SU(3) flavour symmetry in the charmed baryon sector from the lattice

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with

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Science and Technology for FAIR in Europe

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Outline

- Lattice QCD
- Setting the stage
- Results
- Summary

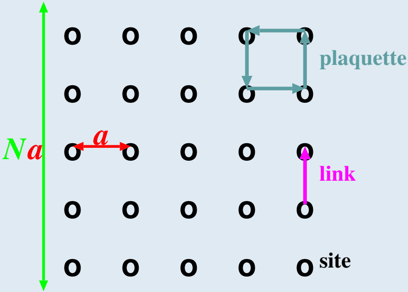
Why Worms?

photo by Christian Fischer

What I will (not) cover

- RQCD: P Pérez-Rubio, 1302.5774 and in prep $n_f = 2+1$, stout link NP clover-Wilson (SLiNC), $a \approx 0.076\,\mathrm{fm}$, $m_\pi > 250$ MeV.
- Also shown: M Padmanath et al [HSC], 1311.4806, $n_f = 2+1$ anisotropic clover-Wilson, $a_s \approx 0.12\,\mathrm{fm} \approx 4.5a_t$, $m_\pi \approx 390$ MeV.
- Recent results not covered (only in summary plots):
  L Liu et al 0909.3294 clover-Wilson and DW on $n_f = 2+1$ asqtad
  C Alexandrou et al [ETMC] 1205.6856 $n_f = 2$ Twisted Mass, 2 a's
  R Briceño et al 1207.3536 clover on $n_f = 2+1+1$ HISQ, 3 a's
  S Dürr et al 1208.6270 Brilloin on $n_f = 2$ NP clover-Wilson
  S Basak et al [ILGTI] 1211.6277, 1312.3050 overlap on $n_f = 2+1+1$ HISQ
  Y Namekawa et al [PACS-CS] 1301.4743 $n_f = 2+1$ NP clover-Wilson
  R Horsley et al [QCDSF] 1311.1510 $n_f = 2+1$ SLiNC
  Z Brown et al 1409.0497 FNAL clover-Wilson on $n_f = 2$ domain wall

Lattice QCD



typical values:

$a^{-1} = 1.5$–4 GeV, $Na = 1.5$–6 fm

continuum limit: $a \to 0$, Na fixed

infinite volume: $Na \to \infty$

$$\langle O \rangle = \frac{1}{Z} \int [dU]\,[d\psi][d\bar{\psi}]\; O[U] e^{-S[U,\psi,\bar{\psi}]}$$

"Measurement": average over a representative ensemble of gluon configurations $\{U_i\}$ with probability $P(U_i) \propto \int [d\psi][d\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$

$$\langle O \rangle = \frac{1}{n} \sum_{i=1}^{n} O(U_i) + \Delta O \qquad \Delta O \propto \frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0$$

Input: $\mathcal{L}_{QCD} = -\frac{1}{16\pi\alpha_L} FF + \bar{q}_f(\not{D} + m_f)q_f$

$$\begin{aligned} m_N^{\text{latt}} &= m_N^{\text{phys}} &\longrightarrow&\quad a \\ m_\pi^{\text{latt}}/m_N^{\text{latt}} &= m_\pi^{\text{phys}}/m_N^{\text{phys}} &\longrightarrow&\quad m_u \approx m_d \\ &\ldots \end{aligned}$$

Output: hadron masses, matrix elements, decay constants, etc...

Extra- & interpolations:

1. $a \to 0$: $\mathcal{O}(a^2)$ or $\mathcal{O}(\alpha_s a)$, depending on the lattice action.
   charm quark mass: $a^{-1} \gg m_c$
   fine structure: $a^{-1} \gg m_c$ for $c\bar{\ell}, c\ell\ell, (cc\ell)$, $a^{-1} \gg m_c v$ for $c\bar{c}, cc\ell$
   level splittings: $a^{-1} \gg \bar{\Lambda}$ for $c\bar{\ell}, c\ell\ell, (cc\ell)$, $a^{-1} \gg m_c v^2$ for $c\bar{c}, cc\ell$
2. $L = Na \to \infty$: FSE suppressed with $\exp(-cLm_\pi)$: harmless but computationally expensive since $V \propto L^4$. High excitations?
3. $m_q^{\text{latt}} \to m_q^{\text{phys}}$: chiral perturbation theory (χPT) helps for $q = ud$ but m_{ud}^{latt} must be sufficiently small to start with.

