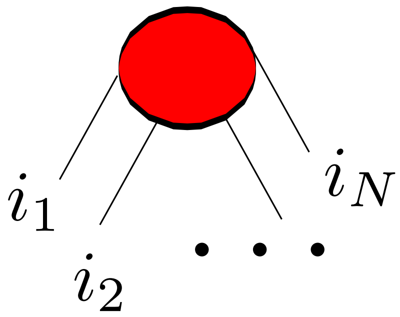

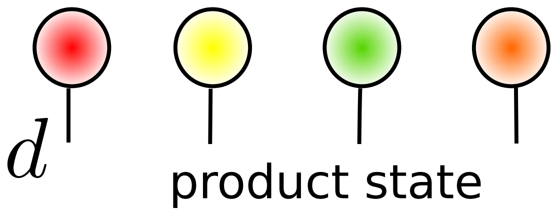

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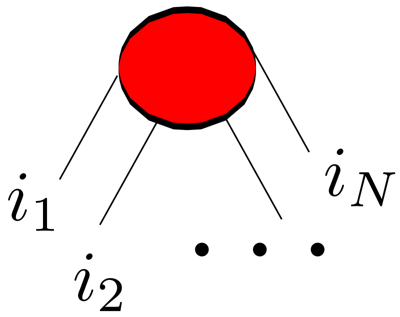
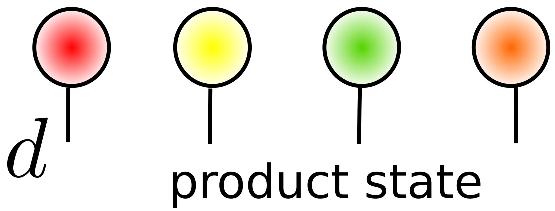
$$\langle\psi|O_iO_j|\psi\rangle = \langle O_i\rangle\langle O_j\rangle$$

Tensor Network states





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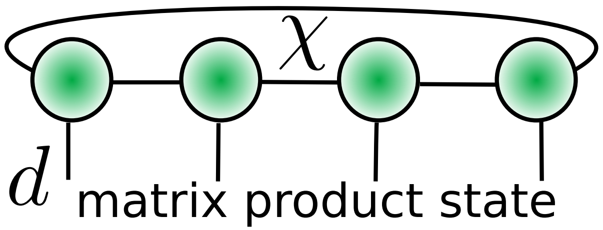
$$|\psi\rangle = \sum_{a,b,c,d} |A_1^{ab}\rangle|A_2^{bc}\rangle|A_3^{cd}\rangle|A_4^{da}\rangle$$



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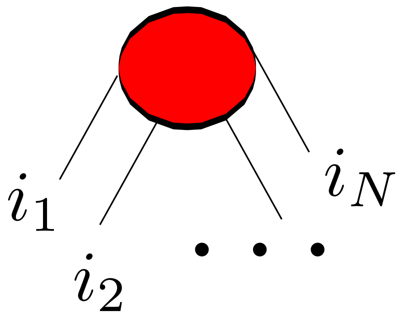
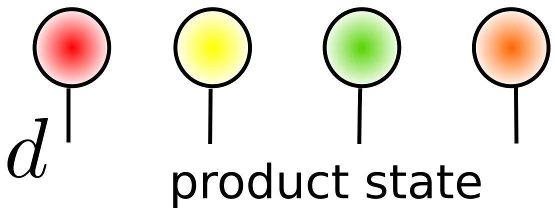
Tensor Network states





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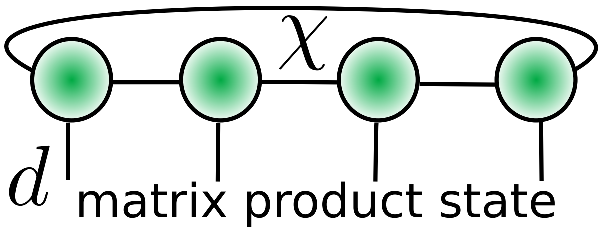

$$d^N \to dN\chi^3$$

$$|A_1^{ab}\rangle = \sum_i A_1^{i,ab}|i\rangle$$

$$\langle\psi|O_i O_j|\psi\rangle \neq \langle O_i\rangle\langle O_j\rangle$$

Tensor Network states

Tensor Network states

Hilbert space

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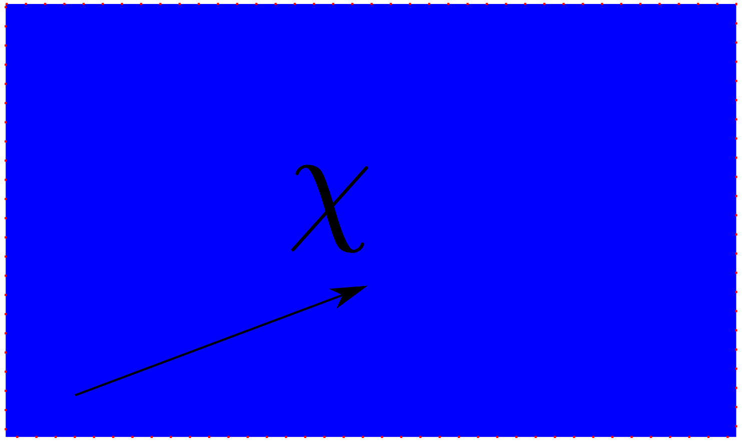
Tensor Network states

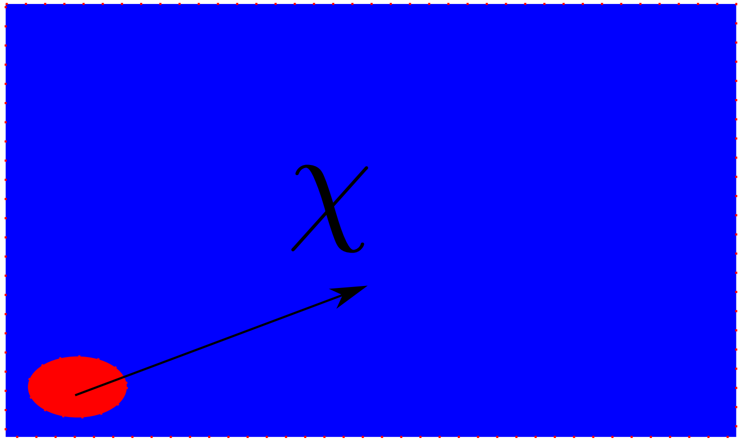
Hilbert space



Refinement parameter χ

Tensor Network states

Hilbert space



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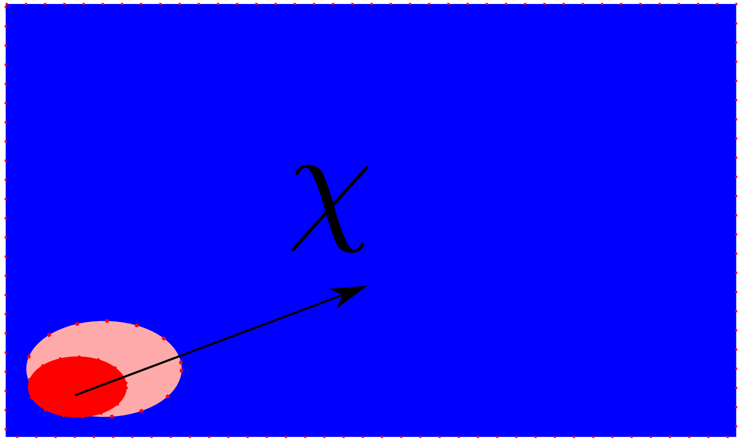

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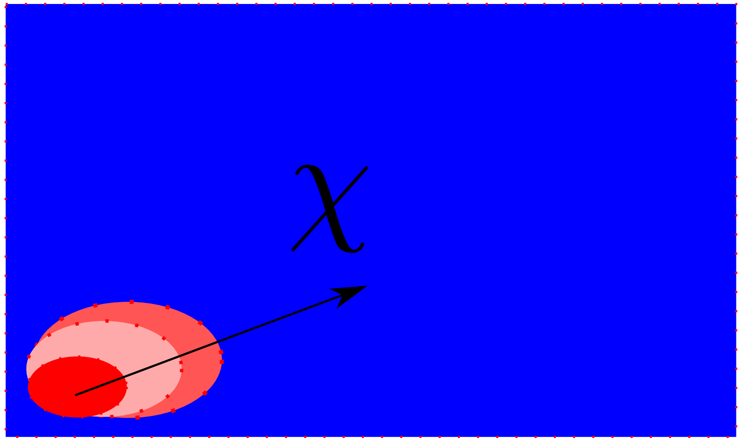

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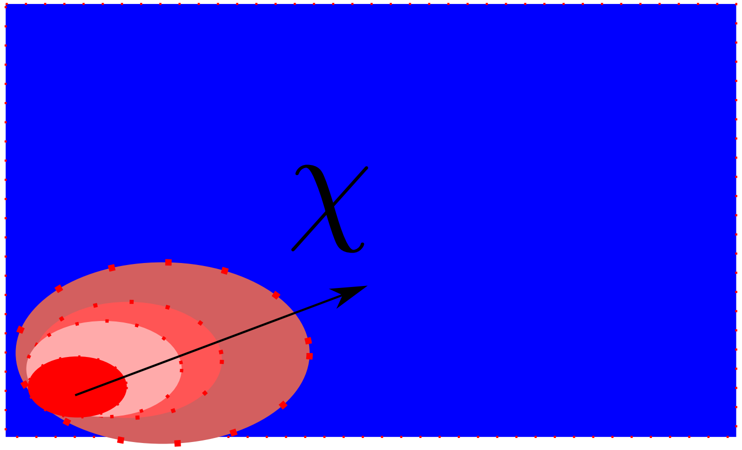

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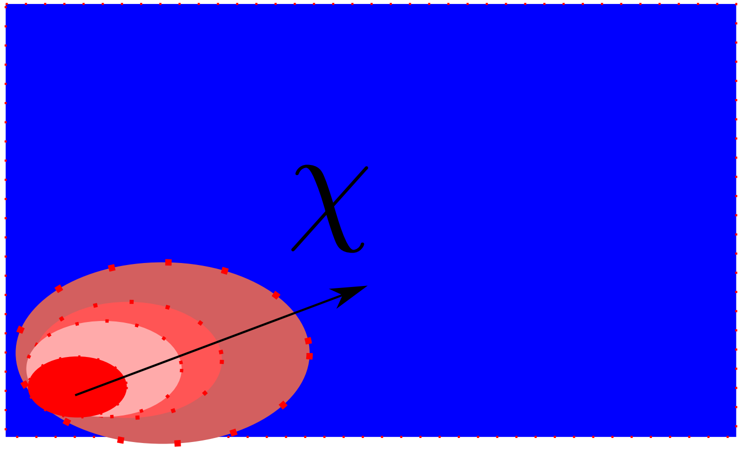

Refinement parameter χ

$$d^N \rightarrow dN\chi^n$$

Tensor Network states

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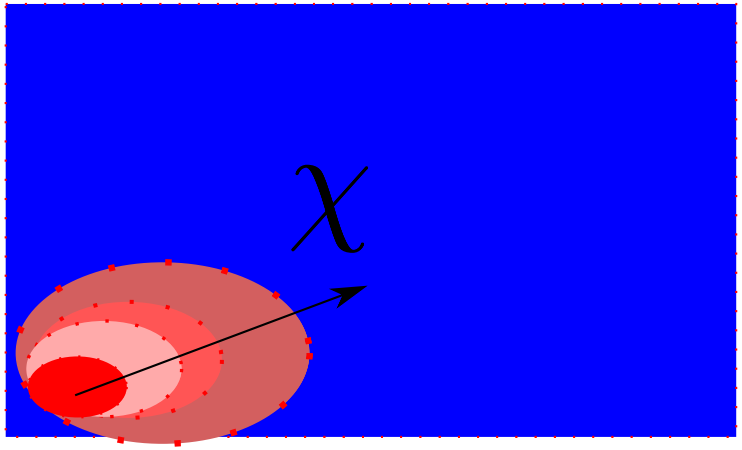
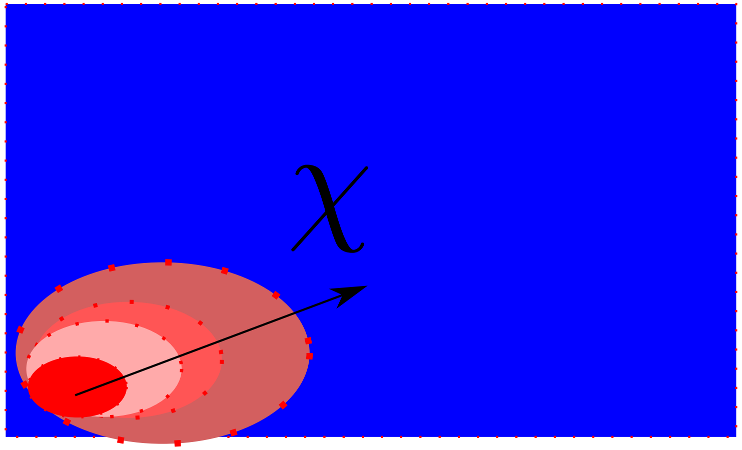


Refinement parameter χ

$$d^N \rightarrow dN\chi^n \quad \chi(N)?$$

Tensor Network states

Hilbert space



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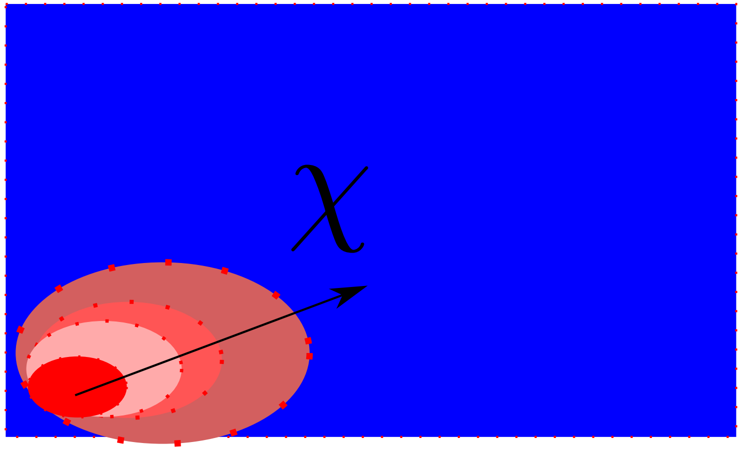
1D

No for gapped systems

Yes for gapless systems

Tensor Network states

Hilbert space



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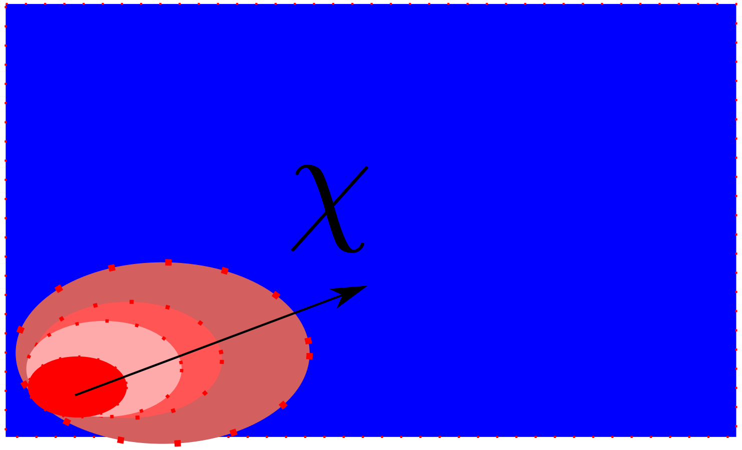

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LT et al.
Phys. Rev. B 78, 024410 (2008)
Phys. Rev. B 86, 075117 (2012)
ArXiv:1401.7654

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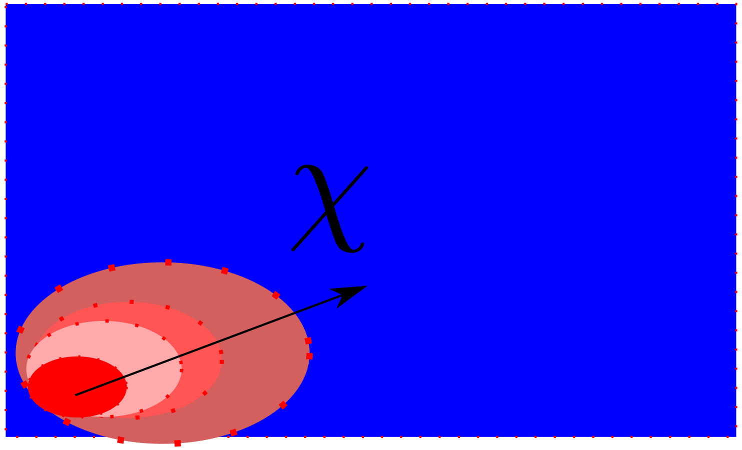

