

Thermodynamics of quark matter with multi-quark clusters

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Dense neutron star matter: Berlin Wall constraint

Towards a unified approach to quark-hadron matter

- Generalized Φ -derivable approach with clusters; cluster virial expansion
- Hadrons (mesons, baryons, multiquark states) as clusters in quark matter; Beth-Uhlenbeck approach
- First results within a schematic model

Outlook: density functionals for quark matter with confinement

- Density functional for warm, dense quark matter; chiral symmetry breaking and color superconductivity
- Quark confinement as density functional → effective Nambu model with density-dependent couplings
- Phase transition construction and hybrid neutron star properties

QCD Phase Diagram

Landscape of our investigations

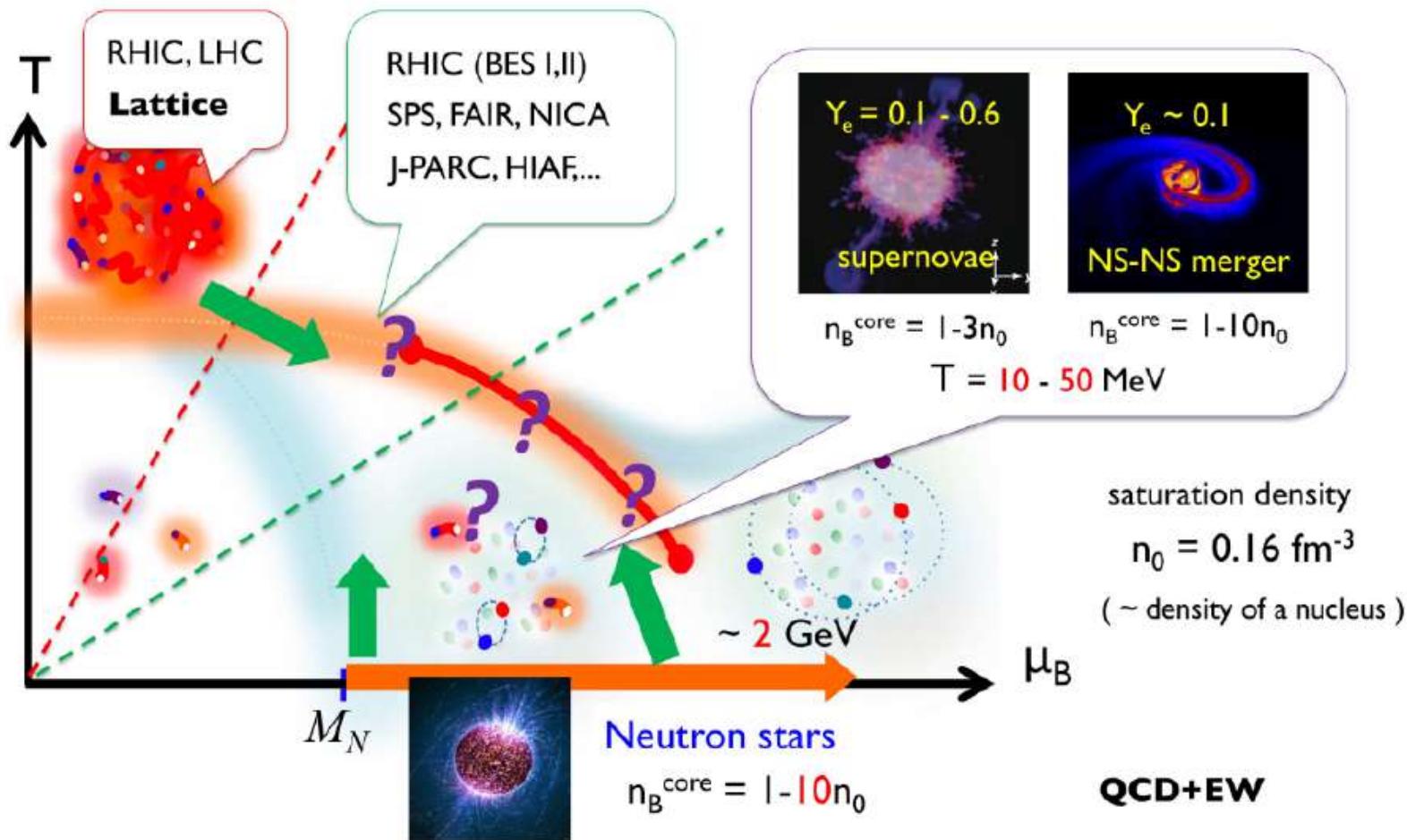
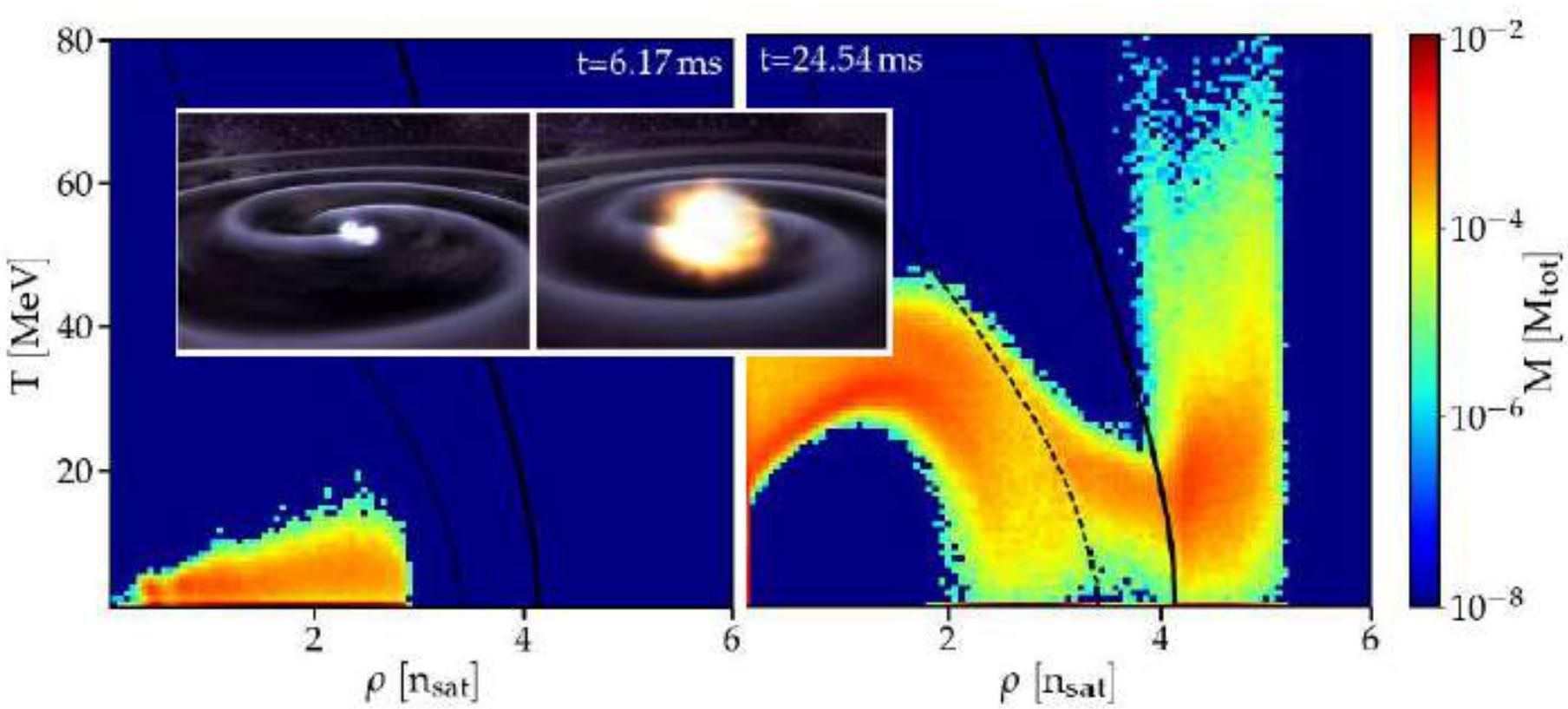


Figure from T. Kojo arXiv:1912.05326 [nucl-th]

Ultra-heavy Nucleus-Nucleus Collisions !

Binary neutron star merger simulation: S. Blacker, A. Bauswein

Population of the QCD phase diagram with mixed phase; time = 6 ... 25 ms

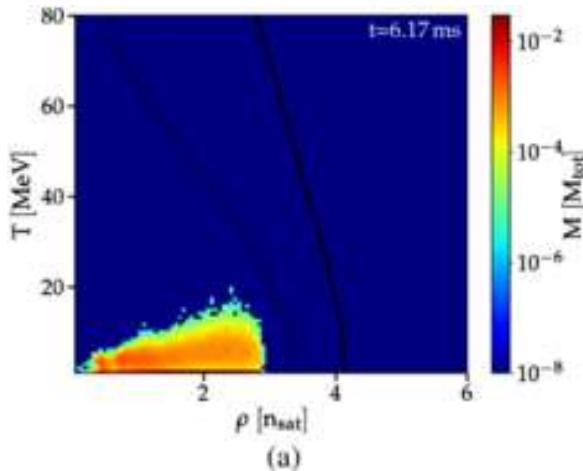


S. Blacker, A. Bauswein et al., Phys. Rev. D 102 (2020) 123023

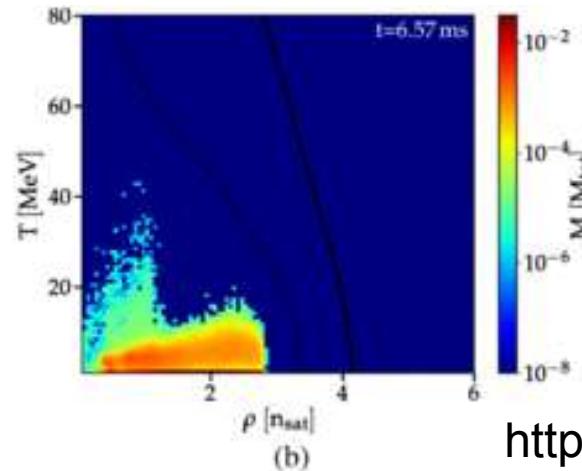
Ultra-heavy Nucleus-Nucleus Collisions !

Mass-radius diagram for purely hadronic EOS

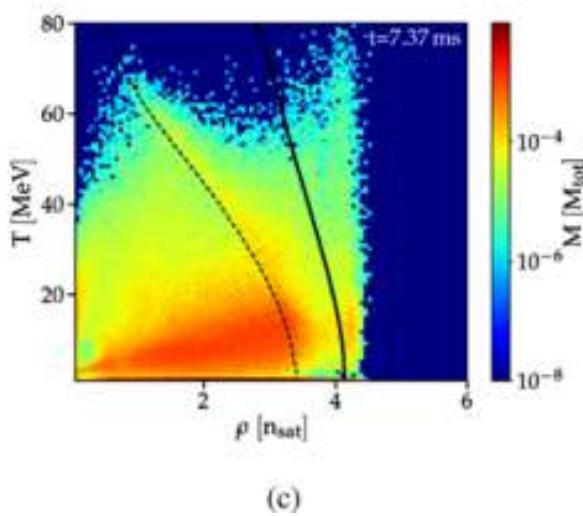
Population of the QCD phase diagram in a merger $1.35 M_{\text{sun}} + 1.35 M_{\text{sun}}$



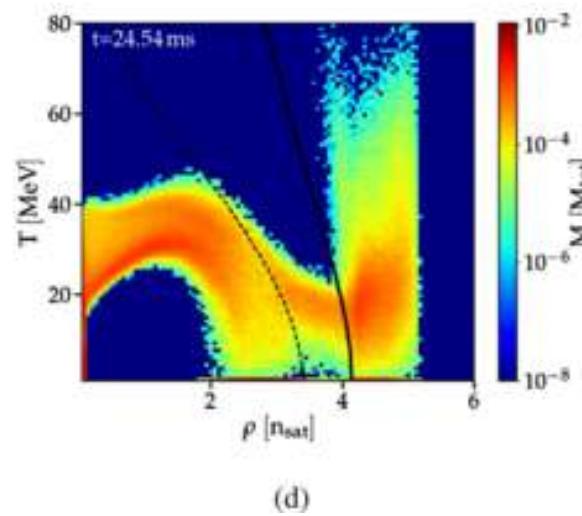
(a)



(b)



(c)



(d)

EoS for applications to supernova and merger Simulation:

CompOSE

With deconfinement:

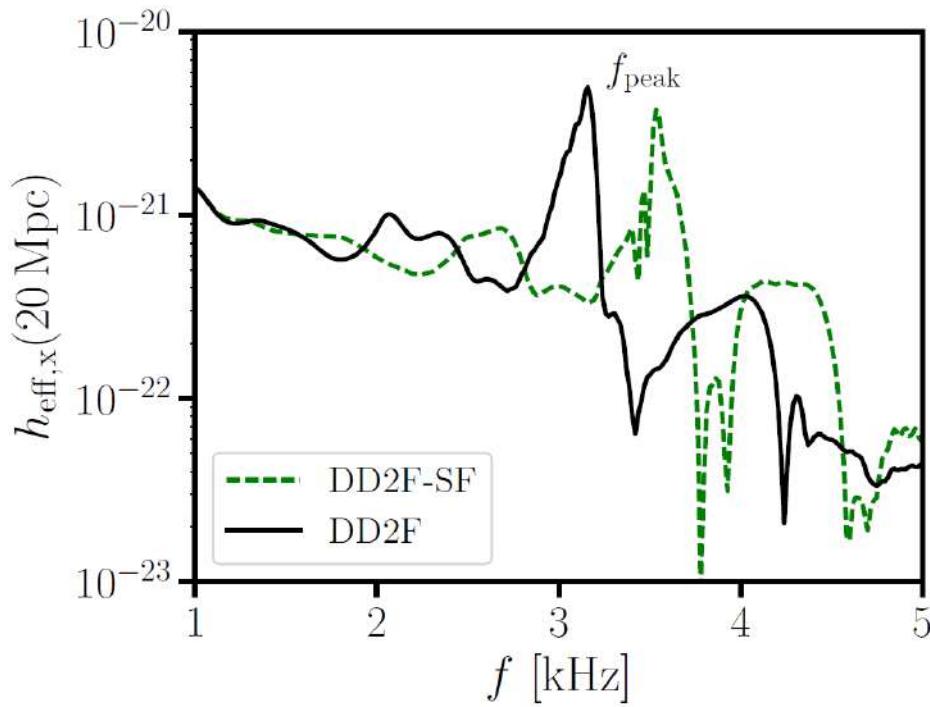
<https://compose.obspm.fr/eos/166>



S. Blacher, A. Bauswein, et al.,
Phys. Rev. D 102 (2020) 123023

Ultra-heavy Nucleus-Nucleus Collisions !

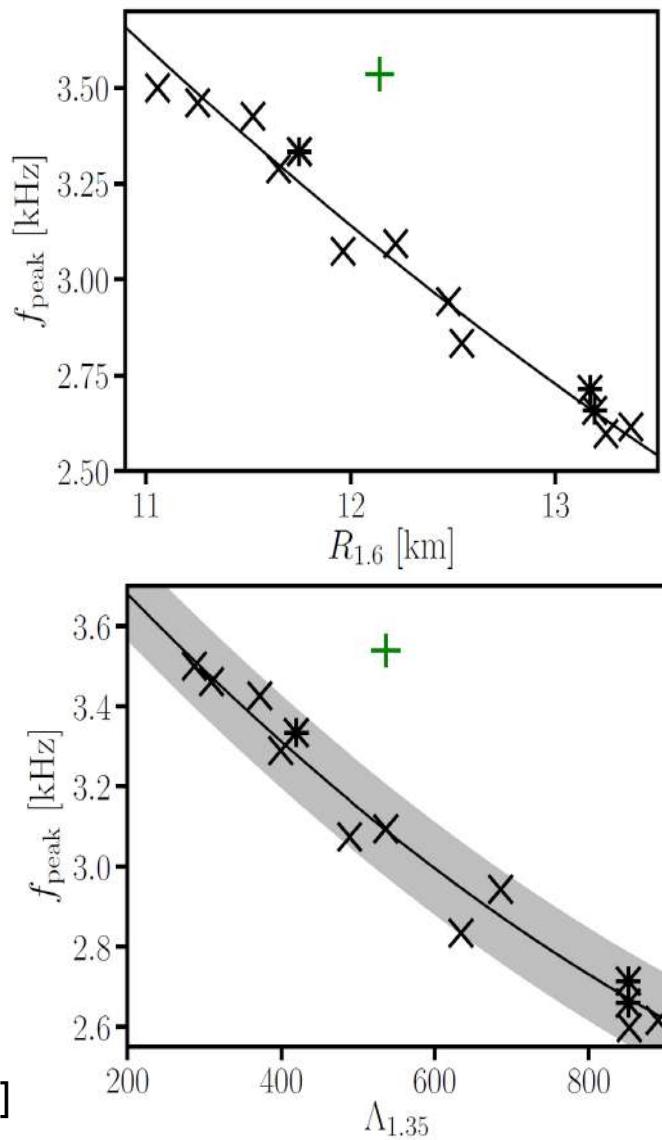
Signal of a deconfinement transition



Strong deviation from $f_{\text{peak}} - R_{1.6}$ relation signals
strong phase transition in NS merger!

Complementarity of f_{peak} from postmerger with
tidal deformability $\Lambda_{1.35}$ from inspiral phase.

A. Bauswein et al., PRL 122 (2019) 061102; [arxiv:1809.01116]



Neutron star phenomenology from TOV eqns.