Some computational challenges

Cost of simulation is proportional to

- number of points: $(L/a)^4$
- condition number of linear system: $1/m_\pi^2$
- $L^{1/2}/m_\pi$ in (Omelyan) time integration within Hybrid Monte Carlo
- $1/a^{\geq 2}$ critical slowing down (autocorrelations)

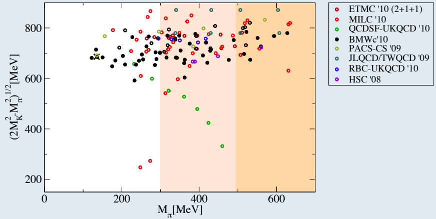
Adjusting $L \propto 1/m_\pi$ this means:

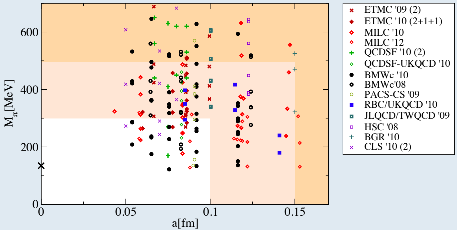
$$\mathrm{cost} \propto \frac{1}{a^{\geq 6}\, m_\pi^{7.5}}$$

In addition: for some observables signal/noise goes down as m_π is reduced.
State of the art: $64^3 \times 128$ sites, corresponding to $\approx (4 \times 10^9)^2$ (sparse) complex Dirac matrices. Inverse matrix: quark propagator.

Huge progress in Hybrid Monte Carlo, solver, source design, “all-to-all”.

