TECHNISCHE UNIVERSITÄT DARMSTADT

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Relativistic laser beam propagation and critical density increase in a plasma

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Preface

“Open Sesame”,

“Ali baba and the forty thieves”

One Thousand and One Nights

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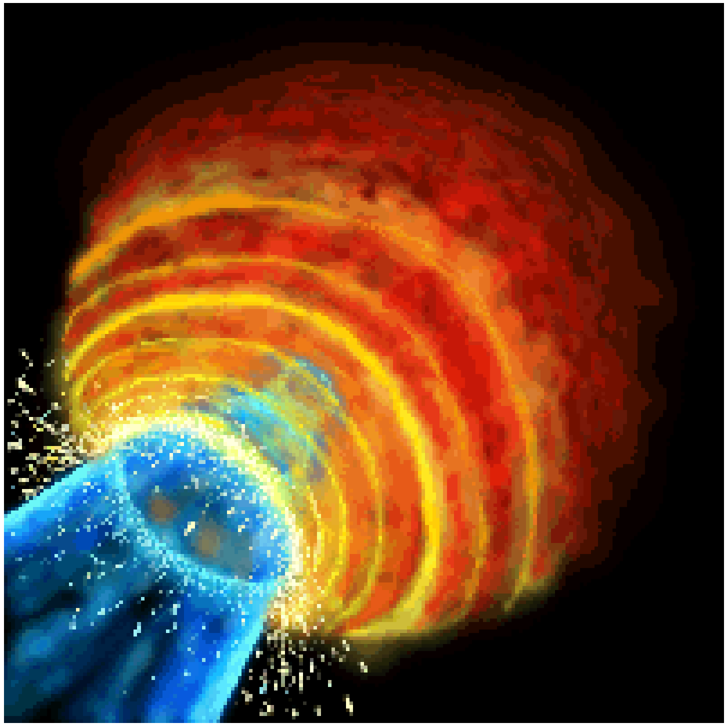
“Open Sesame”,

“Ali baba and the forty thieves”

[One Thousand and One Nights]

Who open the door for ultrahigh intense laser into an overdense plasma?



Outline

- **Theoretical background**
  - Classical eletromagnetic (EM) wave propagation
  - Relativistic induced transparency
- **Numerical simulations**
  - Relativistic critical density increase
  - Relativistic laser beam propagation
- **Potential applications**
  - Fast ignition
  - Relativistic plasma shutter
  - Shortening of laser pulses
- **Conclusion**

Classical EM wave propagation

- **Maxwell's Equations**

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J}$$

current density

- **For a high-frequency monochromatic laser beam in a plasma**

$$\frac{\partial \mathbf{B}}{\partial t} = -i\omega\mathbf{B}, \quad \frac{\partial \mathbf{E}}{\partial t} = -i\omega\mathbf{E}, \quad \mathbf{J} = \frac{i\omega_p^2}{4\pi\omega}\mathbf{E}, \text{ with } \omega_p^2 = 4\pi e^2 n / m_e,$$

plasma frequency

Maxwell's Equations $\Rightarrow$

$$\nabla \times \mathbf{E} = \frac{i\omega}{c}\mathbf{B} \quad (1)$$

$$\nabla \times \mathbf{B} = -\frac{i\omega}{c}\varepsilon\mathbf{E} \quad (2)$$

where ε=1-ω_{pe}^2 / ω^2 is the dielectric function of the plasma.

Eq. (1)-(2) $\Rightarrow$ $\nabla^2\mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) + \frac{\omega^2}{c^2}\varepsilon\mathbf{E} = 0$ (wave equation)

Classical EM wave propagation

- **Dispersion relation**

$\nabla \cdot \mathbf{E} = 0$ holds for a uniform plasma in wave equation $\nabla^2 \mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) + \frac{\omega^2}{c^2} \varepsilon \mathbf{E} = 0$,

and assume that $\mathbf{E} \sim \exp(i\mathbf{k} \cdot \mathbf{x} - i\omega t)$ is a plane wave

$$\frac{\omega^2}{c^2} \varepsilon = k^2 \Leftrightarrow \omega^2 = \omega_p^2 + c^2 k^2, \quad \text{(dispersion relation)}$$

plasma frequency ω_p is the minimum frequency for EM wave propagation in a plasma.

the electrons will shield out the EM field when $\omega < \omega_p$

- **Critical density**

the condition $\omega_p = \omega$ defines the so-called critical density n_c,

$$n_c = m_e \omega^2 / 4\pi e^2 = 1.1 \times 10^{21} / \lambda^2 \ \text{cm}^{-3}$$

- **Group velocity (propagation velocity)**

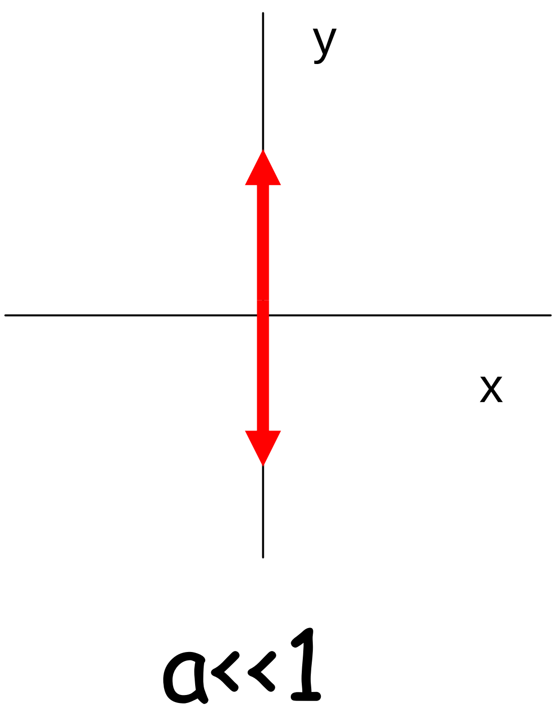
$$\frac{v_g}{c} \equiv \frac{1}{c} \frac{\partial \omega}{\partial k} = \sqrt{\varepsilon} = \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} = \left(1 - \frac{n}{n_c}\right)^{1/2}$$

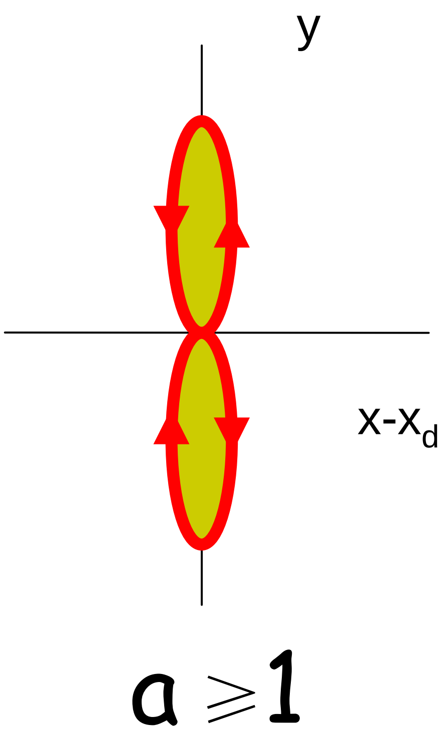
Relativistic induced transparency

- **Dimensionless laser amplitude a:**

$$I\lambda^2 = \frac{\pi}{2} c A^2 = \left[1.37 \times 10^{18} \frac{\mathrm{W}}{\mathrm{cm}^2} \mu\mathrm{m}^2 \right] a^2$$

- **Single particle's 8-like motion for a ⩾ 1**





T. C. Pesch and H. –J. Kull, Phys. Plasmas 14, 083103 (2007).

Relativistic induced transparency

- If |v| ~ c,

$$m_e = (1 - v^2 / c^2)^{1/2} m_{e0} = \gamma m_{e0}$$

- **Relativistic critical density**

$$n_{cr} = m_e \omega^2 / 4\pi e^2 = \langle \gamma \rangle n_c$$

the Lorentz factor averaged from the single particle's 8-like motion

$$\langle \gamma \rangle \approx [1 + a^2 / 2]^{1/2} \quad \text{(Linear Polarization),}$$

$$\langle \gamma \rangle \approx [1 + a^2]^{1/2} \quad \text{(Circular Polarization).}$$

