Determination of the Symmetry Energy

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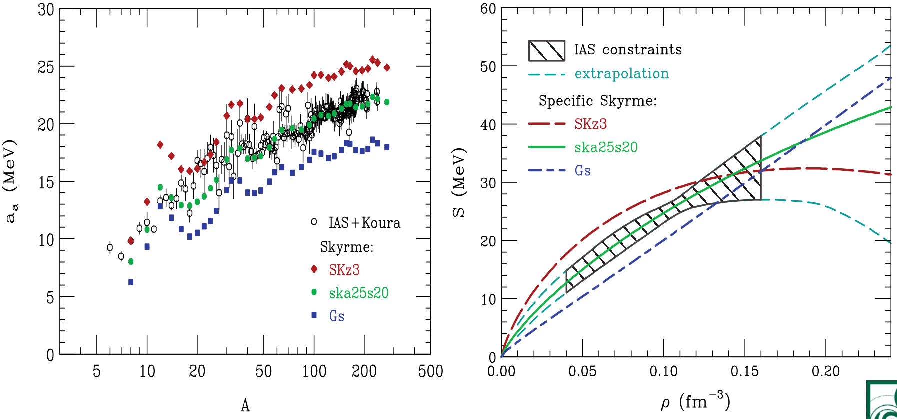
Dense Baryonic Matter
in the Cosmos and the Laboratory

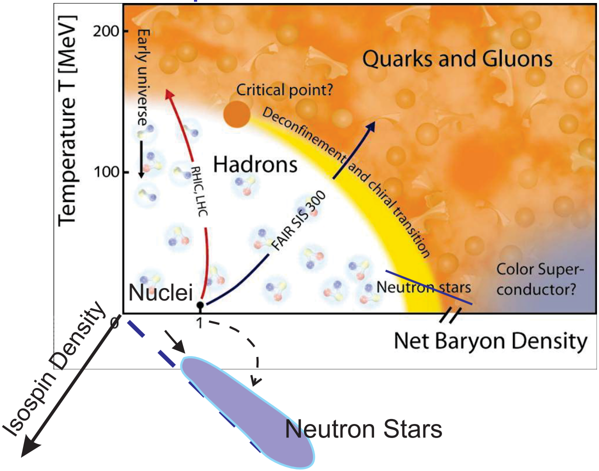
Tübingen, October 11-12, 2012

1-Slide Summary

Symmetry Energy for Finite Nuclei & for Uniform Matter



Equation of State



Reactions - coarse, supranormal densities

Structure - detailed, but subnormal densities, competition of macroscopic & microscopic effects,

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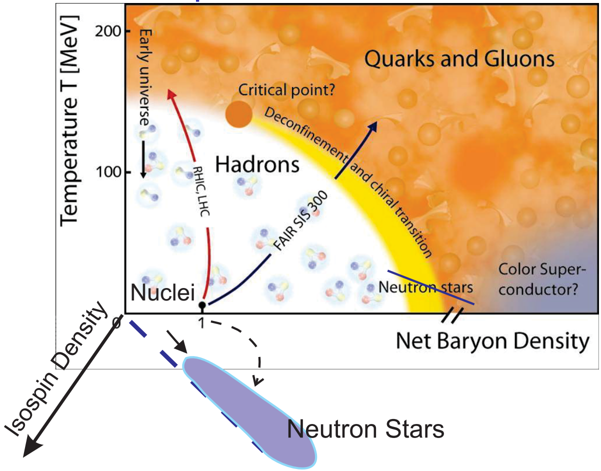

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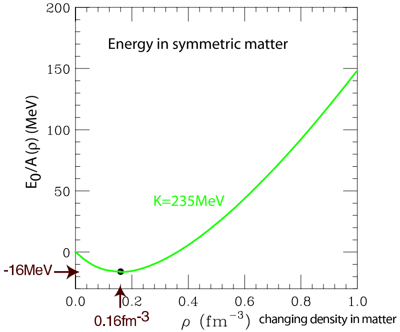
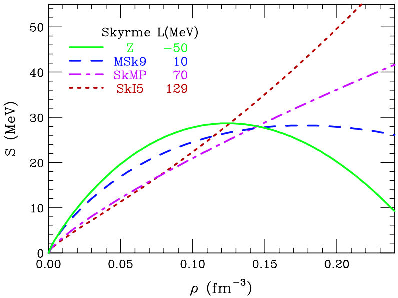
Energy in Uniform Matter

