From isospin mixing to stellar weak interaction rates within beyond-mean-field approach

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Outline

- *complex* **EXCITED VAMPIR beyond-mean-field model**

- **isospin-symmetry-breaking and shape-coexistence effects in A=70 and A=74 isovector triplets on:**
  - ***Coulomb energy differences (CED)***
  - ***mirror energy differences (MED)***
  - ***triplet energy differences (TED)***
  - ***superallowed Fermi β-decay for the Z=N+2 isotopes ^{70}Kr and ^{74}Sr***

- **terrestrial and stellar weak interaction rates for nuclei manifesting shape coexistence :**
  - ***^{70}Kr and ^{74}Sr***
  - ***^{68}Se and ^{72}Kr rp-process waiting points***

A~70 proton-rich nuclei manifest exotic structure and dynamics generated by the interplay of

- ***shape coexistence and shape mixing***
- ***competing T=0 and T=1 pairing correlations***
- ***isospin-symmetry-breaking interactions***

responsible for

drastic changes in structure with number of nucleons, spin, and excitation energy

Challenges for theory

- ***realistic effective Hamiltonians in adequate model spaces, beyond-mean-field methods***
- ***unitary treatment of structure phenomena and β-decay properties***
- ***reliable predictions on stellar weak interaction rates***

based on

self-consistent description of experimentally accessible properties

complex VAMPIR model family

- **the model space is defined by a finite dimensional set of spherical single particle states**

- **the effective many-body Hamiltonian is represented as a sum of one- and two-body terms**

- **the basic building blocks are Hartree-Fock-Bogoliubov (HFB) vacua**

- **the HFB transformations are essentially *complex* and allow for proton-neutron, parity and angular momentum mixing being restricted by time-reversal and axial symmetry**

(T=1 and T=0 neutron-proton pairing correlations already included at the mean field level)

- **the broken symmetries (s=N, Z, I, p) are restored by projection before variation**

* ***The models allow to use rather large model spaces and realistic effective interactions***

Beyond-mean-field variational procedure

complex Vampir

$$E^s[F_1^s] = \frac{\langle F_1^s|\hat{H}\hat{\Theta}_{00}^s|F_1^s\rangle}{\langle F_1^s|\hat{\Theta}_{00}^s|F_1^s\rangle}$$

$\Theta^s{}_{00}$ - symmetry projector

$| F^s{}_1 >$ - HFB vacuum

$$|\psi(F_1^s); sM\rangle = \frac{\hat{\Theta}_{M0}^s|F_1^s\rangle}{\sqrt{\langle F_1^s|\hat{\Theta}_{00}^s|F_1^s\rangle}}$$

complex Excited Vampir

$$|\psi(F_i^s); sM\rangle = \Sigma_{j=1}^i |\phi(F_j^s)\rangle\, \alpha_j^i \qquad \text{for} \quad i = 1, ..., n-1$$

$$|\phi(F_i^s); sM\rangle = \hat{\Theta}_{M0}^s|F_i^s\rangle$$

$$|\psi(F_n^s); sM\rangle = \Sigma_{j=1}^{n-1} |\phi(F_j^s)\rangle\, \alpha_j^n + |\phi(F_n^s)\rangle\, \alpha_n^n$$

$$(H - E^{(n)}N)f^n = 0$$

$$(f^{(n)})^+ N f^{(n)} = 1$$

$$|\Psi_\alpha^{(n)}; sM > = \sum_{i=1}^{n} |\psi_i; sM > f_{i\alpha}^{(n)}, \qquad \alpha = 1, ..., n$$

A ~ 70 mass region

^{40}Ca - core

model space for both: protons and neutrons

$1p_{1/2}$ $1p_{3/2}$ $0f_{5/2}$ $0f_{7/2}$ $1d_{5/2}$ $0g_{9/2}$

(charge-symmetric basis + Coulomb contributions to the π-spe from the core)

$1p_{1/2}$ $1p_{3/2}$ $0f_{5/2}$ $0f_{7/2}$ $2s_{1/2}$ $1d_{3/2}$ $1d_{5/2}$ $0g_{7/2}$ $0g_{9/2}$ $0h_{11/2}$ ***(ext-model space)***

renormalized G-matrix (OBEP- Bonn A/ CD)

- ***pairing properties enhanced by short range Gaussians for:***

  T = 1 : pp *(-35 MeV)*, np *(-20 MeV)*, nn *(-35 MeV)*

  T = 0, S = 0 and S = 1 *(-35 MeV)*

- ***onset of deformation influenced by monopole shifts:***

  $\langle 0g_{9/2}\, 0f;\, T=0\, |G|\, 0g_{9/2}\, 0f; T=0 \rangle$ $(0f_{5/2},\ 0f_{7/2})$

  $\langle 1d_{5/2}\, 1p;\, T=0\, |G|\, 1d_{5/2}\, 1p; T=0 \rangle$ $(1p_{1/2},\ 1p_{3/2})$

- ***Coulomb interaction between valence protons added***

Isospin-symmetry breaking and shape coexistence effects in isovector triplets

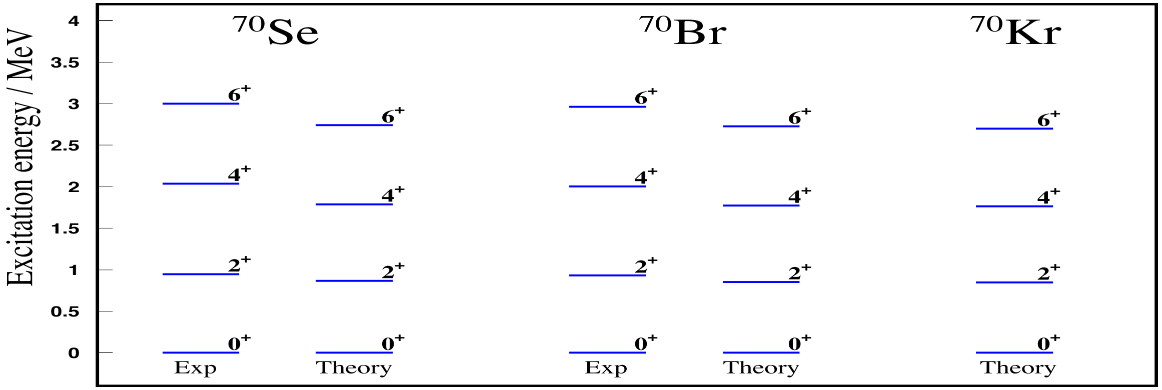
A = 70 : $_{36}Kr_{34} - _{35}Br_{35} - _{34}Se_{36}$

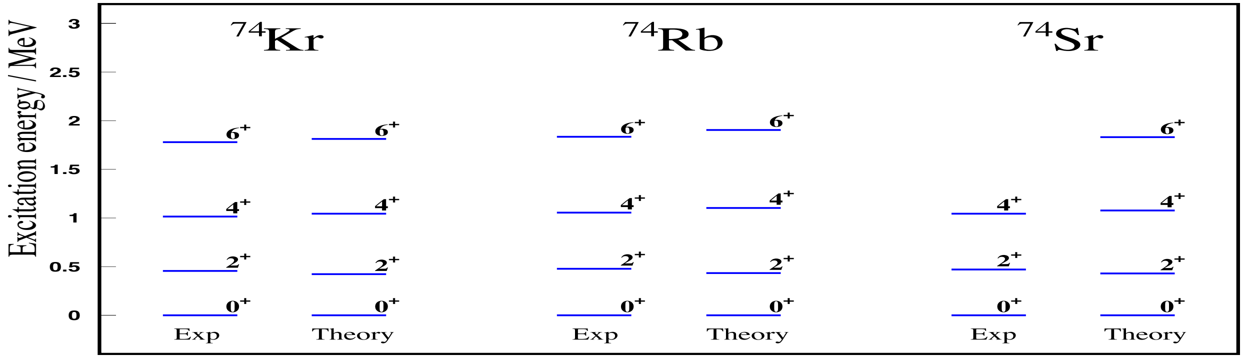
A = 74 : $_{38}Sr_{36} - _{37}Rb_{37} - _{36}Kr_{38}$

***Coulomb Energy Differences:** A = 70 exotic case (anomaly)*

Isospin-symmetry violation induced by Coulomb interaction and strong force

A. Petrovici, Phys. Rev. C 91, 014302 (2015)





Shape mixing manifested in the structure of wave functions   A = 70

- strong oblate-prolate mixing decreasing with increasing spin

^{70}Se

I(ħ)	Prolate content	Oblate content
0^+	41(4)(1)(1) %	51(1) %
2^+	56(2) %	39(2) %
4^+	52(2) %	43(2) %
6^+	76(3)(1)(1) %	17(1) %

- oblate components dominate the ground state in ^{70}Se, but prolate ones in ^{70}Br

^{70}Br

I(ħ)	Prolate content	Oblate content
0^+	68(1) %	26(2)(1) %
2^+	66(2) %	29(1) %
4^+	68(2)(1) %	26(1) %
6^+	81(4)(2)(1)(1) %	10(1) %

- similar structure for ^{70}Br and ^{70}Kr

^{70}Kr

I(ħ)	Prolate content	Oblate content
0^+	69(3) %	24(3) %
2^+	70(3) %	24(1) %
4^+	75(3) %	19(2) %
6^+	86(3)(2) %	7(2) %

Shape mixing manifested in the structure of wave functions A = 74

- *significant, but weaker, oblate-prolate mixing decreasing with increasing spin*

^{74}Kr

$I(\hbar)$	Prolate content	Oblate content
0^+	82(1)(1) %	14(1)(1) %
2^+	92(1)(1) %	6 %
4^+	95(1)(1) %	3 %
6^+	97(1) %	1(1) %

^{74}Rb

$I(\hbar)$	Prolate content	Oblate content
0^+	85(1) %	12(1) %
2^+	94(1) %	4 %
4^+	96(1) %	2 %
6^+	97(1) %	1 %

^{74}Sr

$I(\hbar)$	Prolate content	Oblate content
0^+	77(2) %	19(1) %
2^+	87(1) %	11 %
4^+	90(1) %	8 %
6^+	92(1) %	5(1) %

