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# **$\Lambda$ - $\Lambda$ interactions from lattice QCD and the $H$ dibaryon**

**Hartmut Wittig**

in collaboration with:

J. Green, A. Hanlon, P. Junnarkar, M. Padmanath, S. Paul & Baryon Scattering Collaboration (BaSc)

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**EMMI Workshop: Bound states and particle interactions in the 21st century**

*Department of Physics — University of Trieste*

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# The $H$ Dibaryon

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## Perhaps a Stable Dihyperon\*

R. L. Jaffe†

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics  
and Laboratory of Nuclear Science,‡ Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 1 November 1976)

In the quark bag model, the same gluon-exchange forces which make the proton lighter than the  $\Delta(1236)$  bind six quarks to form a stable, flavor-singlet (with strangeness of  $-2$ )  $J^P = 0^+$  dihyperon ( $H$ ) at 2150 MeV. Another isosinglet dihyperon ( $H^*$ ) with  $J^P = 1^+$  at 2335 MeV should appear as a bump in  $\Lambda\Lambda$  invariant-mass plots. Production and decay systematics of the  $H$  are discussed.

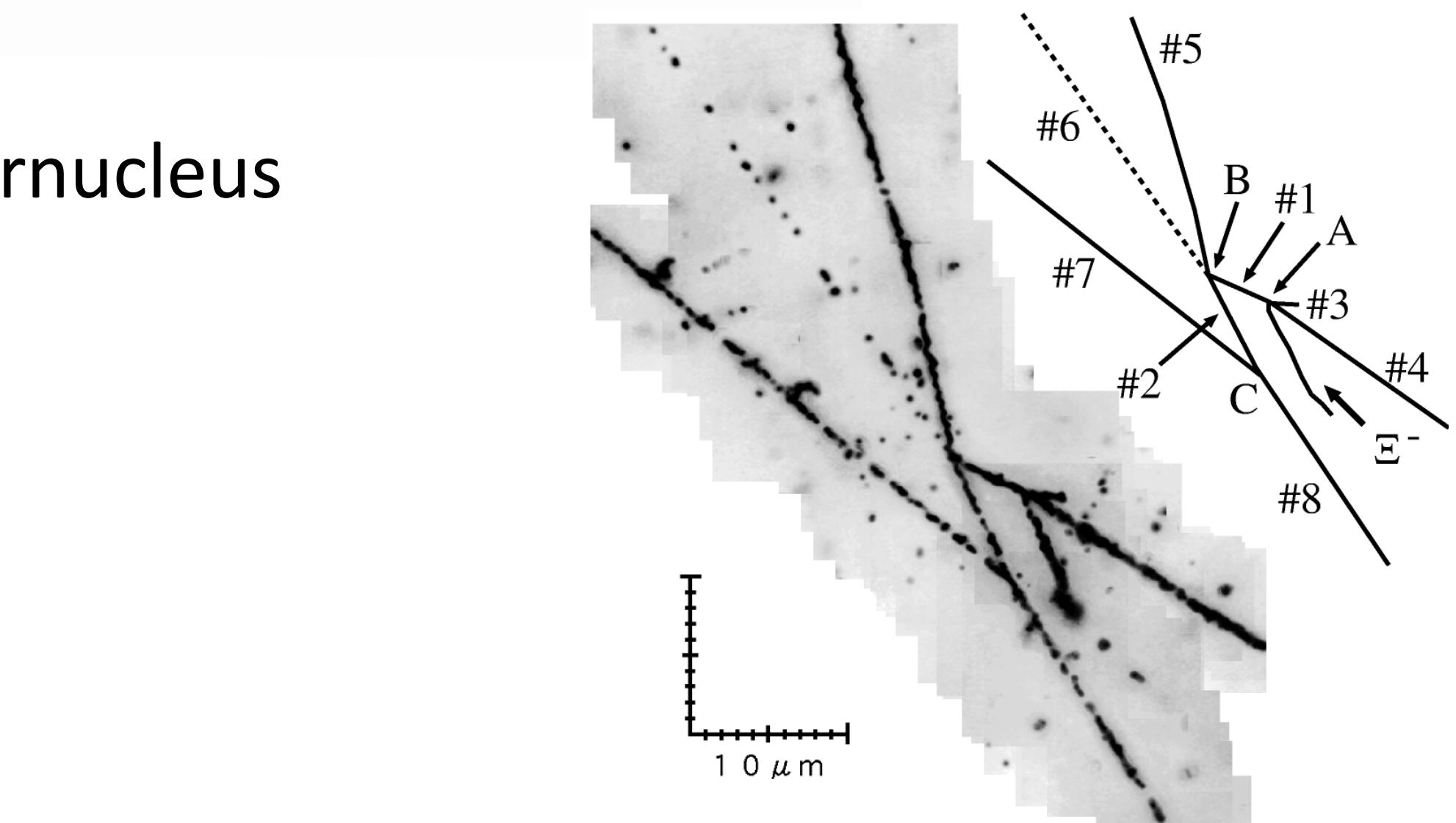
- “Nagara event”: Observation of a  $^{6}_{\Lambda\Lambda}$  He double-hypernucleus

Binding energy:  $B_{\Lambda\Lambda} = 7.25 \pm 0.19 (^{+0.18}_{-0.11})$  MeV

Interpreted as sequential weak decay of  $^{6}_{\Lambda\Lambda}$  He :

$$m_H > 2m_\Lambda - B_{\Lambda\Lambda} = 2223.7 \text{ MeV} \quad @ 90\% \text{ CL}$$

[Takahashi et al., PRL 87 (2001) 212502]



[See Avraham Gal’s talk in this session]

# Hyperon-hyperon interactions and the $H$ Dibaryon

## Hyperon-hyperon interactions

- Relevant for the physics of (double) hyper nuclei, neutron-rich matter, neutron stars
- No experimental information available ( $\Lambda\Lambda$ -scattering not viable experimentally)

Deeply bound *udsuds* state (“sexaquark”) proposed / discussed as dark matter candidate:  
[*G.R. Farrar, A. Strumia et al.,...*]

$$m_H < 2(m_p + m_e) = 1877.6 \text{ MeV} \quad \Rightarrow \quad H \text{ dibaryon absolutely stable}$$

$$m_H > 2(m_p + \text{B.E.}) = 1860 \text{ MeV} \quad \Rightarrow \quad \text{Nuclei absolutely stable}$$

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Recall:  $2m_\Lambda = 2230 \text{ MeV}$

$$m_H = 2150 \text{ MeV} \quad (\text{Jaffe's bag model estimate})$$

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→ Scenario requires very large binding energy of  $\approx 360 \text{ MeV}$

# Outline



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Dibaryons in Lattice QCD



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The  $H$  dibaryon: Lattice results



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Summary & Conclusions

# Dibaryons in Lattice QCD

## Computing the hadron spectrum in Lattice QCD

Spectral information encoded in correlation functions

$$\sum_{\mathbf{x}, \mathbf{y}} e^{i \mathbf{p} \cdot (\mathbf{y} - \mathbf{x})} \langle O_{\text{had}}(\mathbf{y}) O_{\text{had}}^\dagger(\mathbf{x}) \rangle = \sum_n w_n(\mathbf{p}) e^{-E_n(\mathbf{p})(y_0 - x_0)} \xrightarrow{(y_0 - x_0) \rightarrow \infty} w_1(\mathbf{p}) e^{-E_1(\mathbf{p})(y_0 - x_0)}$$

$O_{\text{had}}(x)$  : interpolating operator

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e.g. Nucleon:  $O_N = \epsilon_{abc} (u^a C \gamma_5 d^b) u^c$

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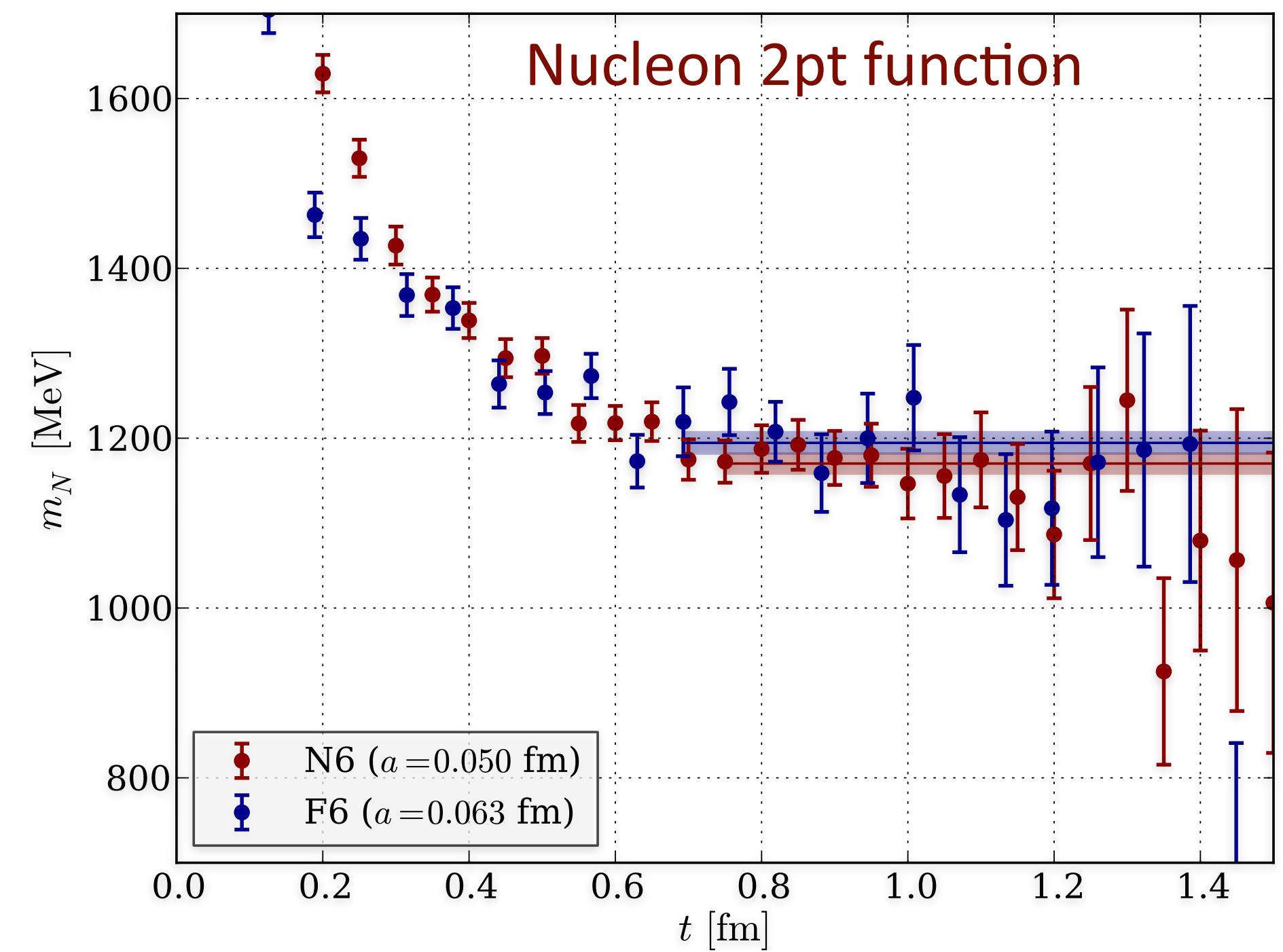
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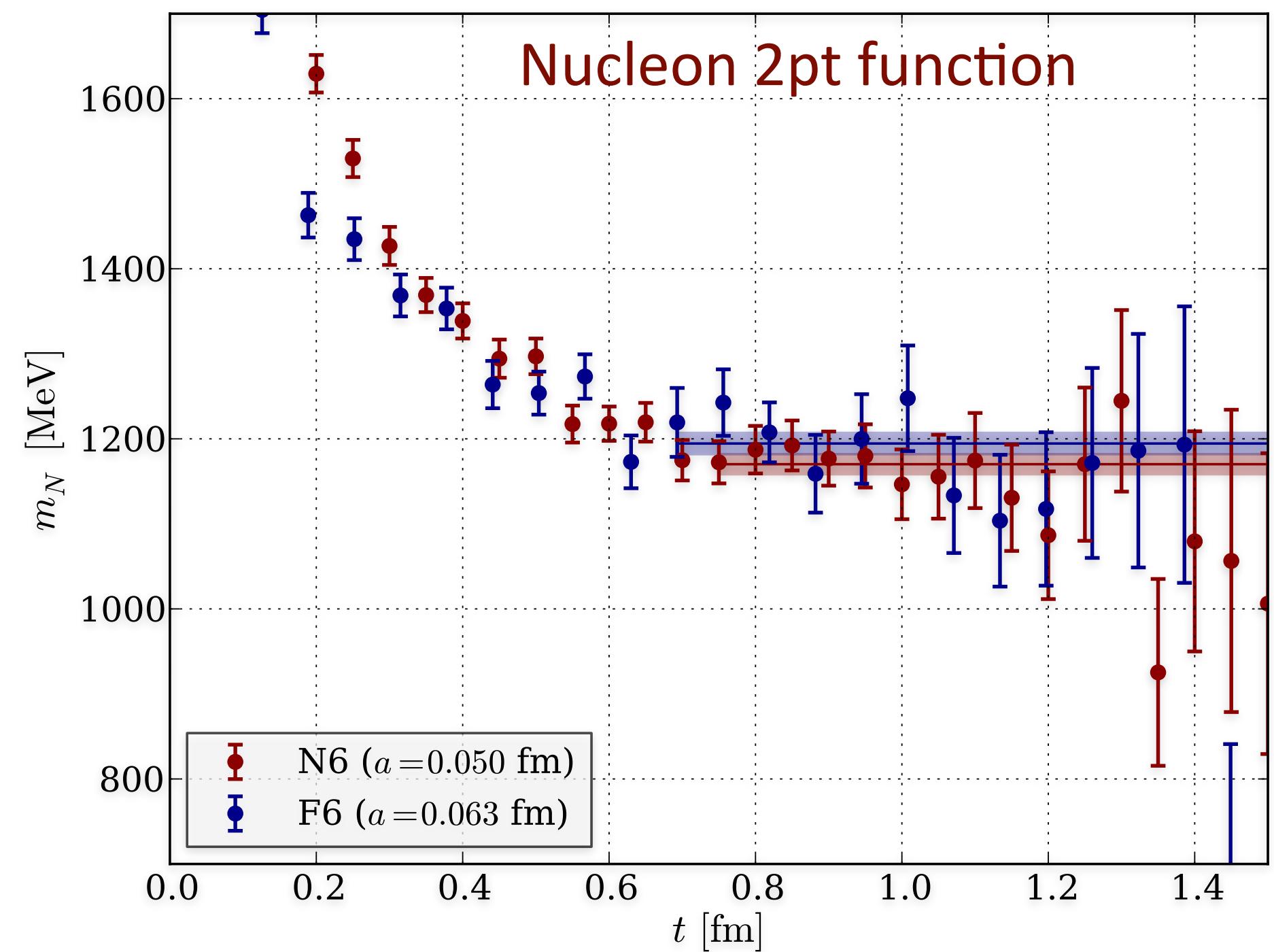
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- Ground state dominates at large Euclidean times:  
 $t \equiv y_0 - x_0 \rightarrow \infty$
- Excited states are **sub-leading** contributions



# Dibaryons in Lattice QCD

**Flavour structure of two octet baryons:**

$$\mathbf{8} \otimes \mathbf{8} = (\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27})_S \oplus (\mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}})_A$$

- $H$  dibaryon lies in **1**-dimensional irrep of  $SU(3)_{\text{flavour}}$
- Upon  $SU(3)$ -symmetry breaking, **8** and **27** mix with singlet
- Singlet, octet and **27**plet operators constructed from linear combinations of  $\Lambda\Lambda$ ,  $\Sigma\Sigma$  and  $N\Xi$  operators

e.g. 
$$[\mathbf{1}] = -\sqrt{\frac{1}{8}}[\Lambda\Lambda]^{I=0} + \sqrt{\frac{3}{8}}[\Sigma\Sigma]^{I=0} + \sqrt{\frac{4}{8}}[N\Xi_s]^{I=0}$$

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**Other dibaryons:**

- Dineutron lies in **27** irrep
- Deuteron lies in **10** irrep with  $J^P = 1^+$

# Dibaryons in Lattice QCD

Interpolating operators for the  $H$  dibaryon

Hexaquark operators (inspired by Jaffe's original bag model calculation):

$$[rstuvw] = \epsilon_{ijk}\epsilon_{lmn} (s^i C\gamma_5 P_+ t^j) (v^l C\gamma_5 P_+ w^m) (r^k C\gamma_5 P_+ u^n)$$

$$H^{(1)} = \frac{1}{48} ([sudsud] - [udusds] - [dudsus])$$

$$H^{(27)} = \frac{1}{48\sqrt{3}} (3[sudsud] + [udusds] - [dudsus])$$

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Momentum-projected two-baryon operators:

$$B_\alpha \equiv [rst]_\alpha = \epsilon_{ijk} (s^i C\gamma_5 P_+ t^j) r_\alpha^k$$

$$(BB)(\mathbf{P}; t) = \sum_{\mathbf{x}} e^{-i\mathbf{p}_1 \cdot \mathbf{x}} B_1(\mathbf{x}, t) (C\gamma_5 P_+) \sum_{\mathbf{y}} e^{-i\mathbf{p}_2 \cdot \mathbf{y}} B_2(\mathbf{y}, t), \quad \mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$$

→ project onto  $(BB)^{(1)}$ ,  $(BB)^{(8)}$ ,  $(BB)^{(27)}$

# Dibaryons in Lattice QCD

Interpolating operators for other dibaryon channels

Building block:  $B_\alpha \equiv [rst]_\alpha = \epsilon_{ijk} \left( s^i C\gamma_5 P_+ t^j \right) r_\alpha^k$

Spin-1 interpolator:

$$(BB)_i(\mathbf{p}_1, \mathbf{p}_2) = \sum_{\mathbf{x}} e^{-i\mathbf{p}_1 \cdot \mathbf{x}} B_1(\mathbf{x}, t) (C\gamma_i P_+) \sum_{\mathbf{y}} e^{-i\mathbf{p}_2 \cdot \mathbf{y}} B_2(\mathbf{y}, t)$$

Deuteron:

$$(BB)_{i; T_1^+}^{(n)} = \frac{1}{N} \sum_{\mathbf{p}; p^2=n} (BB)_i(-\mathbf{p}, \mathbf{p})$$

# Dibaryons in Lattice QCD: Correlator matrices and GEVP

Consider set on  $N_{\text{op}}$  interpolating operators for a given hadron:

Correlation matrix:  $C_{ij}(\mathbf{P}, \tau) = \langle O_i(\mathbf{P}, t) O_j(\mathbf{P}, t')^\dagger \rangle, \quad \tau = t - t', \quad i, j = 1, \dots, N_{\text{op}}$

- Variational method: solve Generalised Eigenvalue Problem (GEVP)

$$\mathbf{C}(t_1) v_n(t_1, t_0) = \lambda_n(t_1, t_0) \mathbf{C}(t_0) v_n(t_1, t_0)$$

$$w_n^\dagger(t_1, t_0) \mathbf{C}(t_1) = \lambda_n(t_1, t_0) w_n^\dagger(t_1, t_0) \mathbf{C}(t_0), \quad n = 1, \dots, N_{\text{op}}$$

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- Distillation: quark propagator with Laplace-Heaviside (LapH) smearing:

$$\mathcal{S} D^{-1} \mathcal{S}, \quad \mathcal{S}^{(t)}(\mathbf{x}, \mathbf{y}) = \sum_{k=1}^{N_{\text{LapH}}} V^{(k)}(\mathbf{x}, t) \otimes V^{(k)}(\mathbf{y}, t)^\dagger \quad (V^{(k)} : k^{\text{th}} \text{ eigenvector of Laplacian } \Delta)$$

# Dibaryons in Lattice QCD

Finite-volume quantisation condition

$$\det(\tilde{\mathcal{K}}^{-1}(p^2) - B(p^2, L)) = 0$$

$\tilde{\mathcal{K}}(p^2)$  :  $2 \rightarrow 2$  scattering amplitude

$B(p^2, L)$  : analytically known function

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S-wave:  $p \cot \delta(p) = \frac{2}{\gamma L \sqrt{\pi}} Z_{00}^D(1, q^2), \quad q = pL/2\pi,$

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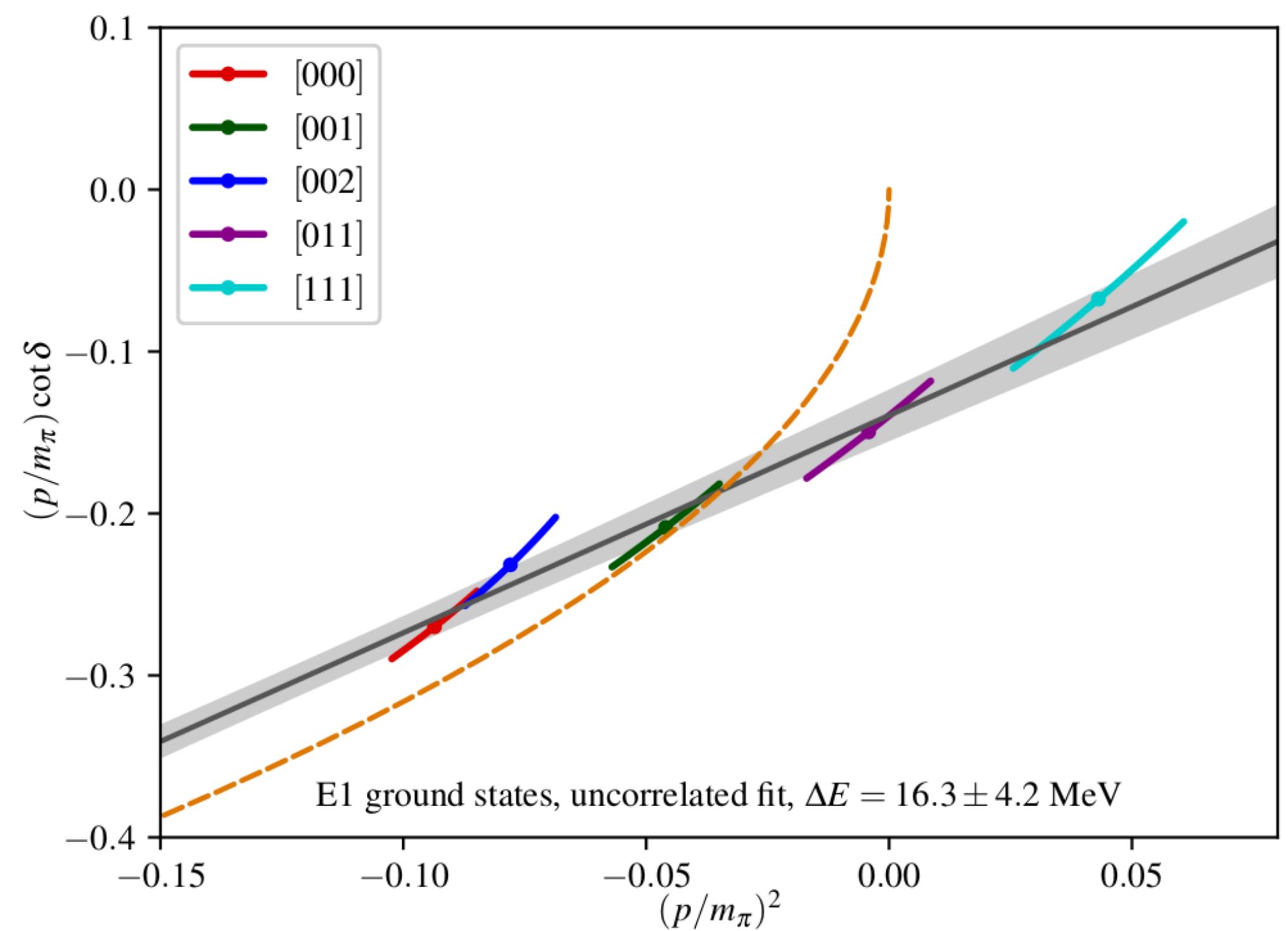
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Fit to effective range expansion:

$$\Rightarrow p \cot \delta_0(p) = A + Bp^2 + \dots \stackrel{!}{=} -\sqrt{-p^2}$$



# The Mainz dibaryon project

## Past and present members:

Anthony Francis, Jeremy Green, Andrew Hanlon, Parikshit Junnarkar, Padmanath Madanagopalan,  
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## Methodology:

- Variational method: point-to-all ( $N_f = 2$ ) and timeslice-to-all propagators
- Exact distillation: timeslice-to-all propagators [Pardon et al., PRD 80 (2009) 054506]
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- Various dibaryon channels — extension to charmed tetraquarks