- **Group velocity (relativistic)**

$$\frac{v_g}{c} = \left(1 - \frac{n}{n_{cr}} \right)^{1/2} = \left(1 - \frac{n}{\langle \gamma \rangle n_c} \right)^{1/2}$$

P. Mulser and D. Bauer, *"HighPowerLaser-Matter Interaction"*, (to be published by Springer).

Particle-in-Cell (PIC) simulation

- Motion equations (particles)

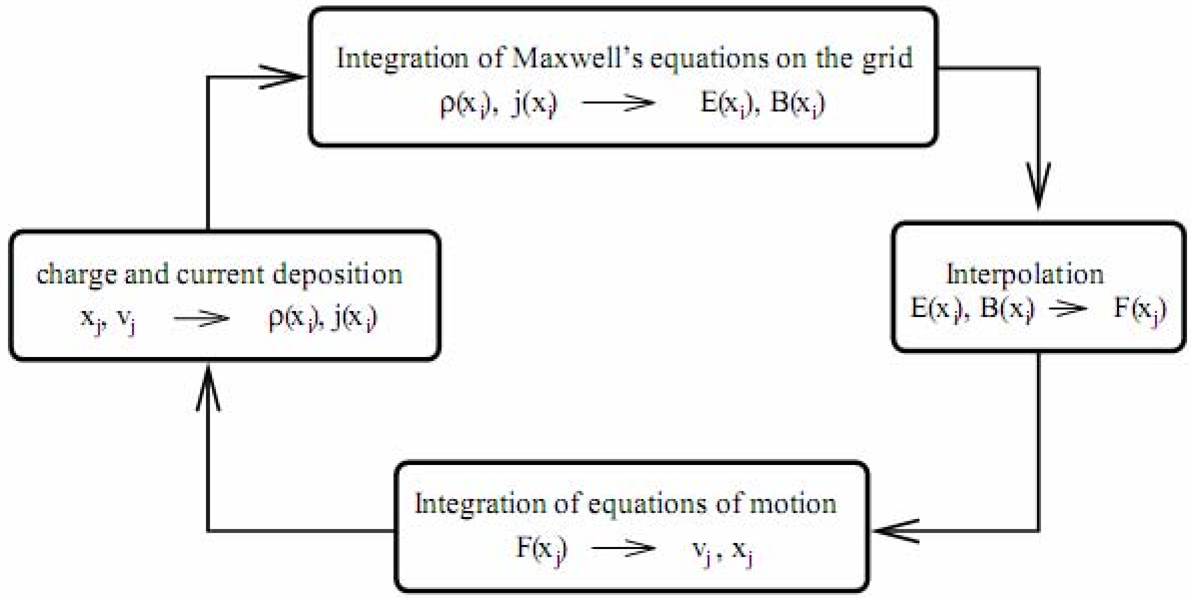
$$\begin{cases} \dfrac{d\mathbf{r}}{dt} = \dfrac{\mathbf{u}}{\gamma}, & \gamma = \sqrt{1+u^2/c^2} \\ m_s \dfrac{d\mathbf{u}}{dt} = q_s(\mathbf{E} + \dfrac{\mathbf{u}}{\gamma}\times\mathbf{B}), & s = e, i \end{cases}$$

+

- Maxwell's equations (fields)

$$\begin{cases} \nabla\times\mathbf{E} = -\partial_t\mathbf{B}, & \nabla\cdot\mathbf{E} = \dfrac{1}{\varepsilon_0}\rho \\ \nabla\times\mathbf{B} = \dfrac{1}{c^2}\partial_t\mathbf{E} + \dfrac{1}{\varepsilon_0 c^2}\mathbf{j}, & \nabla\cdot\mathbf{B} = 0 \end{cases}$$

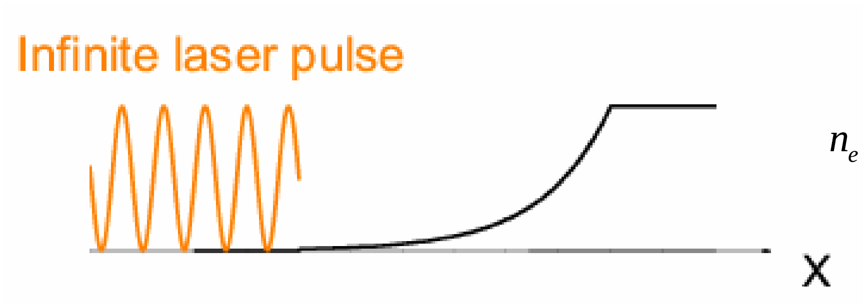
- **PIC simulation cycle**



R. Lichters, *et al.*, LPIC++, (1997).

Determination of critical density

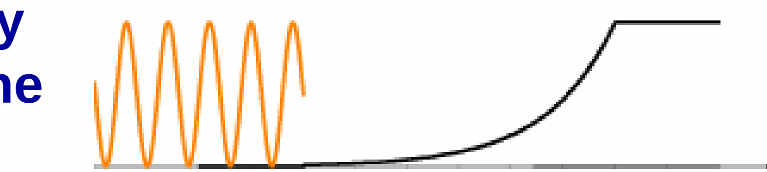
- **Laser and plasma parameters**



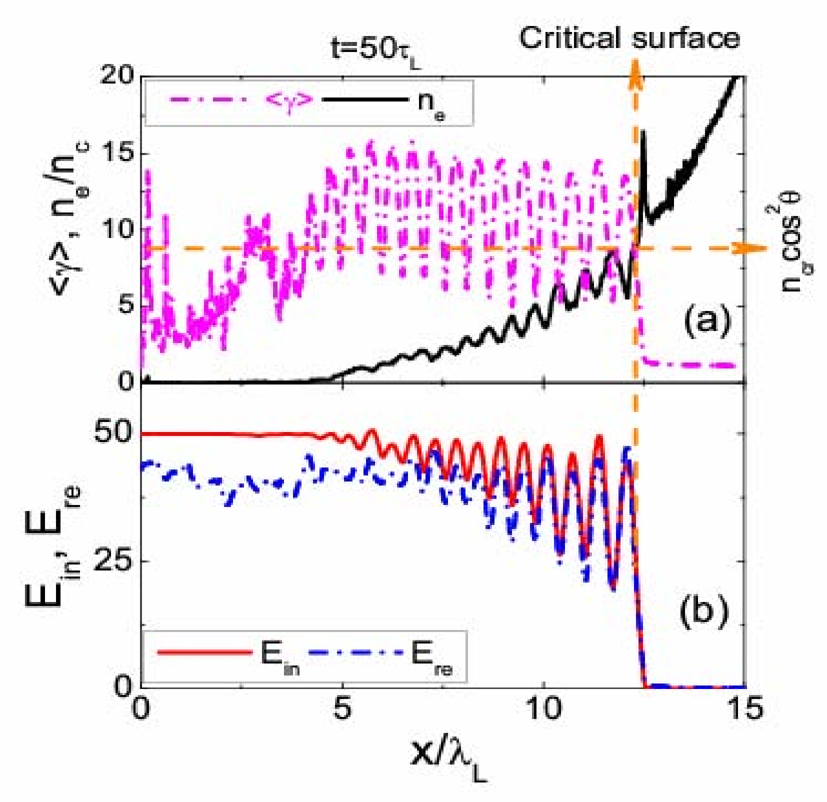
$$n_e = \begin{cases} 0, & x < 5\lambda_L \\ n_{\max}, & x > 20\lambda_L \\ n_{\max}\exp\left[(x-20)/L\right], & \text{otherwise} \end{cases}$$