- *maximum oblate-prolate mixing in* ^{74}Sr

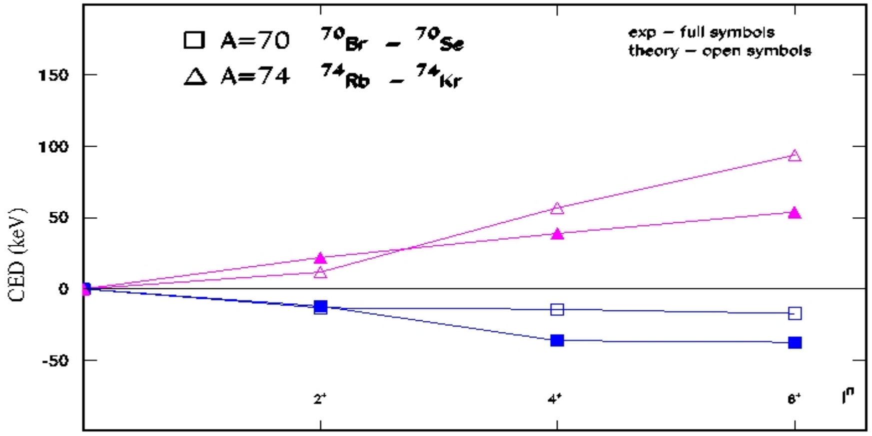
Shape coexistence and deformation revealed by spectroscopic quadrupole moments

$I(\hbar)$	^{70}Se	^{70}Br	^{70}Kr
2_1^+	-7	-18	-25
2_2^+	4	16	18
4_1^+	-7	-30	-42
4_2^+	0	25	33
6_1^+	-49	-59	-65
6_2^+	38	51	53

$I(\hbar)$	^{74}Kr	Exp	^{74}Rb	^{74}Sr
2_1^+	-54	-53(24)	-57	-50
2_2^+	49	24(21)	53	48
4_1^+	-74	-80(40)	-77	-70
4_2^+	68		72	67
6_1^+	-85	-130(50)	-86	-81
6_2^+	78		81	80

Isospin-symmetry-breaking and shape-mixing effects on Coulomb Energy Differences

$$CED_{J,T=1} = E^*_{J,T=1,Tz=0} - E^*_{J,T=1,Tz=+1}$$



Isospin-symmetry-breaking and shape-mixing effects on

Mirror Energy Differences & Triplet Energy Differences

Charge-symmetry breaking:

$$V^{(1)}_{INC} = V_{pp} - V_{nn}$$

(V_{nn} 1% stronger than V_{pp})

$$MED_{J,T=1} = E^*_{J,T=1,Tz=-1} - E^*_{J,T=1,Tz=+1}$$

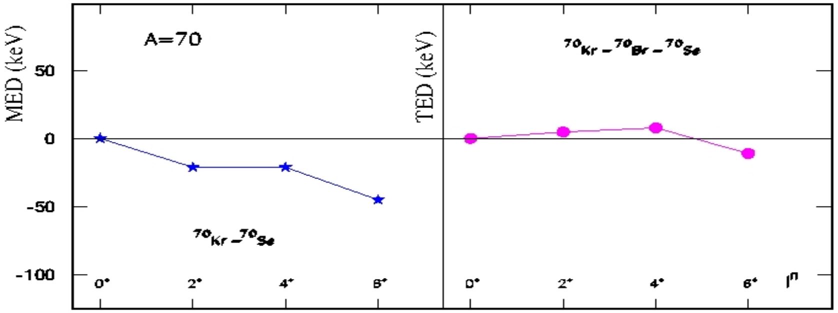
Charge-independence breaking:

$$V^{(2)}_{INC} = V_{pp} + V_{nn} - 2V_{pn}$$

(V_{pn} 2.5% stronger than the average of V_{pp} and V_{nn})

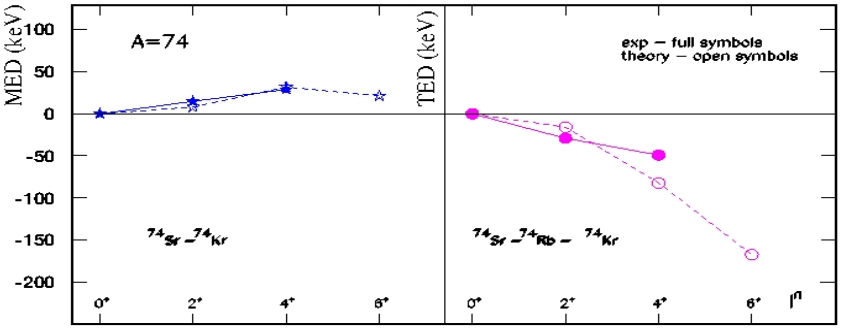
$$TED_{J,T=1} = E^*_{J,T=1,Tz=-1} + E^*_{J,T=1,Tz=+1} - 2E^*_{J,T=1,Tz=0}$$

A=70: *complex* Excited Vampir *predictions*



A=74: *complex* Excited Vampir *predictions*

recent experimental results (J. Henderson, Phys. Rev. C 90, 051303(R) (2014))



Self-consistent terrestrial and stellar weak interaction rates for A~70 nuclei

Fermi transition probabilities

$$B_{if}(F) = \frac{1}{2J_i+1}\frac{g_V^2}{4\pi}|M_F|^2$$

$$\begin{aligned} M_F &\equiv (\xi_f J_f||\hat{1}||\xi_i J_i) \\ &= \delta_{J_i J_f}\sum_{ab} M_F(ab)(\xi_f J_f|||[c_a^\dagger \tilde{c}_b]_0||\xi_i J_i) \end{aligned}$$

$$M_F(ab) = (a||\hat{1}||b)$$

Gamow-Teller transition probabilities

$$B_{if}(GT) = \frac{1}{2J_i+1}\frac{g_A^2}{4\pi}|M_{GT}|^2$$

$$\begin{aligned} M_{GT} &\equiv (\xi_f J_f||\hat{\sigma}||\xi_i J_i) \\ &= \sum_{ab} M_{GT}(ab)(\xi_f J_f|||[c_a^\dagger \tilde{c}_b]_1||\xi_i J_i) \end{aligned}$$

$$M_{GT}(ab) = 1/\sqrt{3}(a||\hat{\sigma}||b)$$

Weak interaction rates and shape coexistence for the Z=N+2 isotopes ^{70}Kr and ^{74}Sr

Isospin-symmetry-breaking and shape coexistence effects on superallowed Fermi β-decay of ^{70}Kr and ^{74}Sr

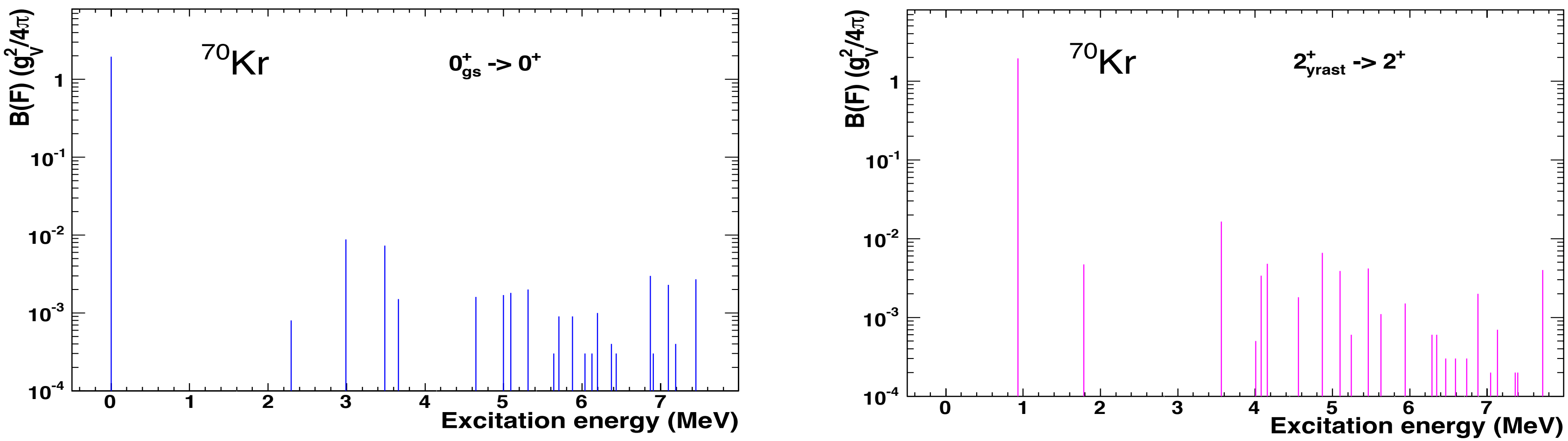
A. Petrovici, J. Phys.: Conf. Series (2016)