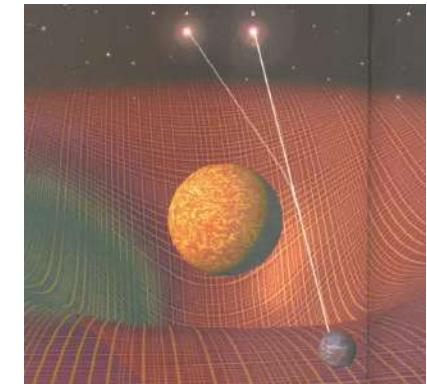
There is a 1:1 correspondence EOS \leftrightarrow M(R)

Tolman-Oppenheimer-Volkoff (TOV) equations



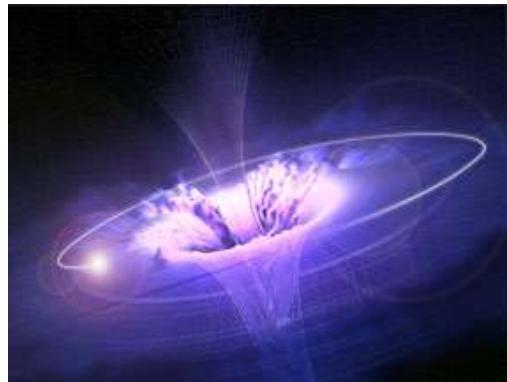
Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$



Non-rotating, spherical masses \rightarrow Schwarzschild Metrics

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + (1 - \frac{2M}{r})^{-1}dr^2 + r^2d\Omega^2$$



Tolman-Oppenheimer-Volkoff eqs.* for
structure and stability of spherical compact stars

$$\frac{dP(r)}{dr} = -G \frac{m(r)\varepsilon(r)}{r^2} \left(1 + \frac{P(r)}{\varepsilon(r)}\right) \left(1 + \frac{4\pi r^3 P(r)}{m(r)}\right) \left(1 - \frac{2Gm(r)}{r}\right)^{-1}$$

Newtonian case GR corrections from EoS and metrics

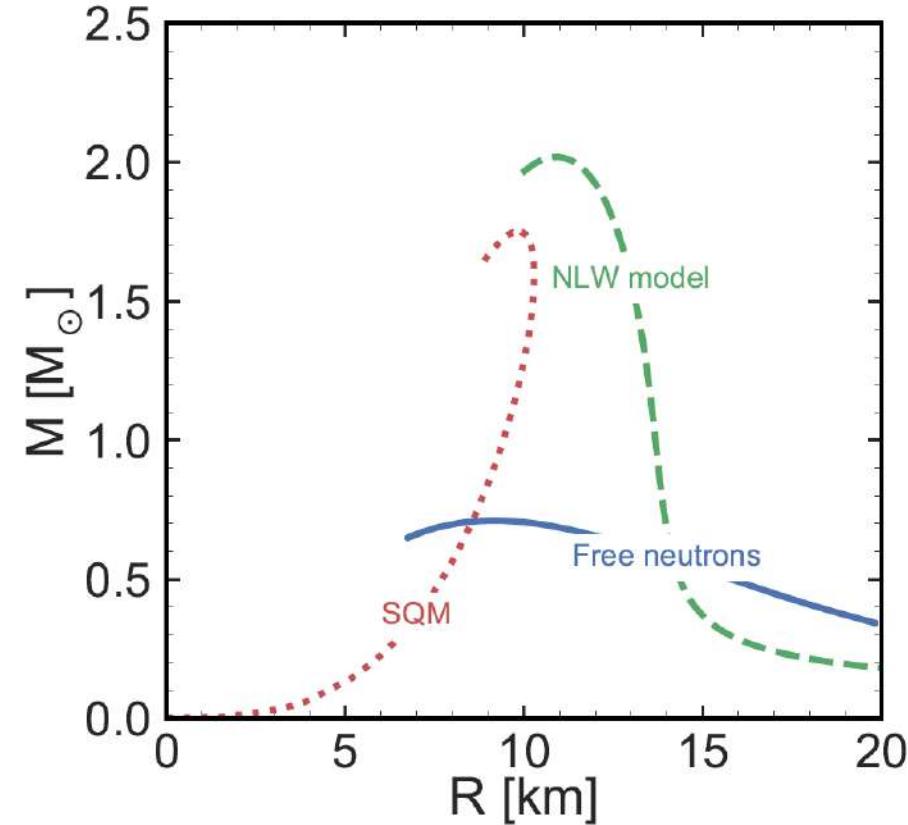
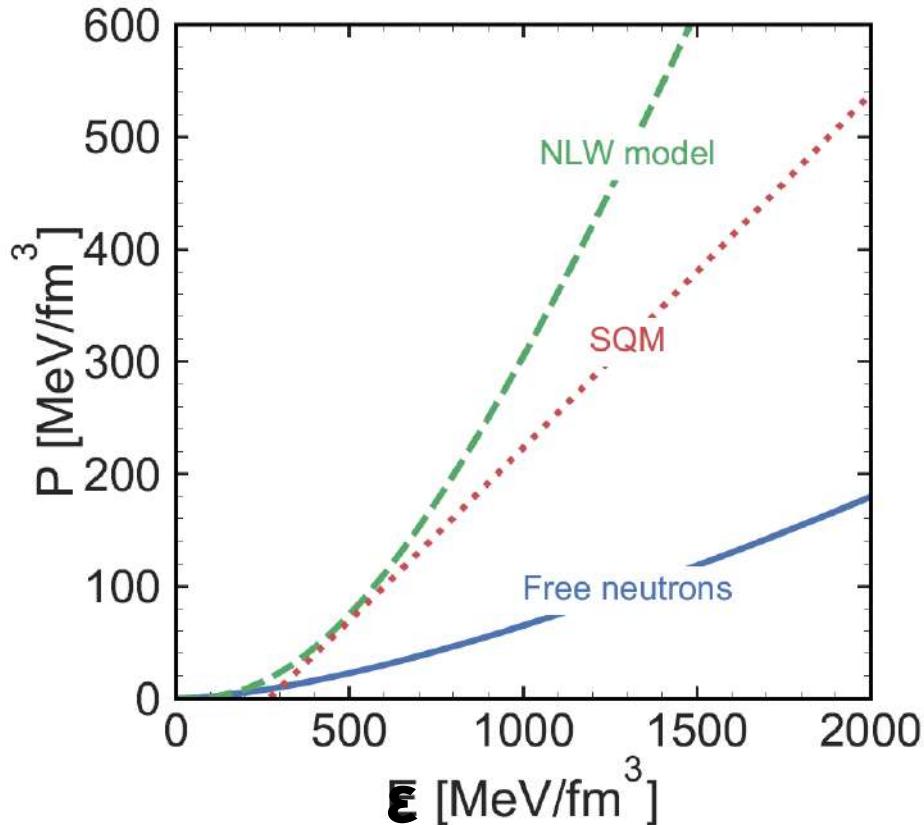
*)R.C. Tolman, Phys. Rev. 55 (1939) 364; J.R. Oppenheimer, G.M. Volkoff, ibid., 374



Neutron star phenomenology from TOV eqns.

There is a 1:1 correspondence EOS $P(\epsilon) \leftrightarrow M(R)$

Tolman-Oppenheimer-Volkoff (TOV) equations - solutions

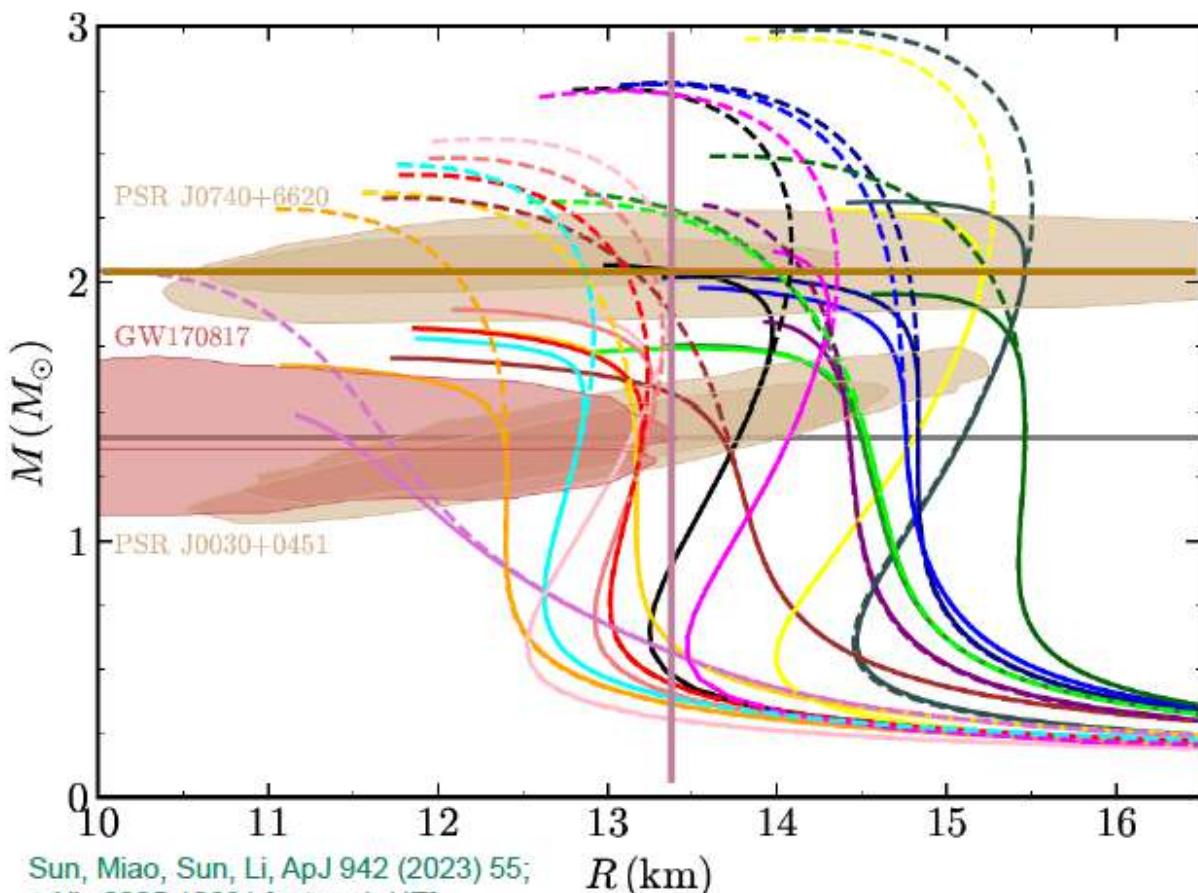


Stiffer equation of state \rightarrow larger radius and larger maximum mass

“Berlin wall” constraint for neutron stars

Realistic hadronic EOS (with strange baryons)

Tension with modern multi-messenger observations by LVC and NICER



Sun, Miao, Sun, Li, ApJ 942 (2023) 55;
arXiv:2205.10631 [astro-ph.HE]

Examples for hadronic EoS without (dashed lines) and with (solid lines) strange baryons. EoS which fulfill the observational constraints should be left of the vertical line at 1.4 M_\odot and should cross the horizontal line for the minimal maximum mass at 2.01 M_\odot . There is no EoS of this sample which fulfills both constraints !!

— LHS	— PK1	— DD2
— RMF 201	— NL3 $\omega\rho$	— PKDD
— NL3	— S271v6	— DD-PC1
— Hybrid	— HC	— FKVW
— TM2	— DD-LZ1	— PC-PK1
— NLSV1	— DD-ME2	— OMEG

From Tab. 2 select EoS which fulfill (w. Y)
 $70 < \Lambda_{1.4} < 580$ and check their M_{max}

EoS	M_{max}	EoS	M_{max}
NL3 $\omega\rho$	1.974	DD2	1.935
DDLZ1	1.989	PKDD	1.781
DD-ME2	1.971	HC	1.828
OMEG	1.862		

“Berlin Wall” constraint for neutron stars?

Mass-radius diagram for purely hadronic EOS

Appearance of hyperons softens the EOS → Limitation for the maximum mass

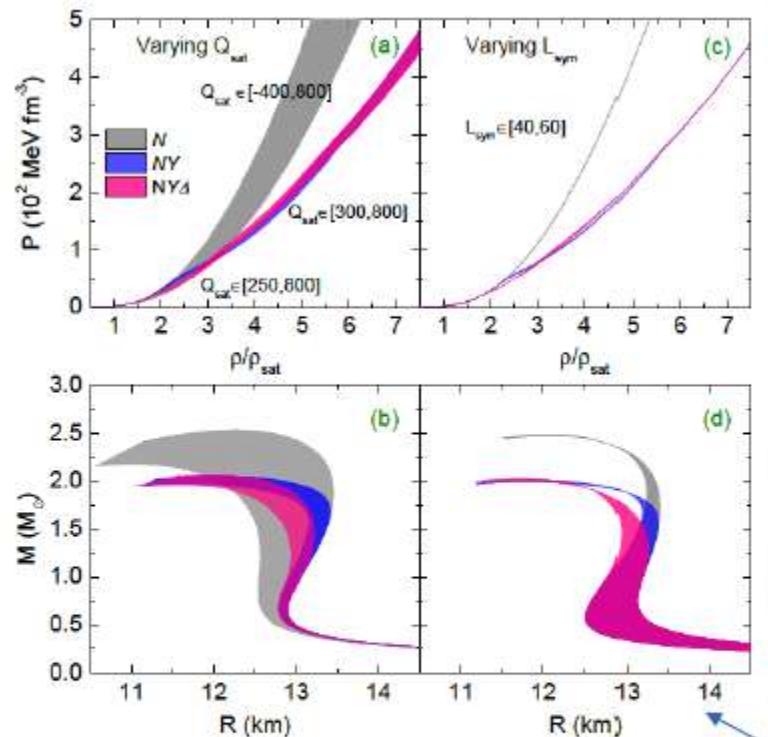


FIG. 4. EoS models and MR relations for N , NY , and $NY\Delta$ compositions of stellar matter. The bands are generated by varying the parameters Q_{sat} [MeV] (a, b) and L_{sym} [MeV] (c, d). The ranges of Q_{sat} and L_{sym} allowed by χ EFT and maximum mass constraints are indicated in the figures.

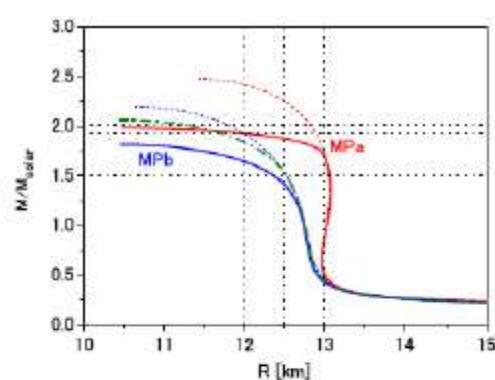


FIG. 7. Neutron-star masses as a function of the radius R . Solid (dashed) curves are with (without) hyperon (Λ and Σ) mixing for ESC+MPa and ESC+MPb. The dot-dashed curve for MPb is with Λ mixing only. Also see the caption of Fig. 3.

Yamamoto et al., Phys.Rev.C 96 (2017) 06580;
arXiv:1708.06163 [nucl-th]

Yamamoto et al., Eur. Phys. J. A 52 (2016) 19;
arXiv:1510.06099 [nucl-th]

Ji & Sedrakian, Phys. Rev. C 100 (2019) 015809;
arXiv:1903.06057 [astro-ph.HE]

Examples for realistic hadronic EoS which suggest a Berlin Wall is inferior to the line $M = 2.0 M_{\odot}$

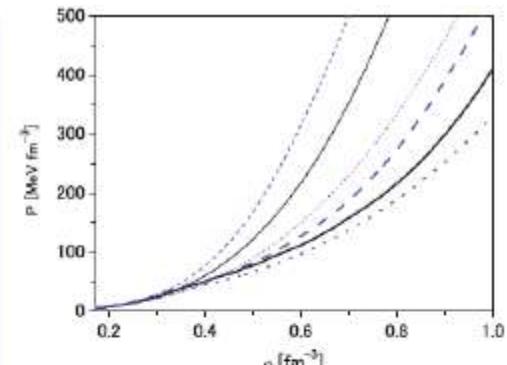


Fig. 8. Pressure P as a function of baryon density ρ . Thick (thin) curves are with (without) hyperon mixing. Solid, dashed and dotted curves are for MPa, MPa⁺ and MPb.

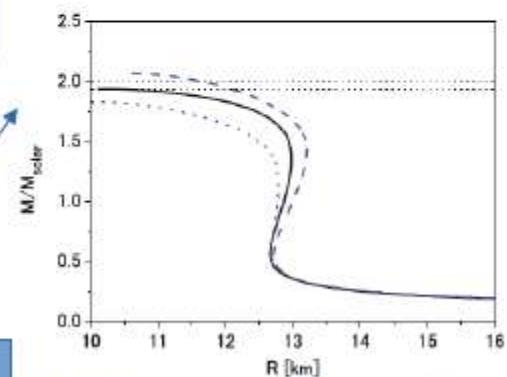
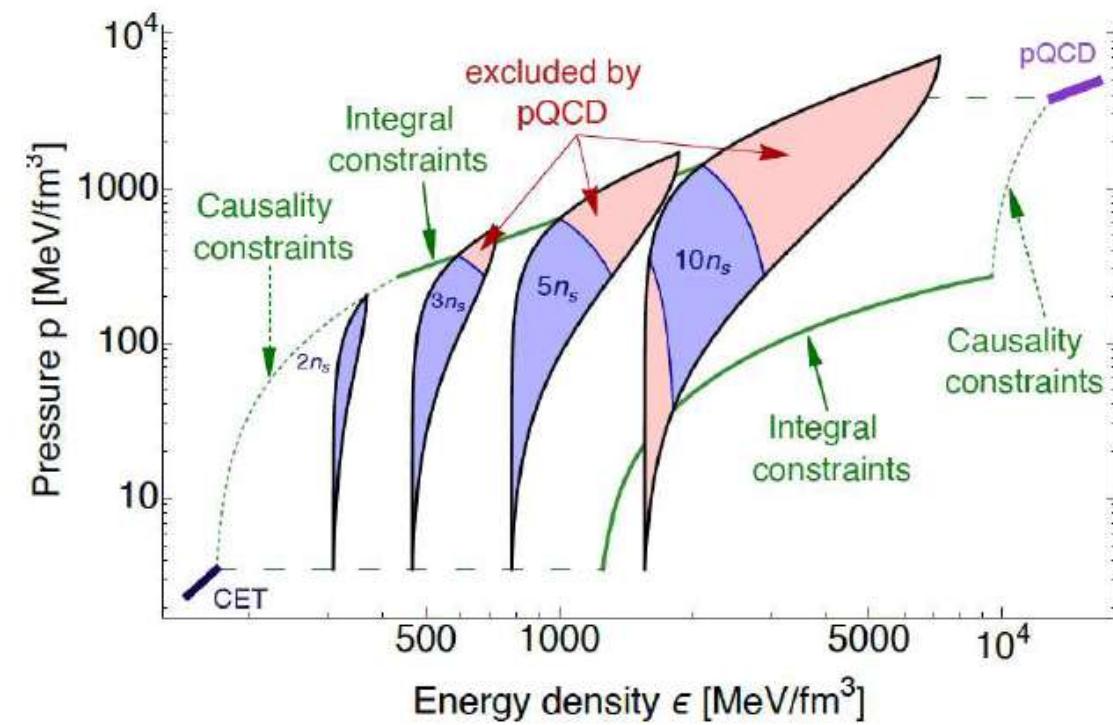
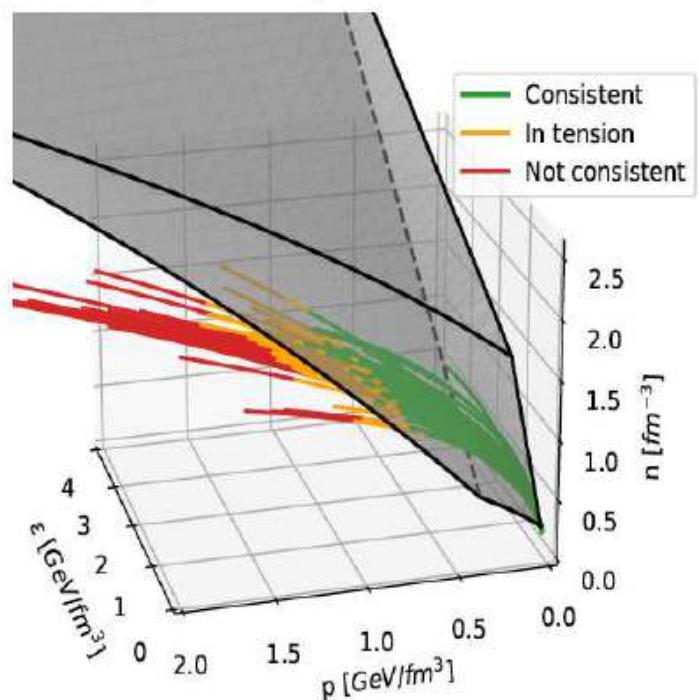


Fig. 9. Neutron-star masses as a function of the radius R . Solid, dashed and dotted curves are for MPa, MPa⁺ and MPb. Two dotted lines show the observed mass $(1.97 \pm 0.04)M_{\odot}$. J1614-2230.

