Trieste, July 5, 2023

Light clusters in hot, dense matter

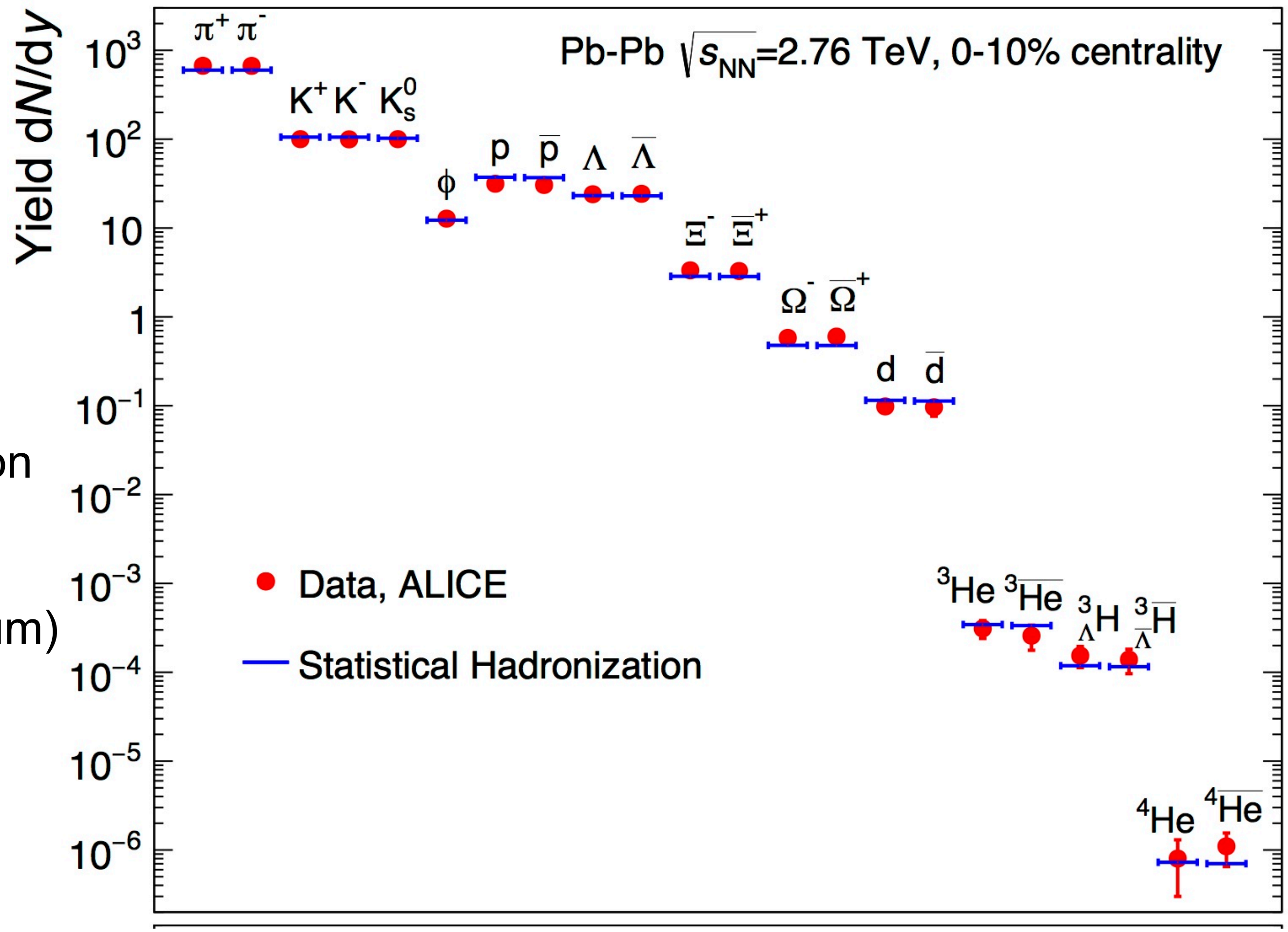
Gerd Röpke, University of Rostock, Germany

Cluster formation at LHC/CERN

ALICE@LHC

Excellent description
Of data by the
Statistical model
(chemical equilibrium)

T = 156 MeV



A. Andronic, P. Braun-Munziger, K. Redlich, J. Stachel, Nature 561, 321 (2018)

Questions

1. Is local thermodynamic equilibrium relevant? – Nonequilibrium process, formation of clusters/ correlations, transport codes for single-particle distribution, coalescence, QMD, FMD/AMD, freeze-out,…

2. Is the statistical equilibrium described by uncorrelated hadrons (statistical hadronization approach) correct?
   – role of multiparticle interactions, dense pion gas,
   medium modifications, continuum correlations, …
   Spectral function: finite life-time, background

Density effects?

The Beth-Uhlenbeck equation is identical

with the Dashen, Ma, Bernstein approach.

Proton anomaly and the Dashen, Ma, Bernstein S-matrix approach

thermal yield of an (interacting) resonance with mass M, spin J, and isospin I

$$\langle R_{I,J} \rangle = d_J \int_{m_{th}}^{\infty} dM \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\pi} B_{I,J}(M) \times \frac{1}{e^{(\sqrt{p^2+M^2}-\mu)/T}+1},$$

need to know derivatives of phase shifts with respect to invariant mass

$$B_{I,J}(M) = 2\,\frac{d\delta_J^I}{dM}.$$

A. Andronic, pbm, B. Friman, P.M. Lo, K. Redlich, J. Stachel, arXiv:1808.03102, Phys.Lett.B792 (2019)304

Problem: Nonequilibrium ?

State of the system in the past $\mathrm{Tr}\{\rho(t)B_n\} = \langle B_n \rangle^t$

Construction of the relevant statistical operator at time t

$$S_{\mathrm{rel}}(t) = -k_{\mathrm{B}}\,\mathrm{Tr}\{\rho_{\mathrm{rel}}(t)\log\rho_{\mathrm{rel}}(t)\}$$

-> maximum

$$\delta[\mathrm{Tr}\{\rho_{\mathrm{rel}}(t)\log\rho_{\mathrm{rel}}(t)\}] = 0$$

$$\mathrm{Tr}\{\rho_{\mathrm{rel}}(t)B_n\} \equiv \langle B_n \rangle^t_{\mathrm{rel}} = \langle B_n \rangle^t$$

Generalized Gibbs distribution

$$\rho_{\mathrm{rel}}(t) = \exp\left\{-\Phi(t) - \sum_n \lambda_n(t) B_n\right\}$$

$$\Phi(t) = \log \mathrm{Tr}\exp\left\{-\sum_n \lambda_n(t) B_n\right\}$$

$$\frac{\partial S_{\mathrm{rel}}(t)}{\partial t} = \sum_n \lambda_n(t) \langle \dot{B}_n \rangle^t$$

But: von Neumann equation?
Entropy?

Nonequilibrium statistical operator (NSO)

principle of weakening of initial correlations (Bogoliubov, Zubarev)

$$\rho_\epsilon(t) = \epsilon \int_{-\infty}^{t} e^{\epsilon(t_1 - t)} U(t, t_1) \rho_{\rm rel}(t_1) U^\dagger(t, t_1)\, dt_1$$

time evolution operator $U(t, t_0)$ relevant statistical operator $\rho_{\rm rel}(t)$

selection of the set of relevant observables $\{B_n\}$

self-consistency relations $\mathrm{Tr}\{\rho_{\rm rel}(t) B_n\} \equiv \langle B_n \rangle^t_{\rm rel} = \langle B_n \rangle^t$

maximum of information entropy $S_{\rm rel}(t) = -k_{\rm B}\, \mathrm{Tr}\{\rho_{\rm rel}(t) \log \rho_{\rm rel}(t)\}$

generalized Gibbs distribution $\rho_{\rm rel}(t) = \exp\left\{-\Phi(t) - \sum_n \lambda_n(t) B_n\right\}$

extended von Neumann equation

$$\frac{\partial}{\partial t} \varrho_\varepsilon(t) + \frac{i}{\hbar}\left[H, \varrho_\varepsilon(t)\right] = -\varepsilon\left(\varrho_\varepsilon(t) - \varrho_{\rm rel}(t)\right)$$

$\varrho(t) = \lim_{\varepsilon \to 0} \varrho_\varepsilon(t)$ after thermodynamic limit

Formation of light clusters in heavy ion reactions, transport codes

Wigner distribution

cluster mean-field potential

$$\partial_t f_X + \{\mathcal{U}_X, f_X\} = \mathcal{K}_X^{\text{gain}}\{f_N, f_d, f_t, \ldots\}(1 \pm f_X) - \mathcal{K}_X^{\text{loss}}\{f_N, f_d, f_t, \ldots\} f_X,$$

$$X = N, d, t, \ldots .$$

loss rate

in-medium
breakup transition operator

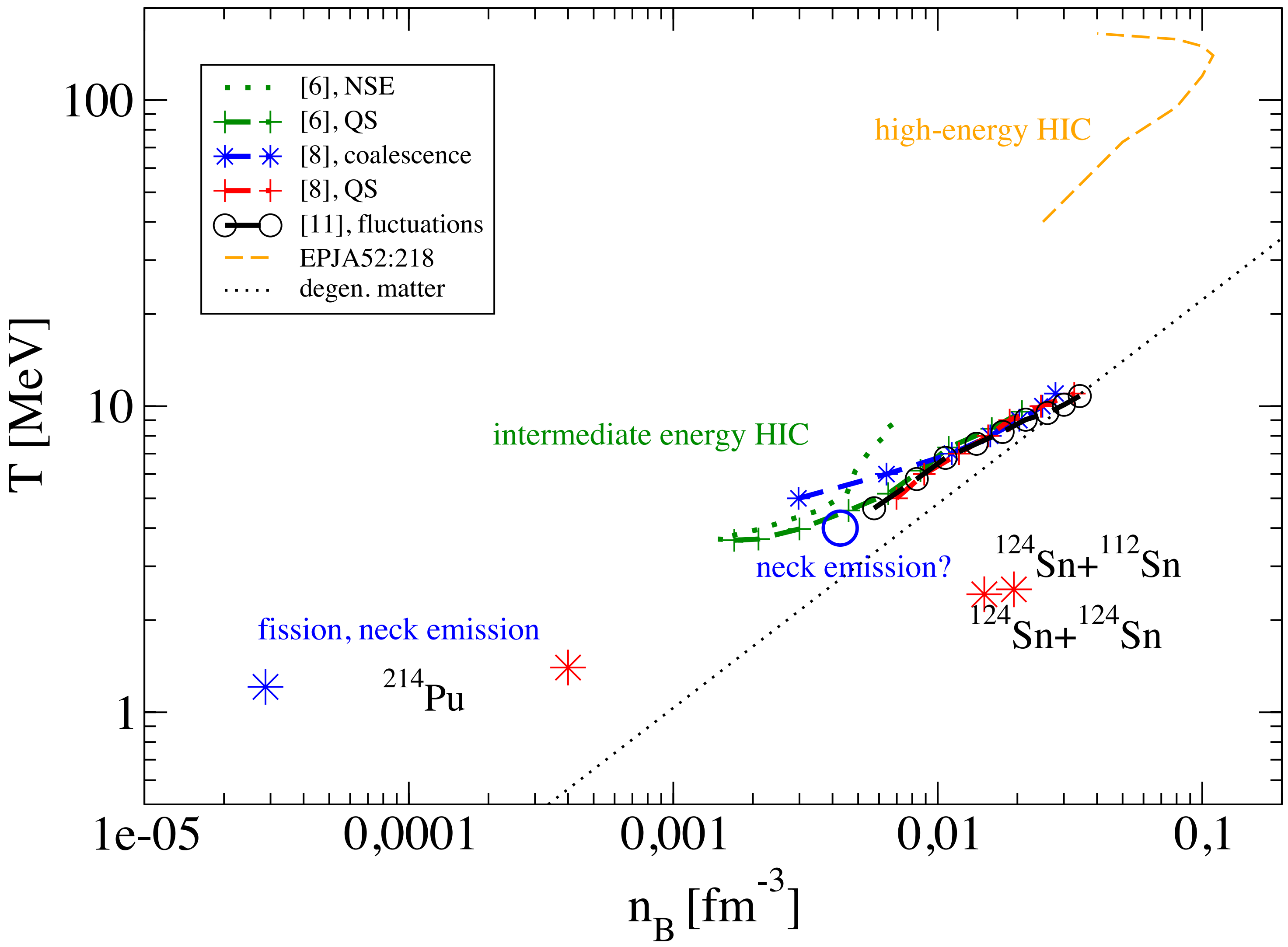
$$\mathcal{K}_d^{\text{loss}}(P,t) = \int d^3k \int d^3k_1\, d^3k_2\, d^3k_3 |\langle k_1 k_2 k_3 | U_0 | kP\rangle|^2_{dN \to pnN} \times f_N(k_1,t) f_N(k_2,t) f_N(k_3,t) f_N(k,t) + \cdots . \quad (3)$$