Determination of critical density

- **Laser and plasma parameters**
- **Cycle-averaged propagation appears very regular, and laser is mainly reflected at the critical surface as that in nonrelativistic regime**



$$n_e = \cos^2\theta n_{cr} = \gamma_e \cos^2\theta n_c,$$



for $a = 10$, incident angle $\theta = 0$

Incident wave energy density

$$E_{in} = (F^+)^2 + (G^-)^2$$

Reflected wave energy density

$$E_{re} = (F^-)^2 + (G^+)^2$$

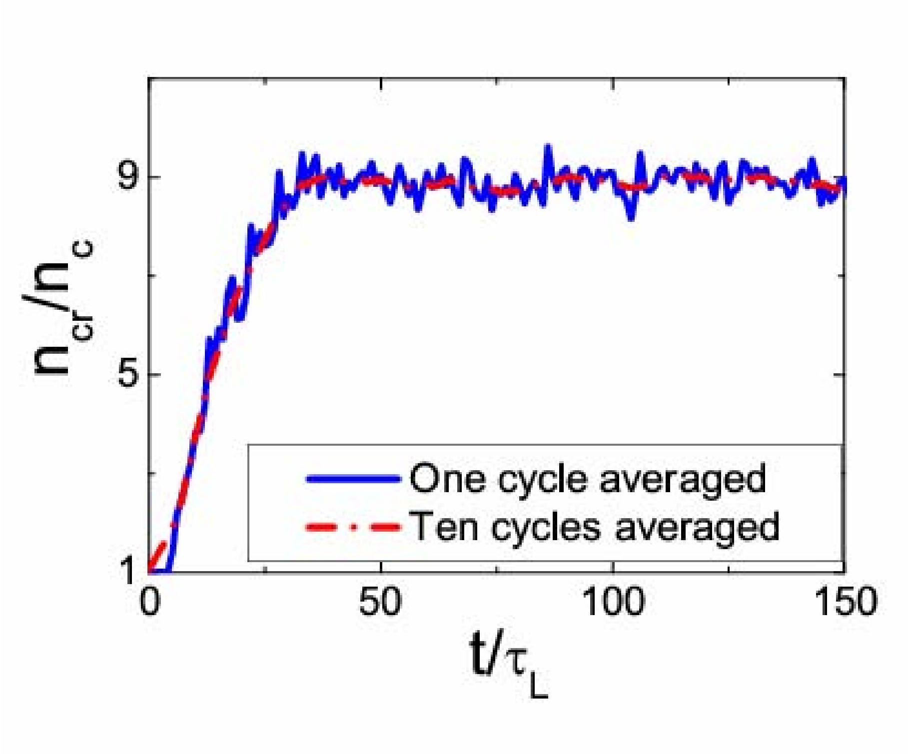
with

$$F^\pm = (E_y \pm B_z)/2$$

$$G^\pm = (E_z \mp B_y)/2$$

Critical density VS laser intensity

- **A very smoothed critical density can be achieved after being averaged over 10 cycles.**



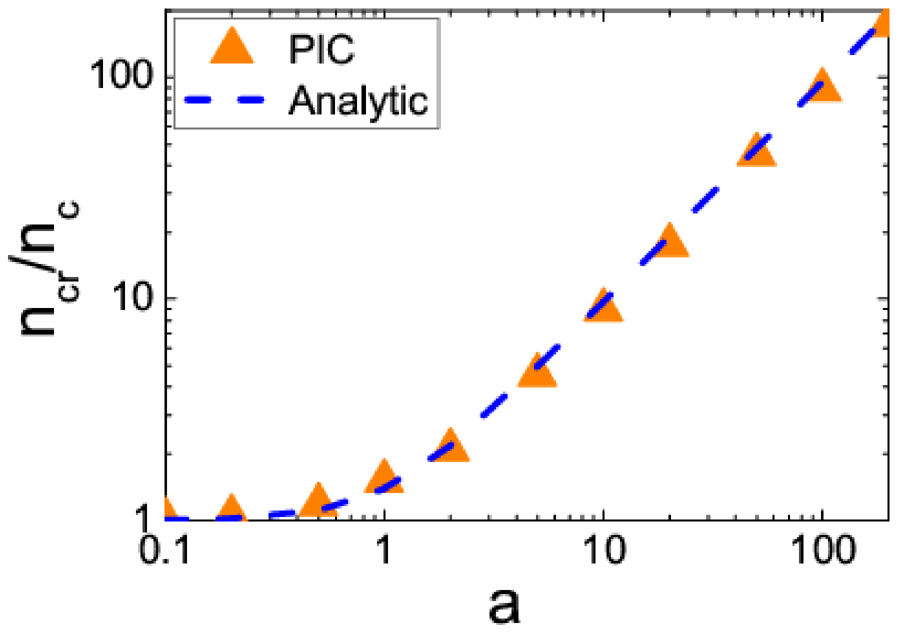
for $a = 10$,

incident angle $\theta = 0$

Critical density VS laser intensity

- **A very smoothed critical density can be achieved after being averaged over 10 cycles.**
- **In a normally incident and linearly polarized laser pulse, relativistic critical density increase is in a good agreement with the averaged value from the single particle's 8-like motion**

$$n_{cr}/n_c = \langle\gamma\rangle = [1+(1+R)a^2/2]^{1/2}, \quad (R \text{ reflectivity}).$$