Neutron star EOS constraint from pQCD



Consistency check for neutron star EoS from the CompOSE library



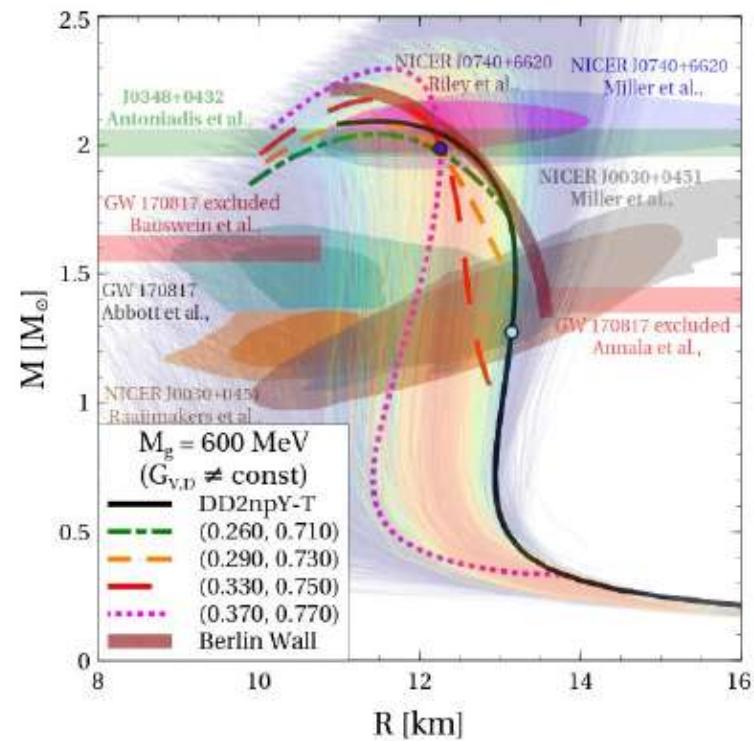
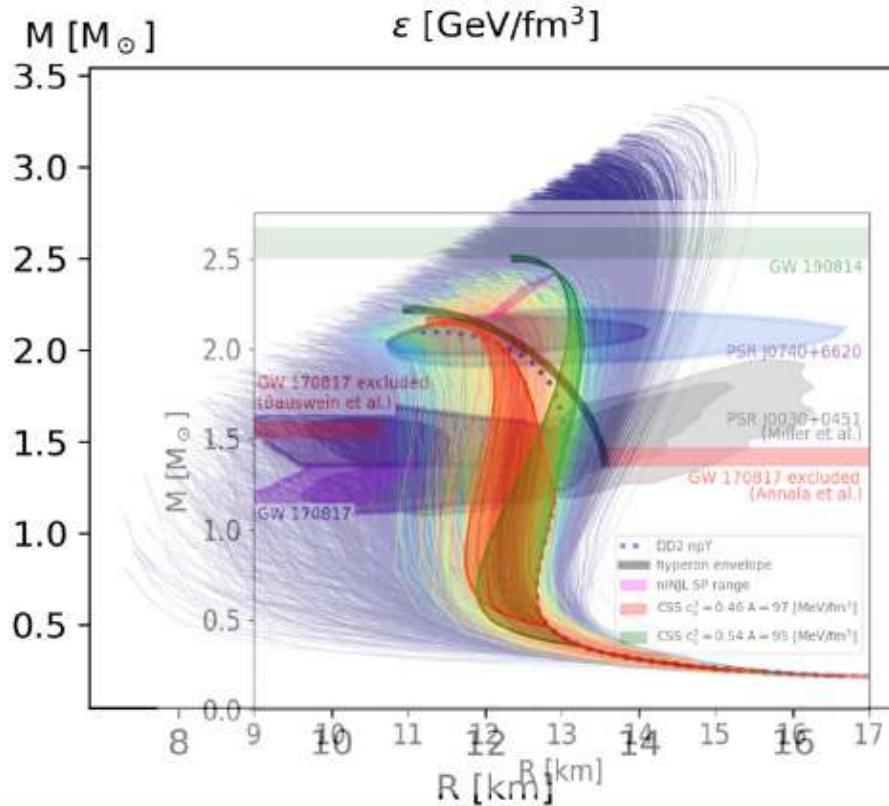
O. Komoltsev and A. Kurkela, Phys. Rev. D 128 (2022) 202701

Result: Not all EoS fulfill the consistency check with pQCD asymptotics! pQCD important for NS!

Breaking the “Berlin wall” constraint

With Bayesian analyses and hybrid EOS

M(R) curves generated by causality, thermodynamic stability and pQCD limit



The conjectured “Berlin Wall” overlaid to the Fig. 2 from Gorda, Komoltsev & Kurkela [2204.11877 [nucl-th]] and hybrid EoS with quark matter described by a CSS model (left) and a confining relativistic density functional (right).

Unified EOS for quark-hadron matter ...

... with multiquark clusters was initiated by Peter Schuck (front right)

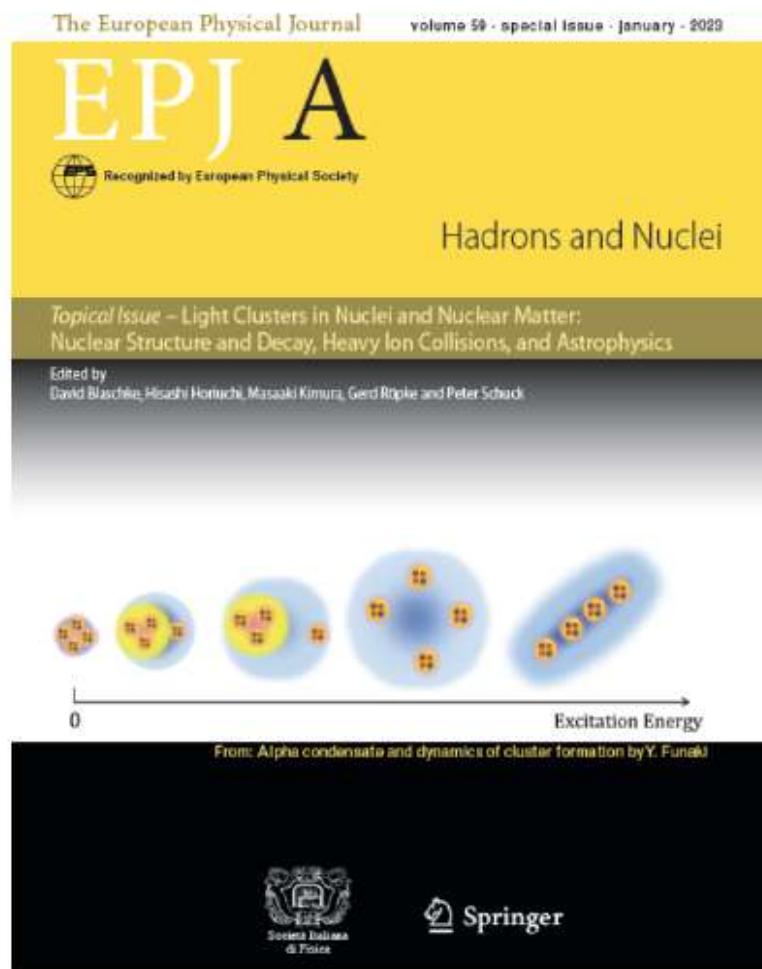


Workshop on "Light Clusters in Nuclei and Nuclear Matter: ..."
ECT* Trento, 2.-6. September 2019

Unified EOS for quark-hadron matter



EPJA Topical Collections (TC)



New TC:

"The Nuclear Many-Body Problem"

Devoted to the legacy of Peter Schuck

Topics:

- The interacting boson model and collective phenomena in nuclear systems
- Nuclear energy density functionals
- Equation of motion method and extended RPA
- Quantum condensates and pairing
- Alpha-particle clustering
- Pions and related experiments, astrophysics
- Applications in solid state physics, quartetting in semiconductor layers, etc.

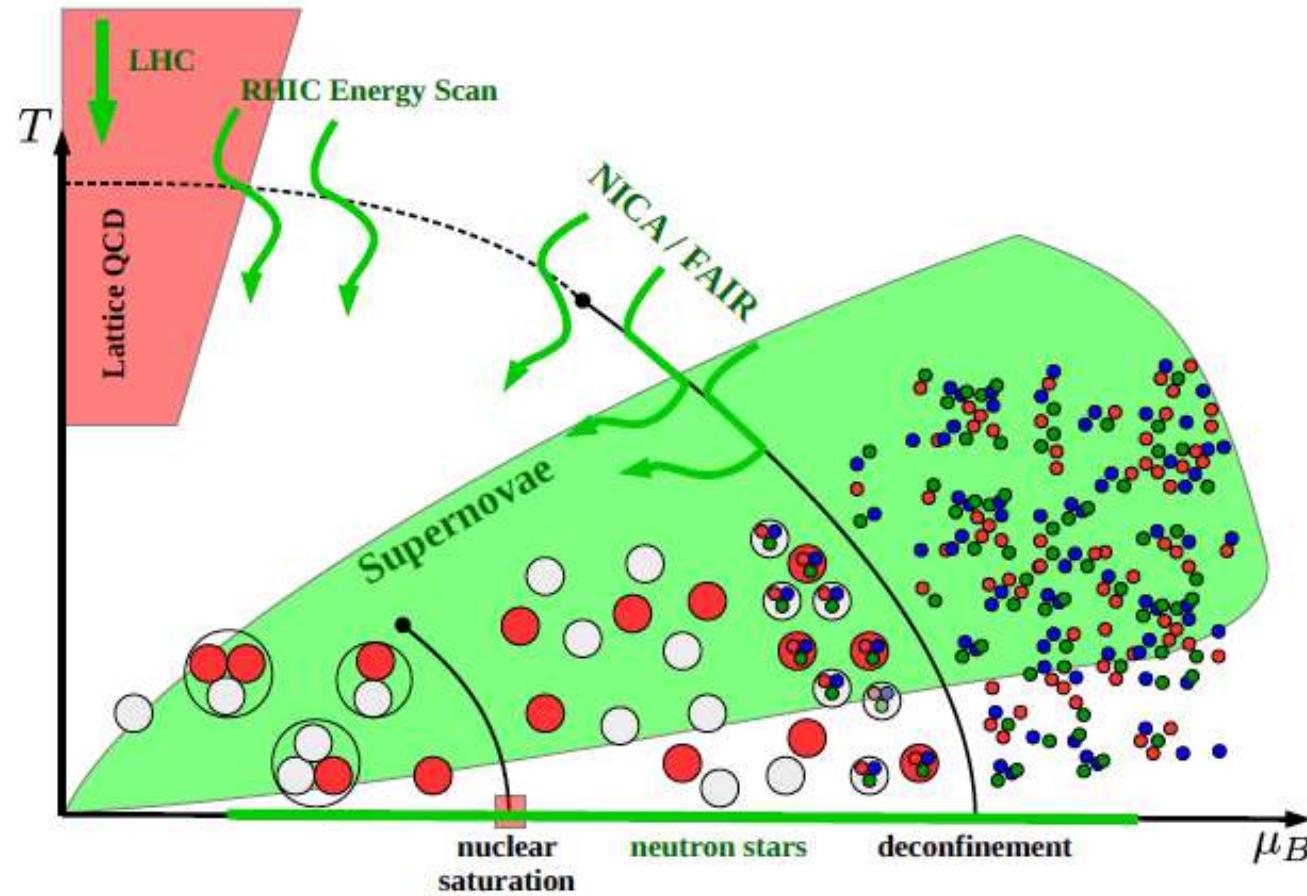
More details on the symposium website and
<https://epja.epj.org/epja-open-calls-for-papers>

New Deadline: 15. August 2023

From ECT* Trento Workshop in 2019

Unified approach to quark-nuclear matter

Clustering aspects in the QCD phase diagram



From: N.-U. Bastian, D.B., et al., Universe 4 (2018) 67; arxiv:1804.10178

Unified approach to quark-nuclear matter

Φ -derivable approach to cluster virial expansion

$$\Omega = \sum_{I=1}^A \Omega_I = \sum_{I=1}^A \left\{ c_I [\text{Tr} \ln (-G_I^{-1}) + \text{Tr}(\Sigma_I G_I)] + \sum_{\substack{i,j \\ i+j=I}} \Phi[G_i, G_j, G_{i+j}] \right\} ,$$

$$G_A^{-1} = G_A^{(0)-1} - \Sigma_A , \quad \Sigma_A(1\dots A, 1'\dots A', z_A) = \frac{\delta \Phi}{\delta G_A(1\dots A, 1'\dots A', z_A)}$$

Stationarity of the thermodynamical potential is implied

$$\frac{\delta \Omega}{\delta G_A(1\dots A, 1'\dots A', z_A)} = 0 .$$

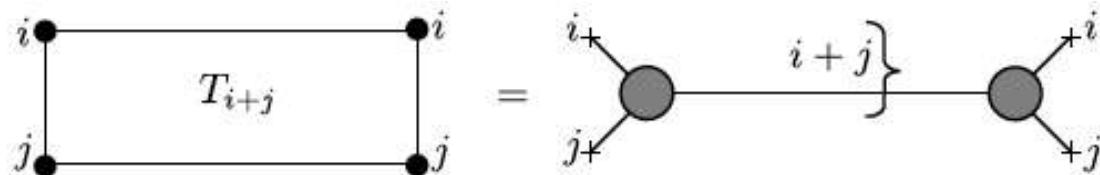
Cluster virial expansion follows for this Φ – functional



Figure: The Φ functional for A –particle correlations with bipartitions $A = i + j$.

Unified approach to quark-nuclear matter

Green's function and T-matrix, separable approx.



The T_A matrix fulfills the Bethe-Salpeter equation in ladder approximation

$$T_{i+j}(1, 2, \dots, A; 1', 2', \dots, A'; z) = V_{i+j} + V_{i+j} G_{i+j}^{(0)} T_{i+j} ,$$

which in the separable approximation for the interaction potential,

$$V_{i+j} = \Gamma_{i+j}(1, 2, \dots, i; i+1, i+2, \dots, i+j) \Gamma_{i+j}(1', 2', \dots, i'; (i+1)', (i+2)', \dots, (i+j)'),$$

leads to the closed expression for the T_A matrix

$$T_{i+j}(1, 2, \dots, i+j; 1', 2', \dots, (i+j)'; z) = V_{i+j} \{1 - \Pi_{i+j}\}^{-1} ,$$

with the generalized polarization function

$$\Pi_{i+j} = \text{Tr} \left\{ \Gamma_{i+j} G_i^{(0)} \Gamma_{i+j} G_j^{(0)} \right\}$$

The one-frequency free i -particle Green's function is defined by the $(i-1)$ -fold Matsubara sum

$$\begin{aligned} G_i^{(0)}(1, 2, \dots, i; \Omega_i) &= \sum_{\omega_1 \dots \omega_{i-1}} \frac{1}{\omega_1 - E(1)} \frac{1}{\omega_2 - E(2)} \dots \frac{1}{\Omega_i - (\omega_1 + \dots + \omega_{i-1}) - E(i)} \\ &= \frac{(1-f_1)(1-f_2)\dots(1-f_i) - (-)^i f_1 f_2 \dots f_i}{\Omega_i - E(1) - E(2) - \dots - E(i)} . \end{aligned}$$

Unified approach to quark-nuclear matter

Useful relationships for many-particle functions

$$G_{i+j}^{(0)} = G_{i+j}^{(0)}(1, 2, \dots, i+j; \Omega_{i+j}) = \sum_{\Omega_i} G_i^{(0)}(1, 2, \dots, i; \Omega_i) G_j^{(0)}(i+1, i+2, \dots, i+j; \Omega_j).$$

Another set of useful relationships follows from the fact that in the ladder approximation both, the full two-cluster ($i + j$ particle) T matrix and the corresponding Greens' function

$$G_{i+j} = G_{i+j}^{(0)} \{1 - \Pi_{i+j}\}^{-1} \quad (1)$$

have similar analytic properties determined by the $i + j$ cluster polarization loop integral and are related by the identity

$$T_{i+j} G_{i+j}^{(0)} = V_{i+j} G_{i+j}. \quad (2)$$

which is straightforwardly proven by multiplying Equation for the T_{i+j} -matrix with $G_{i+j}^{(0)}$ and using Equation (1). Since these two equivalent expressions in Equation (2) are at the same time equivalent to the two-cluster irreducible Φ functional these functional relations follow

$$\begin{aligned} T_{i+j} &= \delta\Phi/\delta G_{i+j}^{(0)}, \\ V_{i+j} &= \delta\Phi/\delta G_{i+j}. \end{aligned}$$

Unified approach to quark-nuclear matter

Generalized Beth-Uhlenbeck EOS from Φ -deriv.