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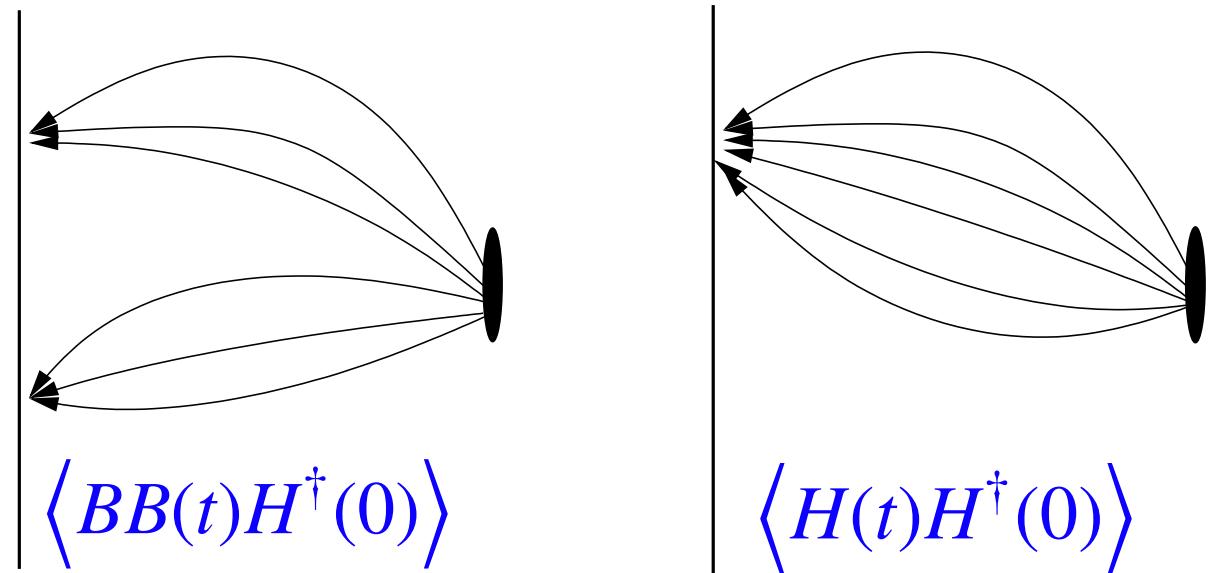
## Collaboration within “Baryon Scattering” (BaSc) Collaboration

- Stochastic LapH on large physical volumes [Morningstar et al., PRD 83 (2011) 114505]
- Alternative discretisations: exponentiated Clover, domain wall

# The $H$ Dibaryon: Pilot study in 2-flavour QCD

- Pion masses match earlier calculations by NPLQCD and HALQCD
- Point-to-all propagators: asymmetric GEVP
- Hexaquark operators have poor overlap onto ground state
- Distillation: much better signal
- Finite-volume quantisation yields smaller binding energy

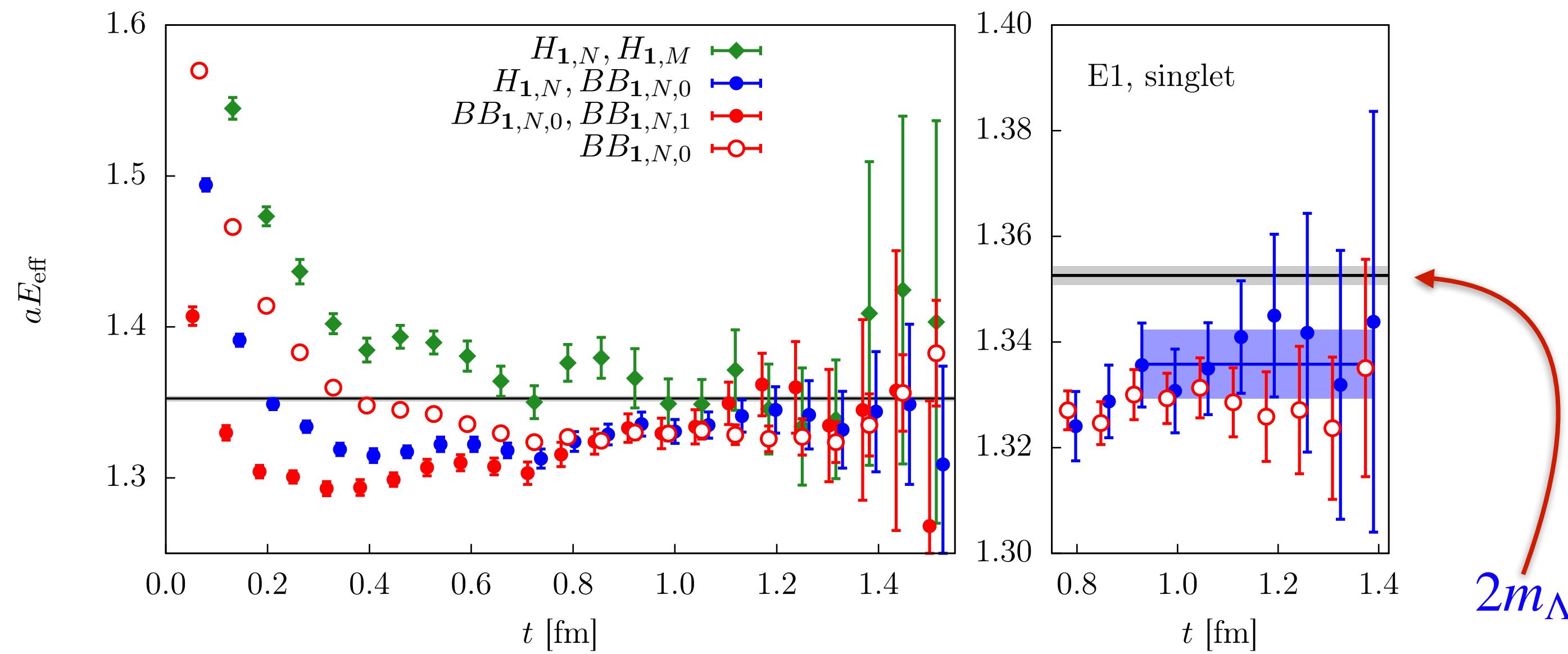
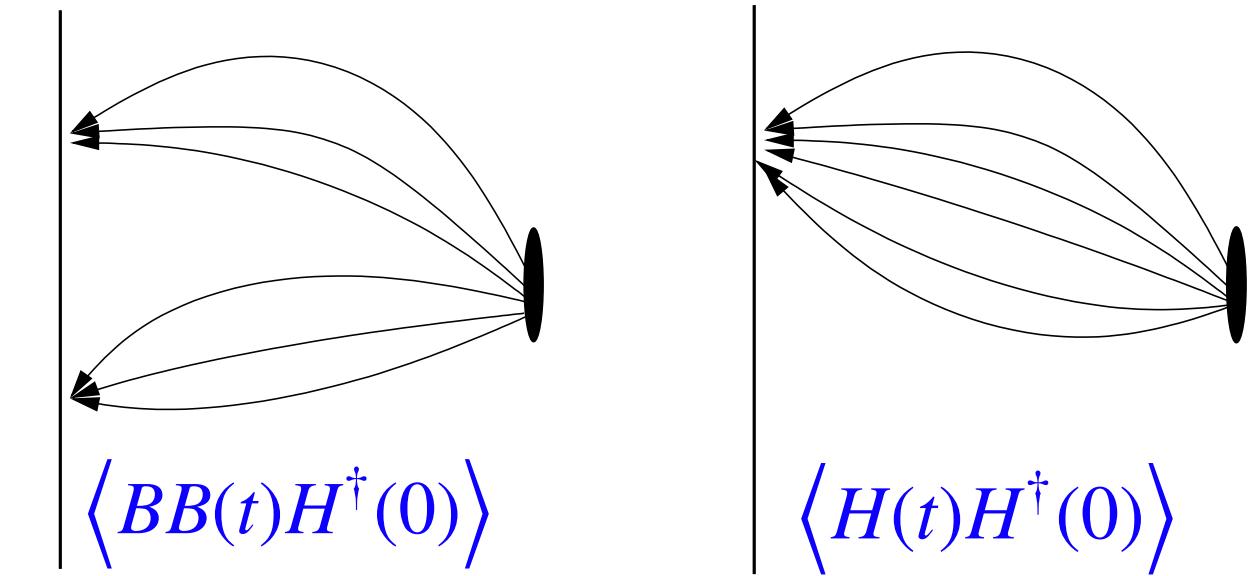
[Francis et al., PRD 99 (2019) 074505]



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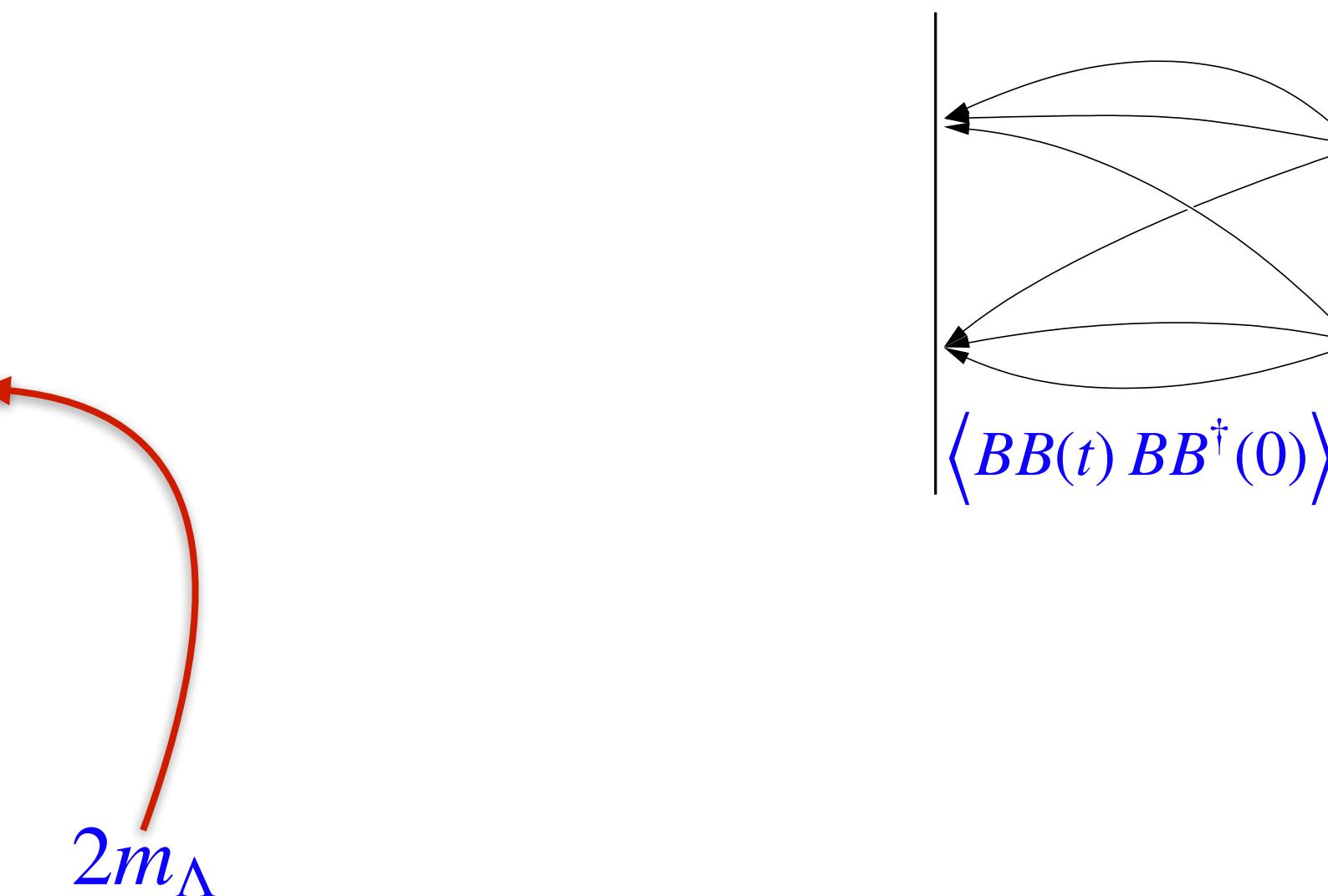
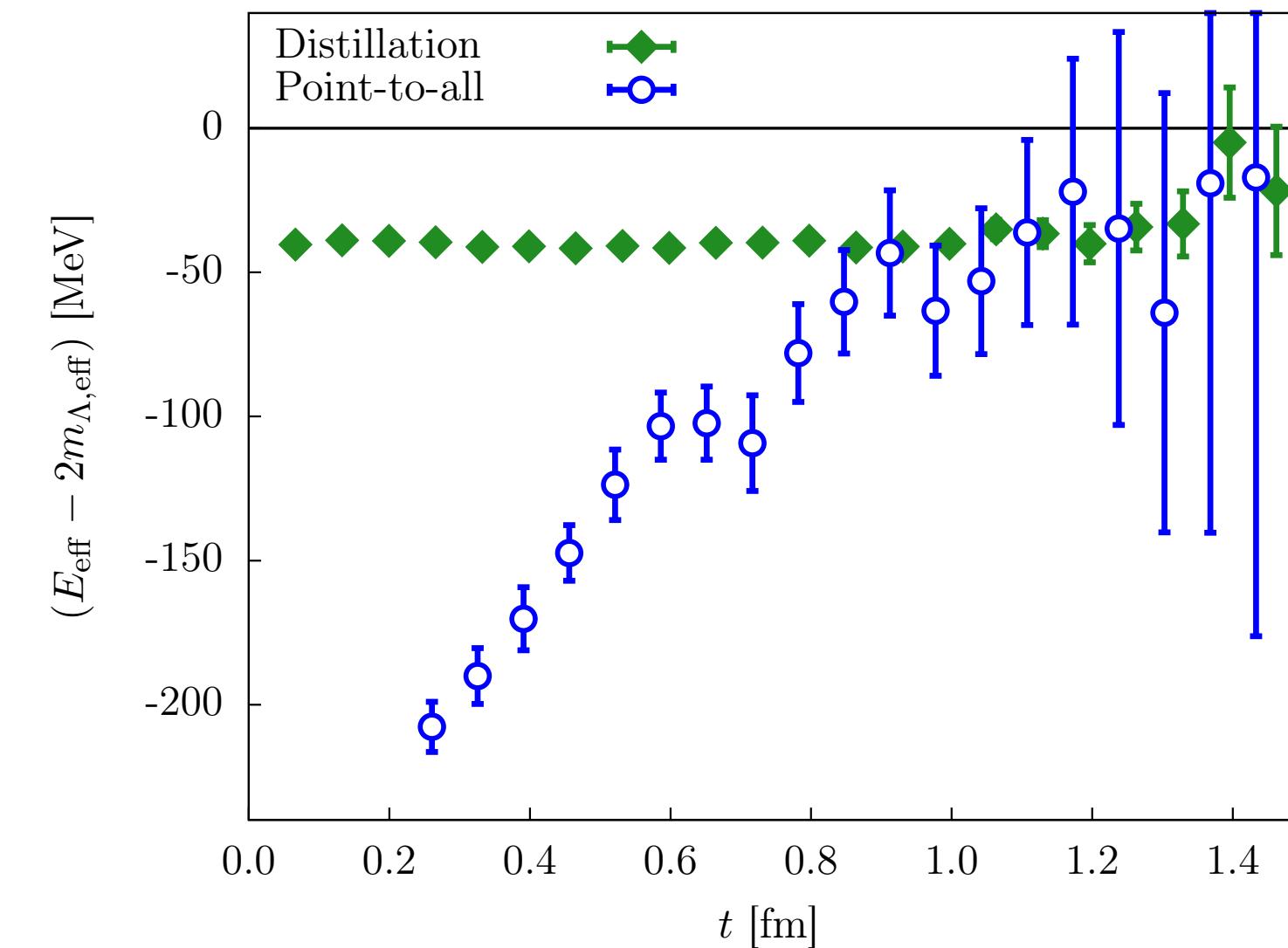
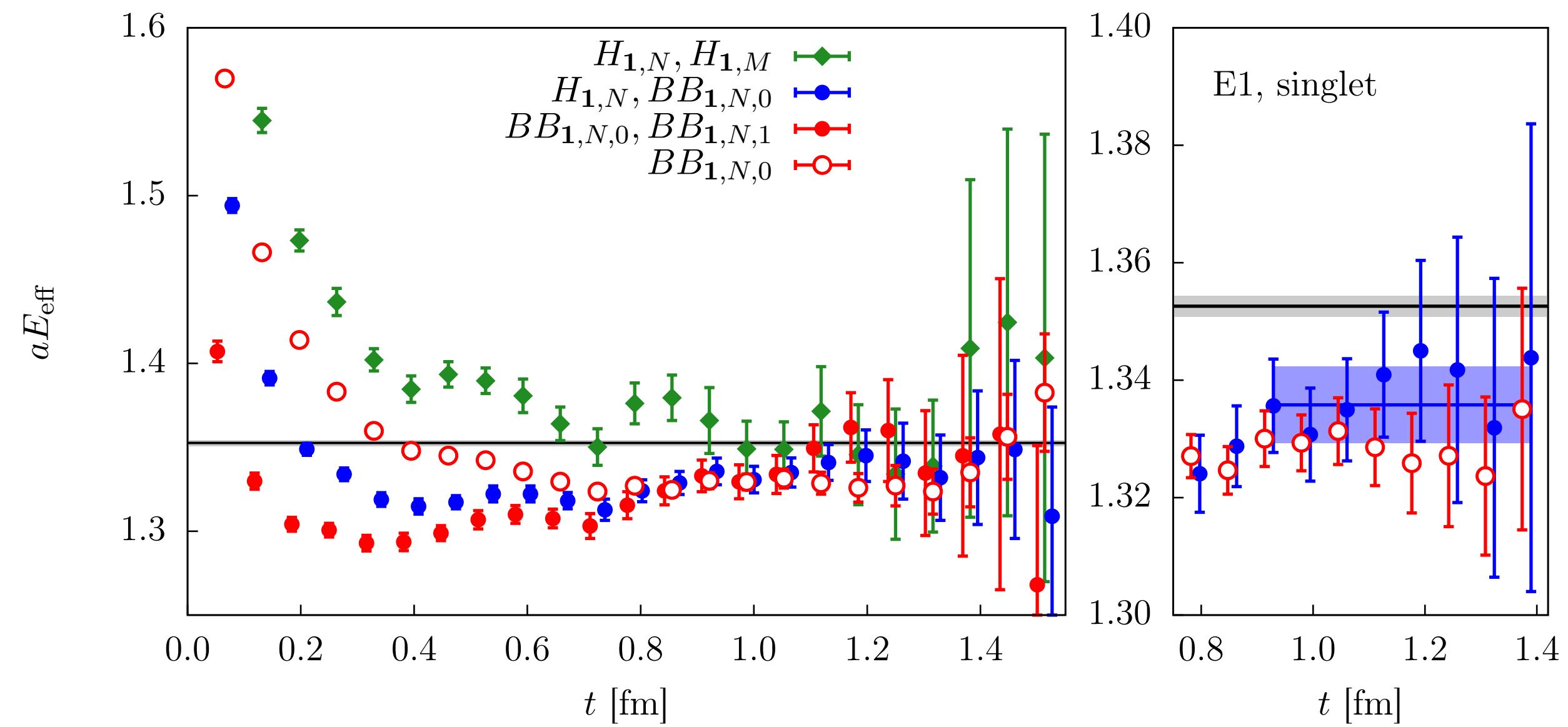
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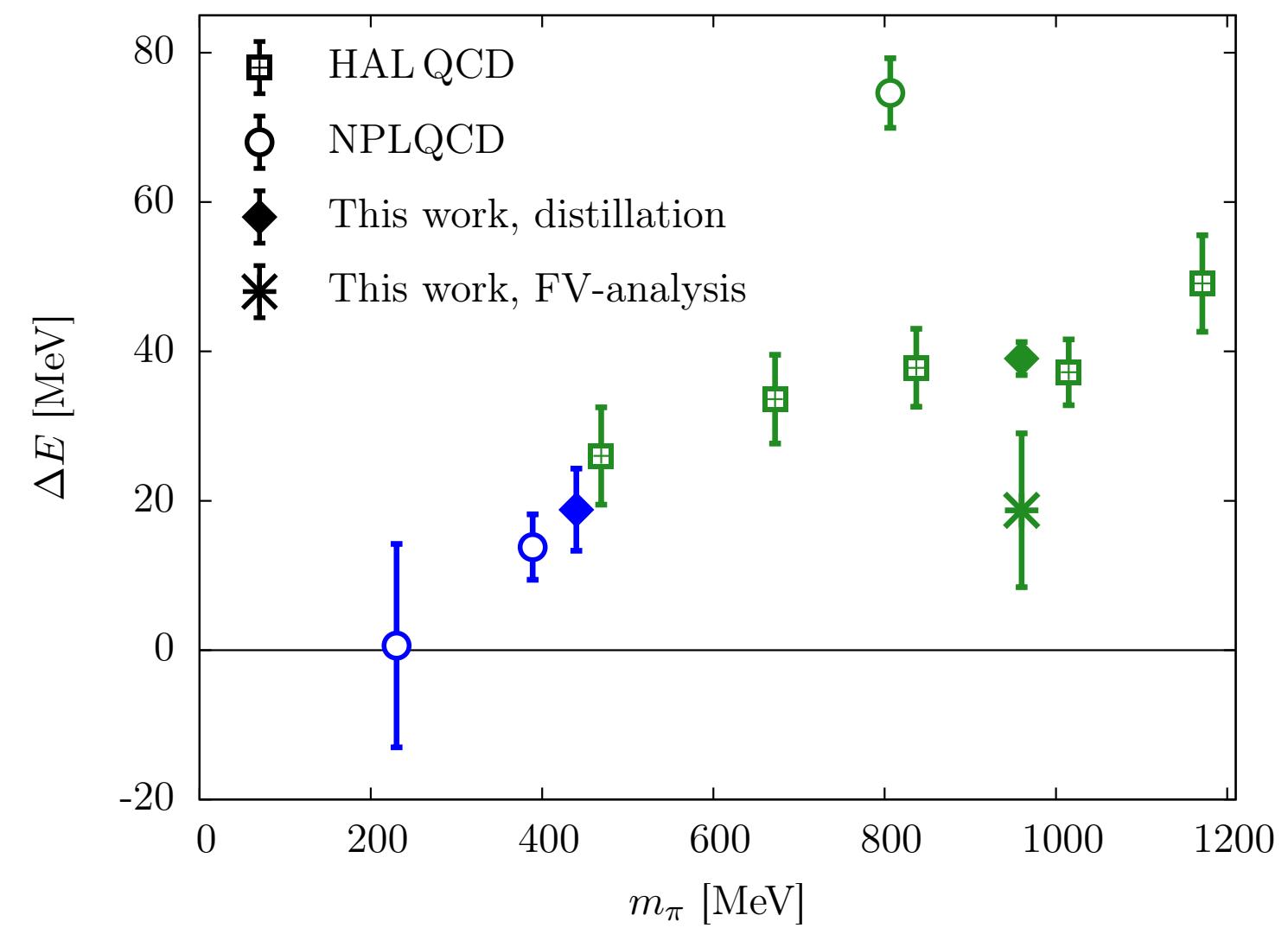
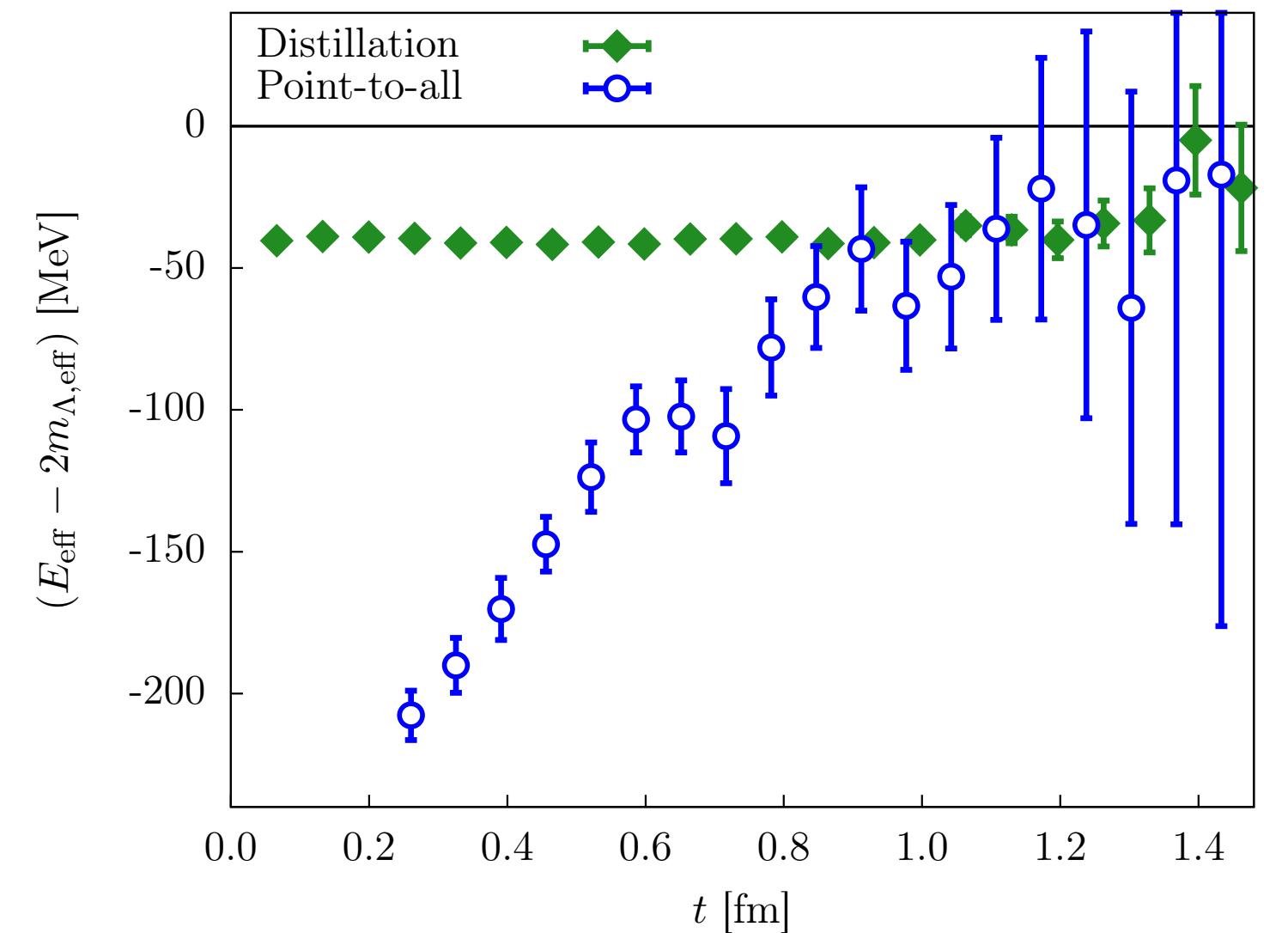
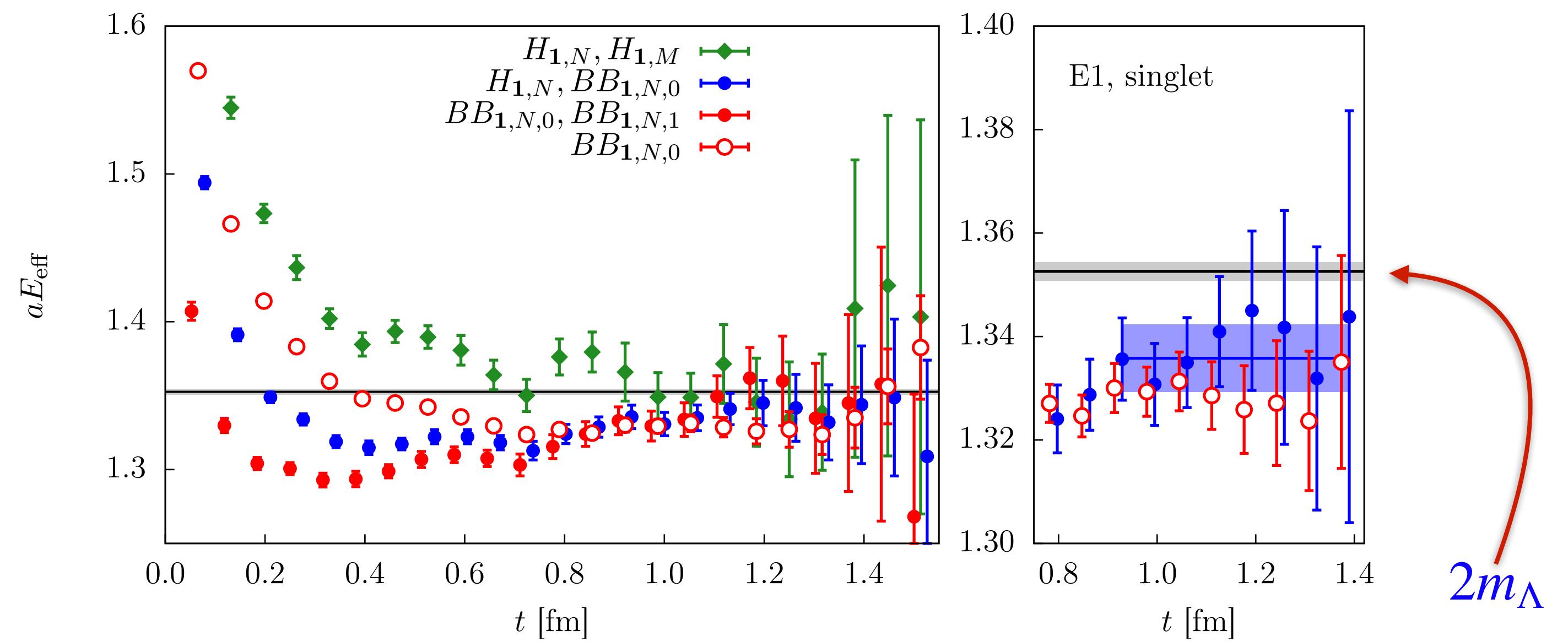
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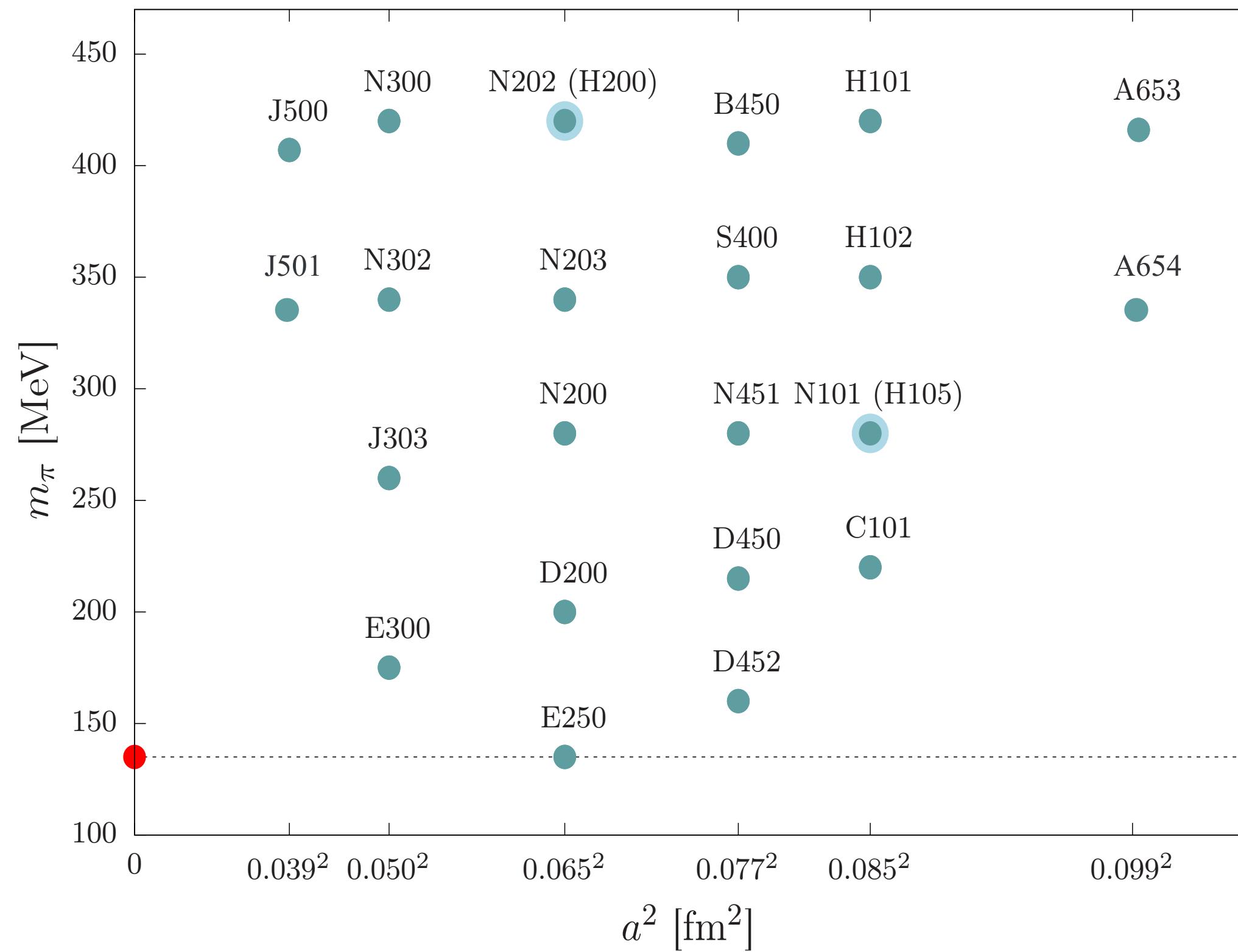
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# Dibaryons in QCD with 3 and 2 + 1 flavours