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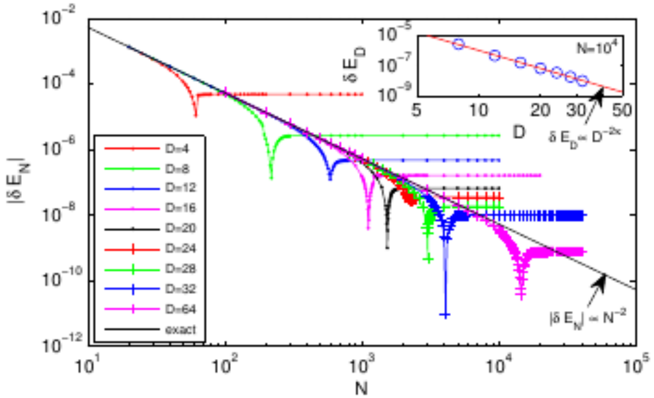
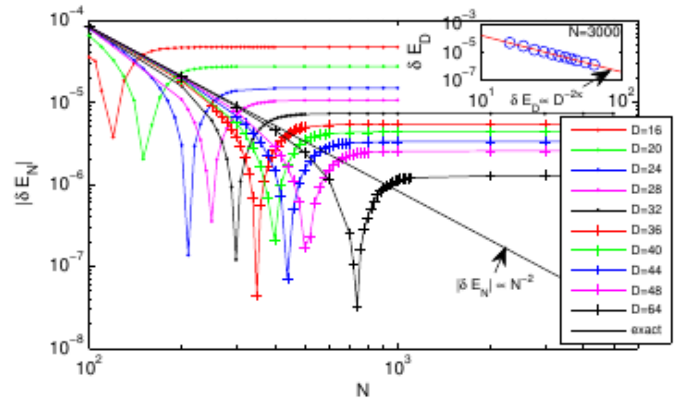


a)



b)

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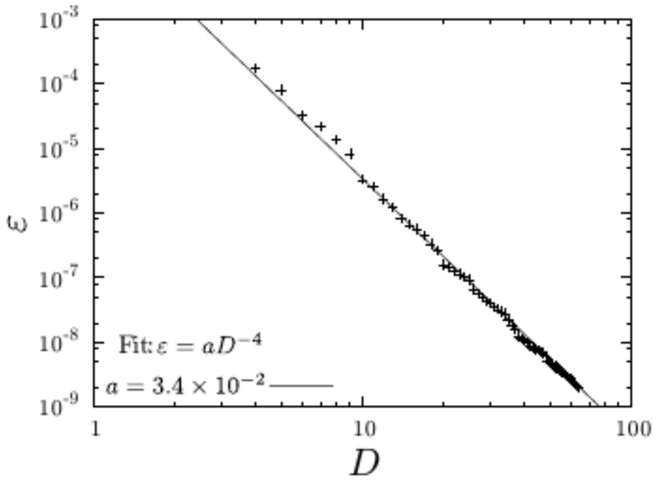

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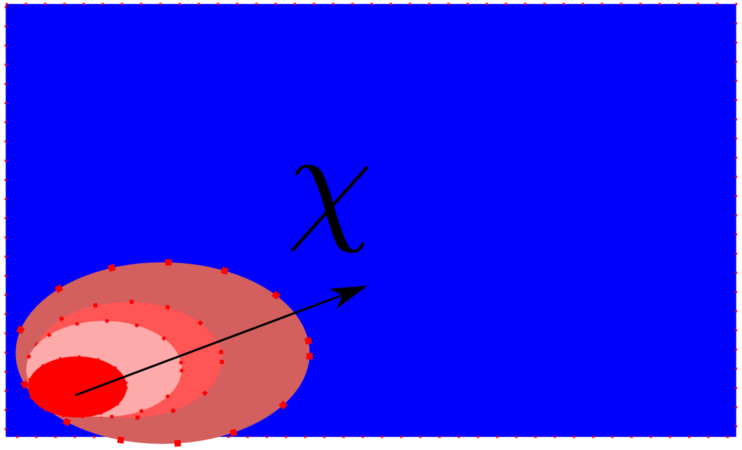

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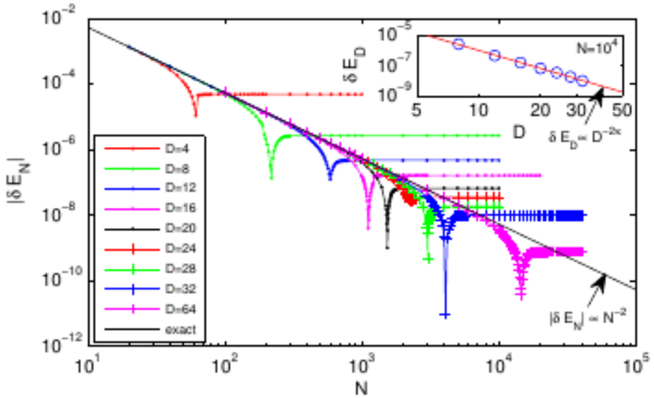

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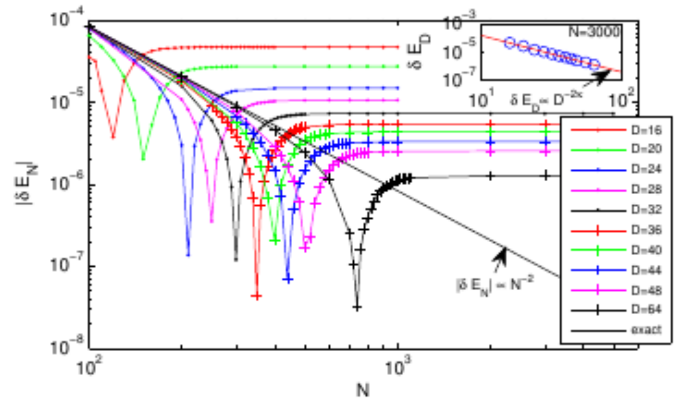

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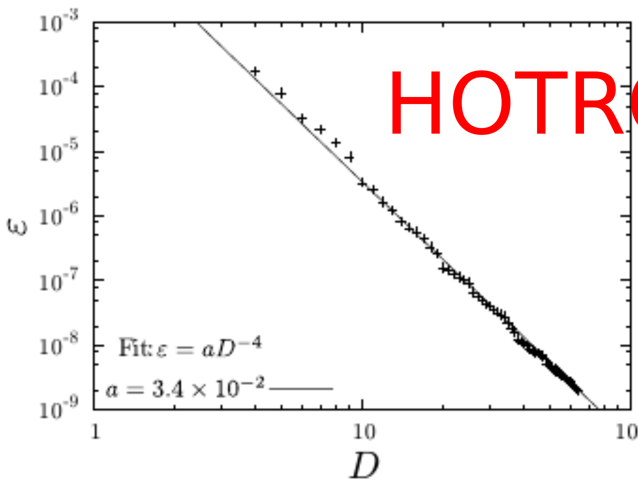

HOTRG

H. Ueda et. al
Phys. Rev. B 89, 075116 (2014)

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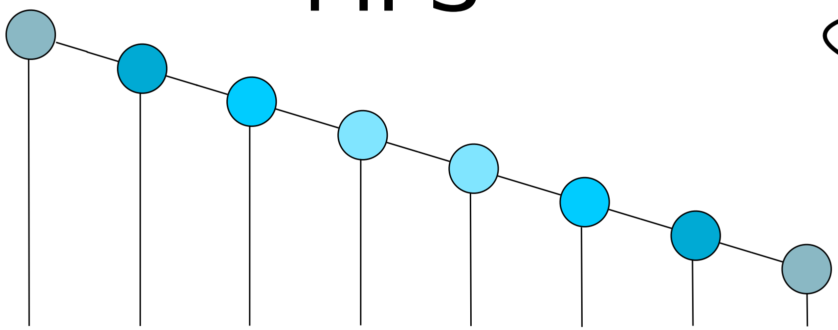
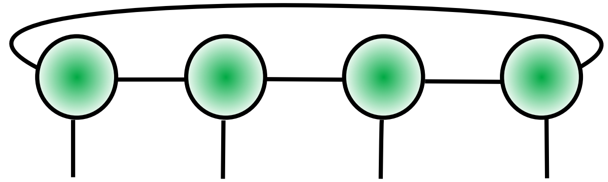
VARIOUS TN STRUCTURES

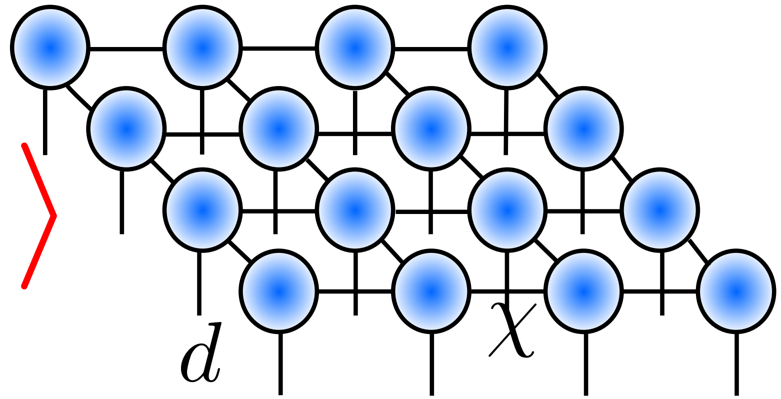
MPS



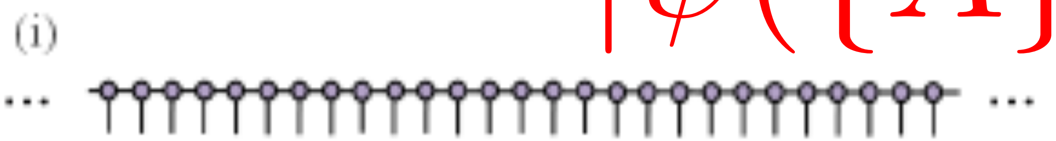


2D PEPS

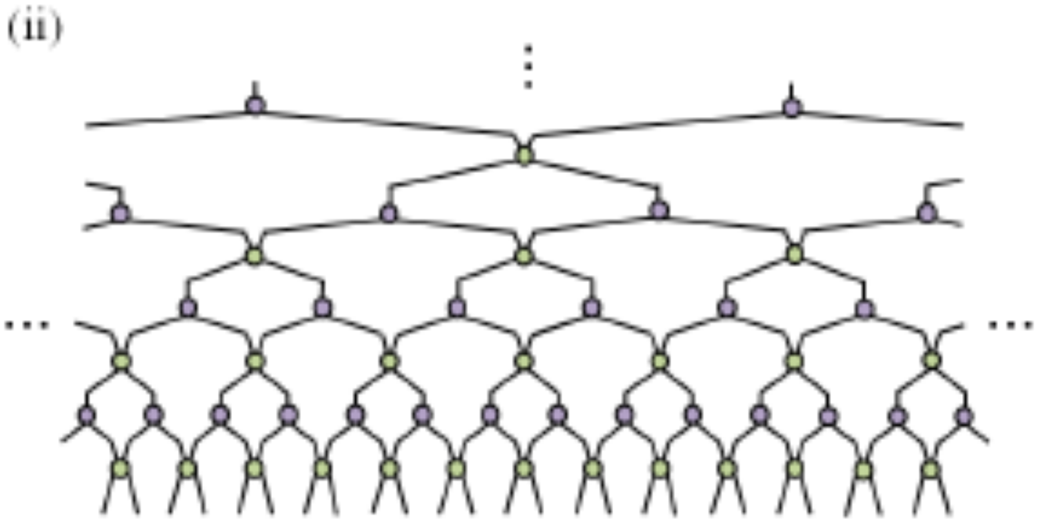
$$|\psi(\{A\})\rangle$$

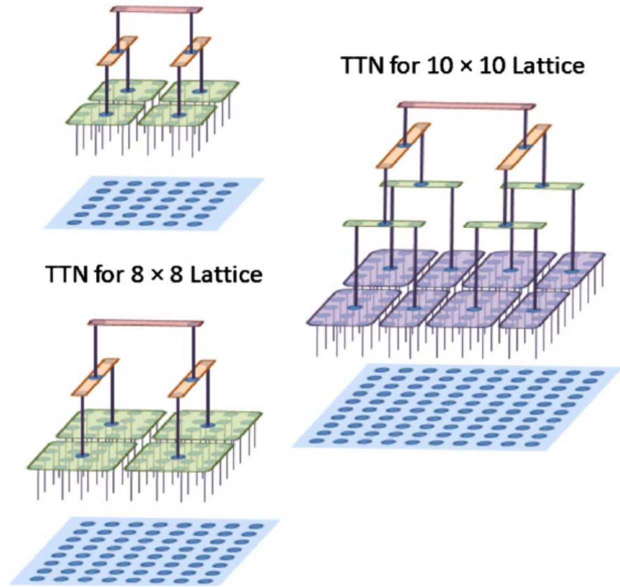


IMPS



MERA





2D TTN

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Motivation

Tensor Networks

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Quantum simulators

Richard Feynman put it in memorable words: “Nature isn’t classical, dammit, and if you want to make a simulation of nature, you’d better make it quantum mechanical, and by golly it’s a wonderful problem, because it doesn’t look so easy.”

Full quantum computers are not there

-proof of principle
-scalability
-decoherence

Dedicated experiments to simulate relevant physical systems

IOP Publishing Reports on Progress in Physics
Rep. Prog. Phys. 75 (2012) 082401 (18pp) doi:10.1088/0034-4885/75/8/082401

Can one trust quantum simulators?

Philipp Hauke[1], Fernando M Cucchietti[1,2], Luca Tagliacozzo[1], Ivan Deutsch[3,4] and Maciej Lewenstein[1,5]

Macroscopic quantum effects

Macroscopic quantum effects

Neutral atoms, diluted gas

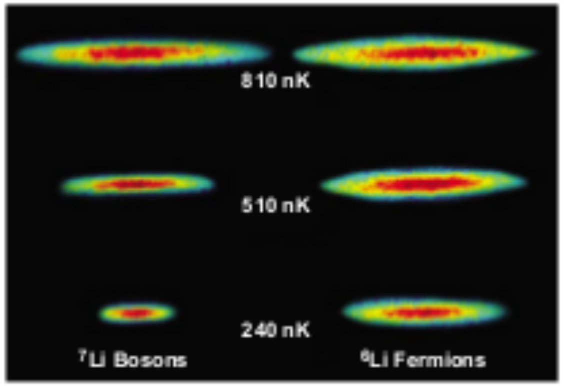

FIG. 1. (Color online) Simultaneous cooling of a bosonic and fermionic quantum gas of ^{7}Li and ^{6}Li to quantum degeneracy. In the case of the Fermi gas, the Fermi pressure prevents the atom cloud from shrinking further in space as quantum degeneracy is approached. From Truscott *et al.*, 2001.

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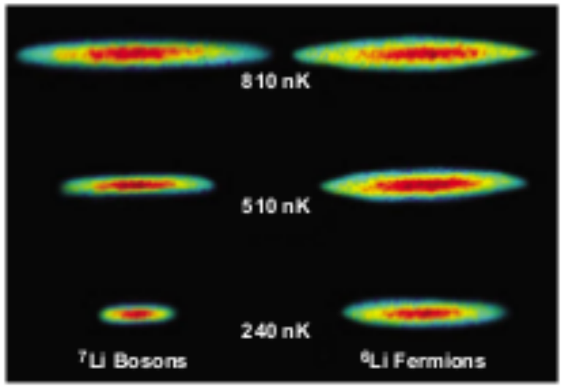

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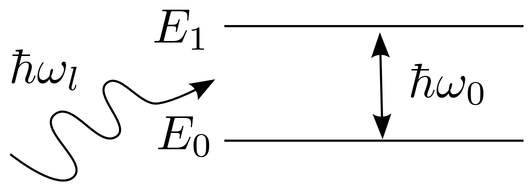
AC Stark effect

$$\mathbf{F} = \frac{1}{2}\alpha(\omega_L)\,\boldsymbol{\nabla}\left[|\mathbf{E}(\mathbf{r})|^2\right]$$

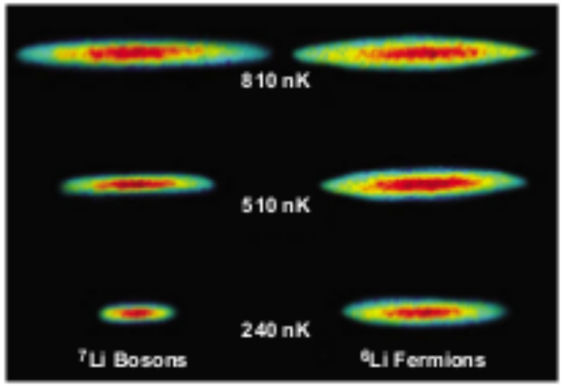
$$\Delta = \omega_L - \omega_0$$



$$V_{\text{dip}}(\mathbf{r}) = \frac{3\pi c^2}{2\omega_0^3}\frac{\Gamma}{\Delta} I(\mathbf{r}),$$

Macroscopic quantum effects

Neutral atoms, diluted gas



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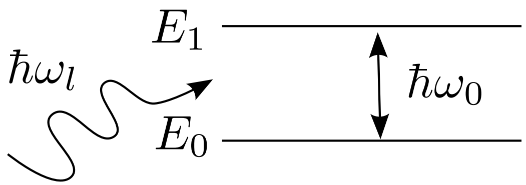

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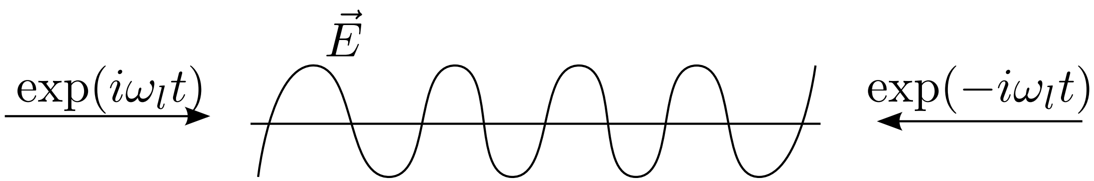

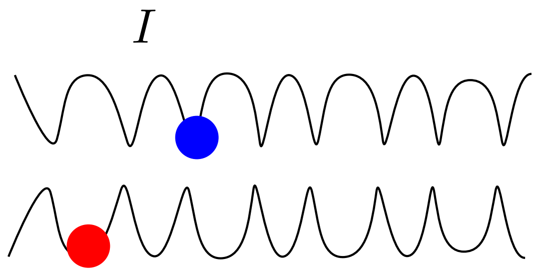

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Optical lattices: strong correlations

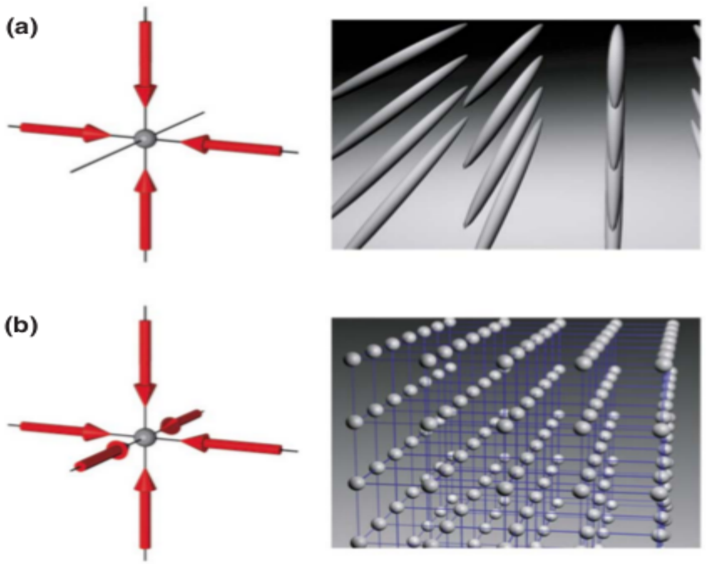


FIG. 4. (Color online) Optical lattices. (a) Two- and (b) three-dimensional optical lattice potentials formed by superimposing two or three orthogonal standing waves. For a two-dimensional optical lattice, the atoms are confined to an array of tightly confining one-dimensional potential tubes, whereas in the three-dimensional case the optical lattice can be approximated by a three-dimensional simple cubic array of tightly confining harmonic-oscillator potentials at each lattice site.