$$\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + S(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)^2 + \mathcal{O}(\ldots^4)$$

symmetric matter (a)symmetry energy $\rho = \rho_n + \rho_p$





$$\frac{E_0}{A}(\rho) = -a_V + \frac{K}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \ldots \qquad S(\rho) = -a_a^V + \frac{L}{3}\frac{\rho - \rho_0}{\rho_0} + \ldots$$

Known: $a_a \approx 16$ MeV $K \sim 235$ MeV Unknown: a_a^V ? L ?

Nuclear Masses

Mass formula

$$E = E_{\mathrm{nucl}} + E_{\mathrm{Coul}} = E_0 + E_1 + E_{\mathrm{mic}} + E_{\mathrm{Coul}}$$

Bulk contribution to the energy of a symmetric nucleus:

$$E_0(A) = -a_V\, A + a_S\, A^{2/3} + \ldots$$

Symmetry energy:

$$E_1(N,Z) = a_a(A)\, \frac{(N-Z)^2}{A} = 4a_a(A)\, \frac{T_z^2}{A}$$

Isospin invariance (charge invariance):

$$E_1 = 4a_a\, \frac{T_z^2}{A} \quad \longrightarrow \quad 4a_a \frac{\boldsymbol{T}^2}{A} = 4a_a\, \frac{\boldsymbol{T}_\perp^2 + T_z^2}{A} = 4a_a\, \frac{T(T+1)}{A}$$

e.g. Jänecke *et al.*, NPA728(03)23

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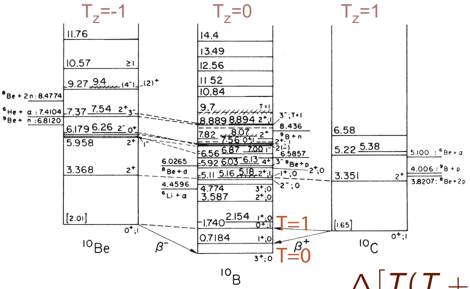
Symmetry Coefficient Nucleus-by-Nucleus

Mass formula generalized to the lowest state of a given T:

$$E(A, T, T_z) = E_0(A) + 4a_a(A)\,\frac{T(T+1)}{A} + E_{\mathrm{mic}} + E_{\mathrm{Coul}}$$

In the ground state T takes on the lowest possible value $T = |T_z| = |N - Z|/2$. Through '+1' most of the Wigner term absorbed.

?Lowest state of a given T: isobaric analogue state (IAS) of some neighboring nucleus ground-state.



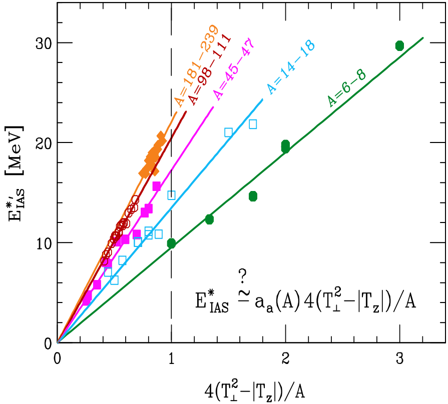
Study of changes in the symmetry term possible nucleus by nucleus

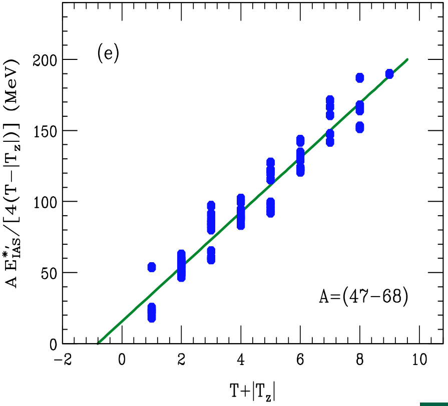
$$E^*_{\mathrm{IAS}} = \Delta E = a_a\,\frac{\Delta\left[T(T+1)\right]}{A} + \Delta E_{\mathrm{mic}}$$

