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On the enhancement of characteristic x-ray emission from a target covered with spherical clusters irradiated by a femtosecond laser pulse

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Scope

- We consider the model of hot electrons generation according to vacuum heating mechanism for calculations of *Kα* yield from a massive targets with a flat and covered with spherical clusters surfaces
- Comparison of model calculations and experimental results on *Kα* yield

Analytical model of vacuum heating of electrons

- **F. Brunel**. Not-So-Resonant, Resonant Absorption // Phys. Rev. Lett. **59** (1987) 52.

Driving electric field

$$E_{\text{ext}} = E_0 \sin \omega t$$

The surface density of absorbed energy during one cycle

$$W_a = \eta N \frac{m\upsilon_{os}^2}{2}, \quad \upsilon_{os} = \frac{eE_0}{m\omega}, \quad N = \frac{E_0}{4\pi e}, \quad \eta = 1.57$$

An average electron energy

$$\varepsilon_e = \frac{\eta}{\gamma} \frac{m\upsilon_{os}^2}{2}, \quad \gamma = 0.7715$$

Electron energy distribution

$$f_h(E) = \frac{1}{\sqrt{\pi E T_h}} \exp\left(-\frac{E}{T_h}\right) \qquad \varepsilon_e = \frac{T_h}{2}$$

Connection between E_0 and laser field E_L

$$E_0 = \alpha |E_L| \sin\theta, \quad \alpha = 1 + \sqrt{1 - f}, \quad f = \frac{W_a \omega}{2\pi I_i}, \quad I_i = \frac{cE_L^2}{8\pi} \cos\theta, \quad a_L \alpha \sin\theta \max\{\sin\theta, \cos\theta\} << 1$$

Conditions of the model applicability

Nonrelativistic intensity

$$\frac{a_L^2}{2} << 1, \quad a_L = \frac{eE_L}{m\omega c}$$

Real reflection coefficient

$$\beta = \frac{a_L \eta}{2\pi} \frac{\sin^3\theta}{\cos\theta} < 1$$

The density scale length

$$L_c << \frac{\upsilon_{os}}{\omega}$$

K_α yield calculation

- **Ch. Reich, P. Gibbon, I. Uschmann, E. Forster.** Yield Optimization and Time Structure of Femtosecond Laser Plasma $K\alpha$ Sources // Phys. Rev. Lett. **84** (2000) 4846.

K_α yield

$$n_\alpha = n_h \int f_h(E) N_g(E) f_e(E) dE$$

The number of K_α photons generated by an electron of energy E (in keV)

$$N_g(E) = 4\times 10^{-3} Z^{-1.67} E_{\mathrm{keV}}^{3/2}$$

The fraction of these photons that escapes from the solid

$$f_e = \theta(l_\gamma - l_e)$$

The hot electron flux

$$S_h = \frac{dn_h}{dtds} = \gamma N \frac{\omega}{2\pi}$$

Laser pulse intensity

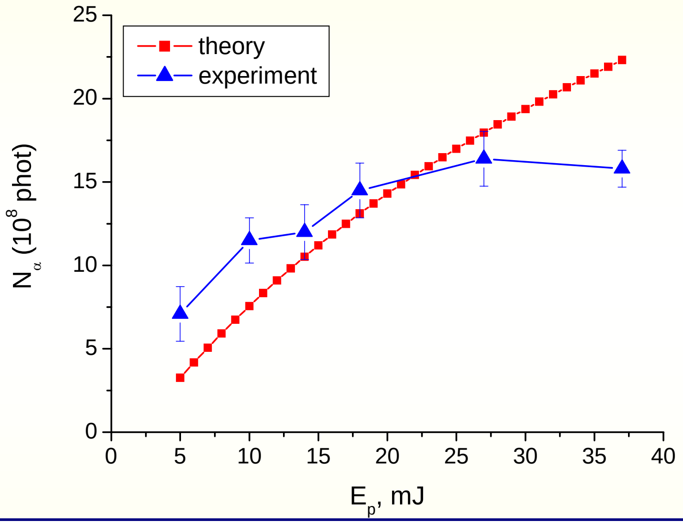
$$I_L(r,t) = I_0 \exp\left(-\frac{r^2}{r_0^2}\right)\exp\left(-\frac{t^2}{\tau^2}\right), \qquad \tau = \frac{\tau_p}{2\sqrt{\ln 2}}, \quad I_0 = \frac{E_p}{\pi^{3/2} r_0^2 \tau}, \qquad \tan\theta << k_0 r_0$$