Bose Hubbard

$$\hat{H} = -J \sum_{\langle \mathbf{R},\mathbf{R}'\rangle} \hat{a}^{\dagger}_{\mathbf{R}}\hat{a}_{\mathbf{R}'} + \frac{U}{2}\sum_{\mathbf{R}} \hat{n}_{\mathbf{R}}(\hat{n}_{\mathbf{R}}-1) + \sum_{\mathbf{R}} \epsilon_{\mathbf{R}}\hat{n}_{\mathbf{R}}.$$

$$U = g\int d^3r|w(\mathbf{r})|^4 = \sqrt{8/\pi}\,ka E_r (V_0/E_r)^{3/4}$$

906 Bloch, Dalibard, and Zwerger: Many-body physics with ultracold gases

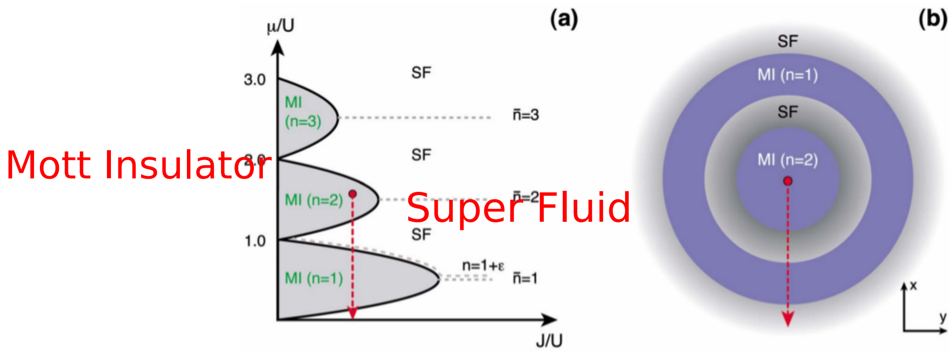
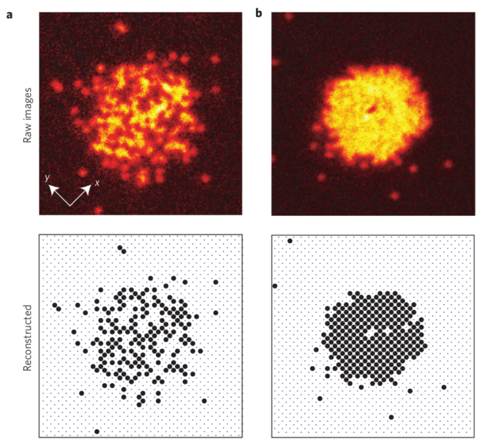


FIG. 13. (Color online) Schematic zero-temperature phase diagram of the Bose-Hubbard model. Dashed lines of constant-integer density $\langle\hat{n}\rangle=1,2,3$ in the SF hit the corresponding MI phases at the tips of the lobes at a critical value of J/U, which decreases with increasing density $\bar{n}$. For $\langle\hat{n}\rangle=1+\varepsilon$ the line of constant density stays outside the $\bar{n}=1$ MI because a fraction ε of the particles remains superfluid down to the lowest values of J. In an external trap with an $\bar{n}=2$ MI phase in the center, a series of MI and SF regions appear on going toward the edge of the cloud, where the local chemical potential has dropped to zero.

I. Bloch et al. Nat. Phys. Insigth 2012



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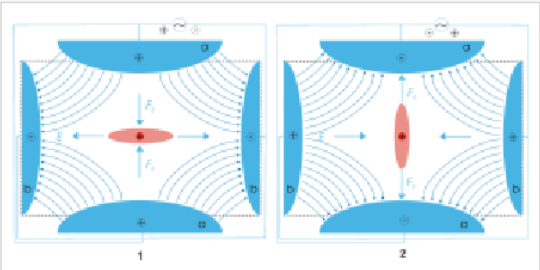
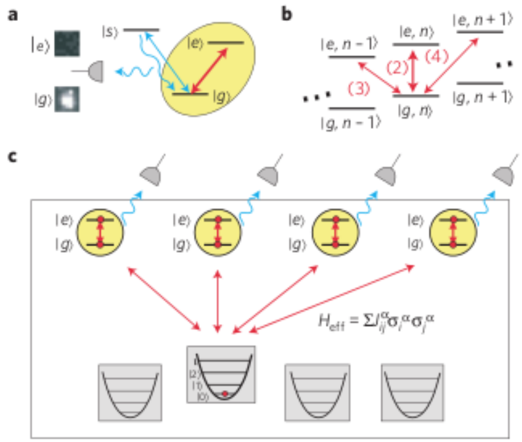
Quantum simulations with ions



Scheme of a Quadrupole ion trap of classical setup with a particle of positive charge (dark red), surrounded by a cloud of similarly charged particles (light red). The electric field E (blue) is generated by a quadrupole of endcaps (a, positive) and a ring electrode (b). Picture 1 and 2 show two states during an AC cycle.

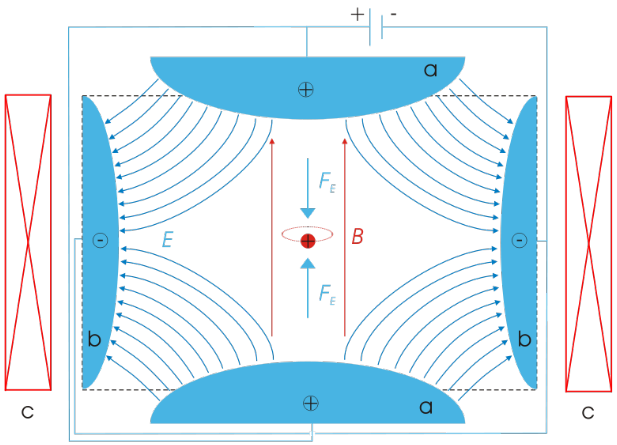
RF traps



$$H_I = \hbar\frac{\Omega}{2}(\sigma^+ e^{i\phi} + \sigma^- e^{-i\phi})$$

$$H_I^{\rm RSB} = \hbar\frac{\eta\Omega}{2} i(a\sigma^+ e^{i\phi} - a^\dagger\sigma^- e^{-i\phi})$$

Blatt, Ross Nat. Phys. Insigth 2012



Penning traps

Wikipedia

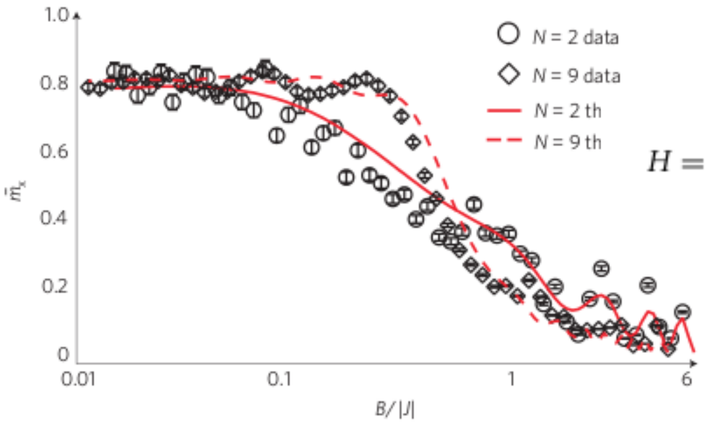


Figure 3 | Magnetization data. The magnetization $\overline{m}_x = \sum_{i=1}^{N}\langle\sigma_i^x\rangle/N$

$$H = \frac{1}{2}\sum_{\alpha,i,j} J_{ij}^{\alpha}\sigma_i^{\alpha}\sigma_j^{\alpha} + \sum_{\alpha,i} B^{\alpha}\sigma_i^{\alpha}$$

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Hilbert space and Operators

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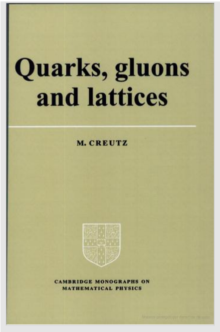
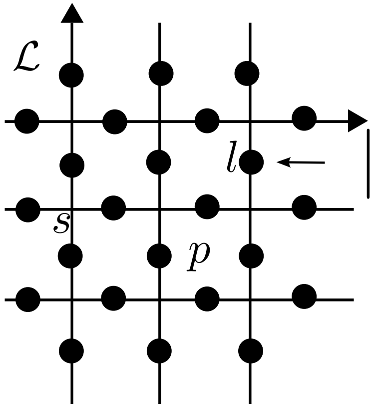


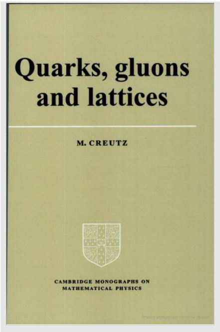


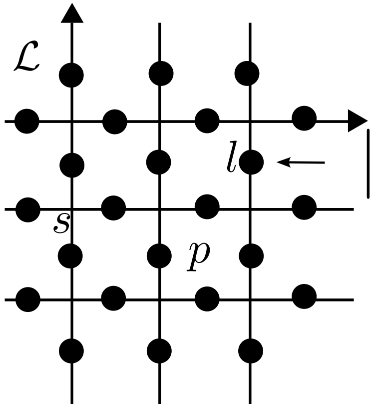
$$|U\rangle \in C(G)$$

$$\langle U|U'\rangle = \delta_U^{U'}$$

Kogut Susskind Lattice Gauge Theory

Hilbert space and Operators





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Kogut Susskind Lattice Gauge Theory

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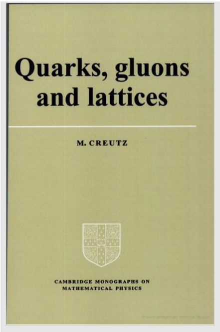

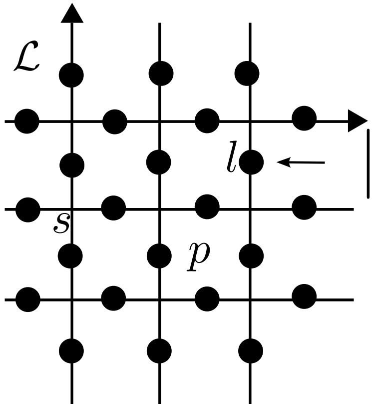

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Example Z2

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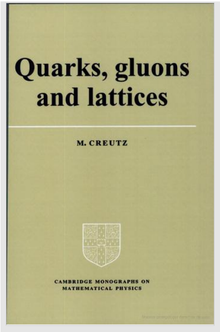
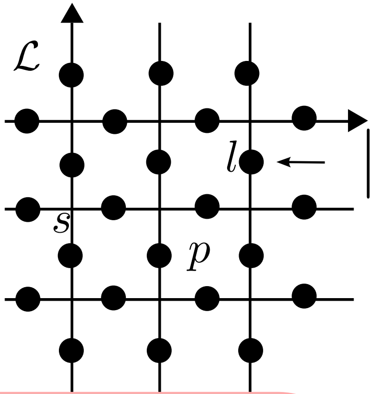
Kogut Susskind Lattice Gauge Theory

Hilbert space and Operators





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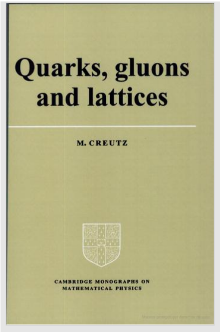
Example Z2

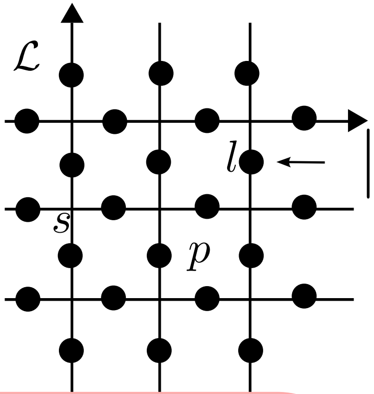
group

$$G:\{1,e\},e^2=1$$

Kogut Susskind Lattice Gauge Theory

Hilbert space and Operators





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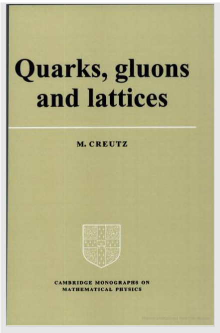
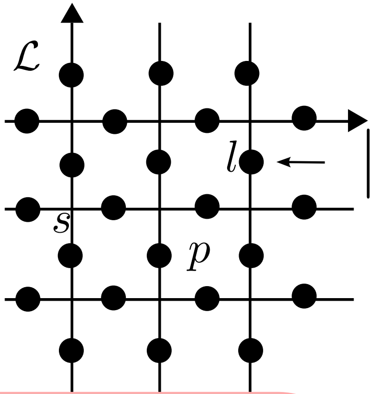
States

$$C(G) : \{|0\rangle, |1\rangle\}$$

Kogut Susskind Lattice Gauge Theory

Hilbert space and Operators





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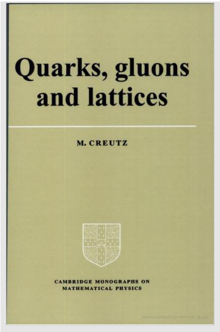

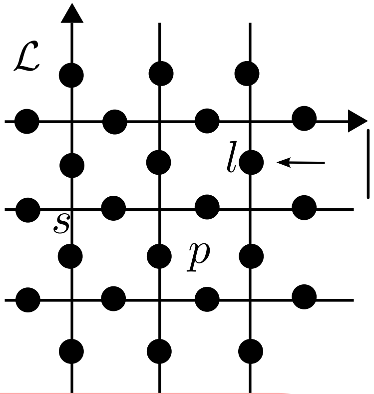

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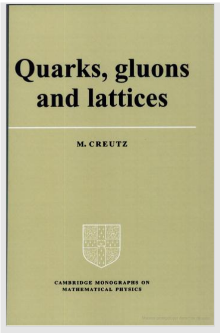

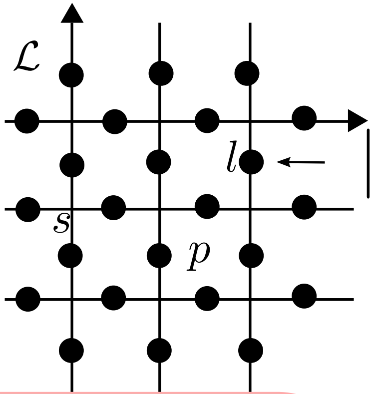

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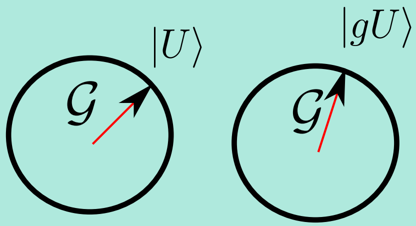
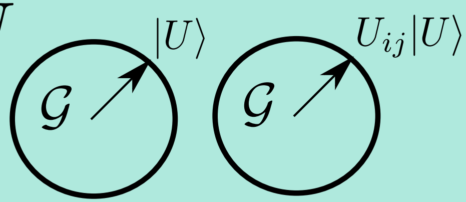
$R(g)$