Peek into IAS Analysis

IAS data: Antony *et al.* ADNDT66(97)1

Shell corrections: Koura *et al.* ProTheoPhys113(05)305





Excitation energies to IAS, E^*_{IAS}, for different A, lumped together in narrow regions of A. Is $E_1 \propto T(T+1)$? YES

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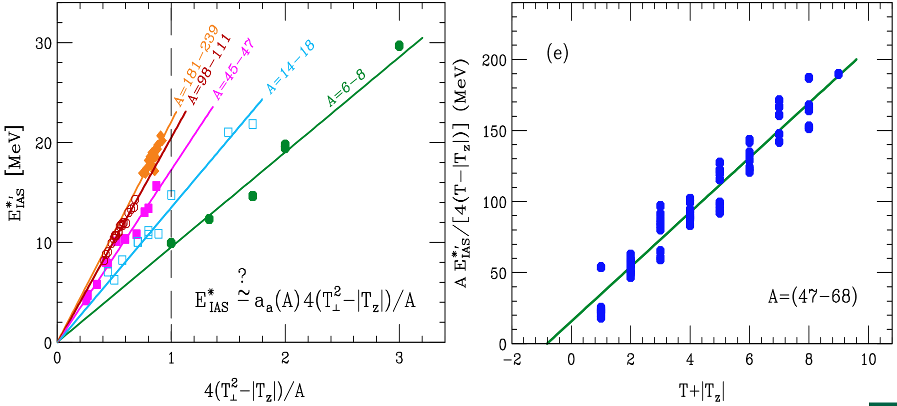

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$a_a(A)$ without Shell Corrections

$$a_a(A) = \frac{A}{4}\,\frac{E^*_{IAS}}{\Delta T^2}$$

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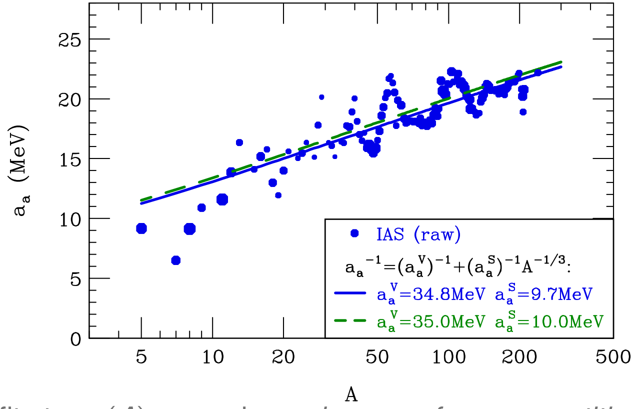
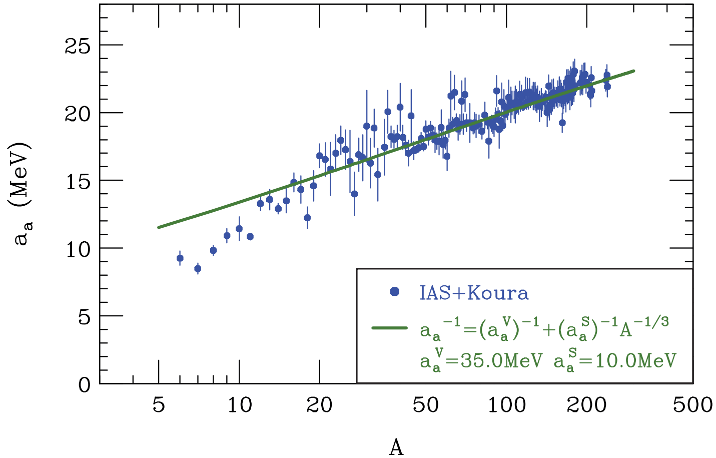


Lines: fits to $a_a(A)$ assuming *volume-surface competition* analogous to that for E_1. ??Fundamental knowledge??