Consider the partial density of the A -particle state defined as

$$n_A(T, \mu) = -\frac{\partial \Omega_A}{\partial \mu} = -\frac{\partial}{\partial \mu} d_A \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left[\ln(-G_A^{-1}) + \text{Tr}(\Sigma_A G_A) \right] + \sum_{\substack{i,j \\ i+j=A}} \Phi[G_i, G_j, G_{i+j}] .$$

Using spectral representation for $F(\omega)$ and Matsubara summation

$$F(i z_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\text{Im} F(\omega)}{\omega - iz_n}, \quad \sum_{z_n} \frac{c_A}{\omega - iz_n} = f_A(\omega) = \frac{1}{\exp[(\omega - \mu)/T] - (-1)^A}$$

with the relation $\partial f_A(\omega)/\partial \mu = -\partial f_A(\omega)/\partial \omega$ we get for Equation (3) now

$$n_A(T, \mu) = -d_A \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_A(\omega) \frac{\partial}{\partial \omega} \left[\text{Im} \ln(-G_A^{-1}) + \text{Im} (\Sigma_A G_A) \right] + \sum_{\substack{i,j \\ i+j=A}} \frac{\partial \Phi[G_i, G_j, G_A]}{\partial \mu} ,$$

where a partial integration over ω has been performed. For two-loop diagrams of the sunset type holds a cancellation³ which generalize here for cluster states

$$d_A \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_A(\omega) \frac{\partial}{\partial \omega} (\text{Re} \Sigma_A \text{Im} G_A) - \sum_{\substack{i,j \\ i+j=A}} \frac{\partial \Phi[G_i, G_j, G_A]}{\partial \mu} = 0 .$$

Using generalized optical theorems we can show that ($G_A = |G_A| \exp(i\delta_A)$)

$$\frac{\partial}{\partial \omega} \left[\text{Im} \ln(-G_A^{-1}) + \text{Im} \Sigma_A \text{Re} G_A \right] = 2 \text{Im} \left[G_A \text{Im} \Sigma_A \frac{\partial}{\partial \omega} G_A^* \text{Im} \Sigma_A \right] = -2 \sin^2 \delta_A \frac{\partial \delta_A}{\partial \omega} .$$

The density in the form of a generalized Beth-Uhlenbeck EoS follows

$$n(T, \mu) = \sum_{i=1}^A n_i(T, \mu) = \sum_{i=1}^A d_i \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_i(\omega) 2 \sin^2 \delta_i \frac{\partial \delta_i}{\partial \omega} .$$

³B. Vanderheyden & G. Baym, J. Stat. Phys. (1998), J.-P. Blaizot et al., PRD (2001)

Unified approach to quark-nuclear matter

Example: deuterons in nuclear matter

The Φ -derivable thermodynamical potential for the nucleon-deuteron system reads

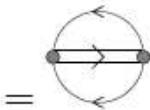
$$\Omega = -\text{Tr}\{\ln(-G_1)\} - \text{Tr}\{\Sigma_1 G_1\} + \text{Tr}\{\ln(-G_2)\} + \text{Tr}\{\Sigma_2 G_2\} + \Phi[G_1, G_2] ,$$

where the full propagators obey the Dyson-Schwinger equations

$$G_1^{-1}(1, z) = z - E_1(p_1) - \Sigma_1(1, z); \quad G_2^{-1}(12, 1'2', z) = z - E_2(p_2) - \Sigma_2(12, 1'2', z),$$

with selfenergies and Φ functional

$$\Sigma_1(1, 1') = \frac{\delta\Phi}{\delta G_1(1, 1')} ; \quad \Sigma_2(12, 1'2', z) = \frac{\delta\Phi}{\delta G_2(12, 1'2', z)} , \quad \Phi = \text{Diagram} ,$$



fulfilling stationarity of the thermodynamic potential $\partial\Omega/\partial G_1 = \partial\Omega/\partial G_2 = 0$.

For the density we obtain the cluster virial expansion

$$n = -\frac{1}{V} \frac{\partial\Omega}{\partial\mu} = n_{\text{qu}}(\mu, T) + 2n_{\text{corr}}(\mu, T) ,$$

with the correlation density in the generalized Beth-Uhlenbeck form

$$n_{\text{corr}} = \int \frac{dE}{2\pi} g(E) 2 \sin^2 \delta(E) \frac{d\delta(E)}{dE} .$$



Cluster virial expansion for nuclear matter

Example: deuterons in nuclear matter

See talk by G. Röpke

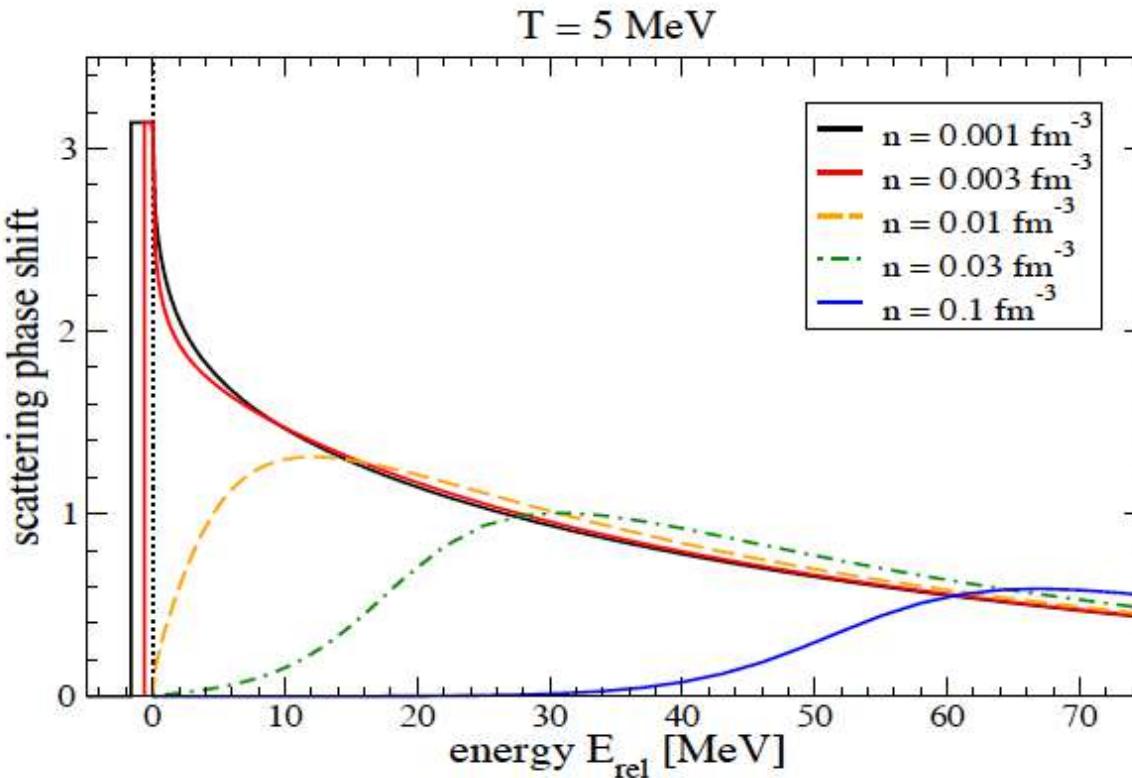
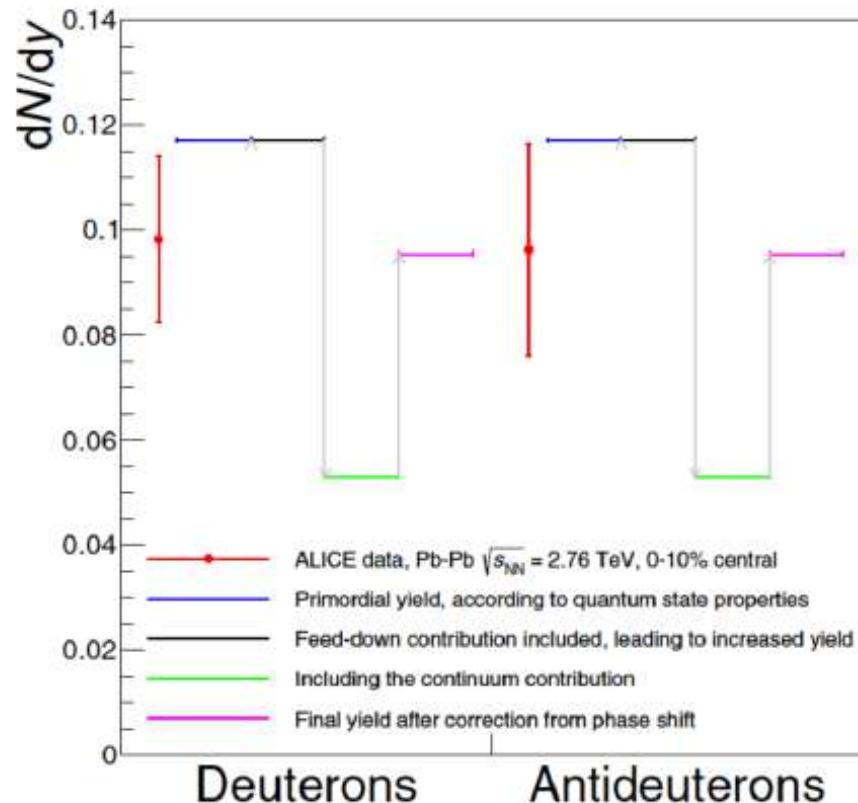


Figure: Integrand of the intrinsic partition function as function of the intrinsic energy in the deuteron channel. Mott dissociation and Levinson's theorem!

Cluster virial expansion for nuclear matter

Example: deuterons in nuclear matter

See talk by G. Röpke



Production of deuterons at the chemical freeze-out temperature $T_{fr} = 156$ MeV in the LHC-ALICE experiment for $\sqrt{s_{NN}} = 2.76$ TeV.

→ "snowballs in hell"

[Oliinychenko et al., PRC 99(2019)]

Important contributions from scattering state continuum in the deuteron channel! Cluster virial approach → Beth-Uhlenbeck EoS

B. Dönigus, G. Röpke, D.B., PRC 106, 044908 (2022)

Unified approach to quark-nuclear matter

Cluster virial expansion for quark-hadron matter

The cluster decomposition of the thermodynamic potential is given as

$$\Omega_{\text{total}}(T, \mu, \phi, \bar{\phi}) = \Omega_{PNJL}(T, \mu, \phi, \bar{\phi}) + \Omega_{\text{pert}}(T, \mu, \phi, \bar{\phi}) + \Omega_{MHRG}(T, \mu, \phi, \bar{\phi}),$$

where the first two terms describe the quark and gluon degrees of freedom via the mean-field thermodynamic potential for quark matter in a gluon background field \mathcal{U}

$$\Omega_{PNJL}(T, \mu, \phi, \bar{\phi}) = \Omega_Q(T, \mu, \phi, \bar{\phi}) + \mathcal{U}(T, \phi, \bar{\phi})$$

with a perturbative correction $\Omega_{\text{pert}}(T, \mu, \phi, \bar{\phi})$.

The Mott-Hadron-Resonance-Gas (MHRG) part for the multi-quark clusters is

$$\Omega_{MHRG}(T, \mu, \phi, \bar{\phi}) = \sum_{i=M, B, \dots} \Omega_i(T, \mu, \phi, \bar{\phi}),$$

where the multi-quark states, described by the GBU formula for color-singlet species:

$$\Omega_i(T, \mu, \phi, \bar{\phi}) = \pm d_i \int_0^\infty \frac{dp}{2\pi^2} p^2 \int_0^\infty \frac{dM}{\pi} \frac{M}{E_p} \left\{ f_\phi^{(a),+} + f_\phi^{(a),-} \right\} \Big|_{\phi=1} \delta_i(M, T, \mu),$$

color-triplet species (color antitriplet is analogous):

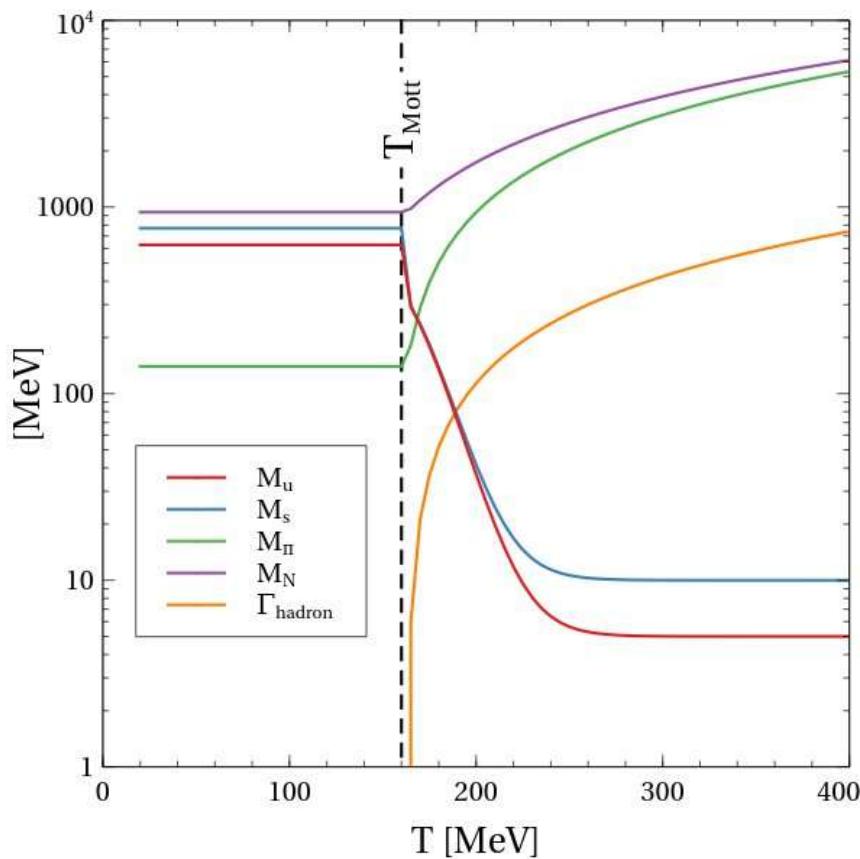
$$\Omega_i(T, \mu, \phi, \bar{\phi}) = \pm d_i \int_0^\infty \frac{dp}{2\pi^2} p^2 \int_0^\infty \frac{dM}{\pi} \frac{M}{E_p} \left\{ f_\phi^{(a),+} + [f_\phi^{(a),-}]^* \right\} \delta_i(M, T, \mu),$$

where d_i is the degeneracy factor, a is the net number of valence quarks in the cluster