breakup cross section

$$\sigma_{\text{bu}}^0(E) = \frac{1}{|v_d - v_N|} \frac{1}{3!} \int d^3k_1\, d^3k_2\, d^3k_3 |\langle kP | U_0 | k_1 k_2 k_3\rangle|^2 \times 2\pi \delta(E' - E)(2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3), \quad (4)$$

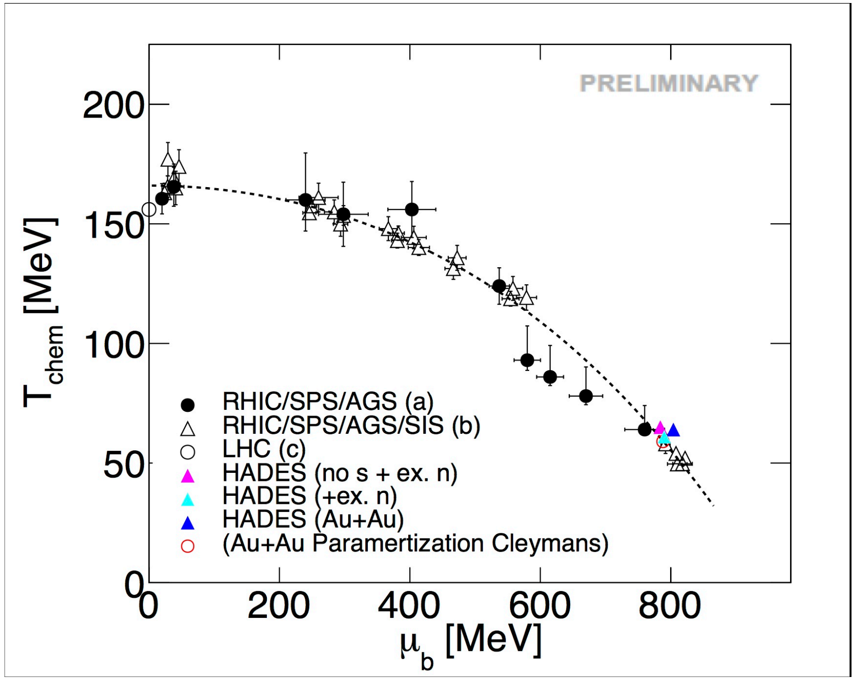
C. Kuhrts, M. Beyer, P. Danielewicz, and G. Ropke, Phys. Rev. C **63**, 034605 (2001)

Freeze-out temperatures and densities

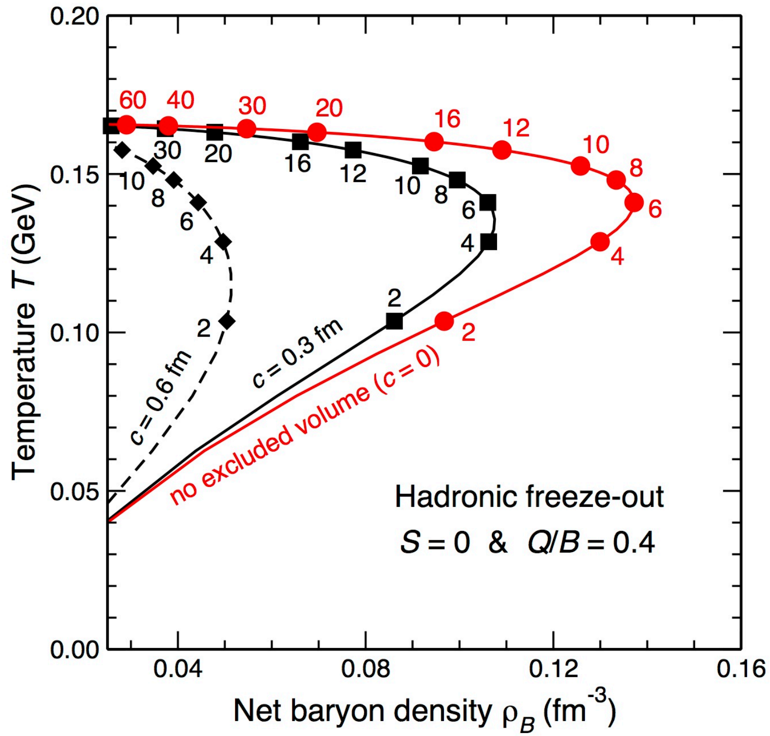


Freeze-out in the phase diagram

from M. Lorenz



Jørgen Randrup, Jean Cleymans:
Freezeout density (2016)



Nonequilibrium evolution of the fireball.
Where the clusters are formed? Very early? Late?

nucleon-nucleon interaction potential

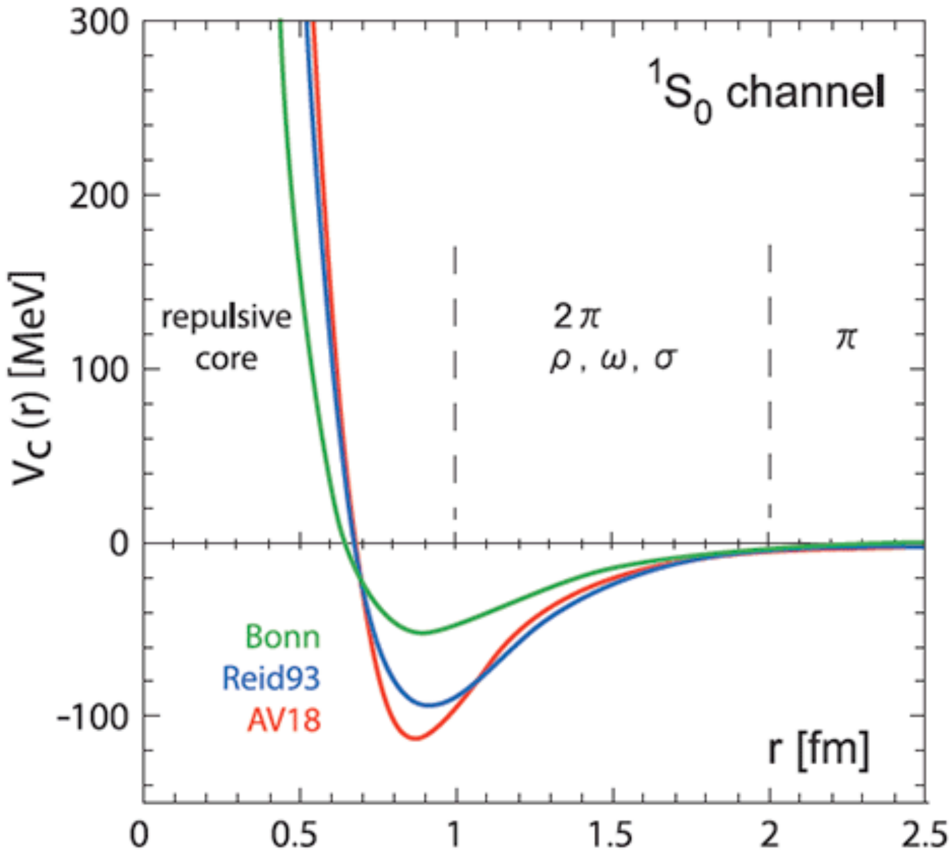
- Effective potentials
  (like atom-atom potential)
  binding energies, scattering

- non-local, energy-dependent?
  QCD?

- microscopic calculations
  (AMD, FMD)

- single-particle descriptions:
  Thomas-Fermi approximation
  shell model
  density functional theory (DFT)

- correlations, clustering
  low-density nα nuclei, Volkov



Many-particle theory, spectral function

Equation of state
$$n_\tau^{\rm tot}(T,\mu_n,\mu_p) = \frac{1}{\Omega}\sum_{p_1,\sigma_1}\int \frac{d\omega}{2\pi}\frac{1}{e^{(\omega-\mu_\tau)/T}+1}S_\tau(1,\omega)$$

Spectral function
$$S_\tau(1,\omega;T,\mu_n,\mu_p) \qquad E(1) = \hbar^2 p_1^2/2m_1$$

Green function G,
Self-energy Σ
$$S(1,\omega) = 2{\rm Im}\, G(1,\omega+i0) = 2{\rm Im}\frac{1}{\omega - E(1) - \Sigma(1,\omega+i0)}$$

$$S_\tau(1,\omega) = \frac{2{\rm Im}\Sigma(1,\omega-i0)}{(\omega-E(1)-{\rm Re}\Sigma(1,\omega))^2+({\rm Im}\Sigma(1,\omega-i0))^2}$$

Expansion for small damping (Im Σ)
$$S(1,\omega) \approx \frac{2\pi\delta(\omega - E^{\rm quasi}(1))}{1-\frac{d}{dz}{\rm Re}\,\Sigma(1,z)|_{z=E^{\rm quasi}(1)}} - 2{\rm Im}\,\Sigma(1,\omega+i0)\frac{d}{d\omega}\frac{\mathcal{P}}{\omega - E^{\rm quasi}(1)}$$

Quasiparticle energy
$$E^{\rm quasi}(1) = E(1) + {\rm Re}\,\Sigma(1,z)|_{z=E^{\rm quasi}(1)}$$

Correlations (bound states) in Im Σ
Cluster decomposition, Bethe-Salpeter equation

