Quantum simulations of thermodynamic and kinetic properties of strongly coupled electromagnetic and quark-gluon plasmas.

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OUTLINE

- Phase diagram of strongly coupled quantum Coulomb systems
- Basic assumptions of semi-classical theory for non-Abelian plasma and limits of applicability
- Fast simulation of thermodynamics of quantum many-particle systems by Feynman path integral Monte Carlo method
- Wigner approach to fast simulations of quantum dynamics
- Applications to the semi-classical models of quark-gluon plasma
- Applications to the strongly coupled electromagnetic plasma

Interaction and quantum effects in strongly coupled Coulomb systems with different masses of particles.

Classical one-component plasma - COCP

Quantum one-component plasma - QOCP

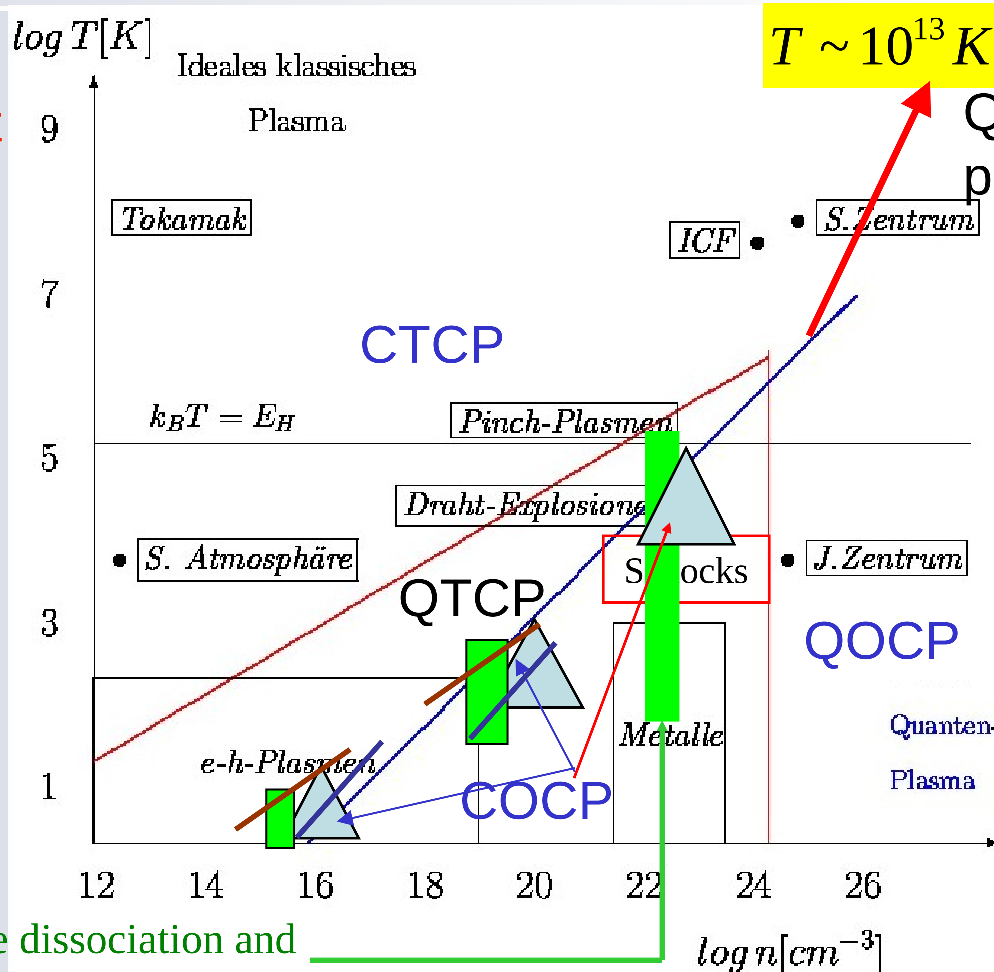
Classical two-component plasma - CTCP

Quantum two-component plasma model - QTCP

Coulomb interaction: $U_{ab}(r) = e_a e_b / r$

$T \sim 10^{13} K, n \sim 10^{45} cm^{-3}$

Quark-gluon plasma- QGP



— Nonideality boundary:

$$< U_{Coul} > = < E_{Kin} >$$

Inside: Strong Coulomb interaction,
Many-body effects
atomes, molecules, clusters

Degeneracy boundary

$$\lambda_e = \bar{r}$$

Below: overlapping electron
Wave functions,
Quantum and spin effects

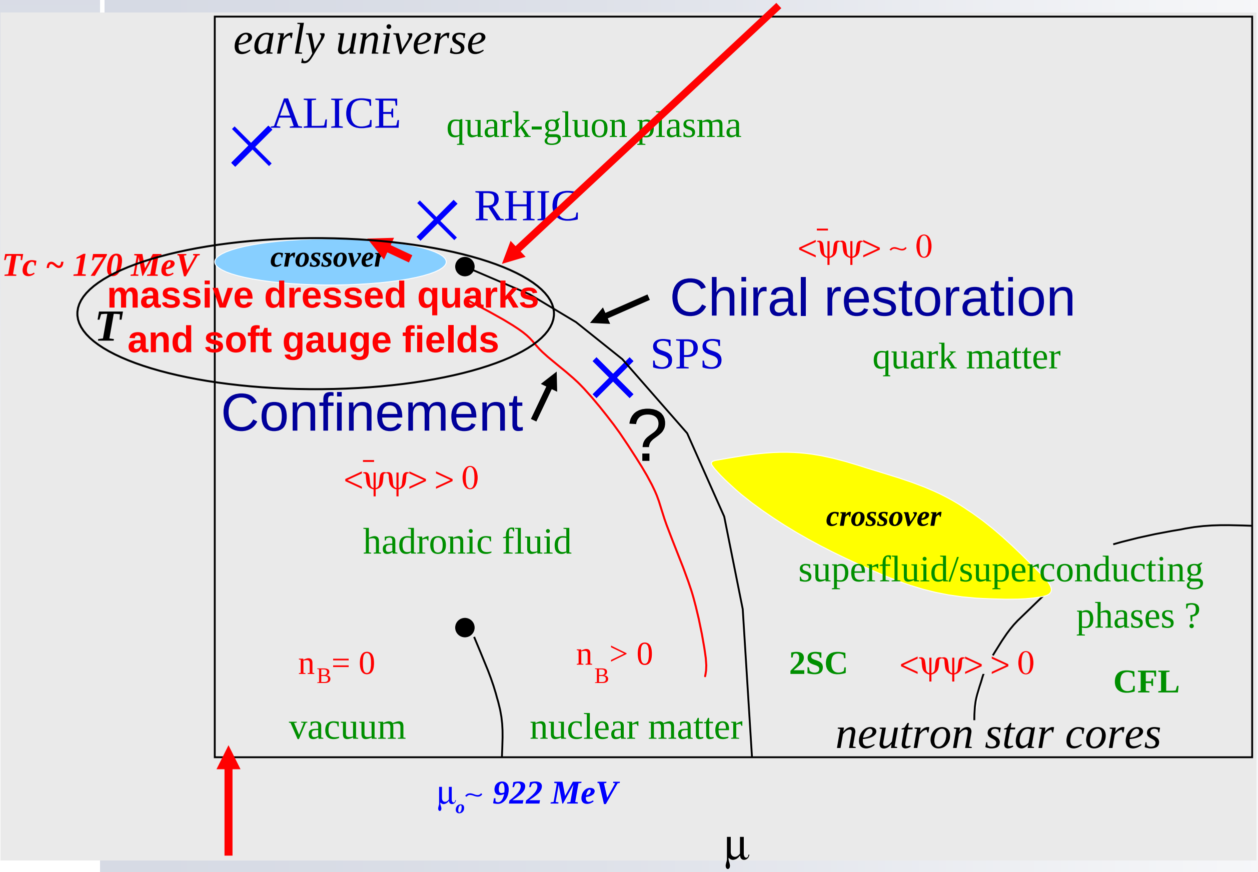
Pressure dissociation and ionization, Mott effect

Semi-classical theory for non-Abelian system of color Coulomb quasi-particles

is based on resummation technique and lattice simulations allowing for consideration of quark-gluon plasma as system of **dressed** quark, antiquark and gluon presented by color Coulomb quasiparticles with T-dependent dispersion curves and width

at and around μ=0 or above T_d and below T_c.

Litim, Manuel, Stoecker,Bleicher,Feinberg, Richardson, Bonasera,Maruyama, Hatsuda,Shuryak, Fukushima,….



Phase diagram
(F.Karsch)

Basic assumptions of the semi-classical quasiparticle model of quark – gluon plasma

is based on resummation technique and lattice simulations allowing for consideration of quark-gluon plasma as system of dressed quark, antiquark and gluon presented by color Coulomb quasiparticles with T-dependent dispersion curves and width.

(Phys.Lett.B478,161(2000), Phys. Rev. C, **74**, 044909, (2006))

- All color quasiparticles are massive (m > T) and move non-relastivistically
- Interparticle interaction is dominated by a color Coulomb potential with distance dependent coupling constant.
- The color operators are substituted by their average values
  - classical color vectors Q in SU(3) (Q is 8D vectors with 2 Casimirs).

The model input requires :

- The temperature dependence of the quasiparticle mass.
- The temperature dependence of the coupling constant.

All the input quantiries should be deduced
from lattice QCD calculations
and substitued in quantum Hamiltonian.

Thermodynamics of quark - gluon plasma in grand canonical ensemble within Feynman formulation of quantum mechanics

$$H_\beta = K_\beta + U_C = \sum_a \sqrt{p_a^2 + m_a^2(\beta)} + U_C \approx$$

$$\approx \sum_a \left(N_a m_a(\beta) + \frac{p_a^2}{2m_a(\beta)}\right) + \sum_{a,b} \frac{g^2(|r_a - r_b|, \beta) < \vec{Q}_a | \vec{Q}_b >}{4\pi |r_a - r_b|}, m_a >> T$$

Grand canonical partition function

$$\Omega\left(\mu, \mu_g = 0, V, \beta\right) =$$

$$= \sum_{N_q, N_{\underline{q}}, N_g} \exp(\beta\mu(N_q - N_{\underline{q}}))Q\left(N_q, N_{\underline{q}} N_g, \beta\right) / N_q! N_{\underline{q}}! N_g!$$

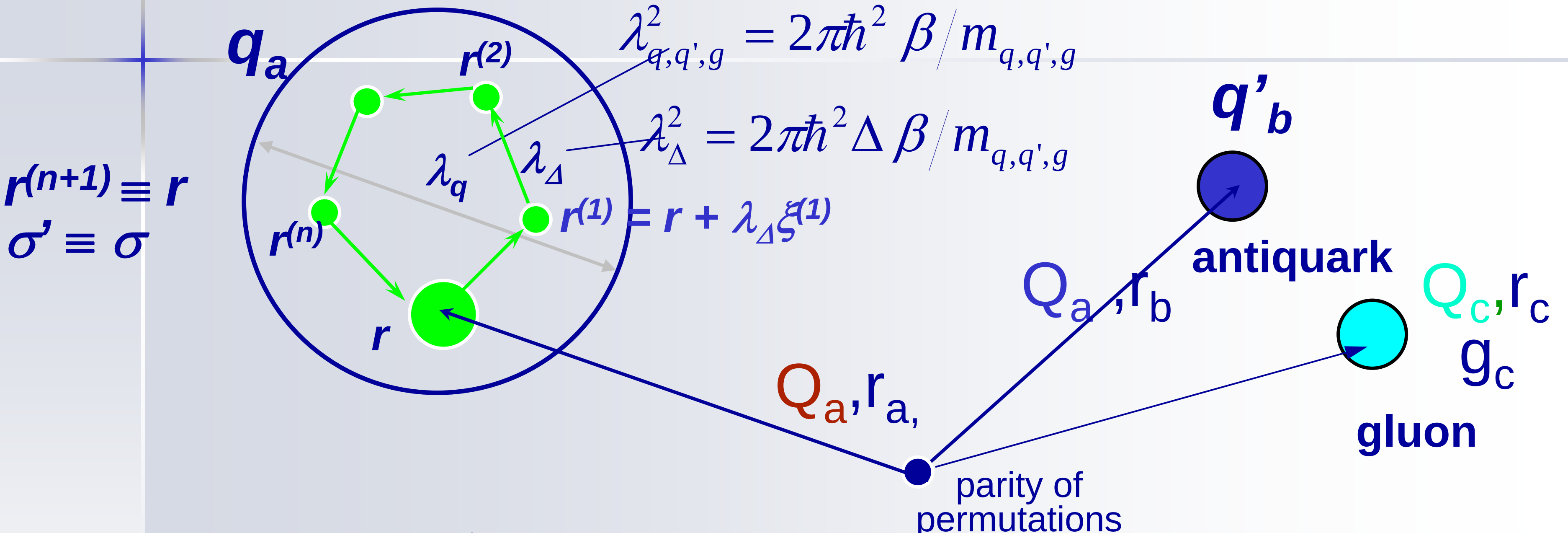
$$Q\left(N_q, N_{\underline{q}}, N_g, \beta\right) = \sum_\sigma \int_V dr d\mathbf{Q} \rho\left(r, \mathbf{Q}, \sigma; \beta\right)$$

$$\rho = \exp\left(-\beta H(\beta)\right) = \underbrace{\exp\left(-\Delta\beta H(\beta)\right) \times \ldots \times \exp\left(-\Delta\beta H(\beta)\right)}_{n+1}$$

$$\beta = 1/kT \qquad \Delta\beta = \beta/(n+1)$$

PATH INTEGRALS
MONTE-CARLO METHOD

quark, antiquark, gluon



$$\rho(r,Q,\sigma;\beta) = \frac{1}{\lambda_{\Delta q}^{3N_q}\lambda_{\Delta q'}^{3N_{q'}}\lambda_{\Delta g}^{3N_g}} \sum_{P=P_q,P_{q'},P_g} (\pm 1)^{\kappa_P} \int_V dr^{(1)}\ldots dr^{(n)} dQ^{(1)}\ldots dQ^{(n)} \times$$

$$\rho\left(r,Q;r^{(1)},Q^{(1)};\Delta\beta\right)\ldots\rho\left(r^{(n)},Q^{(n)};\hat{P}r^{(n+1)},\hat{P}Q^{(n+1)};\Delta\beta\right)S\left(\sigma,\hat{P}\sigma'\right)$$

spin matrix

$$\rho(r^{(l)},Q^{(l)};r^{(l+1)},Q^{(l+1)}) \approx \delta(Q^{(l)}-Q^{(l+1)})\rho(r^{(l)},Q^{(l)};r^{(l+1)},Q^{(l)})$$