$a_a(A)$ with Shell Corrections

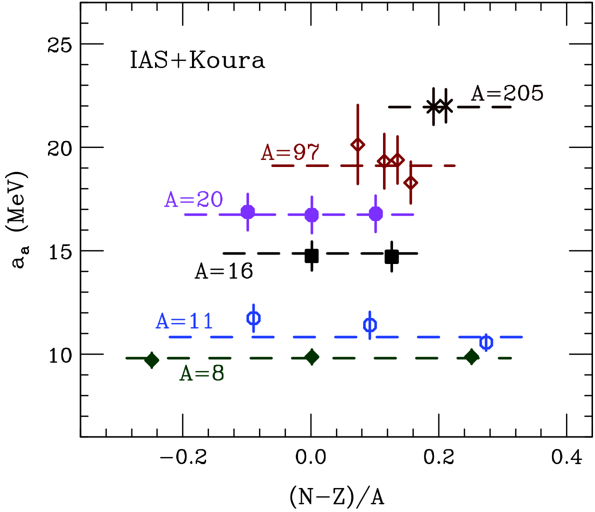
$$a_a(A) = \frac{A}{4} \frac{E^*_{IAS} - \Delta E_{mic}}{\Delta T^2}$$

E_{mic} from: Koura *et al.* ProTheoPhys113(05)305



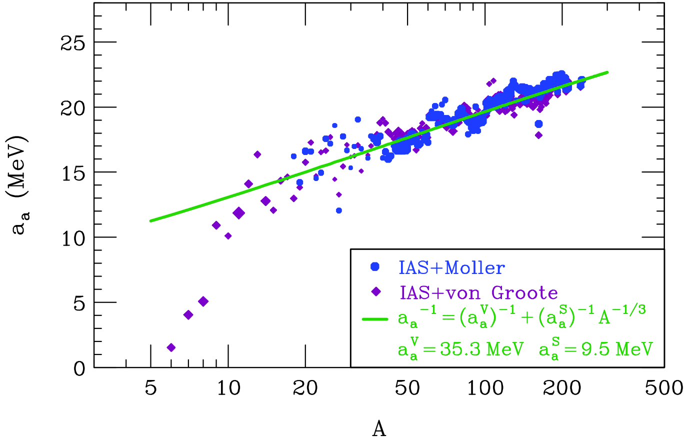
Heavy nuclei $a_a \sim 22$ MeV, light $a_a \sim 10$ MeV

Z-Dependence of Symmetry Coefficients?



Symmetry coefficients on a nucleus-by-nucleus basis

Sensitivity to Shell Corrections

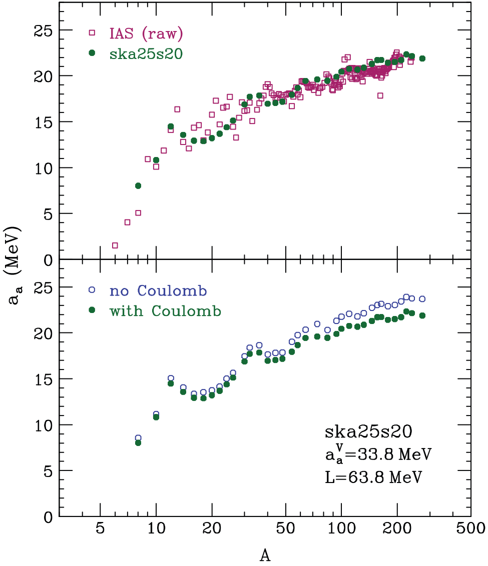


Fit to raw data ($A > 30$) in the middle, but:

Moller *et al.* fit: $a_a^V = 39.73\,\text{MeV}$, $a_a^S = 8.48\,\text{MeV}$

von Groote *et al.*: $a_a^V = 31.74\,\text{MeV}$, $a_a^S = 11.27\,\text{MeV}$

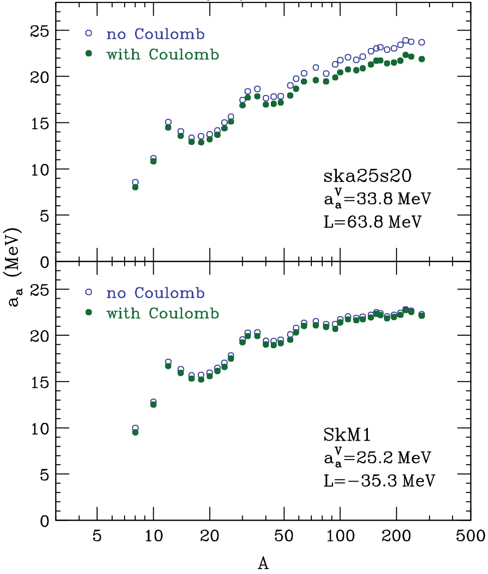
$a_a(A)$ from Skyrme-Hatree-Fock Calculations