Different approximations

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Different approximations

medium effects

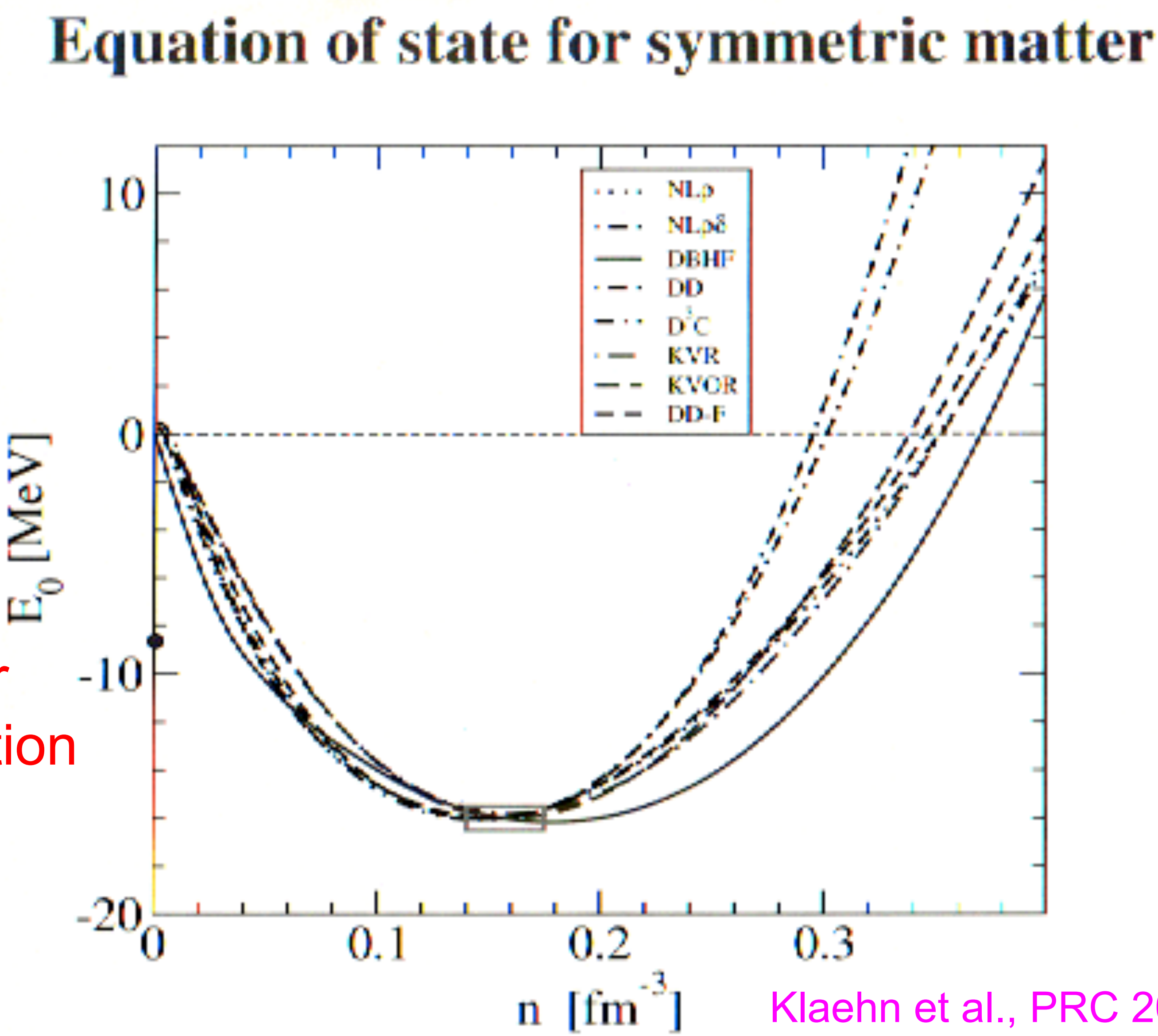
Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

Quasiparticle approximation for nuclear matter



But: cluster formation

Incorrect low-density limit

Klaehn et al., PRC 2006

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

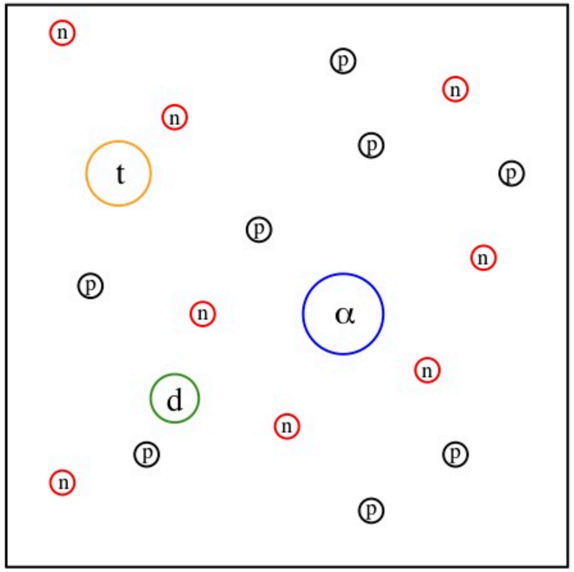
Inclusion of the light clusters (d,t,^{3}He,^{4}He)

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law



Ideal mixture of reacting nuclides

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

ν: internal quantum number
excited states, continuum correlations

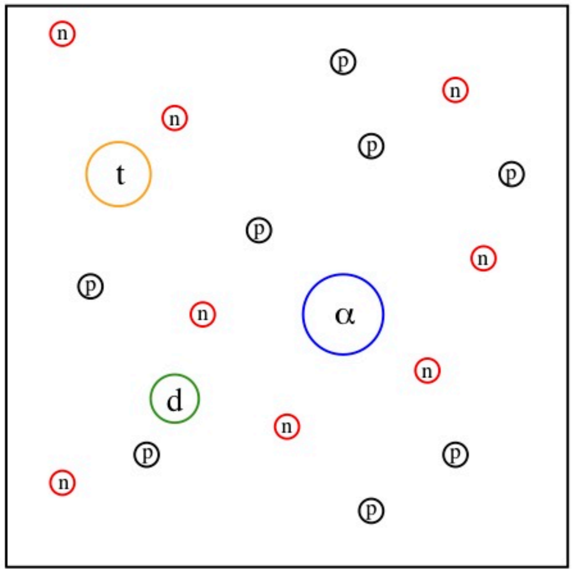
Chemical equilibrium, mass action law,
Nuclear Statistical Equilibrium (NSE)

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

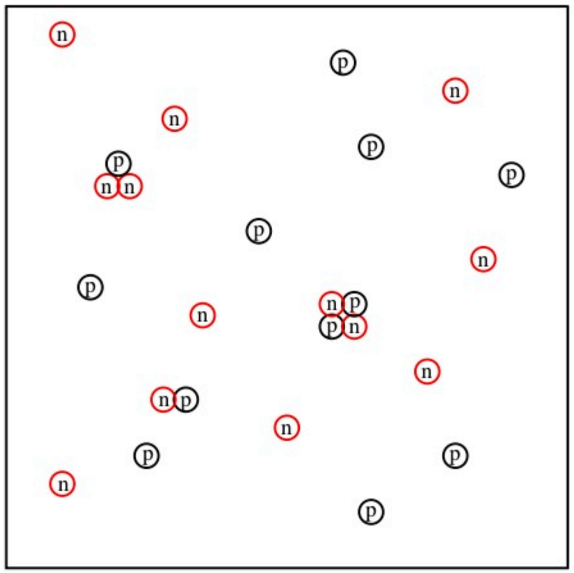
Mass action law



Interaction between the components
internal structure: Pauli principle
"excluded volume"

Physical picture:

"elementary" constituents
and their interaction



Quantum statistical (QS) approach,
quasiparticle concept, virial expansion

Different approximations

medium effects

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:

mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium with quasiparticle clusters:

self-energy and Pauli blocking

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

Add self-energy

Pauli-blocking

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Thouless criterion

$$E_d(T,\mu)=2\mu$$

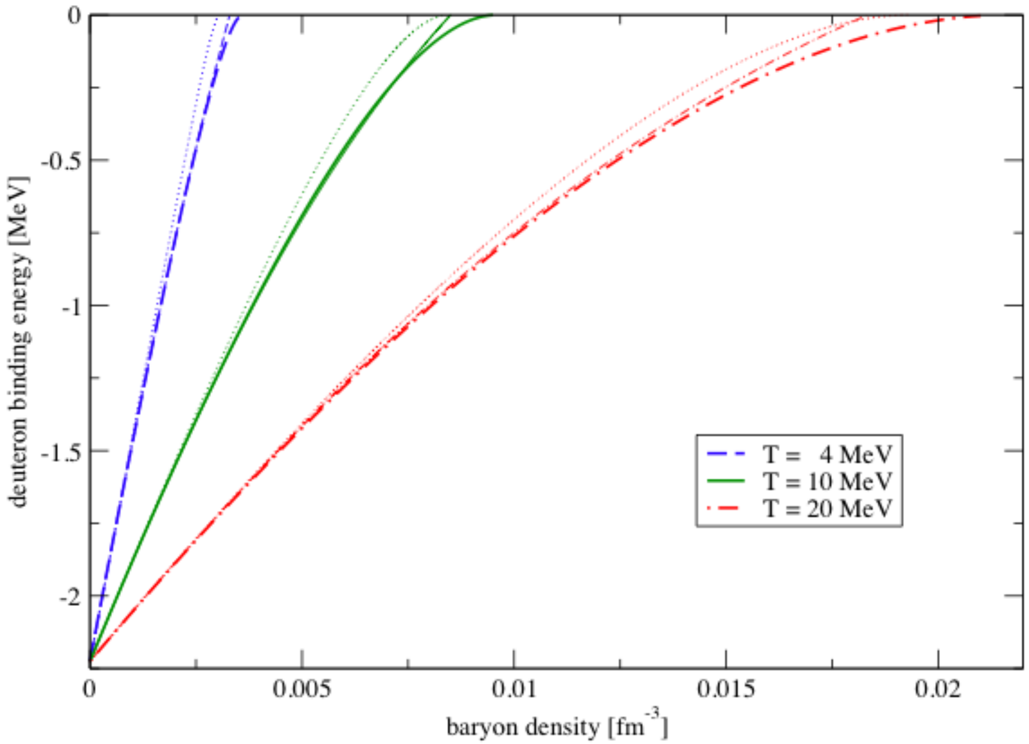
Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

Shift of the deuteron bound state energy

Dependence on nucleon density, various temperatures,
zero center of mass momentum

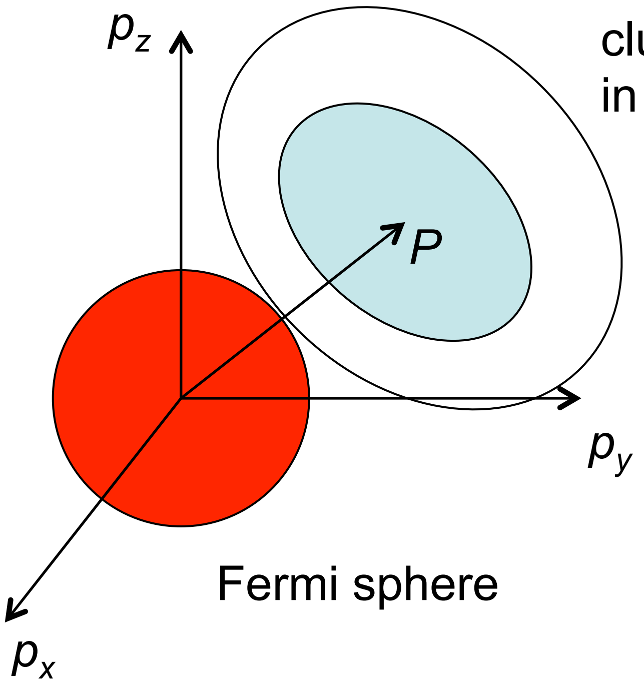


thin lines:

fit formula

G.R., Nucl. Phys. A **867**, 66 (2011)