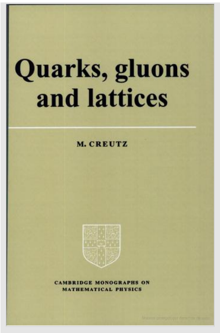
$\hat{U}$

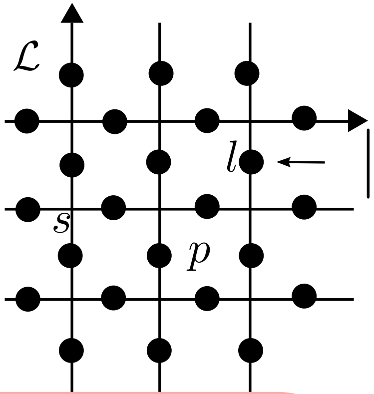


$$U \equiv \sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

Kogut Susskind Lattice Gauge Theory

Hilbert space and Operators





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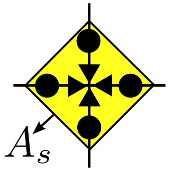

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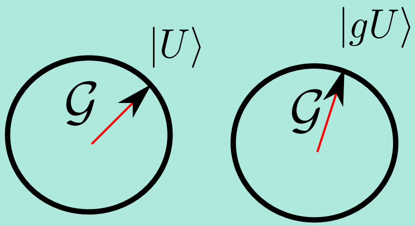

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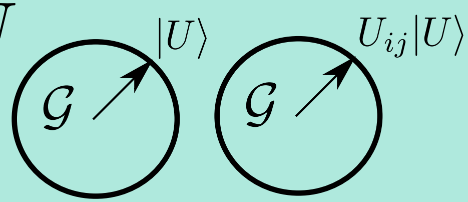

$$R(e) \equiv \sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0| \qquad U \equiv \sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

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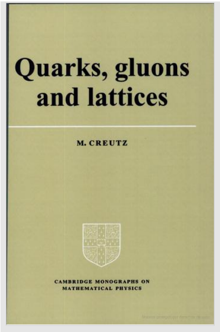

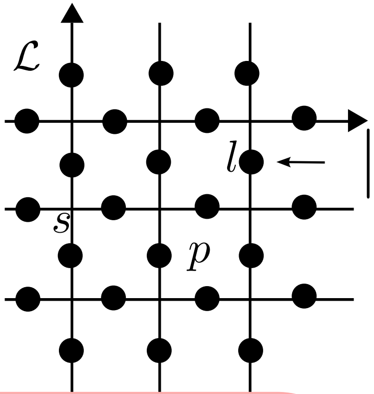

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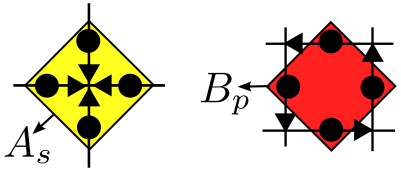

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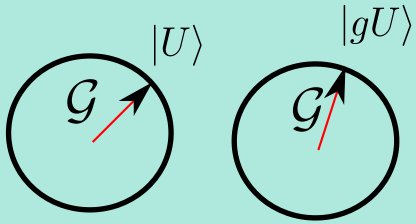

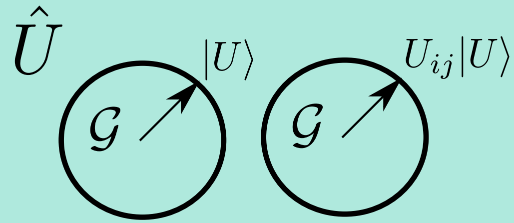

$$R(e) \equiv \sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0| \qquad U \equiv \sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

$$A_s = \sigma_x \otimes \sigma_x \otimes \sigma_x \otimes \sigma_x \qquad B_p = \sigma_z \otimes \sigma_z \otimes \sigma_z \otimes \sigma_z$$

Kogut Susskind Z2 Lattice Gauge Theory

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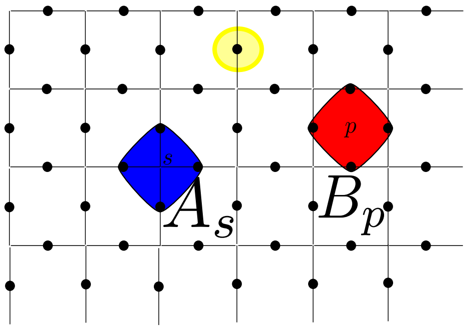
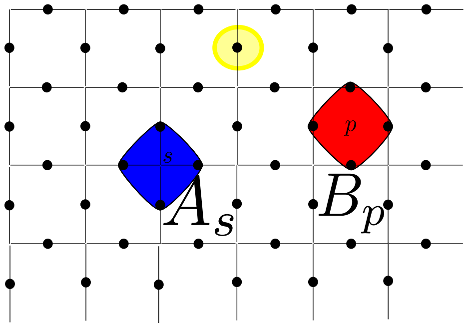
Gauge invariance and Hamiltonian



Kogut Susskind Z2 Lattice Gauge Theory

Gauge invariance and Hamiltonian



GAUSS LAW $\nabla E = 0$

$$V_{LGT} \equiv \{|\psi\rangle : A_s|\psi\rangle = |\psi\rangle \forall s\}$$

Kogut Susskind Z2 Lattice Gauge Theory

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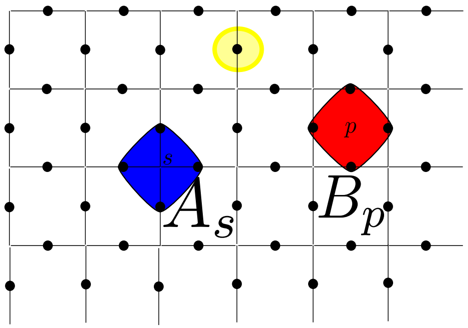

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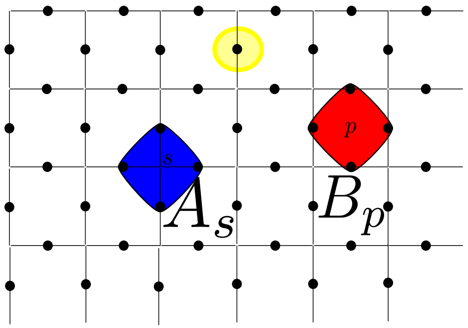
$$[A_s, \sigma_k^x] = 0, \forall s, k \qquad [A_s, B_p] = 0, \forall s, p$$

$$H \equiv - \sum_p B_p - h_x \sum_k \sigma_k^x$$

Kogut Susskind Z2 Lattice Gauge Theory

Gauge invariance and Hamiltonian



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$$H \equiv -\sum_p B_p - h_x \sum_k \sigma_k^x$$

2D spin system with **four body interactions**

Motivation

Tensor Networks

Quantum simulators (QS)

Lattice Gauge Theories (LGT)

Tensor Networks 4 LGT

QS of GT with Rydberg atoms

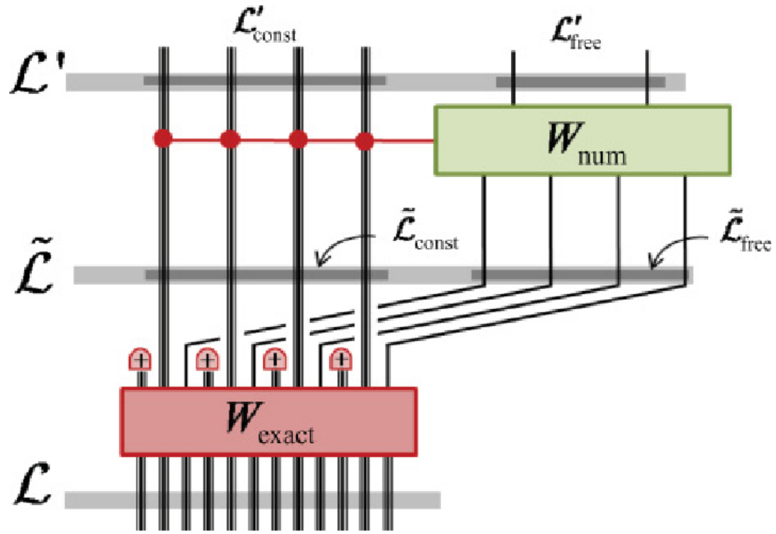
Conclusions and outlook

RESULTS FOR Z2 LGT from TN

LT G. Vidal
PRB 83 115127,2011

RESULTS FOR Z2 LGT from TN

LT G. Vidal
PRB 83 115127,2011



RESULTS FOR Z2 LGT from TN

LT G. Vidal
PRB 83 115127,2011

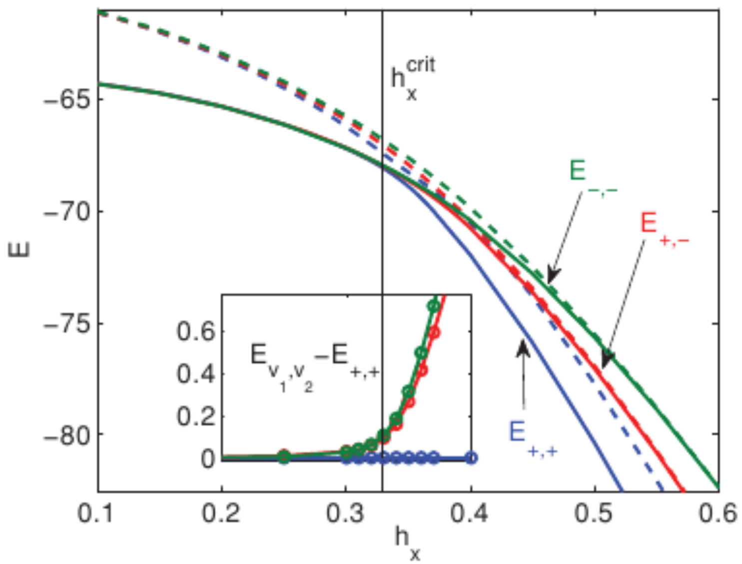
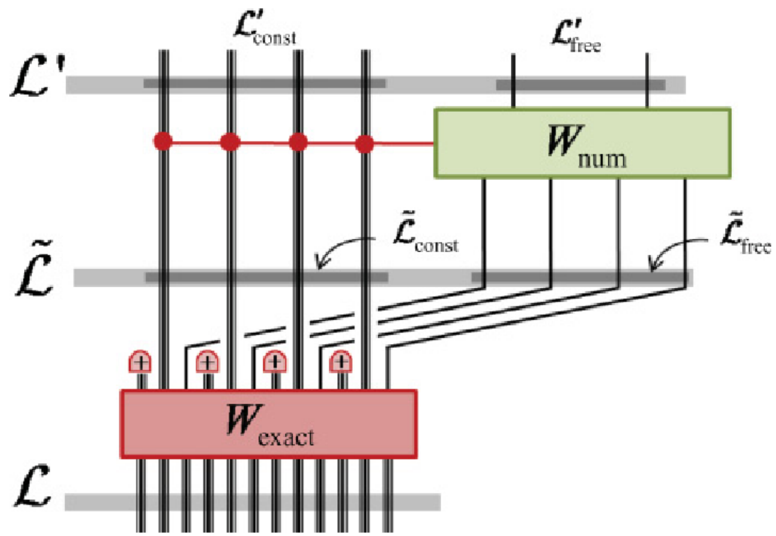
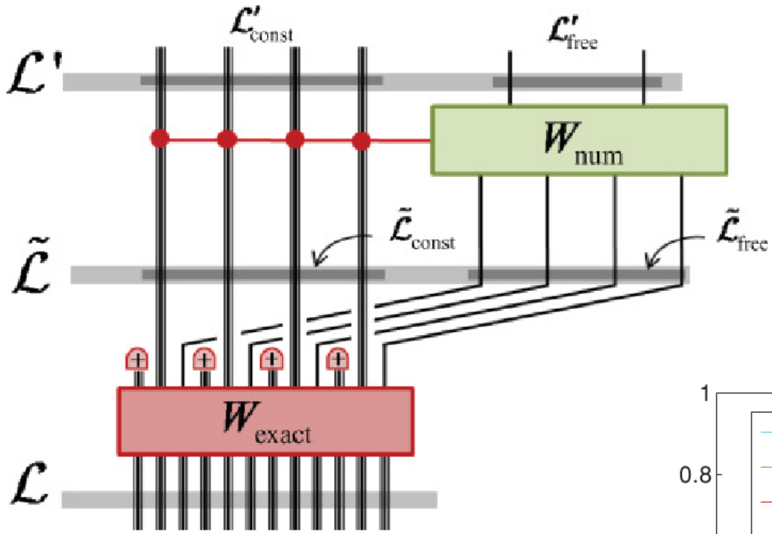


FIG. 33. (Color online) Lowest eigenvalues of H_{TC}^{x} as a function
f the magnetic field h_x, for a system of linear size $L = 8$,
orresponding to $8^2 \times 2 = 128$ spins. The two lowest energies in



RESULTS FOR Z2 LGT from TN

LT G. Vidal
PRB 83 115127,2011



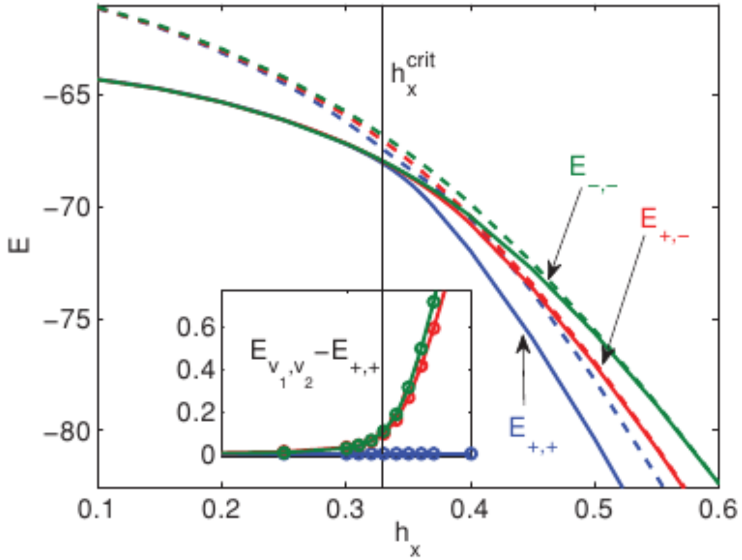


FIG. 33. (Color online) Lowest eigenvalues of H^x_{TC} as a function
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orresponding to $8^2 \times 2 = 128$ spins. The two lowest energies in

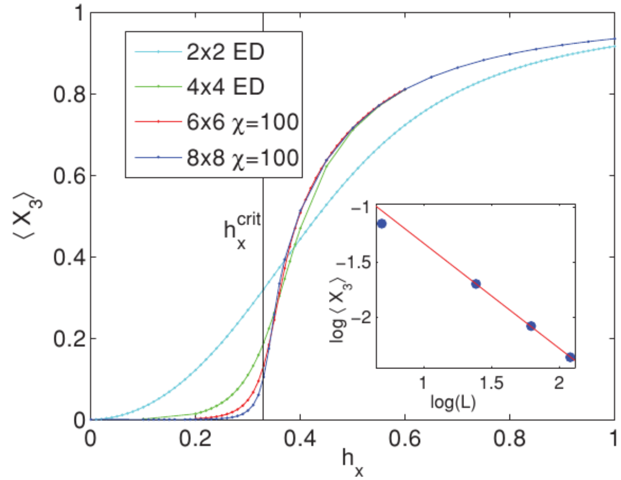
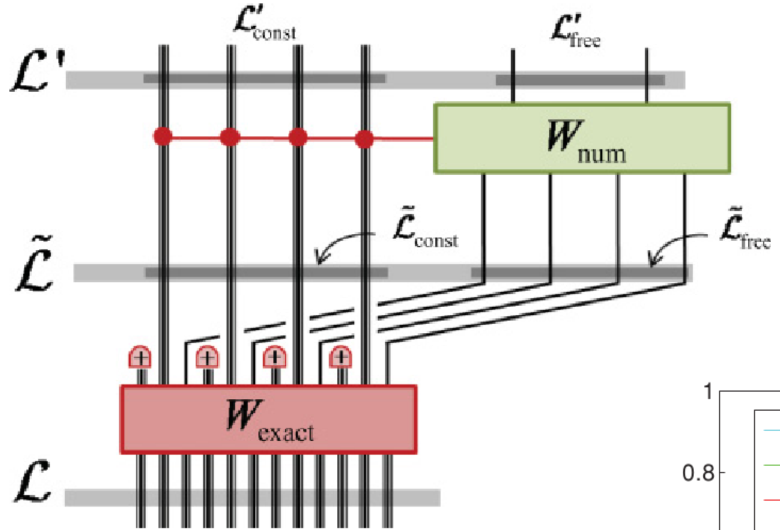
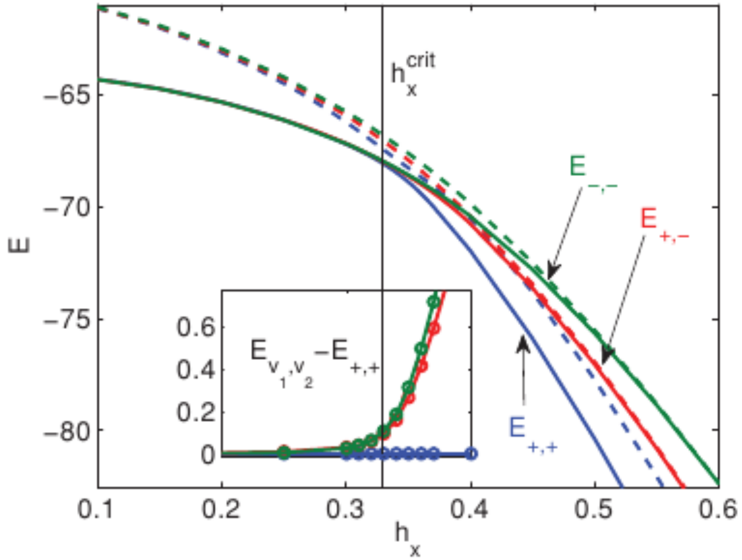