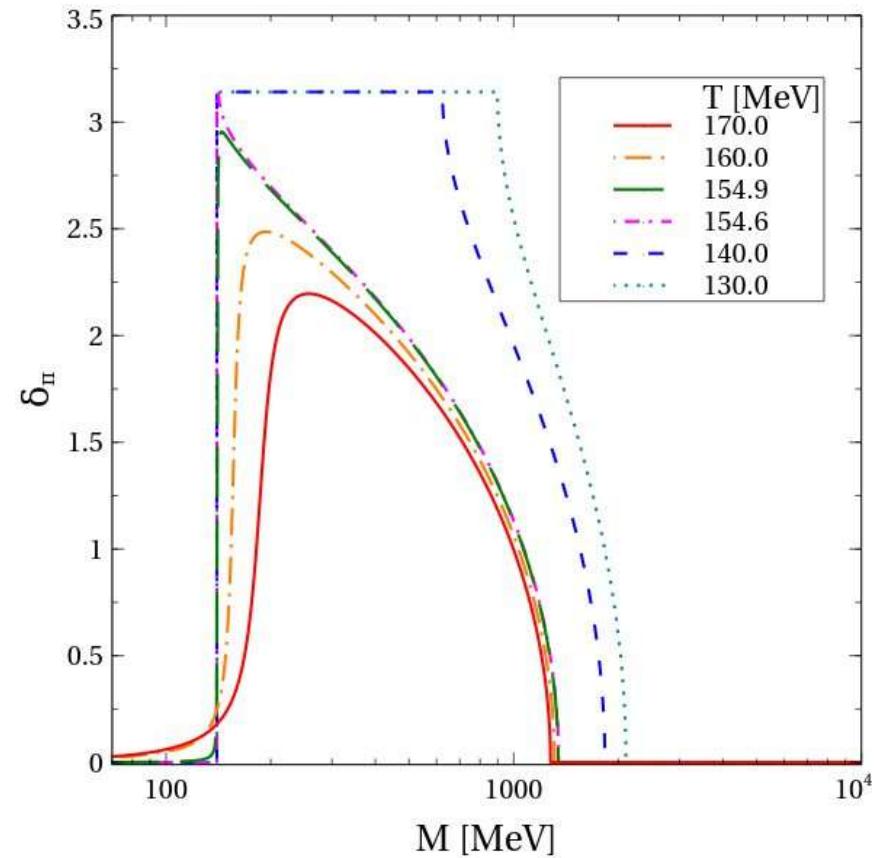
Unified approach to quark-hadron matter

Results for a schematic model

Quark and hadron mass spectrum



Hadron phase shift with Mott effect, Levinson theorem

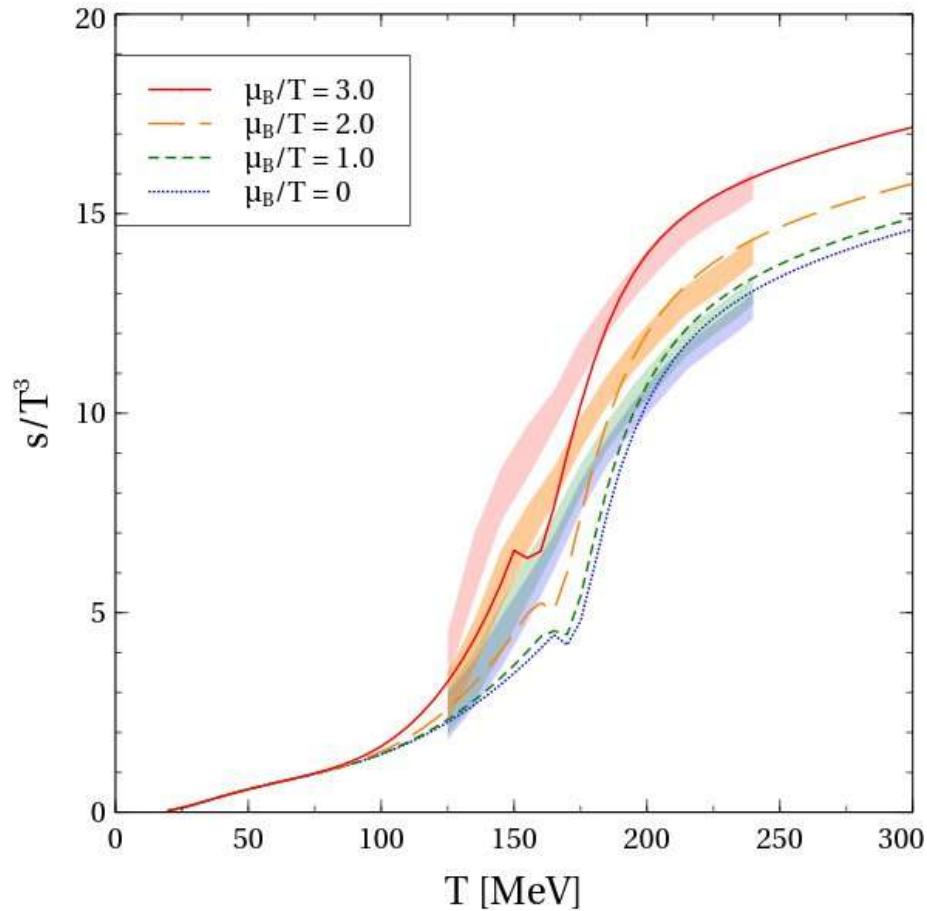
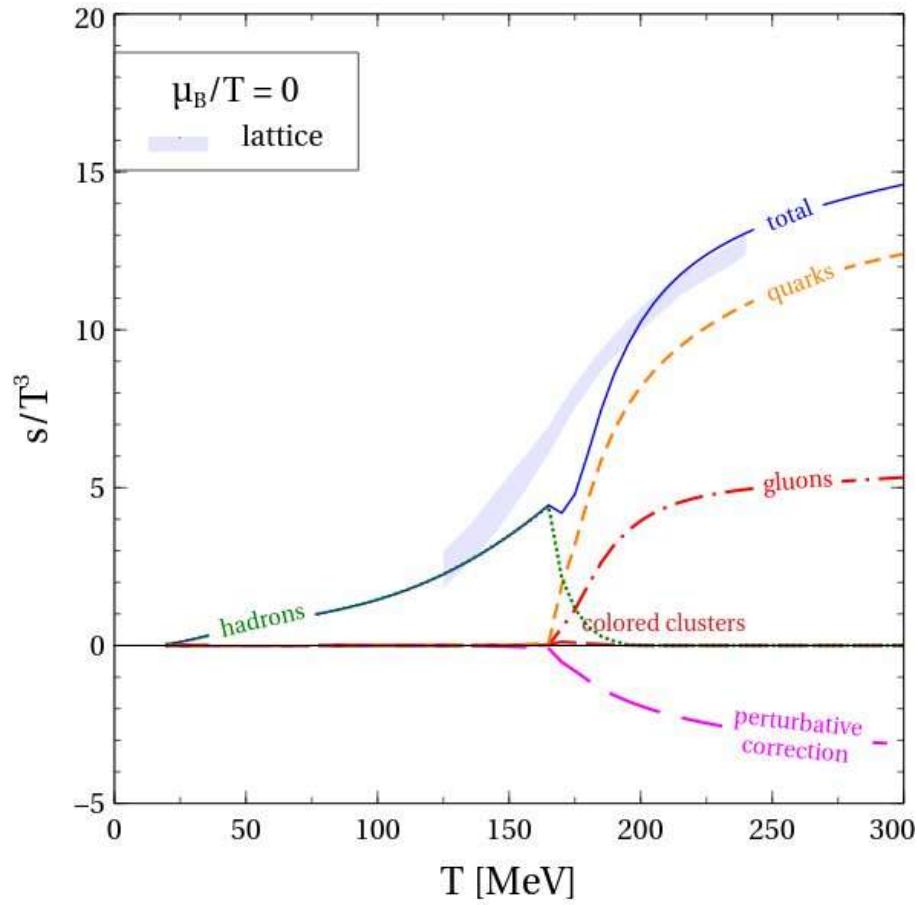


D.B., M. Ciernak, O. Ivanytskyi & G. Röpke, in prep. for EPJA topical collection



Unified approach to quark-hadron matter

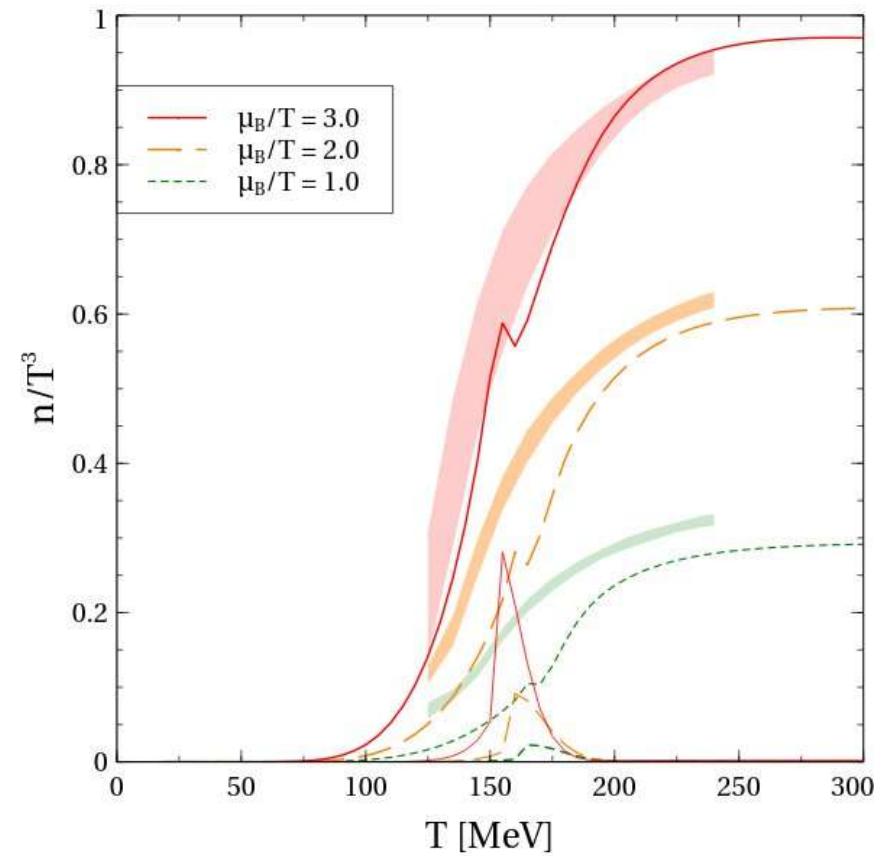
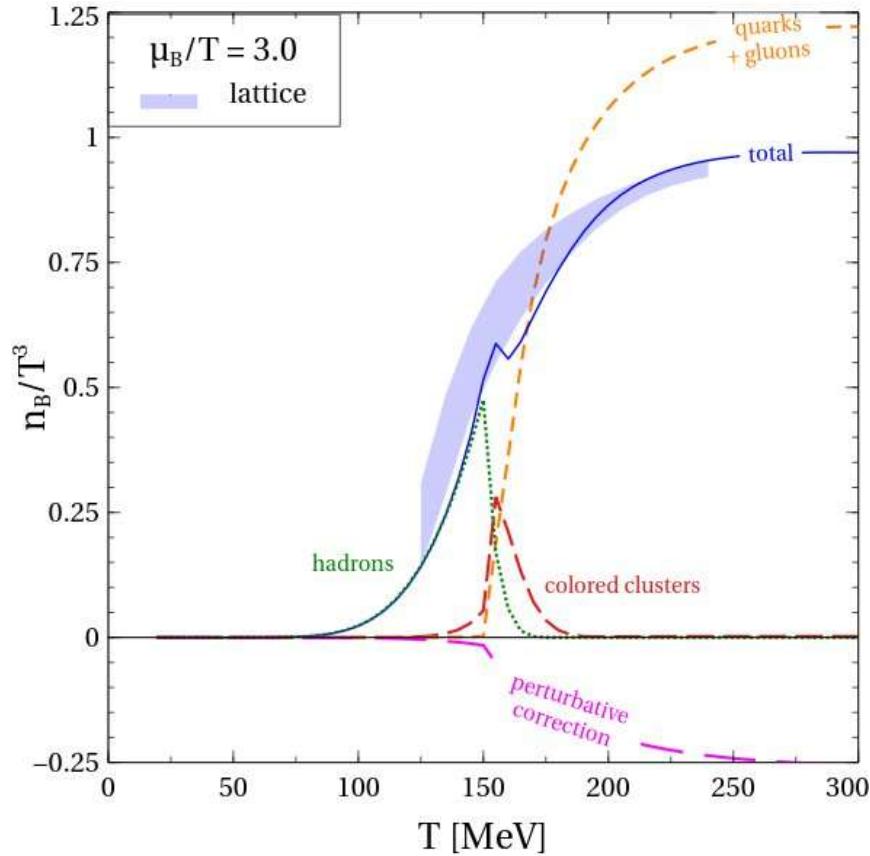
Results for the entropy density at $\mu/T = \text{const}$



D.B., M. Ciernak, O. Ivanytskyi & G. Röpke, in prep. for EPJA topical collection

Unified approach to quark-hadron matter

Results for the baryon density at $\mu/T = \text{const}$



D.B., M. Ciernak, O. Ivanytskyi & G. Röpke, in prep. for EPJA topical collection



Relativistic density functionals for QCD

String-flip model for quark matter



Röpke, Blaschke, Schulz, PRD34 (1986) 3499

$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q \exp \left\{ \int_0^\beta d\tau \int_V d^3x [\mathcal{L}_{\text{eff}} + \bar{q}\gamma_0\hat{\mu}q] \right\}, \quad q = \begin{pmatrix} q_u \\ q_d \end{pmatrix}, \quad \hat{\mu} = \text{diag}(\mu_u, \mu_d)$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{free}} - \boxed{U(\bar{q}q, \bar{q}\gamma_0q)}, \quad \mathcal{L}_{\text{free}} = \bar{q} \left(-\gamma_0 \frac{\partial}{\partial \tau} + i\vec{\gamma} \cdot \vec{\nabla} - \hat{m} \right) q, \quad \hat{m} = \text{diag}(m_u, m_d)$$

General nonlinear functional of quark density bilinears: scalar, vector, isovector, diquark ...
Expansion around the expectation values:

$$U(\bar{q}q, \bar{q}\gamma_0q) = U(n_s, n_v) + (\bar{q}q - n_s)\Sigma_s + (\bar{q}\gamma_0q - n_v)\Sigma_v + \dots,$$

$$\langle \bar{q}q \rangle = n_s = \sum_{f=u,d} n_{s,f} = - \sum_{f=u,d} \frac{T}{V} \frac{\partial}{\partial m_f} \ln \mathcal{Z}, \quad \Sigma_s = \left. \frac{\partial U(\bar{q}q, \bar{q}\gamma_0q)}{\partial (\bar{q}q)} \right|_{\bar{q}q=n_s} = \frac{\partial U(n_s, n_v)}{\partial n_s},$$

$$\langle \bar{q}\gamma_0q \rangle = n_v = \sum_{f=u,d} n_{v,f} = \sum_{f=u,d} \frac{T}{V} \frac{\partial}{\partial \mu_f} \ln \mathcal{Z}, \quad \Sigma_v = \left. \frac{\partial U(\bar{q}q, \bar{q}\gamma_0q)}{\partial (\bar{q}\gamma_0q)} \right|_{\bar{q}\gamma_0q=n_v} = \frac{\partial U(n_s, n_v)}{\partial n_v}$$

$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q \exp \{ \mathcal{S}_{\text{quasi}}[\bar{q}, q] - \beta V \Theta[n_s, n_v] \}, \quad \Theta[n_s, n_v] = U(n_s, n_v) - \Sigma_s n_s - \Sigma_v n_v$$

$$\mathcal{S}_{\text{quasi}}[\bar{q}, q] = \beta \sum_n \sum_{\vec{p}} \bar{q} G^{-1}(\omega_n, \vec{p}) q, \quad G^{-1}(\omega_n, \vec{p}) = \gamma_0(-i\omega_n + \hat{\mu}^*) - \vec{\gamma} \cdot \vec{p} - \hat{m}^*$$

Relativistic density functionals for QCD

$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q \exp \{ \mathcal{S}_{\text{quasi}}[\bar{q}, q] - \beta V \Theta[n_s, n_v] \} , \quad \Theta[n_s, n_v] = U(n_s, n_v) - \Sigma_s n_s - \Sigma_v n_v$$

$$\mathcal{Z}_{\text{quasi}} = \int \mathcal{D}\bar{q}\mathcal{D}q \exp \{ \mathcal{S}_{\text{quasi}}[\bar{q}, q] \} = \det[\beta G^{-1}] , \quad \ln \det A = \text{Tr} \ln A$$

$$\begin{aligned} P_{\text{quasi}} &= \frac{T}{V} \ln \mathcal{Z}_{\text{quasi}} = \frac{T}{V} \text{Tr} \ln [\beta G^{-1}] && \text{"no sea" approximation ...} \\ &= 2N_c \sum_{f=u,d} \int \frac{d^3 p}{(2\pi)^3} \left\{ T \ln \left[1 + e^{-\beta(E_f^* - \mu_f^*)} \right] + T \ln \left[1 + e^{-\beta(E_f^* + \mu_f^*)} \right] \right\} \end{aligned}$$

$$P_{\text{quasi}} = \sum_{f=u,d} \int \frac{dp}{\pi^2} \frac{p^4}{E_f^*} [f(E_f^* - \mu_f^*) + f(E_f^* + \mu_f^*)] \quad \begin{aligned} E_f^* &= \sqrt{p^2 + m_f^{*2}} \\ f(E) &= 1/[1 + \exp(\beta E)] \end{aligned}$$

$$P = \sum_{f=u,d} \int_0^{p_{\text{F},f}} \frac{dp}{\pi^2} \frac{p^4}{E_f^*} - \Theta[n_s, n_v] , \quad p_{\text{F},f} = \sqrt{\mu_f^{*2} - m_f^{*2}}$$

$$\begin{aligned} \hat{m}^* &= \hat{m} + \Sigma_s \\ \hat{\mu}^* &= \hat{\mu} - \Sigma_v \end{aligned}$$

Selfconsistent densities

$$n_s = - \sum_{f=u,d} \frac{\partial P}{\partial m_f} = \frac{3}{\pi^2} \sum_{f=u,d} \int_0^{p_{\text{F},f}} dp p^2 \frac{m_f^*}{E_f^*} , \quad n_v = \sum_{f=u,d} \frac{\partial P}{\partial \mu_f} = \frac{3}{\pi^2} \sum_{f=u,d} \int_0^{p_{\text{F},f}} dp p^2 = \frac{p_{\text{F},u}^3 + p_{\text{F},d}^3}{\pi^2} .$$

Relativistic density functionals for QCD

String-flip model for quark matter

Density functional for the SFM

$$U(n_s, n_v) = D(n_v)n_s^{2/3} + an_v^2 + \frac{bn_v^4}{1 + cn_v^2},$$

Quark selfenergies

$$\Sigma_s = \frac{2}{3}D(n_v)n_s^{-1/3}, \quad \text{Quark "confinement"}$$

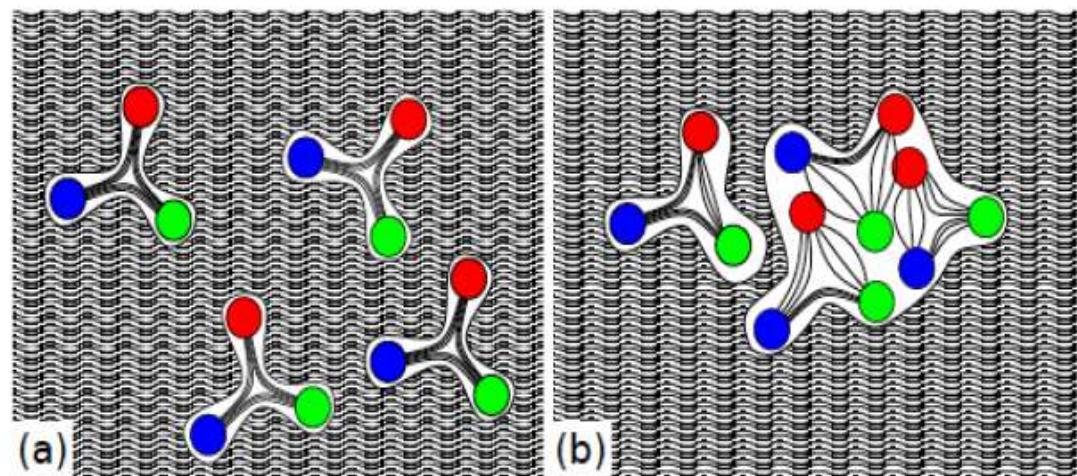
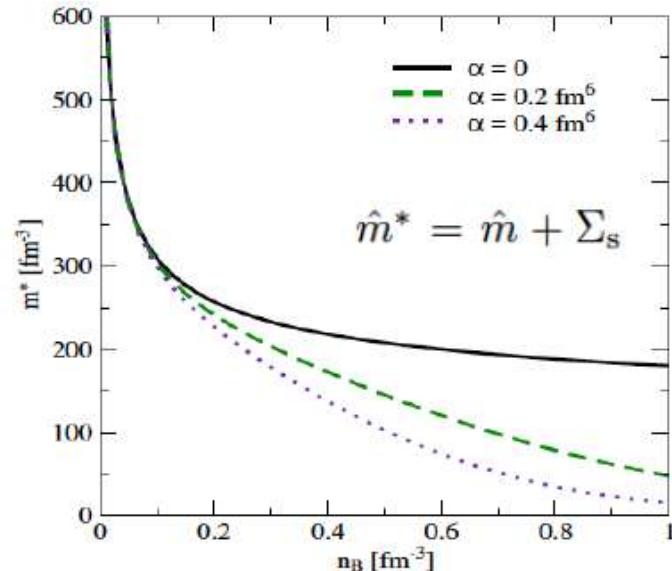
$$\Sigma_v = 2an_v + \frac{4bn_v^3}{1 + cn_v^2} - \frac{2bcn_v^5}{(1 + cn_v^2)^2} + \frac{\partial D(n_v)}{\partial n_v}n_s^{2/3}$$

String tension & confinement
due to dual Meissner effect
(dual superconductor model)

$$D(n_v) = D_0\Phi(n_v)$$

Effective screening of the
string tension in dense matter
by a reduction of the available
volume $\alpha = v|v|/2$