Use CLS ensembles with  $N_f = 2 + 1$  flavours of  $O(a)$  improved Wilson quarks

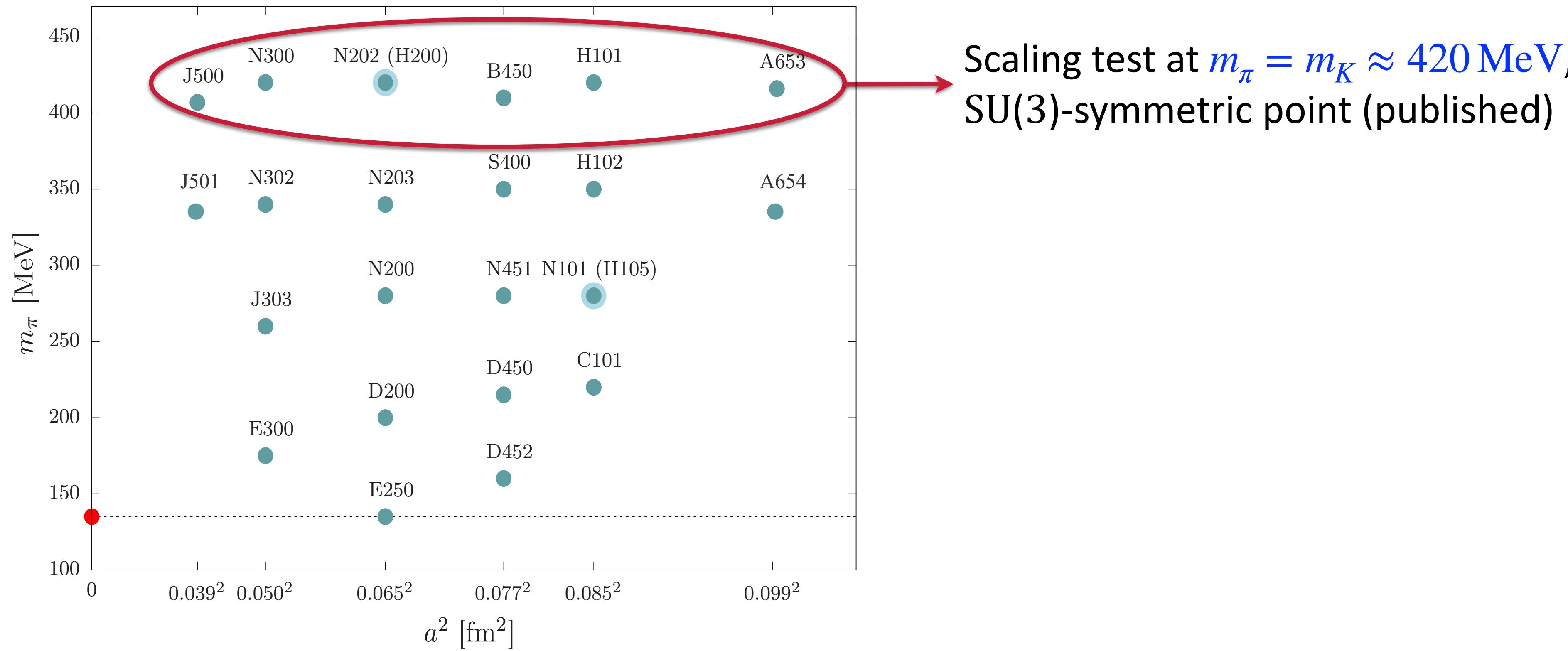
- Six lattice spacings:  $a = 0.099 - 0.039 \text{ fm}$ ; pion masses  $m_\pi = 130 - 420 \text{ MeV}$
- Timeslice-to-all propagators; chiral trajectory:  $\text{Tr } M_q = \text{const.} \Leftrightarrow \frac{1}{2}m_\pi^2 + m_K^2 \approx \text{const.}$



# Dibaryons in QCD with 3 and 2 + 1 flavours

Use CLS ensembles with  $N_f = 2 + 1$  flavours of  $O(a)$  improved Wilson quarks

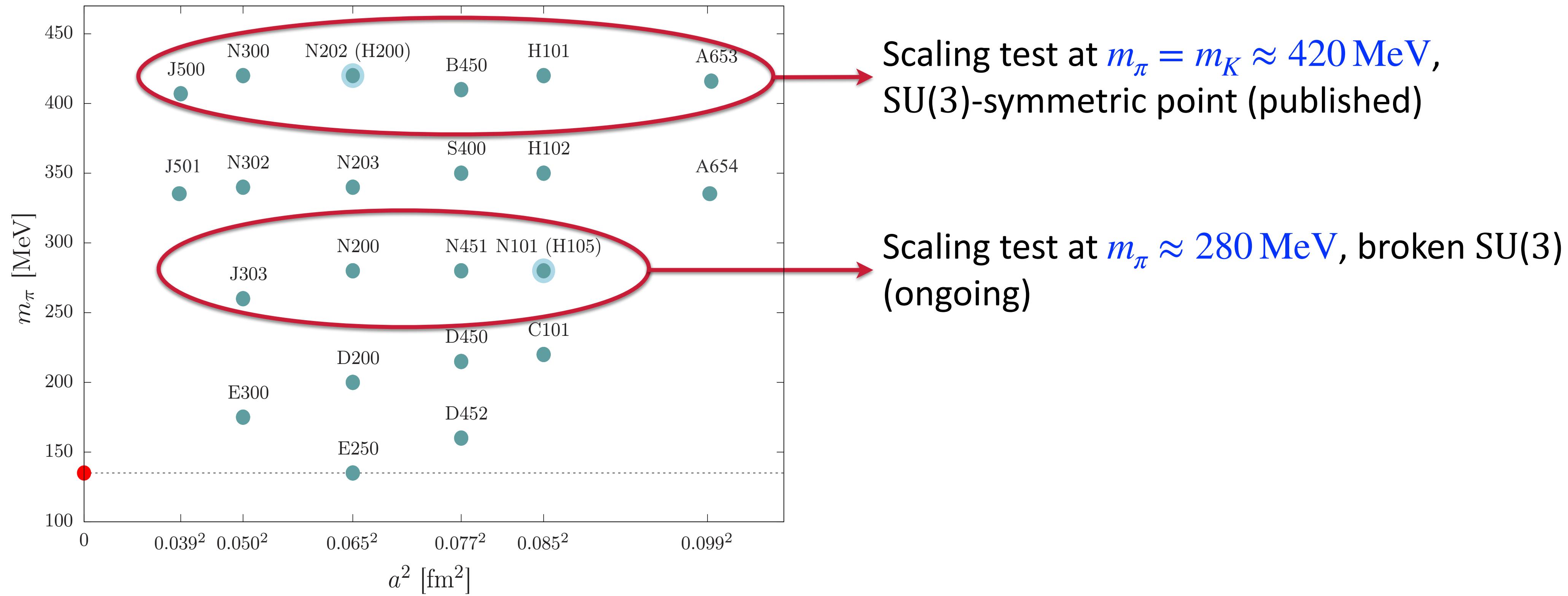
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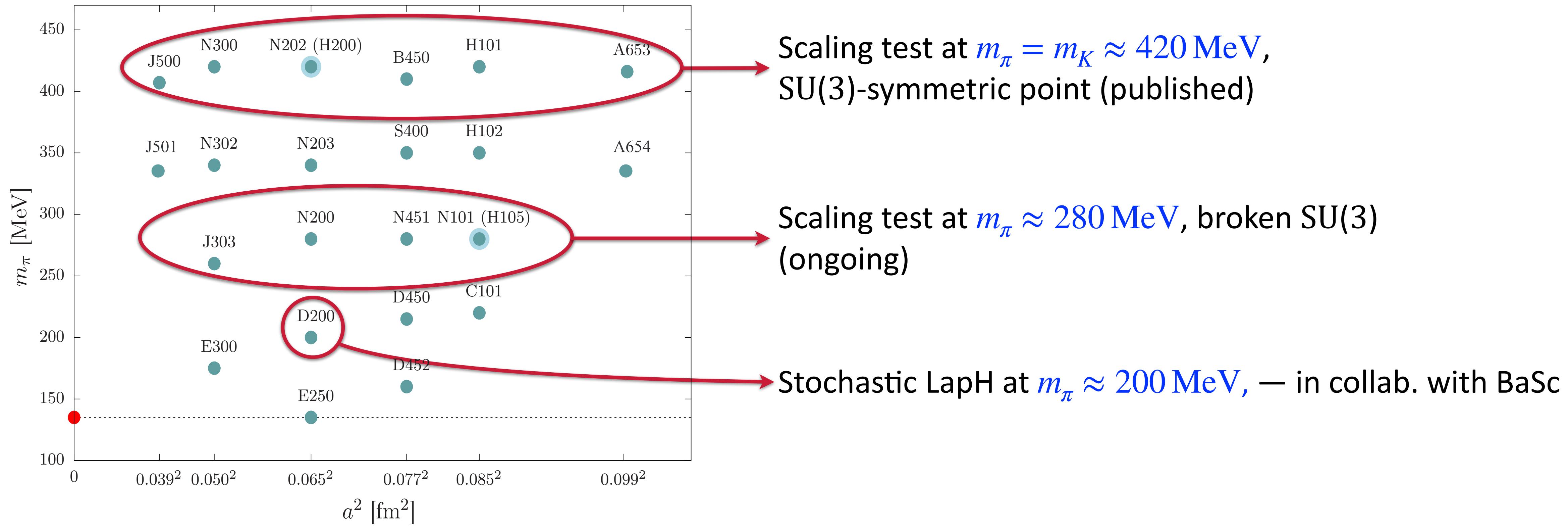
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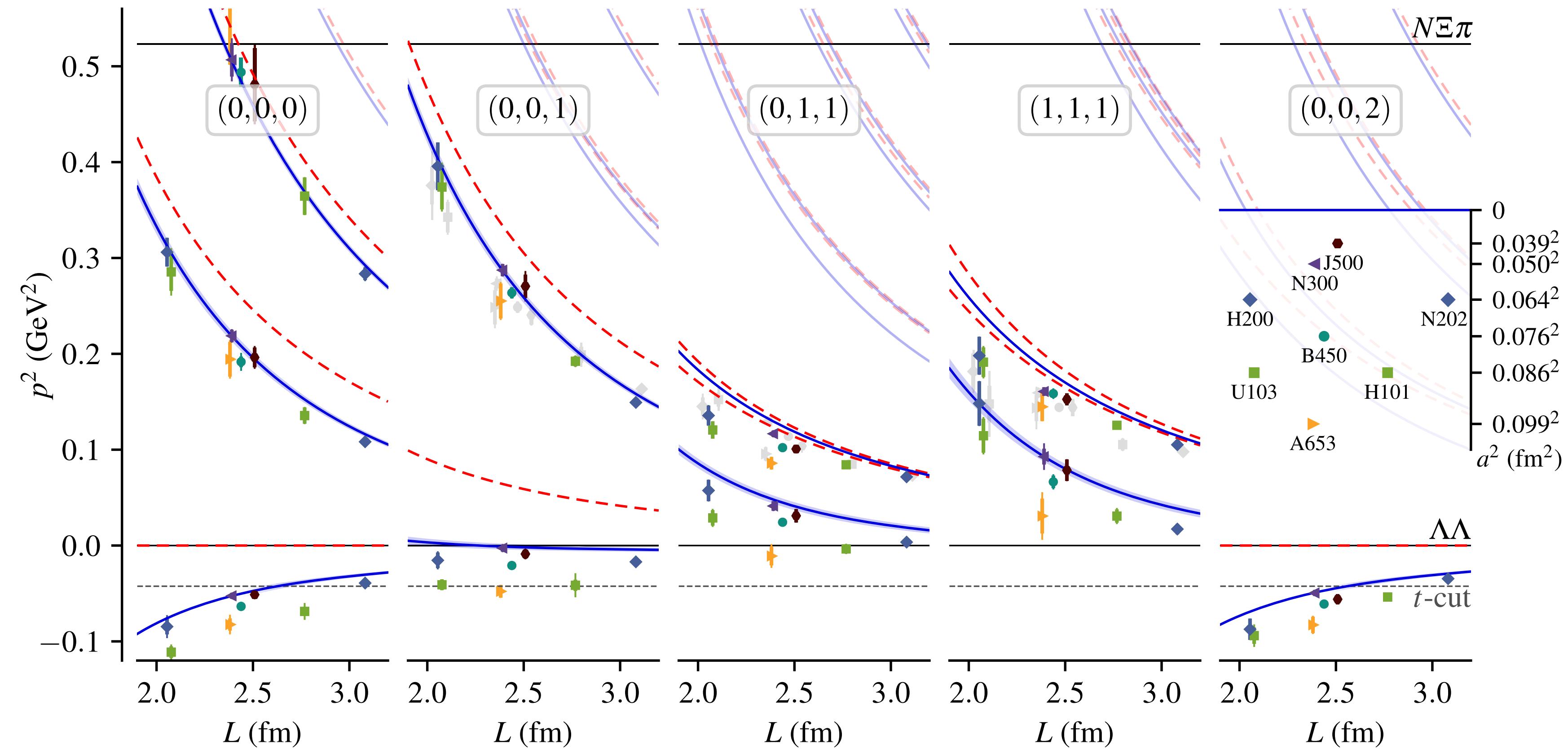
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# $H$ Dibaryon at the SU(3)-symmetric point

Scattering momenta in finite volume in different frames:

$$p^2 = \frac{1}{4}(E_L^2 - \mathbf{P} \cdot \mathbf{P}) - m_\Lambda^2$$



— : interacting spectrum in continuum limit

- - - : non-interacting levels

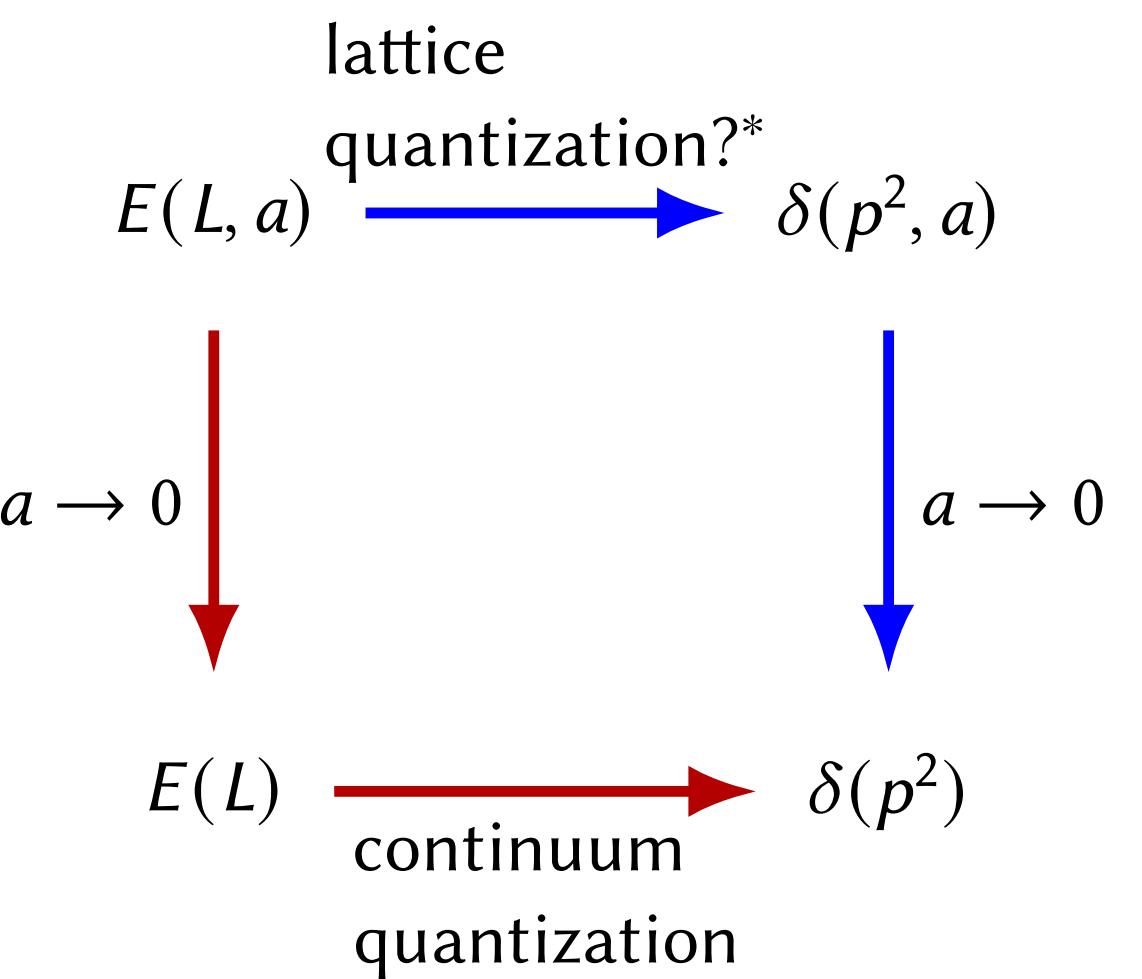
# $H$ Dibaryon at the SU(3)-symmetric point