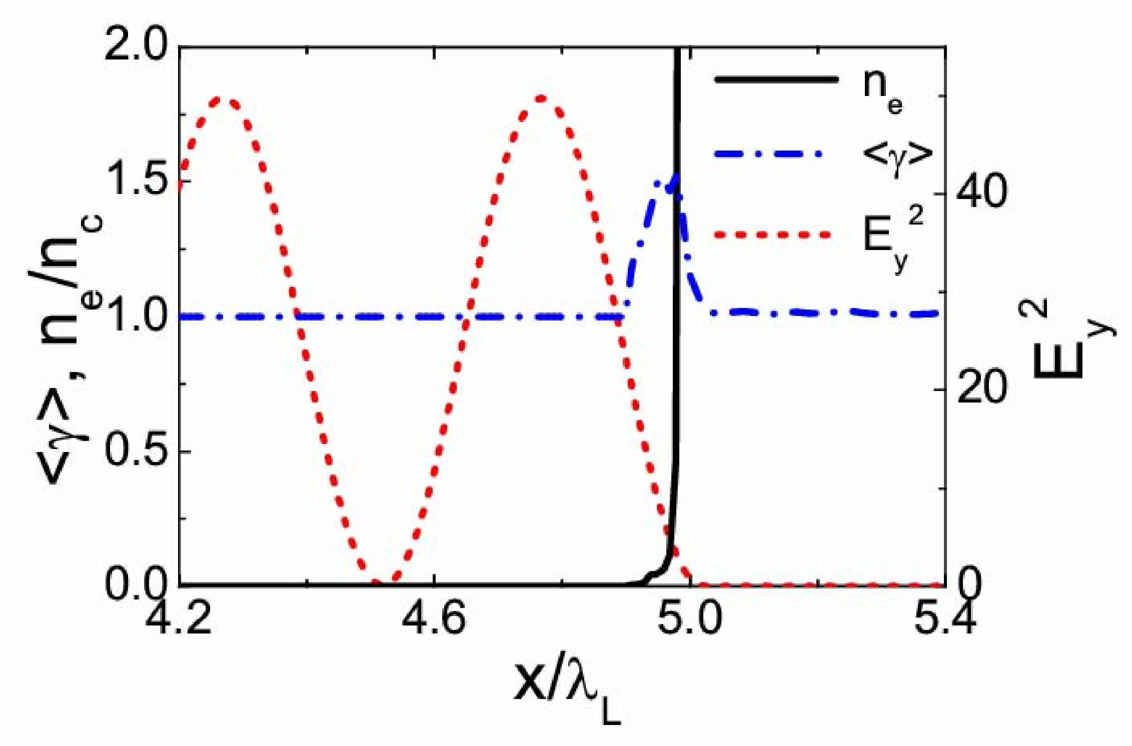
Effect of plasma density profile

- **For normal incident, if density scale length *L*> λ_L,**

  n_{cr} is almost independent of density profile

- **For a very steep and highly overdense plasma.**

  n_{cr} is strongly suppressed



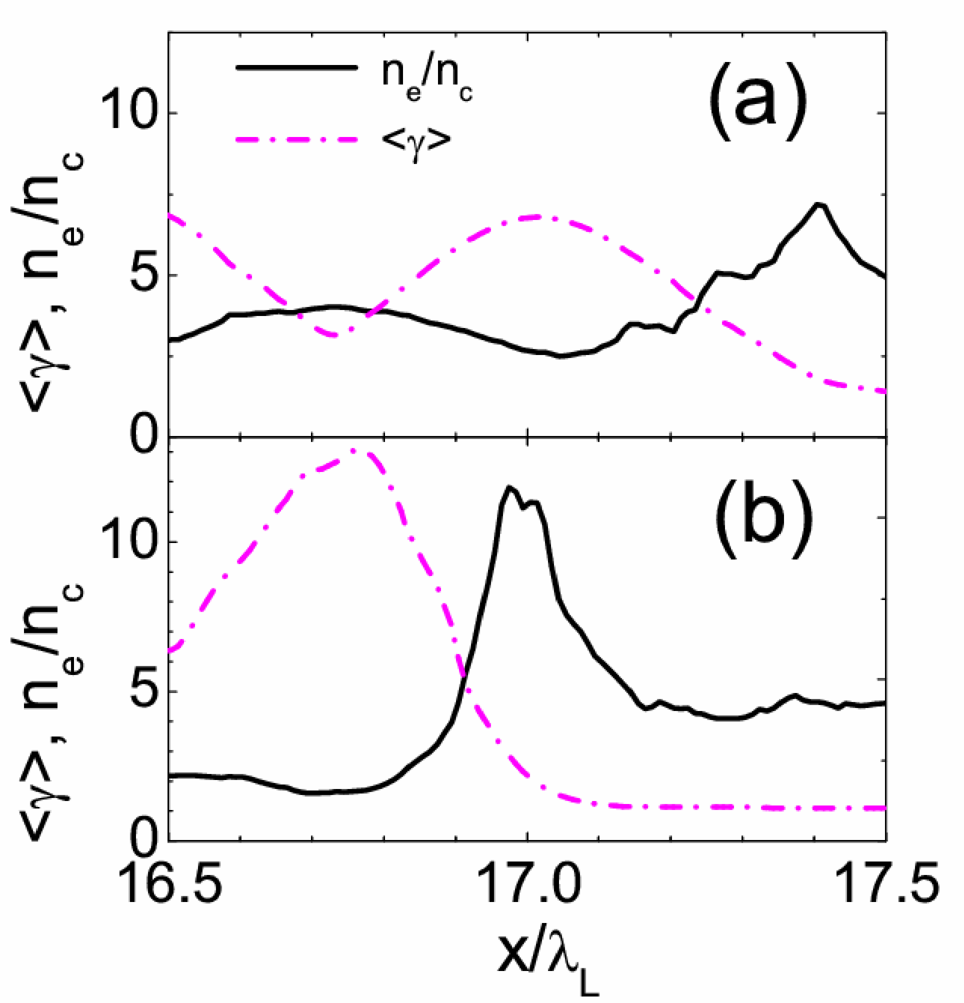
n_{cr} is only about 1.5 for $a = 5$, and

$$n_e = \begin{cases} 0, & x < 5\lambda_L \\ 100 n_c, & x > 5\lambda_L \end{cases}$$

electric field at the surface and skin depth $\propto 1/n_e^{1/2}$

Effect of laser polarization

- For circular polarization, a density ridge prevents the laser from further propagation and restricts the critical density increase



for $a = 5,\ \theta = 0$

(a) Linear polarization

Theoretically

$\langle\gamma\rangle=[1+(1+R)a^2/2]^{1/2}=4.98$,

And from PIC, $\langle\gamma\rangle$ is about 4.55.

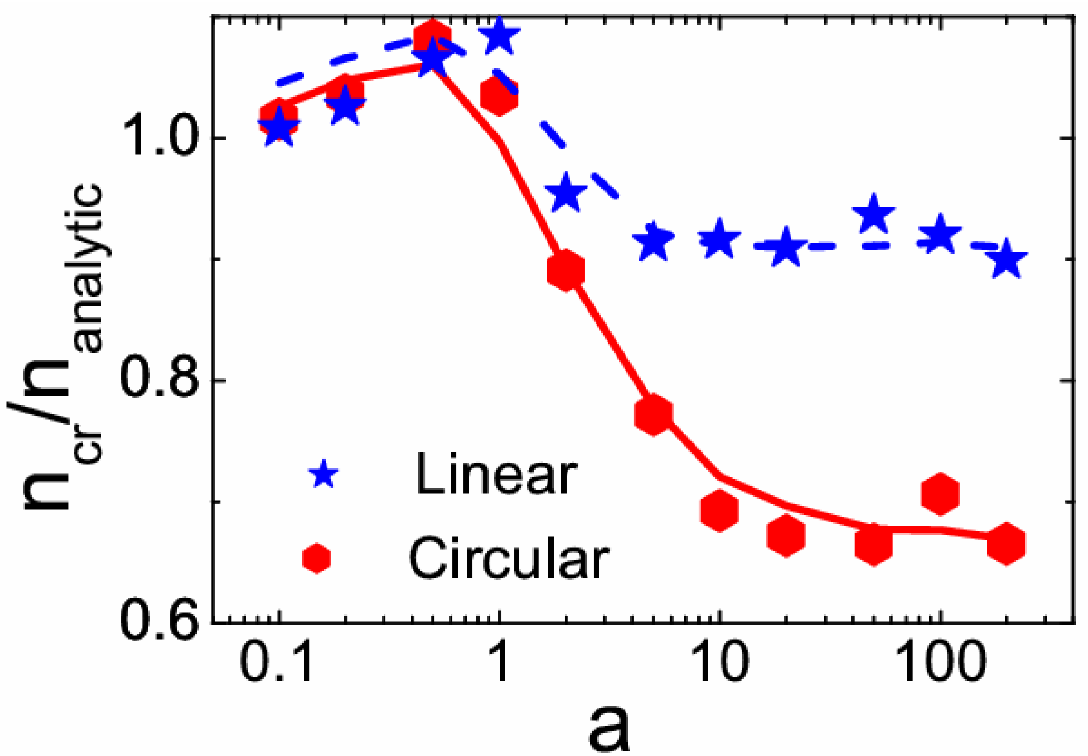
(b) Circular polarization

Theoretically

$\langle\gamma\rangle=[1+(1+R)a^2]^{1/2}=6.91$,

But from PIC $\langle\gamma\rangle$ is about 5.33.

Effect of laser polarization



- For normal incident, the relativistic critical density increase can be well fitted by

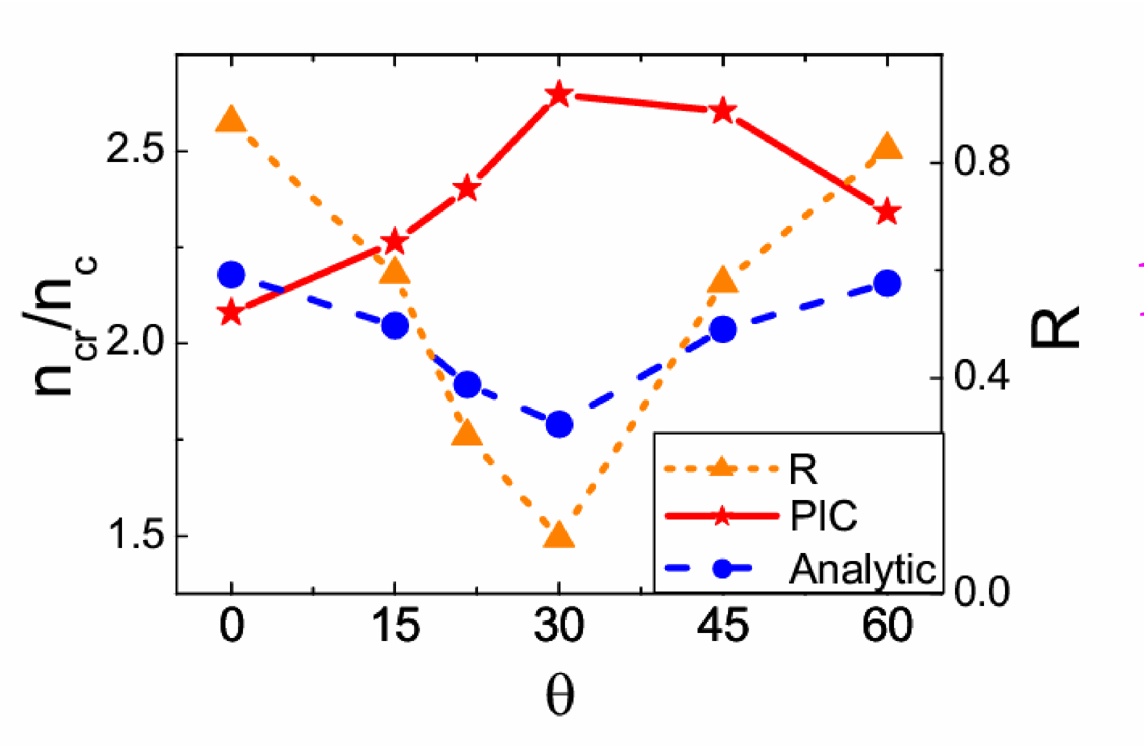
$$\langle\gamma\rangle = [1 + 0.4a^{0.62} + 0.76a^2]^{1/2} \quad \text{(linear polarization)}$$