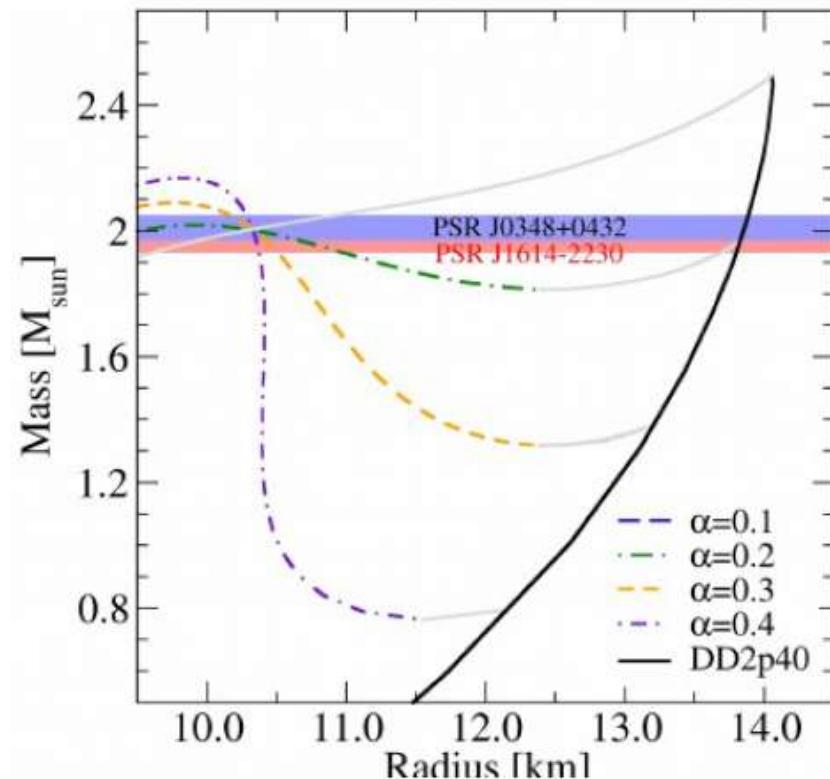
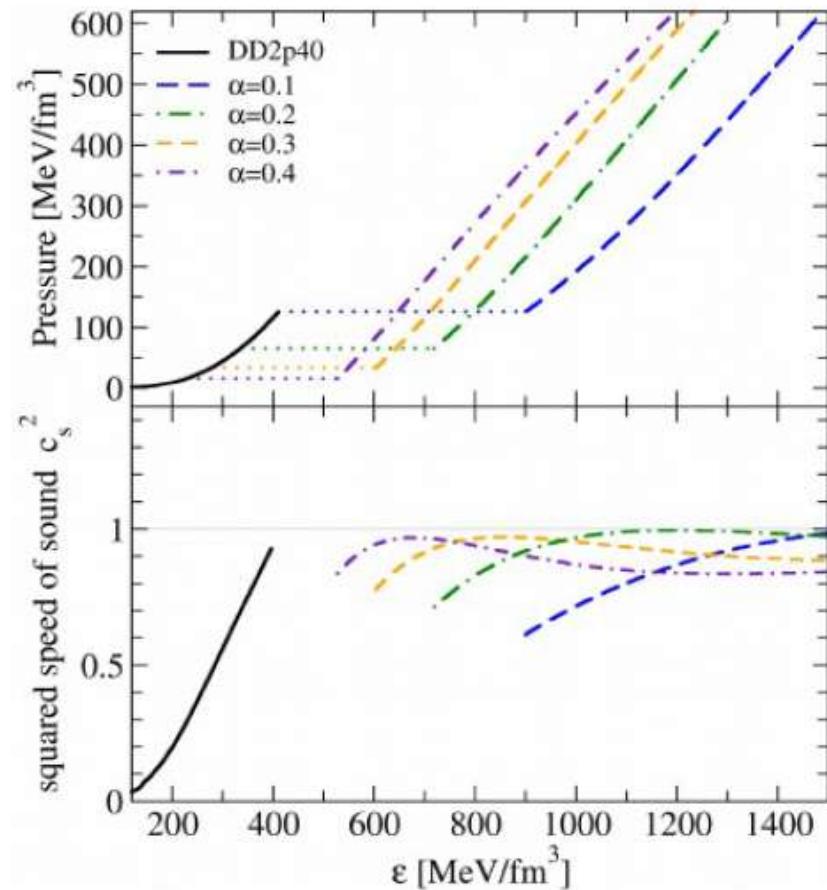
$$\Phi(n_B) = \begin{cases} 1, & \text{if } n_B < n_0 \\ e^{-\alpha(n_B - n_0)^2}, & \text{if } n_B > n_0 \end{cases}$$



Relativistic density functionals for QCD

String-flip model for quark matter

Results for 1st order phase transition by Maxwell construction with DD2p40



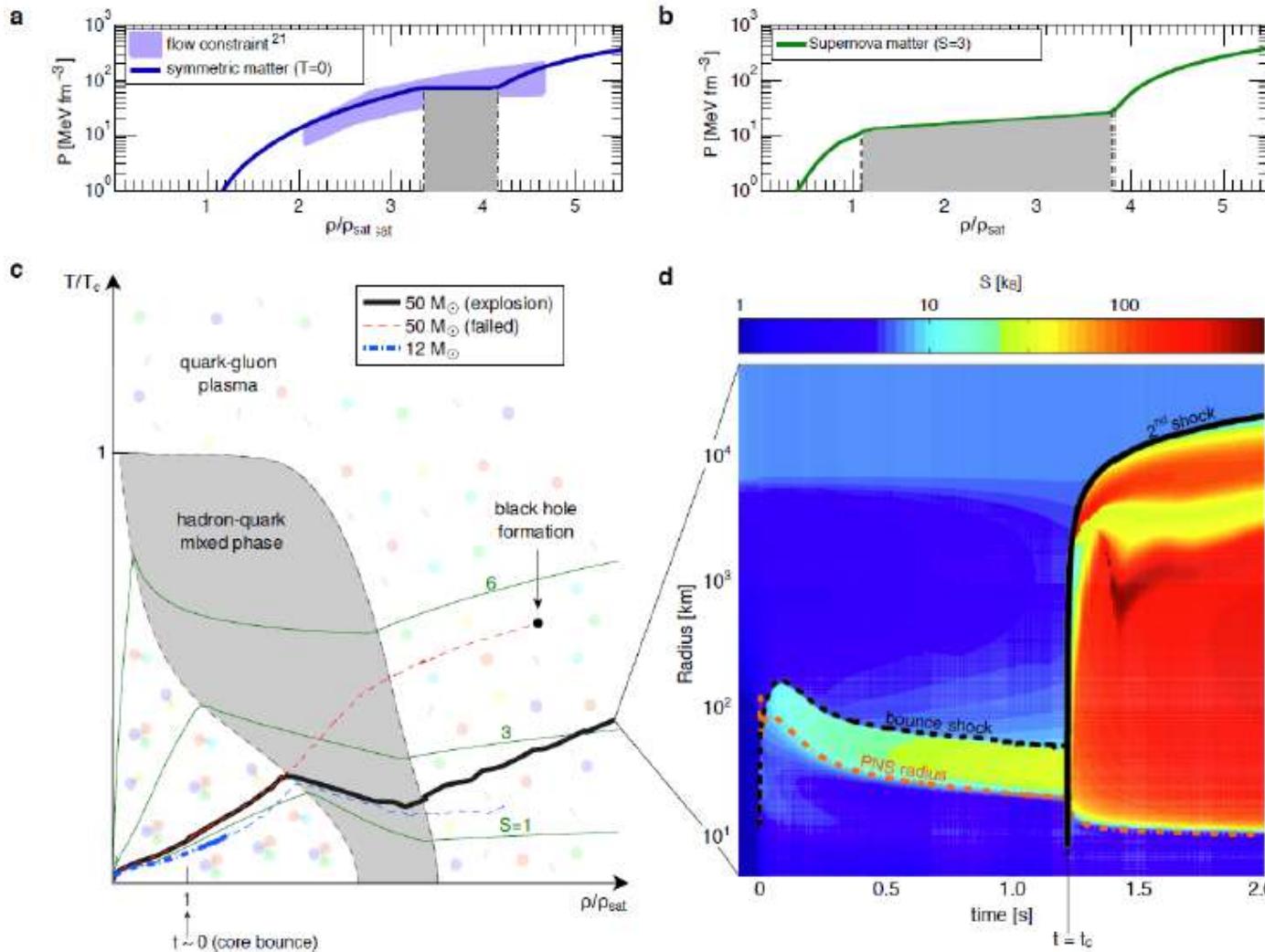
Kaltenborn, Bastian, Blaschke, arXiv:1701.04400



Phys. Rev. D 96, 056024 (2017)

Deconfinement as supernova engine

Of massive blue supergiant star explosions



T. Fischer et al., Nature Astronomy 2, 960 (2018)

Relativistic density functional for quark matter

With chiral symmetry, color SC & confinement

Lagrangian

$$\mathcal{L} = \bar{q}(i\cancel{\partial} - \hat{m})q - \mathcal{U} + \mathcal{L}_V + \mathcal{L}_I + \mathcal{L}_D$$

- **Scalar & pseudoscalar interaction channels**

$$\mathcal{U} = G_0 \left[(1 + \alpha) \langle \bar{q}q \rangle_0^2 - (\bar{q}q)^2 - (\bar{q}i\vec{\tau}\gamma_5 q)^2 \right]^{\frac{1}{3}}$$

(motivated by String Flip Model, χ -dynamics, quark "confinement")

- **Vector-isoscalar interaction channel**

$$\mathcal{L}_V = -G_V(\bar{q}\gamma_\mu q)^2$$

(motivated by gluon exchange, stiff EoS needed to reach $2M_\odot$)

- **Vector-isovector interaction channel**

$$\mathcal{L}_I = -G_I(\bar{q}\gamma_\mu \vec{\tau} q)^2$$

(motivated by gluon exchange, isospin sensitive interaction)

- **Diquark interaction channel**

$$\mathcal{L}_D = G_D \sum_{A=2,5,7} (\bar{q}i\gamma_5 \tau_2 \lambda_A q^c)(\bar{q}^c i\gamma_5 \tau_2 \lambda_A q)$$

(motivated by Cooper theorem, color superconductivity)

Relativistic density functional for quark matter

What is new?

O. Ivanytskyi & D.B., Phys. Rev. D 105 (2022) 114042

Interaction $\mathcal{U} = D_0 [(1 + \alpha)\langle\bar{q}q\rangle_0^2 - (\bar{q}q)^2 - (\bar{q}i\vec{\tau}\gamma_5 q)^2]^{\varkappa}$

- **Parameters**

D_0 - dimensionfull coupling, controls interaction strength

α - dimensionless constant, controls vacuum quark mass

$\langle\bar{q}q\rangle_0$ - χ -condensate in vacuum (introduced for the sake of convenience)

$$\varkappa = 1/3$$



motivated by String Flip model

$$\mathcal{U}_{SFM} \propto \langle q^+ q \rangle^{2/3}$$

$$\varkappa = 1$$



Nambu–Jona-Lasinio model

$$\Sigma_{SFM} = \frac{\partial \mathcal{U}_{SFM}}{\partial \langle q^+ q \rangle} \propto \langle q^+ q \rangle^{-1/3} \propto \text{separation}$$

- **Dimensionality**

$$[\mathcal{U}] = \text{energy}^4$$

$$[\bar{q}q] = \text{energy}^3$$

$$\Rightarrow [D_0]_{\varkappa=1/3} = \text{energy}^2 = [\text{string tension}]$$

self energy = string tension × separation \Rightarrow **confinement**



Relativistic density functional for quark matter

Expansion around mean fields

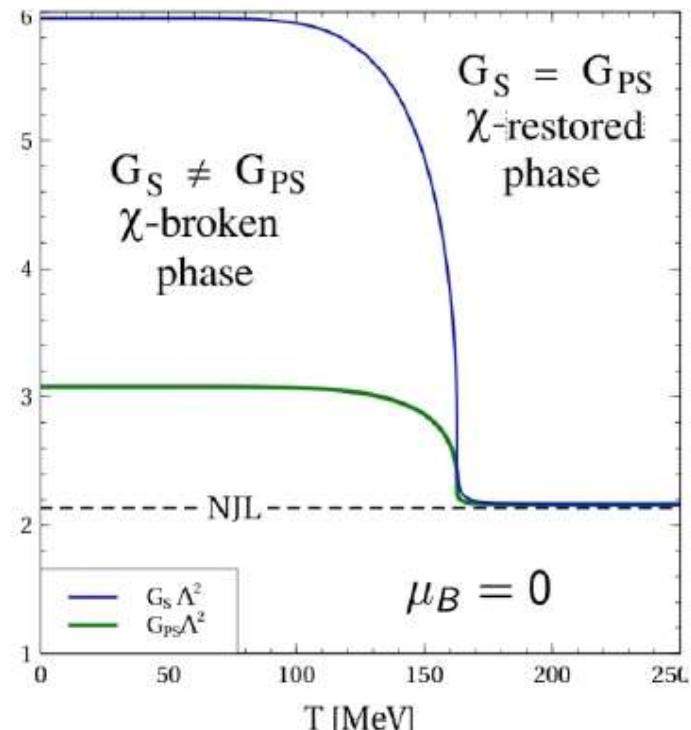
$$\mathcal{U} = \underbrace{\mathcal{U}_{MF}}_{\text{0}^{\text{th}} \text{ order}} + \underbrace{(\bar{q}q - \langle \bar{q}q \rangle) \Sigma_S}_{\text{1}^{\text{st}} \text{ order}} - \underbrace{G_S (\bar{q}q - \langle \bar{q}q \rangle)^2}_{\text{2}^{\text{nd}} \text{ order}} - G_{PS} (\bar{q}i\vec{\tau}\gamma_5 q)^2 + \dots$$

- Mean-field scalar self-energy

$$\Sigma_S = \frac{\partial \mathcal{U}_{MF}}{\partial \langle \bar{q}q \rangle}$$

- Effective medium dependent couplings

$$G_S = -\frac{1}{2} \frac{\partial^2 \mathcal{U}_{MF}}{\partial \langle \bar{q}q \rangle^2}, \quad G_{PS} = -\frac{1}{6} \frac{\partial^2 \mathcal{U}_{MF}}{\partial \langle \bar{q}i\vec{\tau}\gamma_5 q \rangle^2}$$



Relativistic density functional for quark matter

Comparison to Nambu—Jona-Lasinio model

$$\mathcal{L} = \overline{q}(i\cancel{\partial} - \underbrace{(m + \Sigma_S)}_{\text{effective mass } m^*})q + G_S(\overline{q}q)^2 + G_{PS}(\overline{q}i\vec{\tau}\gamma_5 q)^2 + \dots + \mathcal{L}_V + \mathcal{L}_D$$

- **Similarities:**

- current-current interaction
- (pseudo)scalar, vector, diquark, ... channels

- **Differences:**

- high m^* at low $T, \mu \Rightarrow \text{"confinement"}$

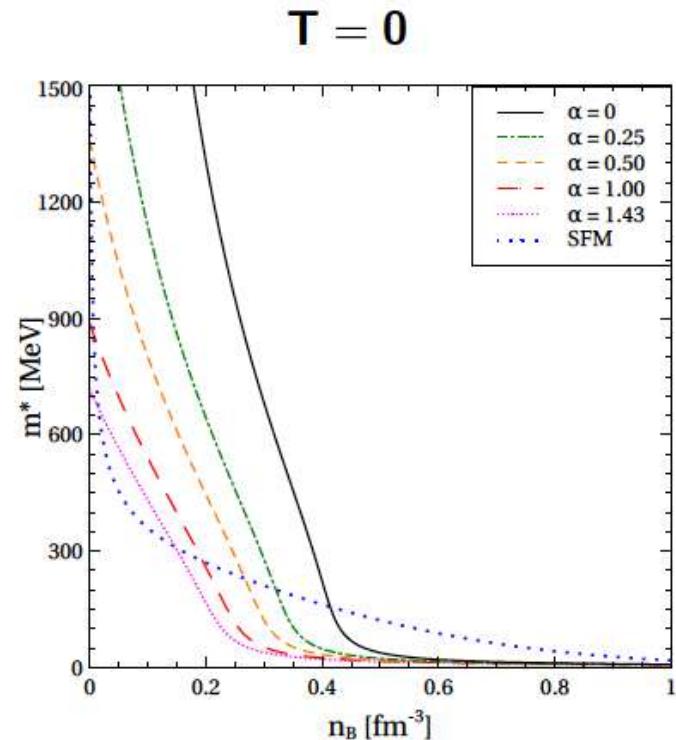
$$\langle \overline{q}q \rangle = \langle \overline{q}q \rangle_0 \Rightarrow m^* = m - \frac{2G_0}{3\alpha^{2/3} \langle \overline{q}q \rangle_0^{1/3}}$$

⇓

$$m^* \rightarrow \infty \text{ at } \alpha \rightarrow 0$$

- medium dependent couplings:

low $T, \mu, \Rightarrow G_S \neq G_{PS} \Rightarrow \chi\text{-broken}$
high $T, \mu, \Rightarrow G_S = G_{PS} \Rightarrow \chi\text{-symmetric}$



Relativistic density functional for quark matter

Bosonisation by HS-transformation

- Hubbard-Stratonovich transformation

$$\exp \left[\int dx \, G(\bar{q} \hat{\Gamma} q)^2 \right] = \int [D\phi] \exp \left[- \int dx \left(\frac{\phi^2}{4G} + \phi \bar{q} \hat{\Gamma} q \right) \right]$$

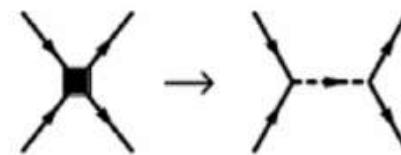
- **Vertexes:** $\hat{\Gamma}_S = 1 \Rightarrow$ scalar-isoscalar σ -field

$\hat{\Gamma}_{PS} = i\gamma^5 \vec{\tau} \Rightarrow$ pseudoscalar-isoscalar $\vec{\pi}$ -field

$\hat{\Gamma}_V^\mu = \gamma^\mu \Rightarrow$ vector-isoscalar ω^μ -field

$\hat{\Gamma}_I^\mu = \gamma^\mu \vec{\tau} \Rightarrow$ vector-isoscalar $\vec{\rho}^\mu$ -field