Continuum extrapolation

Finite-volume quantisation condition only valid in continuum limit

Perform combined fit of  $p \cot \delta(p)$  in both  $p^2$  and  $a$ :

$$p \cot \delta(p) = \sum_{i=0}^{N-1} c_i p^{2i} \stackrel{!}{=} -\sqrt{-p^2}, \quad c_i = c_{i0} + c_{i1} a^2$$



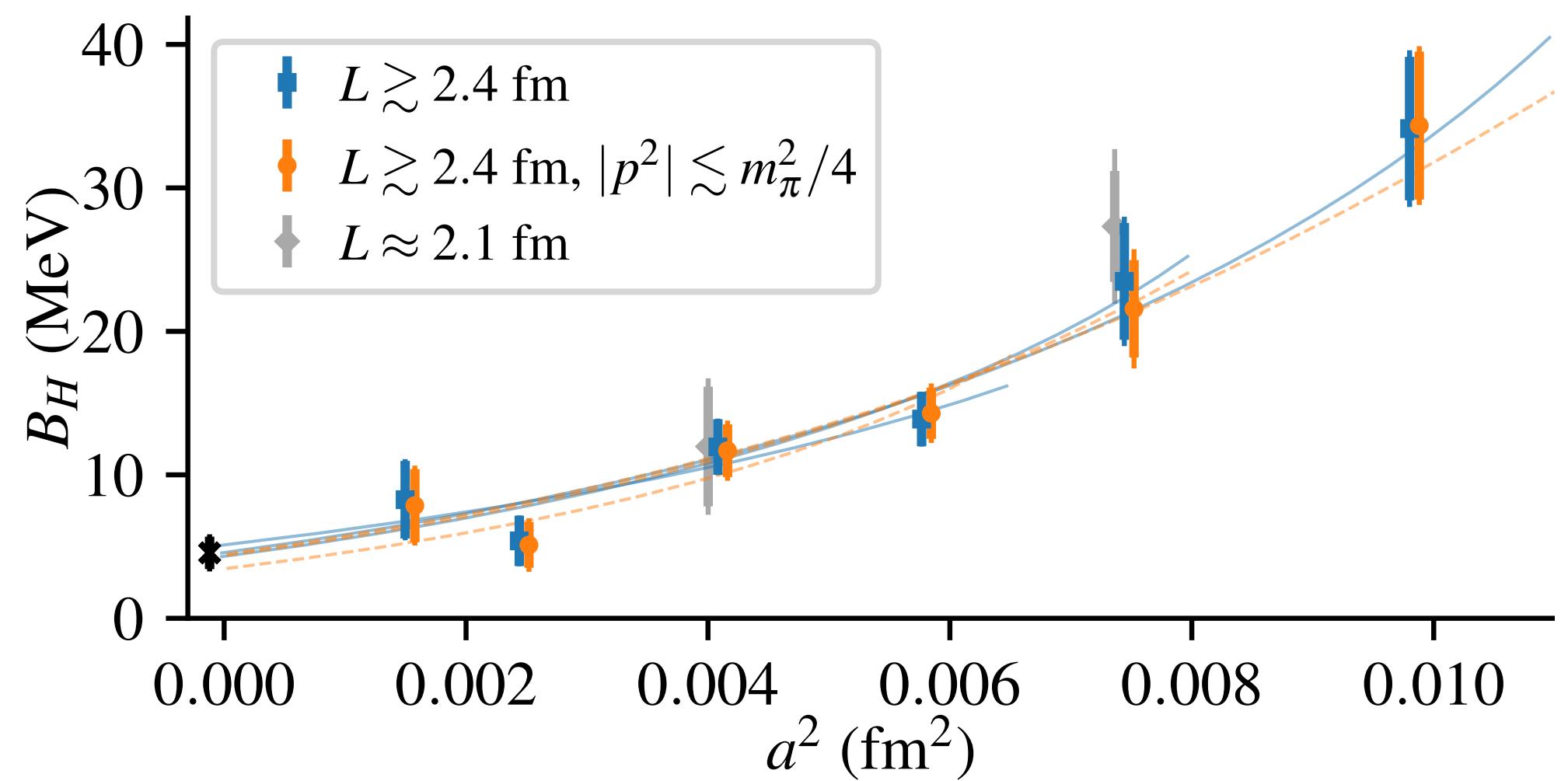
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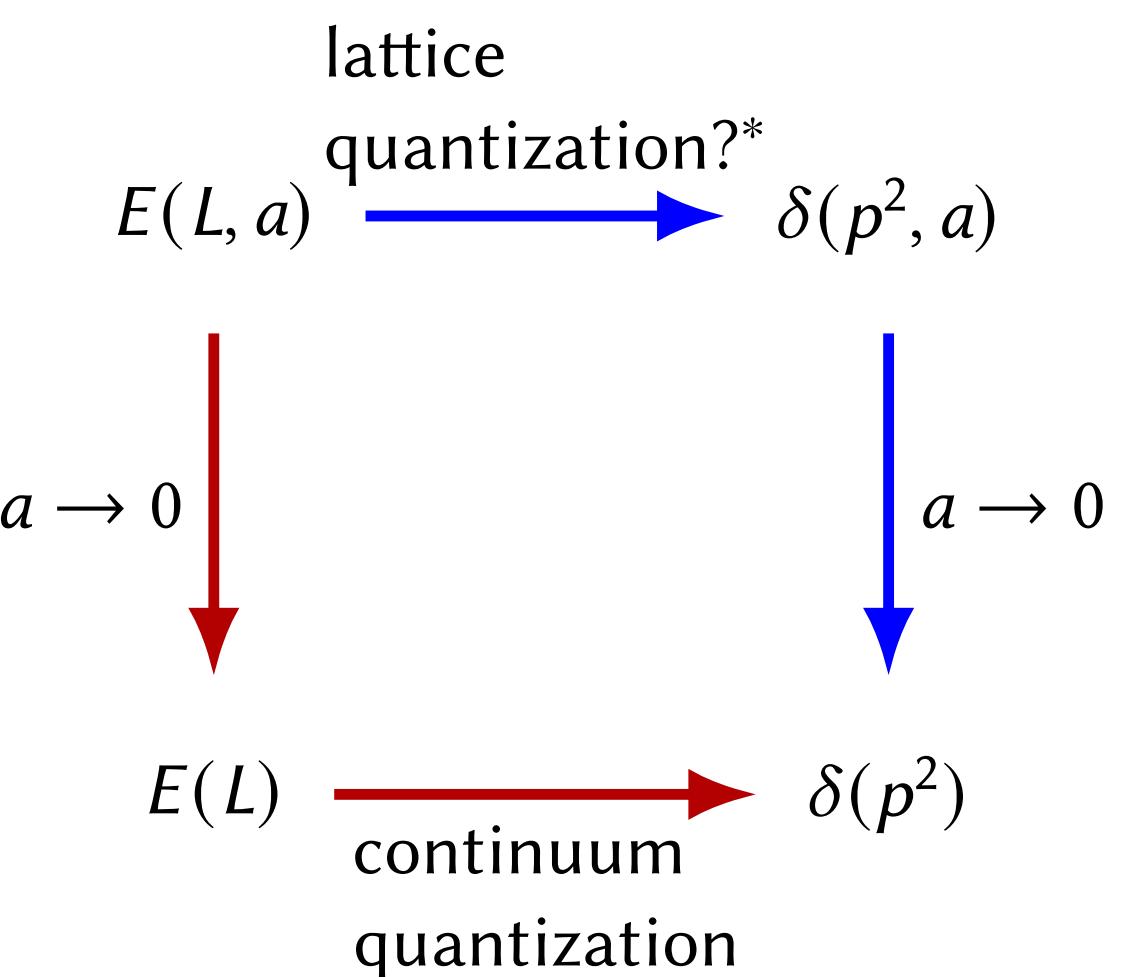
Finite-volume quantisation condition only valid in continuum limit

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$$\Rightarrow B_H^{N_f=3} = 4.56 \pm 1.13 \pm 0.63 \text{ MeV}$$



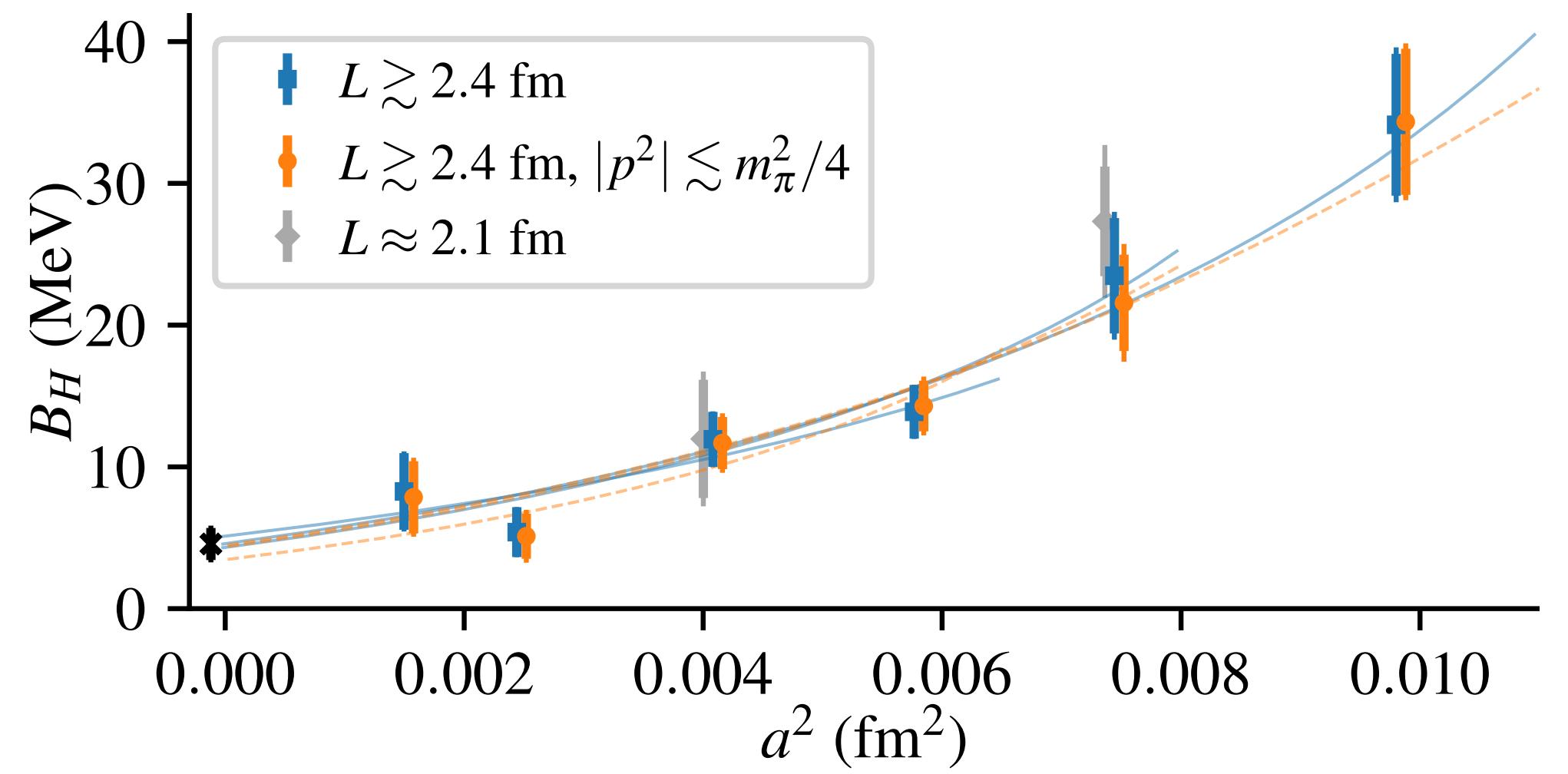
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Continuum extrapolation

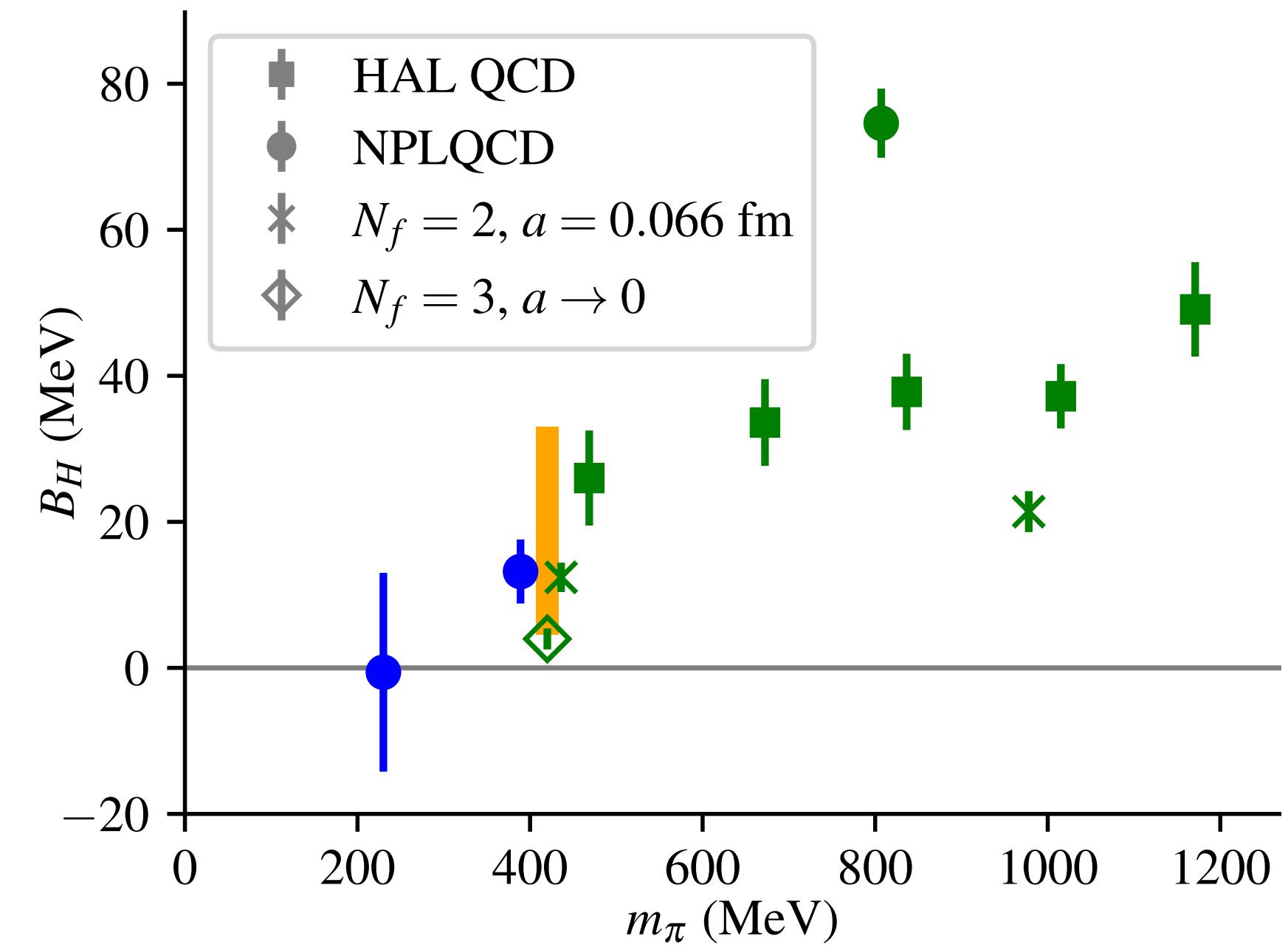
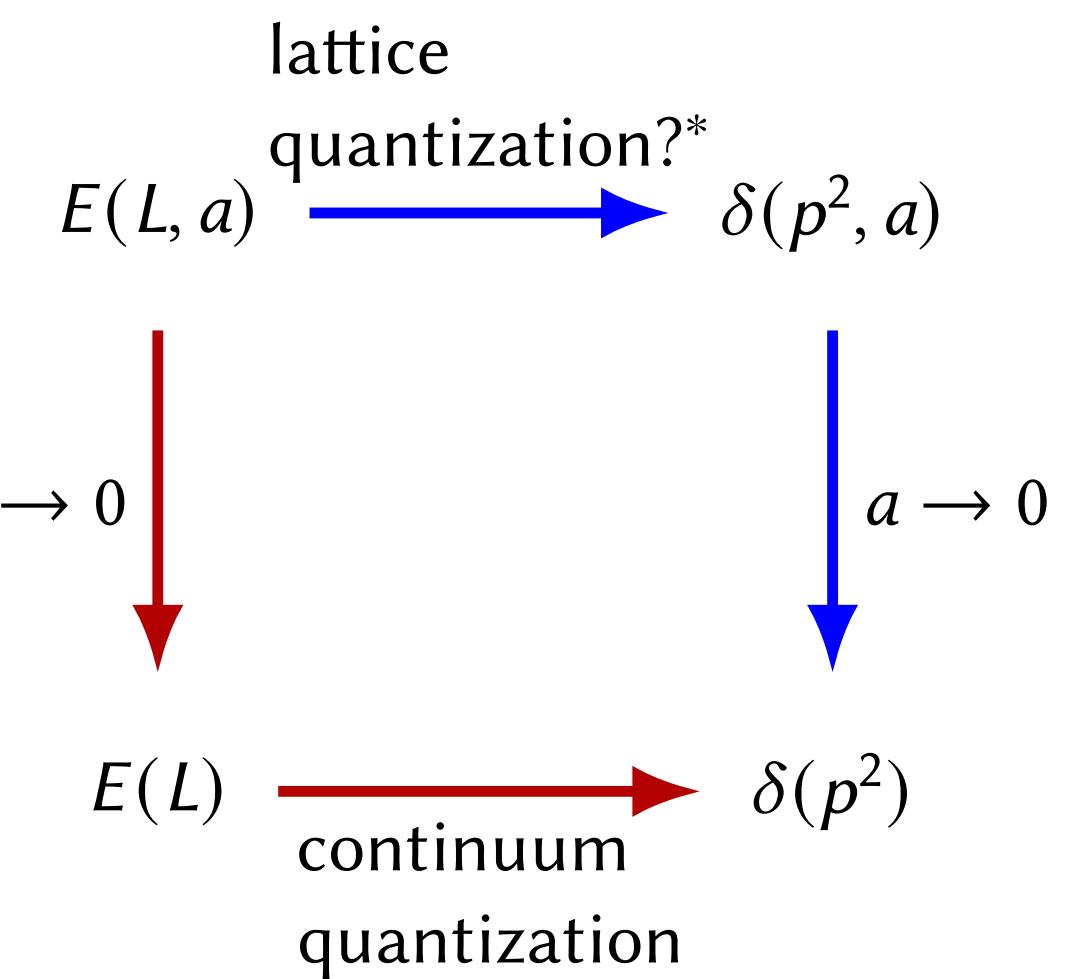
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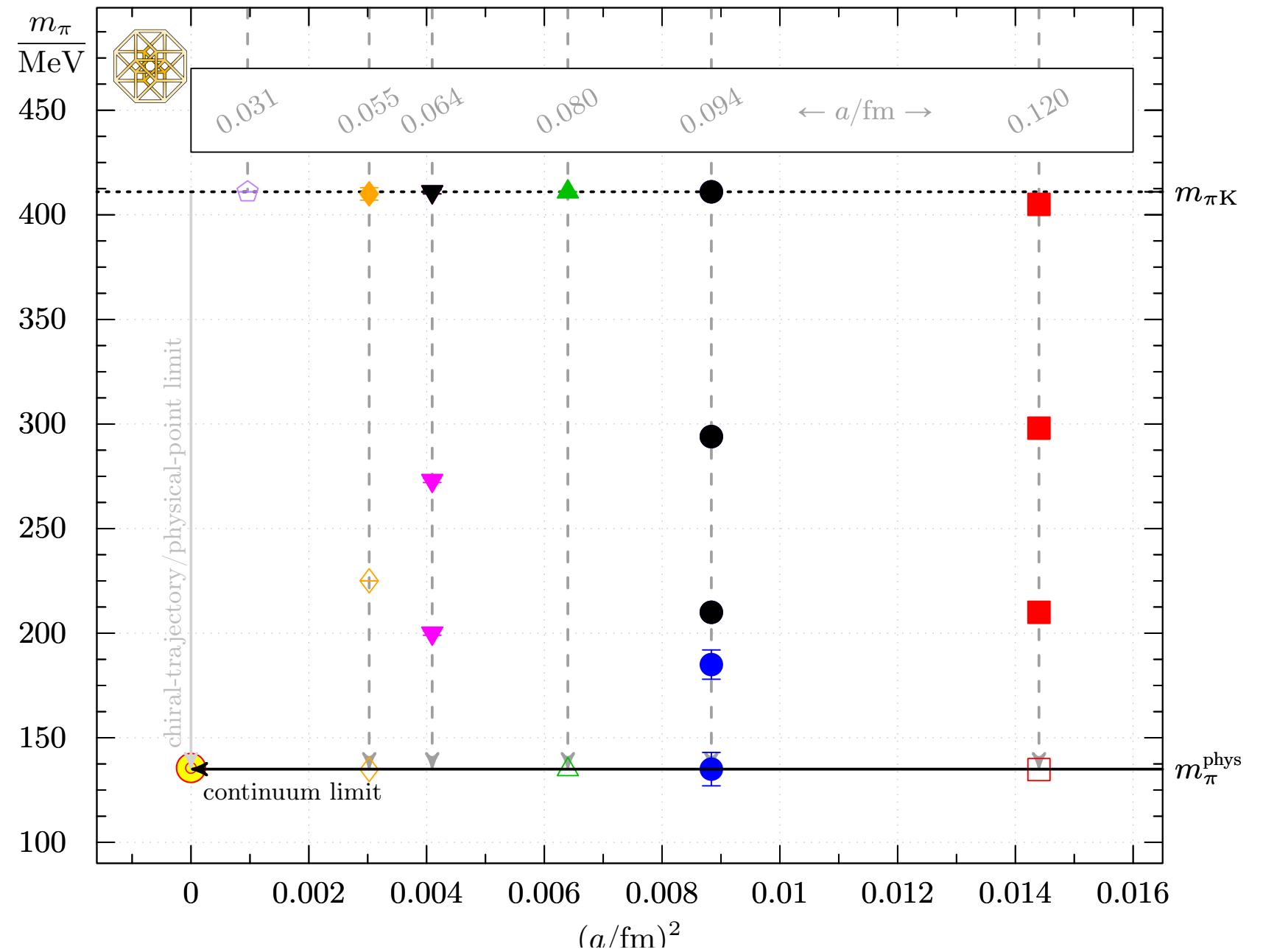


$$\Rightarrow B_H^{N_f=3} = 4.56 \pm 1.13 \pm 0.63 \text{ MeV}$$



# $H$ Dibaryon at the SU(3)-symmetric point

Cross-check using “Stabilised Wilson Fermions” (OpenLat)