test of the CVC hypothesis
test of the unitarity of CKM matrix

$$ft(1+\delta_R)(1-\delta_c) = \frac{K}{2G_v^2(1+\Delta_R^v)}$$

δc – isospin-symmetry-breaking correction

^{70}Kr *Q_{EC} = 10.480 MeV*



$1\% \leq \delta_c \leq 2\%$

Nonanalog branches:
0^+_{IV} , 0^+_{V} $\leq$ 0.4 %

$1\% \leq \delta_c \leq 3\%$

Nonanalog branches:
$2^+_{IV} \leq 1.3\%$

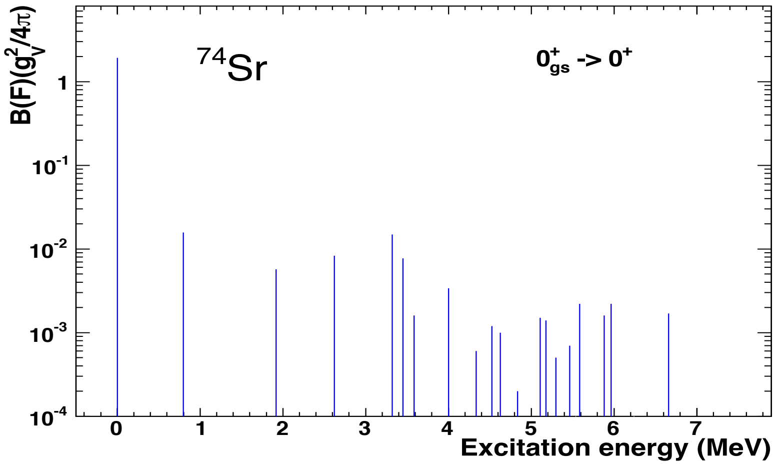
^{74}Sr $Q_{EC} = 11.090$ MeV

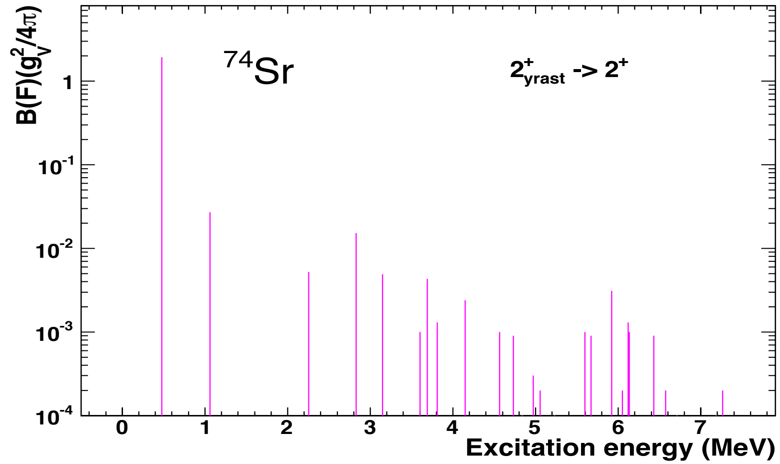
$1\% \leq \delta_c \leq 3\%$

Nonanalog branches:
0^+_{II} , $0^+_{VI} \leq 0.8\%$

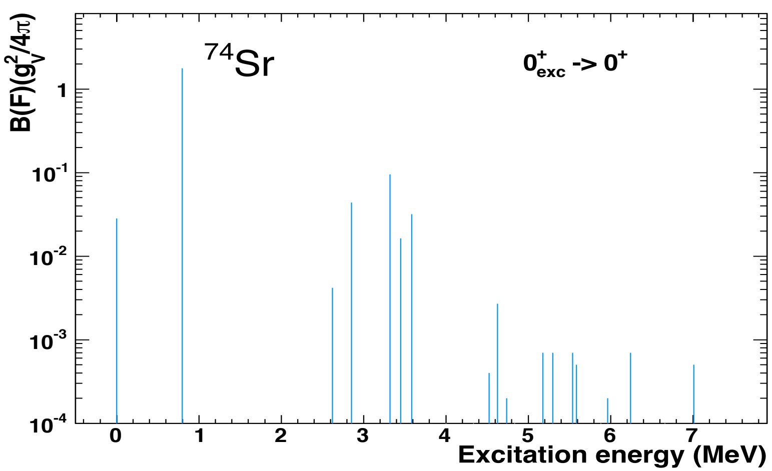
$1\% \leq \delta_c \leq 3.6\%$

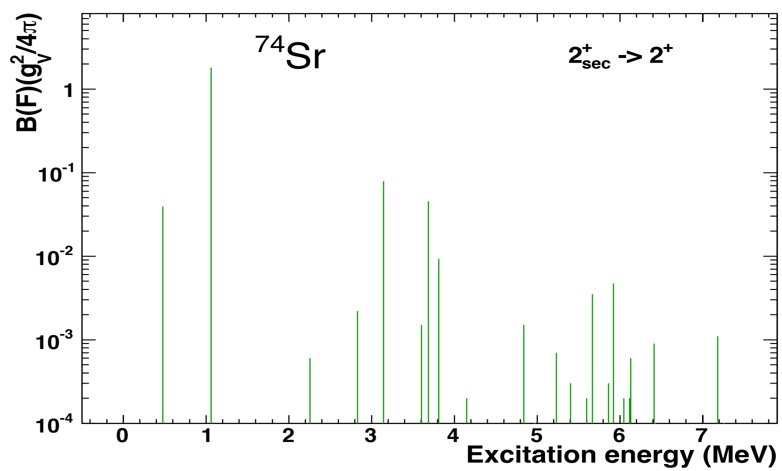
Nonanalog branches:
$2^+_{II} \leq 1.3\%$, $2^+_{IV} \leq 0.8\%$





Relevant for astrophysical scenarios on rp-process path in X-ray burst environment: 0^+_{exc} *and* 2^+_{sec} *decay*



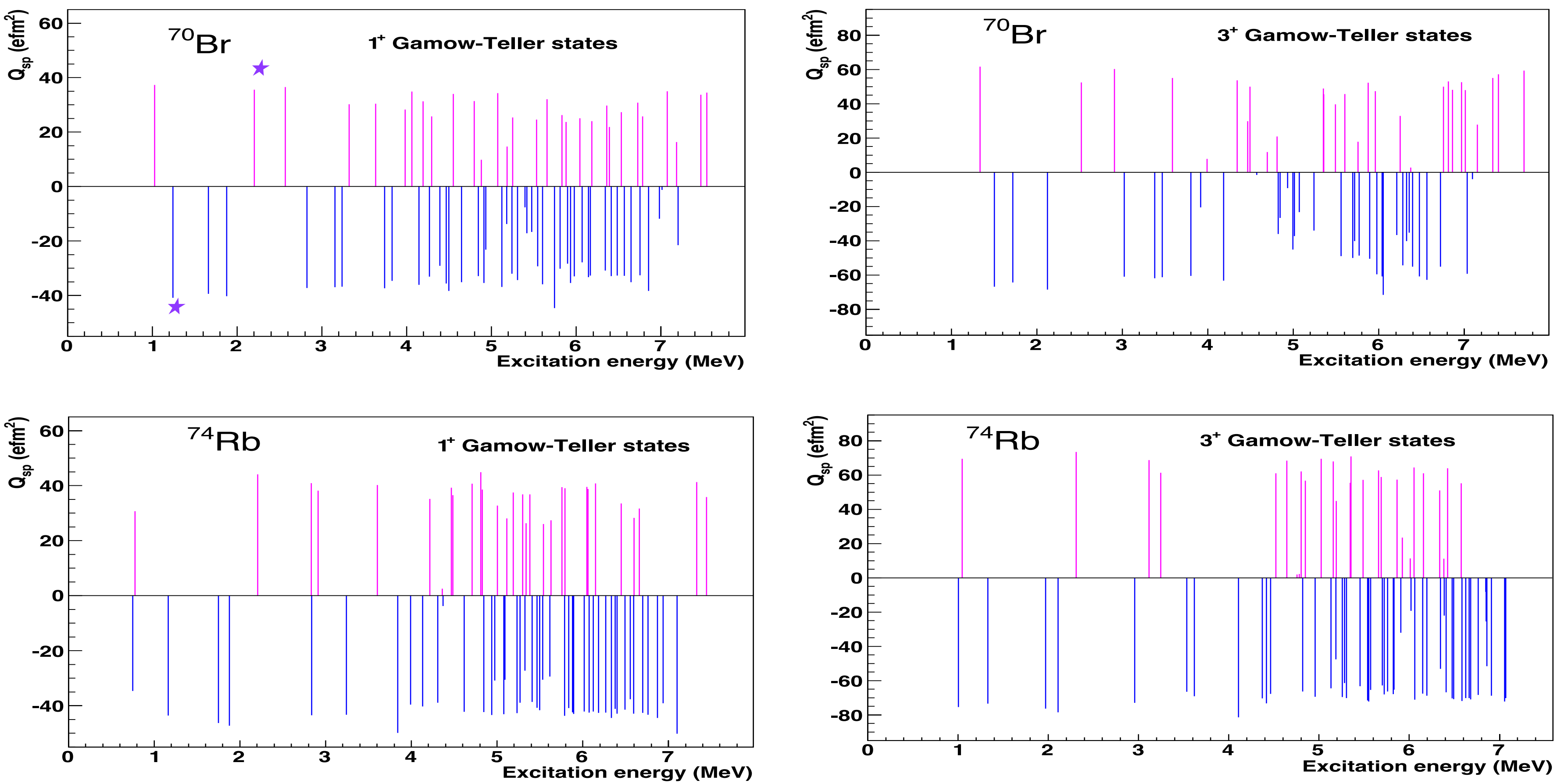


Gamow-Teller β decay and shape coexistence for ^{70}Kr and ^{74}Sr

A. Petrovici and O. Andrei, Phys. Rev. C92, 064305 (2015)

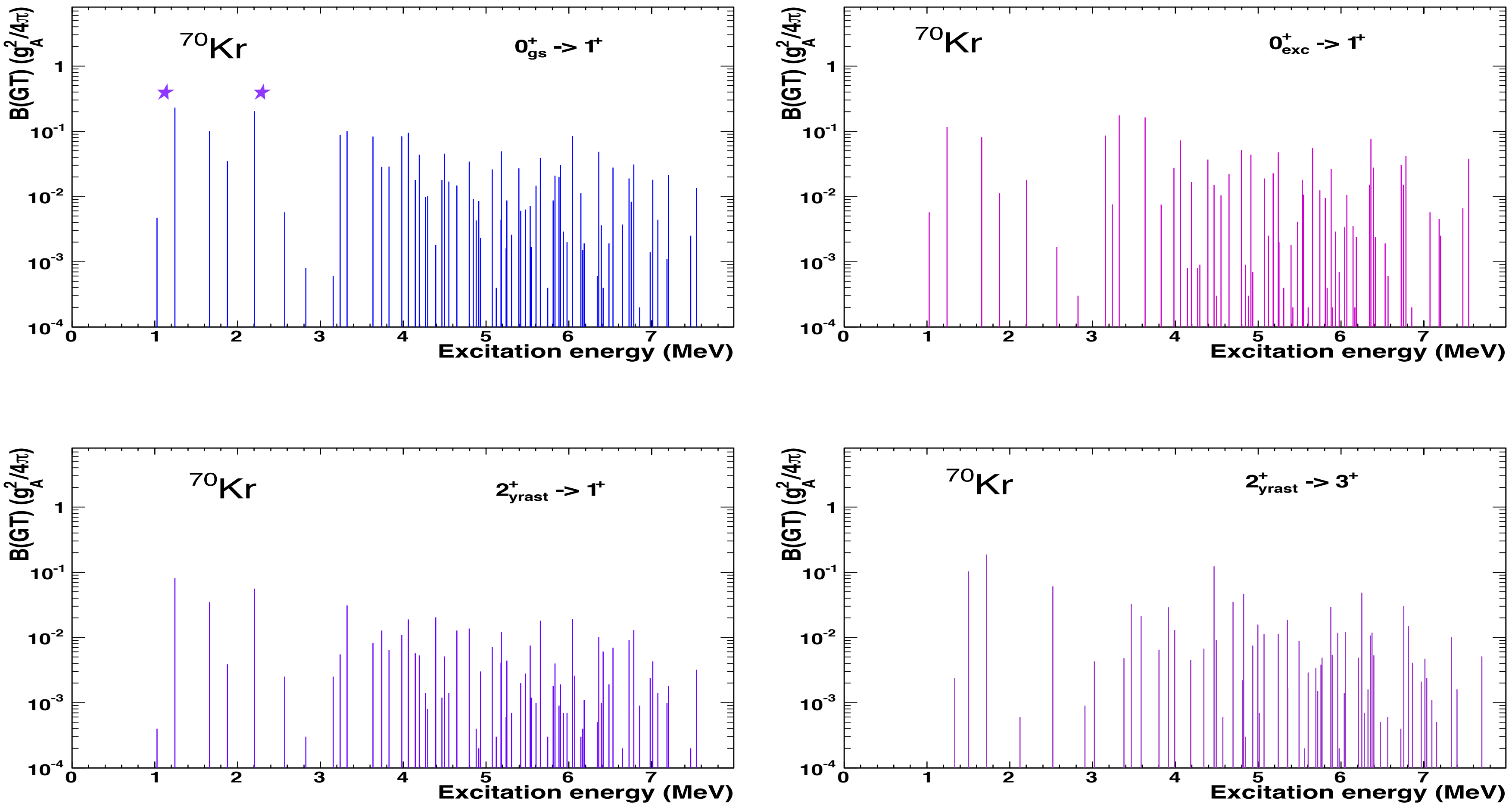
Independent chains of variational calculations in parent and daughter nuclei