FIG. 36. (Color online) Disorder parameter $\langle X_3 \rangle$ as a function of
the magnetic field h_x, for several lattice sizes. The critical magnetic

RESULTS FOR Z2 LGT from TN

LT G. Vidal
PRB 83 115127,2011





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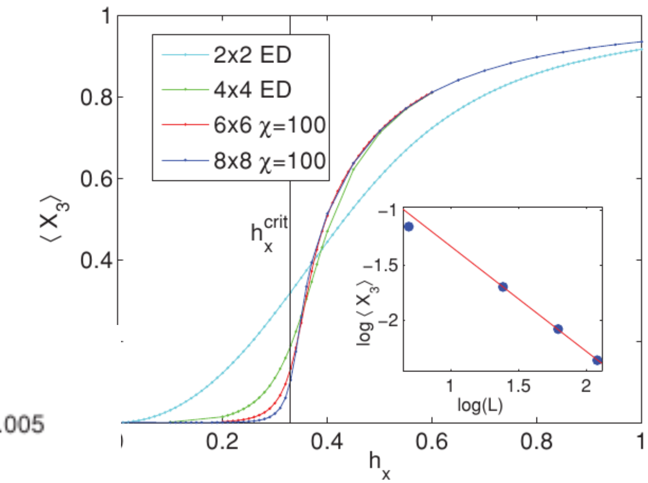

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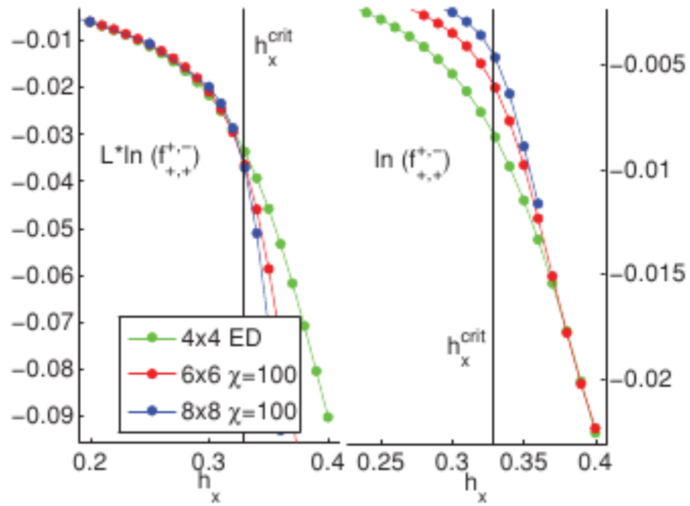

FIG. 39. (Color online) Logarithm of the intensive *topological*
fidelity $f^{+,-}_{+,+}$ as a function of the magnetic field h_x. In the deconfined

SU(2)-U(1) link-model/Gauge magnets

U(1) LGT from spin 1/2 constituents

Horn, D. Phys. Lett. B 100, 149–151 (1981).
Orland, P. & Rohrlich, D. Nucl. Phys. B 338, 647–672 (1990).
Chandrasekharan, S. & Wiese Nucl.Phys. B492, 455–474 (1997)
Lt et al. Annals of Physics (2013), pp. 160-191
LT et al. Nat. Comm 4, 2615 (2013).

Institut de Ciències Fotòniques

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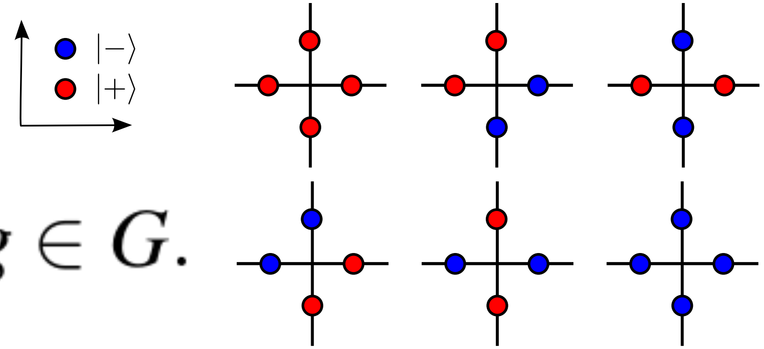

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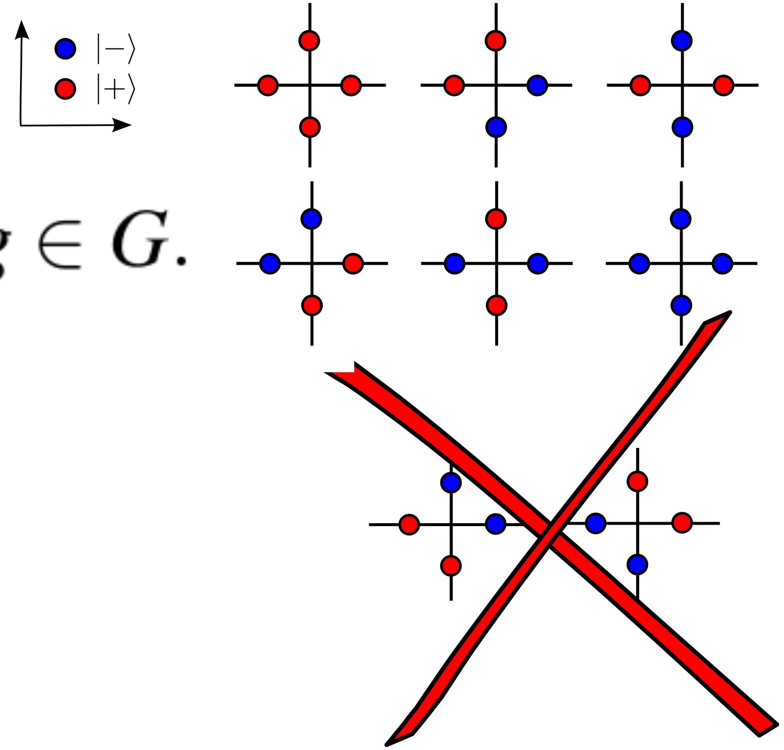

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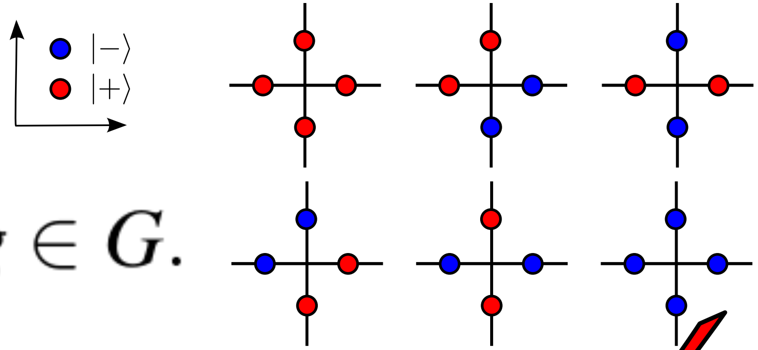

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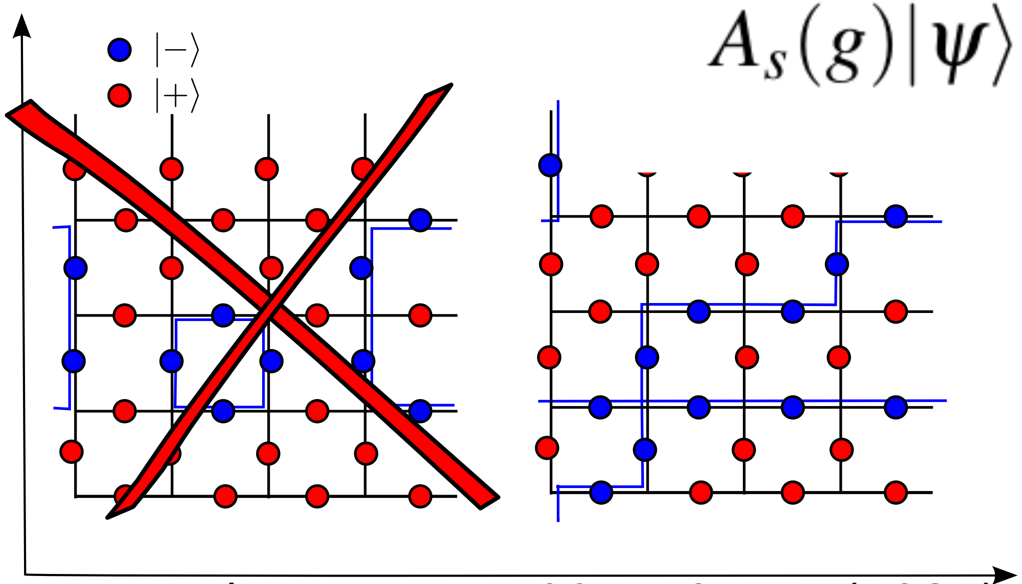

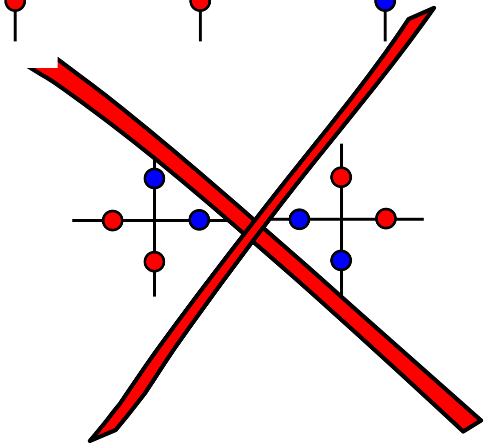

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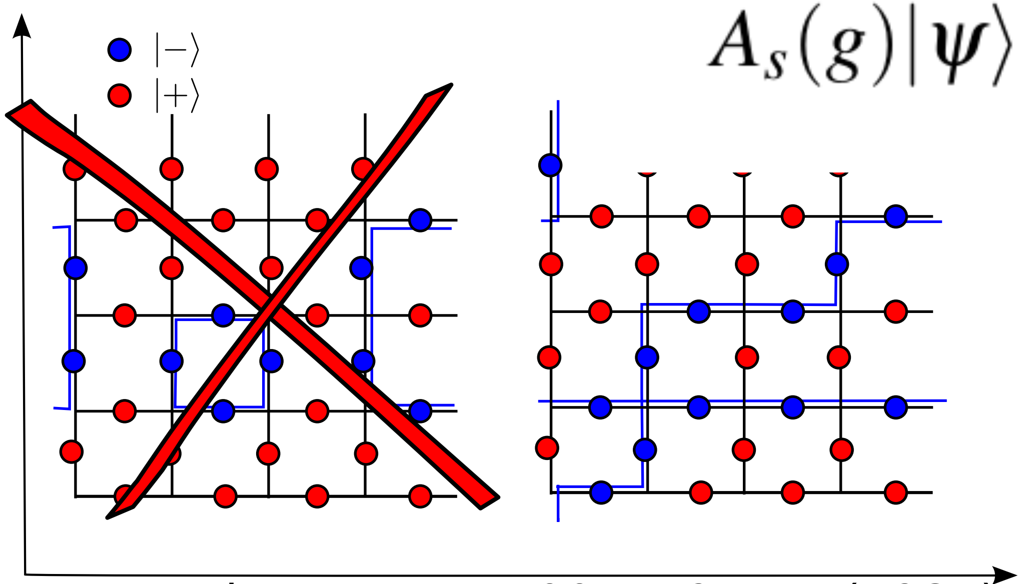

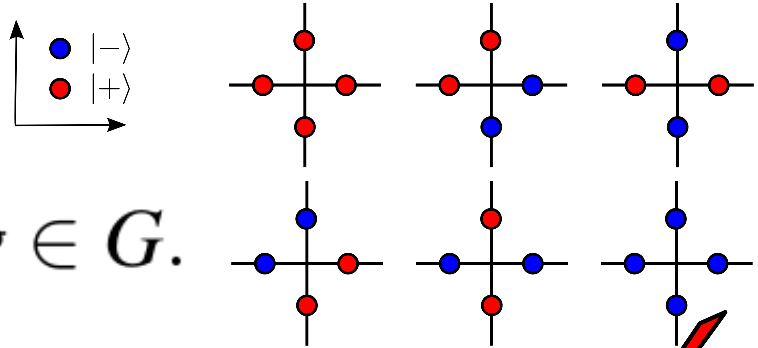

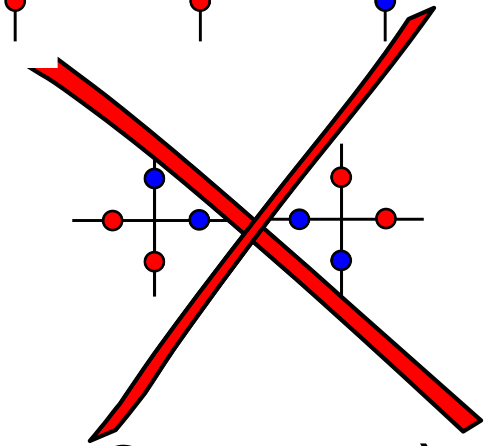

(see Baxter 6 vertex)

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Chandrasekharan, S. & Wiese Nucl.Phys. B492, 455–474 (1997)
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Dynamics

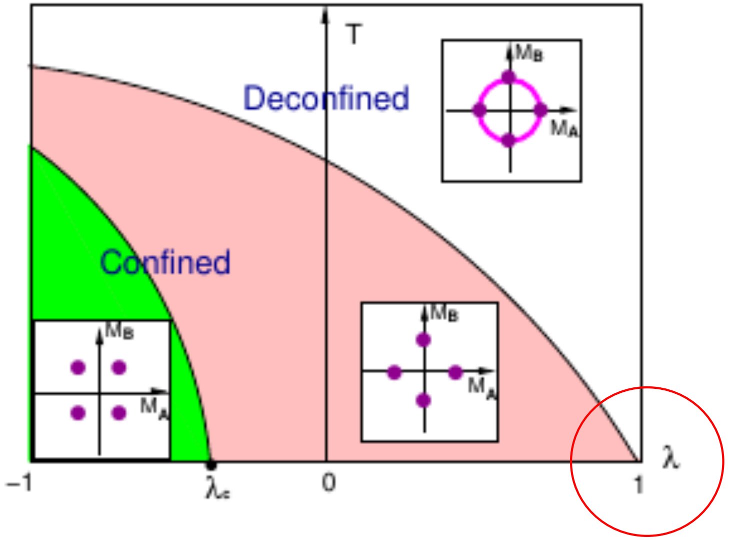


FIG. 1. [Color online] *Schematic sketch of the λ-T phase diagram. The insets indicate the location of the peaks in the probability distribution of the order parameter* $p(M_A, M_B)$.

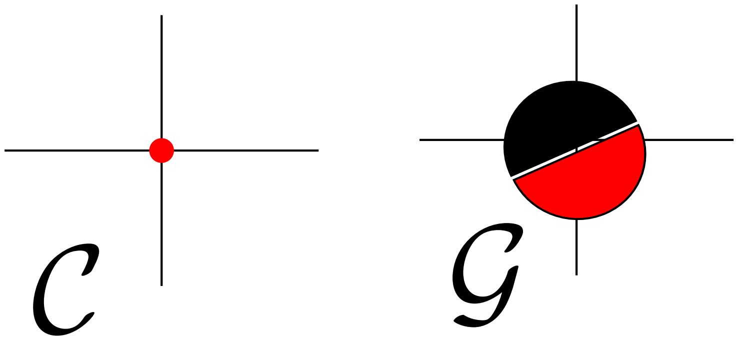
$$H = -J \sum_{\square} \left[U_\square + U_\square^\dagger - \lambda (U_\square + U_\square^\dagger)^2 \right].$$

Lt et al. Annals of Physics (2013), pp. 160-191
Wiese arXiv:1305.1602
D. Banerjee et al. arXiv:1303.6858

U(1)-SU(2) Gauge magnets/ Links model

LT et. al in prep.

U(1)-SU(2) Gauge magnets/ Links model



LT et. al in prep.