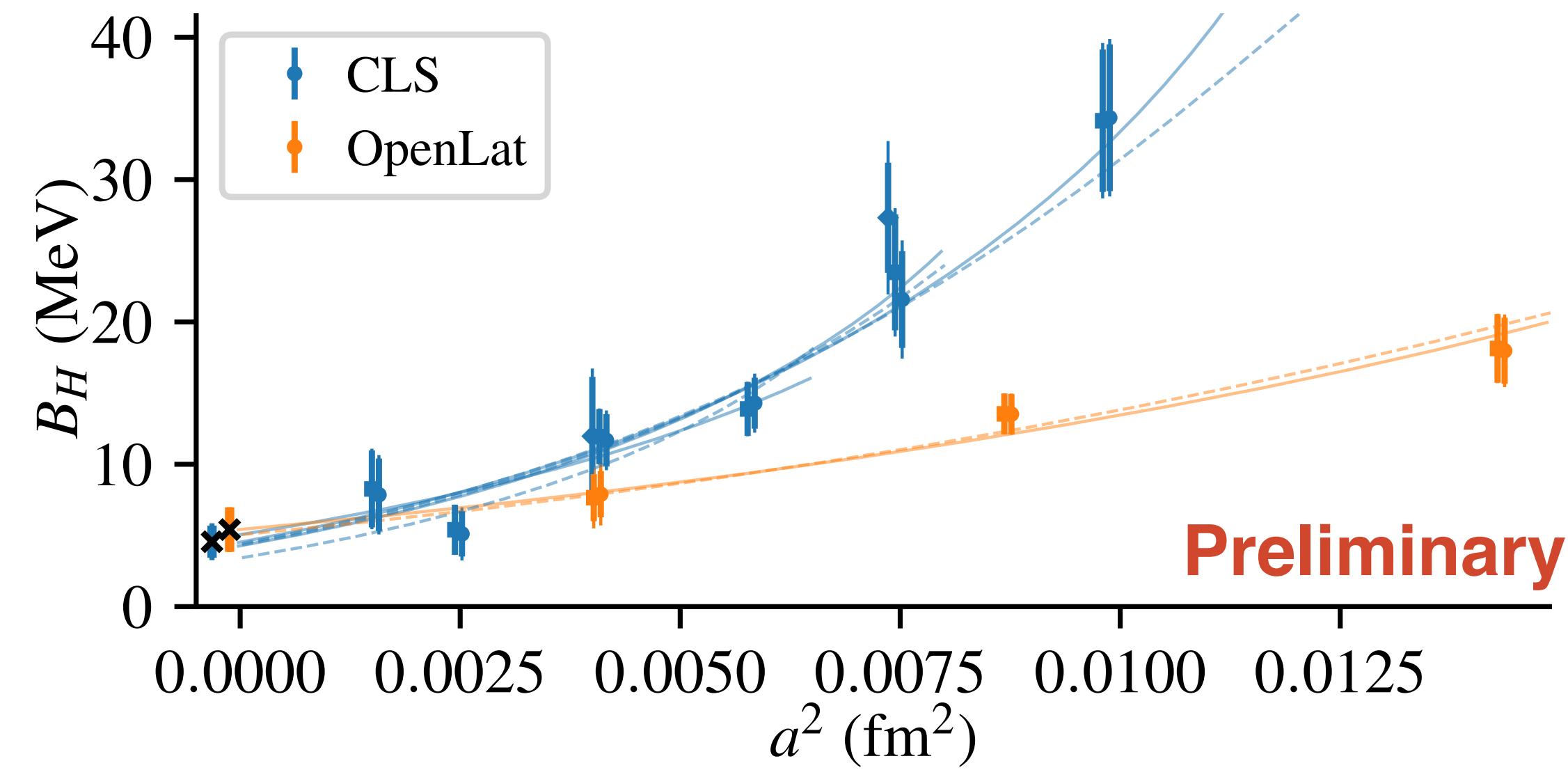
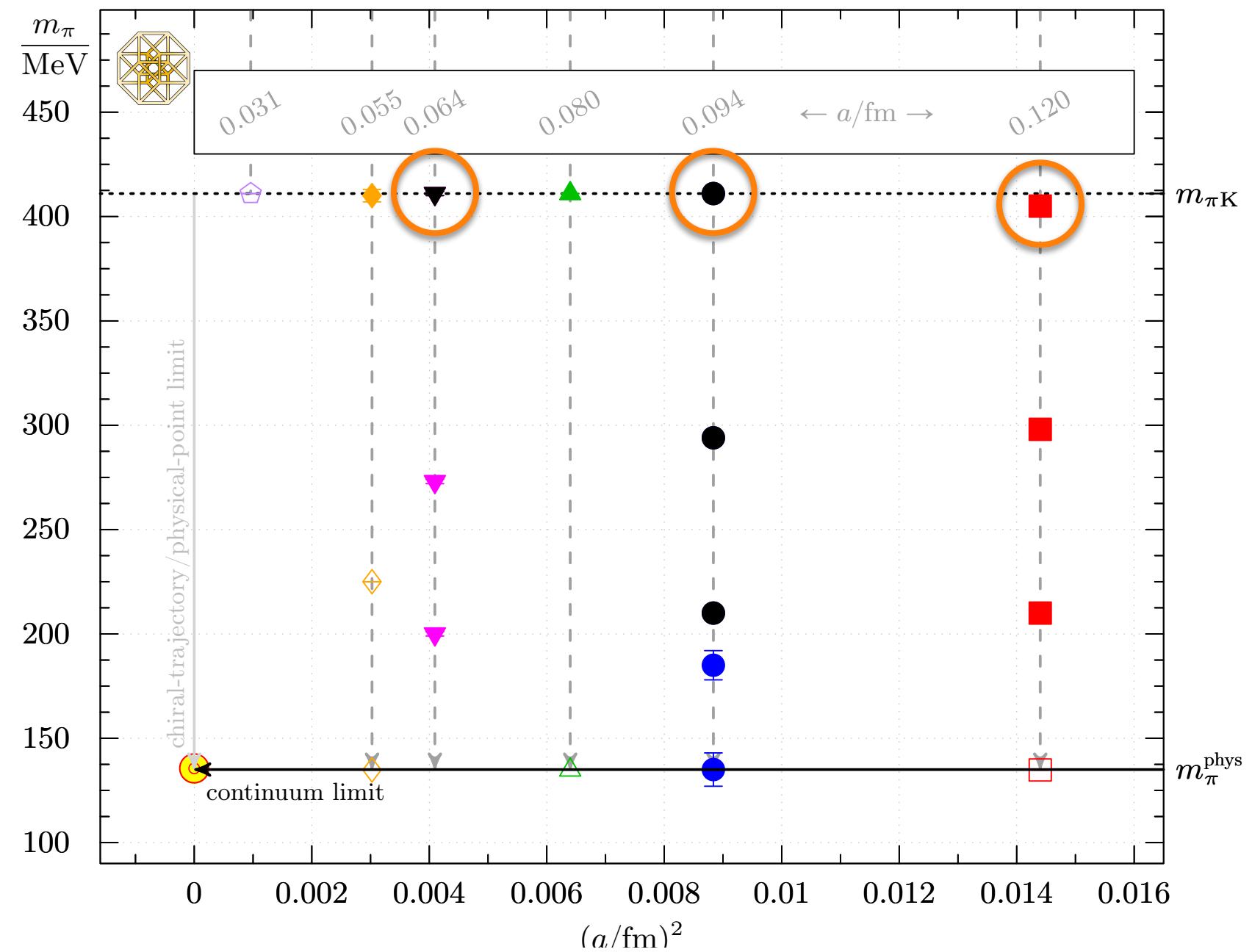


$$D_{\text{wilson}} = \begin{pmatrix} D_{ee} & D_{eo} \\ D_{oe} & D_{oo} \end{pmatrix},$$

$$D_{ee} + D_{oo} = (4 + m_0) \exp \left\{ \frac{c_{\text{sw}}}{4 + m_0} \frac{i}{4} \sigma_{\mu\nu} \widehat{F}_{\mu\nu} \right\}$$

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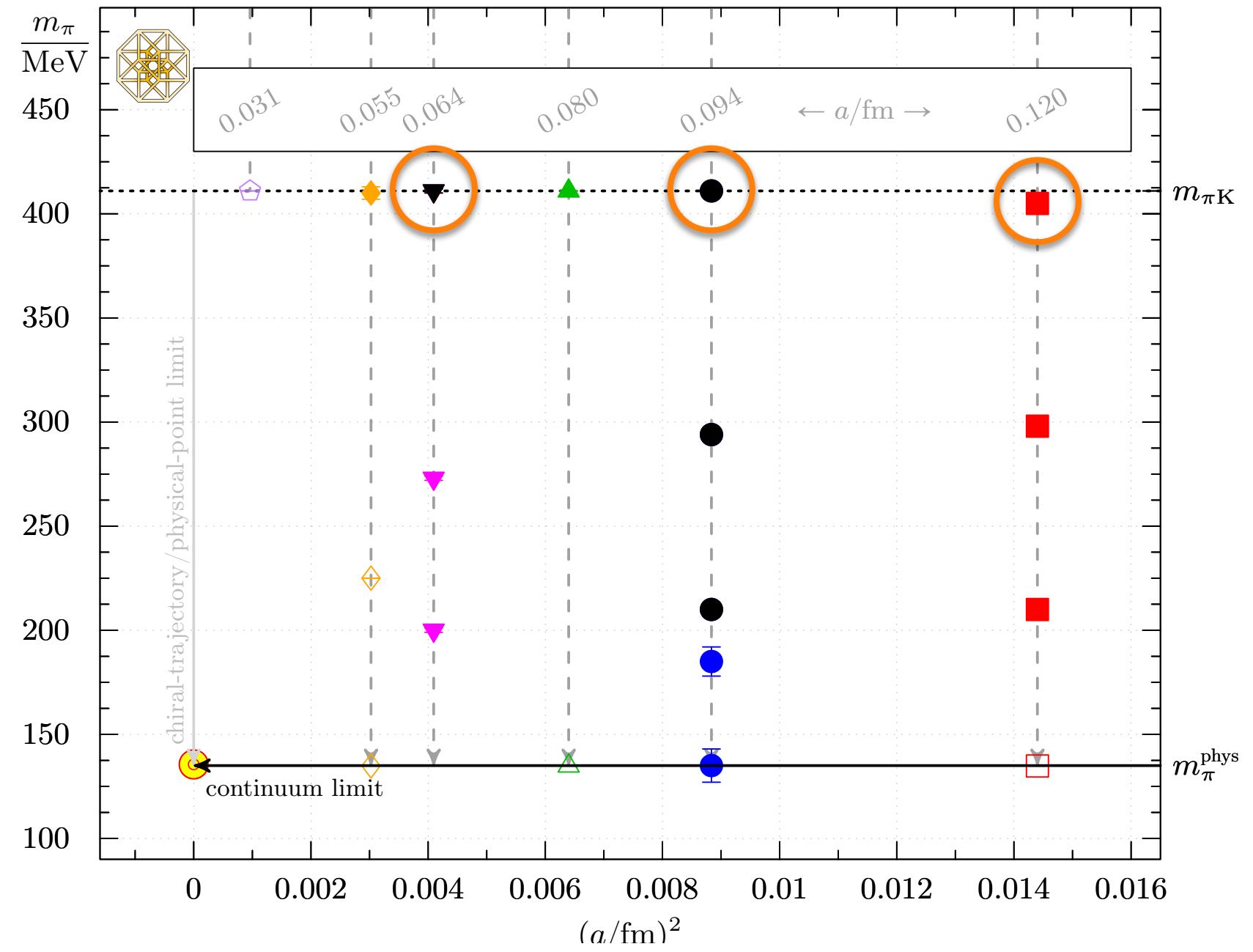


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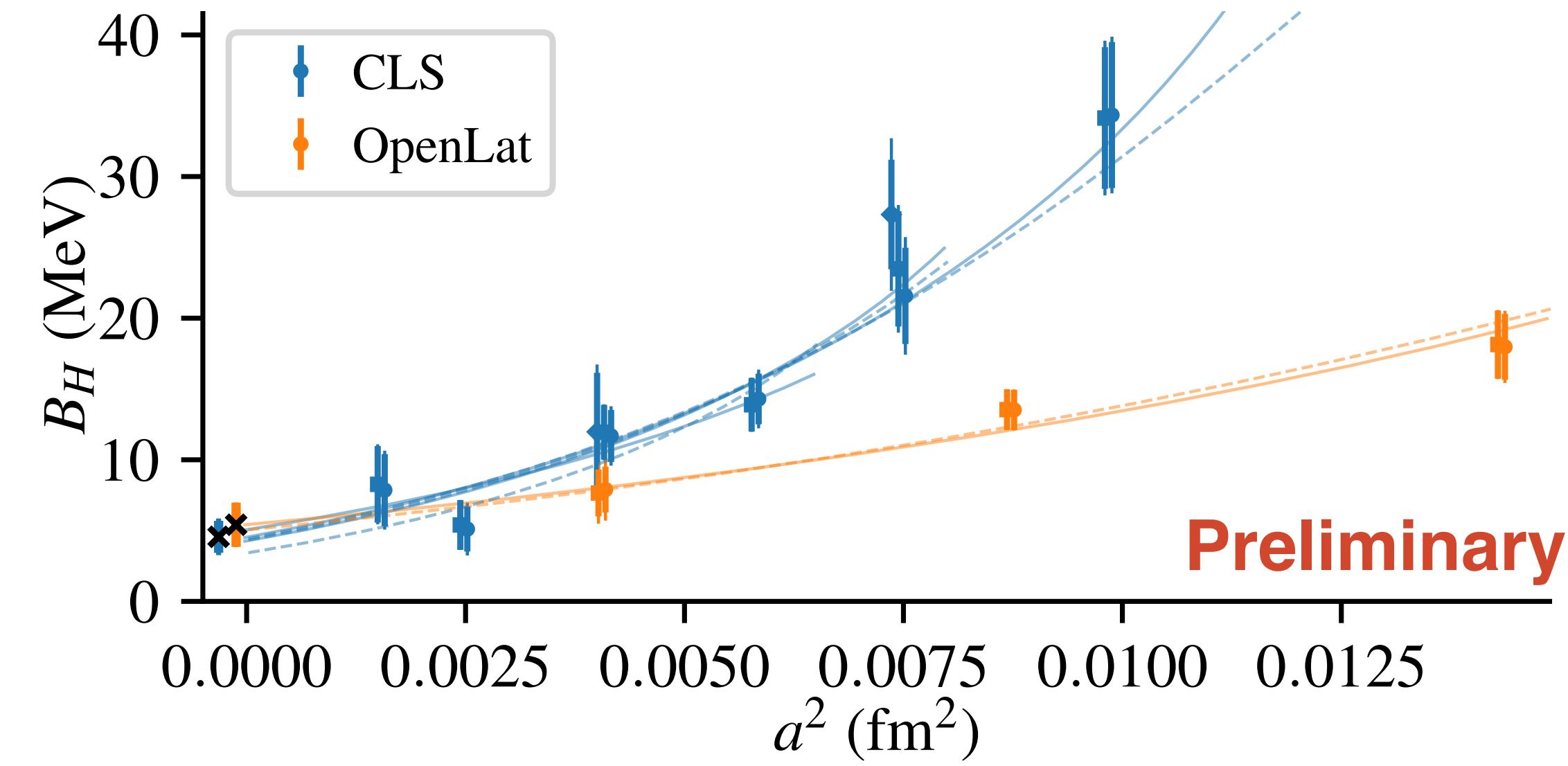
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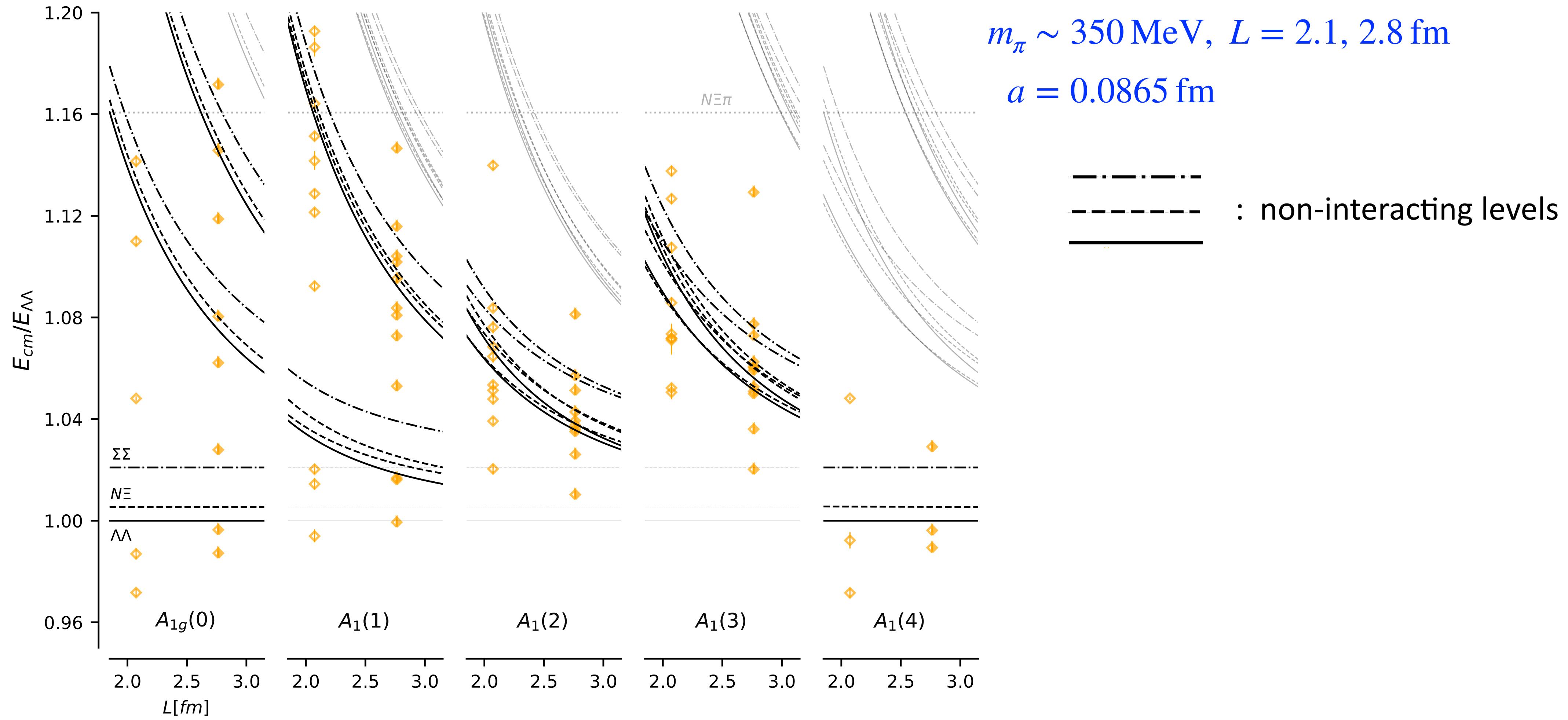
$$B_H^{N_f=3} = \begin{cases} 4.56 \pm 1.13 \pm 0.63 \text{ MeV} & \text{CLS} \\ 5.41 \pm 1.56 \pm 0.24 \text{ MeV} & \text{OpenLat} \end{cases}$$

(CLS: systematic error includes fit error, plus cut in  $a$ ,  $L$  and  $p^2$   
 OpenLat: systematic error from fit uncertainty only)

# The $H$ Dibaryon away from the SU(3)-symmetric point

Finite-volume energy levels at decreasing pion mass:

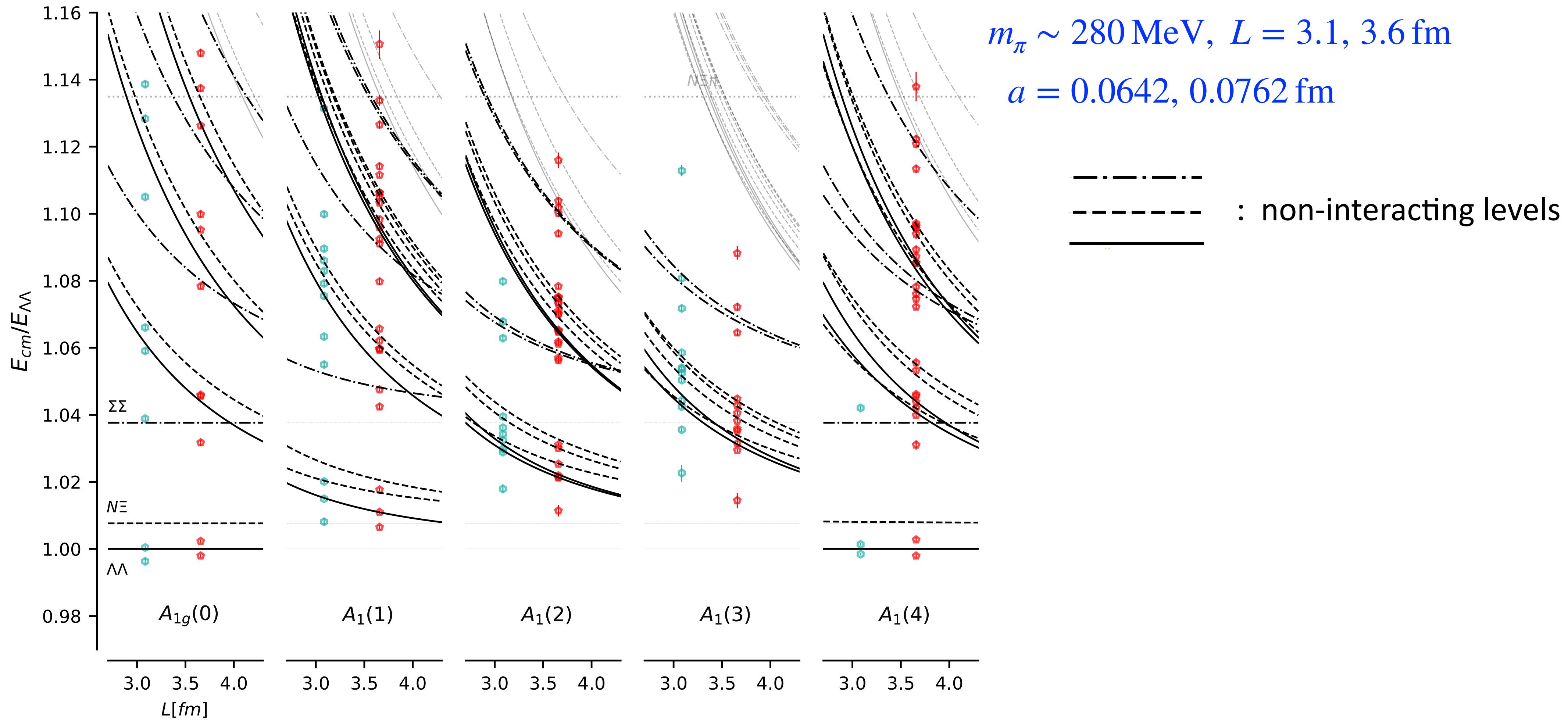
[M. Padmanath et al., arXiv:2111.11541]



