Skyrme-Hartree-Fock Approach for Hypernuclei

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- Λ hypernuclei (Data)
- Extended Skyrme-Hartree-Fock (SHF) approach
- Fit ΛN force parameters, Results
- Deformation and s.o. splitting
- Ξ hypernuclei
- Kaonic nuclei

PRC 62, 064308 (2000)
PRC 76, 034312 (2007)
PTP 123, 569 (2010)
PRC 90, 047301 (2014)
PRC 95, 024323 (2022)
PRC 107, 044317 (2023)

Hypernuclei: Single, Double, Multi-Lambda:

- Λ hyperons (uds, M_Λ=1116 MeV) bound by a nucleus
  Weak decay: $\Lambda \to N + \pi$ etc. ($c\tau \approx 8$ cm)
- Created by (π^+, K^+), (K^-, π^-), $(e, e'K^+)$ reactions
  (BNL, CERN, JLAB, KEK, LNF, GSI, J-PARC, ...)
- Experimentally known (heavy) Λ hypernuclei:
  - Single-lambda: ..., ${}^{13}_{\Lambda}$C, ${}^{16}_{\Lambda}$O, ${}^{28}_{\Lambda}$Si, ${}^{40}_{\Lambda}$Ca, ${}^{89}_{\Lambda}$Y, ${}^{139}_{\Lambda}$La, ${}^{208}_{\Lambda}$Pb,...
  - Double-lambda: ${}^{6}_{\Lambda\Lambda}$He, ${}^{10,11,12}_{\Lambda\Lambda}$Be, ${}^{13}_{\Lambda\Lambda}$B (8 events !)
  - Multi-lambda: None !
- Observables:
  - Λ binding (removal) energy: $B_\Lambda = E({}^{A-1}Z) - E({}^{A}_{\Lambda}Z)$
  - Single-particle levels: e^i_q $(q = n, p, \Lambda;\ i = s, p, d, \ldots)$

Data (light Λ hypernuclei):

Synopsis of the experimental values of B_Λ for A $\leq$ 16 hypernuclei. Column 1: hypernucleus; column 2: emulsions; column 3: KEK-SKS; column 4: revised KEK-SKS; column 5: DAΦNE-FINUDA; column 6: electroproduction. References are in parentheses; [t.w.] stands for this work. In columns 2–6 the first error is statistical, the second one is systematic; in columns 5 and 6 the error quoted for results from Ref. [21] and, respectively, Ref. [15] is total.