The total number of K_α photons produced by the laser pulse

$$N_\alpha = \int ds \int dt S_h \int_{I_k}^{\infty} f_h(E) N_g(E) f_e(E) dE = \frac{0.525}{\cos\theta}\frac{Z^{2.73}}{\lambda_{\mu m}^2} \int r dr_{\mu m} dt_{fs} \int_1^{20} u \exp\left(-\frac{E_k u}{T_h(I_L(r,t))}\right) du$$

Comparison of the results of calculations and measurements

- [1] **Kostenko O.F., Andreev N.E.** Characteristic x rays caused by vacuum heating of electrons // Physics of Extreme States of Matter – 2009. IPCP RAS, Chernogolovka. P. 42.
- [2] **M.B. Agranat, N.E. Andreev, S.I. Ashitkov, A.V. Ovchinnikov, D.S. Sitnikov, V.E. Fortov, A.P. Shevel’ko.** Generation of Characteristic X Rays by a Terawatt Femtosecond Chromium-Forsterite Laser // JETP Letters **83** (2006) 72.



The K_α yield from a massive iron target vs laser pulse energy (τ_p = 80 fs, λ = 1.24 μm, θ = 45°, focal spot diameter 10 μm (FWHM)). Squares – theory [1], triangles – measurements [2] (per one shot in 2π sr). Maximum intensity - 4×10^{17} W/cm^2

There is a satisfactory agreement in the range of intensities $I_0 = (0.5\div3)\times10^{17}$ W/cm^2 for fixed angle of incidence $\theta = 45°$. According to [2], $L_c \approx 10$ nm in the moment of maximum intensity 10^{17} W/cm^2. Then, $\chi = \upsilon_{os}/(\omega L_c) > 8$.

According to Brunel, intensity absorbed by vacuum heating mechanism is greater than the resonance absorption one if $\chi > 2\pi/\eta = 4$. **This indicates determinative role of vacuum heating mechanism in the discussed range of parameters.**

Accelerating electric field at the spherical cluster surface

- Electric field outside the cluster: $\boldsymbol{E} = \boldsymbol{E}_{\mathrm{i}} + \boldsymbol{E}_{\mathrm{s}}$
- The field of a scattered wave is described by real reflection coefficient *r*: $\boldsymbol{E}_{\mathrm{s}} = r\boldsymbol{E}_{\mathrm{s}0}$,

  with $\boldsymbol{E}_{\mathrm{s}0}$ being the electric field of the wave scattered by perfectly conducting sphere.
- The incident laser field: $\boldsymbol{E}_{\mathrm{i}} = \boldsymbol{e}_x E_{\mathrm{L}} \exp\left(\mathrm{i}\, k_0 z - \mathrm{i}\,\omega t\right)$

➢ Then the radial component of electric field at the cluster surface:

$$E_0\left(r,\rho,\theta_1,\varphi_1\right) = -E_{\mathrm{L}}\, \mathrm{e}^{-\mathrm{i}\omega t} \frac{\cos\varphi_1}{\rho} \sum_{n=1}^{\infty} \mathrm{i}^{n+1} \left(2n+1\right) P_n^1\left(\cos\theta_1\right) \left[j_n\left(\rho\right) + r b_n^{\mathrm{r}} h_n^{(1)}\left(\rho\right) \right]$$

Here, $\boxed{\rho = k_0 R}$, with *R* being the cluster radius; $j_n\left(\rho\right)$ and $h_n^{(1)}\left(\rho\right)$ are spherical Bessel functions; $P_n^1\left(\cos\theta_1\right)$ is the associated Legendre function; θ_1 and φ_1 are spherical coordinates.