Density matrix

$$\sum_{\sigma}\rho(r,Q,\sigma;\beta)=\frac{1}{\lambda_{\Delta}^{3N_q}\lambda_{\Delta}^{3N_{q'}}\lambda_{\Delta}^{3N_g}}\sum_{s=0}^{N_q}\sum_{s'=0}^{N_{q'}}\sum_{s''=0}^{N_g}\rho_{ss's''}([rQ],\beta)$$

$$\rho_{ss's''}([rQ],\beta)=\frac{C_{N_q}^{s}}{2^{N_q}}\frac{C_{N_{q'}}^{s'}}{2^{N_{q'}}}\frac{C_{N_g}^{s''}}{2^{N_g}}\exp\{-\beta U([rQ],\beta)\}\times$$

$$\times\prod_{l=1}^{n}\prod_{p=1}^{N_e}\phi_{pp}^{l}\det\left|\psi_{ab}^{n,1}\right|_{s}\prod_{p=1}^{N_i}\tilde{\phi}_{pp}^{l}\det\left|\tilde{\psi}_{ab}^{n,1}\right|_{s'}\prod_{p=1}^{N_i}\tilde{\tilde{\phi}}_{pp}^{l}\,per\left|\tilde{\tilde{\psi}}_{ab}^{n,1}\right|_{s''}$$

$$U([rQ],\beta)=\sum_{l=0}^{n}\frac{U_{l}^{qq'g}([r^{(l)}Q],\beta)}{n+1}$$

Pairwise sum of Kelbg potentials for each l=0,…,n

Exchange matrix

$$\left\|\psi_{ab}^{n,1}\right\|_{s}\equiv\left\|\exp\left\{-\frac{\pi}{\lambda_{\Delta}^{2}}\left|(r_a-r_b)+y_a^n\right|^2\right\}\right\|_{s}$$

COLOR KELBG PSEUDOPOTENTIAL

First order perturbation theory solution of two-particle Bloch equation

$$H_\beta = K_\beta + U_\beta = \frac{p_a^2}{2m_a(\beta)} + \frac{p_b^2}{2m_b(\beta)} + \frac{g^2(|r_a - r_b|,\beta) < \vec{Q}_a | \vec{Q}_b >}{4\pi |r_a - r_b|}, g^2(|r_a - r_b|,\beta) \sim \frac{1}{\ln(|r_a - r_b|)}$$

$$\rho^{(0)} = \exp\left(-\lambda\Delta\beta K_\beta\right)|_{\lambda=1}, \rho = \exp\left(-\lambda\Delta\beta H_\beta\right)|_{\lambda=1}, -\partial\rho/\partial\lambda = \Delta\beta H_\beta$$

$$\rho_{ab}(r_a, r_b, r_a', r_b', \Delta\beta) = \rho_{ab}^{(0)}(R, R', \Delta\beta)\rho_{ab}(r, r', \Delta\beta) \approx \rho_{ab}^{(0)}(R, R', \Delta\beta)[\rho_{ab}^{(0)}(r', r, \Delta\beta) + \rho_{ab}^{(1)}(r', r, \Delta\beta)]$$

$$\rho_{ab} \approx \rho^{(0)}{}_{ab} - \int_0^\lambda d\tau \exp(-(\lambda-\tau)\Delta\beta K_\beta^{ab}) \frac{\Delta\beta g^2(|r''|,\beta) < \vec{Q}_\alpha | \vec{Q}_\beta >}{4\pi |r''|} \exp(-\tau\Delta\beta K_\beta^{ab})$$

$$\rho^{(1)}(r', r, \Delta\beta) = -\int_0^1 d\tau \int dr'' \exp\left(-\frac{\pi |r' - r''|^2}{\tilde{\lambda}^2(1-\tau)}\right) \frac{\Delta\beta g^2(|r''|,\beta) < \vec{Q}_a | \vec{Q}_b >}{4\pi |r''|} \exp\left(-\frac{\pi |r'' - r|^2}{\tilde{\lambda}^2 \tau}\right) / \{\tilde{\lambda}^2 \sqrt{\tau(1-\tau)}\}^3 =$$

$$= \tilde{\lambda}^{-3} \exp\{-\frac{\pi |r' - r|^2}{\tilde{\lambda}^2}\}\{\exp(-\Delta\beta\Phi_{ab}(r', r, \beta)) - 1\} = \rho^{(0)}{}_{ab}(r', r, \Delta\beta)\{\exp\{-\Delta\beta\Phi_{ab}(r', r, \beta)\} - 1\}$$

$$\Delta\beta\Phi_{ab}(r', r, \beta) = \Delta\beta g^2(|\bar{d}_{ab}^{\tilde{\lambda}}|,\beta) < \vec{Q}_a | \vec{Q}_b > \int_0^1 d\tau \frac{\operatorname{erf}\left(\dfrac{d_{ab}(\tau)}{2\tilde{\lambda}_{ab}\sqrt{\tau(1-\tau)}}\right)}{4\pi d_{ab}(\tau)},$$

$$d_{ab}(\tau) = \left|\tau \mathbf{r}_{ab} + (1-\tau)\mathbf{r}'_{ab}\right|$$

$$\tilde{\lambda}_{ab}^2 = \hbar^2 \Delta\beta / 2\mu_{ab}$$

$$\mu_{ab}^{-1} = m_a^{-1} + m_b^{-1}$$

Color Kelbg potential

Richardson, Gelman, Shuryak, Zahed, Harmann, Donko, Leval, Kalman (r=0 ?)

$$x_{ab} = |\mathbf{r}_{ab}| / \tilde{\lambda}_{ab} \qquad \tilde{\lambda}_{ab} = \hbar^2 \Delta\beta / 2\mu_{ab}$$

$$\Phi^{ab}(x_{ab}, \Delta\beta) = \frac{<\vec{Q}_a|\vec{Q}_b> g^2}{4\pi\tilde{\lambda}_{ab} x_{ab}} \left\{ 1 - e^{-x_{ab}^2} + \sqrt{\pi} x_{ab} \left[1 - erf(x_{ab}) \right] \right\}$$

$$|\mathbf{r}_{ab}| \to 0$$

$$\sim \frac{<Q_a|Q_b> g^2 \sqrt{\pi}}{4\pi\tilde{\lambda}_{ab}}$$

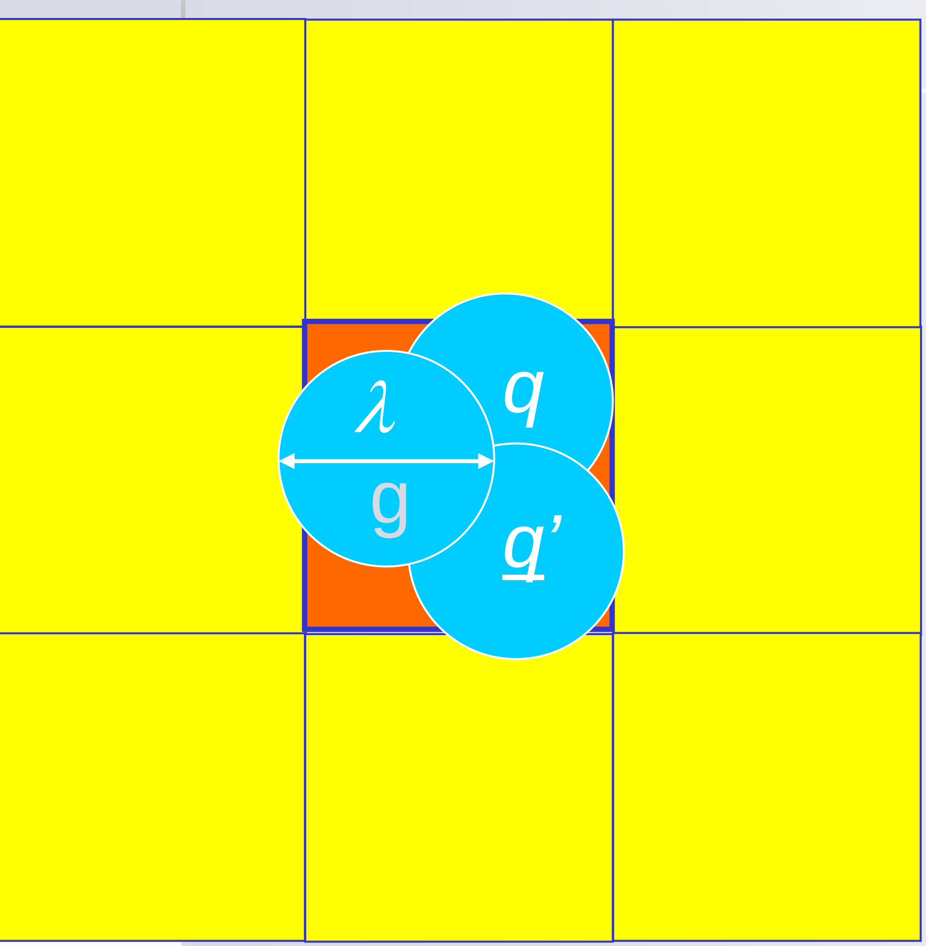
$$|\mathbf{r}_{ab}| >> \tilde{\lambda}_{ab}$$

$$\frac{<Q_a|Q_b> g^2}{4\pi\tilde{\lambda}_{ab} |x_{ab}|}$$

Objects Q are color coordinates of quarks and gluons
There is no divergence at small interparticle distances and it has a true asymptotics (T, x_{ab})

Ha -> $k_B T_c$, T_c =175 Mev,
$T_c < T$, $m_a \sim 5 k_B T_c / c^2$,
$L_o \sim hc/k_B T_c$, $r_s = <r>/L_o < 0.1$,
$L_o \sim 1.2\ 10^{-15}$ m, $x_{ab} \sim 1$

PERIODIC BOUNDAY CONDITIONS TREATMENT OF EXCHANGE EFFECTS



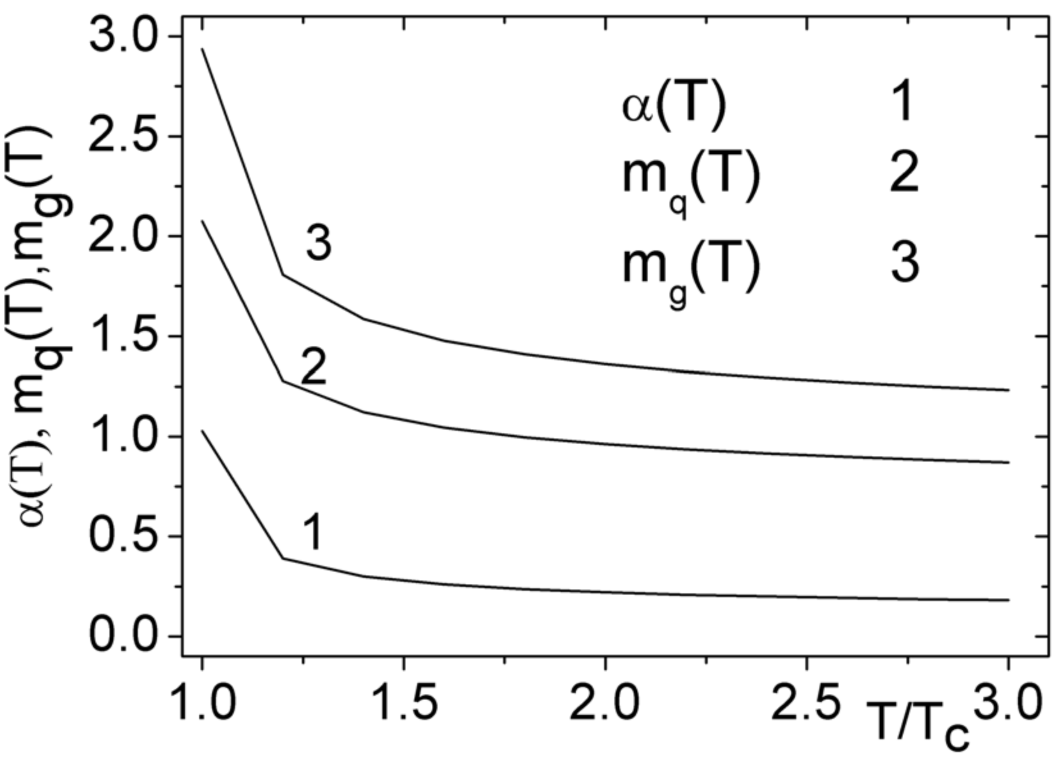
Inside main cell – exchange matrix (exact)