	Emulsions (MeV)	(π^+, K^+) (MeV) KEK-SKS [5]	(π^+, K^+) (MeV) KEK-SKS revised [t.w.]	(K^-_{stop}, π^-) (MeV) DAΦNE-FINUDA	$(e, e'K^+)$ (MeV) JLab, MaMi
${}^{3}_{\Lambda}$H	$0.13 \pm 0.05 \pm 0.04$ [1,2]				
${}^{4}_{\Lambda}$H	$2.04 \pm 0.04 \pm 0.04$ [1,2]				$2.157 \pm 0.005 \pm 0.077$ [16]
${}^{4}_{\Lambda}$He	$2.39 \pm 0.03 \pm 0.04$ [1,2]				
${}^{5}_{\Lambda}$He	$3.12 \pm 0.02 \pm 0.04$ [1,2]				
${}^{6}_{\Lambda}$H				4.0 ± 1.1 [20,28]	
${}^{6}_{\Lambda}$He	4.25 ± 0.10 [1], $4.18 \pm 0.10 \pm 0.04$ [2]				
${}^{7}_{\Lambda}$He					$5.55 \pm 0.10 \pm 0.11$ [11]
${}^{7}_{\Lambda}$Li	$5.58 \pm 0.03 \pm 0.04$ [1,2]	$5.22 \pm 0.08 \pm 0.36$	$5.82 \pm 0.08 \pm 0.08$	$5.85 \pm 0.13 \pm 0.10$ [19], [t.w.], 5.8 ± 0.4 [21]	
${}^{7}_{\Lambda}$Li* [4]	$5.26 \pm 0.03 \pm 0.04$	$4.90 \pm 0.08 \pm 0.36$	$5.50 \pm 0.08 \pm 0.08$	$5.53 \pm 0.13 \pm 0.10$, 5.48 ± 0.40	
${}^{7}_{\Lambda}$Be	$5.16 \pm 0.08 \pm 0.04$ [1,2]				
${}^{8}_{\Lambda}$He	$7.16 \pm 0.70 \pm 0.04$ [1,2]				
${}^{8}_{\Lambda}$Li	$6.80 \pm 0.03 \pm 0.04$ [1,2]				
${}^{8}_{\Lambda}$Be	$6.84 \pm 0.05 \pm 0.04$ [1,2]				
${}^{9}_{\Lambda}$Li	8.53 ± 0.15 [1], $8.51 \pm 0.12 \pm 0.04$ [2]				$8.36 \pm 0.08 \pm 0.08$ [12]
${}^{9}_{\Lambda}$Be	$6.71 \pm 0.04 \pm 0.04$ [1,2],	$5.99 \pm 0.07 \pm 0.36$	$6.59 \pm 0.07 \pm 0.08$	$6.30 \pm 0.10 \pm 0.10$ [19], [t.w.]	
				6.2 ± 0.4 [21]	
${}^{9}_{\Lambda}$B	7.88 ± 0.15 [1], $8.29 \pm 0.18 \pm 0.04$ [2]				
${}^{10}_{\Lambda}$Be	9.30 ± 0.26 [1], $9.11 \pm 0.22 \pm 0.04$ [2]				$8.60 \pm 0.07 \pm 0.16$ [13]
${}^{10}_{\Lambda}$B	$8.89 \pm 0.12 \pm 0.04$ [1,2]	$8.1 \pm 0.1 \pm 0.5$	$8.7 \pm 0.1 \pm 0.08$		
${}^{11}_{\Lambda}$B	$10.24 \pm 0.05 \pm 0.04$ [1,2]			$10.28 \pm 0.2 \pm 0.4$ [t.w.]	
${}^{12}_{\Lambda}$B	$11.37 \pm 0.06 \pm 0.04$ [1,2]				$11.524 \pm 0.019 \pm 0.013$ [14]
${}^{12}_{\Lambda}$C	$10.76 \pm 0.19 \pm 0.04$ [2]	10.80 fixed		$11.57 \pm 0.04 \pm 0.10$ [19], [t.w.]	
				$10.94 \pm 0.06 \pm 0.50$ [18]	
${}^{13}_{\Lambda}$C	11.22 ± 0.08 [1]	$11.38 \pm 0.05 \pm 0.36$	$11.98 \pm 0.05 \pm 0.08$	11.0 ± 0.4 [21]	
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${}^{14}_{\Lambda}$C	$12.17 \pm 0.33 \pm 0.04$ [2]				
${}^{15}_{\Lambda}$N	$13.59 \pm 0.15 \pm 0.04$ [1,2]			$13.8 \pm 0.7 \pm 1.0$ [t.w.]	
${}^{16}_{\Lambda}$N					13.76 ± 0.16 [15]
${}^{16}_{\Lambda}$O		$12.42 \pm 0.05 \pm 0.36$	$13.02 \pm 0.05 \pm 0.08$	13.4 ± 0.4 [21]	

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+0.6 MeV

No new data since!

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Emulsion Data (BNL stopped K^-, ~ 27000 events):