✓ Coefficients of the expansion of the field $\boldsymbol{E}_{\mathrm{s}0}$ in solenoidal vector spherical wave functions:

$$a_n^{\mathrm{r}} = -\frac{j_n\left(\rho\right)}{h_n^{(1)}\left(\rho\right)}, \qquad b_n^{\mathrm{r}} = -\frac{\left[\rho j_n\left(\rho\right)\right]'}{\left[\rho h_n^{(1)}\left(\rho\right)\right]'}$$

Laser power absorption

- At nonrelativistic intensities, the field E_0 can be considered as accelerating field in the context of 1D model, if

$$\frac{\upsilon_0}{\omega} = \frac{e\left|E_0\right|}{m\omega^2} << R$$

➢ Power absorbed by the cluster is equal to

$$P\left(r\right) = \frac{\omega}{2\pi}\int W_{\mathrm{a}} R^2 \sin\theta_1 \,\mathrm{d}\,\theta_1 \,\mathrm{d}\,\varphi_1$$

➢ On the other hand,

$$P\left(r\right) = -\frac{2\pi}{k_0^2} I_{\mathrm{L}} r \sum_{n=1}^{\infty} \left(2n+1\right)\left[\mathrm{Re}\left(a_n^{\mathrm{r}}\right) + \mathrm{Re}\left(b_n^{\mathrm{r}}\right) + r\left(\left|a_n^{\mathrm{r}}\right|^2 + \left|b_n^{\mathrm{r}}\right|^2\right)\right],$$

were I_{L} is the incident laser intensity.

✓ Reflection coefficient was evaluated from solution of above equations

The model of K_α photons generation

- The number of K_α photons generated in a substrate by the fast electrons from one cluster is equal to

$$n_{\mathrm{k}} = R^2 \int_0^{2\pi} \mathrm{d}\varphi_2 \int_0^{\pi/2} \sin\theta_2 \, \mathrm{d}\theta_2 \int S_{\mathrm{h}} \, \mathrm{d}t \int f_{\mathrm{h}}(E) N_{\mathrm{g}}(E) f_{\mathrm{e}}(E) \mathrm{d}E$$

Here, S_{h} is the hot electron flux directed to the cluster centre. The fraction of generated photons that escape from the substrate is approximated by a step function

$$f_{\mathrm{e}}(E) = \max\left[\mathrm{sgn}\left(\frac{20}{\sqrt{\cos\theta_2}} - \frac{E}{I_{\mathrm{k}}}\right), 0\right],$$

where I_{k} is the K-shell ionization energy, and θ_2 is the angle between the direction of fast electron propagation and the normal to the substrate surface.

- Azimuth angle φ_2 is measured from the axis which belongs to the incidence plane of *p*-polarized laser wave, so

$$\cos^2\varphi_1 = \frac{\left(\sin\theta\cos\theta_2 - \sin\theta_2\cos\varphi_2\cos\theta\right)^2}{1-\left(\sin\theta_2\cos\varphi_2\sin\theta + \cos\theta_2\cos\theta\right)^2}, \qquad \cos\theta_1 = -\left(\sin\theta_2\cos\varphi_2\sin\theta + \cos\theta_2\cos\theta\right),$$

where θ is the laser wave angle of incidence. Thus,

$$n_{\mathrm{k}} = 2.1\times10^{-3} Z^{2.73}\rho^2 \int \mathrm{d}t \int_0^{2\pi} \mathrm{d}\varphi_2 \int_0^{\pi/2} \mathrm{d}\theta_2 \,\frac{\sin\theta_2}{\alpha^2}\left[e^{-\alpha}(\alpha+1) - e^{-\alpha A}(\alpha A+1)\right],$$

where $\alpha = \dfrac{I_{\mathrm{k}}}{T_{\mathrm{h}}}$, $A = \dfrac{20}{\sqrt{\cos\theta_2}}$ and *t* is in femtoseconds.

- Taking into account a radial dependence of the laser intensity, we get the total number of K_α photons produced by the laser pulse

$$N_{\mathrm{k}} = \frac{2}{\cos\theta R^2}\int n_{\mathrm{k}} r \,\mathrm{d}r$$

For simplicity sake, we disregard interference of tightly arranged clusters and substrate.

Results

- The dependence of K_α yield from a copper substrate on the cluster size shows a sharp maximum at $\rho \approx 1$ (fig. 1).
- For $\rho \geq 1$, this enhancement is caused by increase of the accelerating field. For $\rho < 1$, strong absorption causes depletion of this field (fig. 2).