Accuracy control of sign problem – comparison with ideal degenerate gas

$$n_e \lambda_e^3 \sim 30$$

Filinov V.S. // J. Phys. A: Math. Gen. ***34****, 1665 (2001)*
Filinov V.S. et al. // J. Phys. A: Math. Gen. ***36****, 6069 (2003)*

Input quantities from Phys.Rev. D66, 094003, (2002)



Coupling constant

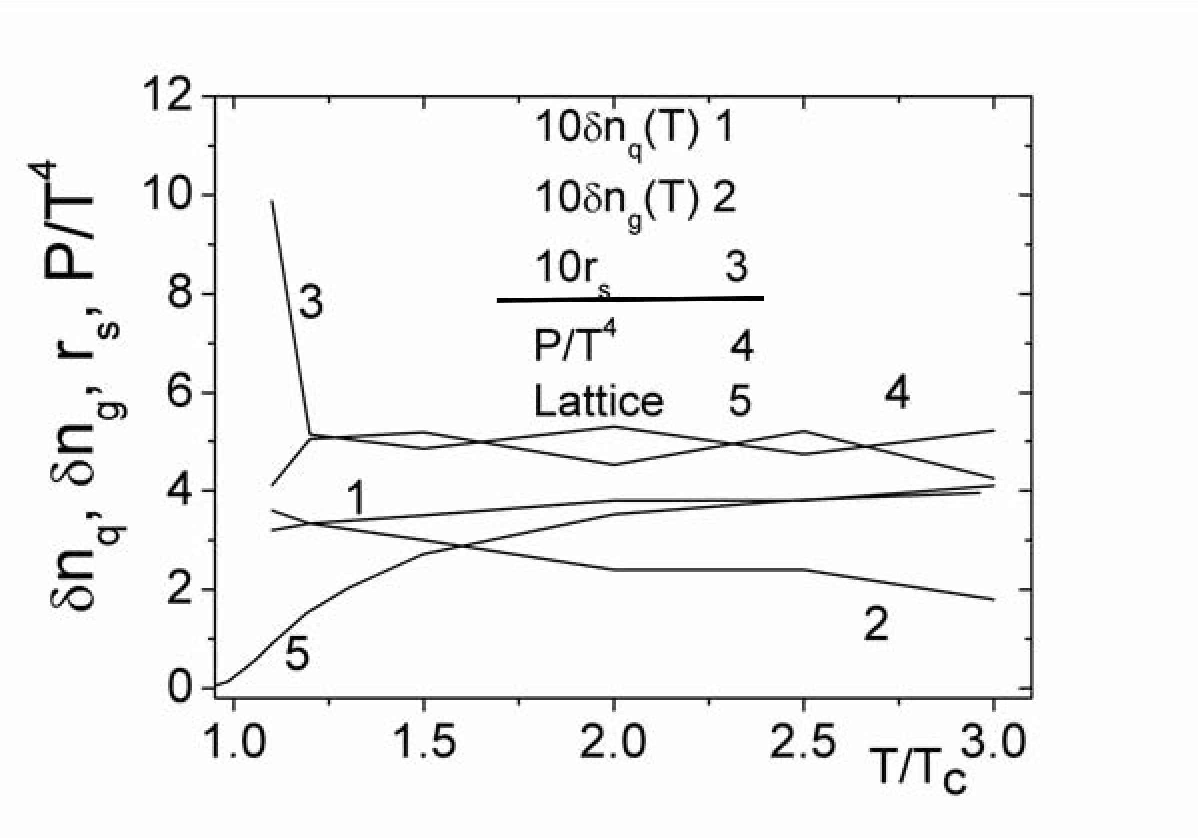
$\sigma \approx 1.1 fm$

Ratio of potential to kinetic energy per quasiparticle

$\Gamma(T) \sim U / K \sim 1$

$\alpha(T) = g^2(T) / 4\pi$

Fractions of quarks, antiquarks and gluons, average distance between particles and equation of state at zero baryon chemical potentials (for SU(2)). Comparison path integral results with lattice (2+1) QCD data.



$$4\pi r_s^3 n\sigma^3/3 = 1$$

Canonical ensemble.

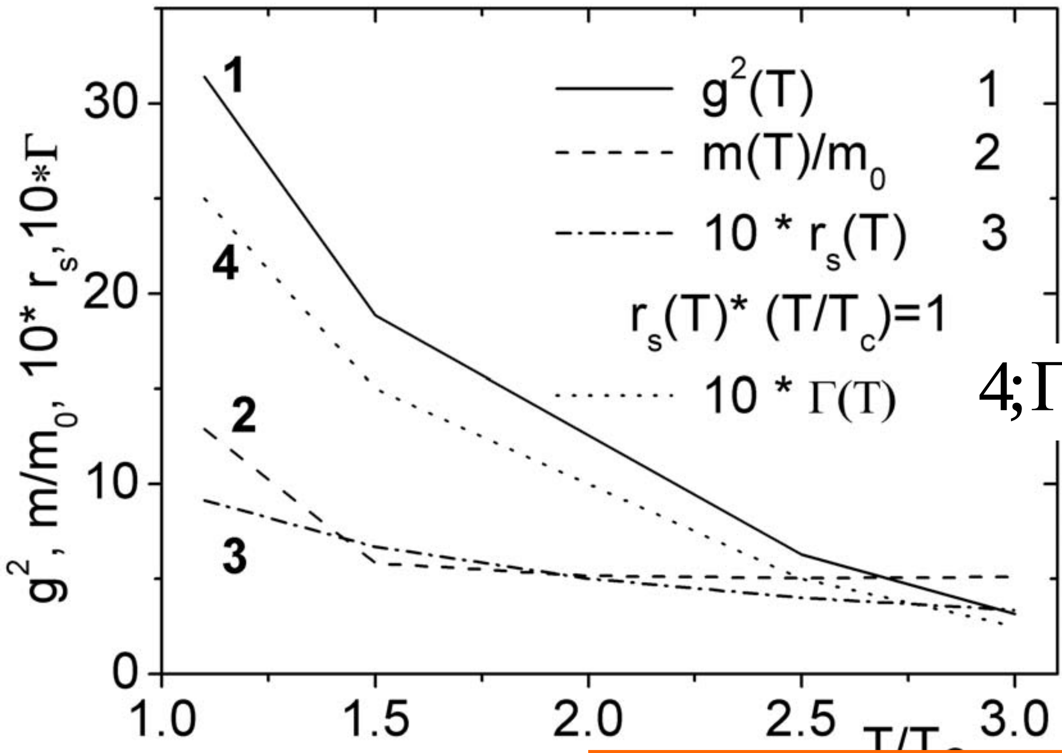
First studies and test calculations for SU(2) **.**

Calculations for SU(3) are in progress.

- The color operators are substituted by their average values
  - – classical color vectors **in SU(2)** (Q is 3D vec.with 1Cas.) instead of SU(3).
- **The model input requires :**
- The temperature dependence of the quasiparticle mass.
- The temperature dependence of the coupling constant.
- **The temperature dependence of the quasiparticle density**

  from grand canonical ensemble or literature

Input quantities from lattice calculations. Canonical ensemble.



Coupling constant

$\alpha(T) = g^2(T)/4\pi \sim 1$

Ratio of potential to kinetic energy per quasiparticle

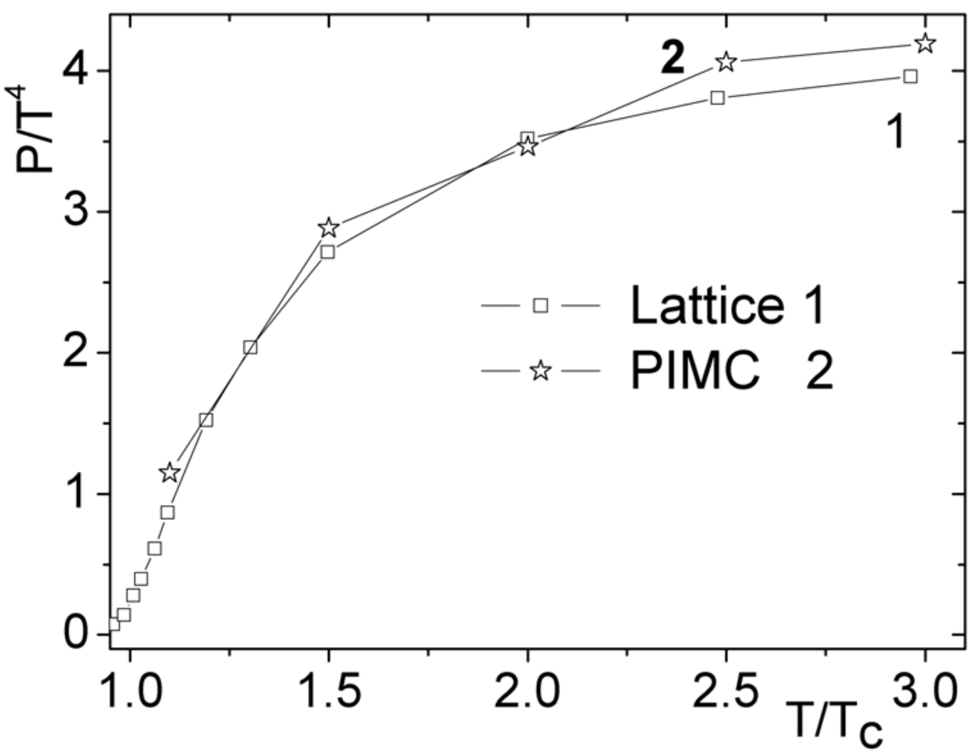
$4; \Gamma(T) \sim U/K \sim 1$

Quasiparticle masses: $m(T)/T_c \approx \frac{0.9}{(T/T_c - 1)} + 3.45 + 0.4T/T_c$

Density: $n\sigma^3 \approx 0.24(T/T_c)^3$ $\quad 4\pi r_s^3 n\sigma^3/3 = 1$ $\quad \sigma \approx 1.1 fm$ $\quad r_s(T) = \langle r \rangle/\sigma \approx 1/(T/T_c)$

[Phys. Rev. C, **74**, 044909, (2006), Phys. Rev. D, 73, 014509, (2006)]

Equation of State. Comparison path integral results with lattice (2+1) QCD



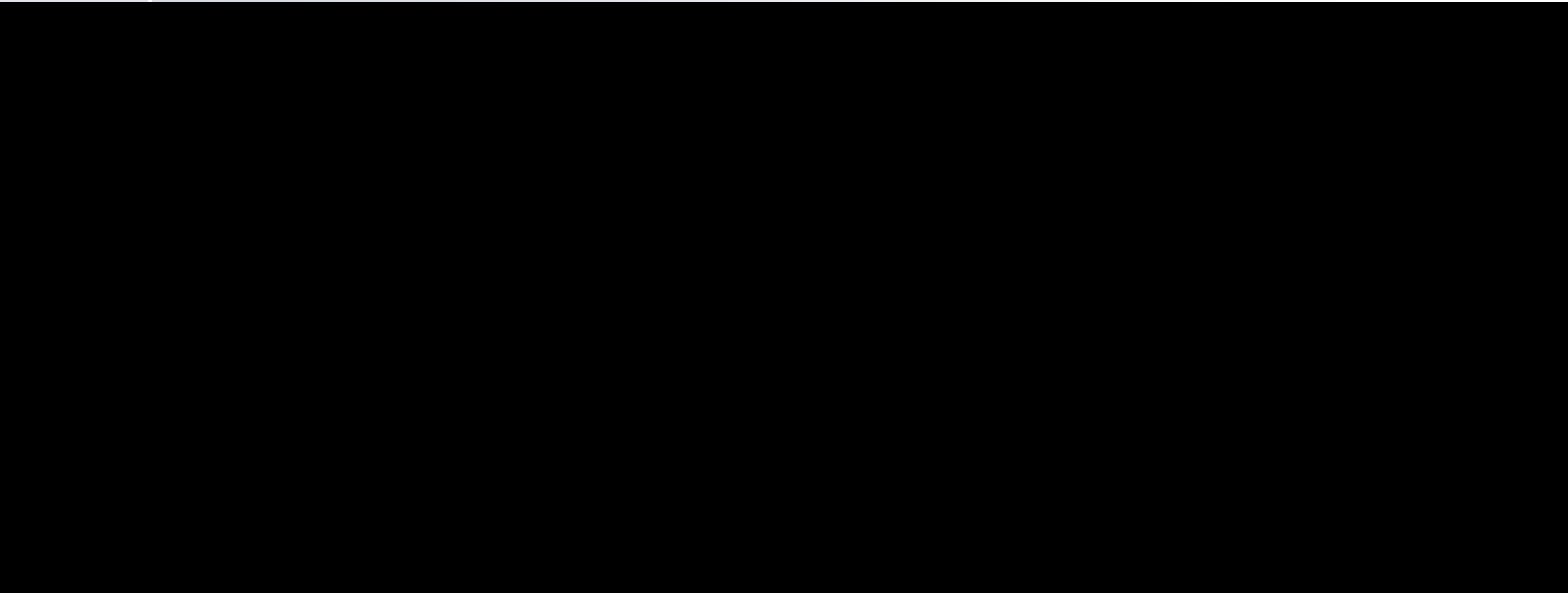
Snapshots of typical configurations

$T=1.1T_0$

Gas-like rarefied system of 3-4 quasiparticle clusters

$T=3T_0$

Liquid-like dense system of individual quasiparticles



Spatial and color correlation of quasi-particles.