We employ codes by P.-G. Reinhard, assuming spherical symmetry

Similar behavior with A as for IAS

$a_a(A)$ from Different Skyrmes



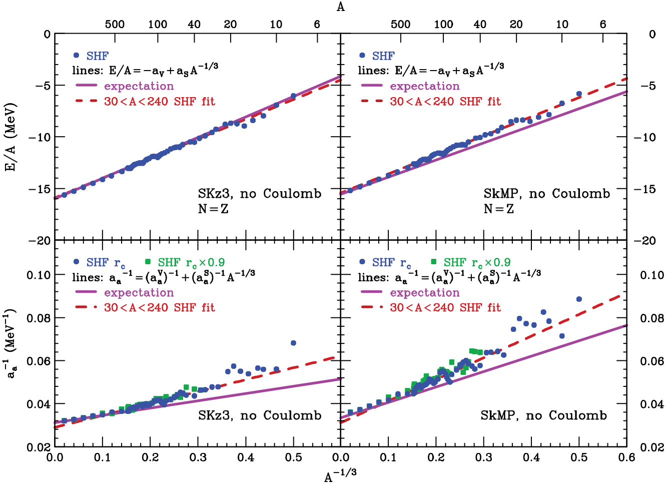
Less impact of the slope L at ρ_0 than expected

??Difficulty for L determination??

Test of Large-A Expansion

Symbols: results of spherical no-Coulomb SHF calcs

⇒ Lines: volume-surface decomposition - expectation vs fit



→Symmetric matter energy f/sample Skyrmes

∼ Works

→Symmetry coefficient

∼ Not. . .

Expectations from half-∞ matter.

Can $S(\rho)$ Be Constrained??!

Pearson correlation coefficient

$$r_{XY} = \frac{\langle (X - \langle X \rangle)(Y - \langle Y \rangle) \rangle}{\sqrt{\langle (X - \langle X \rangle)^2 \rangle \langle (Y - \langle Y \rangle)^2 \rangle}}$$

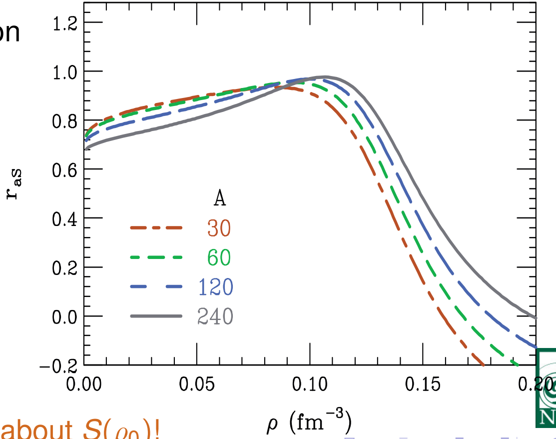
$|r| \sim 1$ - strong correlation

$r \sim 0$ - no correlation

$X \equiv a_a(A)$

$Y \equiv S(\rho)$

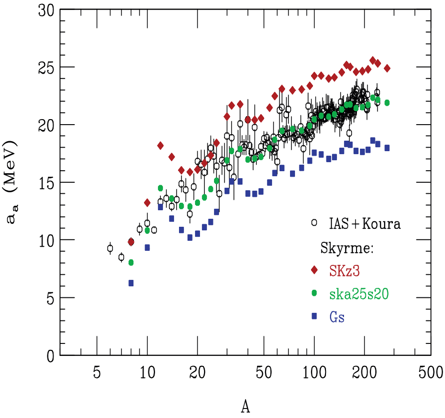
Ensemble of Skyrmes

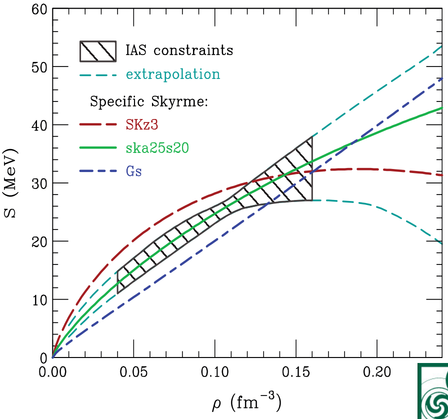


Nearly no information about $S(\rho_0)$!