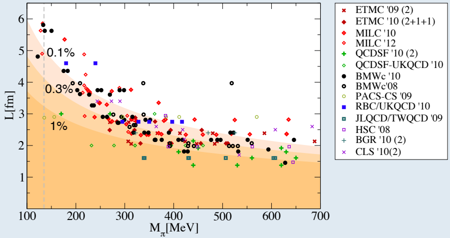
Landscape of recent lattice simulations



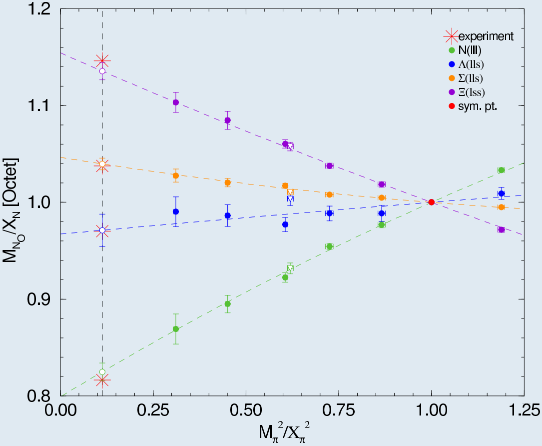


Figures taken from

C Hoelbling, arXiv:1410.3403



Strategy: $m_u + m_d + m_s \propto m_\pi^2 + 2m_K^2 \approx \text{const.}$



QCDSF + UKQCD: W Bietenholz et al. 10

$n_f = 2 + 1$ configurations

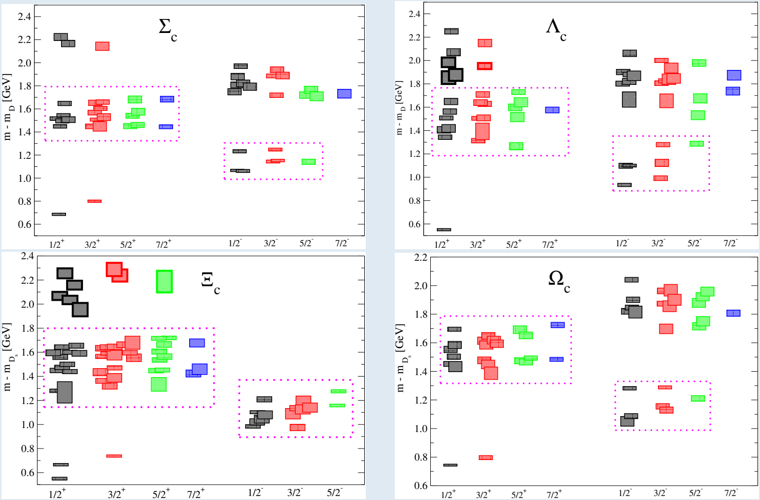
- **RQCD: SLiNC action**
  $m_u + m_s + m_d$ kept constant $(2m_\pi^2 + m_K^2 \approx \mathrm{const})$,
  so far only $a \approx 0.076\,\mathrm{fm} \approx (2.6\,\mathrm{GeV})^{-1}$.

$L/a \times T/a$	m_π (MeV)	Lm_π	n_{conf}
24×48	458.9	4.0	4676
32×64	458.9	5.3	1764
24×48	397.9	3.6	2126
24×48	354.3	3.2	2391
32×64	354.3	4.3	1880
32×64	254.6	3.1	2002

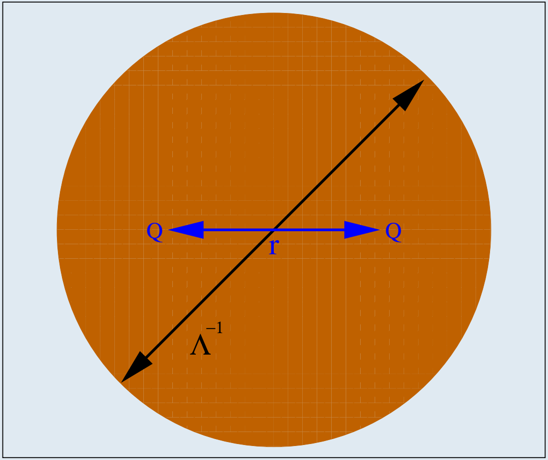
- **HSC (Padmanath et al): anisotropic clover-Wilson**
  $a_s \approx 0.12$ fm, $a_t \approx 0.035$ fm,

  $L/a \times T/a$: $16^3 \times 128$, $m_\pi \approx 390$ MeV.

Singly charmed baryons: Hadron Spectrum Collaboration

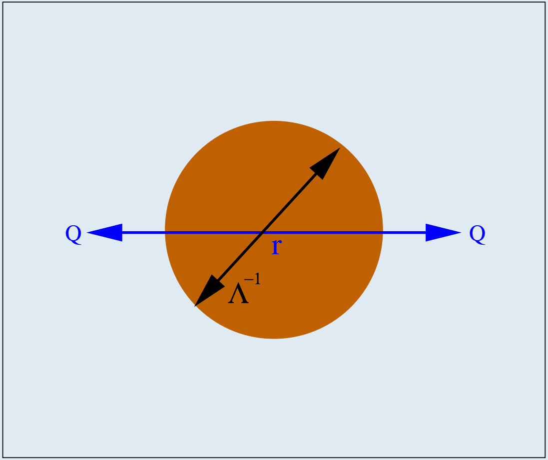


What is interesting about doubly charmed baryons?



HQET picture for $r \ll \overline{\Lambda}^{-1}$.

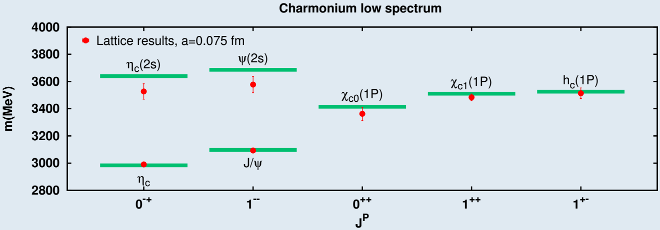
What is interesting about doubly charmed baryons?