Pair distribution functions in canonical emsemble

Color correlation functions

$$H_\beta \approx \sum_a (N_a m_a(\beta) + \frac{p_a^2}{2m_a(\beta)}) + \sum_{a,b} \frac{g^2(|r_a - r_b|, \beta) C_{ab} < \vec{Q}_a | \vec{Q}_b >}{4\pi |r_a - r_b|}$$

$$Z(N_q, N_{q'} N_g, V, \beta) = Q(N_q, N_{q'} N_g, \beta) / N_q! N_{q'}! N_g!$$

$$Q(N_q, N_{q'}, N_g, \beta) = \sum_\sigma \int_V drdQ \rho(r, Q, \sigma; \beta)$$

$$g_{ab}(|R_1 - R_2|) = g_{ab}(R_1, R_2) = \frac{1}{Q(N_q, N_{q'}, N_g)} \times$$

$$\sum_\sigma \int_V drd\,Q\, \delta(R_1 - r^a{}_1) \delta(R_2 - r^b{}_2) \rho(r, Q, \sigma; \beta),$$

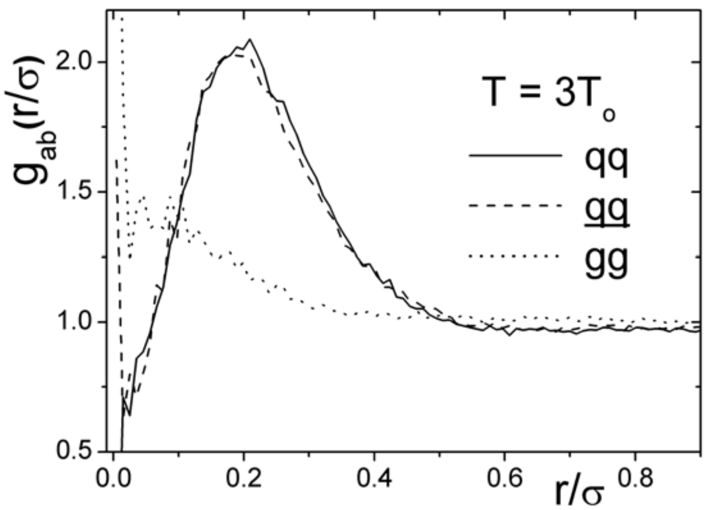
$$c_{ab}(R_1 - R_2)_{Def} = \frac{1}{Q(N_q, N_{q'}, N_g)} \sum_\sigma \int_V drd\,Q \times$$

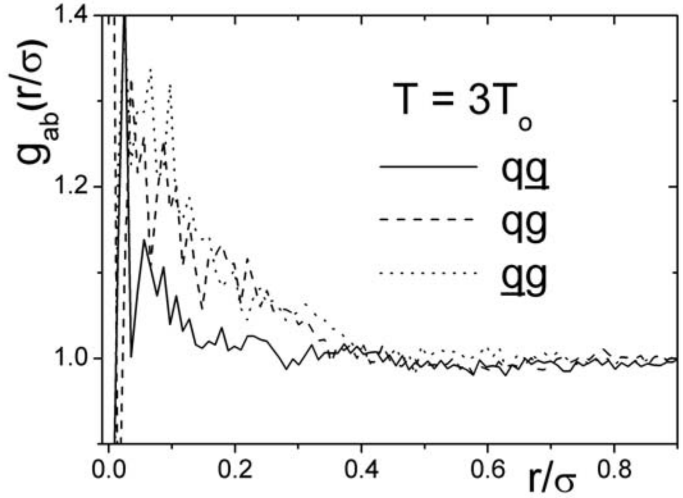
$$\delta(R_1 - r^a{}_1) \delta(R_2 - r^b{}_2) < Q^1{}_a | Q^2{}_b > \rho(r, Q, \sigma; \beta)$$

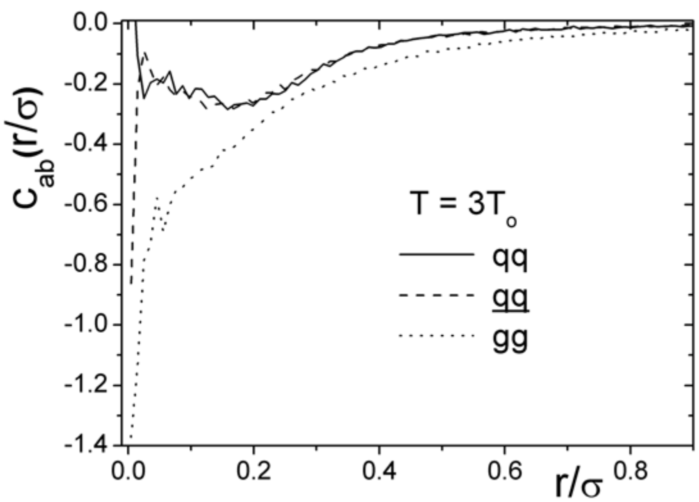
PAIR DISTRIBUTION AND COLOR CORRELATION FUNCTIONS

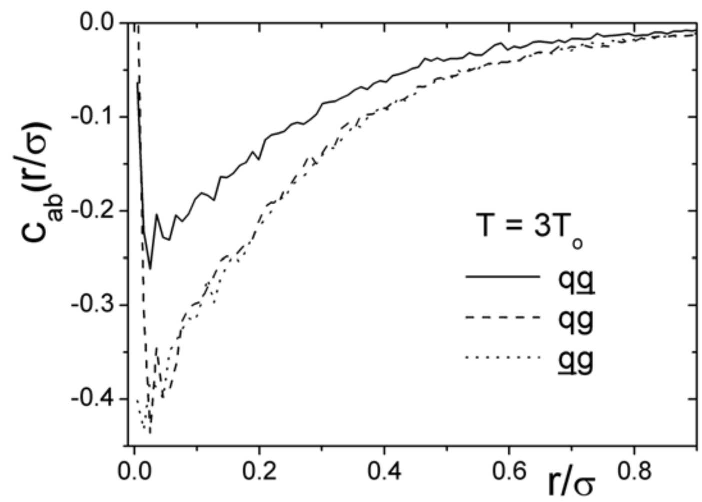
Similar quasiparticles

Different quasiparticles





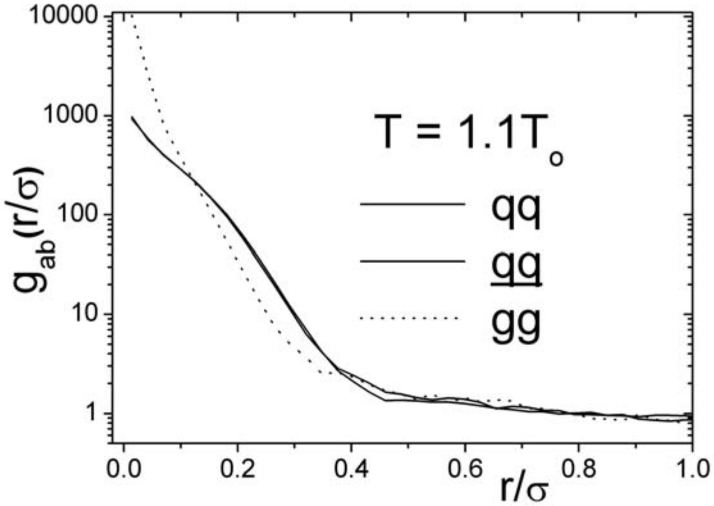


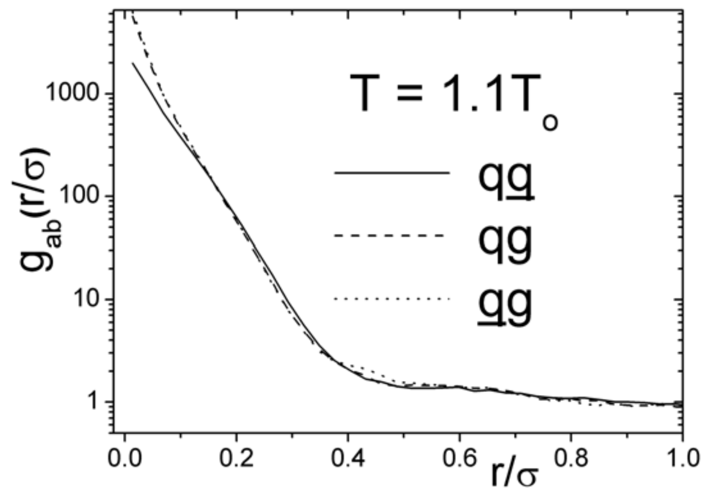


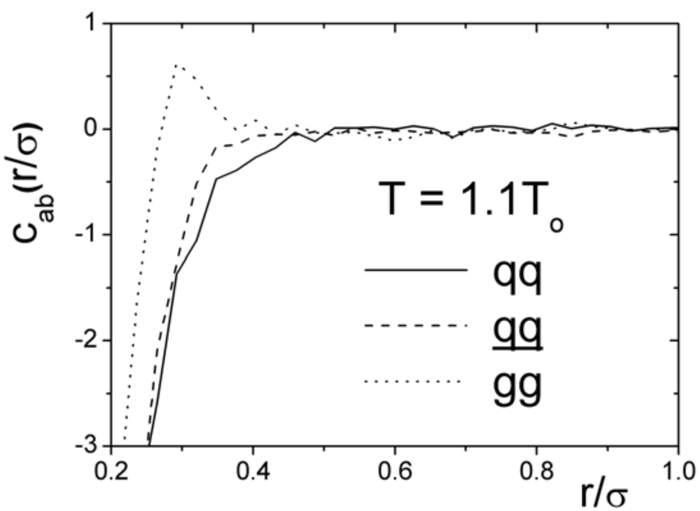
PAIR DISTRIBUTION AND COLOR CORRELATION FUNCTIONS

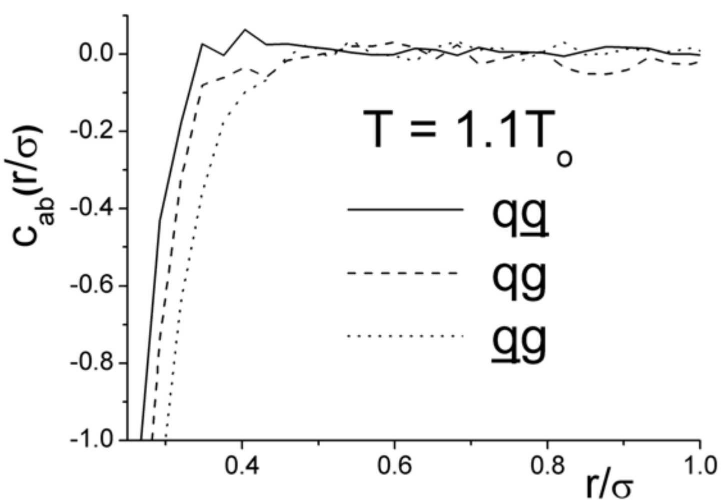
Similar quasiparticles

Different quasiparticles









Color bound states.
Estimation of the bound states in electromagnetic plasma

The product $r^2 g_{ab}(r)$ has the physical meaning of a probability to find an two quasipartices at a distance |r| from each other. On the other hand, the corresponding quantum mechanical probability is the product of r^2 and the two-particle Slater sum $\sum_{ab}$

$$\sum_{ab} = 8\pi^{3/2}\lambda_{ab}^3 \sum_{E_\alpha}^{\infty} |\Psi_\alpha(r)|^2 \exp(-\beta E_\alpha) = \sum_{ab}{}^d + \sum_{ab}{}^c$$

$$\sum_{ab}{}^d = 8\pi^{3/2}\lambda_{ab}^3 \sum_{E_\alpha}^{E'} |\Psi_\alpha(r)|^2 \exp(-\beta E_\alpha)$$

$$r^2 g(r) \sim r^2 \left(\sum_{ab}{}^d + \sum_{ab}{}^c\right)$$

$$r^2 g(r) \sim r^2 \sum_{ab}{}^d > r^2, r < a_b$$

$$r^2 g(r) \sim r^2 \sum_{ab}{}^c \sim r^2, r > a_b$$

$$\sum_{ab}{}^c >> \sum_{ab}{}^d \Rightarrow r^2 * \sum_{ab}{}^c \sim r^2$$

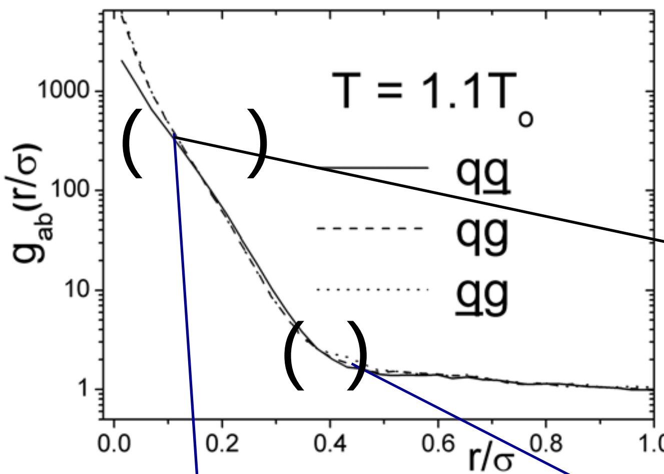
$$\sum_{ab}{}^c << \sum_{ab}{}^d \Rightarrow r^2 * \sum_{ab}{}^d >> r^2$$