Experimental Λ separation energies, B_Λ from emulsion studies

Hypernucleus	# events	$B_\Lambda \pm \Delta B_\Lambda$ MeV	Hypernucleus	# events	$B_\Lambda \pm \Delta B_\Lambda$ MeV
${}^{3}_{\Lambda}$H$(1/2^+)$	204	0.13 ± 0.05	${}^{9}_{\Lambda}$Li	8	8.50 ± 0.12
${}^{4}_{\Lambda}$H(0^+)	155	2.04 ± 0.04	${}^{9}_{\Lambda}$Be	222	6.71 ± 0.04
${}^{4}_{\Lambda}$He	279	2.39 ± 0.03	${}^{9}_{\Lambda}$B	4	8.29 ± 0.18
${}^{5}_{\Lambda}$He	1784	3.12 ± 0.02	${}^{10}_{\Lambda}$Be	3	9.11 ± 0.22
${}^{6}_{\Lambda}$He	31	4.18 ± 0.10	${}^{10}_{\Lambda}$B	10	8.89 ± 0.12
${}^{7}_{\Lambda}$He	16	not averaged	${}^{11}_{\Lambda}$B$(5/2^+)$	73	10.24 ± 0.05
${}^{7}_{\Lambda}$Li	226	5.58 ± 0.03	${}^{12}_{\Lambda}$B(1^-)	87	11.37 ± 0.06
${}^{7}_{\Lambda}$Be	35	5.16 ± 0.08	${}^{12}_{\Lambda}$C	6	10.80 ± 0.18
${}^{8}_{\Lambda}$He	6	7.16 ± 0.70	${}^{13}_{\Lambda}$C	6	11.69 ± 0.12
${}^{8}_{\Lambda}$Li(1^-)	787	6.80 ± 0.03	${}^{14}_{\Lambda}$C	3	12.17 ± 0.33
${}^{8}_{\Lambda}$Be	68	6.84 ± 0.05	${}^{15}_{\Lambda}$N	14	13.59 ± 0.15

Jurič et al., NPB 52 1 (1973)

Courtesy of J. Millener

Caution: In some cases only few events

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+0.6 MeV ?

Jurič et al., NPB 52 1 (1973)

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Caution: In some cases only few events

Data (heavy Λ hypernuclei):

TABLE IV. B_Λ values from a variety of sources for Λ single-particle states.

Hypernucleus	s_Λ	p_Λ	d_Λ	f_Λ	g_Λ
			(π^+, K^+)		
${}^{208}_{\Lambda}$Pb	26.9(8)	22.5(6)	17.4(7)	12.3(6)	7.2(6)
${}^{139}_{\Lambda}$La	25.1(12)	21.0(6)	14.9(6)	8.6(6)	2.1(6)
${}^{89}_{\Lambda}$Y	23.6(5)	17.7(6)	10.9(6)	3.7(6)	−3.8(10)
${}^{51}_{\Lambda}$V	21.5(6)	13.4(6)	5.1(6)		
${}^{28}_{\Lambda}$Si	17.2(2)	7.6(2)	−1.0(5)		
${}^{16}_{\Lambda}$O	13.0(2)	2.5(2)			
${}^{13}_{\Lambda}$C	12.0(2)	1.1(2)			
${}^{12}_{\Lambda}$C	11.36(20)	0.36(20)			
${}^{10}_{\Lambda}$B	8.7(3)				
			$(e, e'K^+)$		
${}^{52}_{\Lambda}$V	21.8(3)				
${}^{16}_{\Lambda}$N	13.76(16)	2.84(18)			
${}^{12}_{\Lambda}$B	11.52(2)	0.54(4)			
${}^{10}_{\Lambda}$Be	8.55(13)				
${}^{7}_{\Lambda}$He	5.55(15)				
			Emulsion		
${}^{13}_{\Lambda}$C	11.69(12)	0.8(3)			
${}^{12}_{\Lambda}$B	11.37(6)				
${}^{12}_{\Lambda}$C		0.14(5)			
${}^{8}_{\Lambda}$Li	6.80(3)				
${}^{7}_{\Lambda}$Be	5.16(8)				
			(K^-, π^-)		
${}^{40}_{\Lambda}$Ca		11.0(5)	1.0(5)		
${}^{32}_{\Lambda}$S	17.5(5)	8.2(5)	−1.0(5)		

Gal, Hungerford, Millener
Rev. Mod. Phys., Vol. 88, No. 3, July–September 2016

+0.6 MeV

Lambda Hypernuclear Chart:

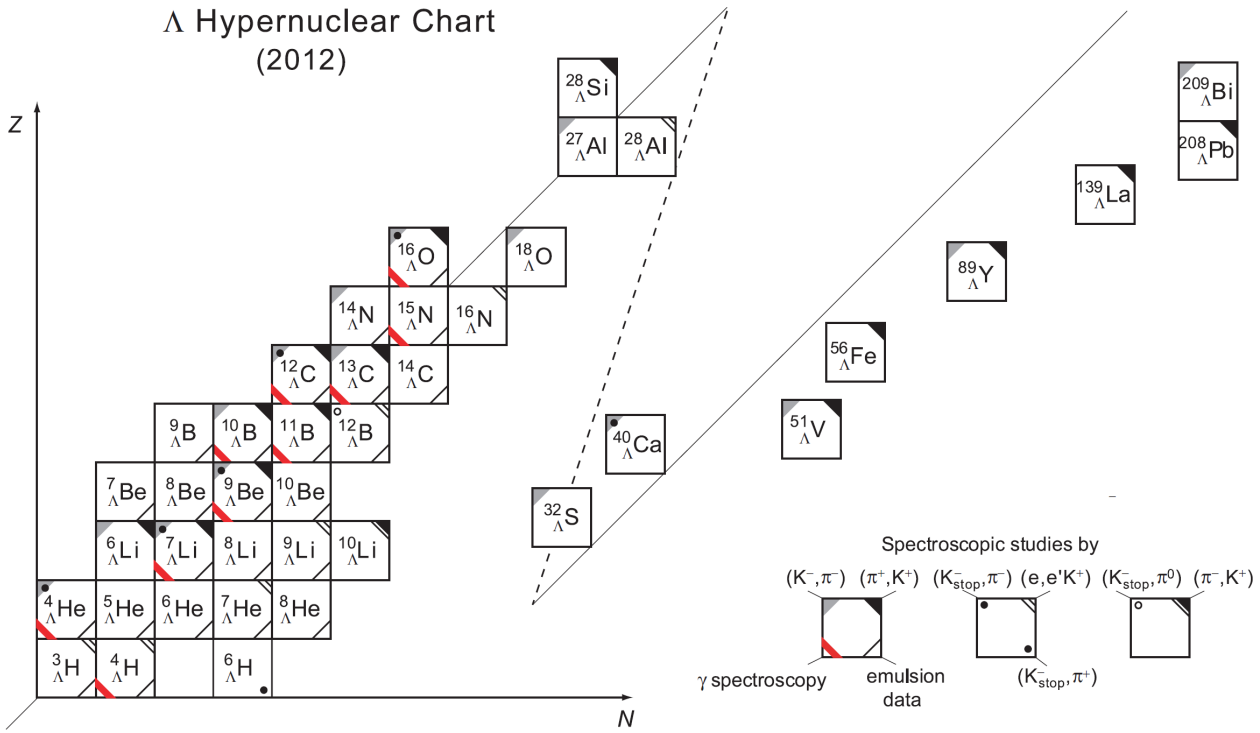
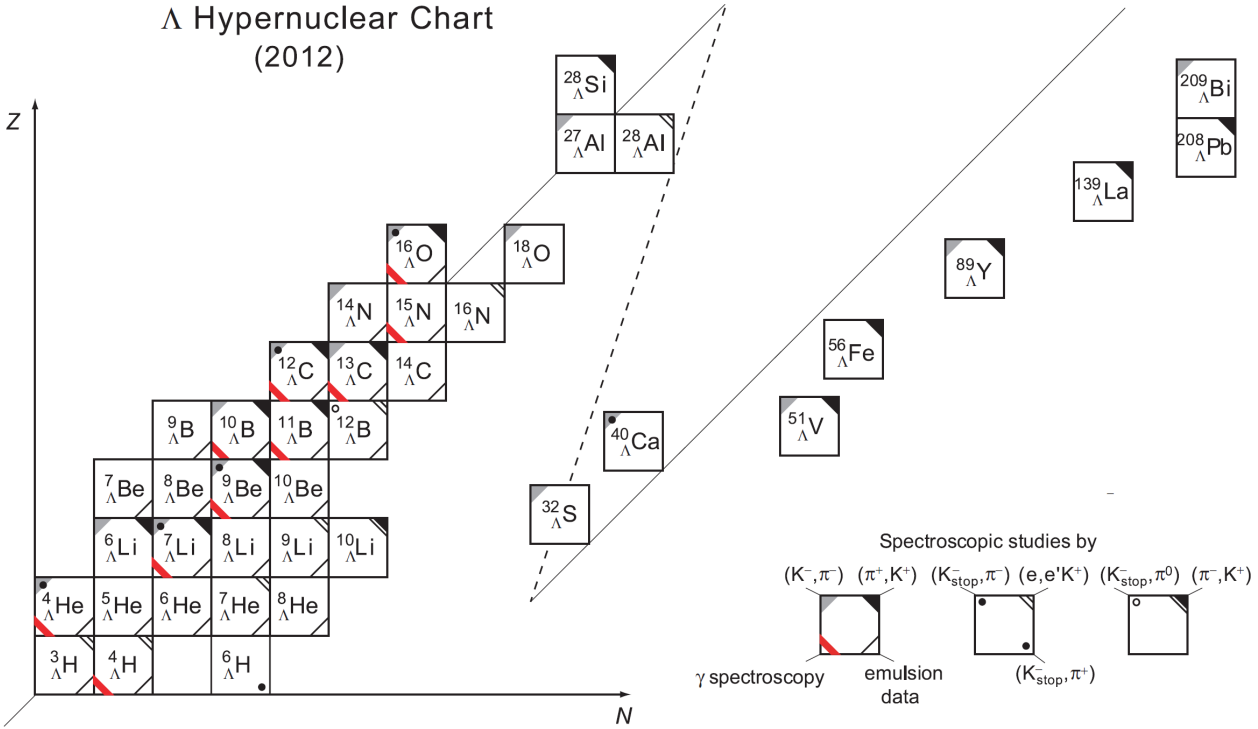