Constraints on Symmetry Energy $S(\rho)$

Demand that Skyrme approximates IAS results at $A > 30$ produces a constraint area for $S(\rho)$:





Conclusions

- Symmetry-energy term weakens as nuclear mass number decreases: from $a_a \sim 23\,$Mev to $a_a \sim 9\,$MeV for $A \lesssim 8$.
- For $A \gtrsim 25$, $a_a(A)$ may be fitted with $a_a^{-1} = (a_a^V)^{-1} + (a_a^S)^{-1}\, A^{-1/3}$, where $a_a^V \approx 35\,$MeV and $a_a^S \approx 10\,$MeV.
- Weakening of the symmetry term can be tied to the weakening of $S(\rho)$ in uniform matter, with the fall of ρ.
- In spite of difficulties, significant, $\pm$(1-2) MeV, constraints on $S(\rho)$ at densities $\rho =$(0.05-0.13) fm^{-3}.
- Forthcoming: charge radii, skins, $S(\rho \sim \rho_0)$ ☺.

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NSCL

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Symmetry Energy in the Binding Formula

Bethe-Weizsäcker formula:

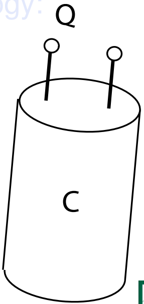
$$E = -a_V\,A + a_S\,A^{2/3} + a_C\,\frac{Z^2}{A^{1/3}} + a_a(A)\,\frac{(N-Z)^2}{A} + E_{\text{mic}}$$

In the standard formula $a_a(A) \equiv a_a^V \simeq 21$ MeV, the symmetry term has purely volume character.

A-dependent symmetry coefficient?? Capacitor analogy:

Nuclear: $$E = -a_V\,A + a_s\,A^{2/3} + \frac{a_a}{A}\,(N-Z)^2$$
$$= E_0(A) + \frac{a_a}{A}\,(N-Z)^2$$

Electrostatic: $$E = E_0 + \frac{Q^2}{2C} \Rightarrow \begin{cases} Q \equiv N - Z \\ C \equiv \dfrac{A}{2a_a} \end{cases}$$



Note: For coupled capacitors, capacitances add up. Contributions to C with different A-dependence??

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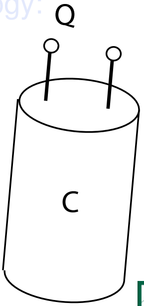

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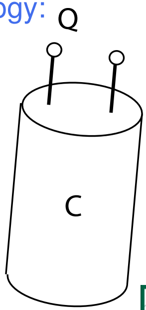

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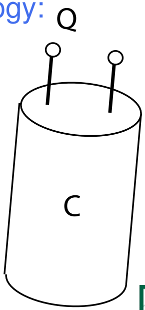

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A-Dependence of Symmetry Coefficient

E.g. volume-surface breakdown of energy & asymmetry:

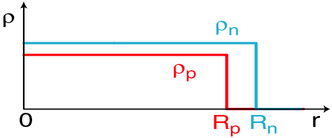
$$E = E_S + E_V \qquad N - Z = (N_S - Z_S) + (N_V - Z_V)$$

$$E_V = a_V\, A + a_a^V\, \frac{(N_V - Z_V)^2}{A} \qquad E_S = a_S\, A^{2/3} + a_a^S\, \frac{(N_S - Z_S)^2}{A^{2/3}}$$

under charge symmetry, i.e. $N \leftrightarrow Z$ invariance.