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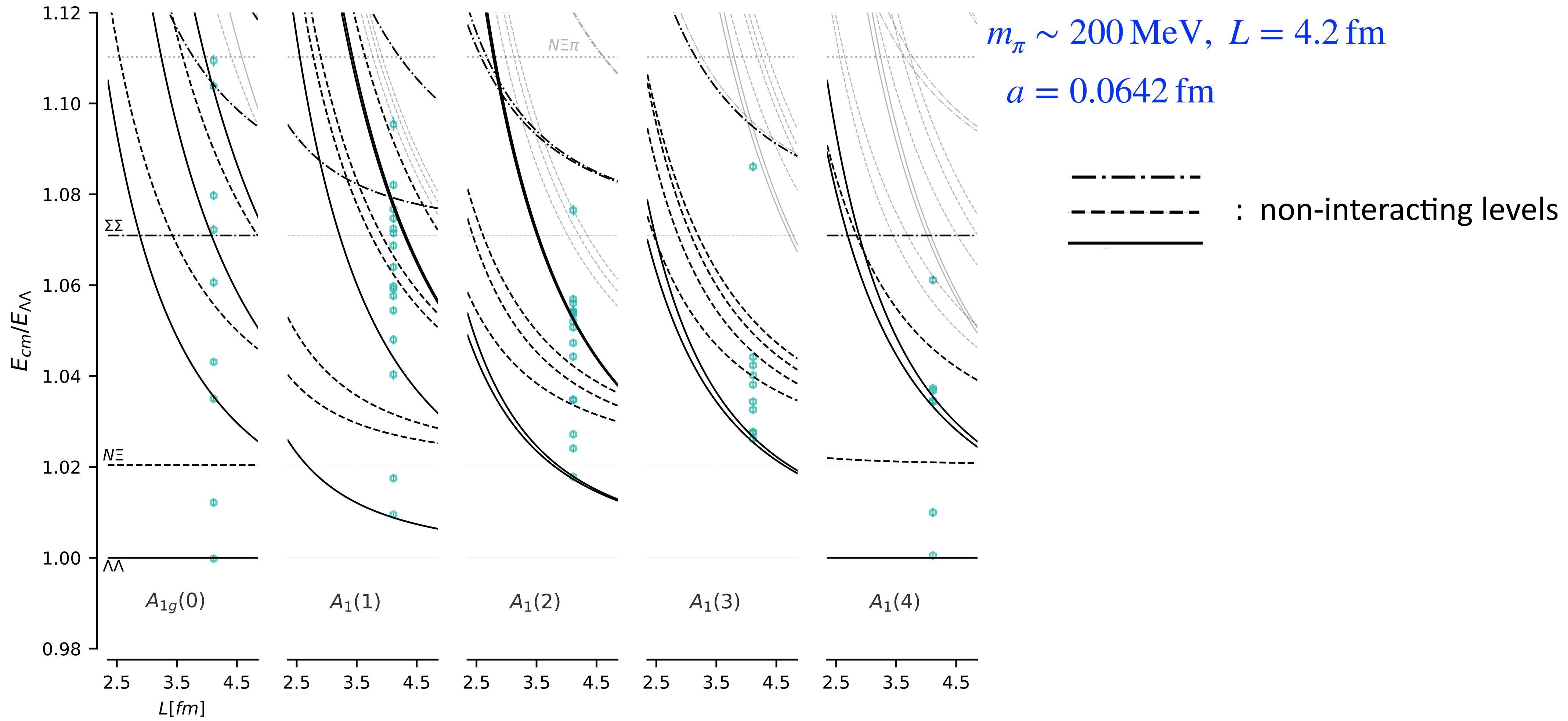
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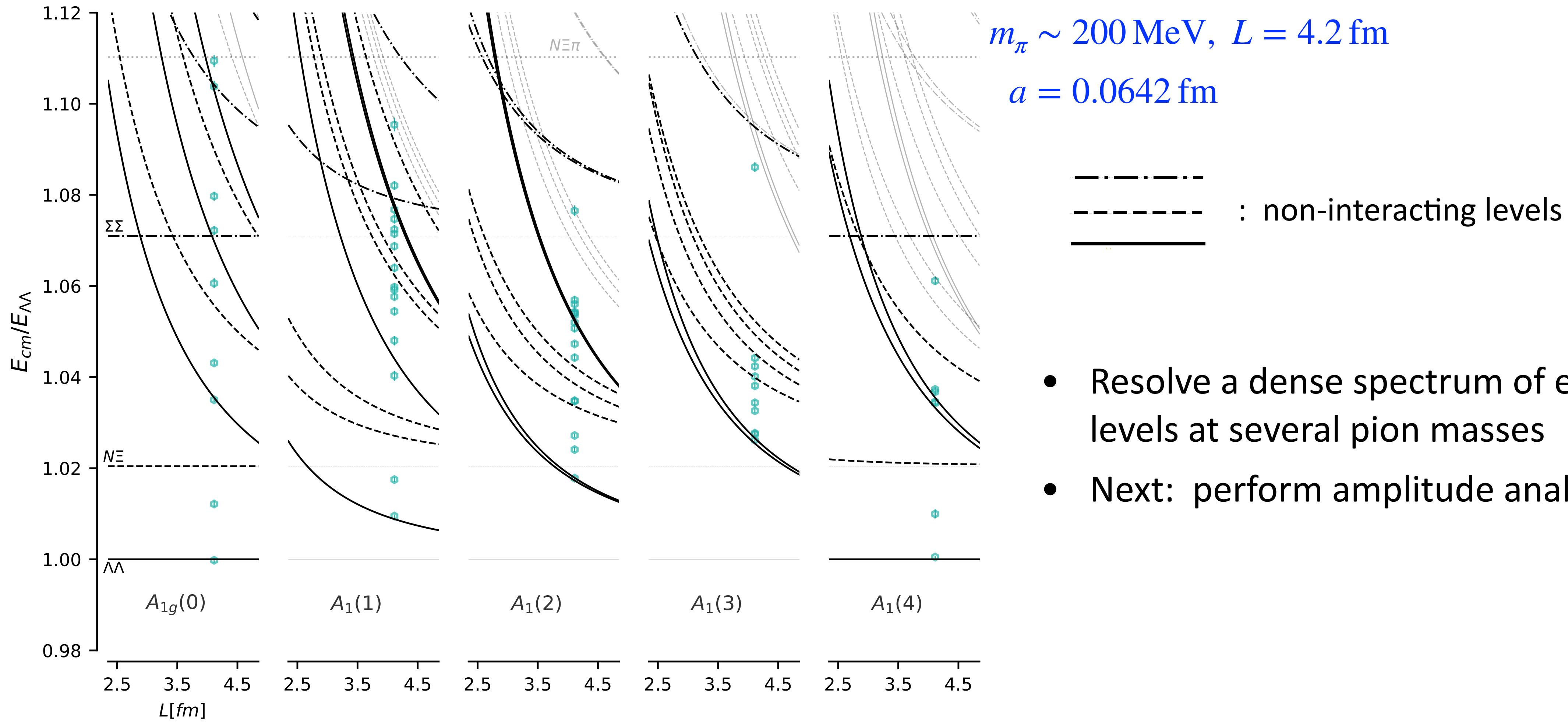
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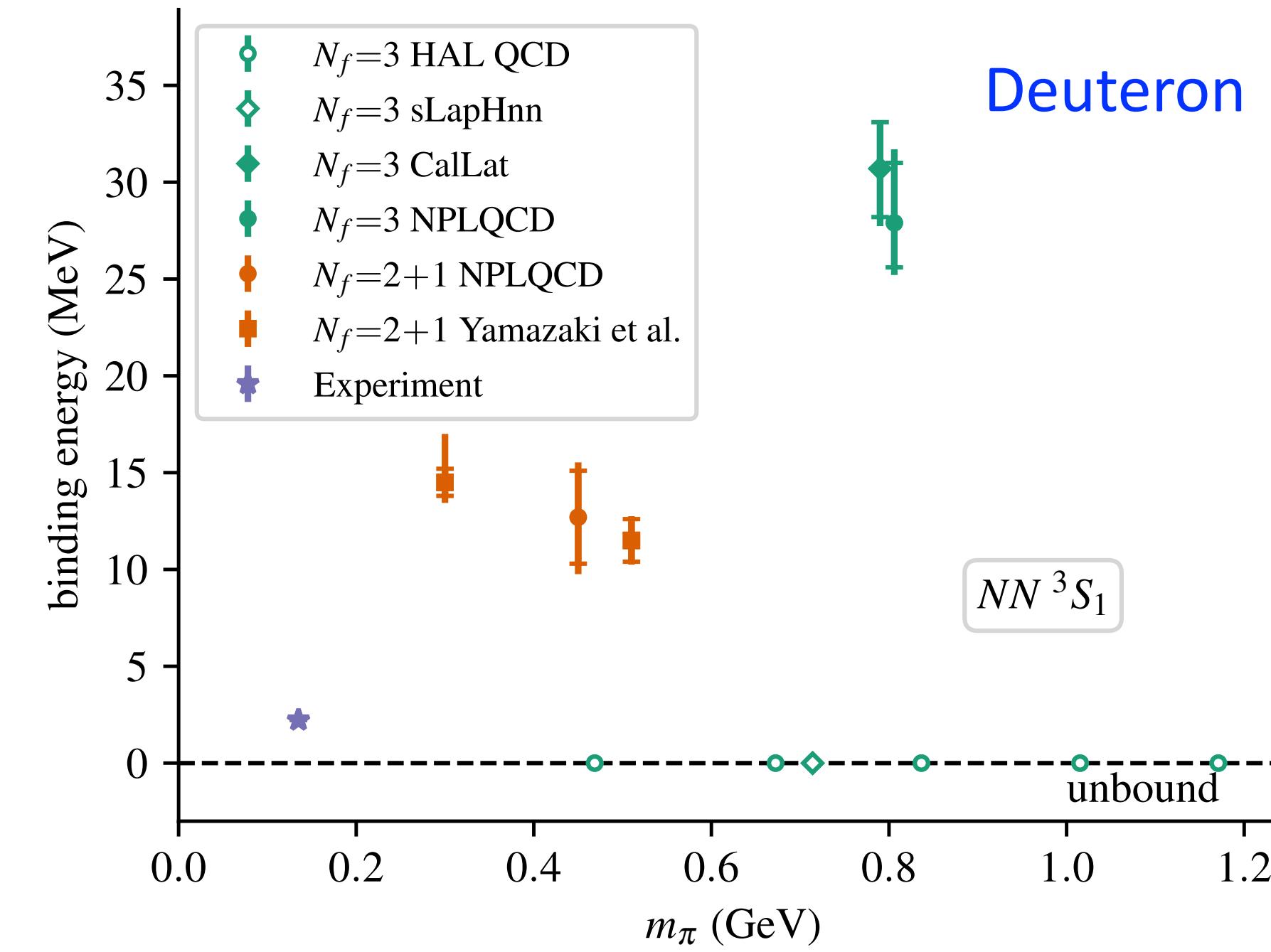
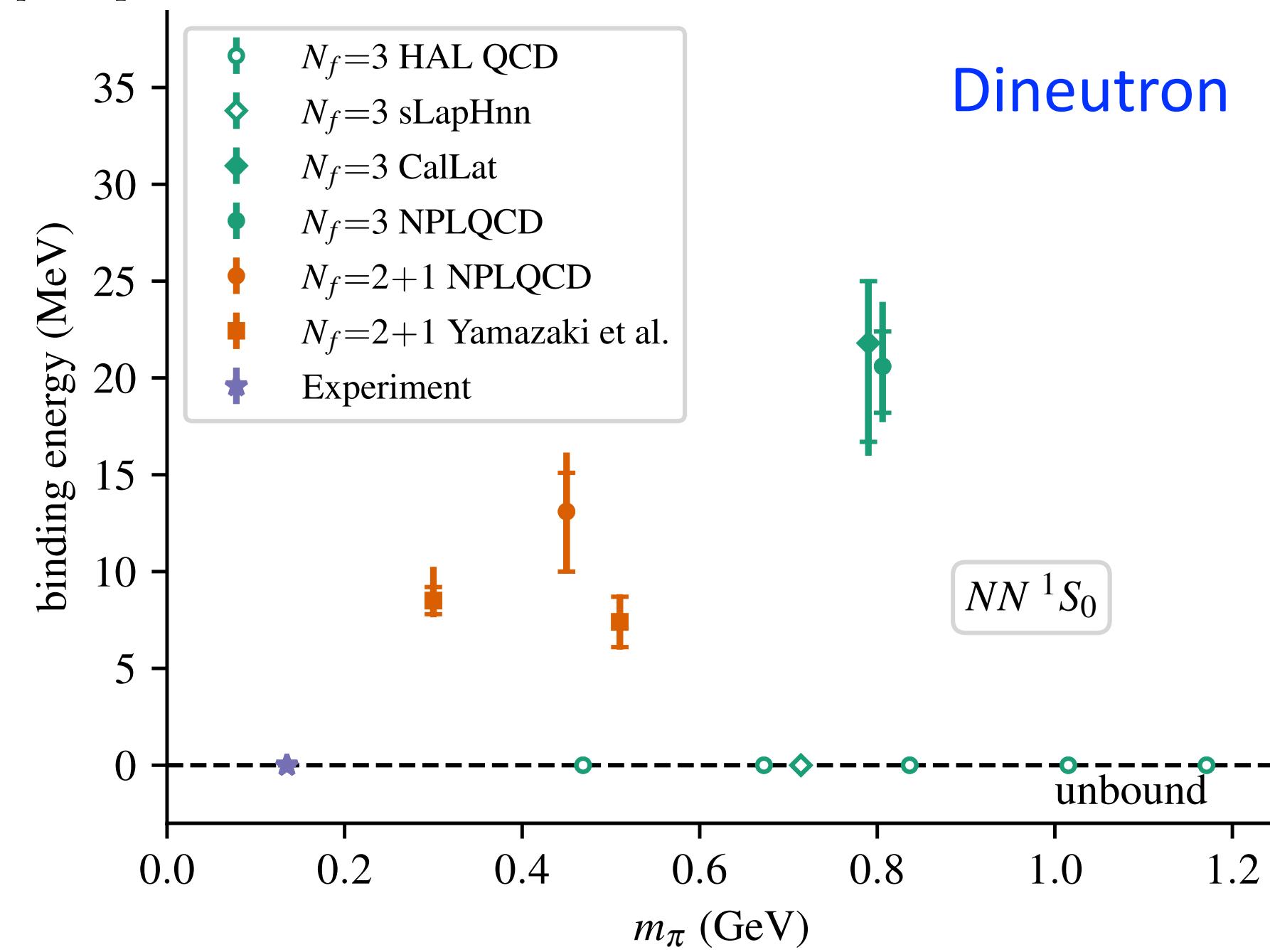
[M. Padmanath et al., arXiv:2111.11541]



# Nucleon-nucleon interactions

[Green, Hanlon, Junnarkar, HW (BaSc), arXiv:2212.09587]

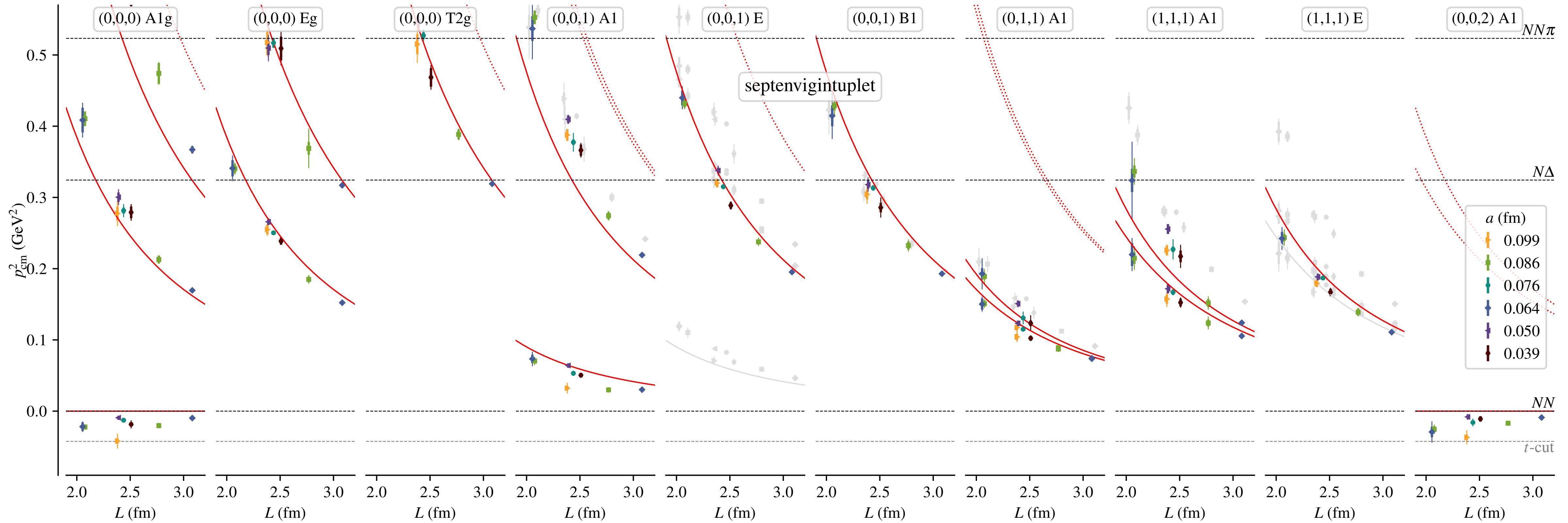
Inconclusive results on existence of bound states at unphysical pion masses:



Study the dineutron and deuteron channels at SU(3)-symmetric point

- Employ distillation and symmetric GEVP
- Study dependence on lattice spacing

# 27-plet ( $NN$ , $I = 1$ ): spin-0 spectrum

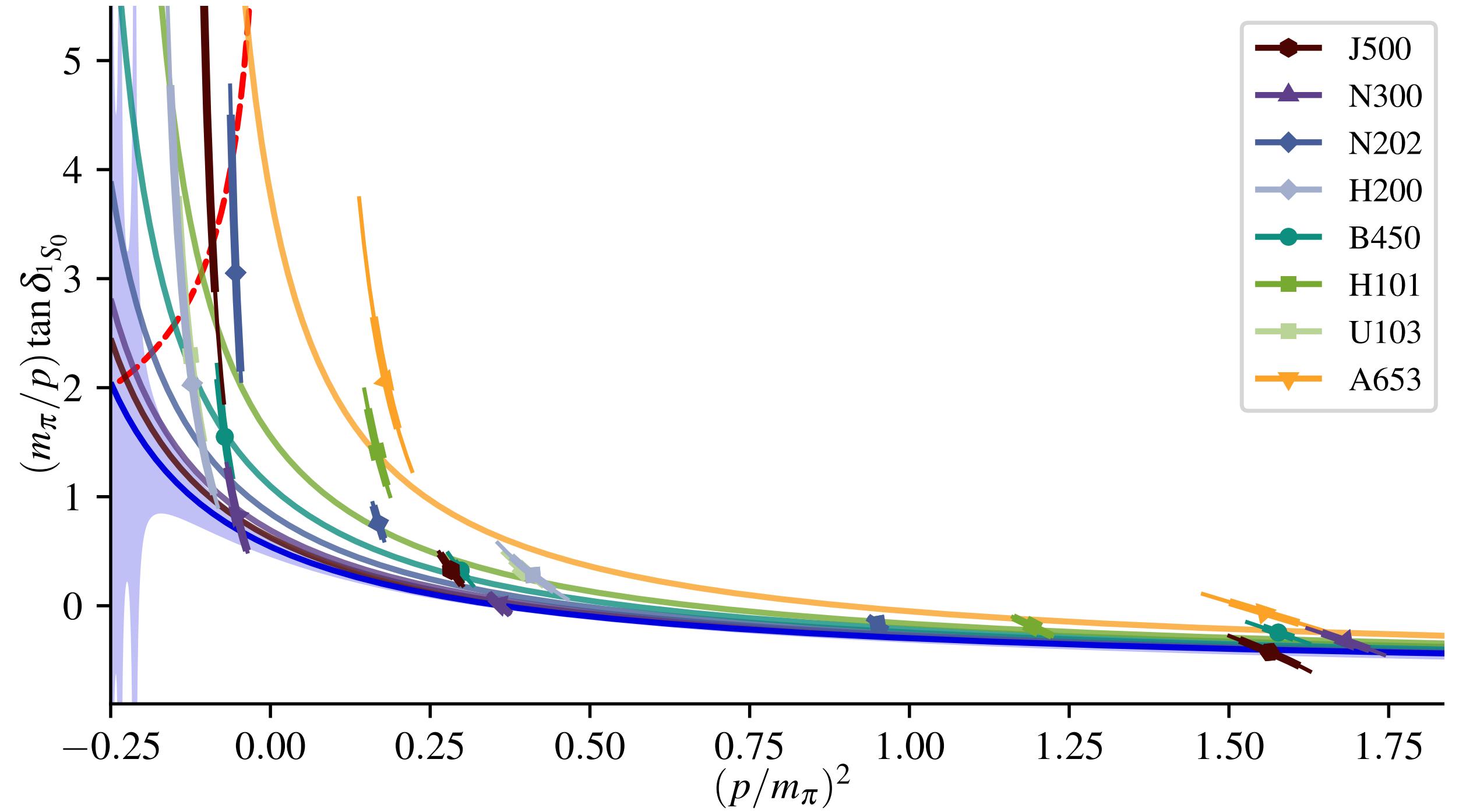


- Spin-1 states (grey) identified by overlaps
- Quantisation condition factorises in spin;  $^1S_0$  and  $^1D_2$  are relevant

: non-interacting levels

# 27-plet ( $NN$ , $I = 1$ ): spin-0

Phase shift analysis:  $^1S_0$

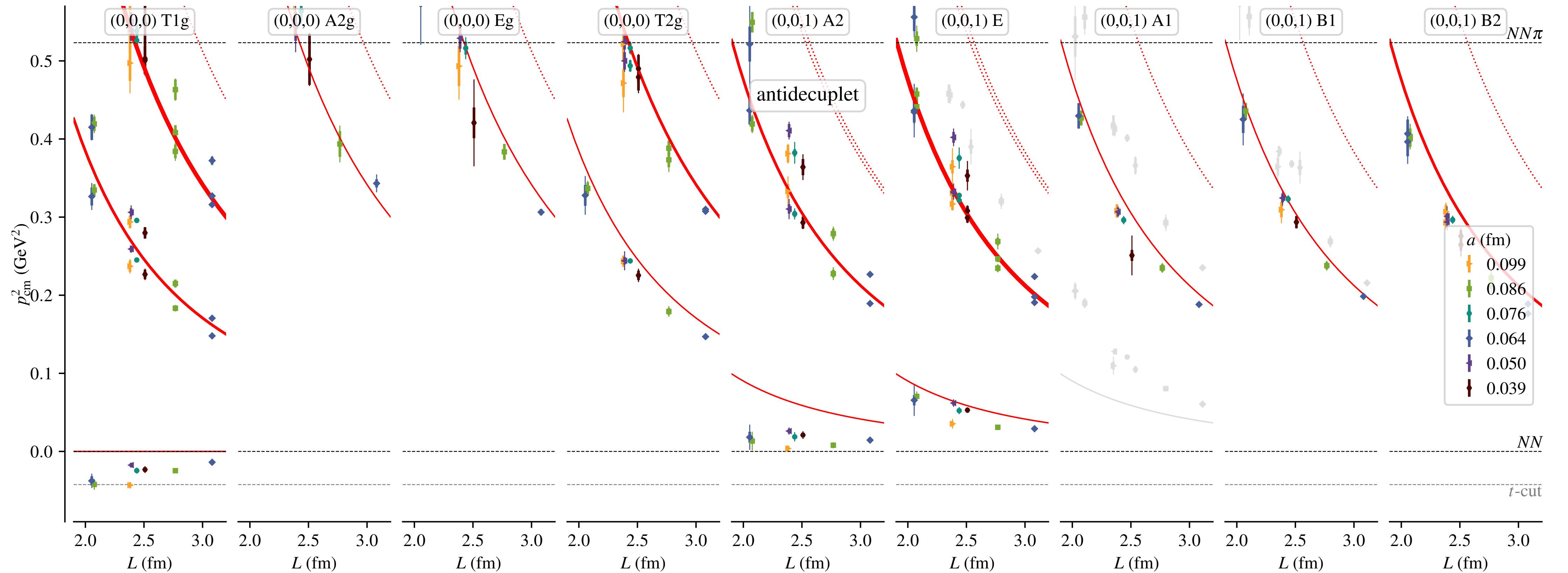


- Levels from rest frame and first moving frame
- Fit to rational function:

$$p \cot \delta(p) = \frac{c_0 + c_1 p^2}{1 + c_2 p^2}$$

- Observe virtual bound state
- Phase shift decreases towards continuum limit
  - Discretisation effects enhance baryon-baryon interactions

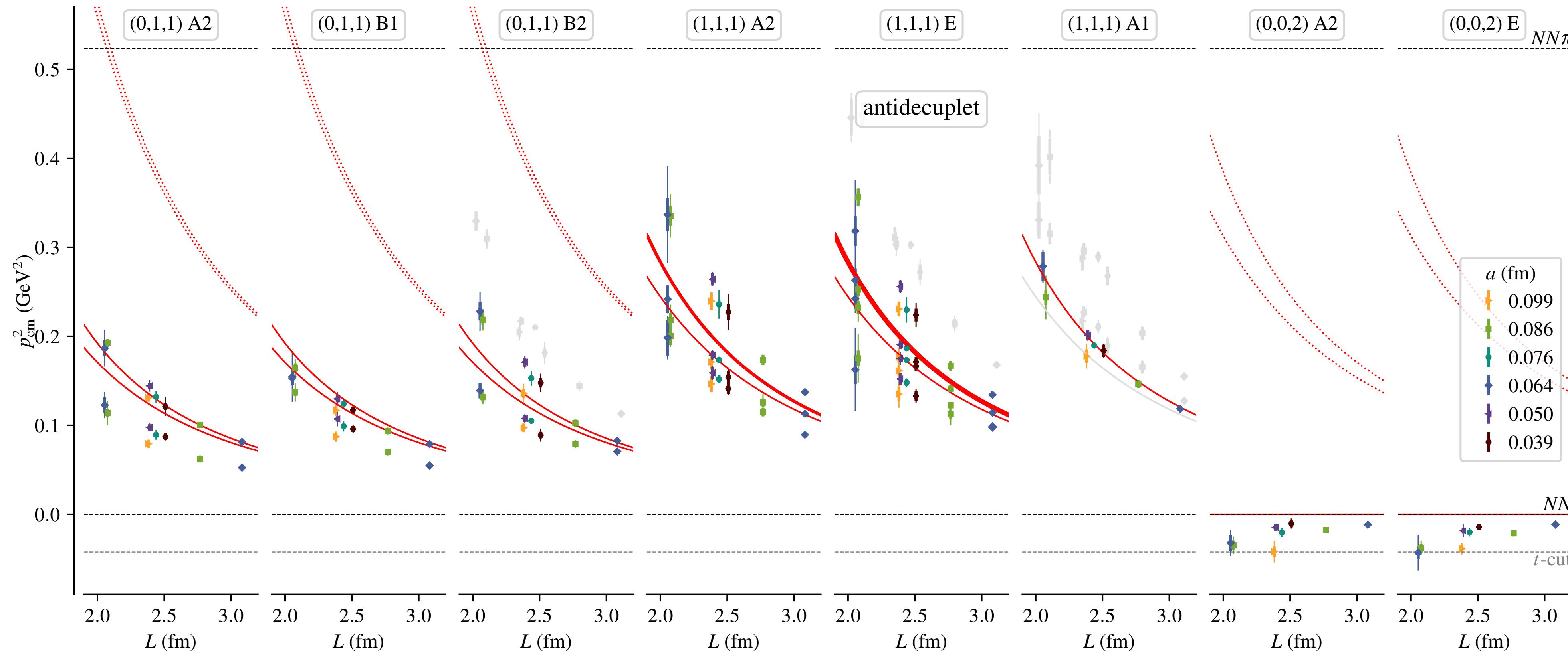
# Anti-decuplet ( $NN$ , $I = 0$ ): spin-1 spectrum



- Resolve  $\approx 300$  energy levels
- ${}^3S_1$ ,  ${}^3D_1$ ,  ${}^3D_2$  and  ${}^3D_3$  can be relevant

— : non-interacting levels  
 (thickness proportional to degeneracy)  
 (Spin-0 states (grey) identified by overlaps)