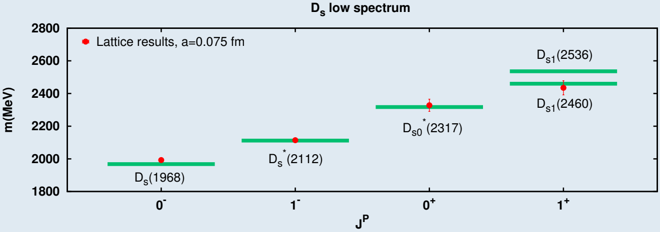


$r \gg \overline{\Lambda}$: NRQCD/Charmonium-alike?

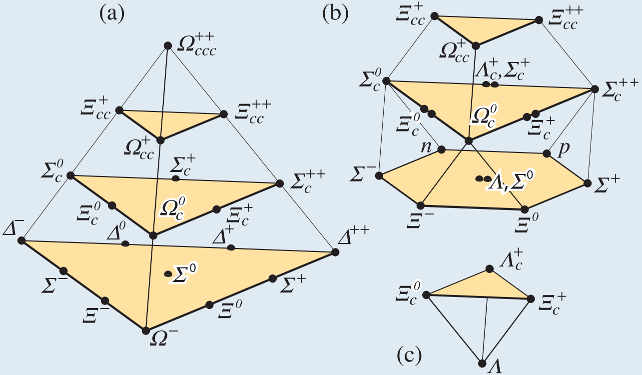
RQCD: Setting the charm quark mass

$$a^{-1} \approx 2.6 \text{ GeV from nucleon octet, } m_c \text{ from } m(\eta_c) + 3m(J/\psi)$$

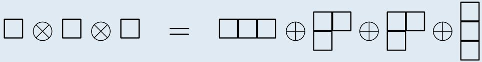




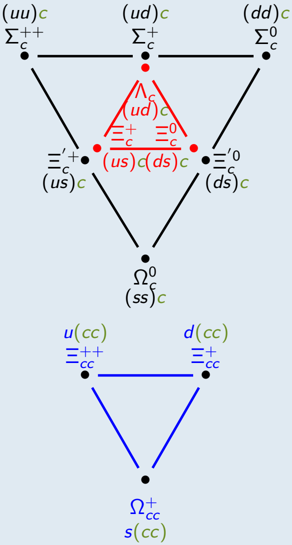
- SU(4) representations



- Flavour symmetry is not respected but
- simplest way to see which baryons should exist.
- SU(4): $\quad 4 \otimes 4 \otimes 4 \quad = \quad 20 \;\oplus\; 20 \oplus 20 \;\oplus\; \overline{4}$



Gell-Mann Okubo formulae



Sextet

$$m_{\Sigma_c^{(*)}} = m_0 - \frac{2}{3}A\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$
$$m_{\Xi_c'^{(*)}} = m_0 + \frac{1}{3}A\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$
$$m_{\Omega_c^{(*)}} = m_0 + \frac{4}{3}A\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

Anti-triplet

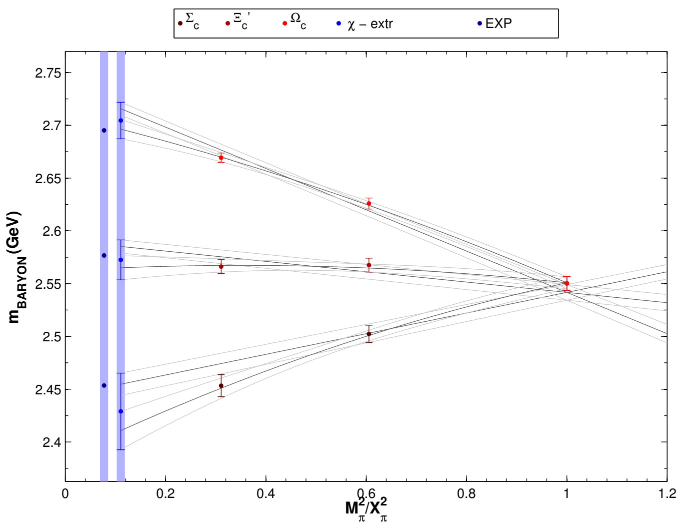
$$m_{\Lambda_c} = m_0 + \frac{2}{3}B\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$
$$m_{\Xi_c} = m_0 - \frac{1}{3}B\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