$$\langle\gamma\rangle = [1 + a^{1.2} + 0.84a^2]^{1/2} \quad \text{(circular polarization)}$$

Critical density VS incident angle

- **The highest critical density coincides with the lowest reflectivity in the case of 30º incidence.**

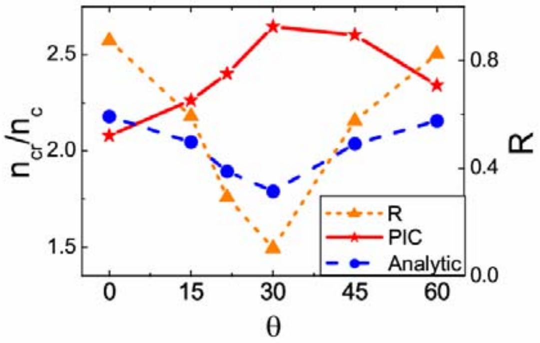
conflicts with $n_{cr}/n_c = \langle\gamma\rangle = [1+(1+R)a^2/2]^{1/2}$.



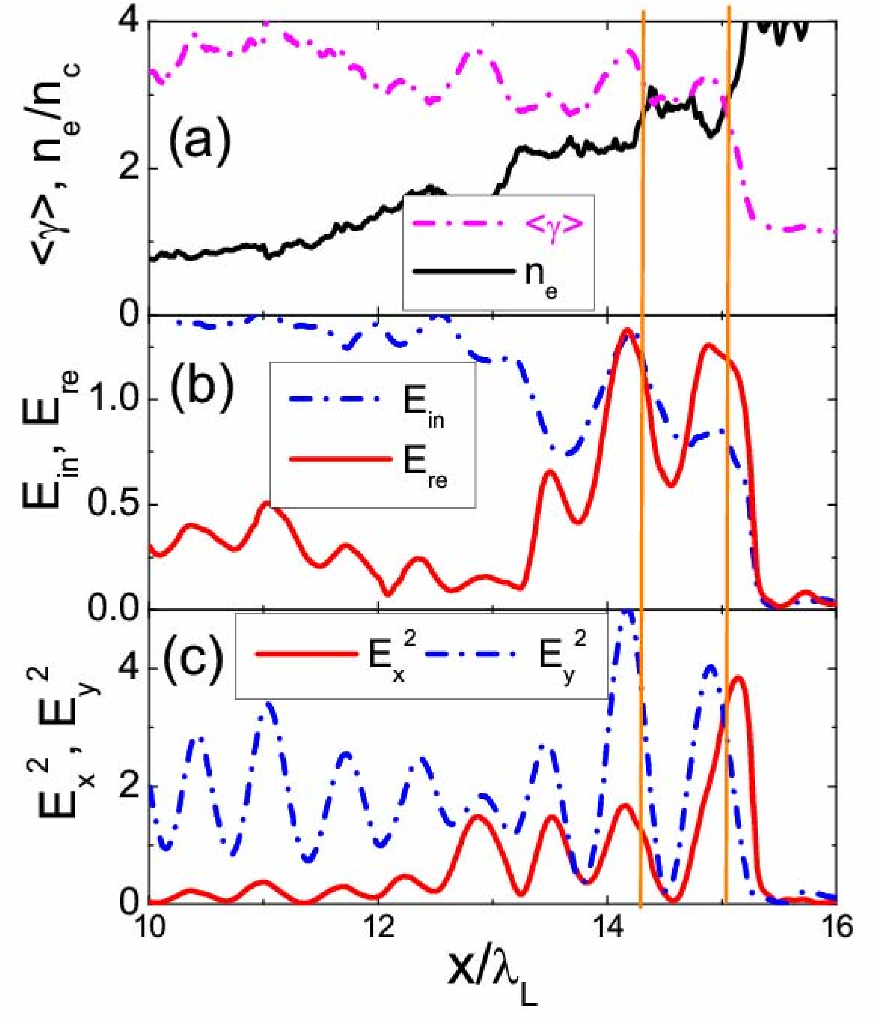
for $a = 2$,

Critical density VS incident angle

- **The highest critical density coincides with the lowest reflectivity in the case of 30º incidence.**



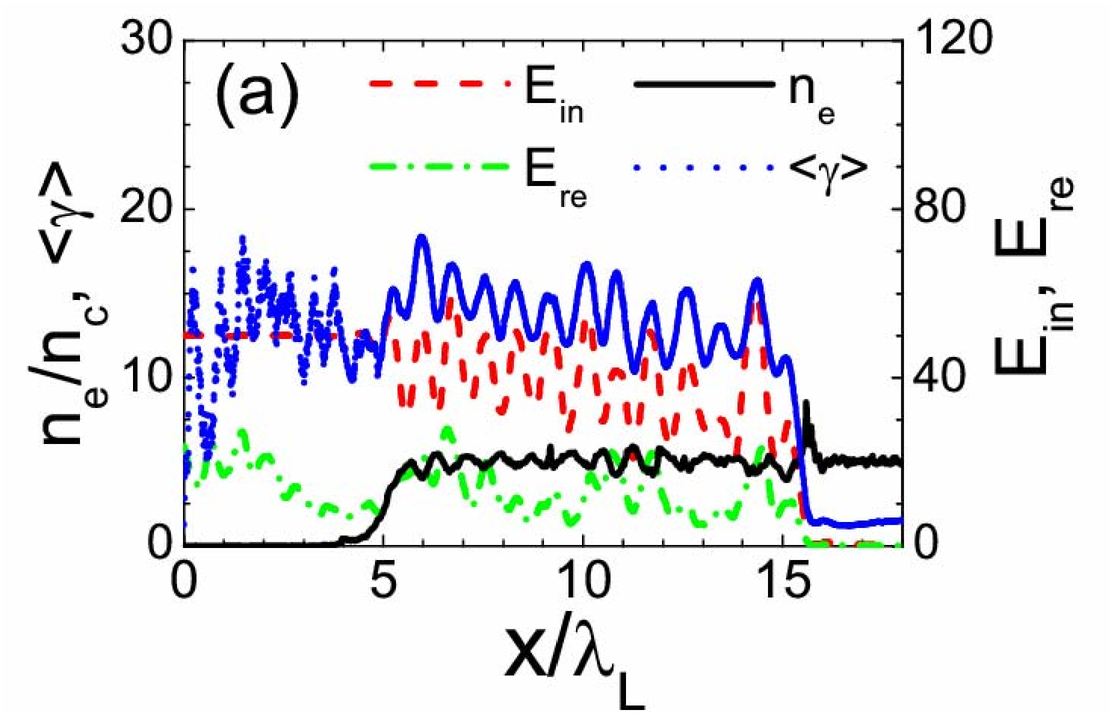
for



for $a = 2,\ \theta = 30^{o}$

This phenomenon is due to the formation of semi-black density cavity and resonance.