Large variety of deformations in daughter states revealed by spectroscopic quadrupole moments



Gamow-Teller strength distributions for the decay of low-lying 0^+ and 2^+ states in ^{70}Kr

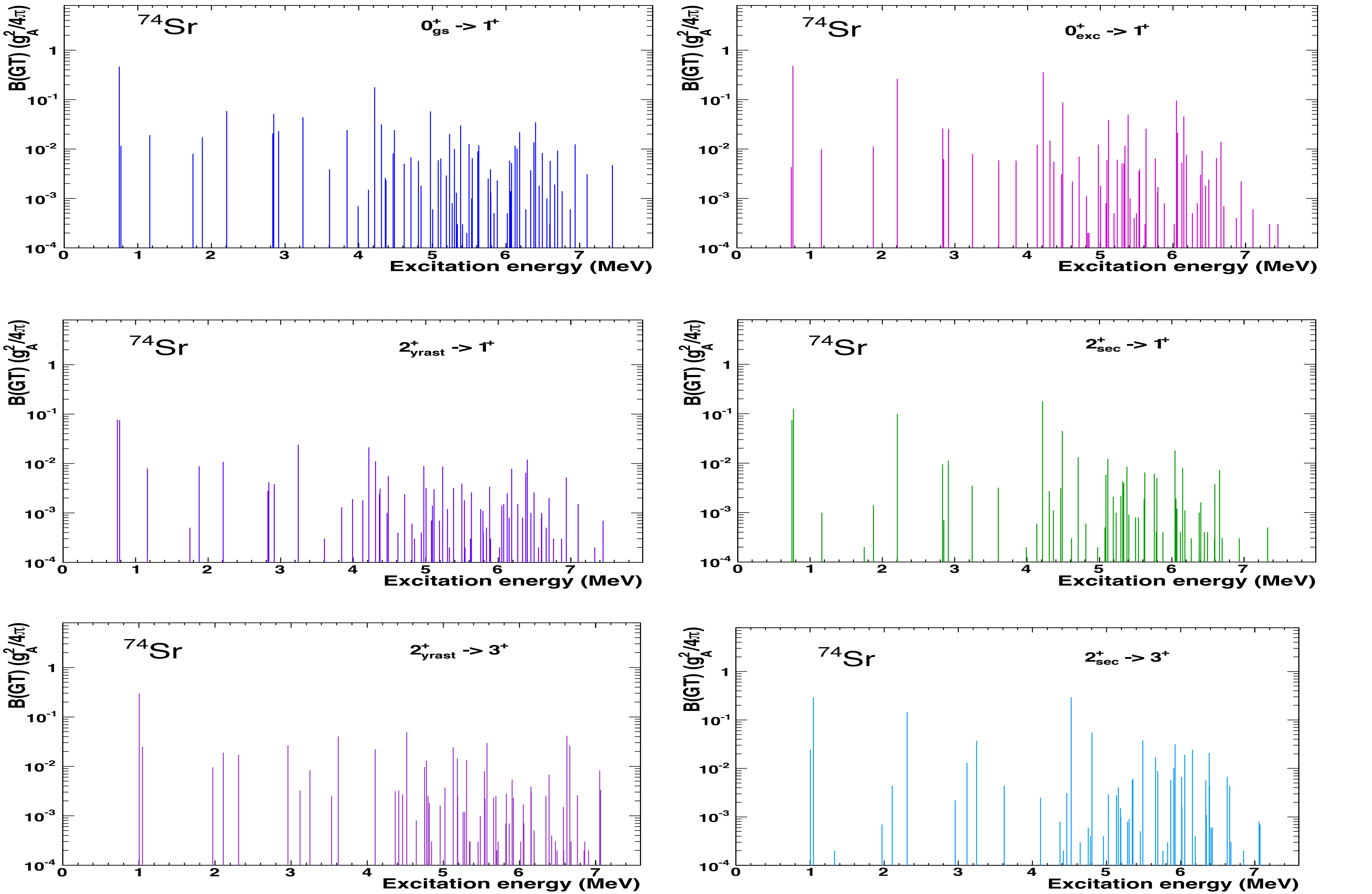
Specific shape mixing for each parent state influences the strength distributions

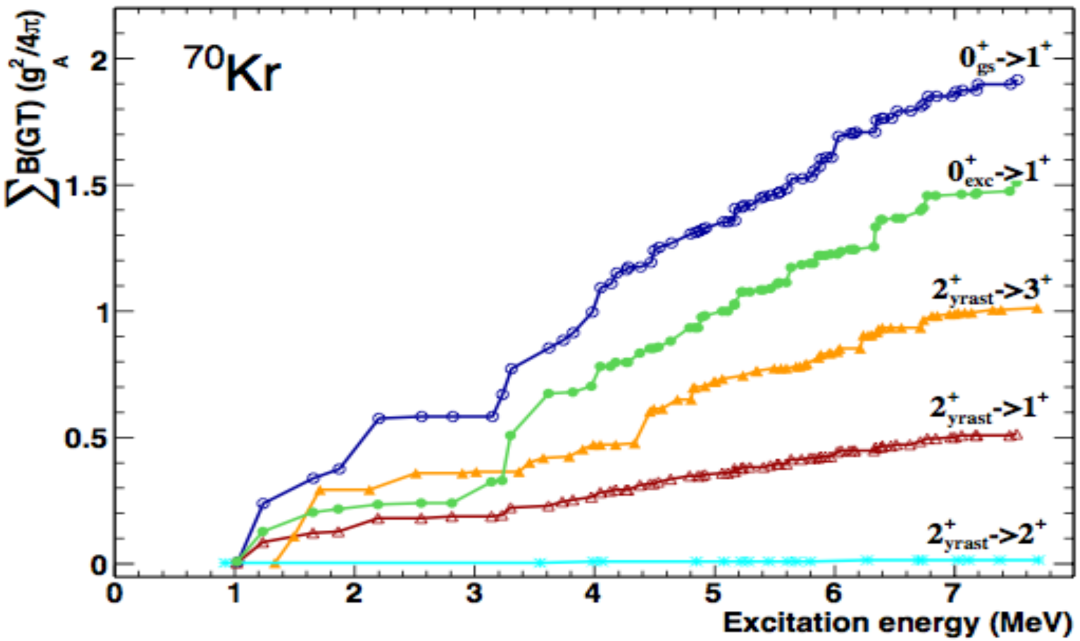


Contributions from $p^{\nu(\pi)}_{1/2}p^{\pi(\nu)}_{3/2}$, $p^{\nu}_{3/2}p^{\pi}_{3/2}$, $f^{\nu}_{5/2}f^{\pi}_{5/2}$, $f^{\nu(\pi)}_{5/2}f^{\pi(\nu)}_{7/2}$, $g^{\nu}_{9/2}g^{\pi}_{9/2}$ matrix elements

(coherent / cancelling effect)

Gamow-Teller strength distributions for the decay of low-lying 0^+ and 2^+ states in ^{74}Sr





Terrestrial half-lives

$$\frac{1}{T_{1/2}} = \frac{1}{D} \sum_{0<E_f<Q_{EC}} f(Z,E_f)[B_{if}(GT) + B_{if}(F)]$$