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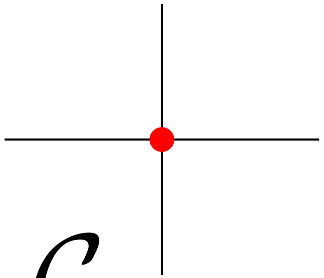
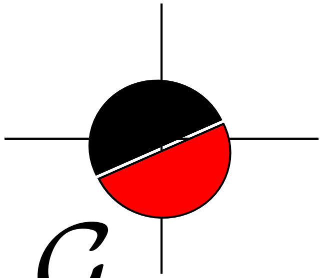
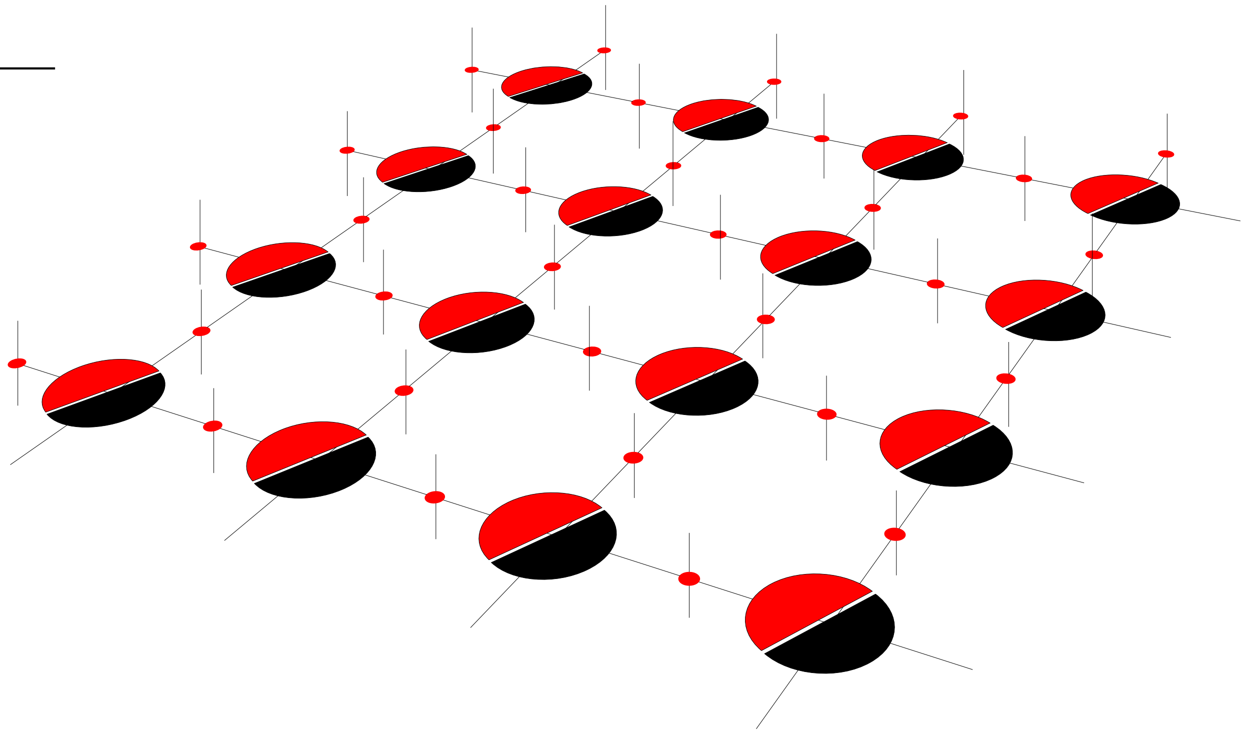
U(1)-SU(2) Gauge magnets/ Links model



$\mathcal{C}$



$\mathcal{G}$



$$\mathcal{P}_{V_{LGT}}$$

LT et. al in prep.

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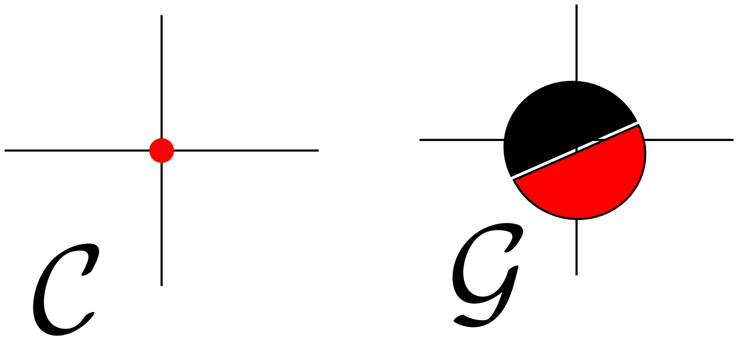
U(1)-SU(2) Gauge magnets/ Links model



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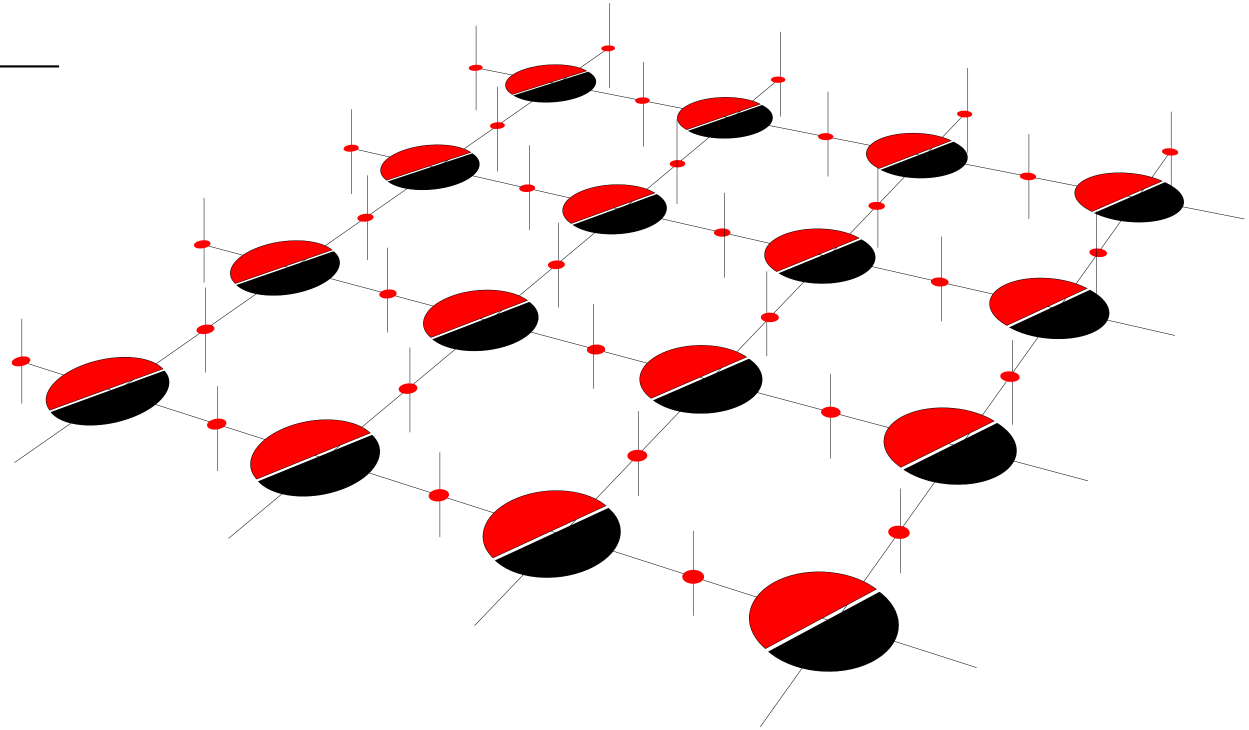

Exact projection on the gauge invariant Hilbert space
as a Tensor Network

LT et. al in prep.

RK state
and string tension states

LT et. al in prep.

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RK state
and string tension states

$$|\beta\rangle_i = e^{\beta}|0\rangle_i + e^{-\beta}|1\rangle_i$$

LT et. al in prep.

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RK state
and string tension states

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LT et. al in prep.

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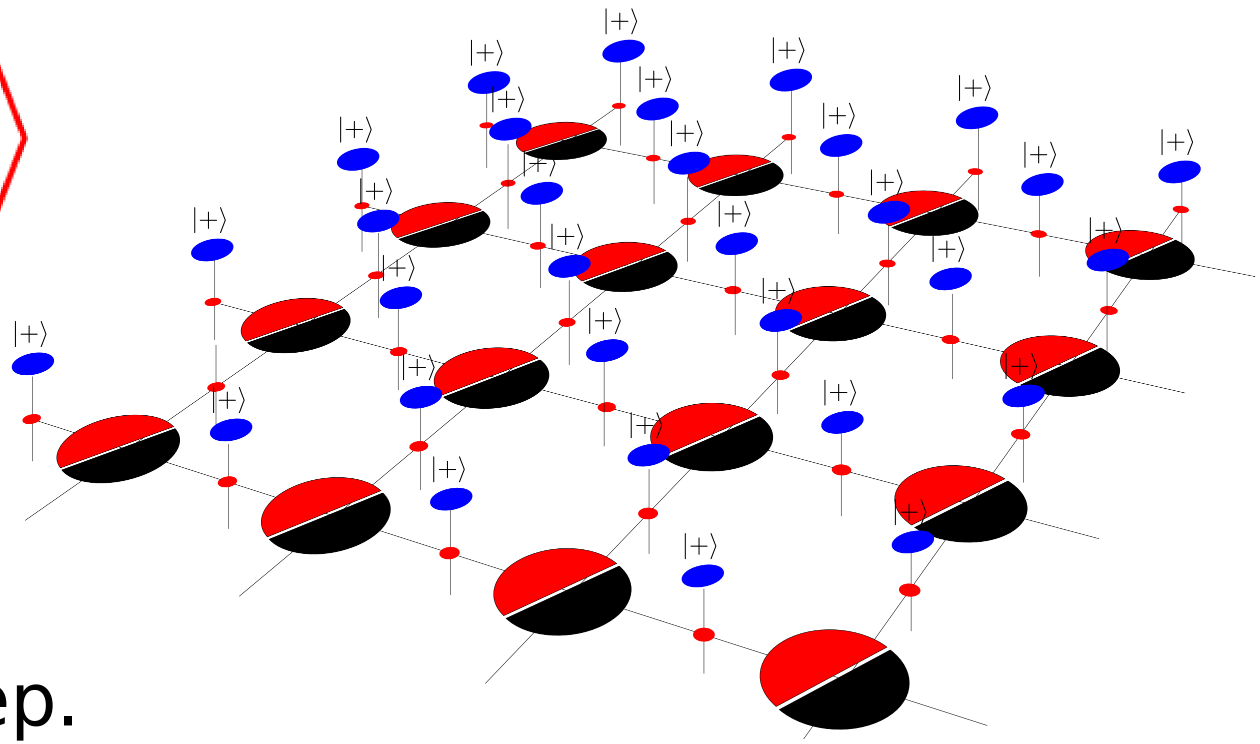
RK state and string tension states

$$|\beta\rangle_i = e^{\beta}|0\rangle_i + e^{-\beta}|1\rangle_i \qquad \mathcal{P}_{\mathrm{LGT}} \prod_i (|\beta\rangle_i)$$

$$|\psi(\beta)\rangle$$



LT et. al in prep.

$$Z(\beta) = \langle\psi(\beta)|\psi(\beta)\rangle$$

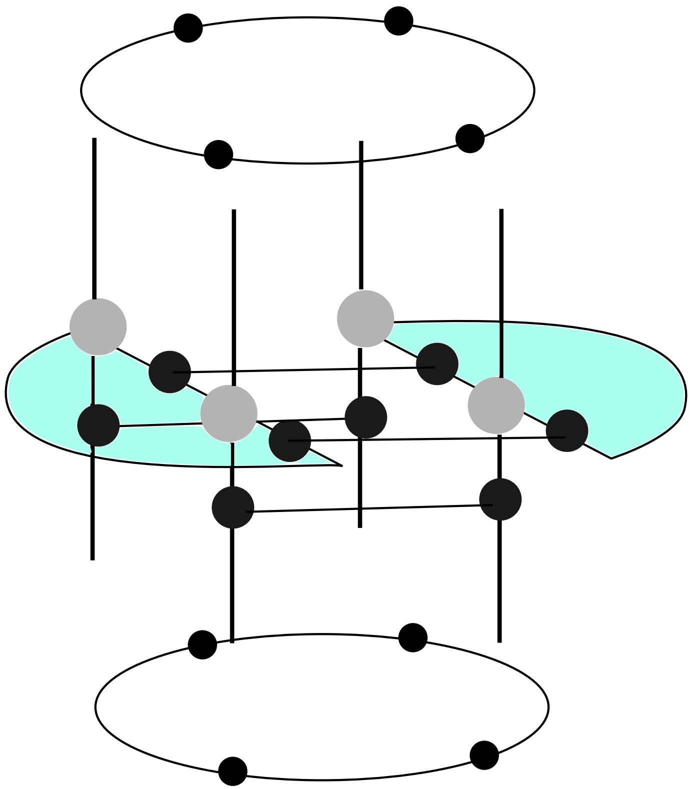
2D Cylinder

Interesting observables

$$Z(\beta) = \langle \psi(\beta) | \psi(\beta) \rangle$$

2D Cylinder

Interesting observables



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2D Cylinder

Interesting observables

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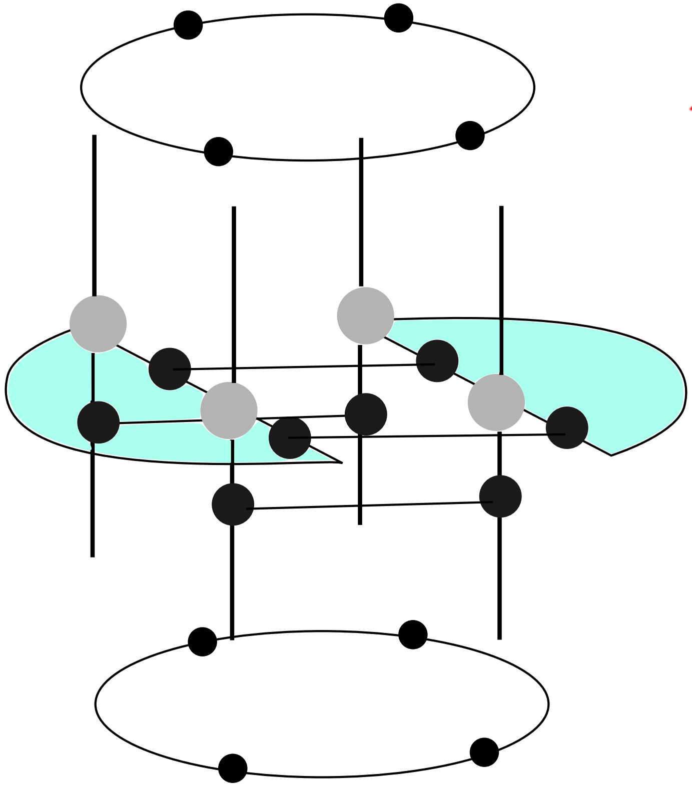

$$T = \exp - H_t$$

Classical to quantum correspondence

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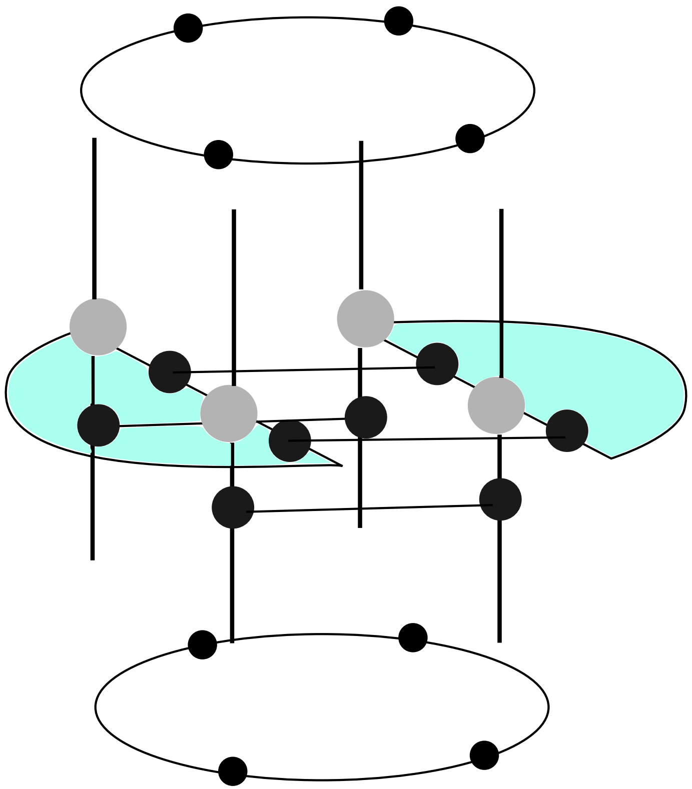
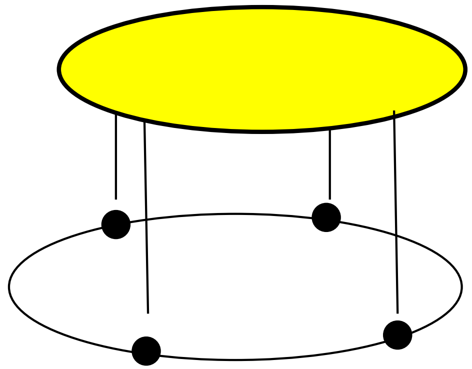
2D Cylinder