Relativistic laser beam propagation (LP)



linear polarization $\theta = 0,$

$a = 10,$ at t=35τ_L, and

$$n_e = \begin{cases} 0, & x < 5\lambda_L \\ 5n_c, & x > 5\lambda_L \end{cases}$$

Theoretically

$$v_{prop} \approx v_g = (1 - n/n_{cr})^{1/2} c = 0.66c,$$

but from PIC

$$v_{prop} \approx \frac{(15.5-5)\lambda_L}{(35-5)\tau_L} = 0.35c,$$

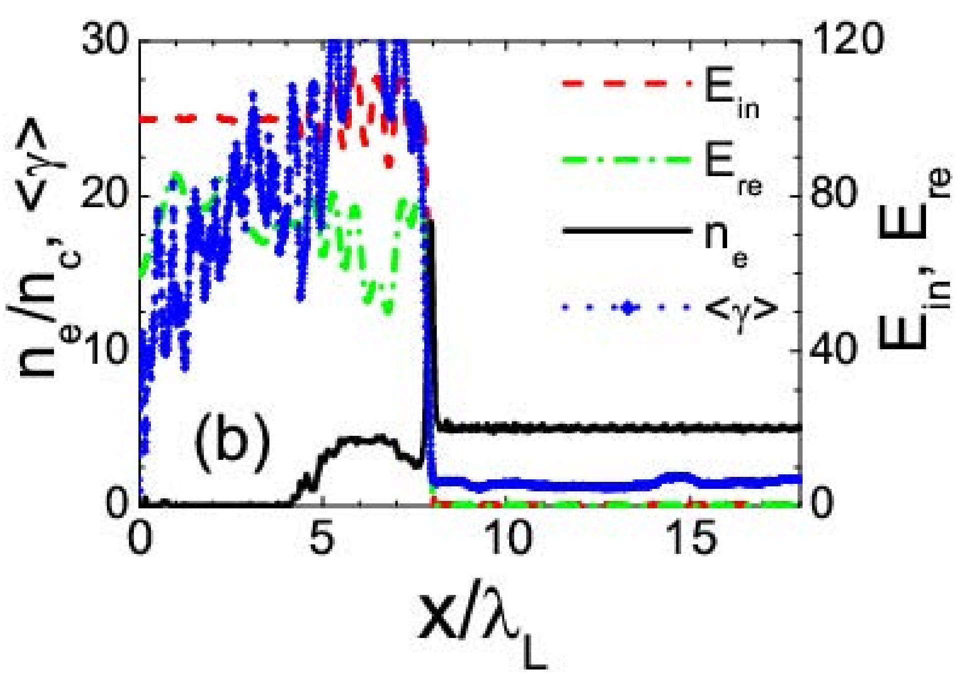
- **Sakagami and Mima attributed the inhibition of the propagation velocity to the oscillation of the ponderomotive force and hence the oscillation of electron density at the laser front.**

H. Sakagami, K. Mima, Phys. Rev. E 54, 1870 (1996).

Relativistic laser beam propagation (CP)

- **Ponderomotive force for circular polarized laser**

$$f_p = -\frac{m}{4}\frac{\partial}{\partial x} v_{os}^2(x)\hat{x}, \quad v_{os}(x) = eE/m\omega, \text{ without oscillation}$$



Theoretically

$$v_{prop} \approx v_g = \left(1 - n/n_{cr}\right)^{1/2} c = 0.7\text{c},$$

but from PIC

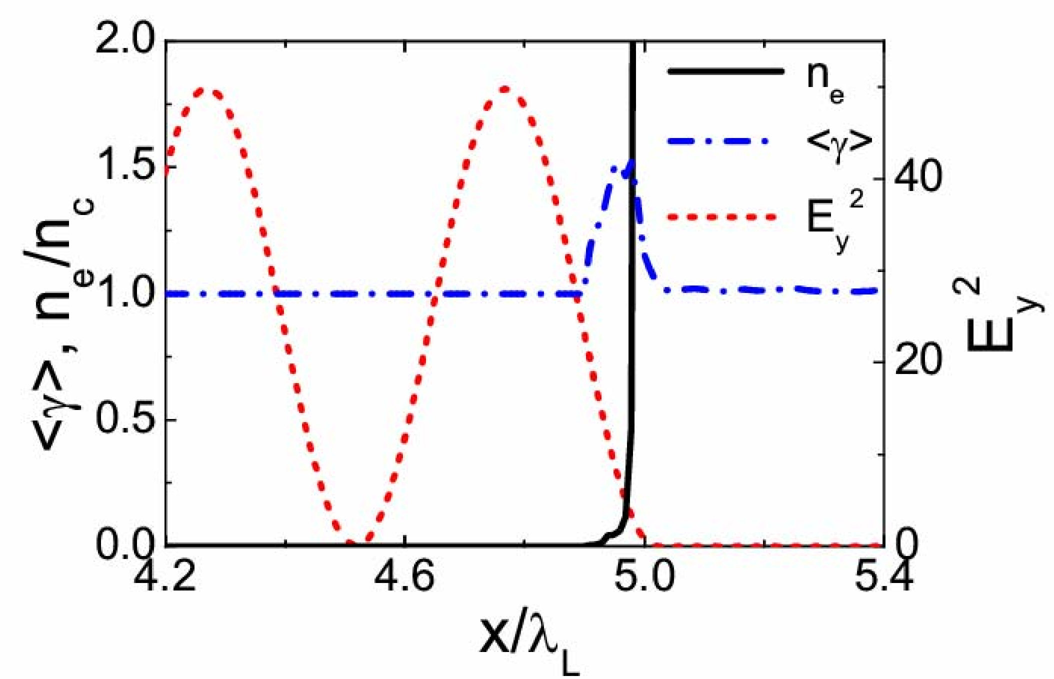
$$v_{prop} \approx \frac{(8.0-5)\lambda_L}{(35-5)\tau_L} = 0.1\text{c}.$$

CP pulse propagates even more slowly than LP pulse.

- **Inhibition of propagation velocity is not attributed to the oscillation of ponderomotive force.**

Relativistic laser beam propagation

- Laser field penetrates into an overdense plasma



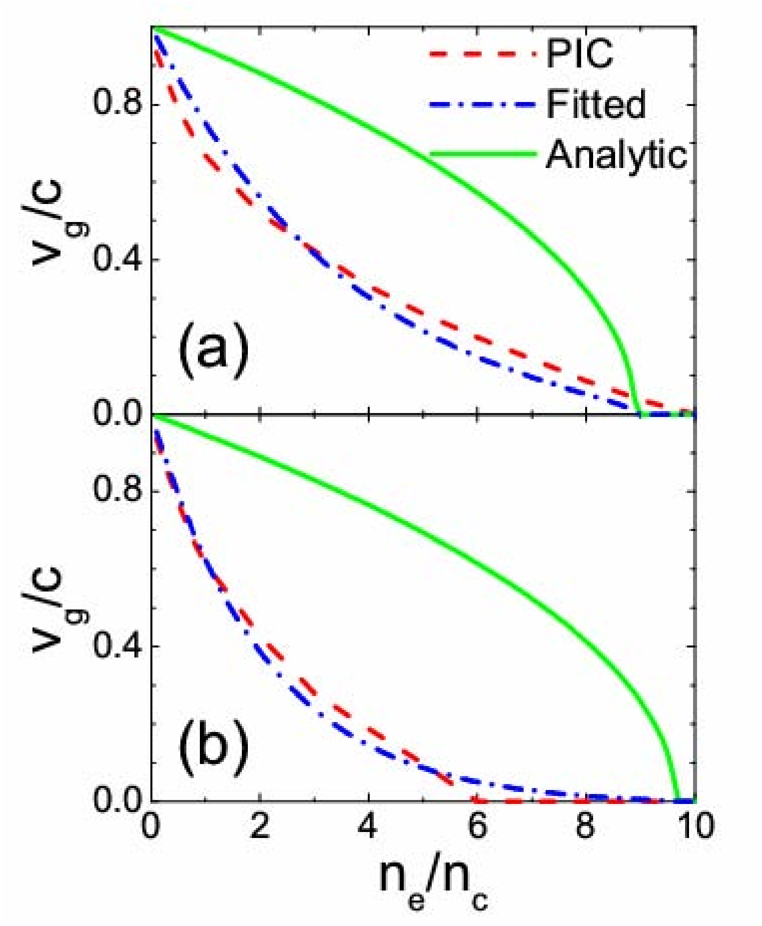
for $a = 5$, and

$$n_e = \begin{cases} 0, & x < 5\lambda_L \\ 100 n_c, & x > 5\lambda_L \end{cases}$$

electric field at the surface and skin depth $\propto 1/n_e^{1/2}$

- Relativistic induced transparency is guided by the skin penetration.
- **Propagation velocity depends on both the group velocity and the efficiency of skin penetration.**