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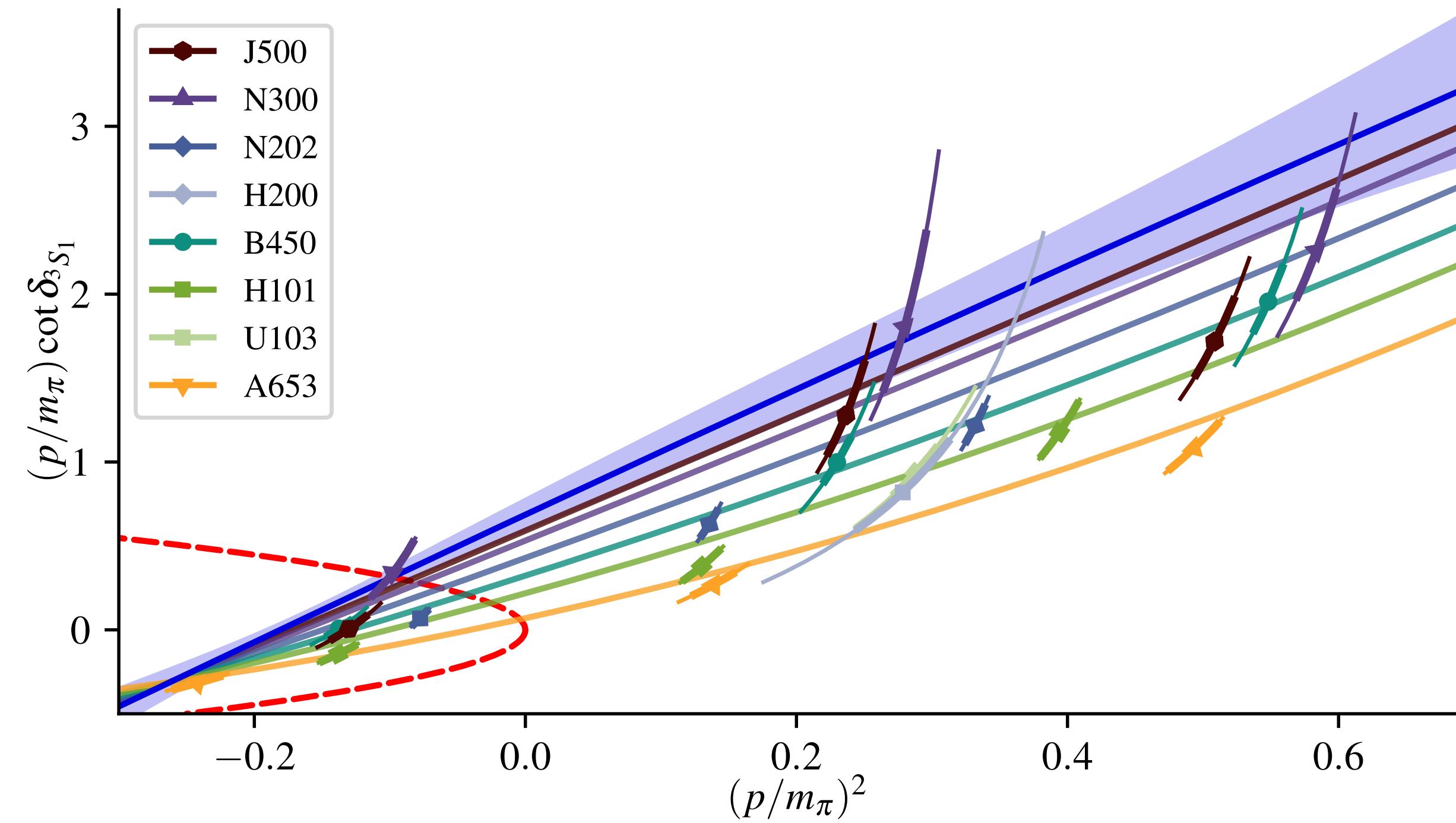


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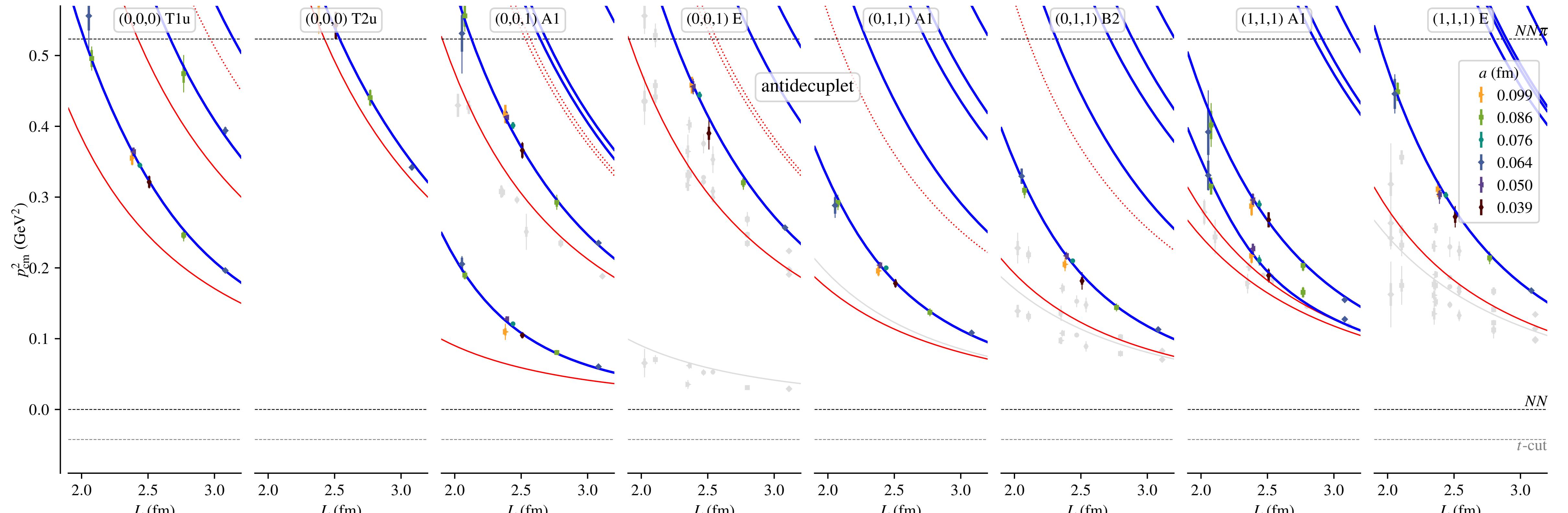
Phase shift analysis:  $^3S_1$



- Helicity-averaged levels from first two moving frames
- Neglect mixing with  $^3D_1$

- Observe virtual bound state  $\rightarrow$  Deuteron not bound at  $m_\pi = m_K \sim 420 \text{ MeV}$
- Phase shift decreases towards continuum limit  $\rightarrow NN$  interaction enhanced by lattice artefacts

# Anti-decuplet ( $NN$ , $I = 0$ ): spin-0 spectrum



— : 4-parameter fit to 70 energy levels

— : non-interacting levels

$$p^3 \cot \delta_{1P_1} = c_0 + c_1 p^2, \quad p^3 \cot \delta_{1F_3} = c_2 + c_3 p^8$$

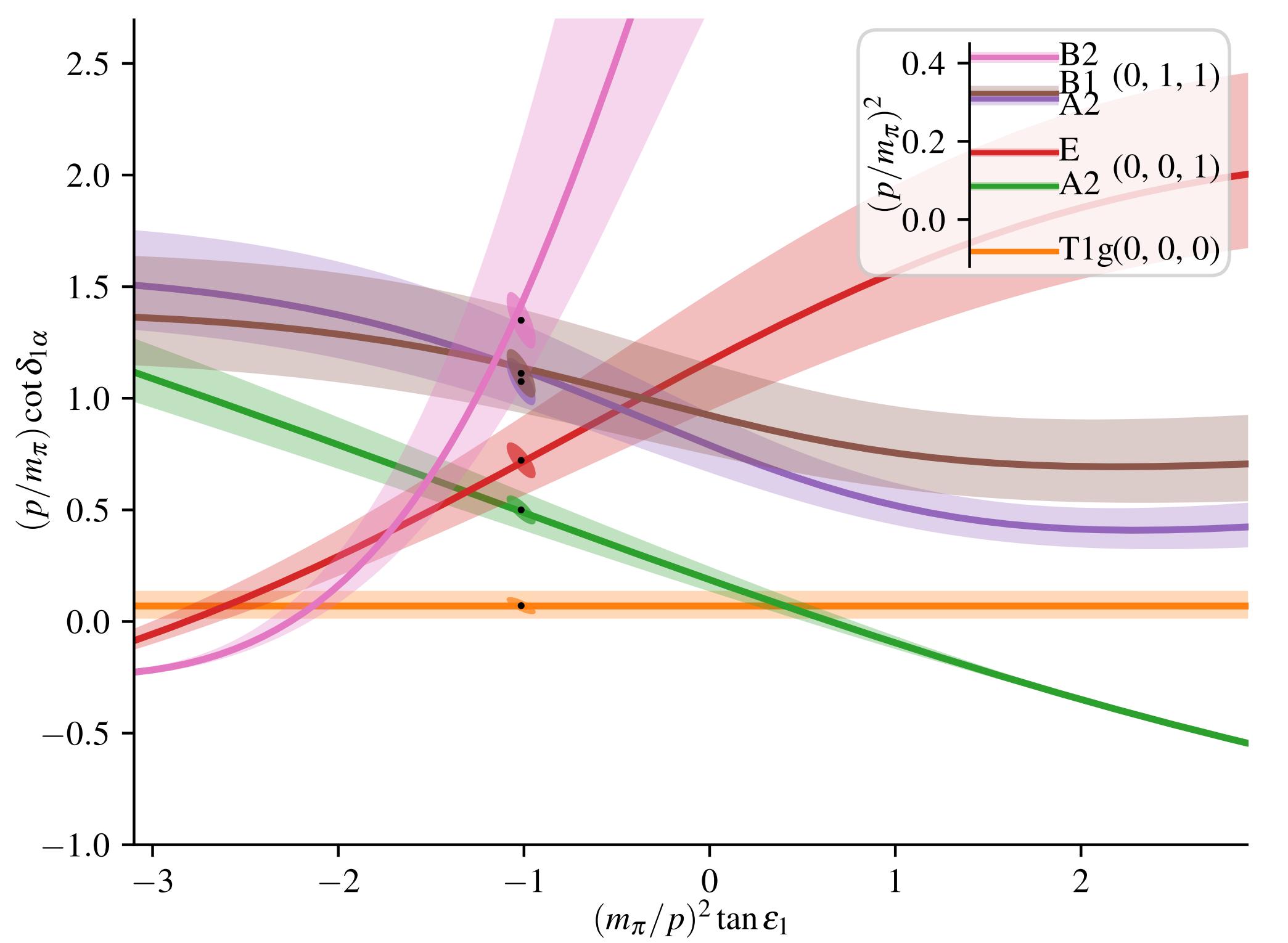
(good fit quality without terms describing lattice artefacts)

# Coupled partial waves

[Green, Hanlon, Junnarkar, HW (BaSc), arXiv:2212.09587]

$^3S_1 - ^3D_1$  : Ansatz for  $K$ -matrix: Blatt-Biedenharn parameterisation

$$\tilde{K}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & p^2 \end{pmatrix} \begin{pmatrix} \cos \epsilon_1 & -\sin \epsilon_1 \\ \sin \epsilon_1 & \cos \epsilon_1 \end{pmatrix} \begin{pmatrix} p \cot \delta_{1\alpha} & 0 \\ 0 & p \cot \delta_{1\beta} \end{pmatrix} \begin{pmatrix} \cos \epsilon_1 & \sin \epsilon_1 \\ -\sin \epsilon_1 & \cos \epsilon_1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & p^2 \end{pmatrix}$$



$$\epsilon_1 = 0 \Leftrightarrow \alpha \sim ^3S_1, \beta \sim ^3D_1$$

$\delta_{1\beta} = 0$  : energy levels impose constraints

$$p \cot \delta_{1\alpha} = \frac{B_{00} + (B_{01} + B_{10})x + B_{11}x^2}{1 + p^4 x^2}, \quad x = p^{-2} \tan \epsilon_1$$

Fit spectrum on N202 to

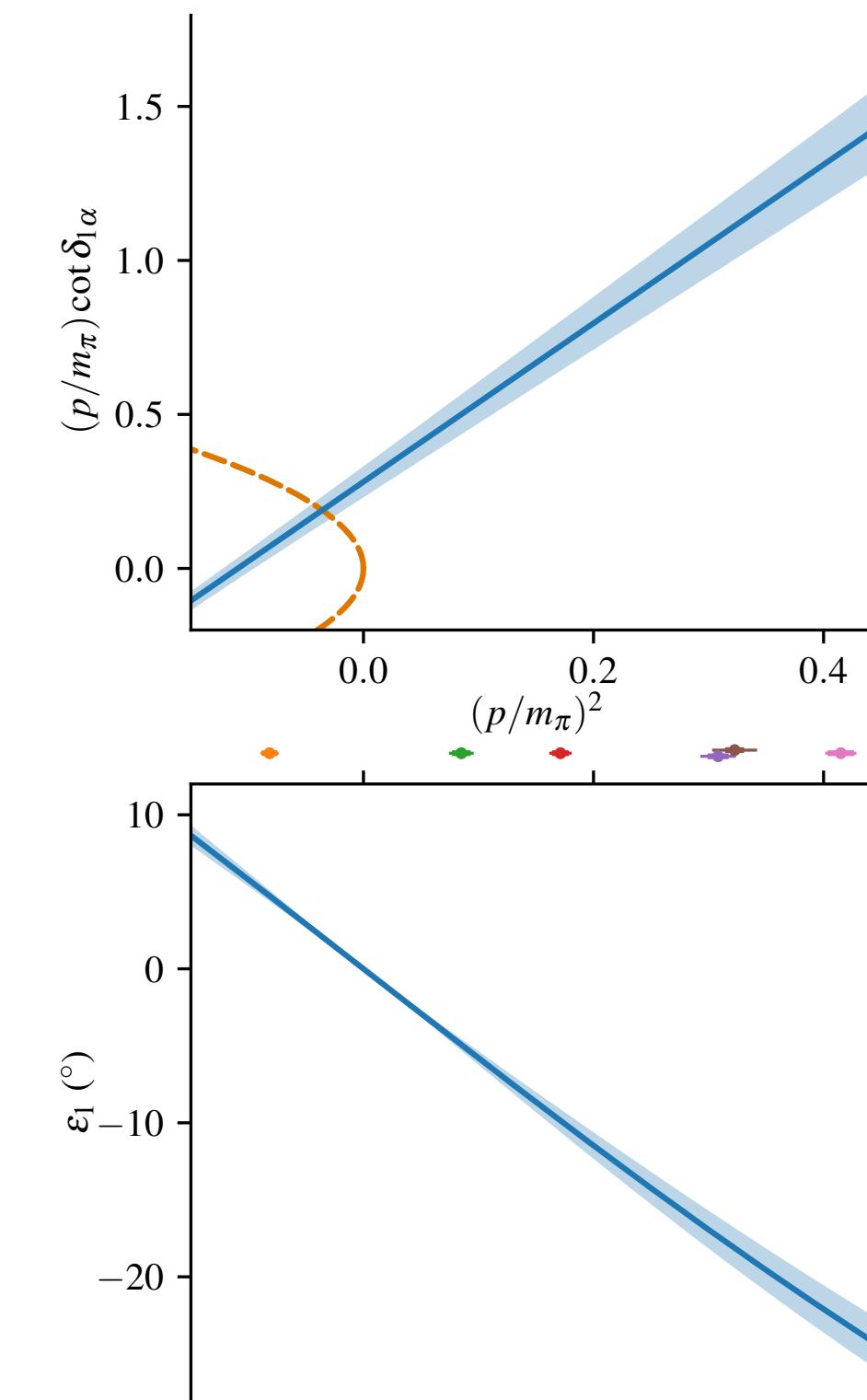
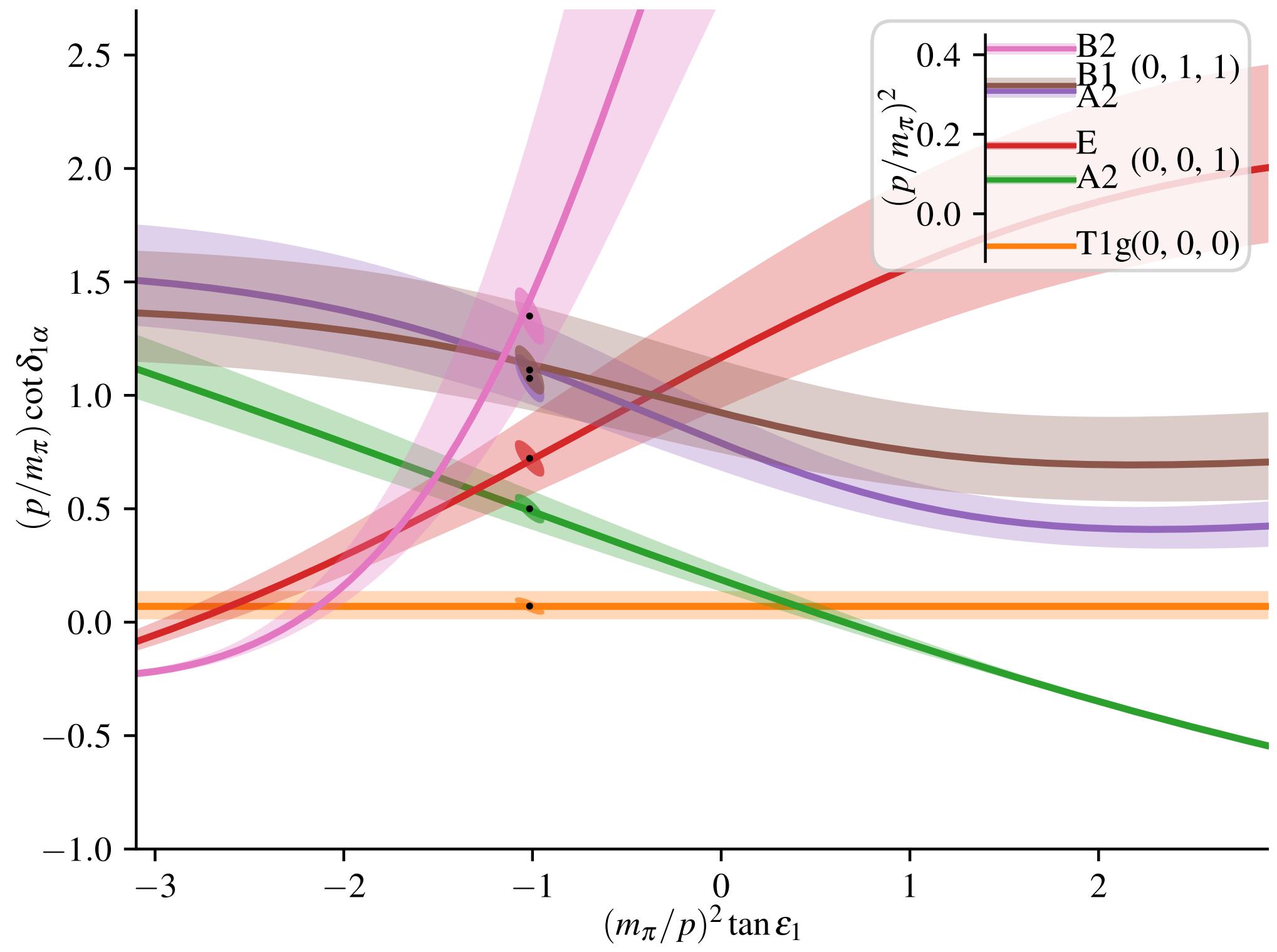
$$p \cot \delta_{1\alpha} = c_1 + c_2 p^2, \quad p^{-2} \tan \epsilon_1 = c_3$$

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Fitted  $p \cot \delta_{1\alpha}$  and  $\epsilon_1$  versus  $p^2$ :  
Sign of  $\epsilon_1$  opposite to experiment

## Conclusions — Outlook

- \* Distillation, GEVP and Finite-volume Quantisation:  
Detailed and precise studies of two-particle interactions
- \* Discretisation effects are sizeable:
  - Binding energy of  $H$  dibaryon much smaller in continuum limit:  $O(5 \text{ MeV})$
  - Lattice artefacts enhance strength of hadron-hadron interactions
  - Confirmed using different lattice actions
- \* No bound states in dineutron and deuteron channels observed at  $m_\pi = m_K \sim 420 \text{ MeV}$
- \* SU(3)-flavour breaking makes amplitude analysis much more complicated
- \* Mixing with higher partial waves under study

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Thank you!

# Backup

# The $H$ Dibaryon in 3-flavour QCD

## Source positions on ensembles with SU(3) symmetry

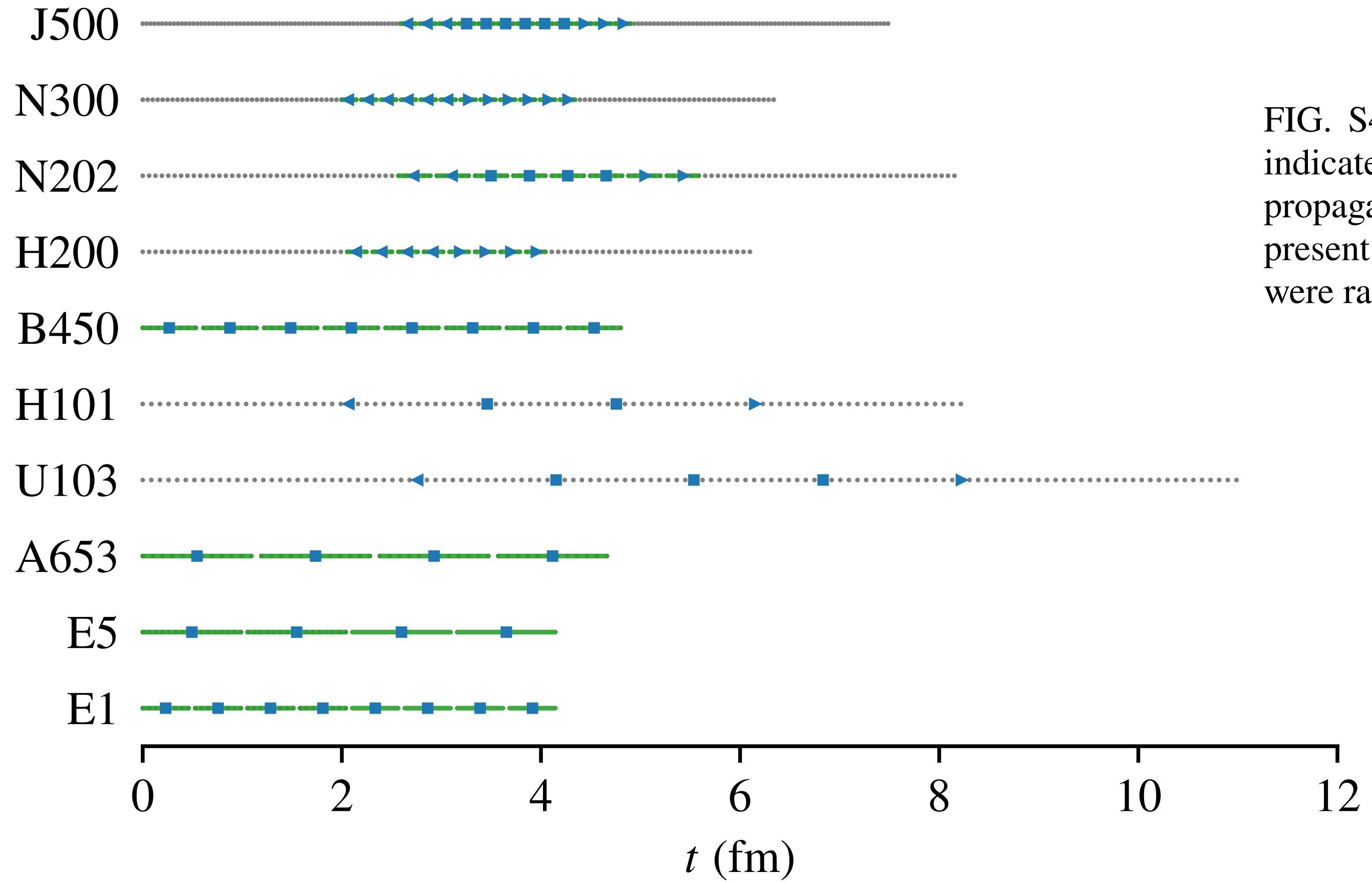
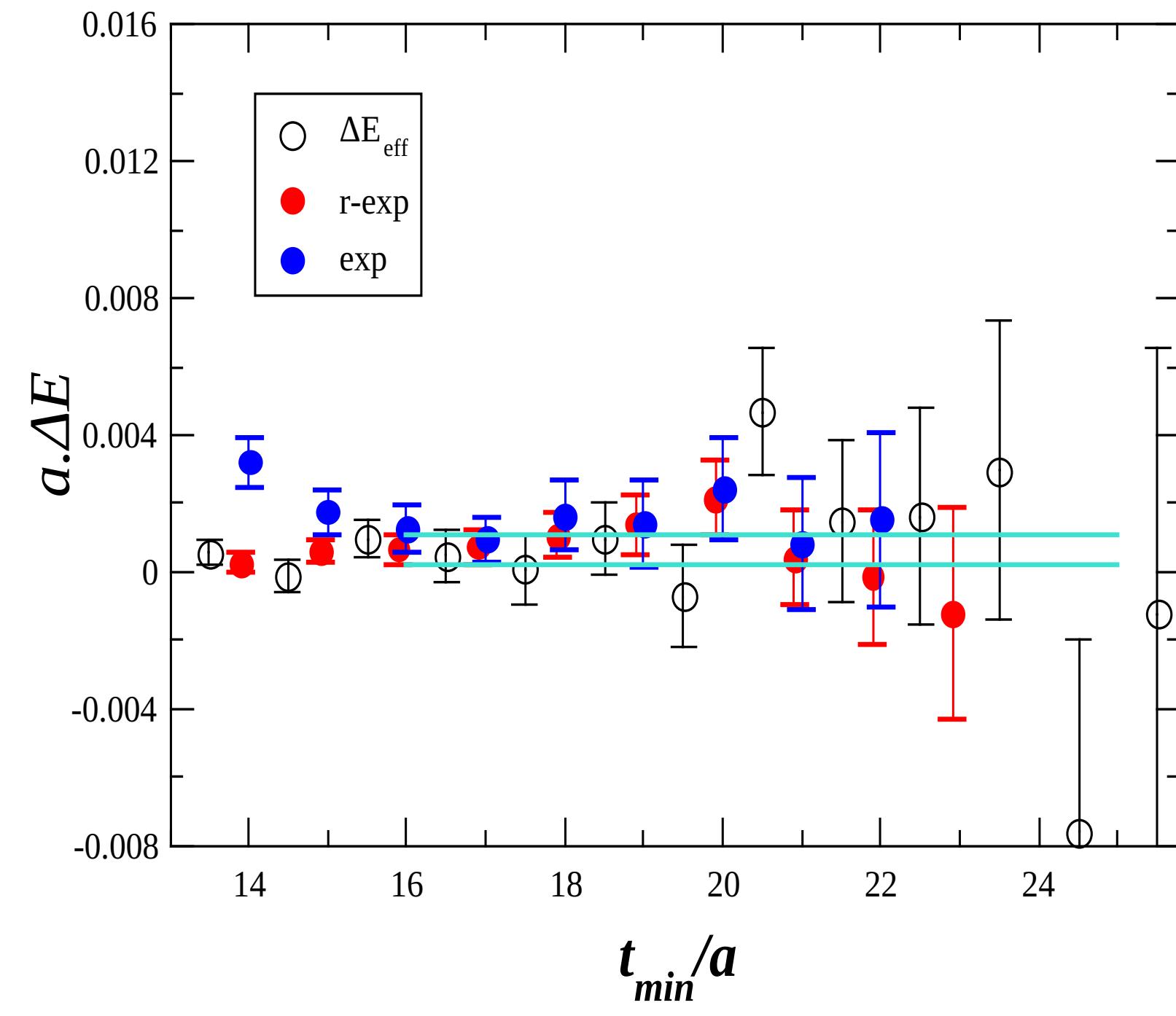