Peak related to bound states at interparticle distances of order one Bohr radius exists if discrete bound states in electron-hole or hydrogen plasma are well populated (low temperatures and small densities)

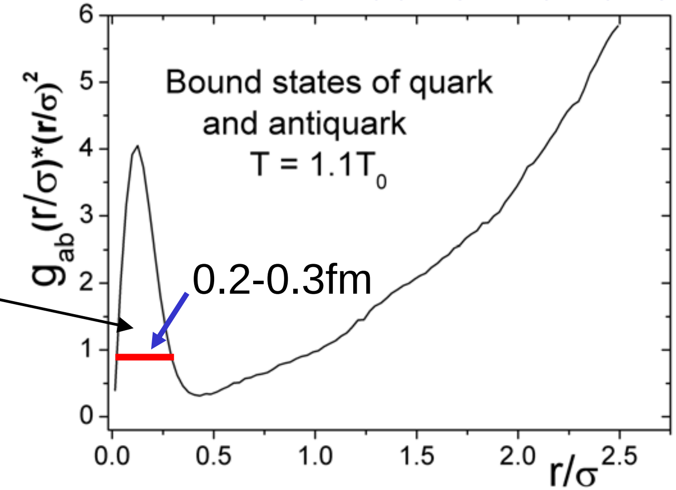
For low densities it is reasonable to choose $E' > -1/\beta$ while for high densities is appropriate $E' = -Ry/r_s$ since the quasiparticle in states with energy $E_\alpha > E'$ can be considered as free particles.

Color bound states and mean force potential ($T = 1.1\,T_c$)

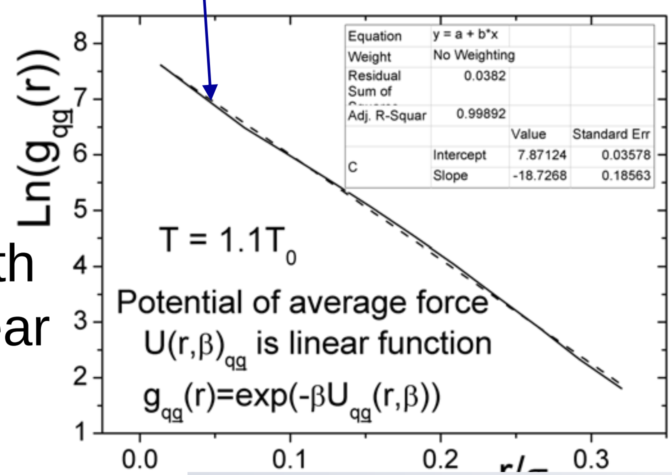
Distribution functions



r^2 * Distribution function

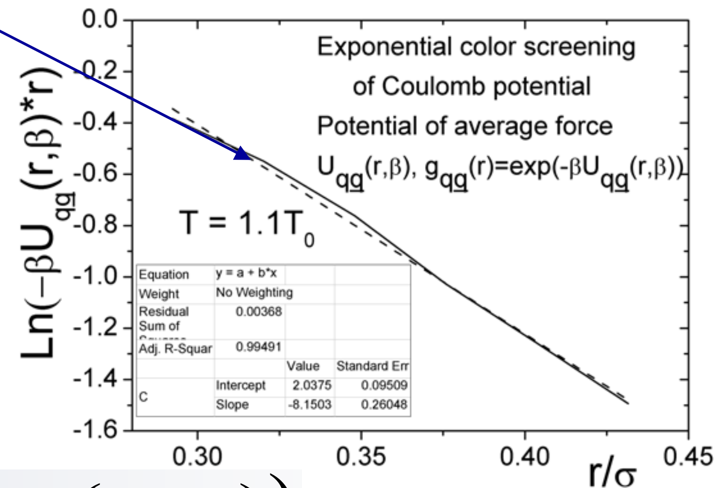


Linear part of the mean force potential



Depth U > 1 GeV agree with lattice near T_c

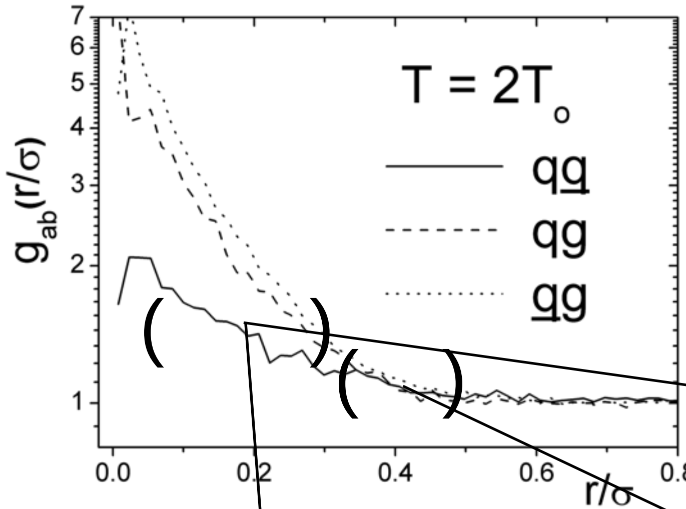
Color screening part of mean force potential



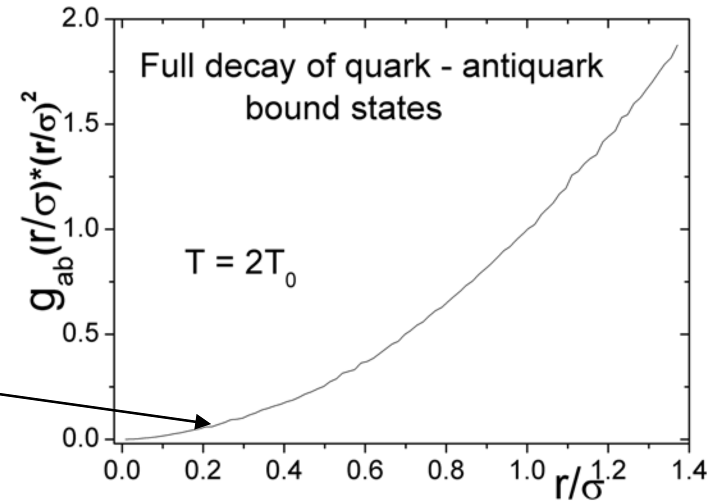
$$g_{q\bar{q}}(r) = \exp\left(-\beta U_{q\bar{q}}(r,\beta)\right)$$

Decay of color bound states and mean force potential ($T = 2T_c$)

Distribution functions

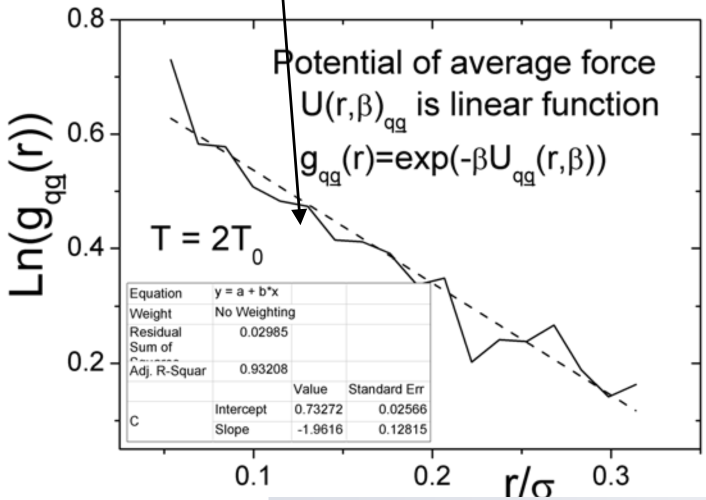


r^2 * Distribution function

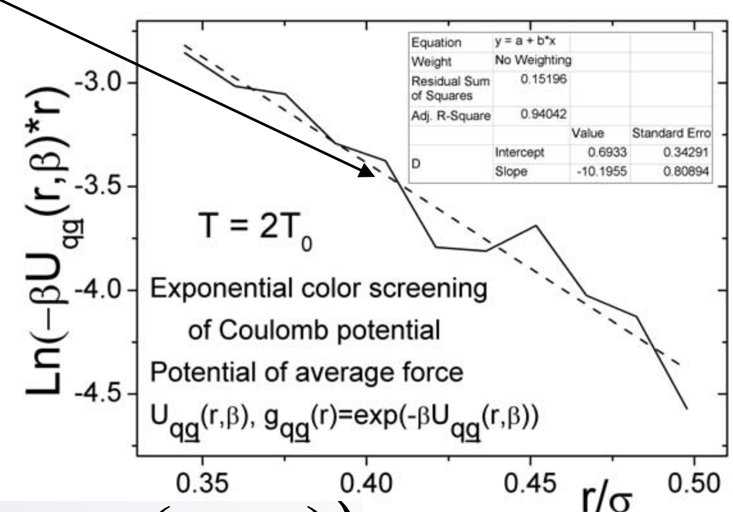


Linear part of the mean force potential

Depth~ 175 MeV



Color screening part of mean force potential



$$g_{q\bar{q}}(r) = \exp\left(-\beta U_{q\bar{q}}(r,\beta)\right)$$

Kinetic properties of quark – gluon plasma in canonical ensemble

$$C_{FA}(t) = Z^{-1} Tr\{F \exp(i\frac{Ht_c}{h}) A \exp(-i\frac{Ht_c}{h})\};$$

$$H = K + V(qQ), t_c = t - i\frac{\beta h}{2}, \beta = \frac{1}{kT},$$

$$Z = Tr\{\exp(-\beta H)\}$$

$$C_{FA}(t) = \frac{1}{(2\pi h)^{2\nu}} \iint dQ_1 dp_1 dq_1 dp_2 dq_2 F(p_1, q_1) A(p_2, q_2) \times$$

$$W(p_1, q_1, Q_1; p_2, q_2, Q_1; t; i\beta h),$$

In this model we use approximation

$$\delta(Q_1 - Q_1^{'})\delta(Q_2 - Q_2^{'})\delta(Q_1 - Q_2)$$

$$A(p,q) = \iint d\xi \exp(-i\frac{p\xi}{h}) < q - \frac{\xi}{2} | A | q + \frac{\xi}{2} >$$

Weil symbols of operators

$$W(p_1, q_1, Q_1; p_2, q_2, Q_1; t; i\beta h) = Z^{-1} \iint d\xi_1 d\xi_2 \exp(i\frac{p_1\xi_1}{h}) \exp(i\frac{p_2\xi_2}{h}) \times$$

$$< q_1 + \frac{\xi_1}{2} | \exp(i\frac{Ht_c}{h}) | q_2 - \frac{\xi_2}{2} >< q_2 + \frac{\xi_2}{2} | \exp(-i\frac{Ht_c}{h}) | q_1 - \frac{\xi_1}{2} >$$

Integral equation

$$W(p_1,q_1,Q_1;p_2,q_2,Q_1;t;i\beta h)=\bar{W}(p^0{}_1,q^0{}_1,Q_1^0;p^0{}_2,q^0{}_2,Q_2^0;0;i\beta h)+$$

$$+\int_0^t d\tau\iint ds\iint d\eta W(p_1^\tau-s,q_1^\tau,Q_1^\tau;p_2^\tau-\eta,q_2^\tau,Q_2^\tau;\tau;i\beta h)\gamma(s,q_1^\tau,Q_1^\tau;\eta,q_2^\tau,Q_2^\tau),$$

$$\gamma(s,q_1^\tau,Q_1^\tau;\eta,q_2^\tau,Q_2^\tau)=\frac{1}{2}\{\omega(s,q_1^\tau,Q_1^\tau)\delta(\eta)-\omega(\eta,q_2^\tau,Q_2^\tau)\delta(s)\},F(q,Q)=-\nabla_q V(q,Q)$$

$$\omega(\eta,q,Q)=\frac{4}{(2\pi h)^\nu h}\iint dq' V(q-q',Q)Sin(\frac{2sq'}{h})+F(q,Q)\bullet\frac{d\delta(s)}{ds}$$