Pauli blocking – phase space occupation



cluster wave function (deuteron, alpha,…)
in momentum space

P - center of mass momentum

The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum P

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$

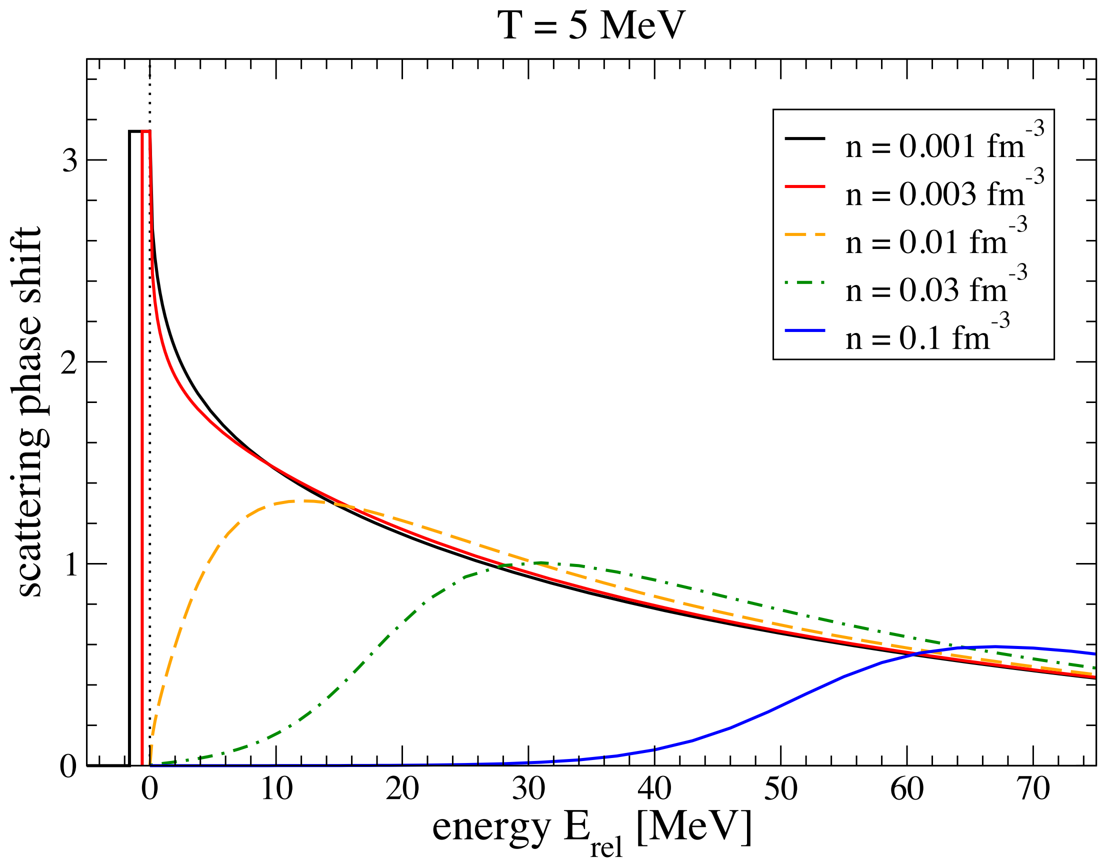
$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

ν: internal quantum number
excited states, continuum correlations

- Medium effects: correct behavior near saturation
self-energy and Pauli blocking shifts of binding energies,
Coulomb corrections due to screening (Wigner-Seitz, Debye)

Deuteron-like scattering phase shifts

Virial coeff. $\propto e^{-E_d^0/T} - 1 + \frac{1}{\pi T}\int_0^\infty dE\, e^{-E/T}\left\{\delta_c(E) - \frac{1}{2}\sin[2\delta_c(E)]\right\}$

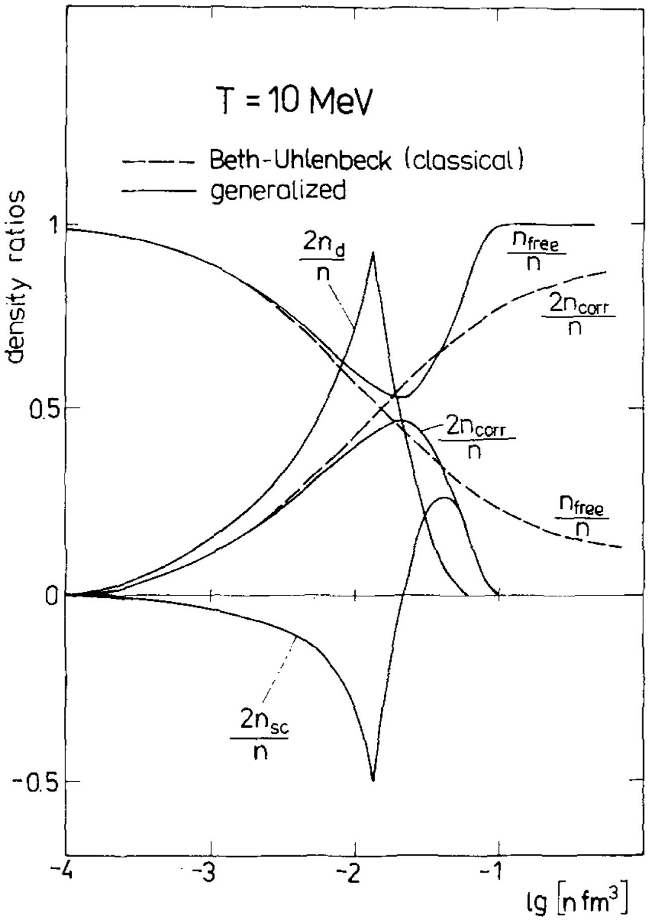


deuteron bound state -2.2 MeV

G. R., J. Phys.: Conf. Series 569, 012031 (2014)
Phys. Part. Nucl. 46, 772 (2015) [arXiv:1408.2654]

Two-particle correlations

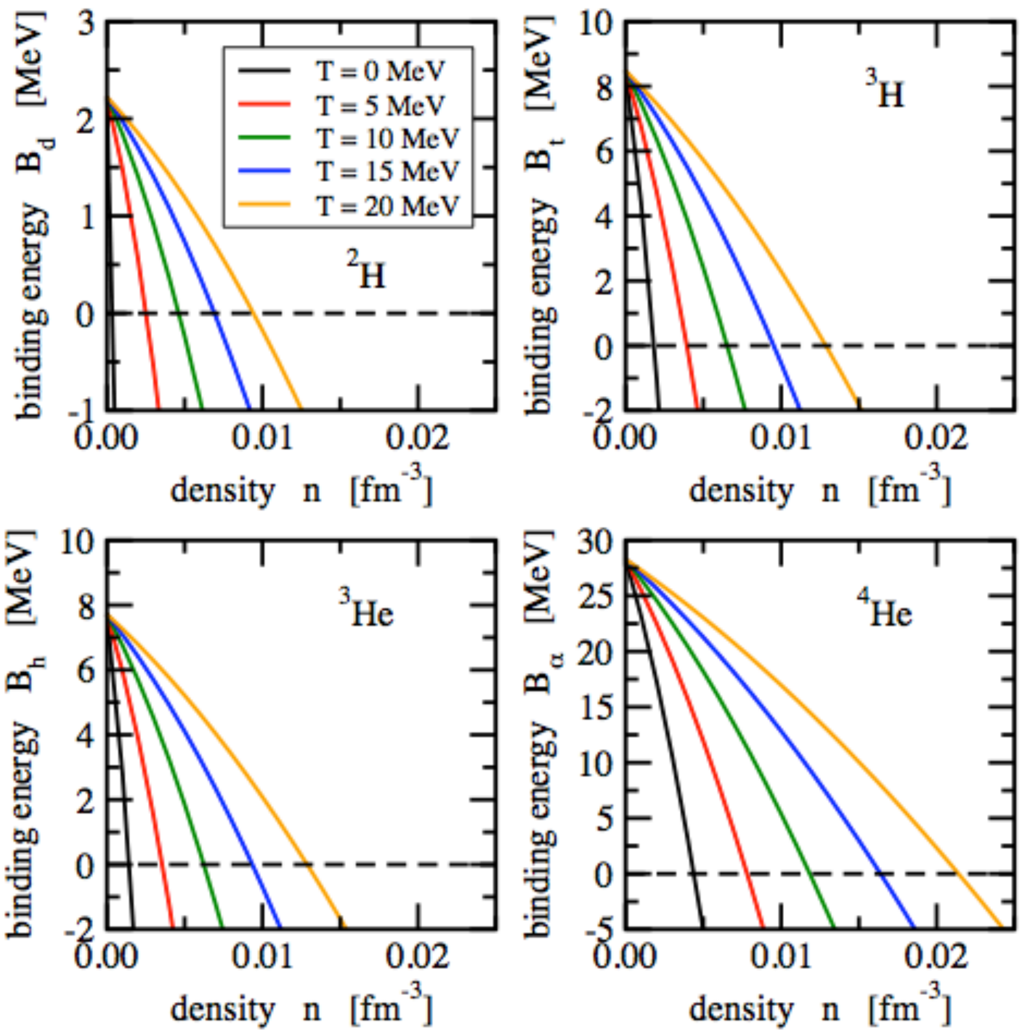
Generalized
Beth-Uhlenbeck Approach
for Hot Nuclear Matter



M. Schmidt, G.R., H. Schulz
Ann. Phys. **202**, 57 (1990)

FIG. 7. The composition of nuclear matter as a function of the density n for given temperature $T = 10$ MeV. The solid and dashed lines show the results of the generalized and classical Beth–Uhlenbeck approach, respectively. Note the distinct behavior of n_{free} and n_{corr} predicted by the two approaches in the low and high density limit!

Shift of Binding Energies of Light Clusters



Symmetric matter

G.R., PRC **79**, 014002 (2009)
S. Typel et al.,
PRC **81**, 015803 (2010)

EoS at low densities from HIC

PRL **108**, 172701 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

Laboratory Tests of Low Density Astrophysical Nuclear Equations of State

L. Qin,[1] K. Hagel,[1] R. Wada,[2,1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[6] M. Huang,[6] J. Wang,[6] H. Zheng,[1] S. Kowalski,[7] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] Z. Majka,[11] and Y. G. Ma[12]