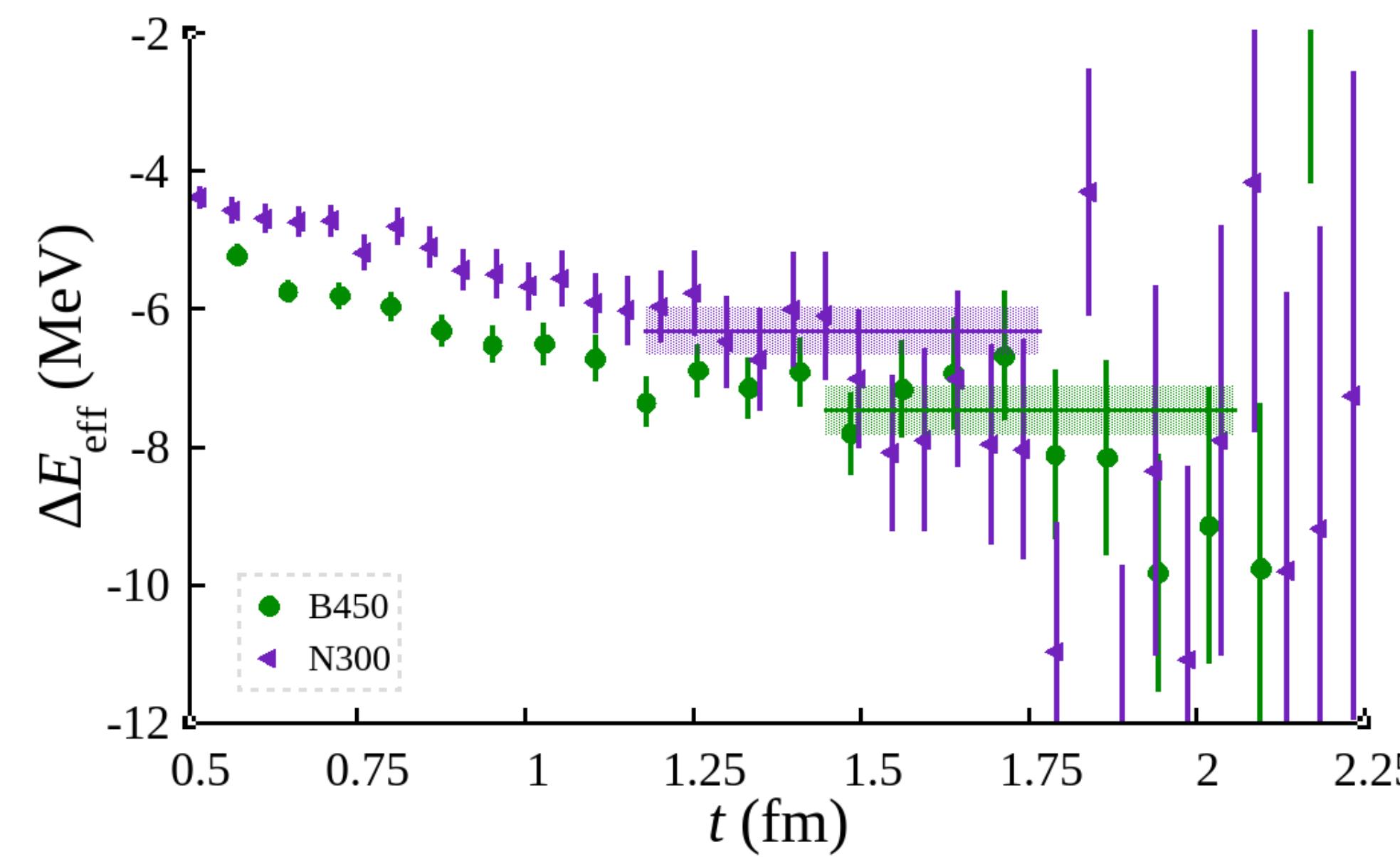
FIG. S4. Location of source times on all ensembles. Triangles indicate sources used for only forward-propagating or backward-propagating states, and squares indicate sources used for both. When present, green line segments indicate the range over which sources were randomly shifted on each gauge configuration.

# Charmed tetraquarks

## Consistency of different fit procedure to extract ground state



**Figure 2:** Comparative study of single exponential fits to  $\lambda^{(n)}(t)$  [exp] and  $r^{(n)}(t)$  [r-exp] for the first excited state in the  $A_1$  irrep of  $P^2 = 2$  moving frame in the N200 ensemble.  $\Delta E_{\text{eff}}$  is the effective energy difference and  $t_{\text{min}}$  refers to the boundary of the chosen fit range close to the source time slice. The cyan horizontal line indicates the chosen fit.



# The HAL QCD Method

Baryon-baryon potential from **Nambu-Bethe-Salpeter** amplitude computed on the lattice

$$G_4(\mathbf{r}, t - t_0) = \langle 0 | (BB)^{(\alpha)}(\mathbf{r}, t) (\overline{B}\overline{B})^{(\alpha)}(\mathbf{r}, t_0) | 0 \rangle = \phi(\mathbf{r}, t) e^{-2M(t-t_0)}$$

$(BB)^{(\alpha)}(\mathbf{r}, t)$  : 2-baryon interpolating operator; flavour irrep.  $\alpha$

$\phi(\mathbf{r}, t)$  : NBS wave function

$M$  : single baryon mass

- Determine potential via  $V(r) = \frac{[-H_0 - (\partial/\partial t)] \phi(\mathbf{r}, t)}{\phi(\mathbf{r}, t)}$
- Solve Schrödinger equation → determine binding energies and scattering phase shifts

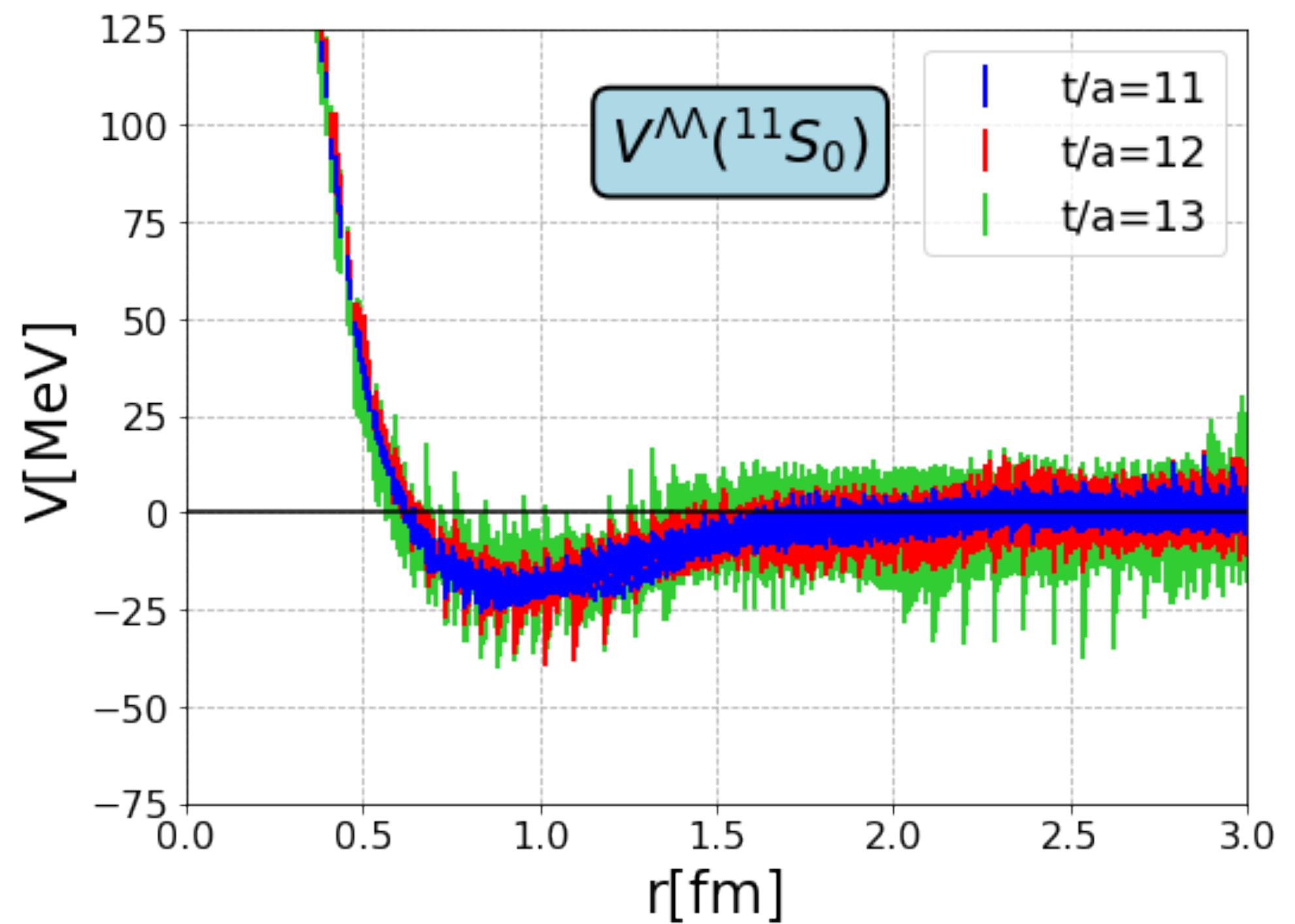
# The HAL QCD Method

Details of the calculation:

$N_f = 2 + 1$ ,  $O(a)$  improved Wilson fermions

Single lattice spacing:  $a = 0.0846$  fm; Volume:  $L \approx 8.1$  fm

Near physical point:  $m_\pi = 146$  MeV,  $m_K = 525$  MeV

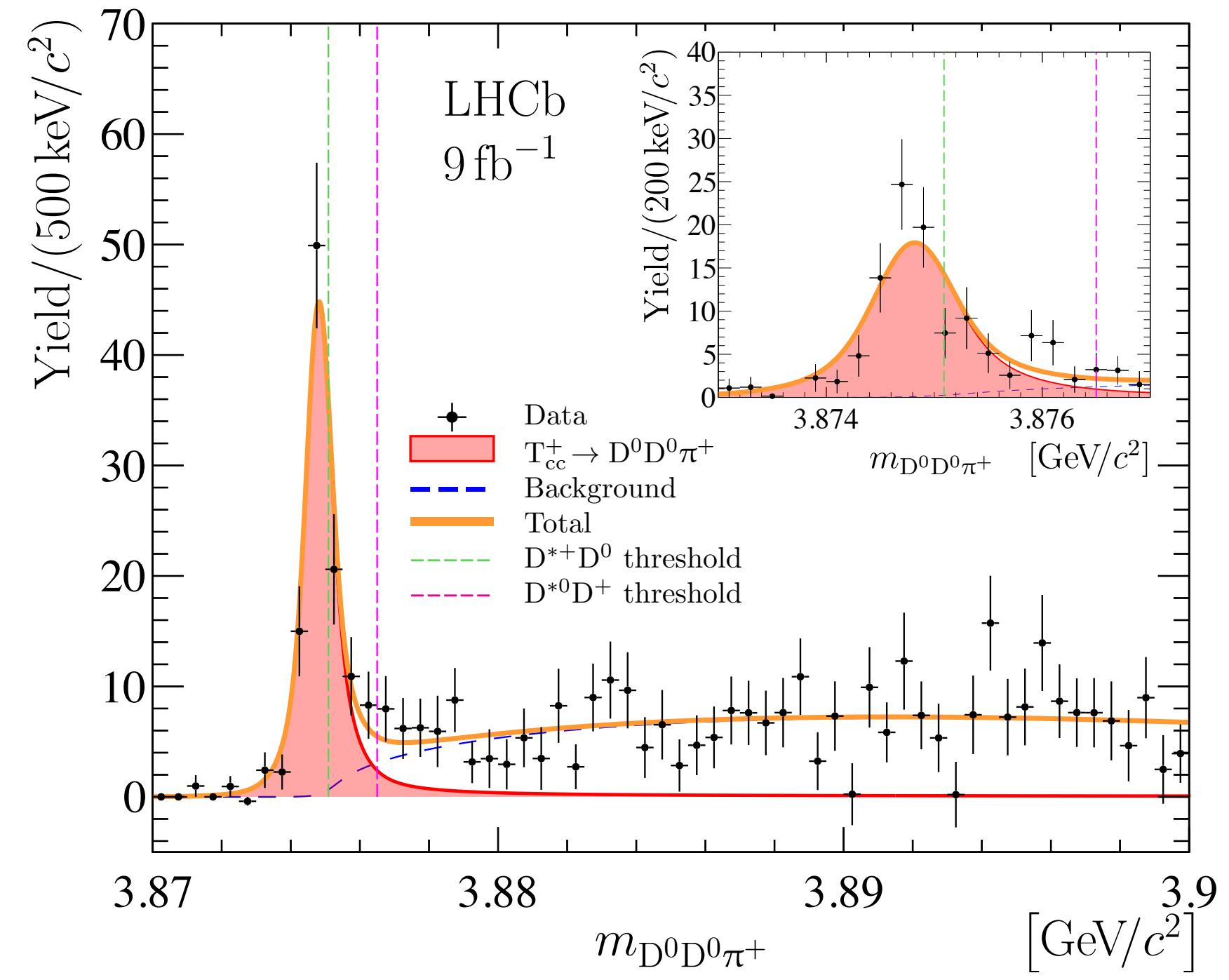


- $\Lambda\Lambda$  interaction weakly attractive
- No bound or resonant dihyperon near  $\Lambda\Lambda$  threshold observed at the physical point

[Sasaki et al., Nucl Phys A998 (2020) 121737, arXiv:1912.08630]

# Charmed tetraquarks

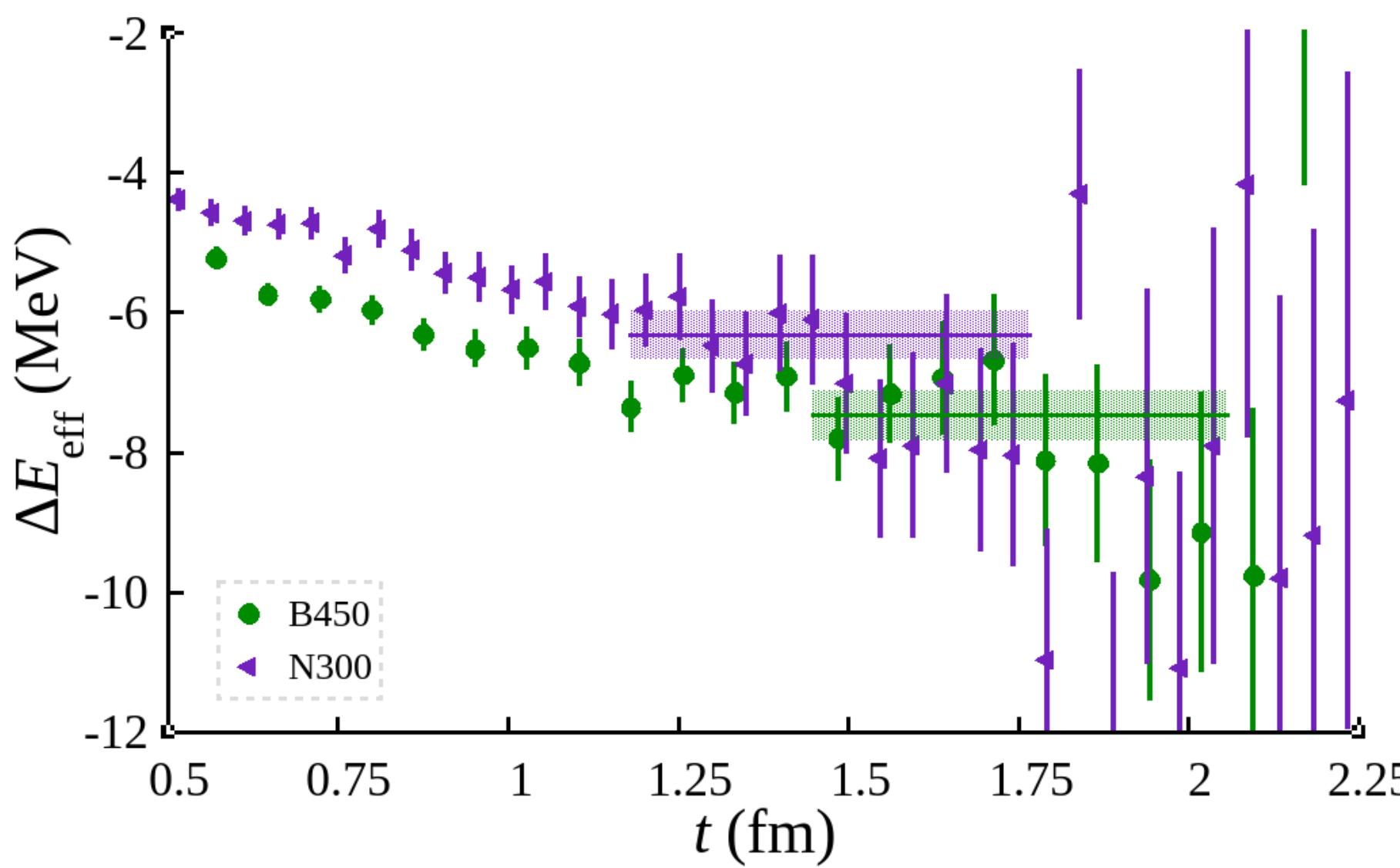
LHCb: observation of doubly charmed tetraquark  $T_{cc}^+$  with  $I = 0, J^P = 1^+$  close to  $D^{*+}D^0$  threshold



$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0} \text{ keV}$$

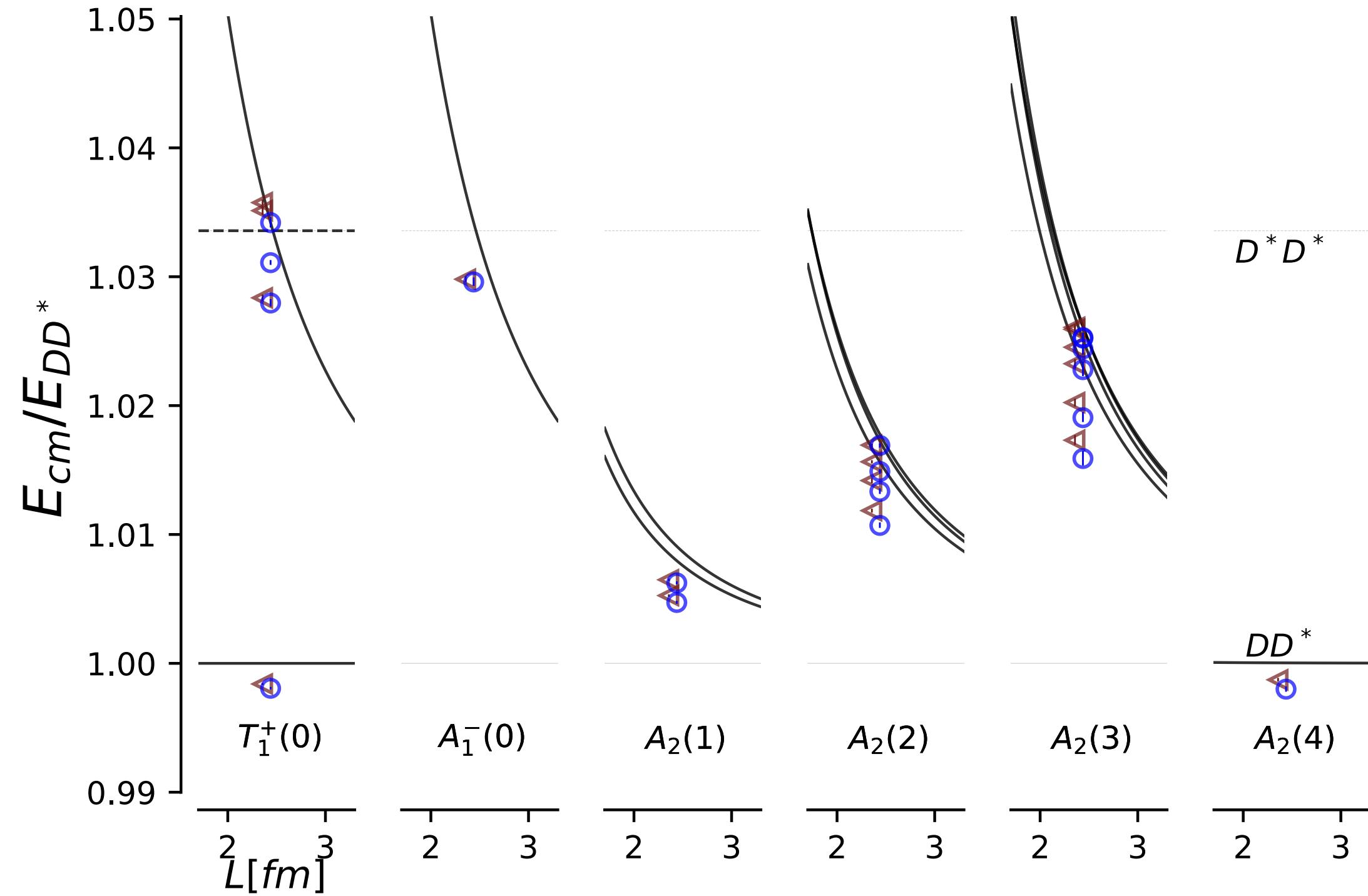
$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14} \text{ keV}$$

- Lattice QCD: discretisation effects may be large for heavy quark systems
- Perform scaling test for different lattice spacings



# Charmed tetraquarks: Finite-volume energy levels

Perform scaling test in SU(3)-symmetric limit



$$m_\pi \sim 420 \text{ MeV}, L = 2.4 \text{ fm}$$

$$a = 0.0498, 0.0762 \text{ fm}$$

- Preliminary amplitude analysis suggests virtual bound state
- Complementary to calculation at 280 MeV pion mass

[Padmanath & Prelovsek, PRL 129 (2022) 032002]