Interesting observables



$$Z(\beta) = \langle \psi(\beta) | \psi(\beta) \rangle$$



$$|GS\rangle$$

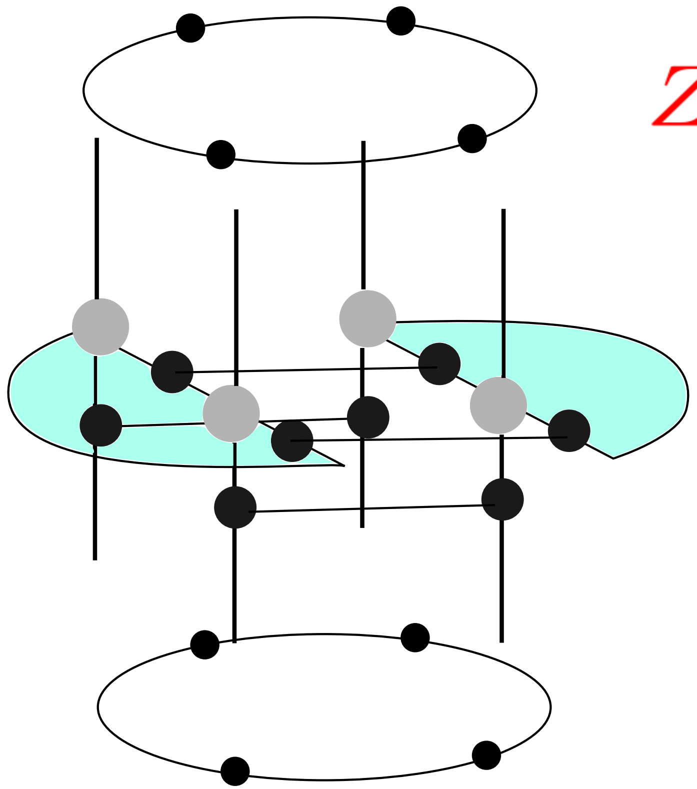
$$T = \exp - H_t$$

Classical to quantum correspondence

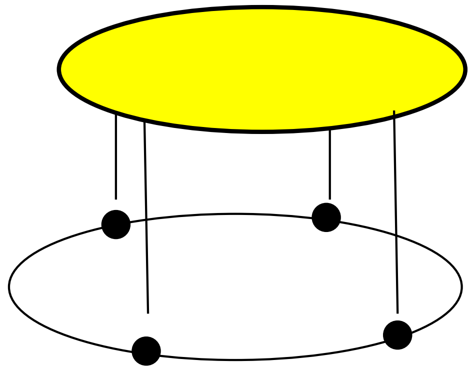
Institut de Ciències Fotòniques

2D Cylinder

Interesting observables



$$Z(\beta) = \langle \psi(\beta) | \psi(\beta) \rangle$$



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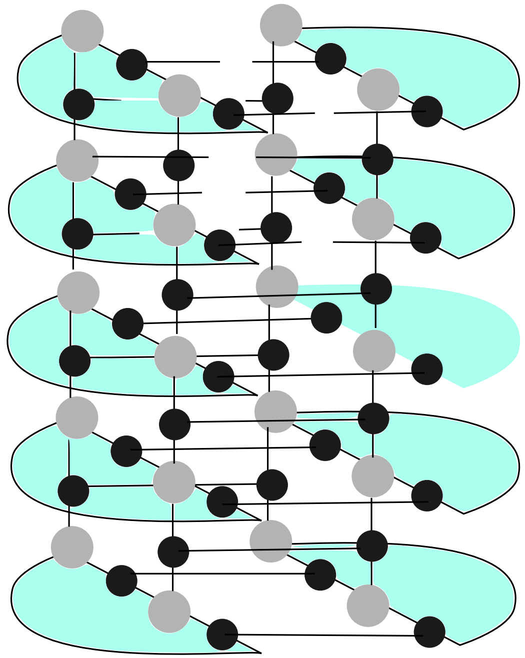
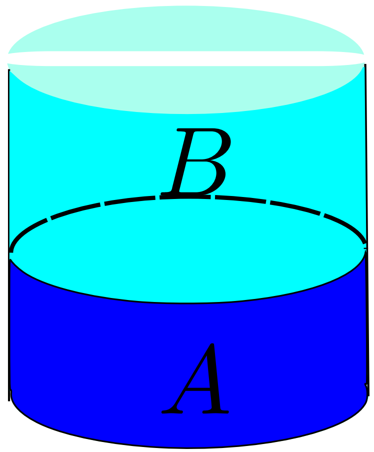


$$T = \exp - H_t$$

Classical to quantum correspondence

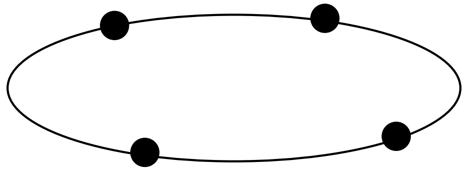
$$\rho_A$$

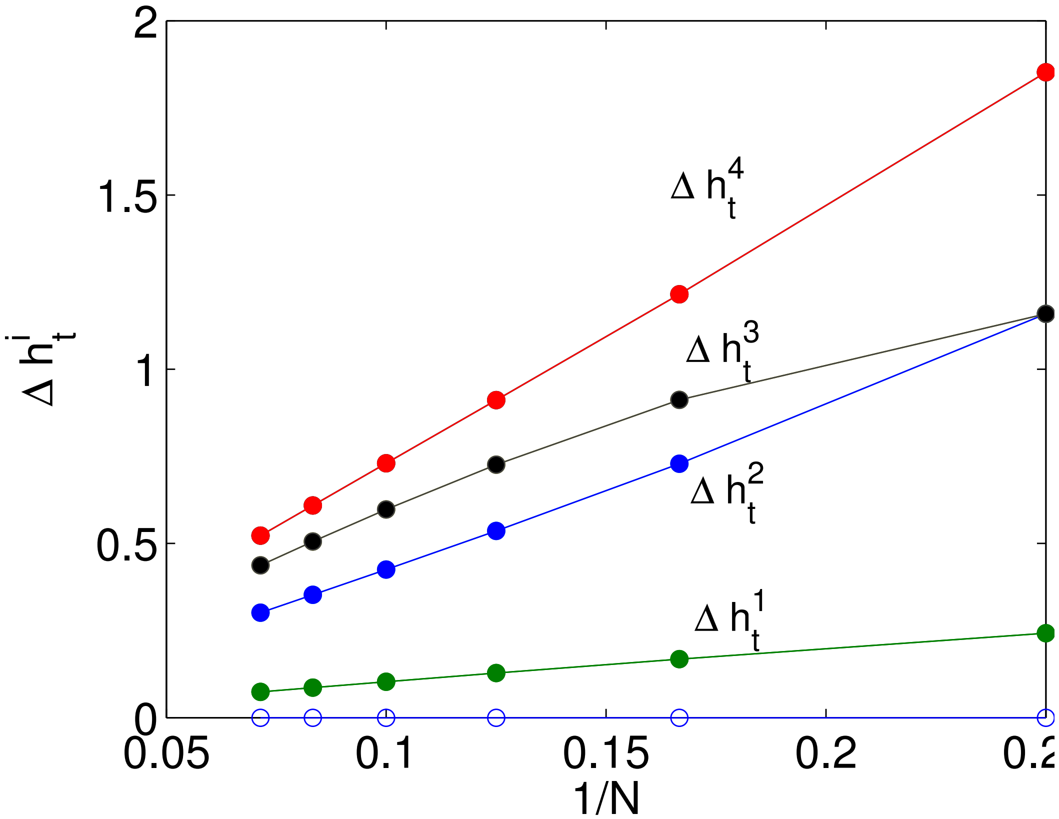


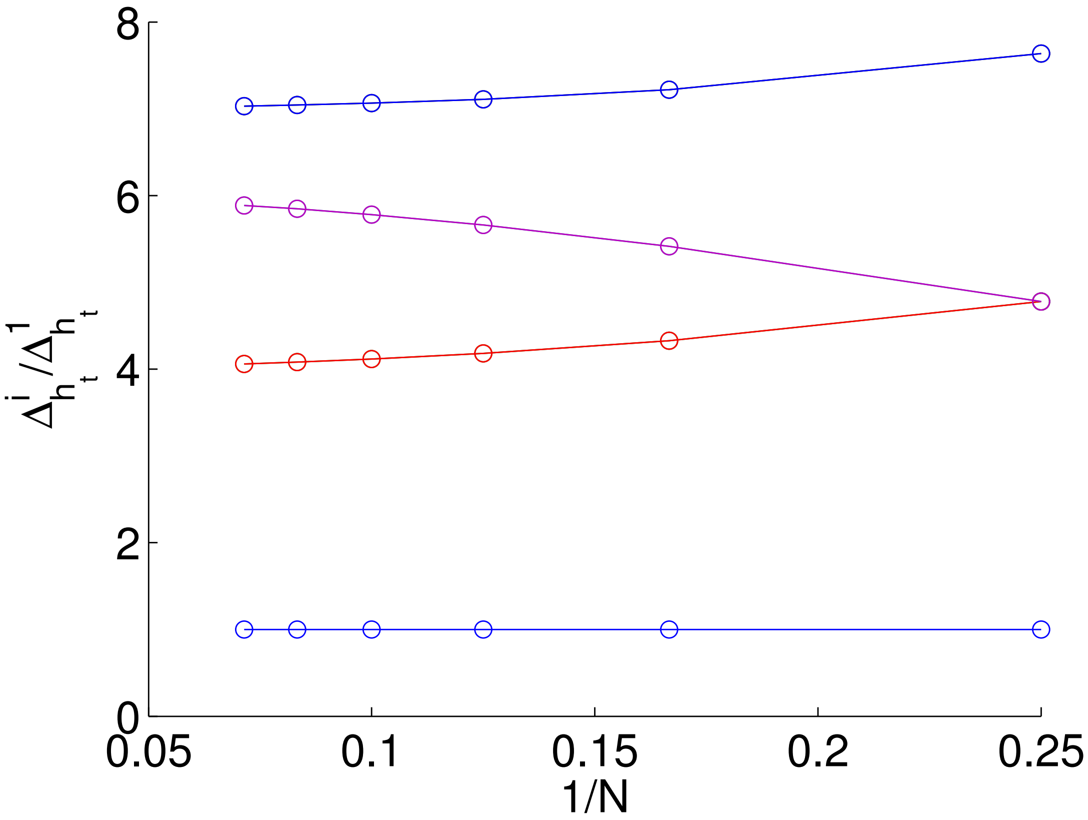
Holographic correspondence

2D Cylinder

Transfer matrix







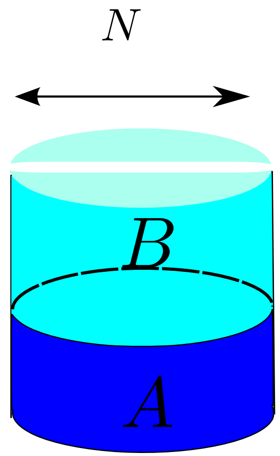
GAPLESS 1D SYSTEM

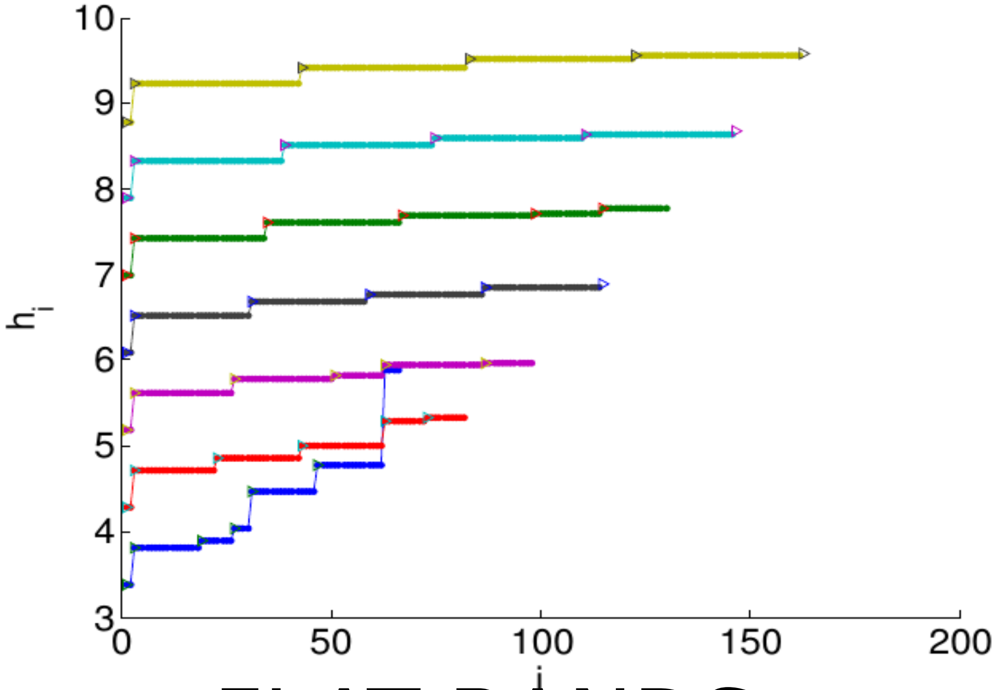
Ratio between critical exp

LT et. al in prep. xxz model at $\Delta = 1/2$

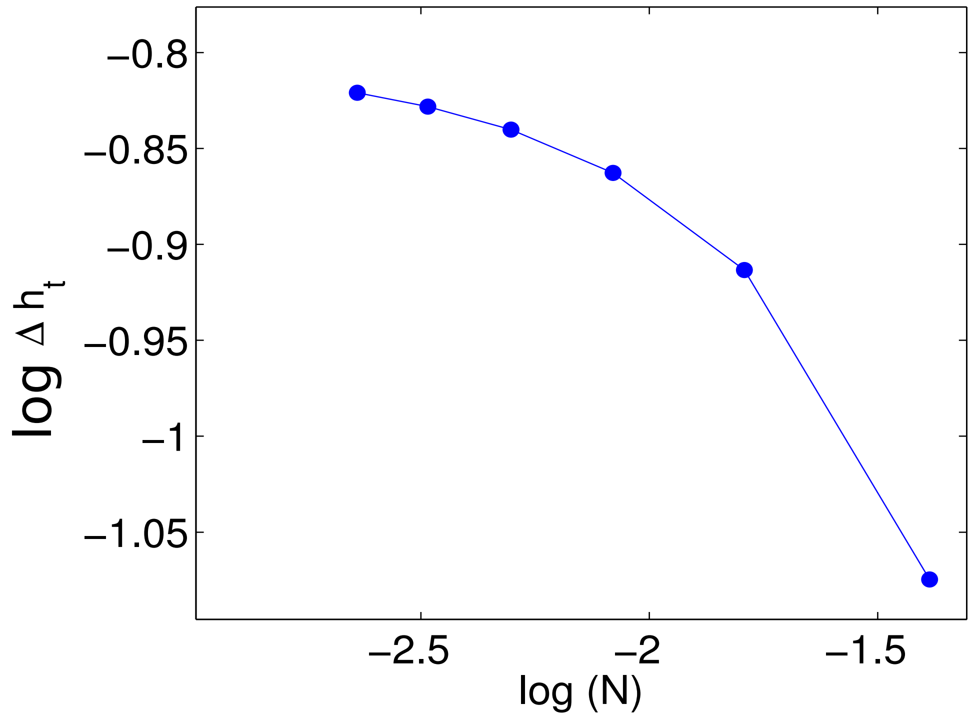
2D Cylinder Entanglement spectrum

$$\rho_B = \exp(-H_e)$$





FLAT BANDS

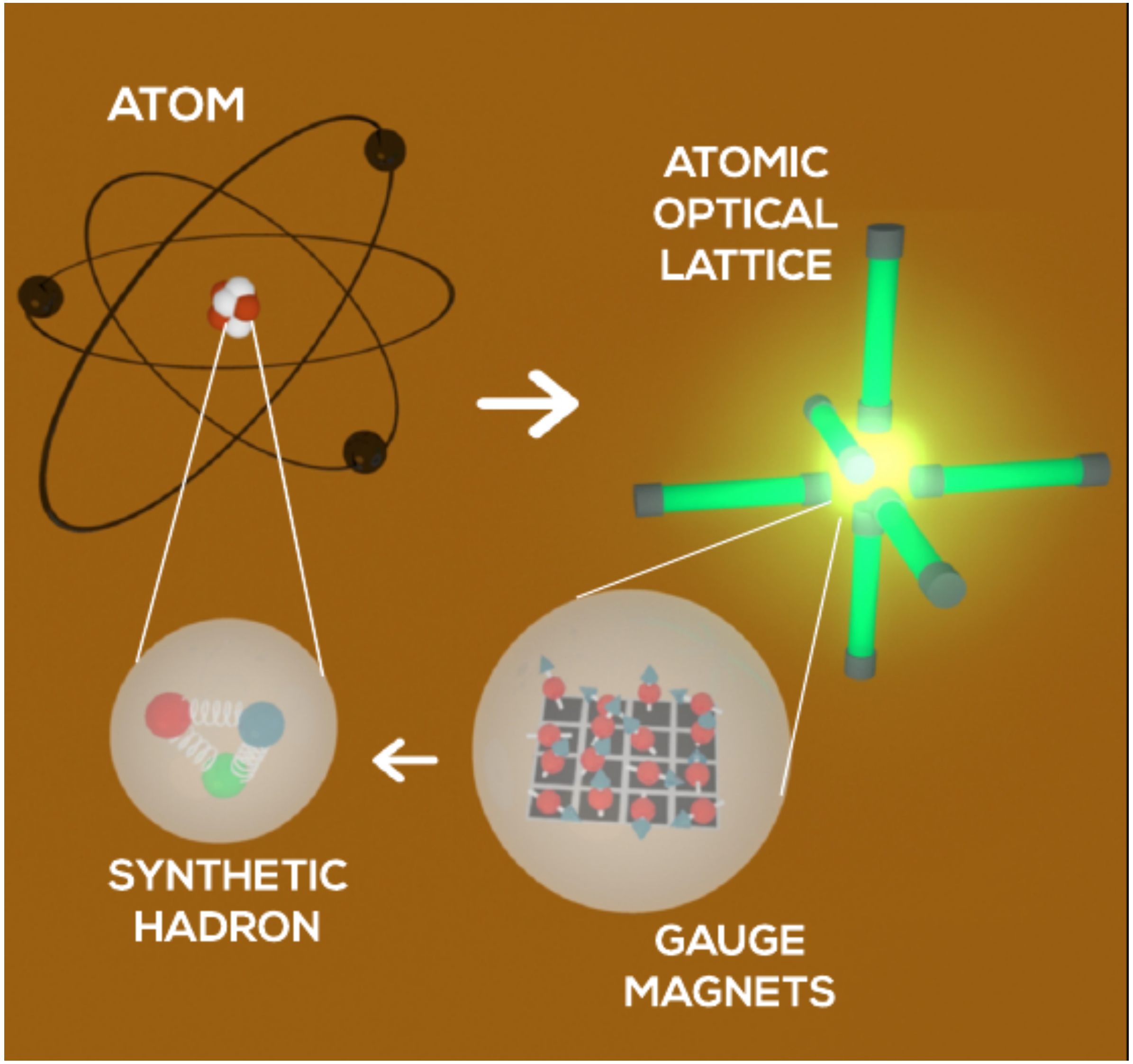


FINITE GAP

Lt et al in prep.
Stephan et al. PRB 80, 184421 (2009)