Positive time direction

$$\frac{dq_1^t}{dt}=\frac{1}{2m}p_1^t,\frac{dp_1^t}{dt}=\frac{1}{2}F(q_1^t,Q_1^t),$$

$$\frac{dQ_{1,i}^{t,a}}{dt}=\frac{1}{2}\sum_{b,c}f^{abc}Q_{1,i}^b\nabla_{Q_{1,i}^c}V(q_1^t,Q_1^t),$$

Color dynamics in SU(2) or SU(3)

$$p_1^t(t,p_1,q_1,Q_1)=p_1,q_1^t(t,p_1,q_1,Q_1)=q_1,Q_1^t(t,p_1,q_1,Q_1)=Q_1$$

Initial conditions

$$\frac{dq_2^t}{dt}=-\frac{1}{2m}p_2^t,\frac{dp_2^t}{dt}=-\frac{1}{2}F(q_2^t,Q_2^t),$$

Hamiltonian eqations

$$\frac{dQ_{2,i}^{t,a}}{dt}=-\frac{1}{2}\sum_{b,c}f^{abc}Q_{2,i}^b\nabla_{Q_{2,i}^c}V(q_2^t,Q_2^t),$$

Negative time direction

$$p_2^t(t,p_2,q_2,Q_1)=p_2,q_2^t(t,p_2,q_2,Q_1)=q_2,Q_2^t(t,p_2,q_2,Q_1)=Q_1$$

Initial conditions

$$\exp(-\frac{\beta}{2}H) = \exp(-\Delta\beta H)\exp(-\Delta\beta H)\ldots\exp(-\Delta\beta H), \Delta\beta = \beta/2M, t=0$$

$$\exp(-\Delta\beta H) = \exp(-\Delta\beta K)\exp(-\Delta\beta V)\exp(-\Delta\beta^2[K,V]/2)\ldots,$$

$$\bar{W}(p_1,q_1,Q_1;p_2,q_2,Q_1;0;i\beta h) \approx \iint d\bar{q}_1 d\bar{q}_2\ldots d\bar{q}_M d\tilde{q}_1 d\tilde{q}_2\ldots d\tilde{q}_M \times$$

$$\Psi\{p_1,q_1,Q_1;p_2,q_2,Q_1;\bar{q}_1,\bar{q}_2\ldots\bar{q}_M;\tilde{q}_1,\tilde{q}_2\ldots\tilde{q}_M;i\beta h\},$$

$$\Psi\{p_1,q_1,Q_1;p_2,q_2,Q_1;\bar{q}_1,\bar{q}_2\ldots\bar{q}_M;\tilde{q}_1,\tilde{q}_2\ldots\tilde{q}_M;i\beta h\} =$$

$$Z^{-1} < q_1|\exp(-\Delta\beta K)|\bar{q}_1 > \exp(-\Delta\beta V(\bar{q}_1,\bar{Q}_1)) < \bar{q}_1|\exp(-\Delta\beta K)|\bar{q}_2 >$$

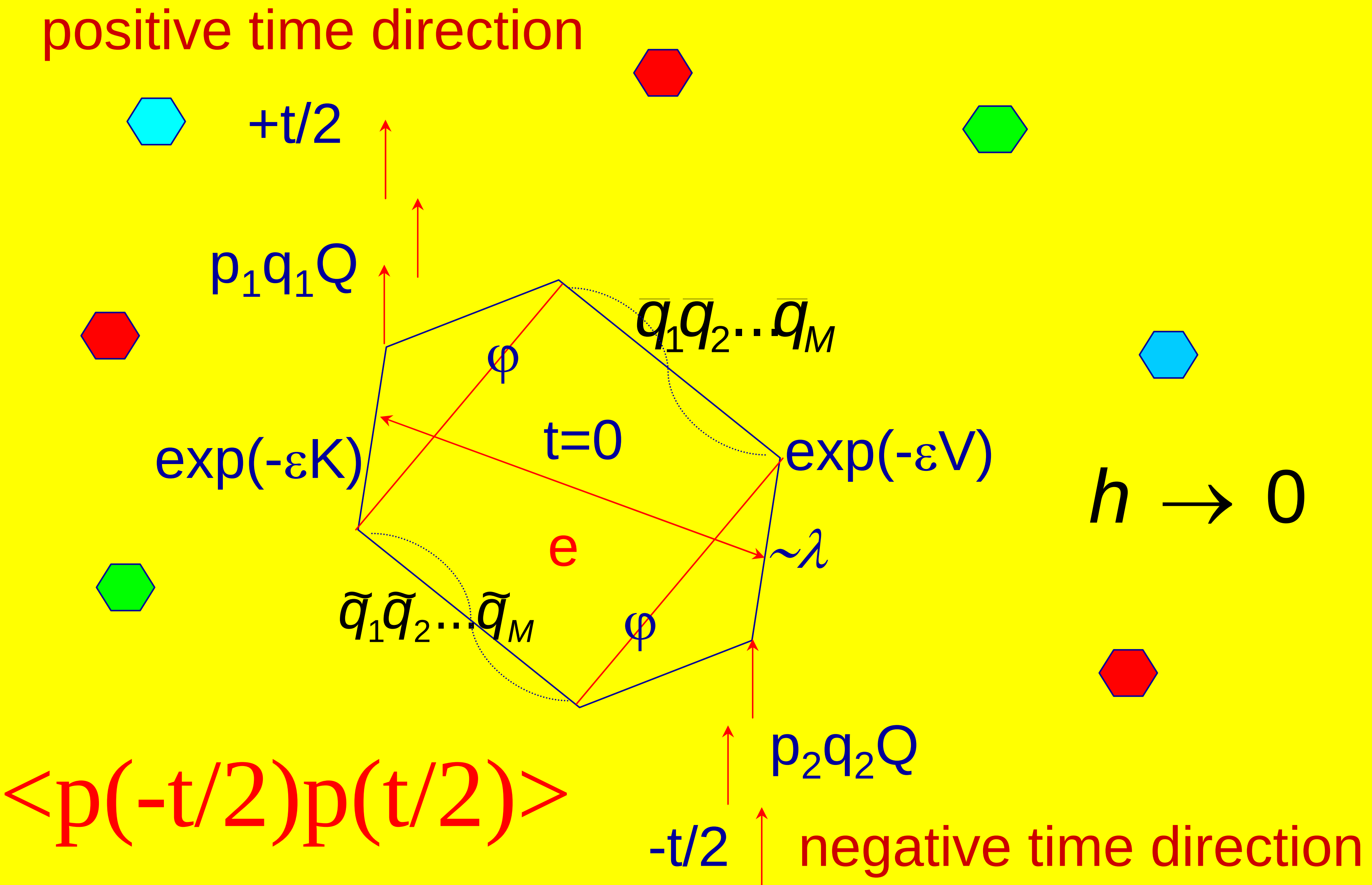
$$\exp(-\Delta\beta V(\bar{q}_2,\bar{Q}_1))\ldots\exp(-\Delta\beta V(\bar{q}_M,\bar{Q}_1)) < \bar{q}_M|\exp(-\Delta\beta K)|q_2 > \phi(p_2,\bar{q}_M,\tilde{q}_1)\times$$

$$< q_2|\exp(-\Delta\beta K)|\tilde{q}_1 > \exp(-\Delta\beta V(\tilde{q}_1,\bar{Q}_1)) < \tilde{q}_1|\exp(-\Delta\beta K)|\tilde{q}_2 >$$

$$\exp(-\Delta\beta V(\tilde{q}_2,\bar{Q}_1))\ldots\exp(-\Delta\beta V(\tilde{q}_M,\bar{Q}_1)) < \tilde{q}_M|\exp(-\Delta\beta K)|q_1 > \phi(p_1,\tilde{q}_M,\bar{q}_1)$$

$$\phi(p,\bar{q},\tilde{q}) = \lambda^{\nu}\exp\left(\frac{< \frac{p\lambda}{h} + i\pi\frac{\bar{q}-\tilde{q}}{\lambda} | \frac{p\lambda}{h} + i\pi\frac{\bar{q}-\tilde{q}}{\lambda} >}{2\pi}\right), \lambda^2 = \frac{2\pi h^2\beta}{2Mm},$$

Schematic snapshot for color phase space dynamics

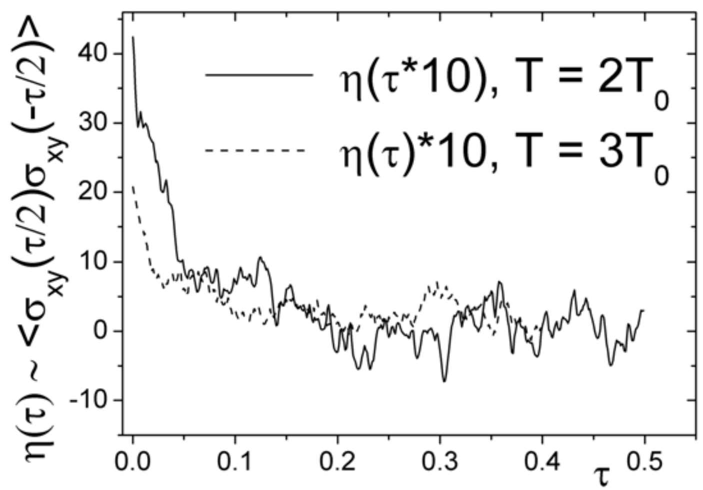


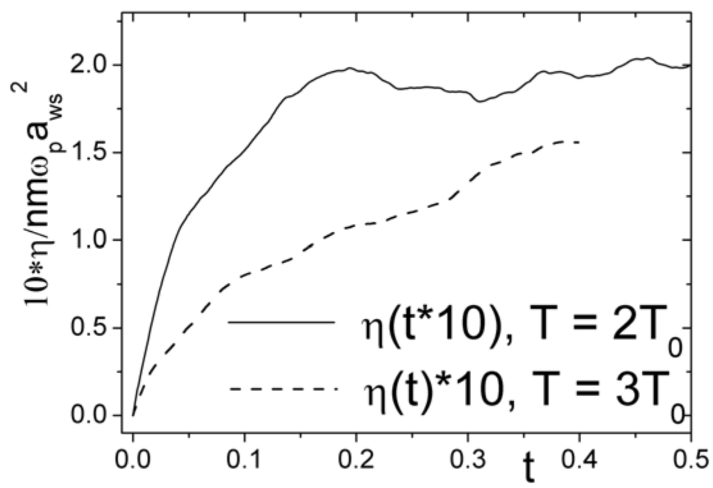
Time autocorrelation function of the stress energy tensor and shear viscosity of quark –gluon plasma

$$\eta(\tau) = \frac{n}{3k_B T}\left\langle \sum_{X<Y} \sigma_{XY}(\tau/2)\sigma_{XY}(-\tau/2) \right\rangle$$

$$\sigma_{XY}(\tau) = \frac{1}{N}\left(\sum_{i=1}^{N} m_i v_{ix} v_{iy} + \frac{1}{2}\sum_{i \neq j} r_{ij,x} F_{ij,y} \right)$$

$$\eta = \lim_{t\to\infty} \eta(t) = \lim_{t\to\infty} \int_0^t d\tau \eta(\tau)$$