Yields of clusters from HIC: p, n, d, t, h, α

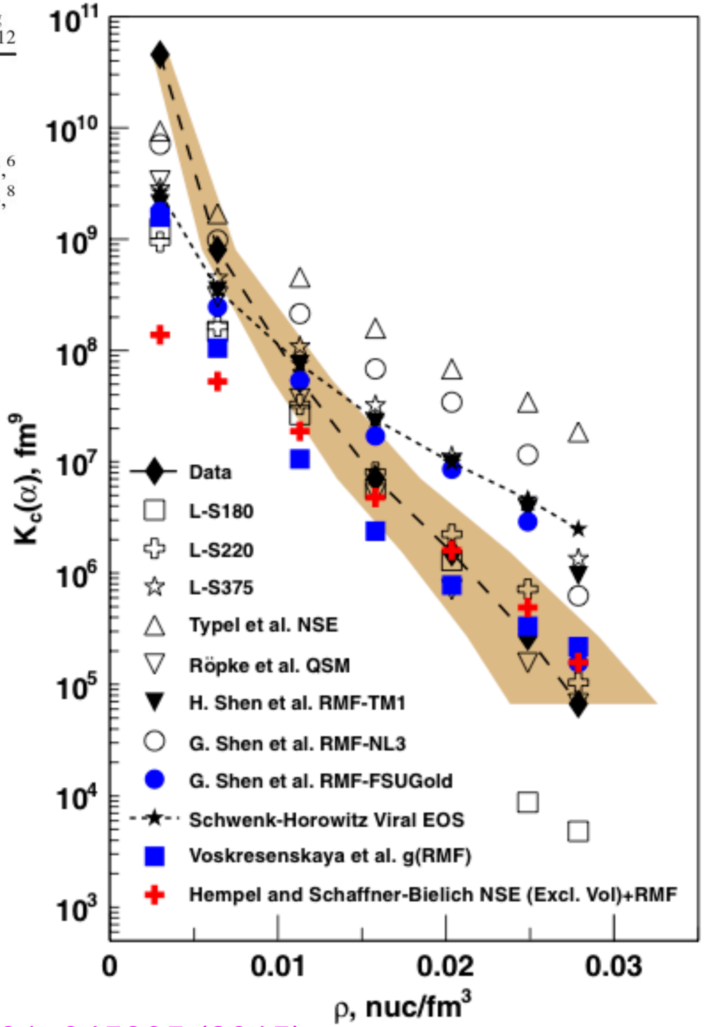
chemical constants

$$K_c(A,Z) = \rho_{(A,Z)}/[(\rho_p)^Z(\rho_n)^N]$$

inhomogeneous,
non-equilibrium

QS, excluded volume

M. Hempel, K. Hagel, J. Natowitz, G. Ropke, S. Typel, Phys. Rec. C 91, 045805 (2015)



$\chi-\alpha$ scattering phase shifts

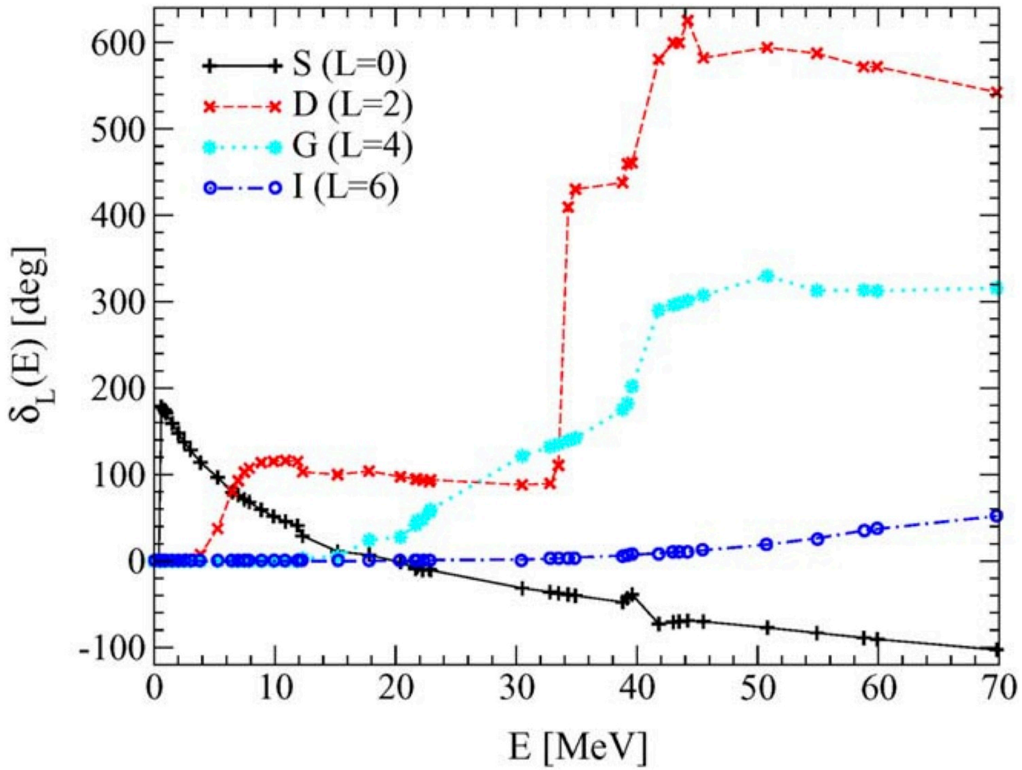


Fig. 3. (Color online.) The phase shifts for elastic $\alpha\alpha$ scattering $\delta_L(E)$ versus laboratory energy E. As discussed in the text, the phase shifts are taken from Afzal et al. [40] and from Bacher et al. [41].

C.J. Horowitz, A. Schwenk / Nuclear Physics A 776 (2006) 55–79

Example: ^{5}He

Partial density
$$n_{^5\mathrm{He}} = 8\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2} b_{\alpha n}(T) e^{(-E_\alpha+3\mu_n+2\mu_p)/T}$$

virial coefficient

nuclear stat. equ.
$$b_{\alpha n}^{\mathrm{NSE}}(T) = \frac{5^{3/2}}{2} e^{(-E_{^5\mathrm{He}}+E_{^4\mathrm{He}})/T}$$

generalized Beth-Uhlenbeck
$$b_{\alpha n}^{\mathrm{gBU}}(T) = \left(\frac{5}{4}\right)^{1/2} \frac{1}{\pi T}\int_0^\infty dE_{\mathrm{lab}}\, e^{-4E_{\mathrm{lab}}/5T}\left\{\delta_{\alpha n}^{\mathrm{tot}}(E_{\mathrm{lab}}) - \frac{1}{2}\sin[2\delta_{\alpha n}^{\mathrm{tot}}(E_{\mathrm{lab}})]\right\}$$

B(^{5}He)-B(^{4}He)= - 0.735 MeV

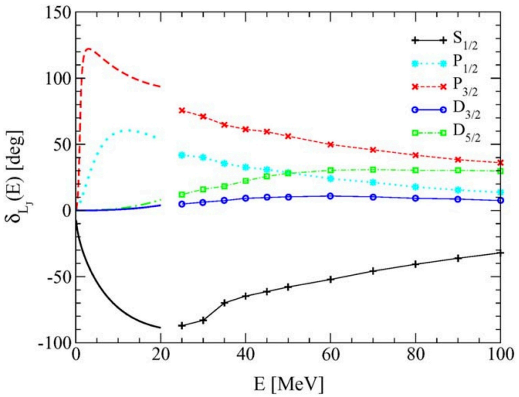
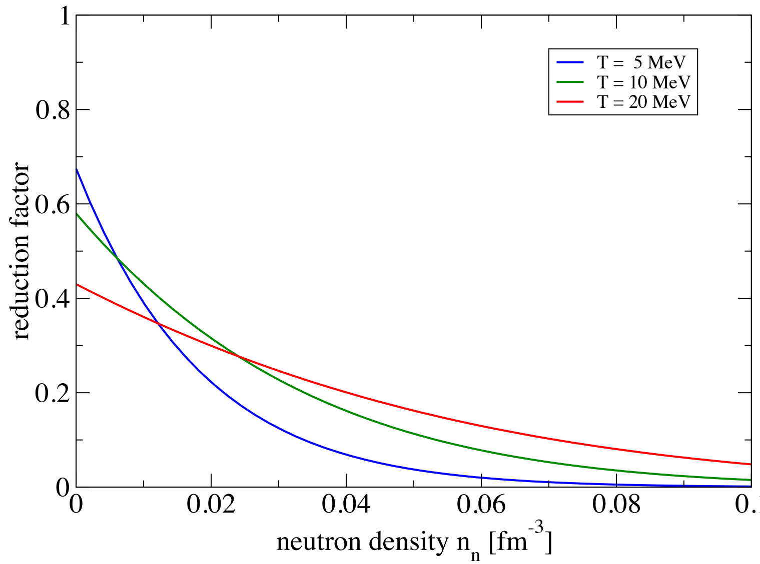


Fig. 2. (Color online.) The phase shifts for elastic neutron-alpha scattering $\delta_{L_J}(E)$ versus laboratory energy E. As discussed in the text, the solid lines are from Arndt and Roper [37] and the symbols are from Amos and Karataglidis [38]. For clarity, we do not show the F-waves included in our results for $b_{\alpha n}$.

C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)



ratio generalized Beth-Uhlenbeck/NSE

Oct. 2017 update: excellent description of ALICE@LHC data

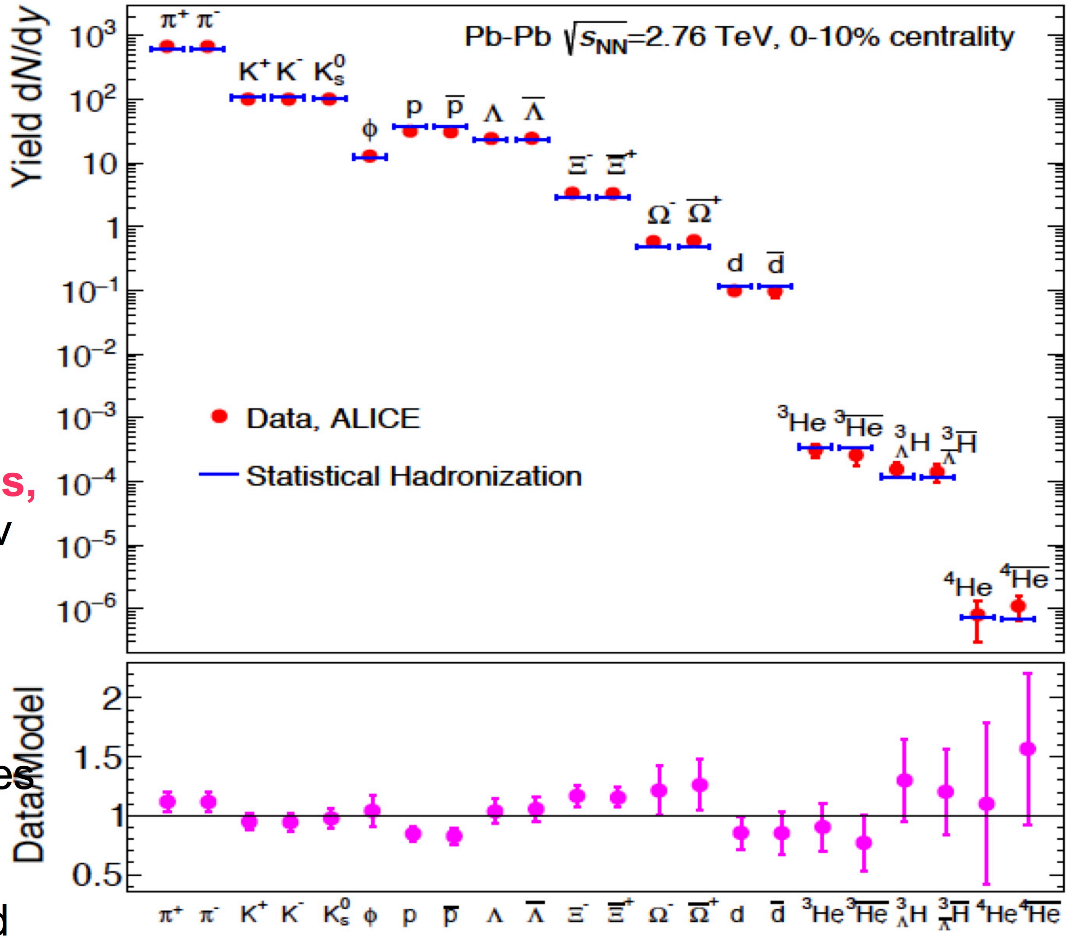
fit includes loosely bound systems such as deuteron and hypertriton
how light nuclei emerge from LQCD see Detmold et al., Eur.Phys.J. A55 (2019) 193

hypertriton is bound-state of (Λ,p,n), Λ separation energy about 130 keV
size about 10 fm, the **ultimate halo nucleus,** produced at T=156 MeV. close to an Efimov state

proton discrepancy about 2.8 sigma

agreement with hyper-triton yield also implies that hyper-triton has no excited states

for an excited state with J=3/2 the total yield would triple, inconsistent with data