$T_{1/2}^{exp} = 52(17)$ ms

$T_{1/2}^{GT} = 258$ ms $\quad T_{1/2}^{F} = 63$ ms

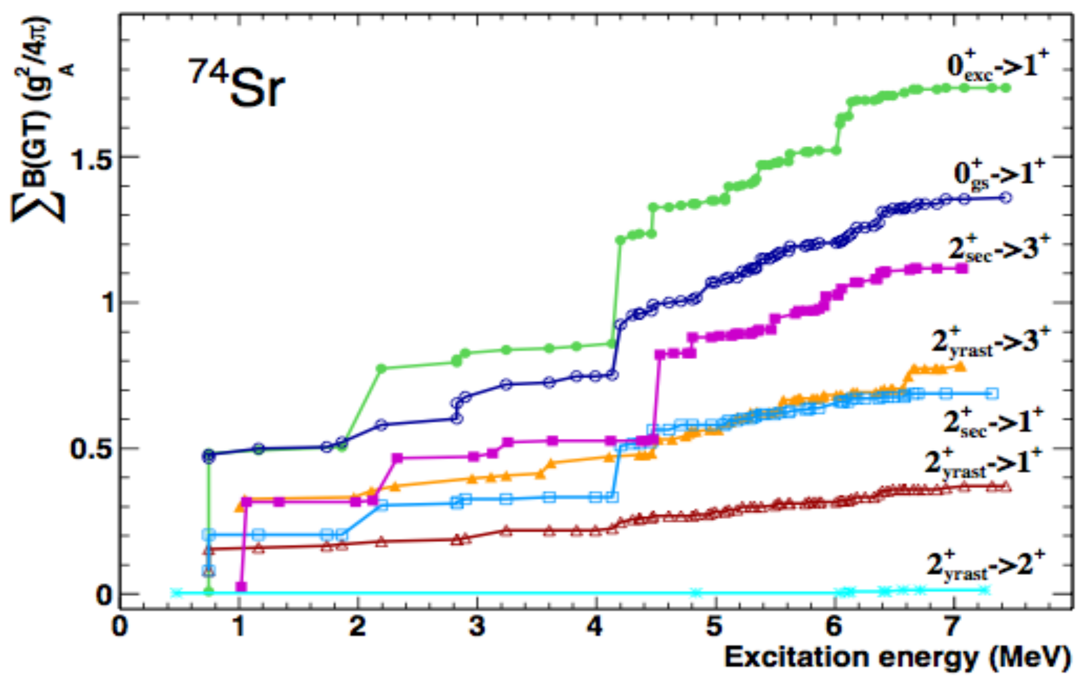
$T_{1/2}^{EXVAM} = 51$ ms

Terrestrial half-lives

$T_{1/2}^{exp} = 27(8)$ ms

$T_{1/2}^{GT} = 137$ ms $\quad T_{1/2}^{F} = 48$ ms

$T_{1/2}^{EXVAM} = 36$ ms



Weak interaction rates in X-ray burst astrophysical environment

In the X-ray burst stellar environment at densities (~10^6 mol/cm^3) and temperatures (~10^9K) typical for the rp-process the contribution of thermally populated low-lying 0^+ and 2^+ states may be relevant.

(H. Schatz et al., Phys. Rep. 294, 167 (1998))

$$\lambda^\alpha = \frac{ln2}{K} \sum_i \frac{(2J_i+1)e^{-E_i/(kT)}}{G(Z,A,T)} \sum_j B_{ij}\phi_{ij}^\alpha$$

$$G(Z,A,T) = \sum_i (2J_i+1)exp(-E_i/(kT))$$

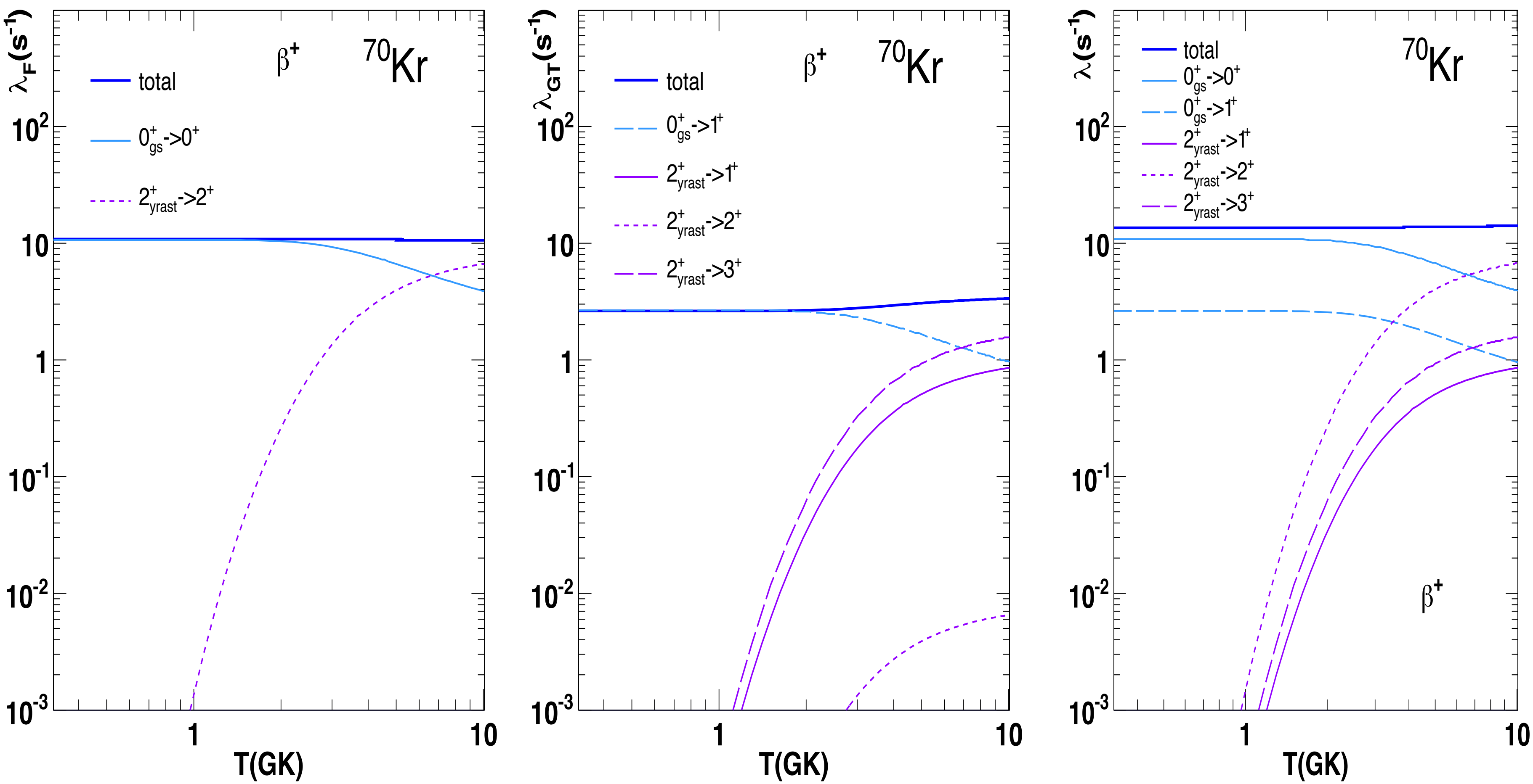
$$B_{ij} = B_{ij}(F) + B_{ij}(GT)$$

$$\phi_{ij}^{ec} = \int_{w_l}^{\infty} wp(Q_{ij}+w)^2 F(Z,w) S_e(w)(1-S_\nu(Q_{ij}+w))dw$$

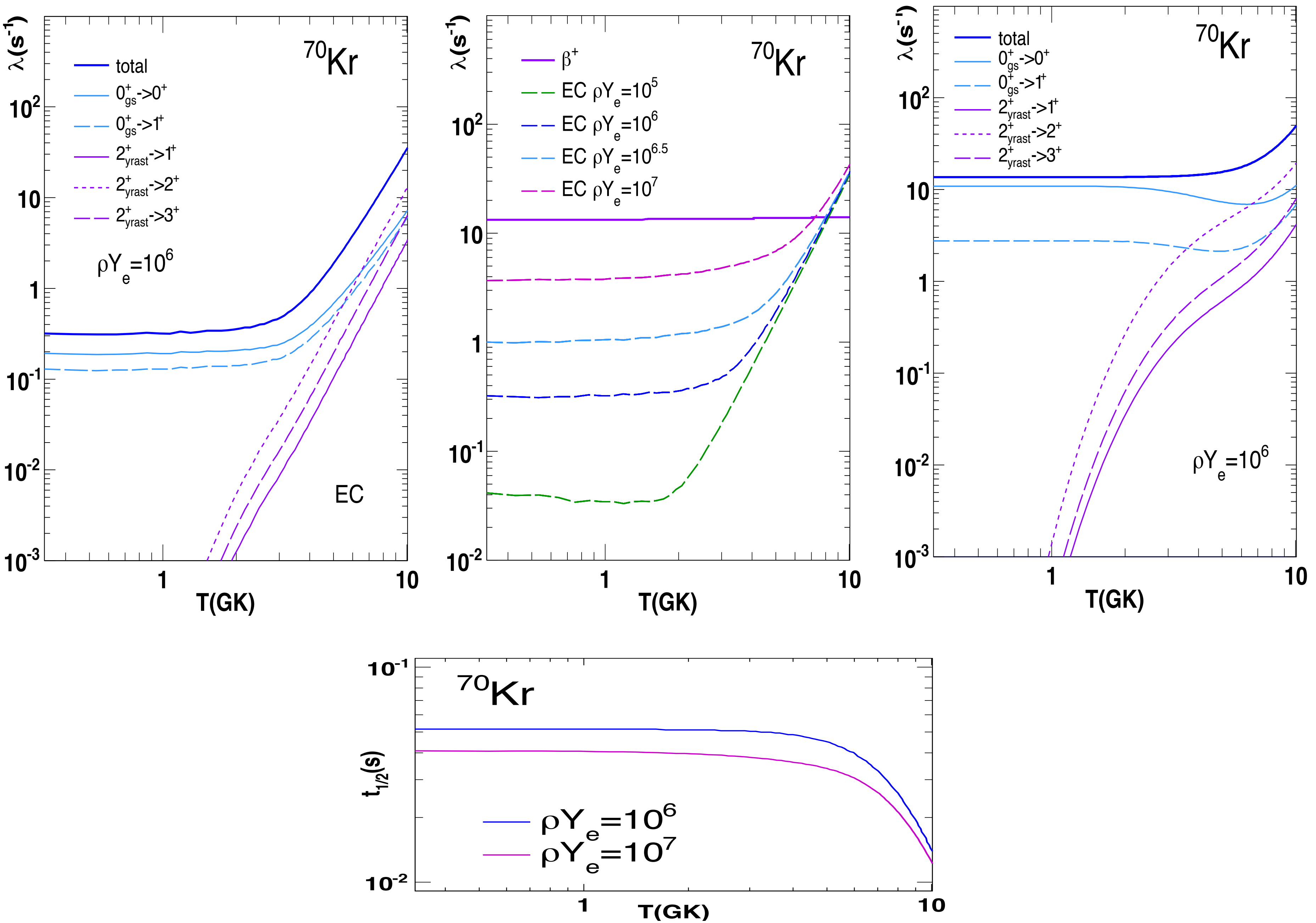
$$\phi_{ij}^{\beta^+} = \int_1^{Q_{ij}} wp(Q_{ij}-w)^2 F(-Z+1,w)(1-S_p(w))(1-S_\nu(Q_{ij}-w))dw$$