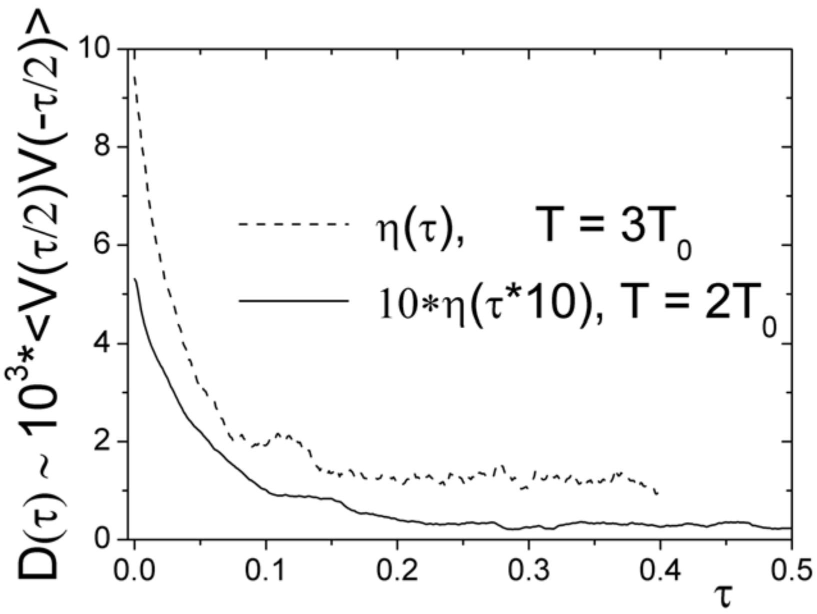


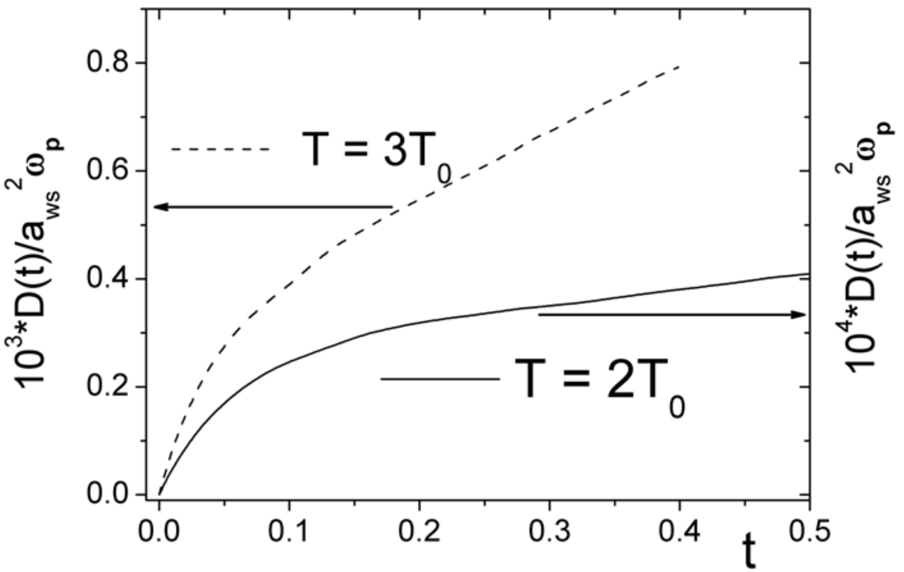
Velocity autocorrelation function and diffusion constant QGP

$$D(\tau) = \langle v(\tau/2)v(-\tau/2)\rangle =$$

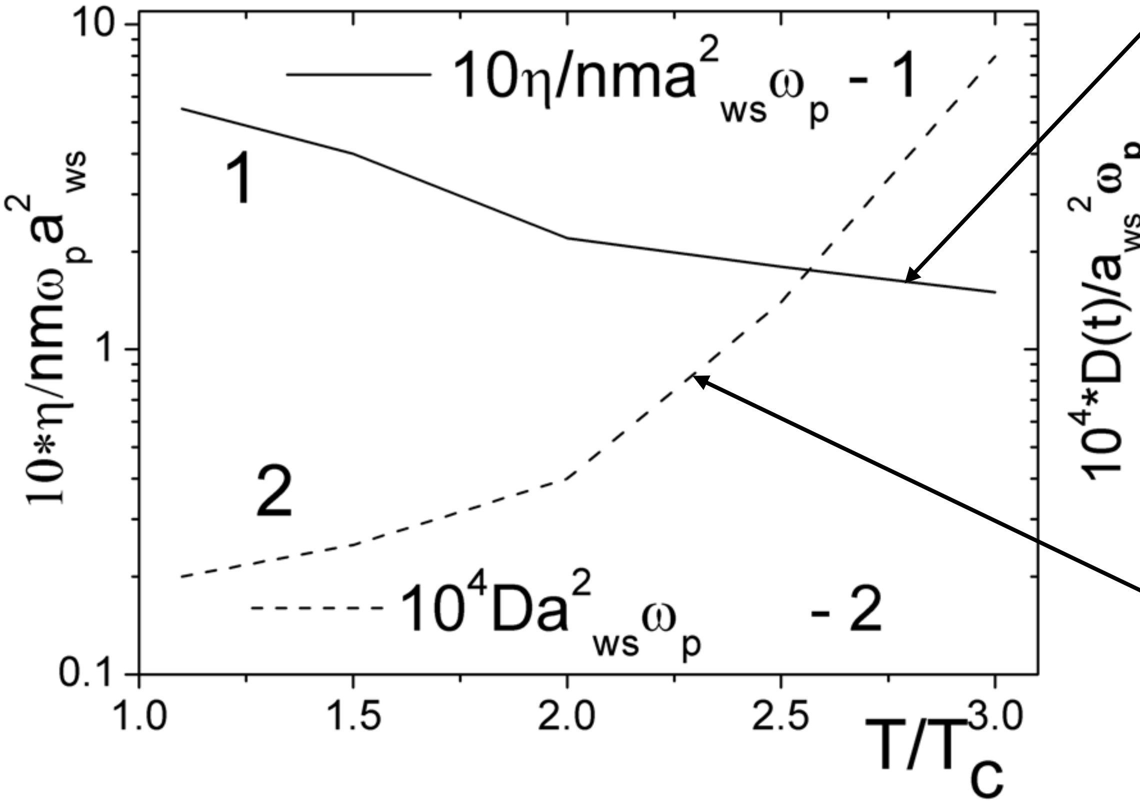
$$= \frac{1}{3N}\langle \sum_{i=1}^{N} \vec{v}_i(\tau/2) \bullet \vec{v}_i(-\tau/2) \rangle$$

$$D = \lim_{t\to\infty} D(t) = \lim_{t\to\infty} \int_0^t d\tau D(\tau)$$





Diffusion coefficient and shear viscosity



Shear viscosity agrees with Gelman et al., 2006

Diffusion coefficient is ~10^3 lower in comparison with Gelman et al., 2006

Electromagnetic plasma
Crystallization of protons

HYDROGEN, PIMC-SIMULATION,

$n = 10^{25}$ cm^{-3}, T=10 000 K^{o}

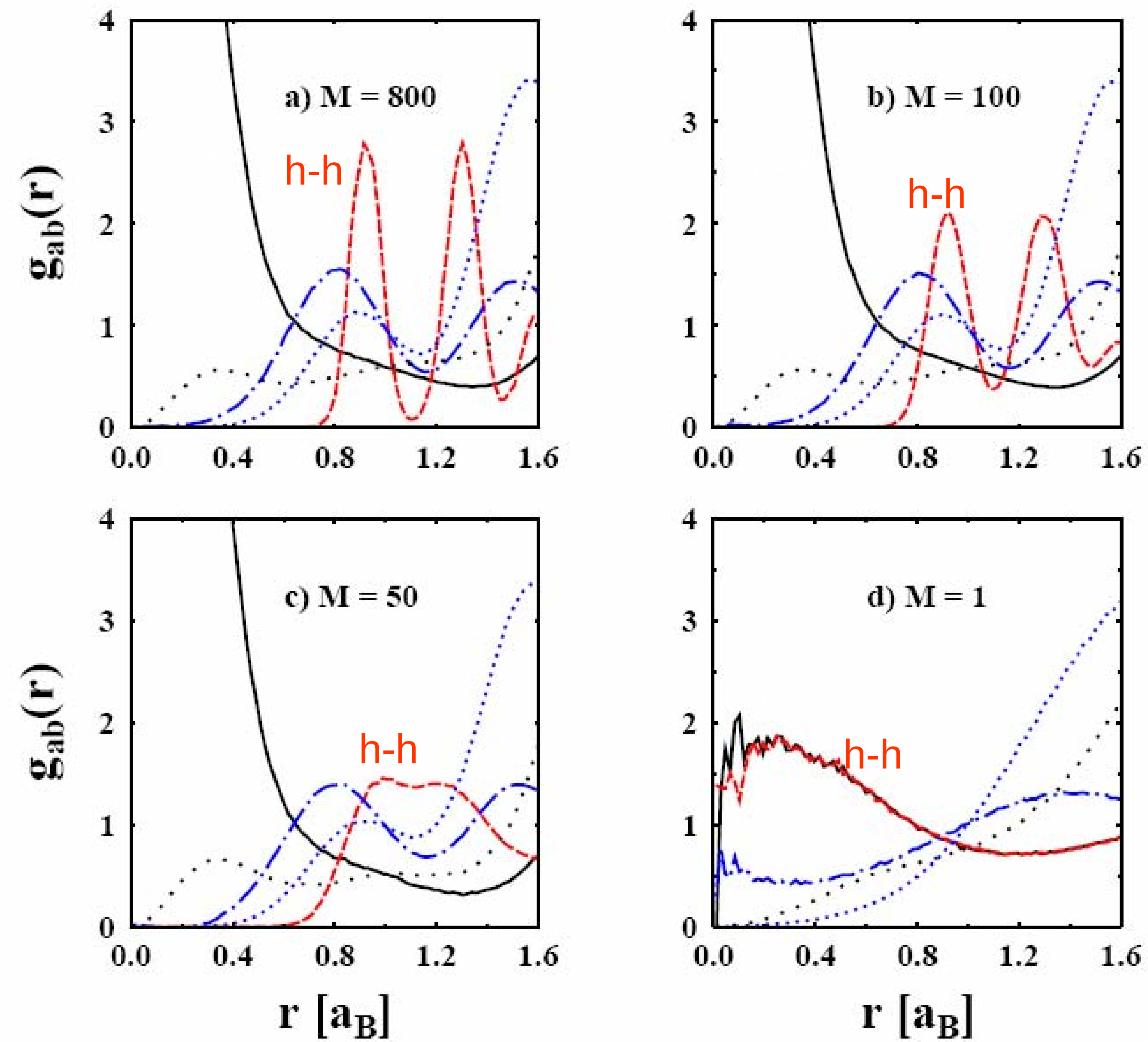
Bloch oscillation of electron density in periodic potential