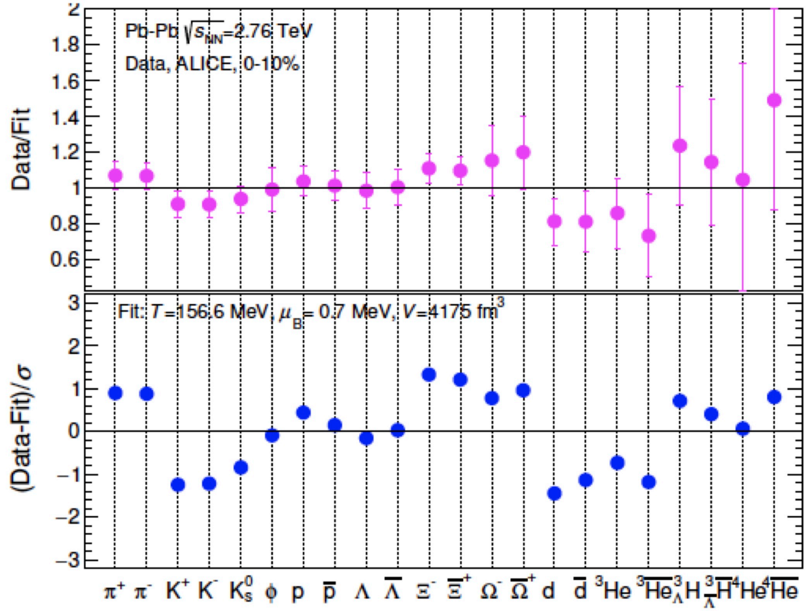


Andronic, pbm, Redlich, Stachel, arXiv:1710.09425, Nature 561 (2018) 321

Jan. 2019 update: excellent description of ALICE@LHC data

proton discrepancy of 2.8 sigma is now explained in arXiv:1808.03102
explicit phase shift description of baryon resonance region
(Andronic, pbm, Friman, Lo, Redlich, Stachel Phys.Lett.B792 (2019)304)

Contributions of three- and higher resonances and inelastic channels are taken into account with normalization with normalization to LQCD susceptibilities



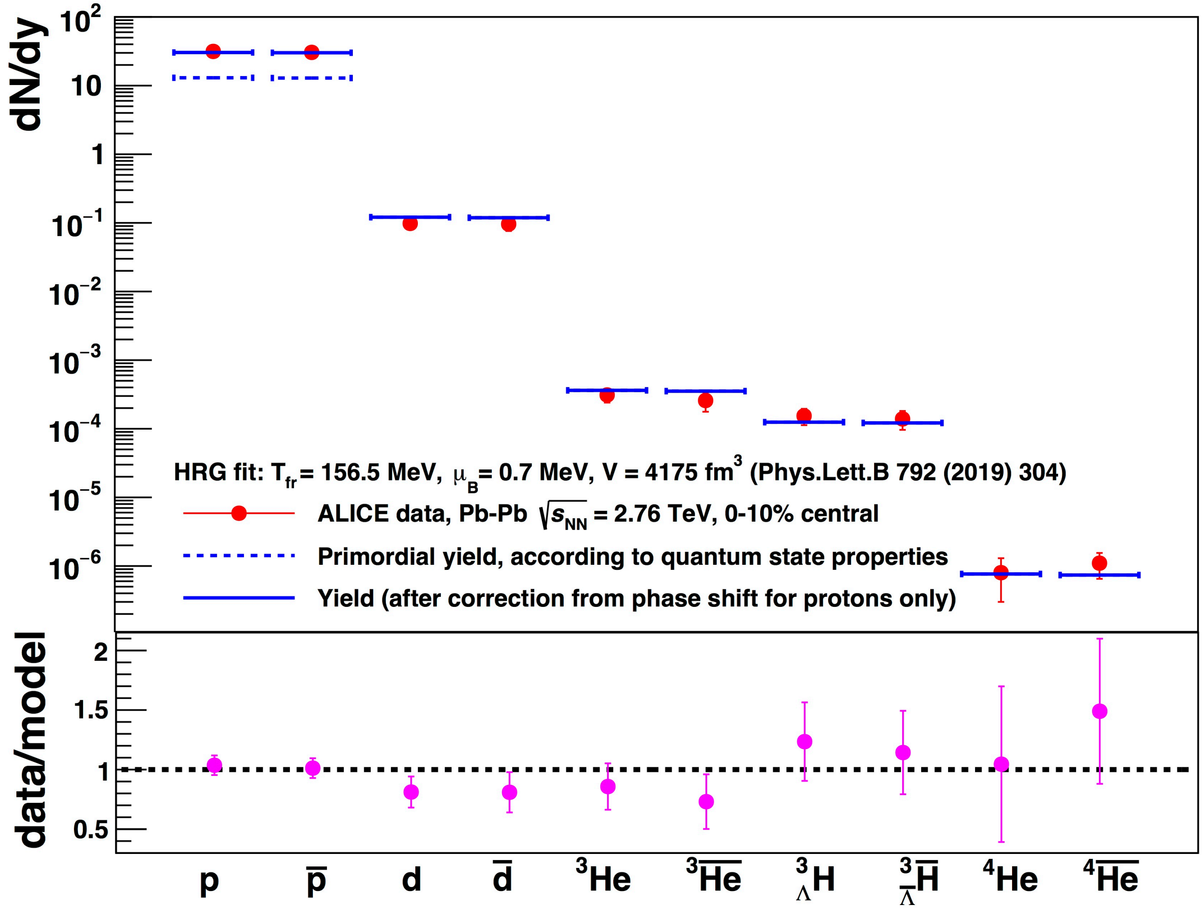
$$\chi^2 = 19.7 \text{ per } 19 \text{ dof}$$

very good fit!

p well described, but d, ^{3}He ?

Cluster formation at LHC/CERN

ALICE@LHC

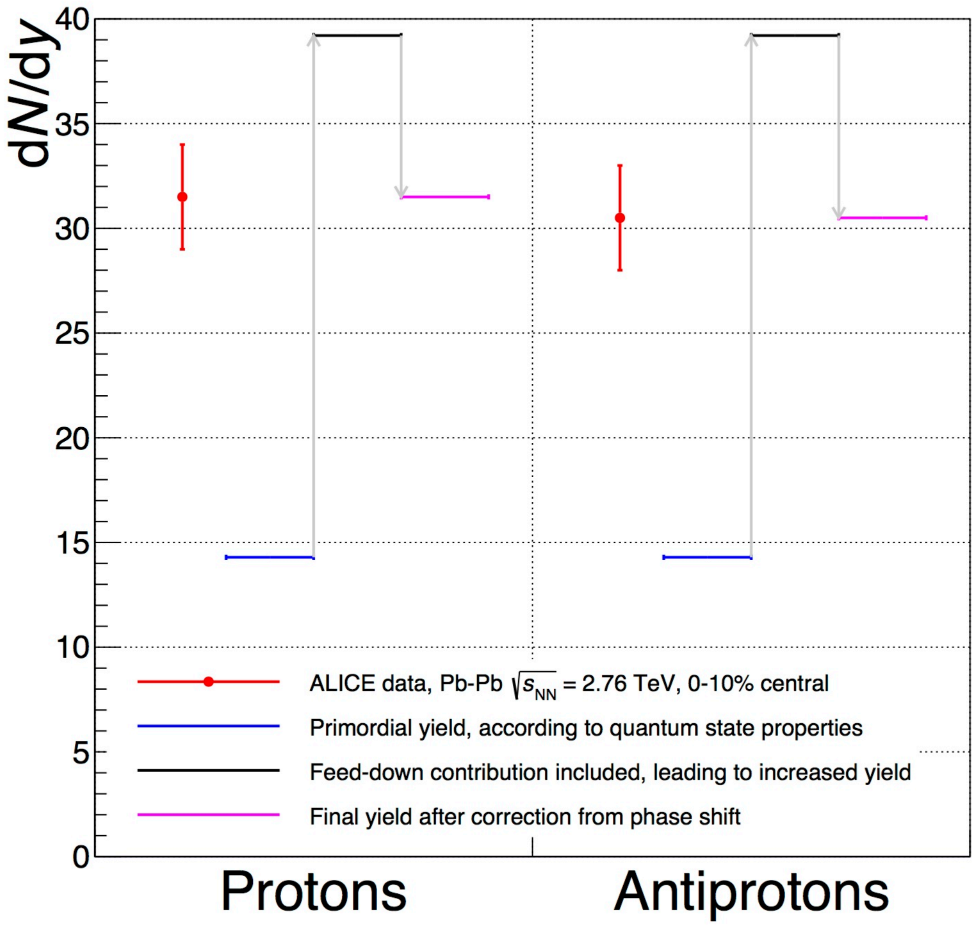


Excellent description
Of data by the
Statistical model
(chemical equilibrium)

T = 156 MeV

B. Doenigus, G.R., D. Blaschke, Phys. Rev. C 106, 044908 (2022)

Proton yields at LHC - ALICE

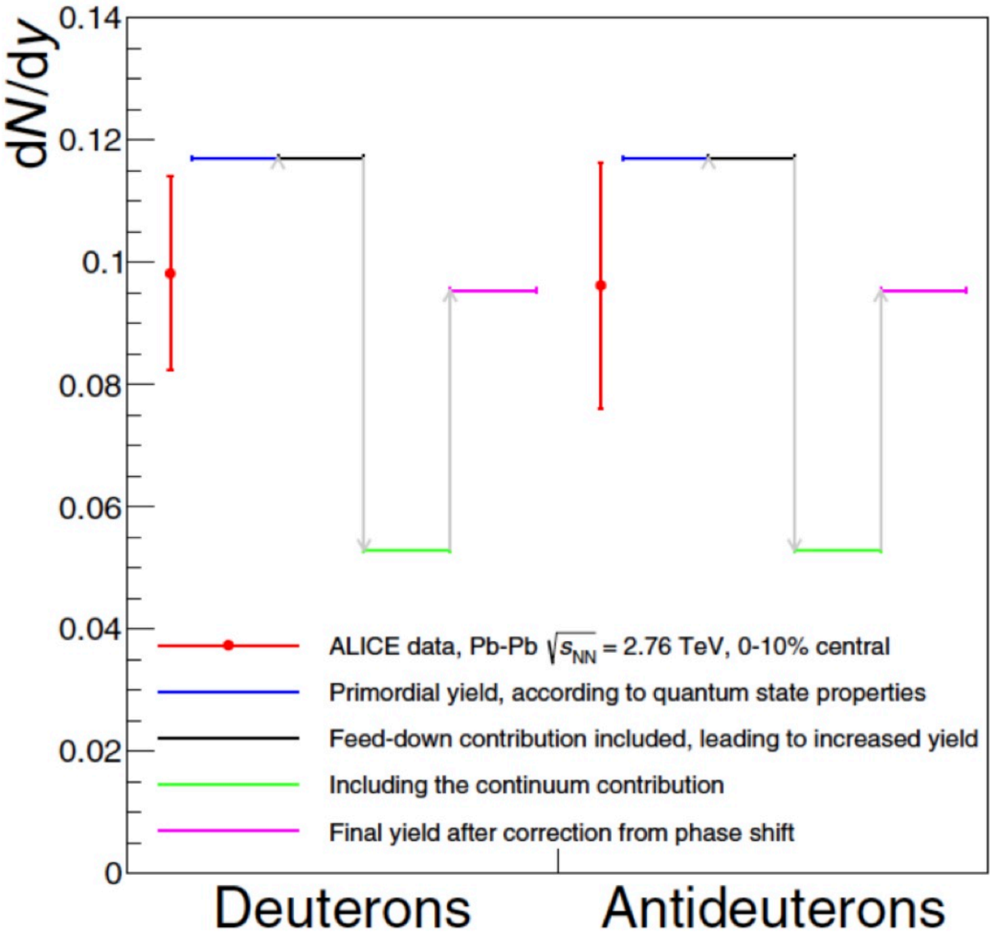


Production of protons at chemical freeze-out temperature T = 156 MeV for $s_{NN}^{1/2}$=2.76 TeV