Fig. 2. Λ hypernuclear chart as of 2012.

Lambda Hypernuclear Chart:

PTEP **2012**, 02B012 H. Tamura

few-body (cluster) models <— —> mean-field (eff.int.) models



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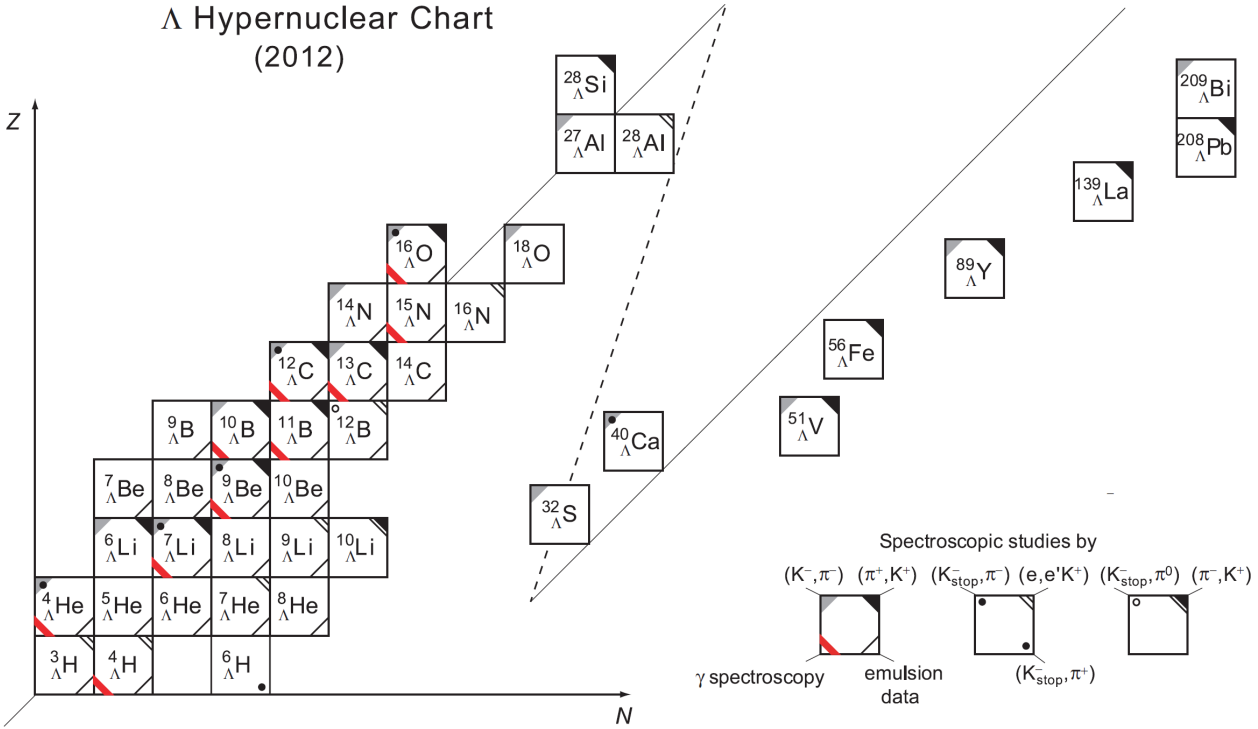