Stellar rates for ^{70}Kr : β^+ - decay

0^+_{gs} and 2^+_{yrast} - parent states

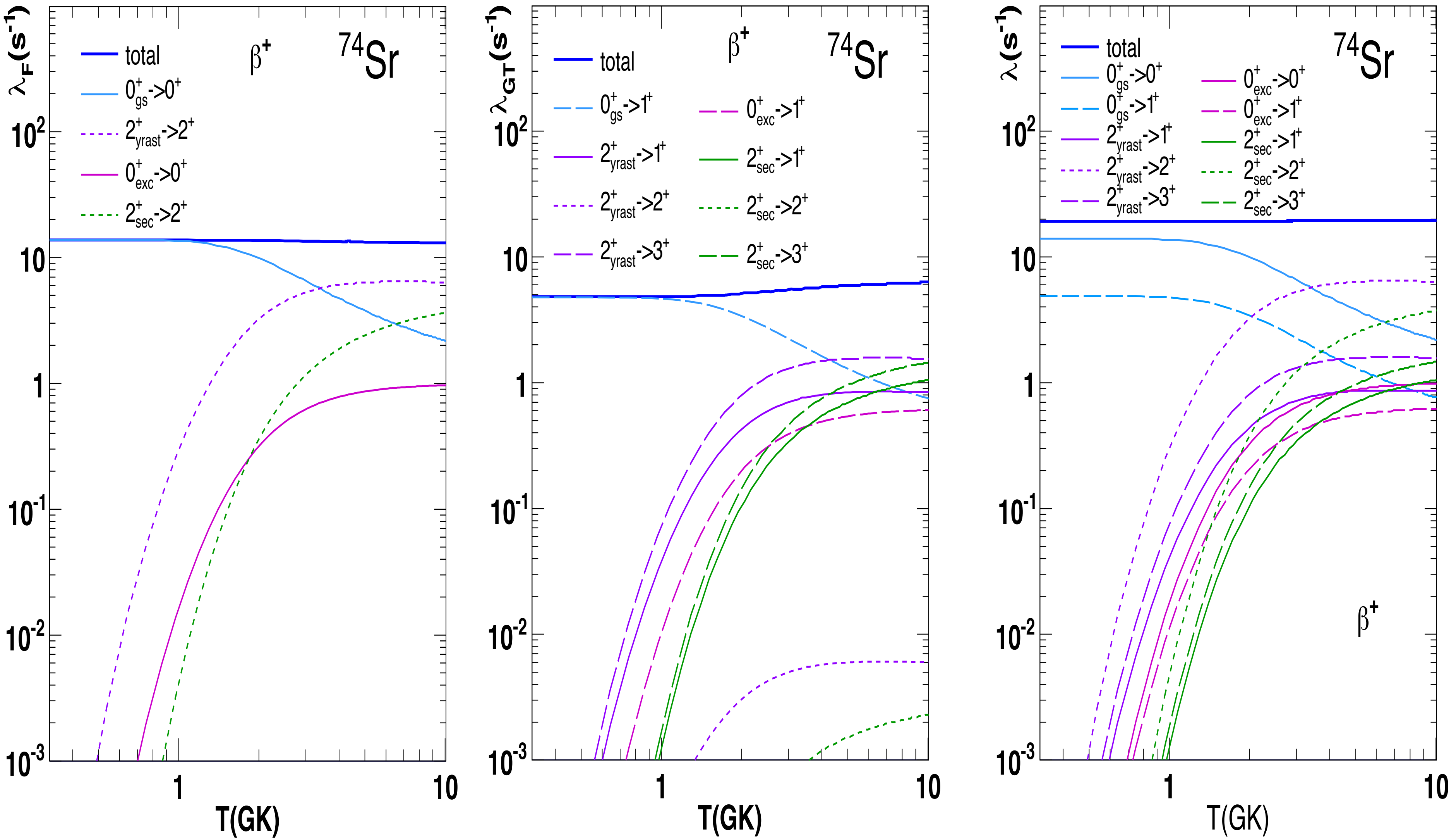


β⁺ and electron capture rates for ^{70}Kr

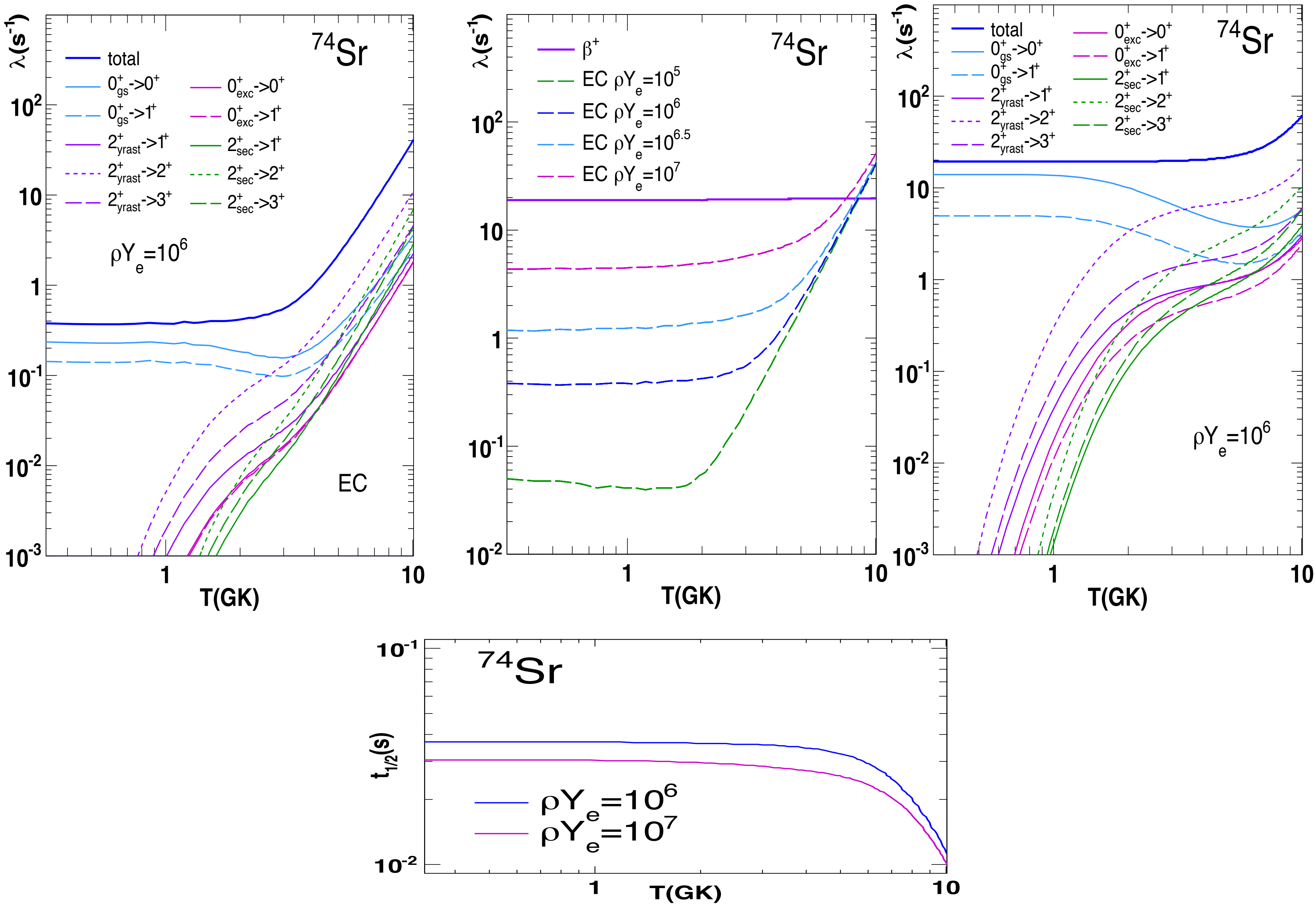


Stellar rates for ^{74}Sr : β^+ - decay

$E^{th}_{0^+_{exc}} = 0.564$ MeV $E_{2^+_{yrast}} = 0.471$ MeV $E^{th}_{2^+_{sec}} = 0.823$ MeV



β^+ and electron capture rates for ^{74}Sr

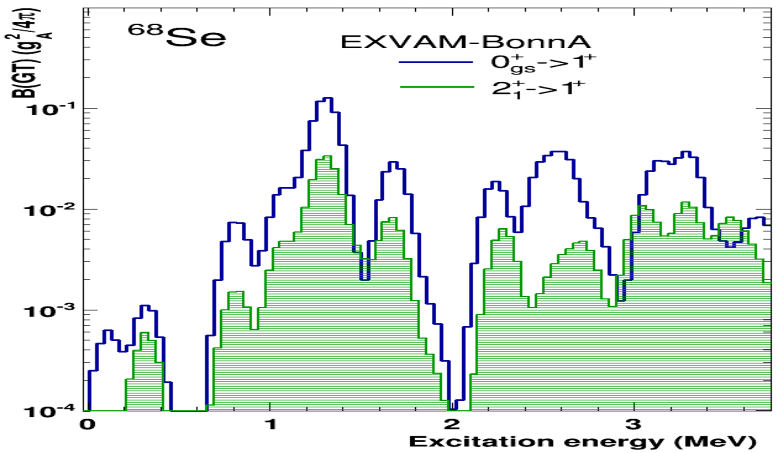


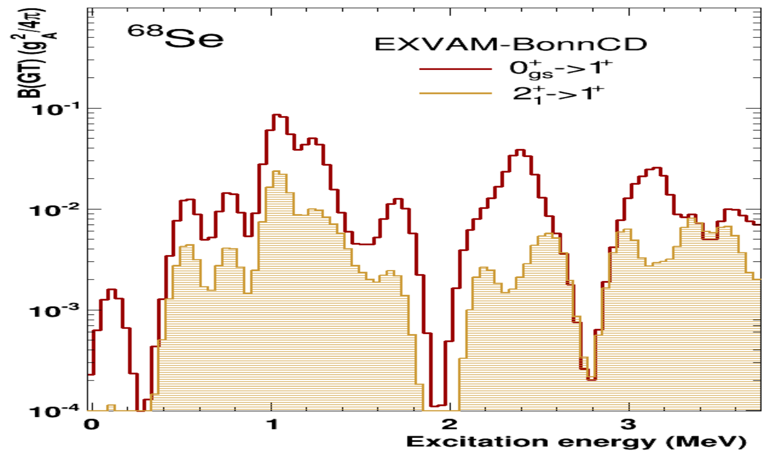
Weak interaction rates and shape coexistence for ^{68}Se and ^{72}Kr waiting points