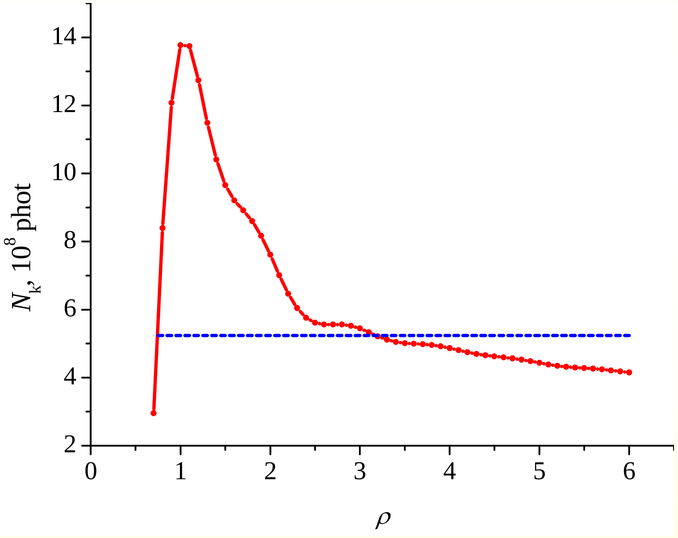


Fig. 1. The K_α yield from a copper substrate covered with clusters versus ρ (red solid line) and the K_α yield from a copper solid (blue dashed line) for laser pulse intensity $I_L = 2\times10^{17}$ W/cm^2 and the angle of incidence θ = 45°. Laser wave-length $\lambda = 0.4\ \mu$m, focal spot radius $r_0 = 5\ \mu$m, laser pulse duration $\tau_p = 40$ fs.

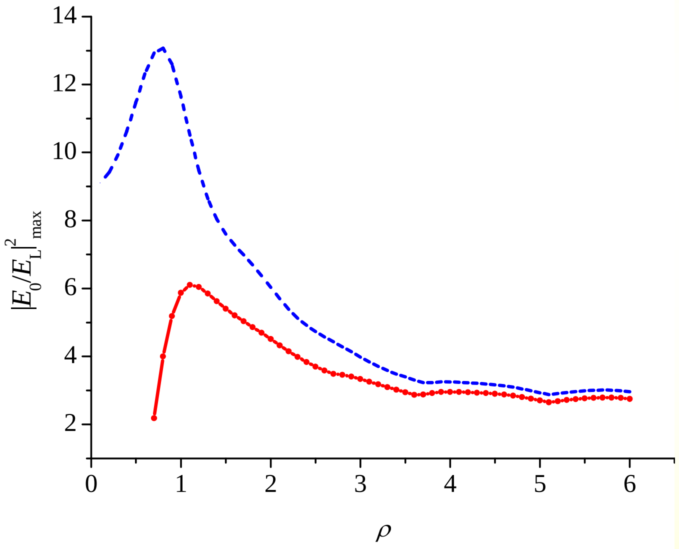


Fig. 2. Maximum value of $|E_0/E_L|^2$ versus ρ, allowing for absorption (red solid line) and neglecting absorption (blue dashed line). $I_L = 2\times10^{17}$ W/cm^2, λ = 0.4 μm.

- Growth of N_k with the increase of the laser pulse intensity from 7×10^{16} to 2×10^{17} W/cm^2, given $\rho = 1$, comes to 4 times (fig. 3).
- Increase of K_α yield with growth of the angle of incidence is the most pronounced for $\rho \approx 1$ (fig. 4).