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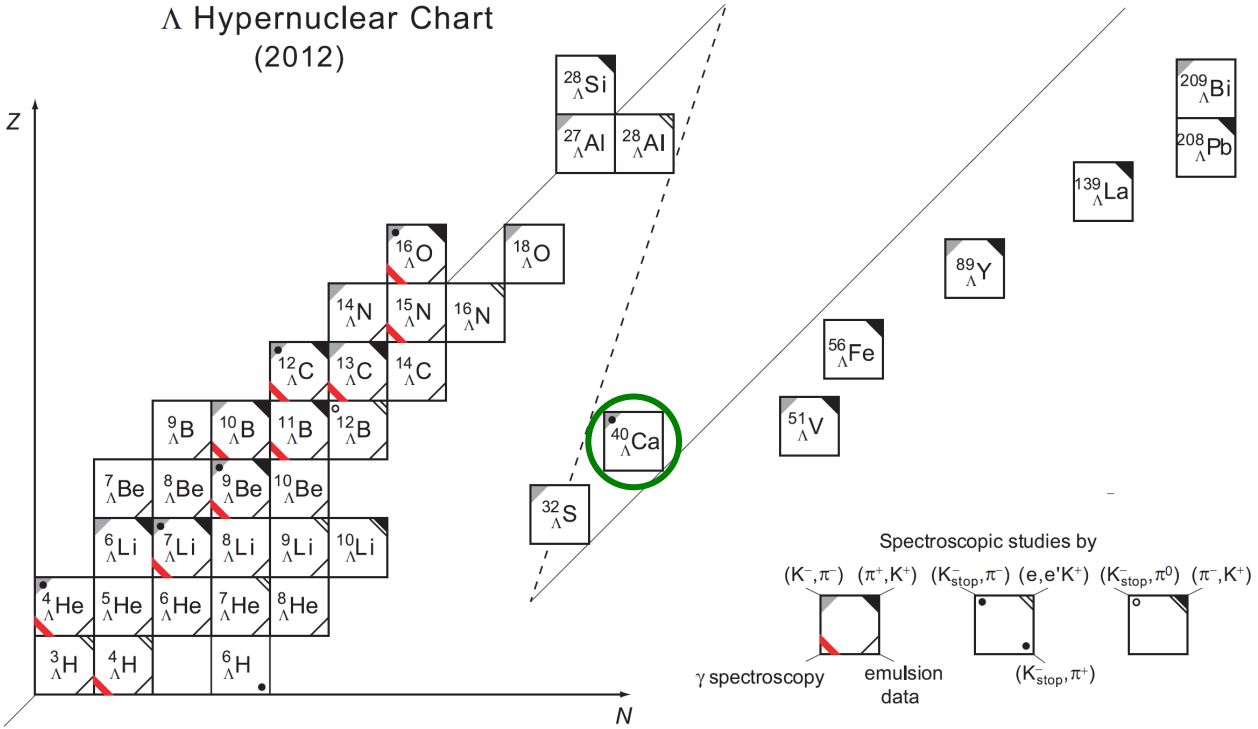