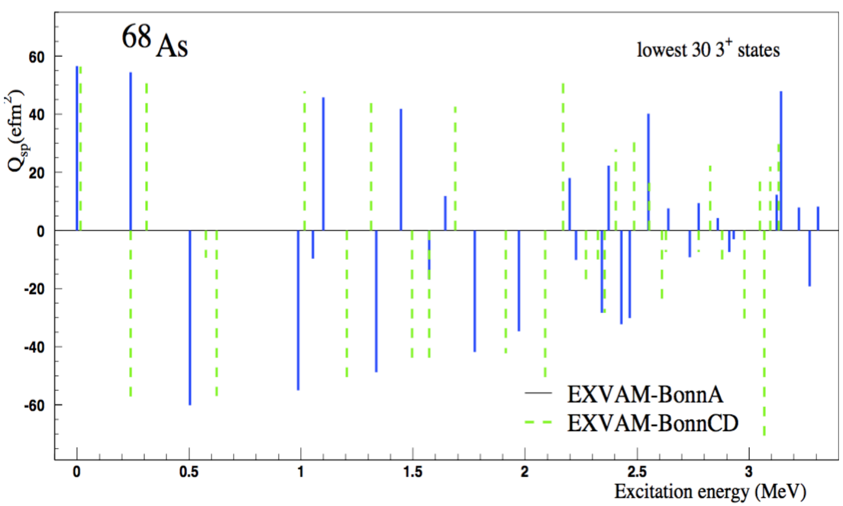
A. Petrovici and O. Andrei, Eur. Phys. J. A51, 133 (2015)

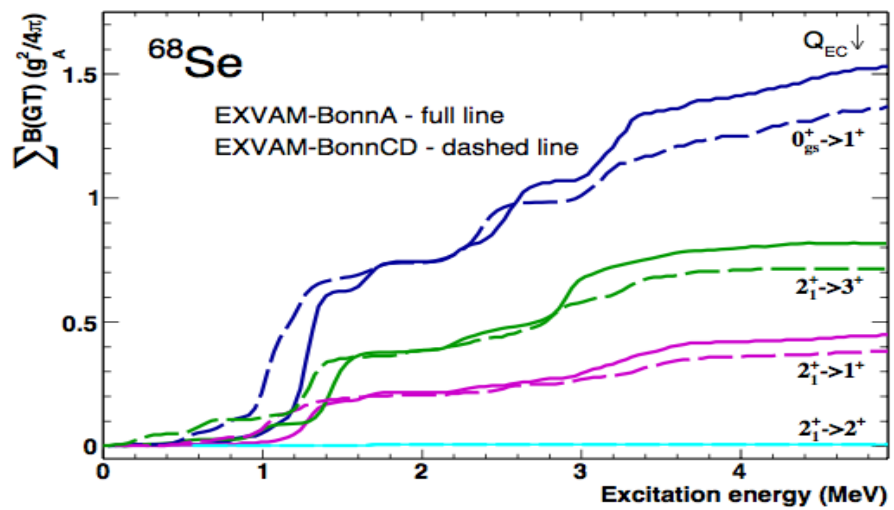
Shape coexistence and mixing in both parent and daughter nuclei

^{68}Se: $E_{2^+_{yrast}}$ = 0.854 MeV $Q^{sp}_{2^+_{yrast}}$ = 3.5 efm^2(A); -7.1 efm^2 (CD) B(E2;$2^+ \to 0^+$) ~ 500 e^{2}fm^4 (Exp.: 430(60) e^{2}fm^4)





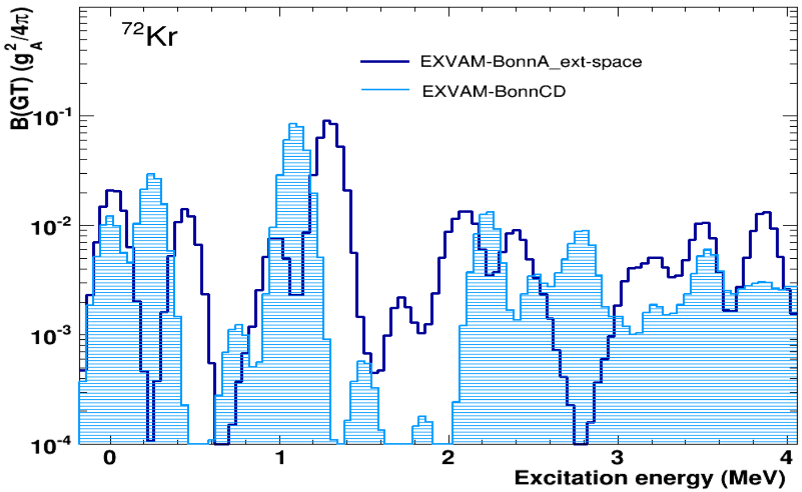


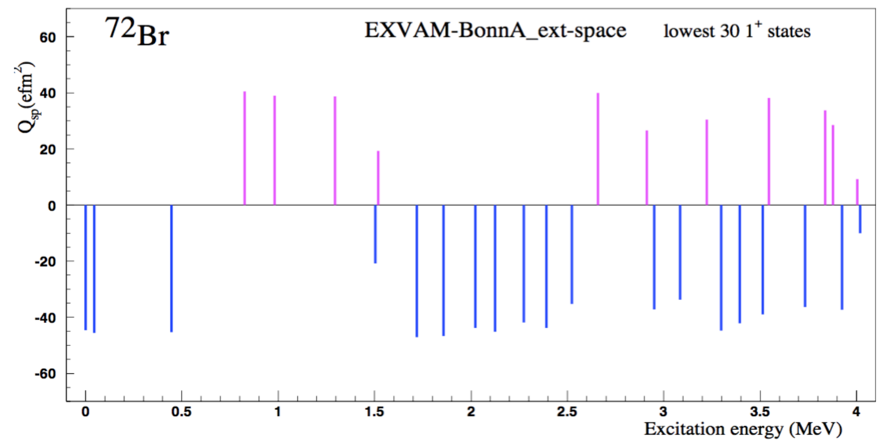


$T_{1/2}^{exp}$ = 35.5(7) ms $T_{1/2}^{EXVAM}$ = 48.8 ms (BonnA) $T_{1/2}^{EXVAM}$ = 33.5 ms (BonnCD)

^{72}Kr: $E_{0^+_{gs}}$ = 0.0 MeV [*60/40(%) – p/o mixing*] $E_{0^+_{exc}}$ = 0.671 MeV [*38/62(%) – p/o mixing*]

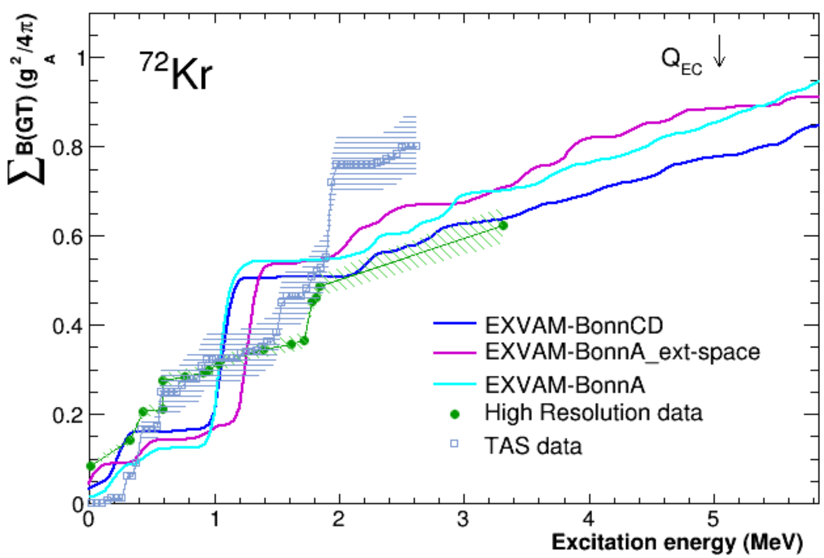
$E_{2^+_{yrast}}$ = 0.710 MeV [*41/59(%) – p/o mixing*] (BonnA-ext-space)

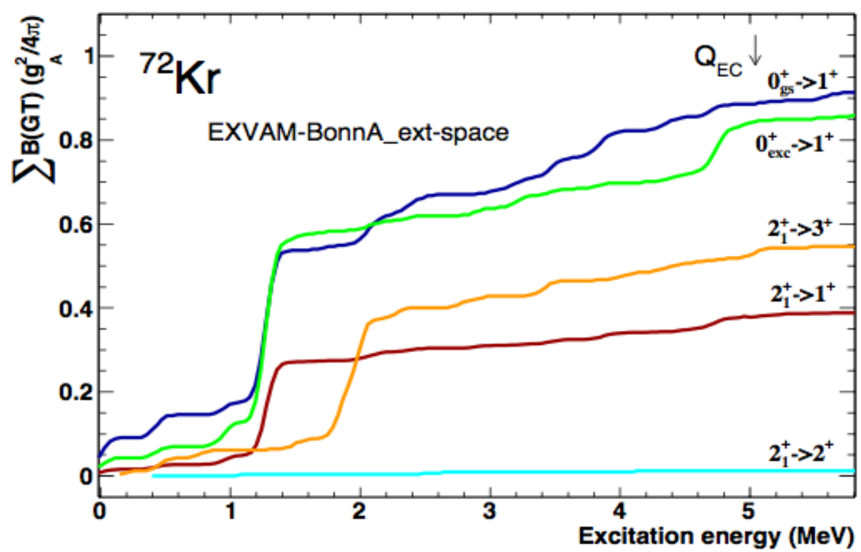




Contributions: - $p^{\nu(\pi)}{}_{1/2}p^{\pi(\nu)}{}_{3/2}$, $p^{\nu}{}_{3/2}p^{\pi}{}_{3/2}$, $f^{\nu}{}_{5/2}f^{\pi}{}_{5/2}$, $f^{\nu(\pi)}{}_{5/2}f^{\pi(\nu)}{}_{7/2}$, $g^{\nu}{}_{9/2}g^{\pi}{}_{9/2}$ ***matrix elements (decay to 1^+ states)***