Relativistic propagation velocity



for $a = 10$, and $\theta = 0$

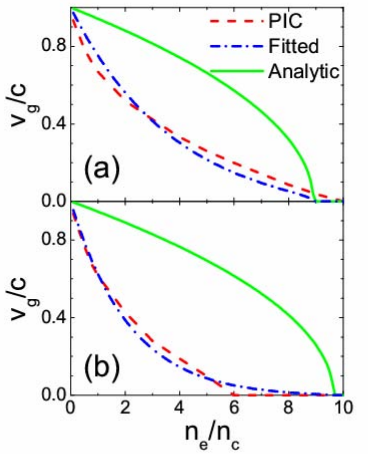
(a) linear polarization

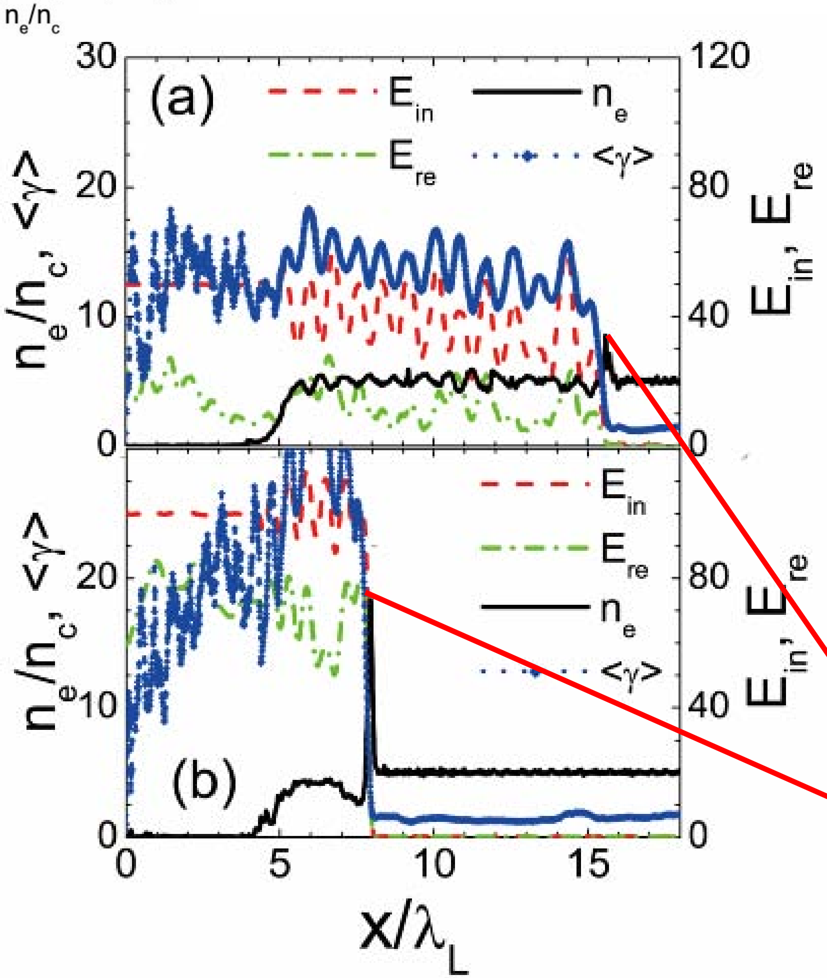
$$\frac{v_{prop}}{c} = \exp\left(-\frac{2n_0}{\langle\gamma\rangle n_c}\right)\left(1 - \frac{n_0}{\langle\gamma\rangle n_c}\right)^{1/2}$$

(b) circular polarization

$$\frac{v_{prop}}{c} = \exp\left(-\frac{4n_0}{\langle\gamma\rangle n_c}\right)\left(1 - \frac{n_0}{\langle\gamma\rangle n_c}\right)^{1/2}$$

Relativistic propagation velocity





for $a = 10$, and $\theta = 0$

(a) linear polarization

$$\frac{v_{prop}}{c} = \exp\left(-\frac{2n_0}{\langle\gamma\rangle n_c}\right)\left(1-\frac{n_0}{\langle\gamma\rangle n_c}\right)^{1/2}$$

(b) circular polarization

$$\frac{v_{prop}}{c} = \exp\left(-\frac{4n_0}{\langle\gamma\rangle n_c}\right)\left(1-\frac{n_0}{\langle\gamma\rangle n_c}\right)^{1/2}$$

the different heights of density ridge formed before the front of laser

$\Rightarrow$

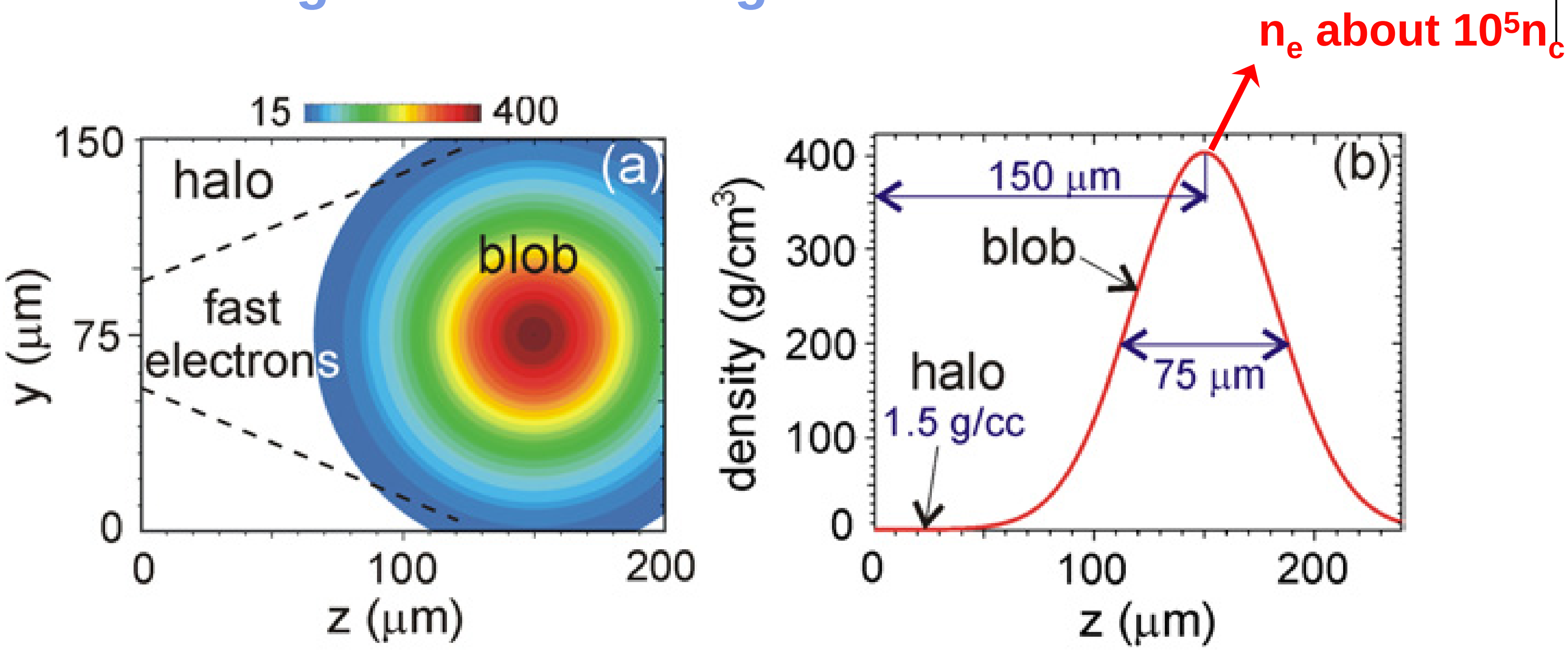
different reduced factors

$\exp\left(-2n_0/\langle\gamma\rangle n_c\right)$ (linear polarized)