Minimization of the energy E with respect to $(N - Z)$ partition between volume and surface yields:

$$E = E_0 + E_a = E_0 + \frac{(N-Z)^2}{\frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S}}$$



Capacitance for asymmetry:

$$2C \equiv \frac{A}{a_a(A)} = \frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S}$$

volume capacitance   surface

E.g. droplet model
different radii for *n&p*
Myers&Swiatecki
AnnPhys55(69)395

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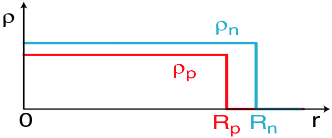

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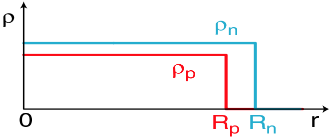

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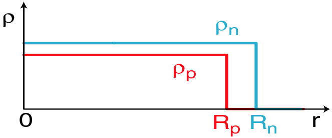
Minimization of the energy E with respect to $(N - Z)$ partition between volume and surface yields:

$$E = E_0 + E_a = E_0 + \frac{(N-Z)^2}{\frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S}}$$



Capacitance for asymmetry:

$$2C \equiv \frac{A}{a_a(A)} = \frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S}$$

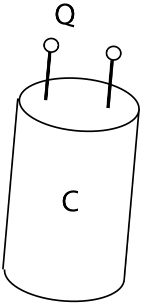
volume capacitance surface

E.g. droplet model different radii for n&p Myers&Swiatecki AnnPhys55(69)395

More on the Analogy

Asymmetry chemical potential
($\propto$ difference of *n* & *p* separation energies)

$$\mu_a = \frac{\partial E}{\partial(N-Z)} = \frac{1}{2}(\mu_n - \mu_p)$$
$$= \frac{2a_a(A)}{A}(N-Z)$$



Analogy: Voltage

$$V = \frac{\partial E}{\partial Q} = \frac{1}{C}Q \quad \Rightarrow \quad C \leftrightarrow \frac{A}{2a_a}$$

Connected capacitors end up at the same voltage;
charge distributes itself in proportion to capacitance.

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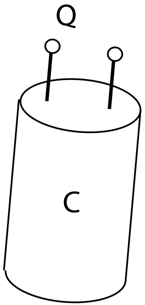

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Invariant Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N - Z)$ for given A. [Coulomb suppressed...]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A\,\rho_{np}(r)/(N - Z)$ isoscalar!
$A/(N - Z)$ normalizing factor global... Similar local normalizing factor, in terms of intense quantities, $2a_a^V/\mu_a$, where $a_a^V \equiv S(\rho_0)$

Asymmetric density (formfactor for isovector density) defined:

$$\rho_a(r) = \frac{2a_a^V}{\mu_a}\,[\rho_n(r) - \rho_p(r)]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on η!

In any nucleus:

$$\rho_{n,p}(r) = \frac{1}{2}\left[\rho(r) \pm \frac{\mu_a}{2a_a^V}\rho_a(r)\right]$$

where $\rho(r)$ & $\rho_a(r)$ have universal features! (subject to shell effects)

No shell-effects, ρ's as dynamic vbles: Hohenberg-Kohn functional

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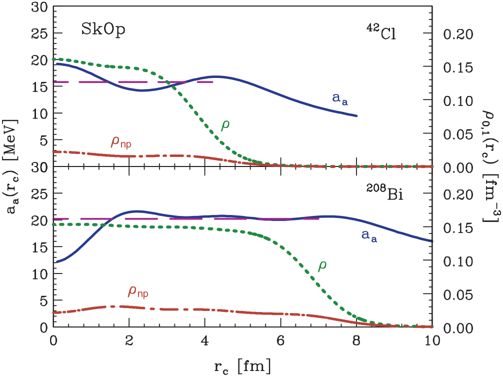
Comparisons to SHF

Issues in data-theory comparisons (codes by P.-G. Reinhard):

1. No isospin invariance in SHF - impossible to follow the procedure for data
2. Shell corrections not feasible at such scrutiny as for data
3. Coulomb effects.



Solution: Procedure that yields the same results as the energy, in the bulk limit, but is weakly affected by shell effects:

$$\frac{(N-Z)_{r<r_c}}{N-Z} = \frac{C_{r<r_c}}{C} = \frac{a_a}{A\,a_a^V}\int_{r<r_c}\frac{\rho}{S(\rho)}$$