Triplet

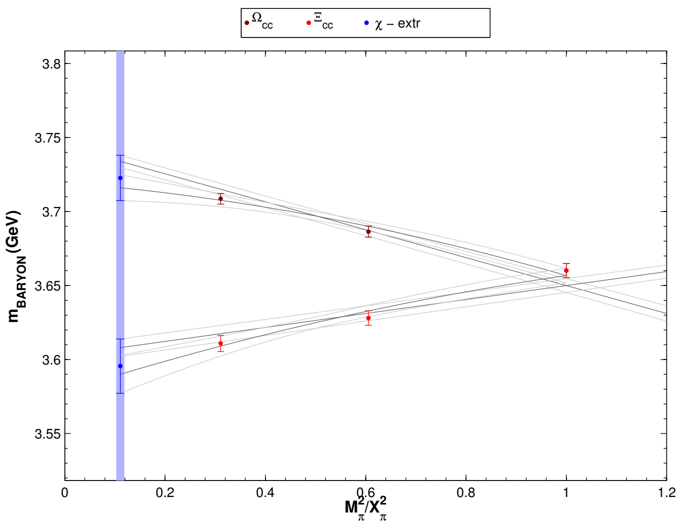
$$m_{\Xi_{cc}^{(*)}} = m_0 + \frac{1}{3}C\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$
$$m_{\Omega_{cc}^{(*)}} = m_0 - \frac{2}{3}C\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

$$\delta m_\ell = m_s - m_{ud}$$

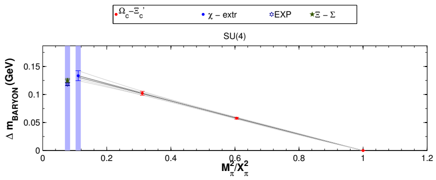
Spin 1/2 sextet baryons

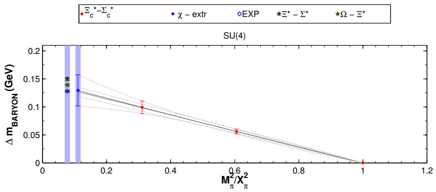


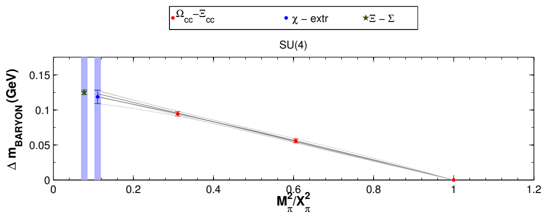
Spin 1/2 triplet baryons (doubly charmed)