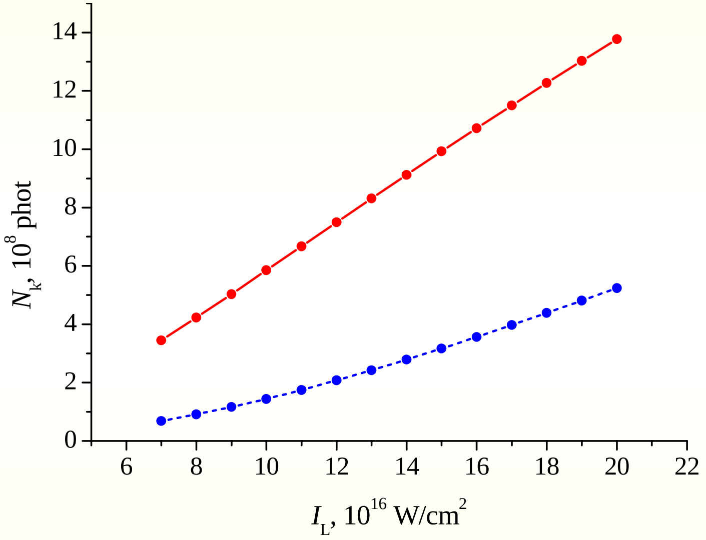


Fig. 3. The K_α yield from a copper target versus laser pulse intensity in the cases of the surface covered with clusters, given ρ = 1, (red solid line) and the flat target (blue dashed line). Laser pulse parameters as in fig. 1.

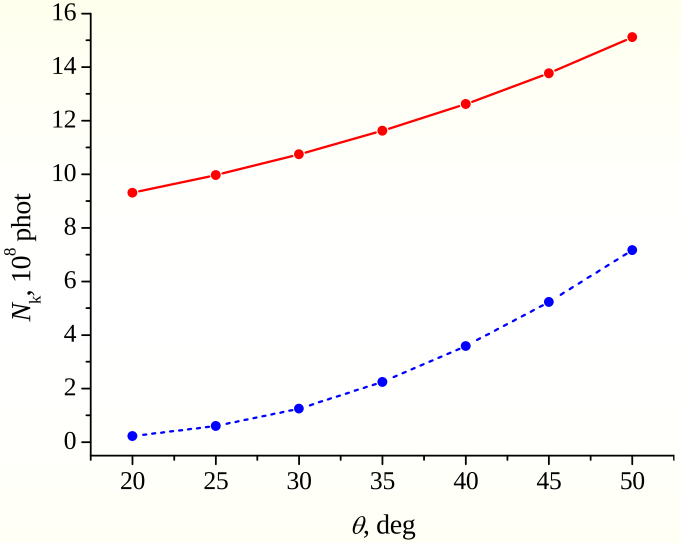


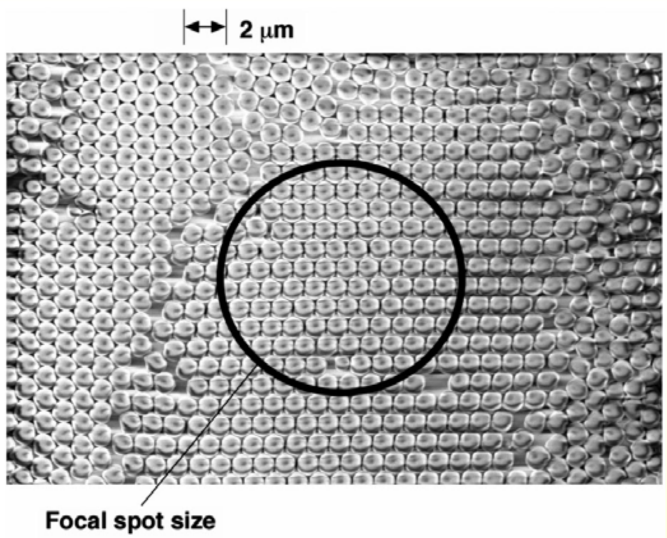
Fig. 4. The K_α yield from a copper target versus laser pulse angle of incidence in the cases of the surface covered with clusters, given ρ = 1, (red solid line) and the flat target (blue dashed line). Laser pulse parameters as in fig. 1.

The experiment [1]