HOLE CRYSTALLIZATION AND QUANTUM MELTING

$\langle r \rangle / a_B = 0.63$

$T = 0.064 E_b$

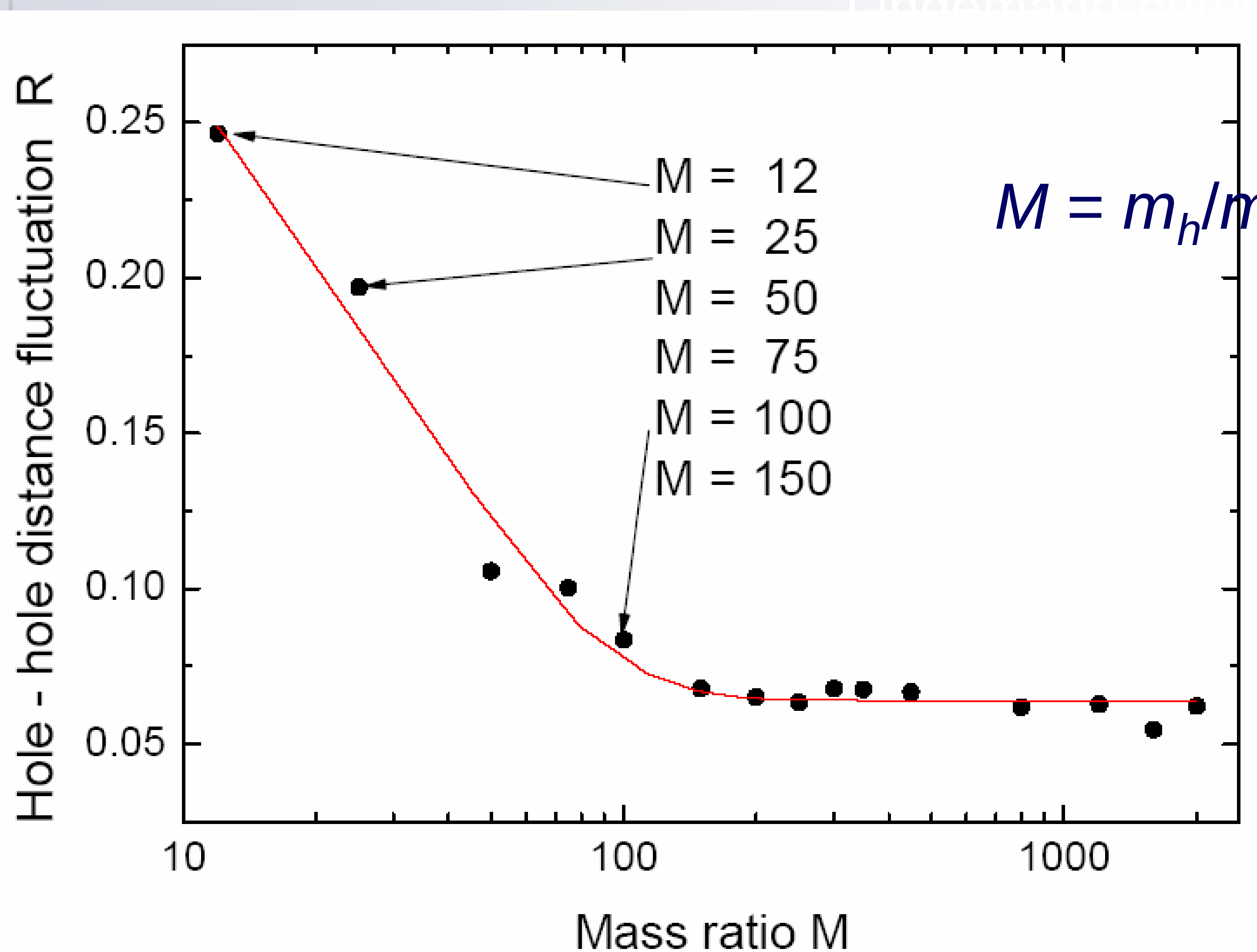
$M = m_h / m_e$



e-e
h-h
e-h
(e-e) · r^2
(e-h) · r^2

QUANTUM MELTING

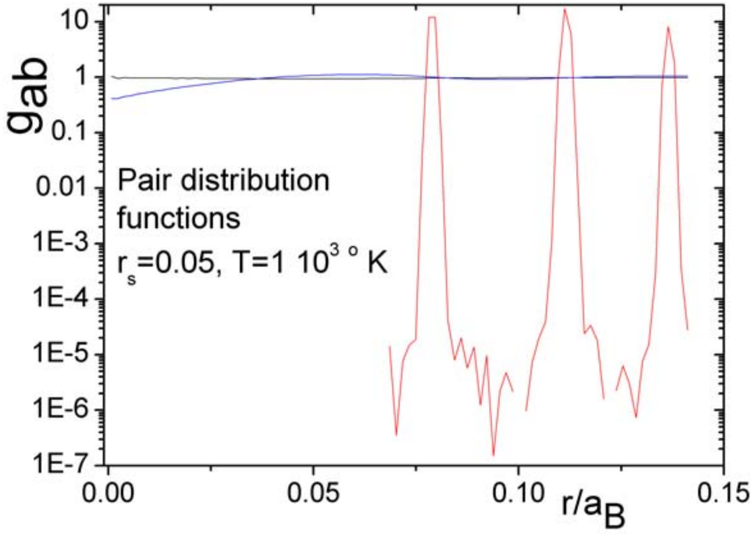
HOLE-HOLE DISTANCE FLUCTUATIONS

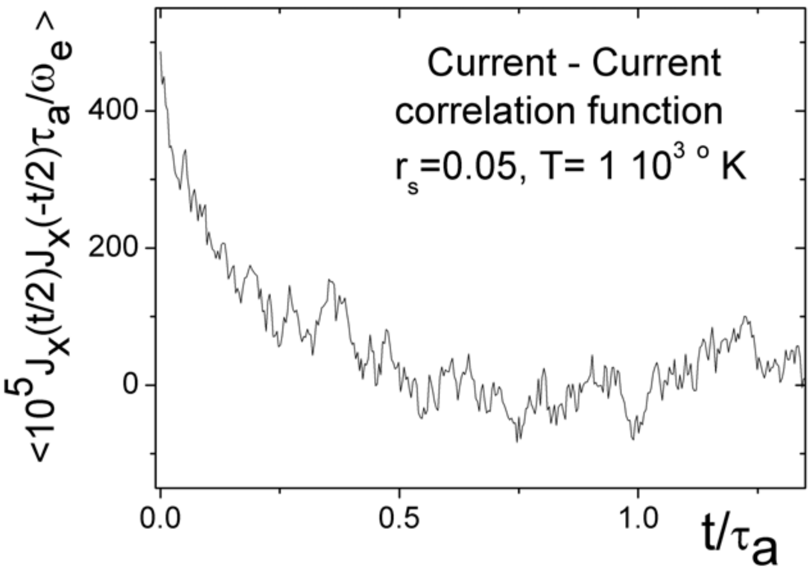


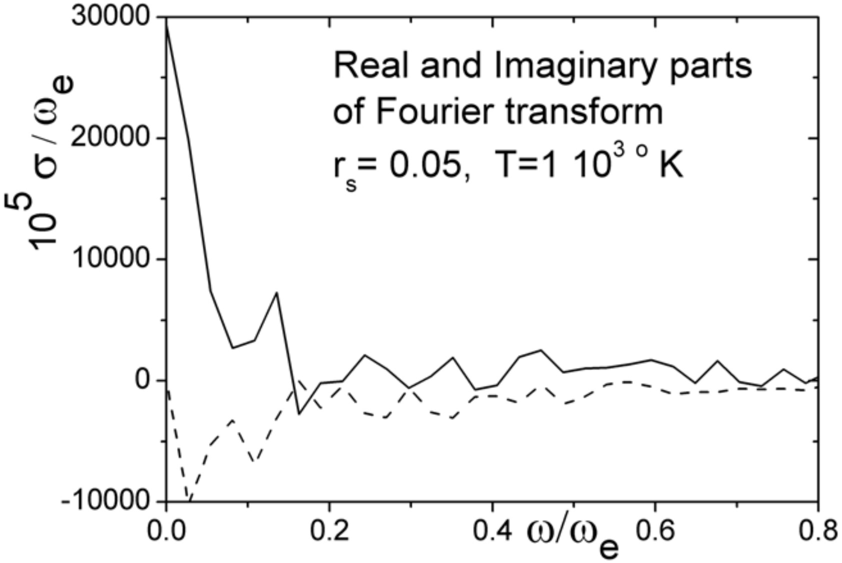
$M = m_h/m_e$

Correlation functions and transport coefficients EMP

T = 10 000º K

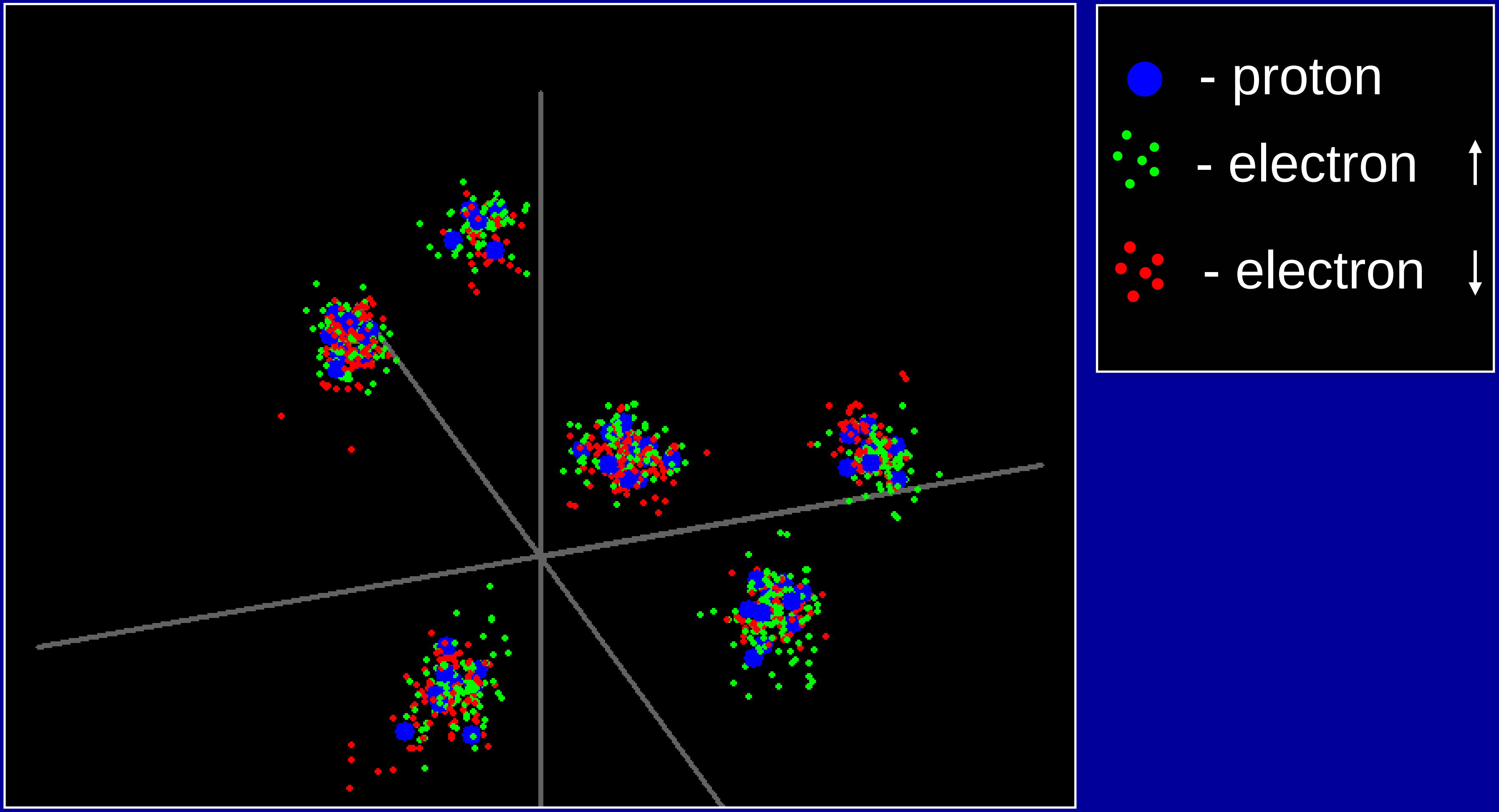






Phase transition to metallic state
Metallic drops and many particle clusters in hydrogen plasma

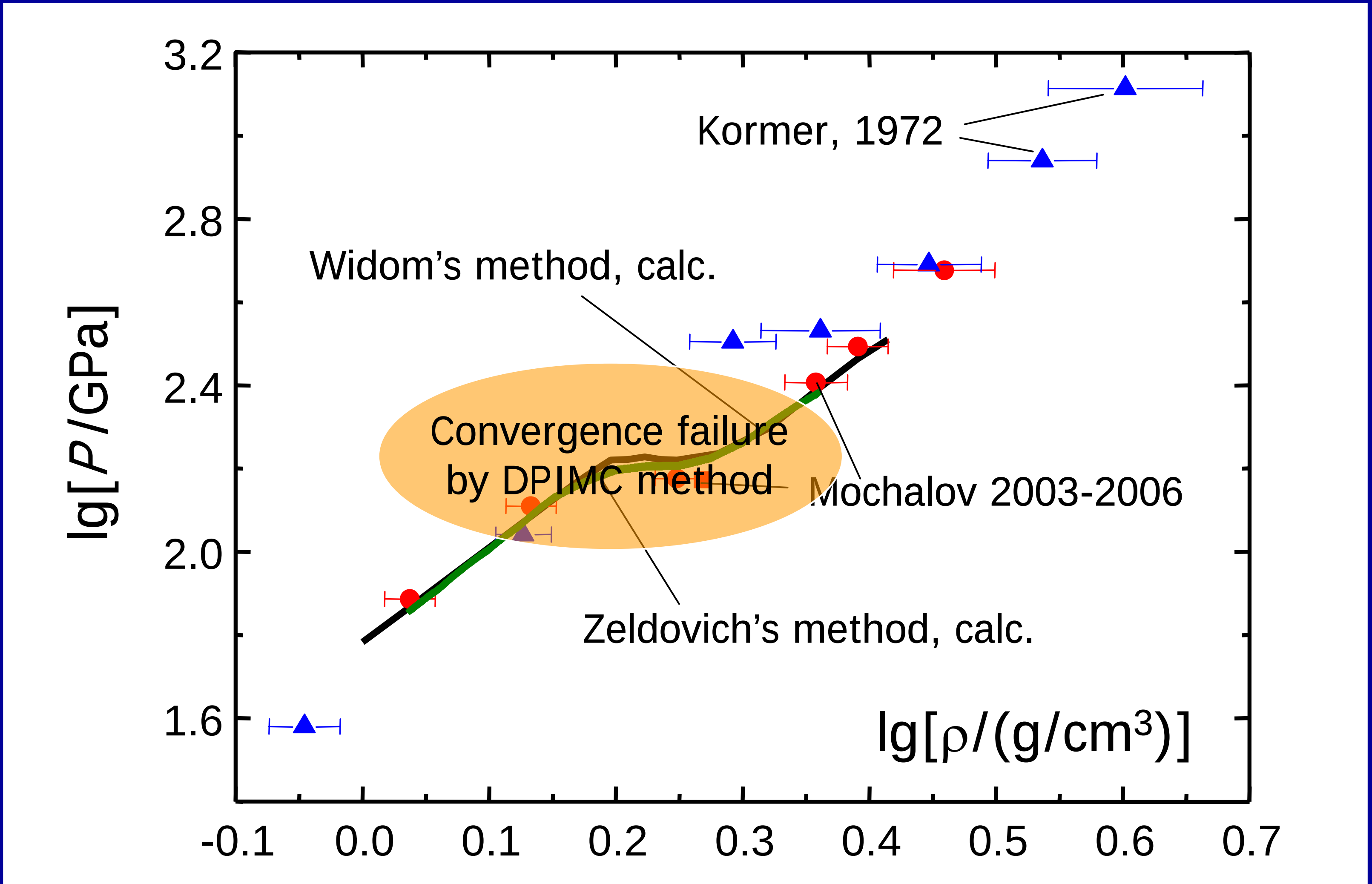
3D quantum two-component plasma.



T = 10000 K, n = 10^{22} cm^{-3}, ρ = 0.0167 g/cm^3

GASEOUS DEUTERIUM QUASI-ISENTROPE

Kormer et al., 1972; Mochalov et al., 2006



CONCLUSIONS

- Path integral Monte Carlo is a reliable and very fast method of simulation thermodynamic properties in a wide range of plasma parameters
- Fast quantum dynamics can be constructed on the basis of Feynman and Wigner formulation of quantum mechanics
- The developed numerical approach can be applied to consideration of EM and QG plasmas.
- Results of simulations agree with available theoretical and experimental data.

Thank you for attention.

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