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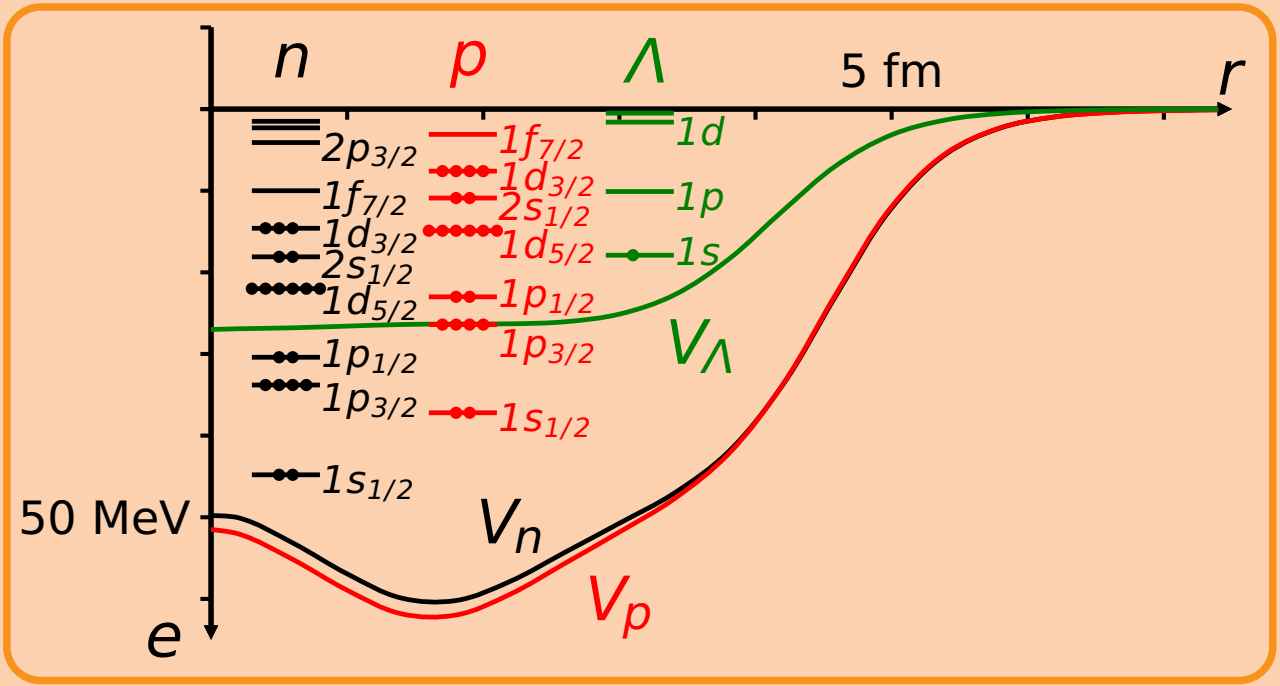
Hypernuclei: Typical Example: $^{40}_{\Lambda}$Ca:



- Theoretical model:
  - Skyrme-Hartree-Fock (SHF) Vautherin & Brink, PRC 5 626 (1972)
  - Standard *NN* force: SIII, SGII, SkI4, **SLy4**, . . .
  - Optimize *N*Λ Skyrme force: **SLL4**, . . .

Extended SHF Model for Hypernuclei:

M. Rayet, Ann. Phys. 102 226 (1976); Nucl. Phys. A367 381 (1981)
D. Vautherin, PRC 7 296 (1973)

- Total energy of the hypernucleus:

$$E = \int \mathrm{d}^3\boldsymbol{r}\, \epsilon(\boldsymbol{r})$$

Energy density functional:

$$\epsilon = \epsilon_N[\tau_n, \tau_p, \rho_n, \rho_p, \boldsymbol{J}_n, \boldsymbol{J}_p] + \epsilon_\Lambda[\tau_\Lambda, \rho_\Lambda, \rho_N]$$