$\hat{\Gamma}_D^A = i\gamma^5 \lambda_A \tau_2 \Rightarrow$ scalar diquark Δ_A -field



- **Bosonized Lagrangian ($m^* = m + \Sigma_S$ - effective mass, $\mathbf{Q}^T = (\mathbf{q} \ \mathbf{q}^c)/\sqrt{2}$)**

$$\mathcal{L} + q^+ \hat{\mu} q = \overline{Q} \hat{S}^{-1} Q - \frac{\sigma^2}{4G_S} - \frac{\vec{\pi}^2}{4G_{PS}} + \frac{\omega^2}{4G_V} + \frac{\vec{\rho}^2}{4G_I} - \frac{\Delta_A \Delta_A^*}{4G_D} - \mathcal{U}_{MF} + \langle \bar{q} q \rangle (\Sigma_S + \sigma)$$

$$\hat{S}^{-1} = \begin{pmatrix} \hat{S}_+^{-1} & i\Delta_A \gamma_5 \tau_2 \lambda_A \\ i\Delta_A^* \gamma_5 \tau_2 \lambda_A & \hat{S}_-^{-1} \end{pmatrix}, \quad \hat{S}_\pm^{-1} = i\cancel{\partial} - m^* - \sigma - i\gamma^5 \vec{\pi} \cdot \vec{\tau} \pm \gamma_0 \hat{\mu} \pm \psi \pm \vec{\rho} \cdot \vec{\tau}$$

Mean field approximation

- Field equations for σ and $\vec{\pi}$

$$\begin{cases} \sigma = 2G_S(\langle\bar{q}q\rangle - \bar{q}q) \\ \vec{\pi} = -2G_{PS}\bar{q}i\vec{\tau}\gamma_5 q \end{cases} \Rightarrow \langle\sigma\rangle = \langle\vec{\pi}\rangle = 0 \Rightarrow \sigma, \vec{\pi} - \text{beyond MF}$$

comment: $\langle\sigma\rangle = 0$ does not assume χ -symmetry since $\langle\bar{q}q\rangle \neq 0$

- Thermodynamic potential

$$\langle\omega_\mu\rangle = \delta_{\mu 0}\omega, \quad \langle\rho_\mu^a\rangle = \delta_{\mu 0}\delta_{a3}\rho, \quad |\langle\Delta_A\rangle| = \delta_{A2}\Delta$$

⇓

$$\Omega = -\frac{1}{2\beta V} Tr \ln(\beta \hat{S}^{-1}) - \frac{\omega^2}{4G_V} - \frac{\rho^2}{4G_I} + \frac{\Delta^2}{4G_D} + \mathcal{U}_{MF} - \langle\bar{q}q\rangle\Sigma_S$$

- Vector fields, diquark gap, χ -condensate

$$\frac{\partial\Omega}{\partial\omega} = 0, \quad \frac{\partial\Omega}{\partial\rho} = 0, \quad \frac{\partial\Omega}{\partial\Delta} = 0, \quad \langle\bar{q}q\rangle = \sum_f \frac{\partial\Omega}{\partial m_f}$$

Relativistic density functional for quark matter

Define the couplings in mesonic channels

- Mesonic correlations

$$\mathcal{L} = \dots + \bar{q}(\sigma + i\gamma_5 \vec{\pi} \cdot \vec{\tau} + \psi + \vec{\rho} \cdot \tau)q - \frac{\sigma^2}{4G_S} - \frac{\vec{\pi}^2}{4G_{PS}} + \frac{\omega^2}{4G_V} + \frac{\vec{\rho}^2}{4G_I}$$

$$D_i^{-1}(p^2) = \frac{1}{2G_i} - \text{wavy line} \circlearrowleft \text{wavy line} \quad - \text{one-loop mesonic propagator}$$

$$D_i^{-1}(M_i^2) = 0 \Rightarrow \text{mesonic masses}$$

- Fierz transformation - rearrangement of the Dirac, color and flavor indexes

$$\begin{aligned} (\gamma^\mu)_{mn}(\gamma_\mu)_{m'n'} &= \mathbf{1}_{mn'}\mathbf{1}_{m'n} + (i\gamma_5)_{mn'}(i\gamma_5)_{mn'} \\ &\quad - \frac{1}{2}(\gamma^\mu)_{mn'}(\gamma_\mu)_{m'n} \\ &\quad - \frac{1}{2}(\gamma^\mu\gamma_5)_{mn'}(\gamma_\mu\gamma_5)_{m'n} \end{aligned}$$

$$\begin{aligned} \mathbf{1}_{ij}\mathbf{1}_{kl} &= \frac{1}{3}\mathbf{1}_{il}\mathbf{1}_{kj} + \frac{1}{2}(\tau_a)_{il}(\tau_a)_{kj} \\ \lambda_\alpha^{ab}\lambda_\alpha^{a'b'} &= \frac{16}{9}\mathbf{1}_{ab'}\mathbf{1}_{a'b} - \frac{1}{3}\lambda_\alpha^{ab'}\lambda_\alpha^{a'b} \end{aligned}$$

coefficients - proportional to couplings

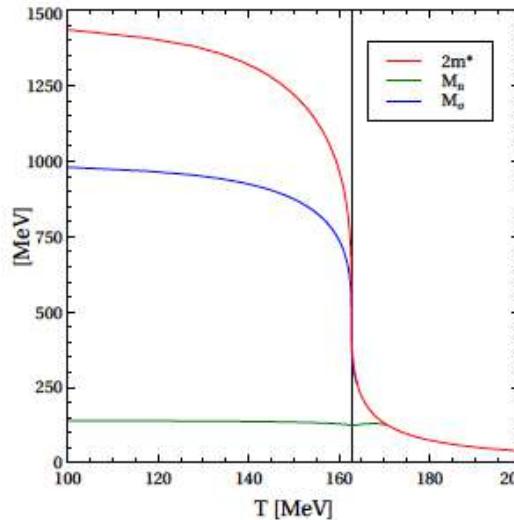
$$\mathbf{G}_S : \mathbf{G}_V : \mathbf{G}_I : \mathbf{G}_D = 1 : 0.5 : 0.5 : 0.75$$

Relativistic density functional for quark matter

Model setup – parameter fixing with observables

- (Pseudo)scalar interaction channels
(chiral condensate & π , σ mesons)

m [MeV]	Λ [MeV]	α	$D_0 \Lambda^{-2}$
4.2	573	1.43	1.39
M_π [MeV]	F_π [MeV]	M_σ [MeV]	$\langle \bar{I} I \rangle_0^{1/3}$ [MeV]
140	92	980	-267



Pseudocritical temperature

$$T_c = 163 \text{ MeV}$$

- low T : $2m_{\text{quark}} > M_\pi, M_\sigma$
(stable mesons, confined quarks)
- high T : $2m_{\text{quark}} < M_\pi, M_\sigma$
(unstable mesons, deconfined quarks)

- Vector-isoscalar & vector-isovector channels (ω , ρ mesons)

$$M_\omega = 783 \text{ MeV} \Rightarrow \eta_V \equiv \frac{G_{V0}}{G_{S0}} = 0.452, \quad M_\rho = 775 \text{ MeV} \Rightarrow \eta_I \equiv \frac{G_{I0}}{G_{S0}} = 0.454$$

- Diquark pairing channel (Fierz transformation) $\eta_D \equiv \frac{G_{D0}}{G_{S0}} = 1.5\eta_V = 0.678$

Relativistic density functional for quark matter

Onset of color superconductivity

- Single quark energy and distribution

$$E_f^\pm = \text{sgn}(E_f \mp \mu_f) \sqrt{(E_f \mp \mu_f)^2 + \Delta^2}$$

$$f_f^\pm = [\exp(E_f^\pm / T) + 1]^{-1}$$

- Gap equation

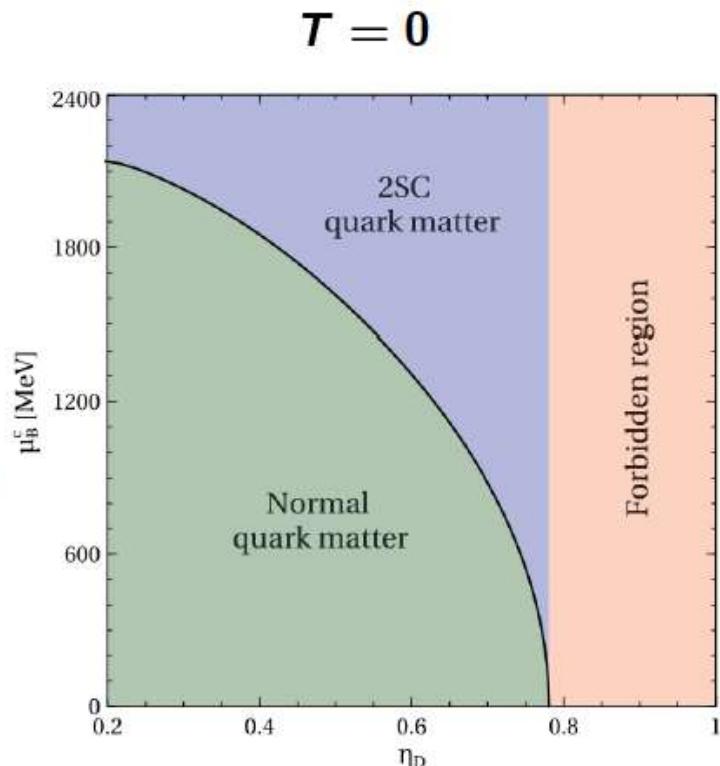
$$\frac{\partial \Omega}{\partial \Delta} = \frac{\Delta}{2G_D} - 2\Delta \sum_{f,a=\pm} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1 - 2f_f^a}{E_f^a} = 0$$

↓

two solutions : $\Delta = 0$ or $\Delta \neq 0$

- Two solutions coincide \Rightarrow SC onset

$$\frac{\partial^2 \Omega}{\partial \Delta^2} \Big|_{\Delta=0} = 0 \quad \Rightarrow \quad \mu_B = \mu_B(G_D)$$



No vacuum superconductivity

↓

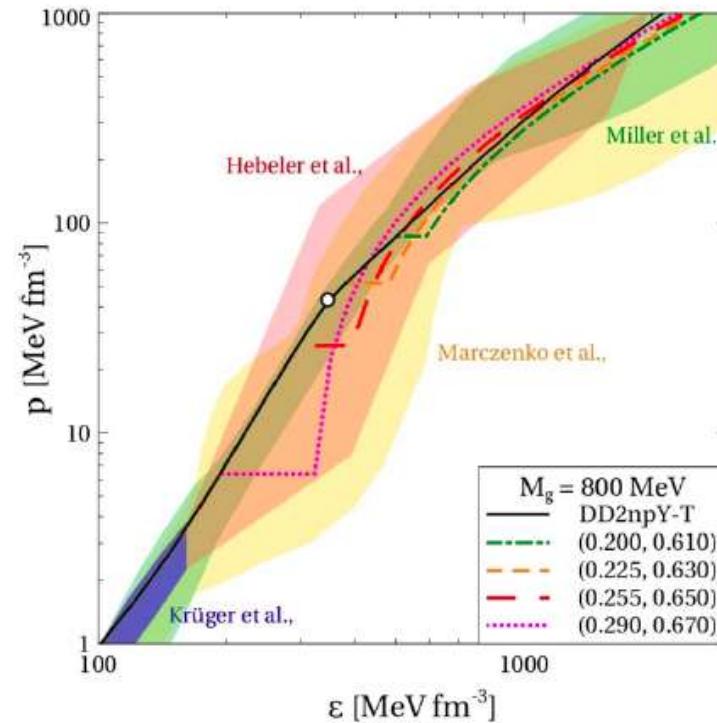
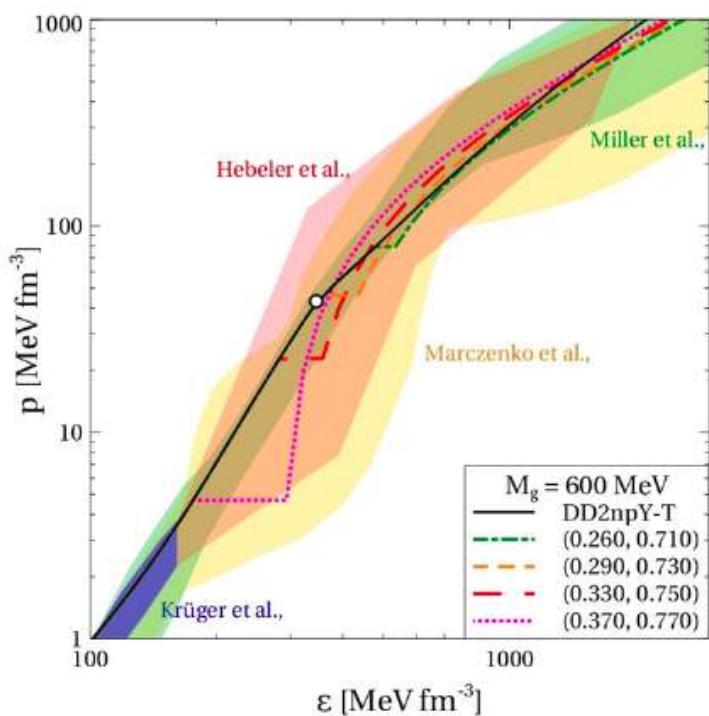
$$\eta_D \lesssim 0.78$$

(agrees with the Fierz value)

Relativistic density functional for quark matter

Asymptotically conformal EOS for neutron stars

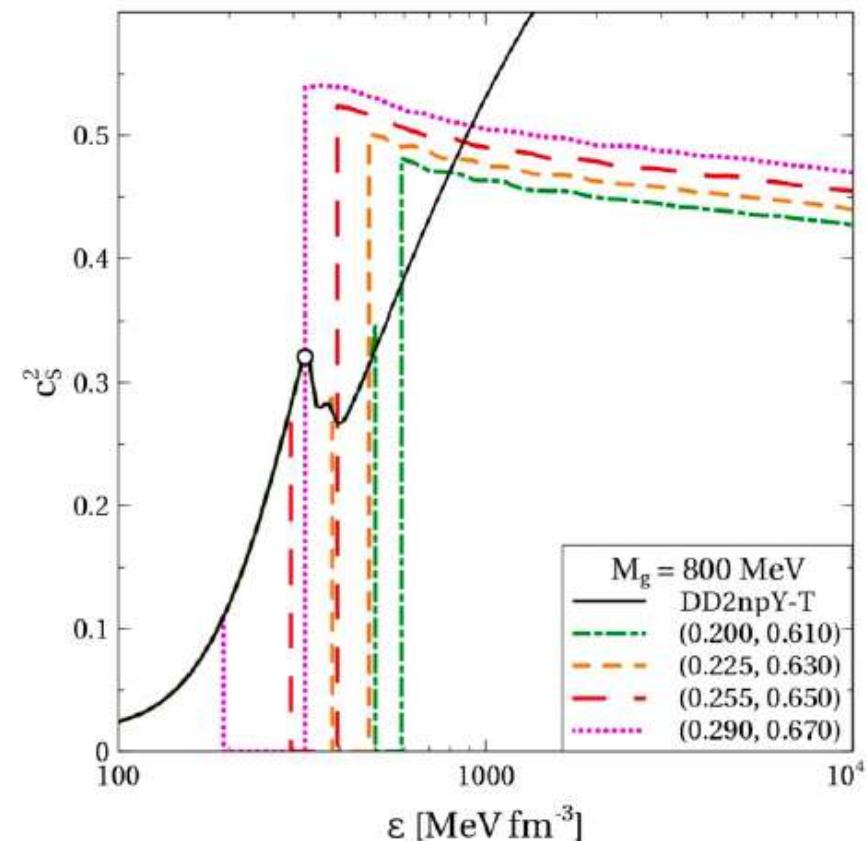
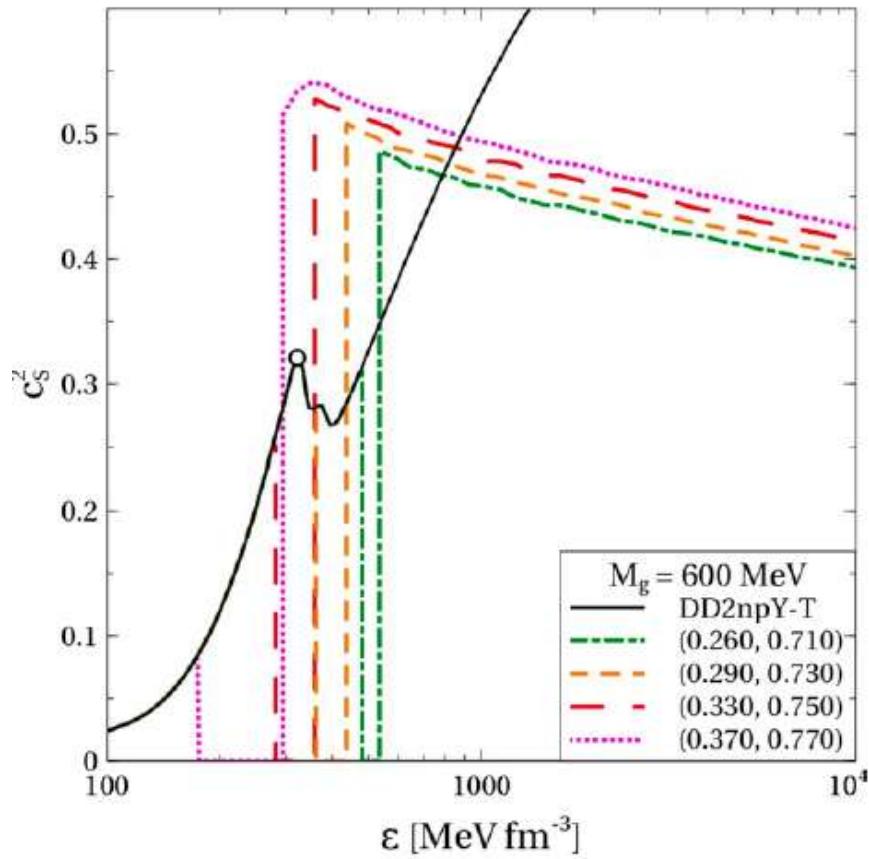
- **Setup:** electric neutrality, β -equilibrium, Maxwell construction with DD2 EoS
- **Scanning over η_V and η_D at $M_{gD} = M_{gV}$**



The ω -meson value of η_V and the Fierz value of η_D
prefer early deconfinement?