Primordial,
Feed-down from resonances,
Scattering phase shifts p-pi

B. Doenigus, G. R., D. Blaschke, Phys. Rev. C **106**, 044908 (2022)

Deuteron yields at LHC - ALICE



Production of deuterons at chemical freeze-out temperature T = 156 MeV for $s_{NN}^{1/2}$=2.76 TeV

"snowballs in the hell": Argand plots, p wave

Primordial, continuum correlations, scattering phase shifts d-pi

B. Doenigus, G. R., D. Blaschke, Phys. Rev. C **106**, 044908 (2022)

Freeze-out at heavy ion collisions

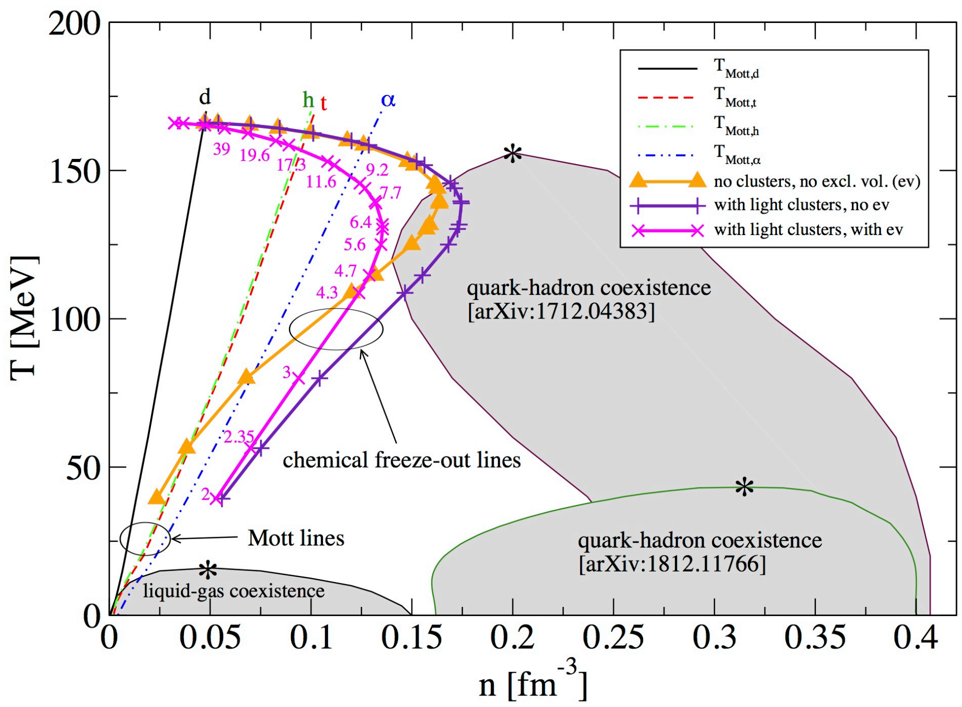


Fig. 1. Chemical freezeout lines in the temperature density plane (phase diagram) together with Mott lines for light clusters. The coexistence regions for the nuclear gas-liquid transition and for two examples of the hadron-quark matter transition are shown as grey shaded regions together with their critical endpoints. For details, see text.

D. Blaschke, G R., Y. Ivanov, M. Kozhenikova, S. Liebig, SQM 2019, Springer Proc. Phys. **250**, 183 (2020)

Challenges

- cluster formation in expanding hot and dense matter: nonequilibium processes, freeze-out concept contains quantum correlations, feed-down, reaction network, vs. kinetic equations and coalescence model, quantum correlation should be included, evolution of the spectral function.

- Improving the ideal mixture of bound states (nuclear statistical equilibrium), account of interactions, quasiparticle concept, weakly bound clusters and resonances, correlations in the continuum.

- formation of quantum condensates, finite systems; transport properties in dense matter, quark substructure.

Thanks

to D. Blaschke, B. Doenigus, A. Sedrakian, P. Schuck,

S. Typel, H. Wolter

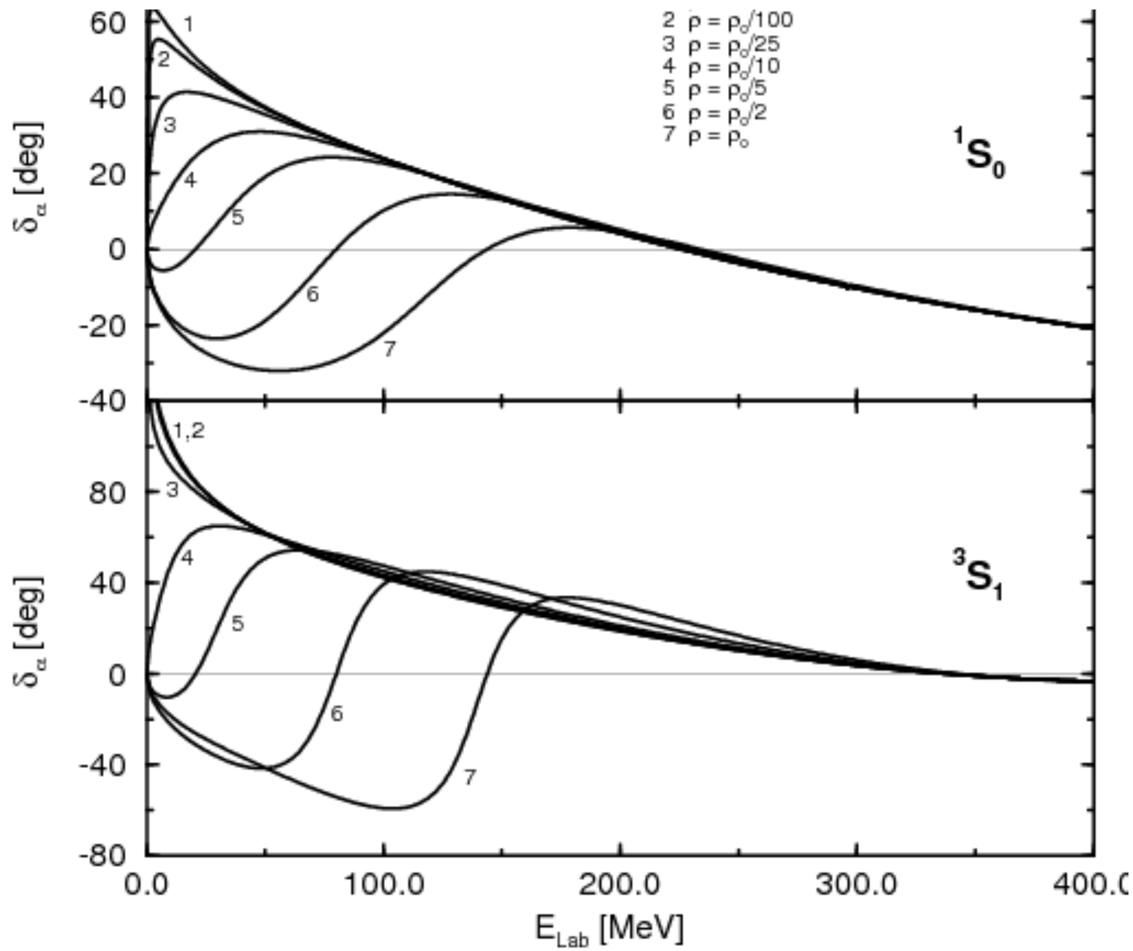
for collaboration

to you

for attention

D.G.

Scattering phase shifts in matter



Total phase shift for neutron matter (n-n) and symmetric matter (n-n + p-n)

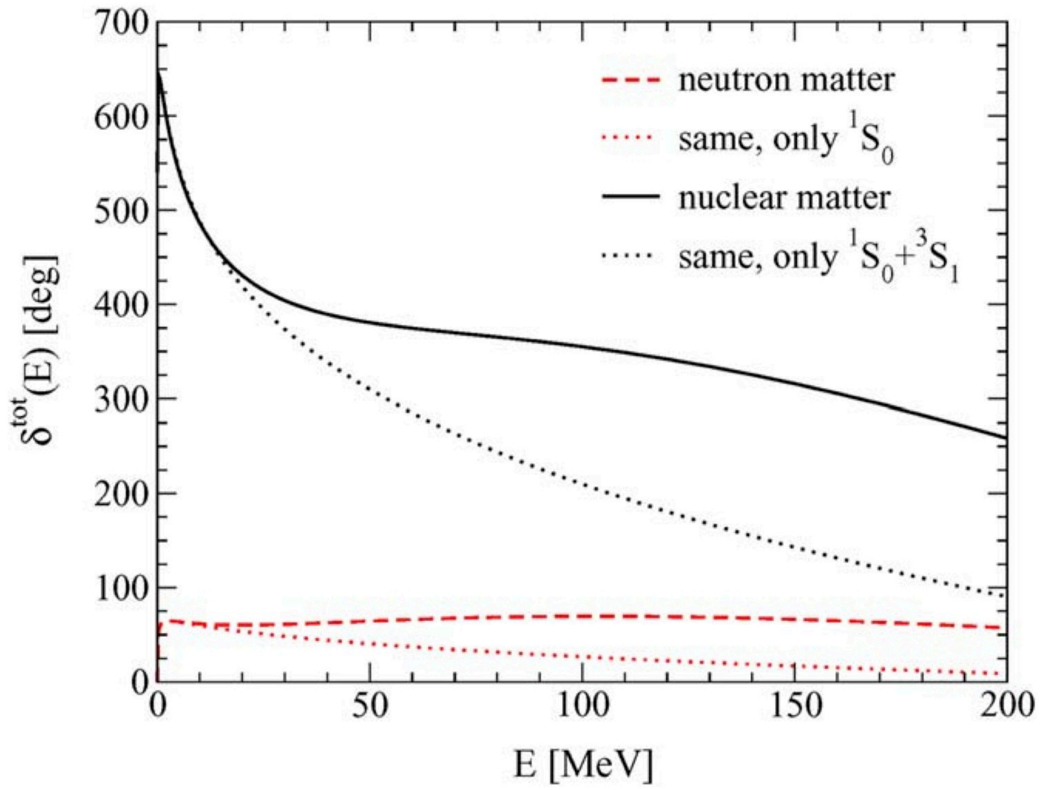


Fig. 1. (Color online.) The total phase shift $\delta_n^{\mathrm{tot}}(E)$ for neutron matter and $\delta_{\mathrm{nuc}}^{\mathrm{tot}}(E)$ for nuclear matter versus laboratory energy E. For reference, we also show the contributions from only the S-wave phase shifts.