$\exp\left(-4n_0/\langle\gamma\rangle n_c\right)$ (circualr polarized)

Application (a): Fast ignition

- The configuration of fast ignition

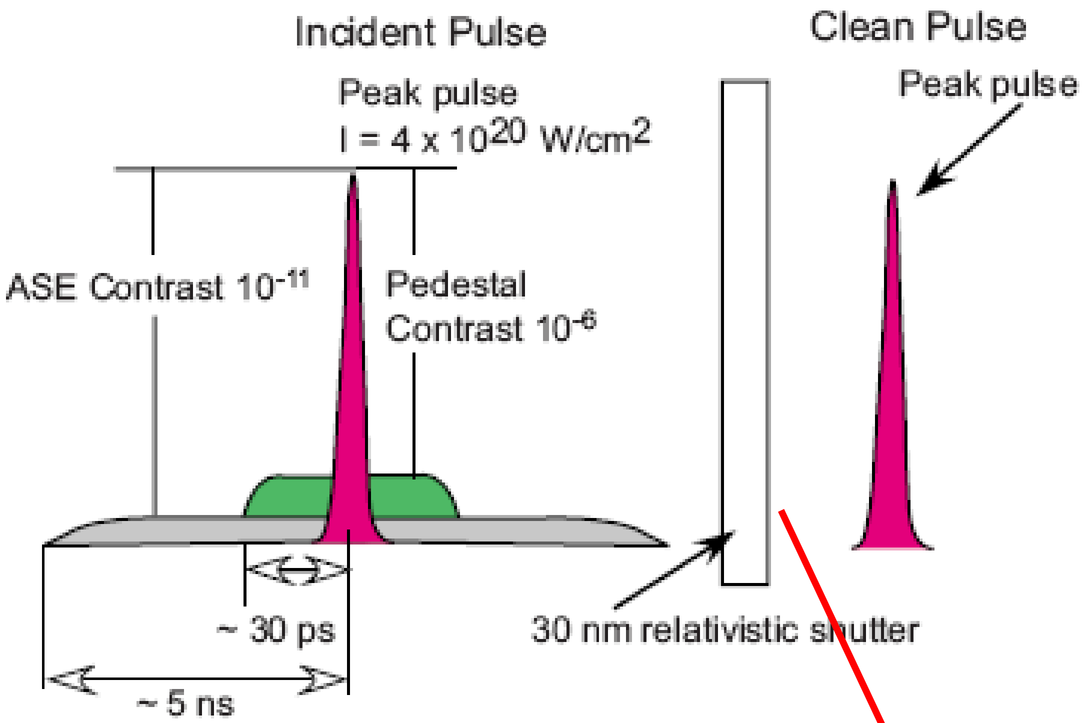


- Relativistic induced transparency, together with cone-guiding, channeling and hole boring may make the deeper penetration into an overdense target possible, and make the fast ignition easier.

J. J. Honrubia et al, Nucl. Fusion 46, L25 (2006)

Application (b): Relativistic plasma shutter

- A relativistic plasma shutter can remove the pre-pulse and produce a clean ultrahigh intensity pulse



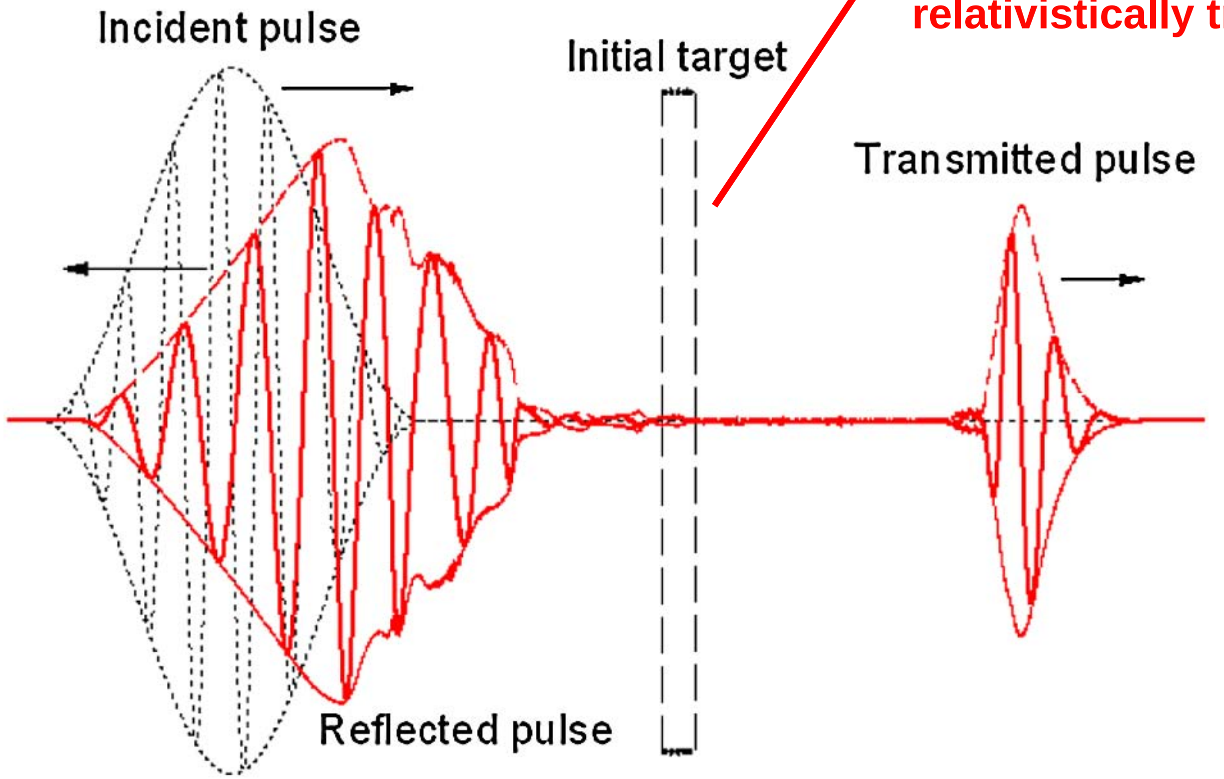
This shutter is overdense but relativistic underdense.

S. A. Reed *et al.*, Appl. Phys. Lett. 94, 201117 (2009).

Application (c): Shortening of laser pulses

- A quasi-single-cycle relativistic pulse can be produced by ultrahigh laser-foil interaction

Initial thin overdense foil evolves into a thick but rare plasma, which is relativistically transparent.



L. L. Ji *et al.*, Phys. Rev. Lett. 103, 215005 (2009).

Conclusion

- Relativistic induced transpancy make the penetration of a laser pulse into a overdense plasma possible.
- For normal incident linearly polarized pulse, the critical density increase approximately follows the theory from a single particle orbit.
- Due to the formation of a density ridge before the laser front, the critical density increase is much lower than predicted for a circular polarized pulse.
- Relativistic induced transparency is guided by the skin penetration; and the propagation velocity depends upon both the group velocity and the efficiency of skin penetration.
- Relativistic induced transparency could find wide applications in fast ignition scheme, relativistic plasma shutter, and shortening of laser pulses.
- Open Sesame!

  Open, skin penetration!


Спасибо!
Vielen Dank!
谢　谢！
Thank You!