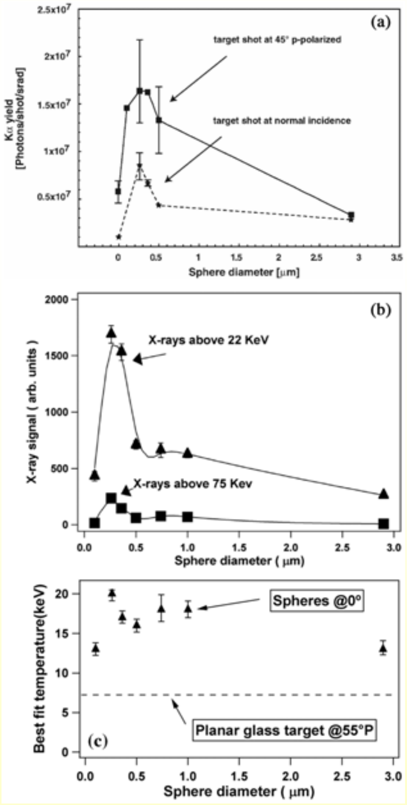
- Experimental investigation of x-ray production from intense short pulse laser irradiation of solid targets coated with dielectric spheres of well-defined sizes exhibited a peak in *Kα* yield when the spheres with diameter roughly half the laser wavelength were employed [1]. This effect was attributed to electric field enhancement at the surface of the clusters and multipass stochastic heating of fast electrons [1,2].

[1] Sumeruk H A *et al* 2007 *PRL* 98 045001

[2] Sumeruk H A *et al* 2007 *Phys. Plasmas* 14 062704



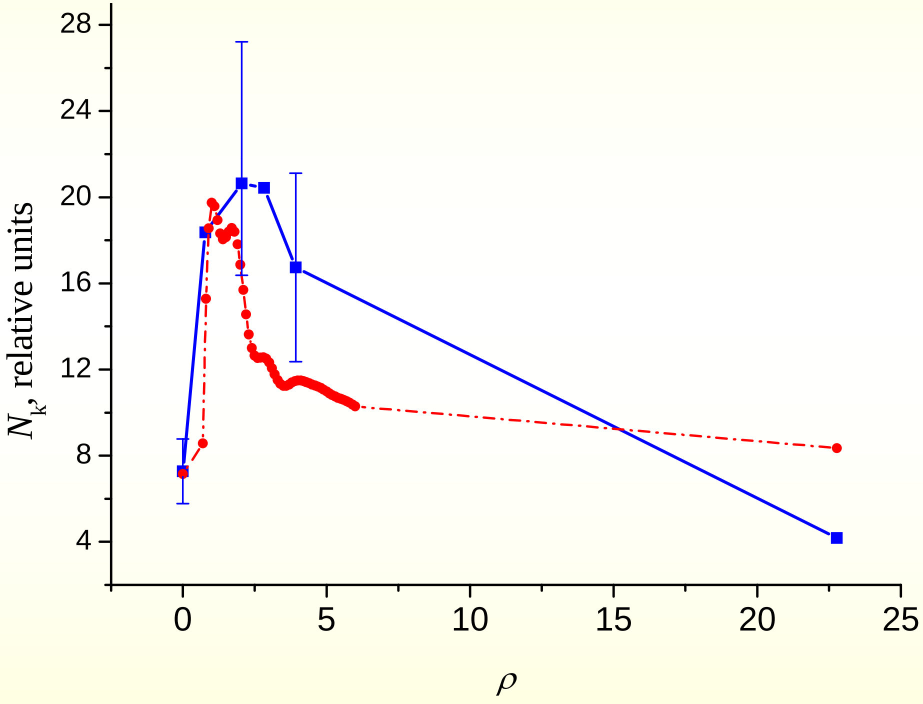
Scanning electron microscope image of 1 μm diameter polystyrene spheres arrayed on a Cu substrate.



a) Si K-alpha yield as a function of sphere size coated on Si wafers.
b) Hard x-ray yield through two different filters as a function of sphere size.
c) Hot electron temperature derived from hard x-ray spectra as a function of sphere size.

- Model calculations for Si wafer with I_L = 2×10^{17} W/cm², λ = 0.4 μm and θ = 45º exhibited a satisfactory agreement with the measurements [1] of the relative enhancement of K_α yield in the range of $\rho \approx 1 - 4$

[1] Sumeruk H A *et al* 2007 *PRL* 98 045001



The theory (red line) and the experiment [1] (blue line)

Conclusion

- Strong dependence of K_α yield on the cluster size is demonstrated by means of modelling that takes into account electric field structure at the cluster surface and its depletion due to laser power absorption by fast electrons, generated according to vacuum (Brunel) heating mechanism.

✓ Kostenko O F and Andreev N E 2010 *Physica Scripta* **81** 055505

Thank you for attention!