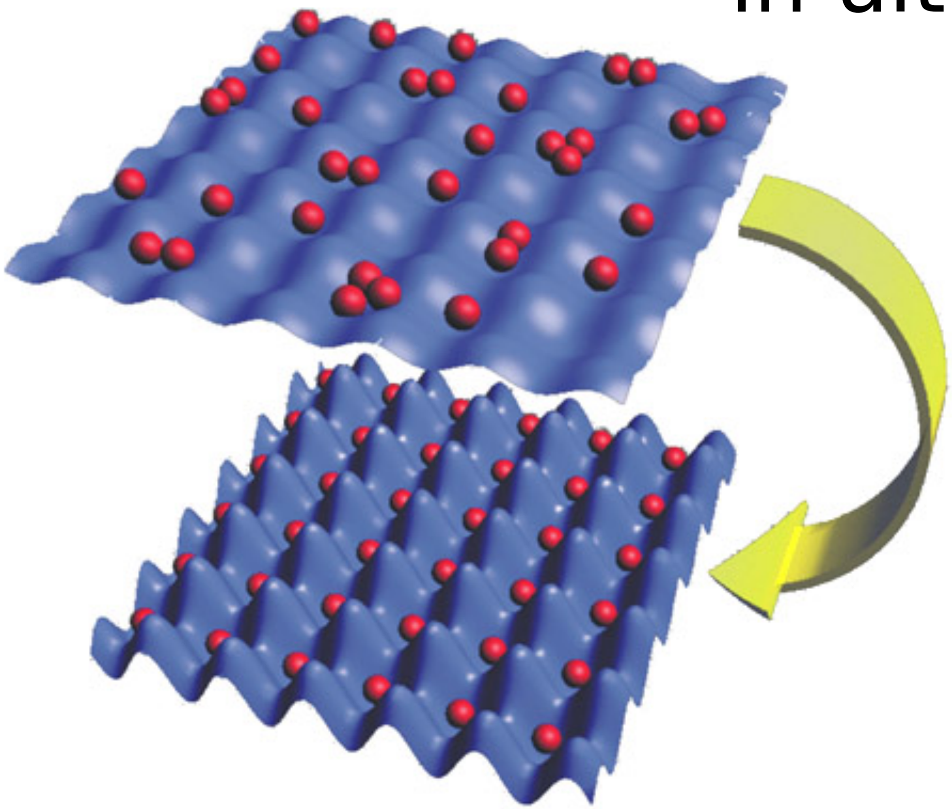
Quantum Simulations of LGT

LT et al. Nat. Comm. 4, 2615 (2013)
LT et al. Ann. Phys. (2013),
E. Zohar et al. PRL 109 (2012)
E. Zohar et al. PRL 110 (2013)
D. Banerjee et al. PRL 109 (2012)
D. Banerjee et al. PRL. 110 (2013)



Natural interactions

in ultra-cold gases



$$c_i c_{i+1}^\dagger$$

$$n_i, n_i^2$$

Difficult to engineer interactions different from

How do we get four body interactions for LGT ?

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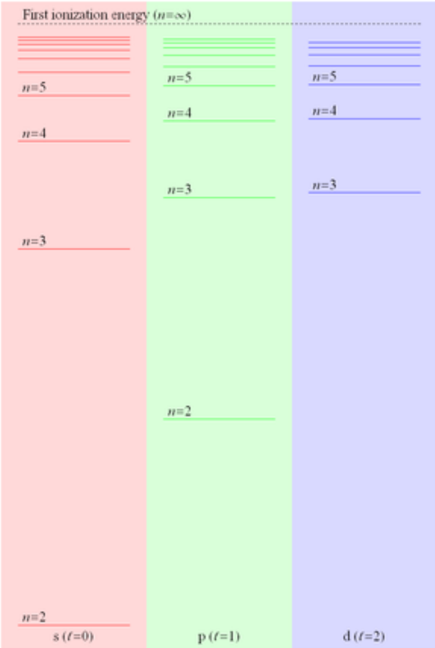
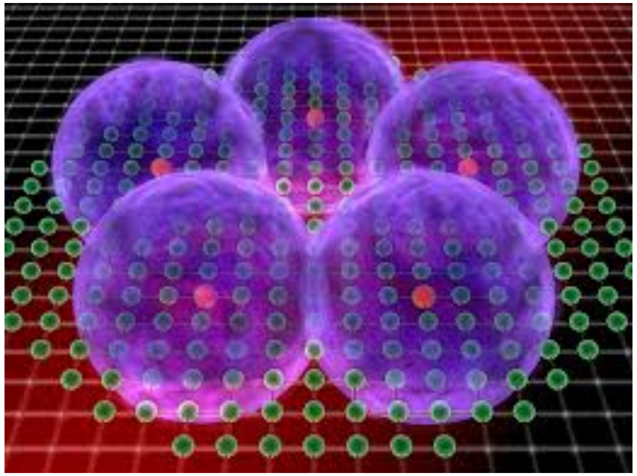
Rydberg atoms



typically n~50

$$V_{ij} \propto 1/r_{ij}^{6}$$



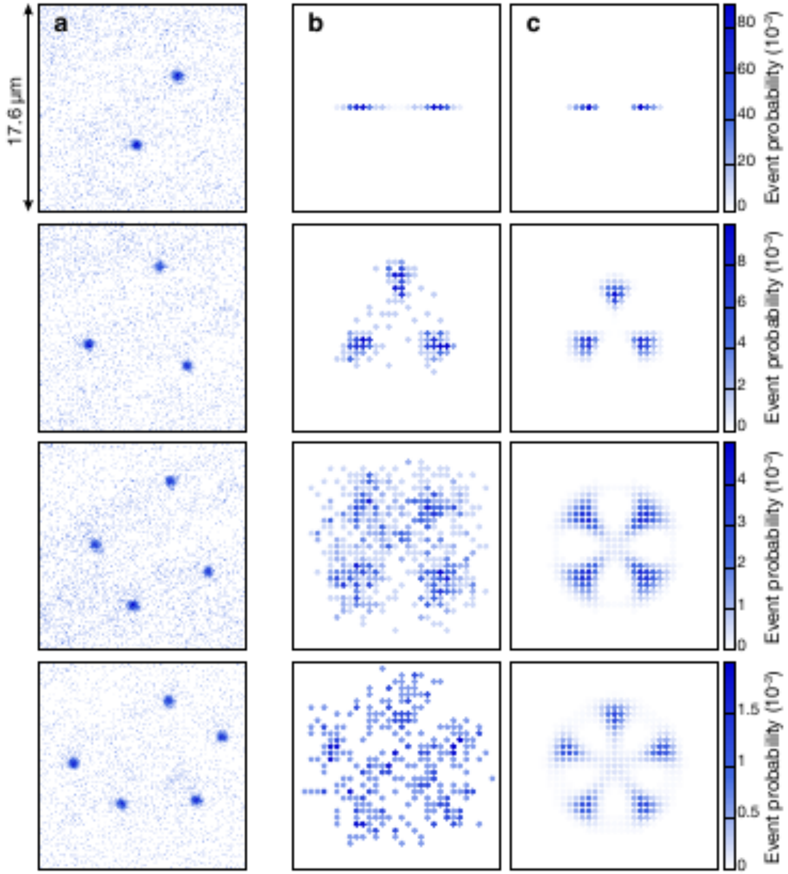


Figure 2. **Mesoscopic crystalline components of the many-body states.** Spatial distribution of excitations for

P. Schauß et al. Nature 491, 87 (2012)

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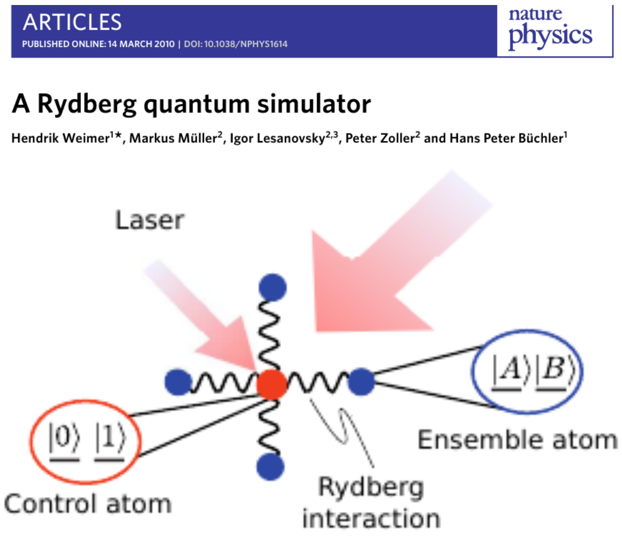
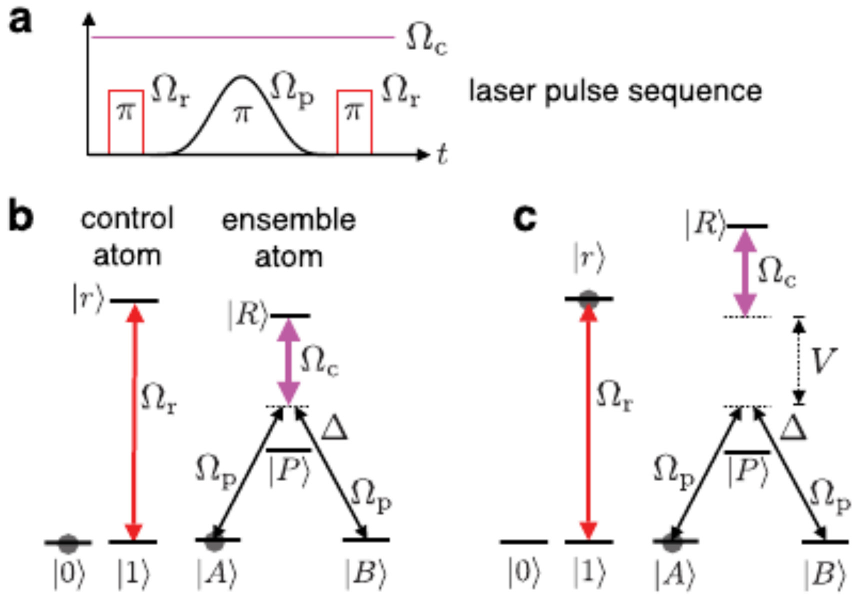
Four Body interaction with Rydberg





$$|0\rangle|A^N\rangle \rightarrow |0\rangle|A^N\rangle, \qquad |0\rangle|B^N\rangle \rightarrow |0\rangle|B^N\rangle,$$

$$|1\rangle|A^N\rangle \rightarrow |1\rangle|B^N\rangle, \qquad |1\rangle|B^N\rangle \rightarrow |1\rangle|A^N\rangle,$$

$$G = U_c(\pi/2)^{-1} U_g U_c(\pi/2)$$

$$U_g = |0\rangle\langle 0|_c \otimes \mathbf{1} + |1\rangle\langle 1|_c \otimes \prod_{i\in p} \sigma_i^x$$

Construct a basis of operators O_i

$$H = \sum_i O_i$$

For technical details please see

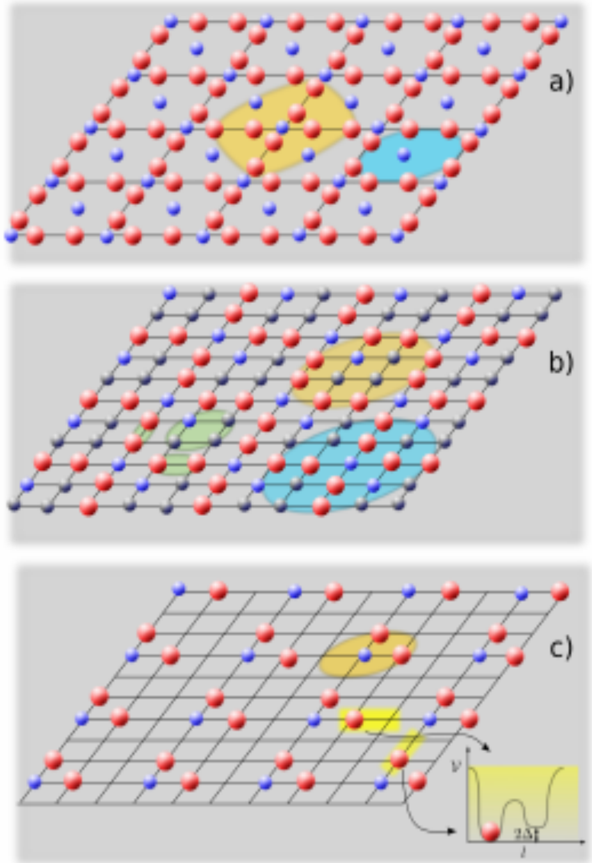
LT et al. arXiv:1211.2704
arXiv:1205.0496

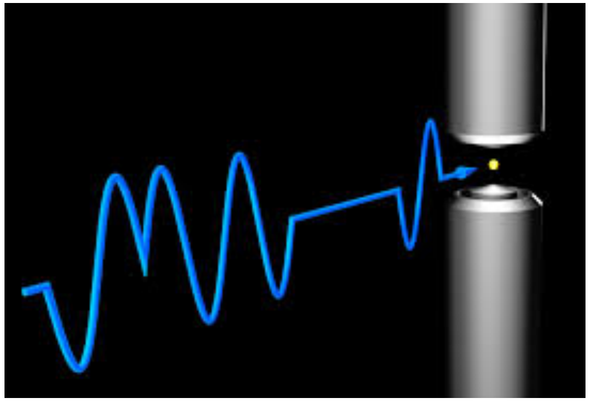
Schematic of the experiment

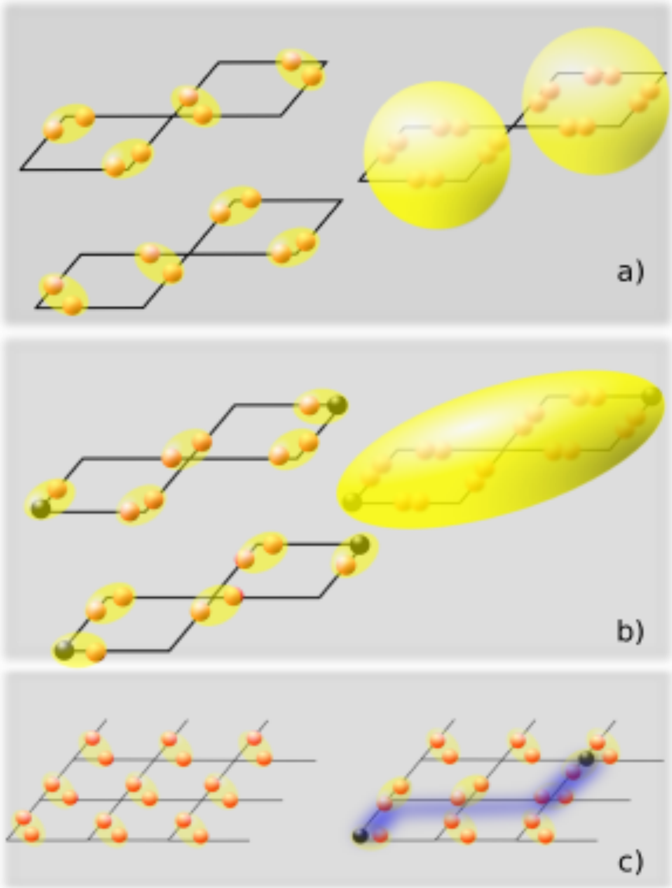
mott insulating

ADIABATIC
Sequence of
laser pulses

strong correlation







LT et al. Nat. Comm. 4, 2615 (2013)

Motivation

Tensor Networks

Quantum simulators (QS)

Lattice Gauge Theories (LGT)

Tensor Networks 4 LGT

QS of LGT with Rydberg atoms

Conclusions and outlook

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lattice gauge theories are specific
MANY BODY STRONGLY CORRELATED SYSTEMS

SYMMETRY GROUP

PROPOSAL for A FULLY SYMMETRIC VARIATIONAL ANSATZ
FOR LGT with both DISCRETE and CONTINOUS GROUPS

PROPOSAL for QUANTUM SIMULATIONS WITH COLD RYDBERG ATOMS
FOR LGT with CONTINOUS GROUPS

NUMERICAL WORK IN PROGRESS

Proof of principle passed, all the hard work still in progress

Thank you for the attention