C.J. Horowitz, A. Schwenk / Nuclear Physics A 776 (2006) 55–79

Loosely bound objects

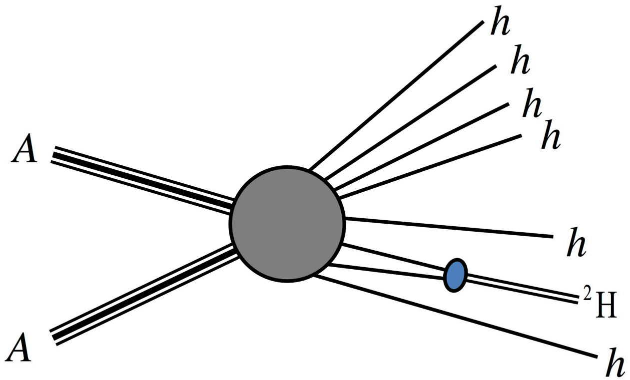
- $\pi - d$ scattering phase shifts?
- Spectral function of the nucleons forming the deuteron
- Sum of the two shifts caused by the interaction with the π system
- Vertex corrections?
- Similar for ^{3}He etc.?

How to form clusters?

Nuclear reactions, nonequilibrium process

Stanisław Mrówczyński

Final state interaction – conventional approach to production of light nuclei



energy scale
of particle production
- particle's mass

>>

energy scale
of formation of a nucleus
- binding energy

fast process

slow process

Binding energy of a deuteron is ε_B = 2.2 MeV.

A characteristic time of deuteron formation is $1/\ \varepsilon_B$ = 100 fm/c.

resonances, ^{4}Li ?

Quantum statistical approach

The total density as well as the DoS are given by the spectral function A,

$$n_e^{\text{total}}(T,\mu_e,\mu_a) = \frac{1}{\Omega}\sum_1 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \hat{f}_e(\omega) A_e(1,\omega) = \int_{-\infty}^{\infty} d\omega \hat{f}_e(\omega) D_e(\omega)$$

$$|1\rangle = |\mathbf{p}_1, \sigma_1\rangle$$

which is related to the Green function and the self-energy as

$$A(1,\omega) = 2\,\mathrm{Im}\,G(1,\omega - i0) = 2\,\mathrm{Im}\frac{1}{\omega - E(1) - \Sigma(1,\omega - i0)}$$

$$E(1) = p_1^2/(2m)$$

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$$E(1) = p_1^2/(2m)$$

A cluster decomposition for the self-energy is possible so that a quasiparticle (free) contribution can be separated,

$$A_e(1,\omega) \approx \frac{2\pi\,\delta(\omega - E_e^{\rm quasi}(1))}{1 - \frac{d}{dz}{\rm Re}\,\Sigma_e(1,z)|_{z=E_e^{\rm quasi}-\mu_e}} - 2{\rm Im}\,\Sigma_e(1,\omega+i0)\frac{d}{d\omega}\frac{\mathcal{P}}{\omega+\mu_e - E_e^{\rm quasi}(1)}$$

$$E^{\rm quasi}(1) = p_1^2/(2m) + {\rm Re}\Sigma(1,\omega)|_{\omega=E^{\rm quasi}(1)}$$

Quantum statistical approach

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$$E^{\mathrm{quasi}}(1) = p_1^2/(2m) + \mathrm{Re}\Sigma(1,\omega)|_{\omega=E^{\mathrm{quasi}}(1)}$$

We obtain the generalized Beth-Uhlenbeck formula (quasiparticles) after calculating the self-energy in ladder approximation.
Bound states appear as solution of an in-medium Schrödinger equation.

Quantum statistical approach

The total density as well as the DoS are given by the spectral function A,

$$n_e^{\mathrm{total}}(T,\mu_e,\mu_a) = \frac{1}{\Omega}\sum_1 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \hat{f}_e(\omega) A_e(1,\omega) = \int_{-\infty}^{\infty} d\omega \hat{f}_e(\omega) D_e(\omega)$$

$$A(1,\omega) = 2\,\mathrm{Im}\,G(1,\omega - i0) = 2\,\mathrm{Im}[\omega - E(1) - \Sigma(1,\omega - i0)]^{-1}$$

A cluster decomposition for the self-energy is possible so that a quasiparticle (free) contribution can be separated,

$$A_e(1,\omega) \approx \frac{2\pi\,\delta(\omega - E_e^{\mathrm{quasi}}(1))}{1 - \frac{d}{dz}\mathrm{Re}\,\Sigma_e(1,z)|_{z=E_e^{\mathrm{quasi}}-\mu_e}} - 2\mathrm{Im}\,\Sigma_e(1,\omega + i0)\frac{d}{d\omega}\frac{\mathcal{P}}{\omega + \mu_e - E_e^{\mathrm{quasi}}(1)}$$

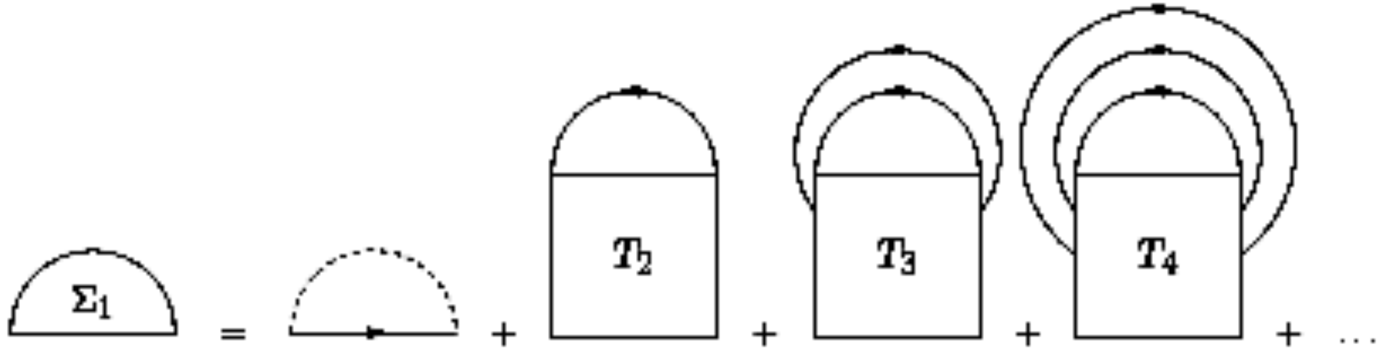
We obtain the generalized Beth-Uhlenbeck formula (quasiparticles)

$$n_e^{\mathrm{total}}(T,\mu_e,\mu_a) = \frac{1}{\Omega}\sum_1 f_e(E^{\mathrm{quasi}}(1))$$

$$+\frac{1}{\Lambda^3}\sum_{i,\gamma} Z_i e^{\beta\mu_i}\left[\sum_{\nu}^{\mathrm{bound}}(e^{-\beta E_{i,\gamma,\nu}} - 1) + \frac{\beta}{\pi}\int_0^{\infty} dE e^{-\beta E}\left\{\delta_{i,\gamma}(E) - \frac{1}{2}\sin[2\delta_{i,\gamma}(E)]\right\}\right]$$

In-medium Schrödinger equation for $E_{i,\gamma,\nu}(T,\mu)$, $\delta_{i,\gamma}(T,\mu)$, channel (spin…) γ

Cluster decomposition of the self-energy



T-matrices: bound states, scattering states
Including clusters like new components
chemical picture,
mass action law, nuclear statistical equilibrium (NSE)

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium
of quasiparticle clusters:
self-energy and Pauli blocking

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.Eq.

Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

chemical & physical picture

Cluster virial approach:
all bound states (clusters)
scattering phase shifts of all pairs

Correlated medium:
phase space occupation by all bound states
in-medium correlations, quantum condensates

Mott effect, in-medium cross section

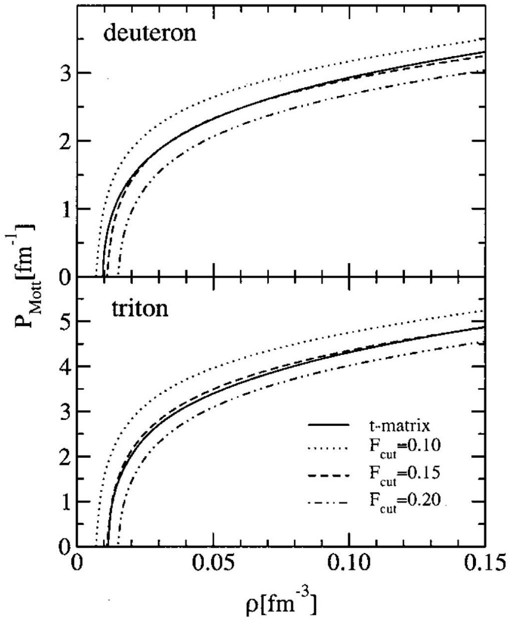


FIG. 1. Deuteron and triton Mott momenta P_{Mott} shown as a function of density ρ at fixed temperature of $T=10$ MeV. The solid line represents results of the t matrix approach. The dashed, dotted, and dashed-dotted lines represent the deuteron Mott momenta from the parametrization given in Eq. (24) for three different cutoff values F_{cut}.

$$\int d^3 q \; f\left(\mathbf{q}+\frac{\mathbf{P}_{\mathrm{c.m.}}}{2}\right) \; |\phi(\mathbf{q})|^2 \leq F_{\mathrm{cut}}$$

C. Kuhrts, PRC **63**,034605 (2001)

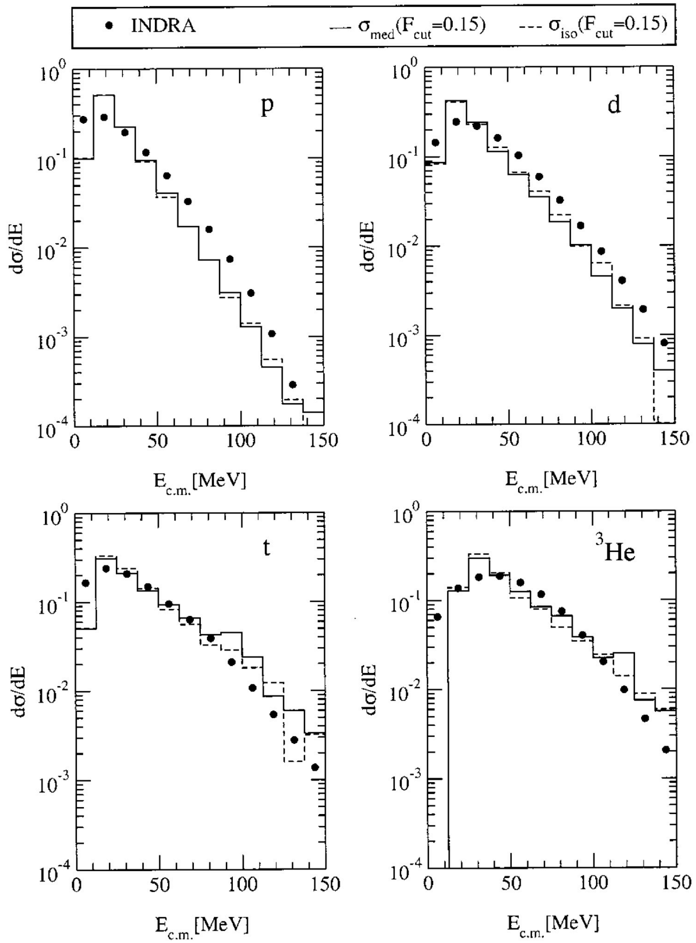


FIG. 5. Renormalized light charged light particle spectra in the center of mass system for the reaction ^{129}Xe+^{119}Sn at 50 MeV/nucleon. The filled circles represent the data of the INDRA Collaboration [21]. The solid line shows the calculations with the in-medium Nd reaction rates, while the dashed line shows a calculation using the isolated Nd breakup cross section; both with $F_{\mathrm{cut}}=0.15$.