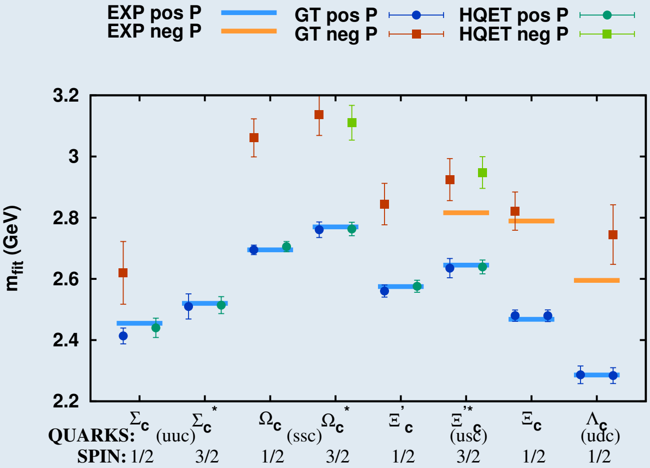
Mass splittings replacing $s \mapsto d$



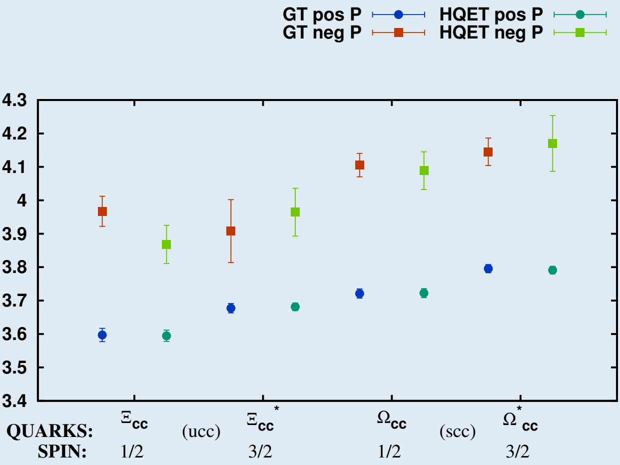




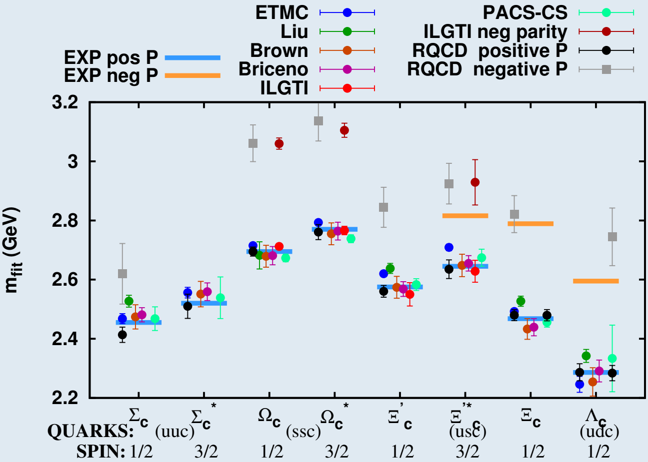
Singly charmed baryons



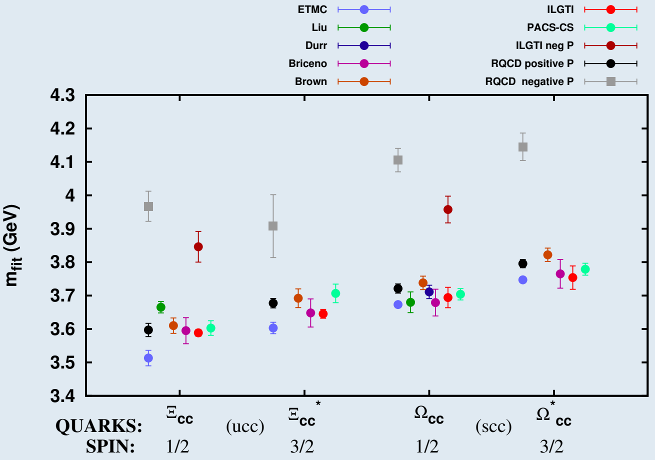
Doubly charmed baryons



Summary plot: singly charmed baryons



Summary plot: doubly charmed baryons



Summary and Outlook

- Hidden and open charm states are narrower and cleaner than many light quark resonances. Theoretically, the heavy quark limit provides guidance. This is a prime arena for addressing "exotic" spectroscopy.
- Unfortunately, the baryonic sisters of charmonium are still awaiting to be discovered (and won't be produced at FAIR).
  Do doubly charmed baryons resemble $(cc)u, (cc)s$ "D, D_s heavy mesons" — or $c(cu), c(cs)$ "charmonia"?
- LHC experiments found many beauty baryons. What about charm? Impressive results on $D_{(s)}$ mesons, $X(3872)$ and $Z_c^+(4430)$!
- Singly charmed baryons could also be studied by PANDA.
- Methodology developed. Now the spectrum of charmed baryons, open charm mesons and charmonia will be computed, down to nearly physical light quark mass, taking the continuum limit.

The future: simultaneous SU(2) χPT and SU(3) GMO extrapolation