- $p^{\nu}{}_{3/2}p^{\pi}{}_{1/2}$, $p^{\nu}{}_{3/2}p^{\pi}{}_{3/2}$, $f^{\nu}{}_{5/2}f^{\pi}{}_{7/2}$ ***matrix elements (decay to 3^+ states)***

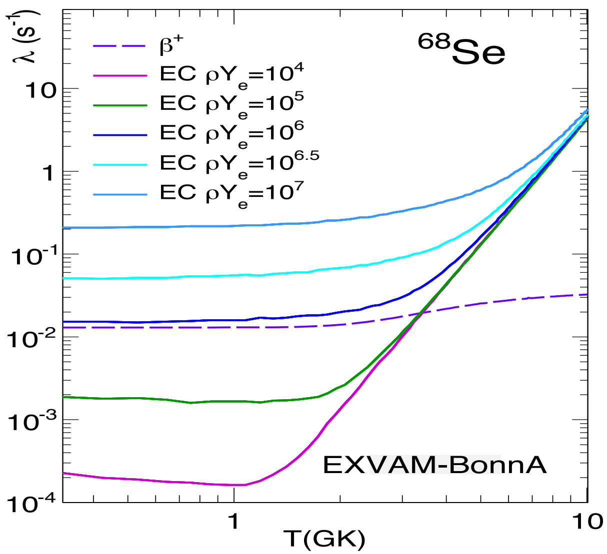


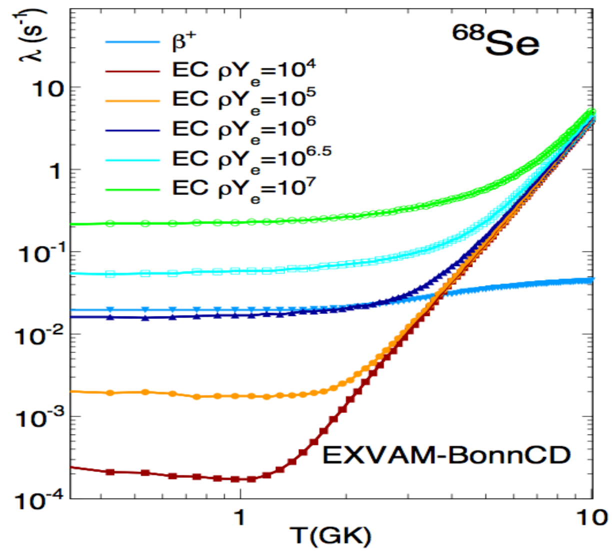


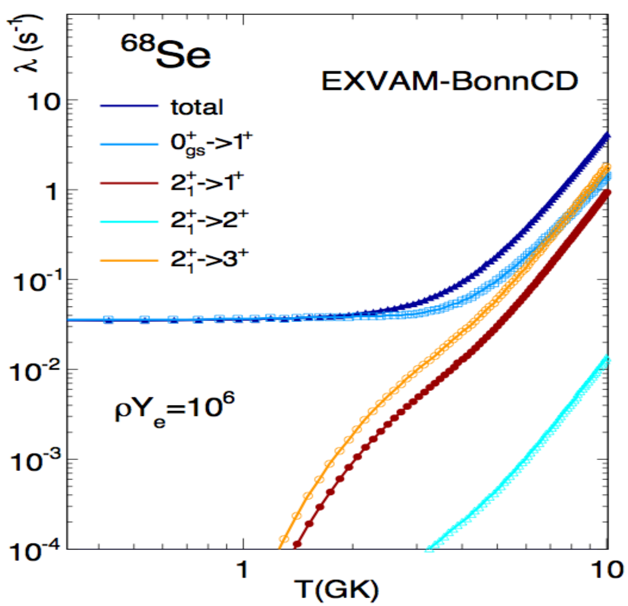
$T_{1/2}^{exp}$ = 17.1(2) ms $T_{1/2}^{EXVAM}$ = 20.8 ms (BonnA) $T_{1/2}^{EXVAM}$ = 20.7 ms (BonnA-ext-space) $T_{1/2}^{EXVAM}$ = 18.9 ms (BonnCD)

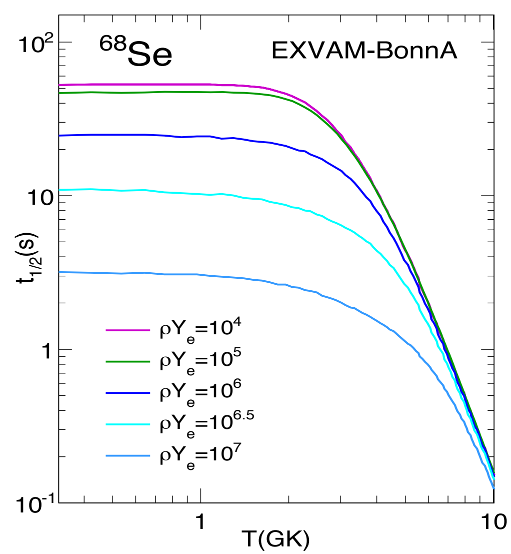
Stellar rates for ^{68}Se : β^{+} and continuum electron capture

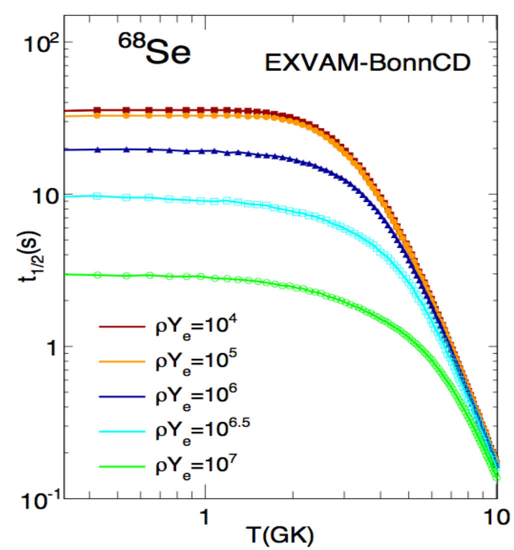
Significant continuum electron capture contribution





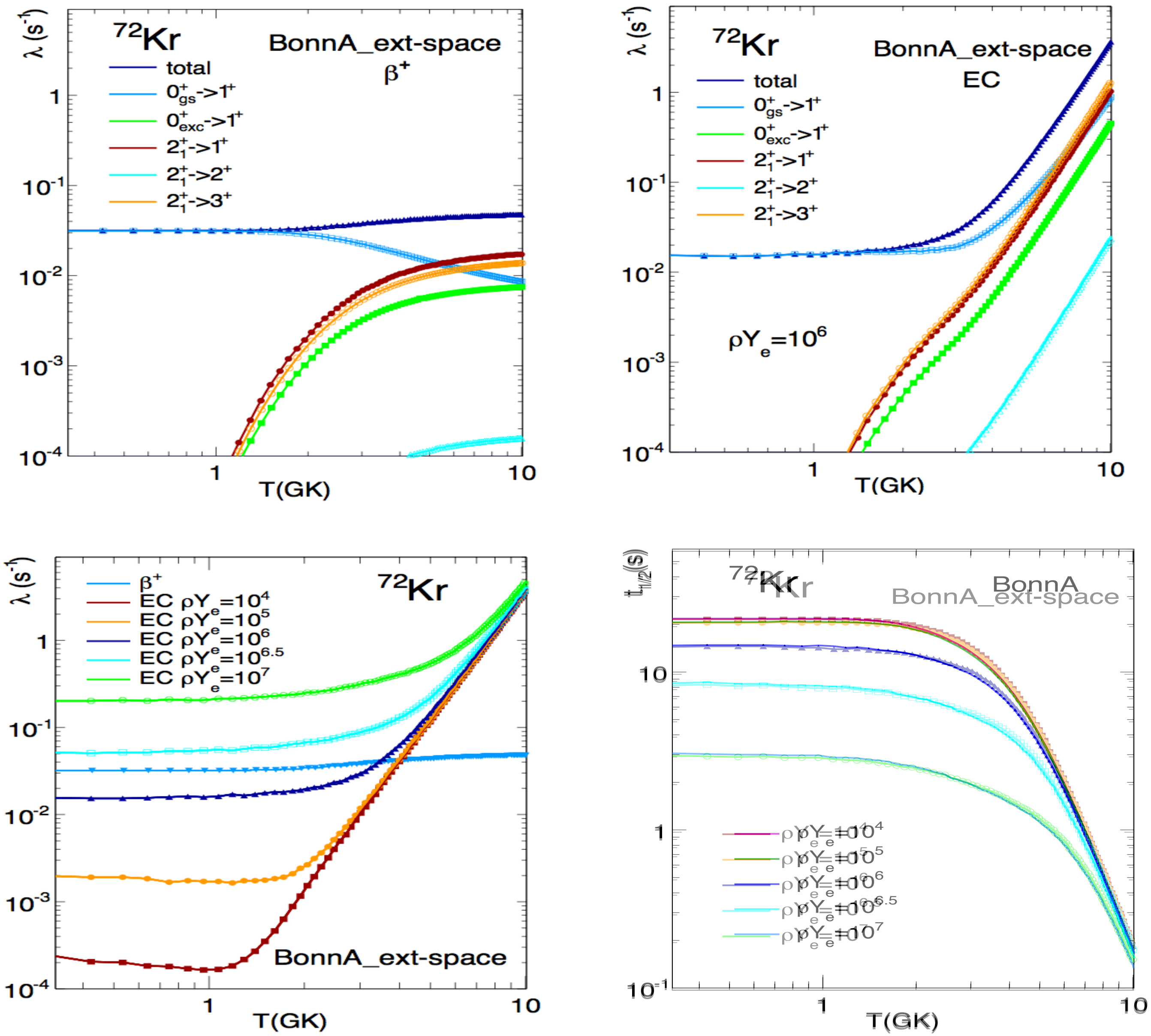






Stellar rates for ^{72}Kr : β^{+} and continuum electron capture

Significant continuum electron capture contribution



Summary

complex **EXCITED VAMPIR beyond-mean-field model** ***self-consistently describes shape-coexistence effects on***

- ***low-spin structure in A=70 and A=74 isovector triplets***
- ***superallowed Fermi β-decay of the Z=N+2 isotopes ^{70}Kr and ^{74}Sr***
- ***terrestrial and stellar weak interaction rates for A~70 proton-rich nuclei***