Relativistic density functional for quark matter

Speed of sound

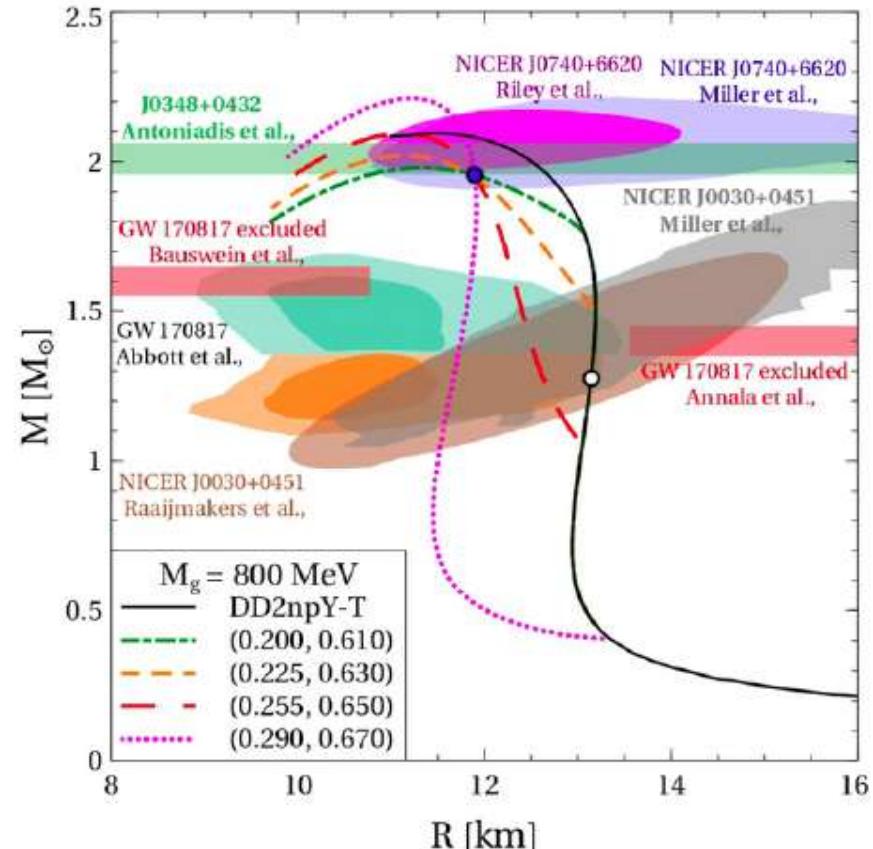
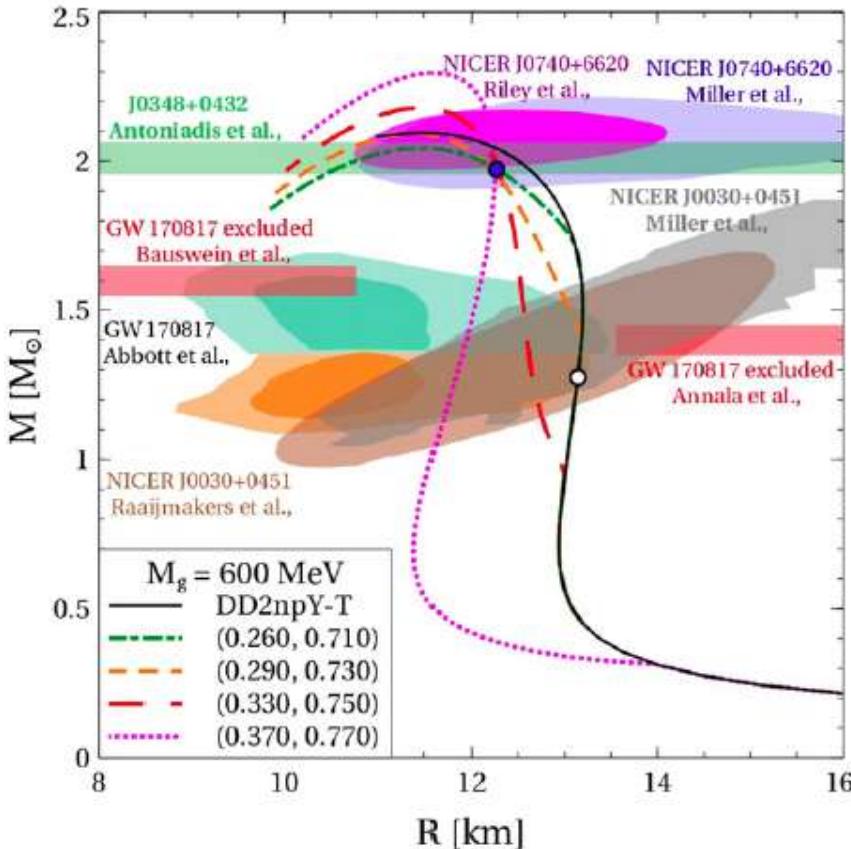


O. Ivanytskyi and D. Blaschke, Particles 5 (2022) 514 - 534



Relativistic density functional for quark matter

Mass-radius diagram for hybrid neutron stars



Observational data prefer early deconfinement?

Relativistic density functional for quark matter

Phase diagram with two-zone interpolation

- **Normal quark matter**

$$2 \text{ spin} \times 2 \text{ flavor} \times 3 \text{ color} = 12$$

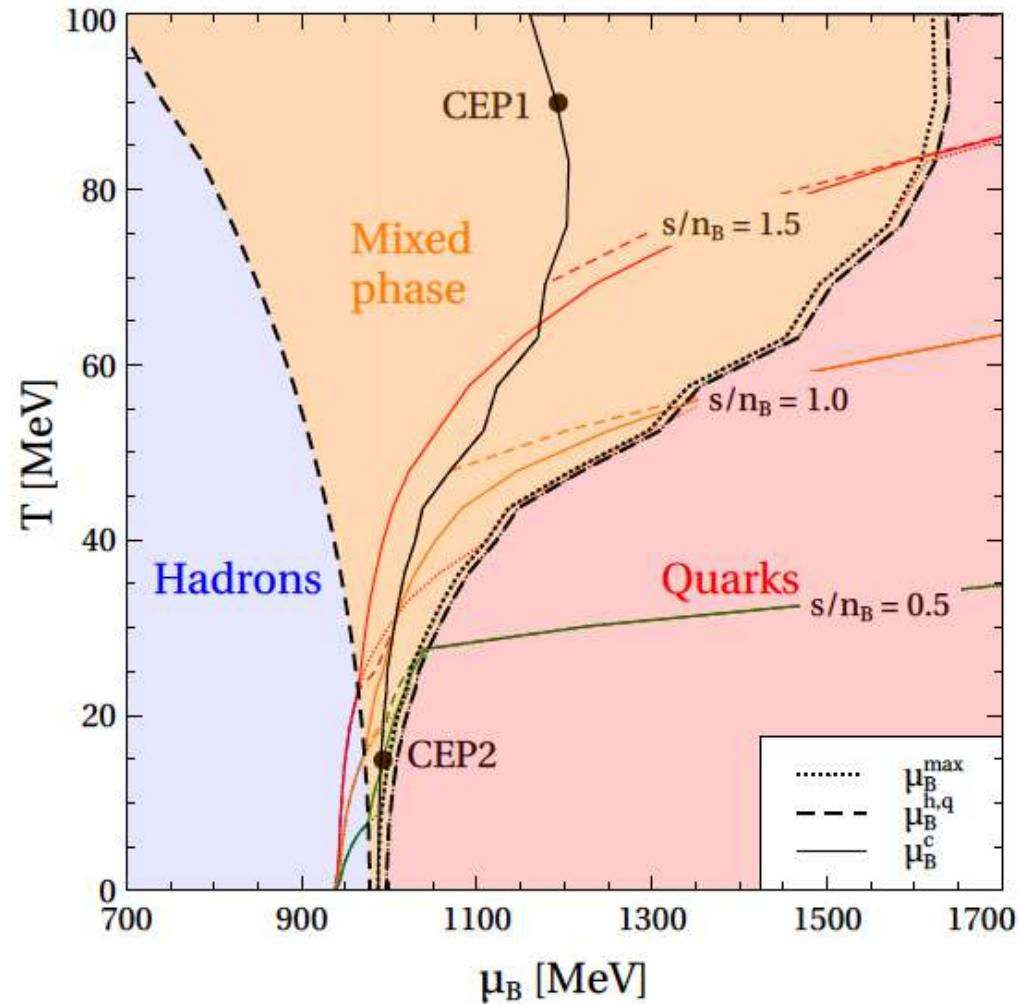
- **2SC quark matter**

$$2 \text{ spin} \times 2 \text{ flavor} \times 1 \text{ color} + 1 = 5$$

Quark pairing reduces
number of quark states



requires higher T
along adiabat



→ EOS tables are prepared for simulation of supernovae and NS mergers

Relativistic density functional for quark matter

Phase diagram with two-zone interpolation

- **Normal quark matter**

$$2 \text{ spin} \times 2 \text{ flavor} \times 3 \text{ color} = 12$$

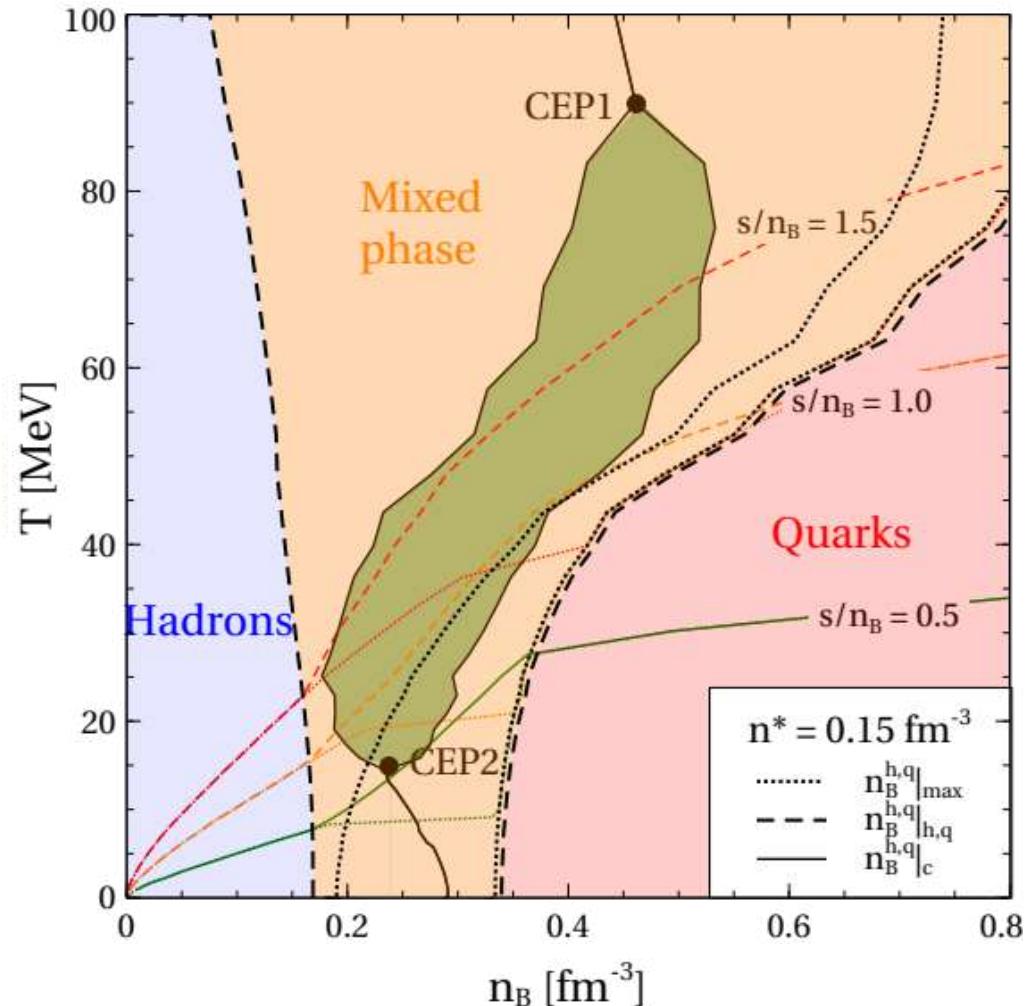
- **2SC quark matter**

$$2 \text{ spin} \times 2 \text{ flavor} \times 1 \text{ color} + 1 = 5$$

Quark pairing reduces
number of quark states



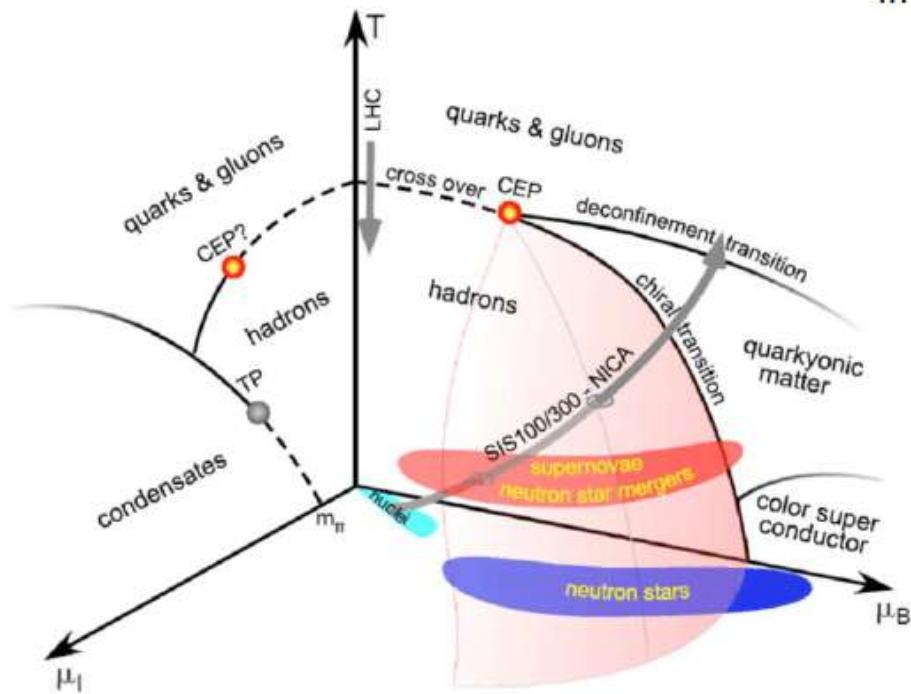
requires higher T
along adiabat



→ EOS tables are prepared for simulation of supernovae and NS mergers

Conclusion

Density functional methods solve obstacles in neutron star astrophysics



Prominent contributions to deconfinement in modern multimessenger Astrophysics:

- Quark deconfinement transition triggers the **supernova explosion** of a very massive ($M = 50M_{\odot}$) blue supergiant progenitor star
T. Fischer et al., Nature Astron. 2 (2018) 960
- Unambiguous signal of a strong phase transition in the postmerger GW from a binary **NS merger** predicted
A. Bauswein et al., Phys. Rev. Lett. 122 (2019) 061102
- Strong deconfinement phase transition in NS can be detected by observing the **mass twin star** phenomenon
D. B. et al., Universe 6 (2020) 81

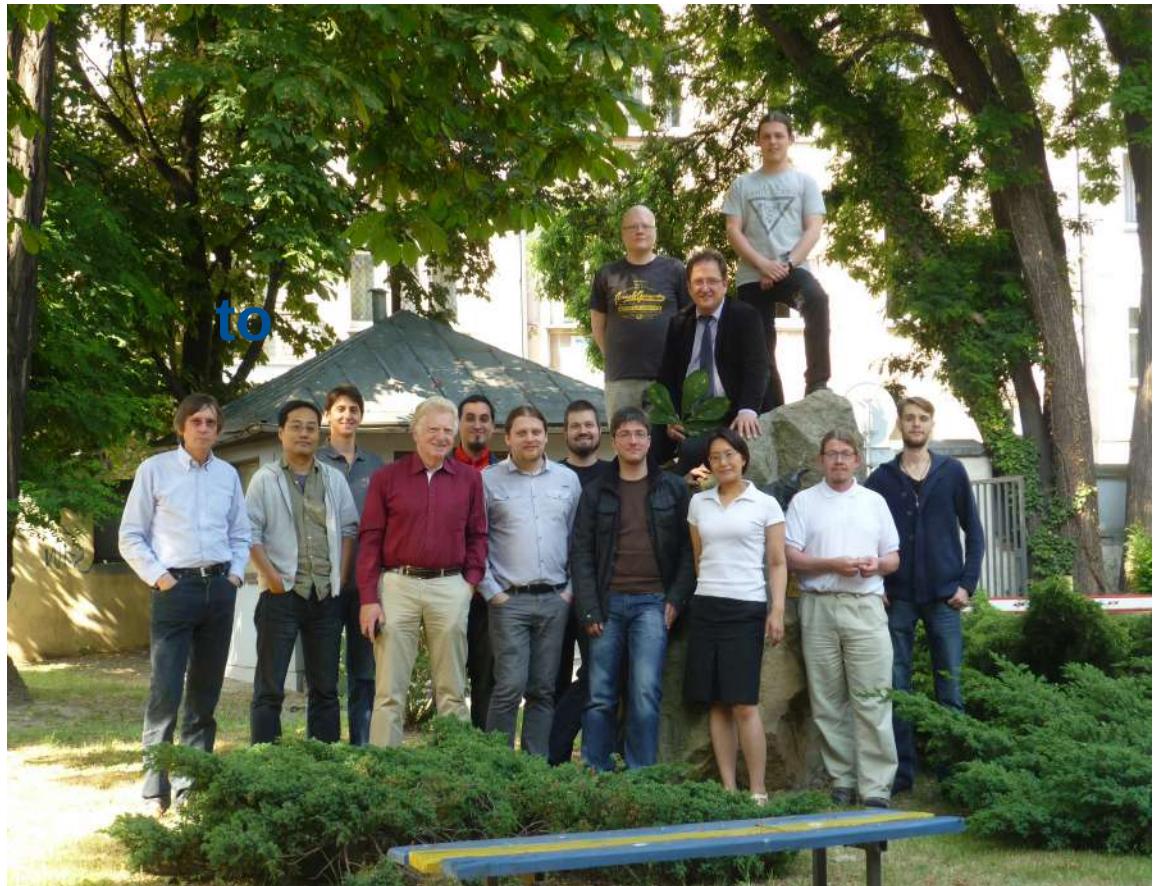
From: NuPECC Long Range Plan 2017

See also: Agnieszka Sorensen et al., Dense nuclear matter EOS from HIC, arXiv:2301.13253

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