Local densities:

$$\rho_q = \sum_{i=1}^{N_q} |\phi_q^i|^2\,, \quad \tau_q = \sum_{i=1}^{N_q} |\nabla\phi_q^i|^2\,, \quad \boldsymbol{J}_q = \sum_{i=1}^{N_q} \phi_q^{i*}(\nabla\phi_q^i \times \boldsymbol{\sigma})/i$$

i: occupied states, N_q: number of particles $q = n, p, \Lambda$

2D model: quadrupole constraint: $\beta_2 = \sqrt{\frac{\pi}{5}}\frac{\langle 2z^2 - r^2\rangle}{\langle z^2 + r^2\rangle}$ fixed

- Nucleonic part: standard Skyrme functional:

$$\begin{aligned}\epsilon_N \;=\; & \frac{1}{2m_N}\tau_N + \Big[b_0\rho_N^2 - d_0(\rho_n^2+\rho_p^2)\Big]/2 \\ & + \; b_1\rho_N\tau_N - d_1(\rho_n\tau_n+\rho_p\tau_p) + d_1(\boldsymbol{J}_n^2+\boldsymbol{J}_p^2)/2 \\ & - \Big[b_2\rho_N\Delta\rho_N - d_2(\rho_n\Delta\rho_n+\rho_p\Delta\rho_p)\Big]/2 \\ & + \Big[b_3\rho_N^2 - d_3(\rho_n^2+\rho_p^2)\Big]\rho_N^\gamma/3 \\ & - \; b_4\rho_N\nabla\cdot\boldsymbol{J}_N - d_4(\rho_n\nabla\cdot\boldsymbol{J}_n+\rho_p\nabla\cdot\boldsymbol{J}_p) + \epsilon_{\text{Coul.}}\end{aligned}$$

- SHF Schrödinger equation:

$$\Big[-\nabla\cdot\frac{1}{2m_q^*(\boldsymbol{r})}\nabla + V_q(\boldsymbol{r}) - i\nabla W_q(\boldsymbol{r})\cdot(\nabla\times\boldsymbol{\sigma})\Big]\phi_q^i(\boldsymbol{r}) = -e_q^i\phi_q^i(\boldsymbol{r})$$

- SHF mean fields:

$$V_N = V_N^{\text{SHF}} + \frac{\partial\epsilon_\Lambda}{\partial\rho_N} \quad , \quad V_\Lambda = \frac{\partial\epsilon_\Lambda}{\partial\rho_\Lambda} \quad , \quad W_\Lambda = 0$$

Empirical Skyrme $N\Lambda$ Force:

M. Rayet, Ann.Phys. 102, 226 (1976); Nucl.Phys. A367, 381 (1981)

$$\epsilon_\Lambda = \frac{\tau_\Lambda}{2m_\Lambda} + a_0\rho_\Lambda\rho_N + a_3\rho_\Lambda\rho_N^{1+\alpha}$$
$$+ a_1(\rho_\Lambda\tau_N + \rho_N\tau_\Lambda) - a_2(\rho_\Lambda\Delta\rho_N + \rho_N\Delta\rho_\Lambda)/2$$
$$- a_4(\rho_\Lambda\nabla\cdot\boldsymbol{J}_N + \rho_N\nabla\cdot\boldsymbol{J}_\Lambda)$$
$$+ c_0\rho_\Lambda^2 + c_3\rho_\Lambda^2\rho_N^\gamma + c_1\rho_\Lambda\tau_\Lambda - c_2\rho_\Lambda\Delta\rho_\Lambda + \ldots$$

Parameters $a_0, \ldots, a_4, \alpha, c_0, \ldots, c_3, \gamma$

(Together with SLy4 NN force)

We use $a_0, a_1, a_2, a_3, \alpha = 1$

Effective mass:

$$\frac{1}{2m_\Lambda^*} = \frac{1}{2m_\Lambda} + a_1\rho_N + c_1\rho_\Lambda$$

Fit to Hypernuclear Data:

- The SHF mean-field approach is not ideal for light nuclei, but one can hope that for $B_\Lambda = E({}^{A-1}Z) - E({}^{A}_{\Lambda}Z)$ important cancellations regarding the *nuclear* core structure occur. We explore this idea pragmatically.

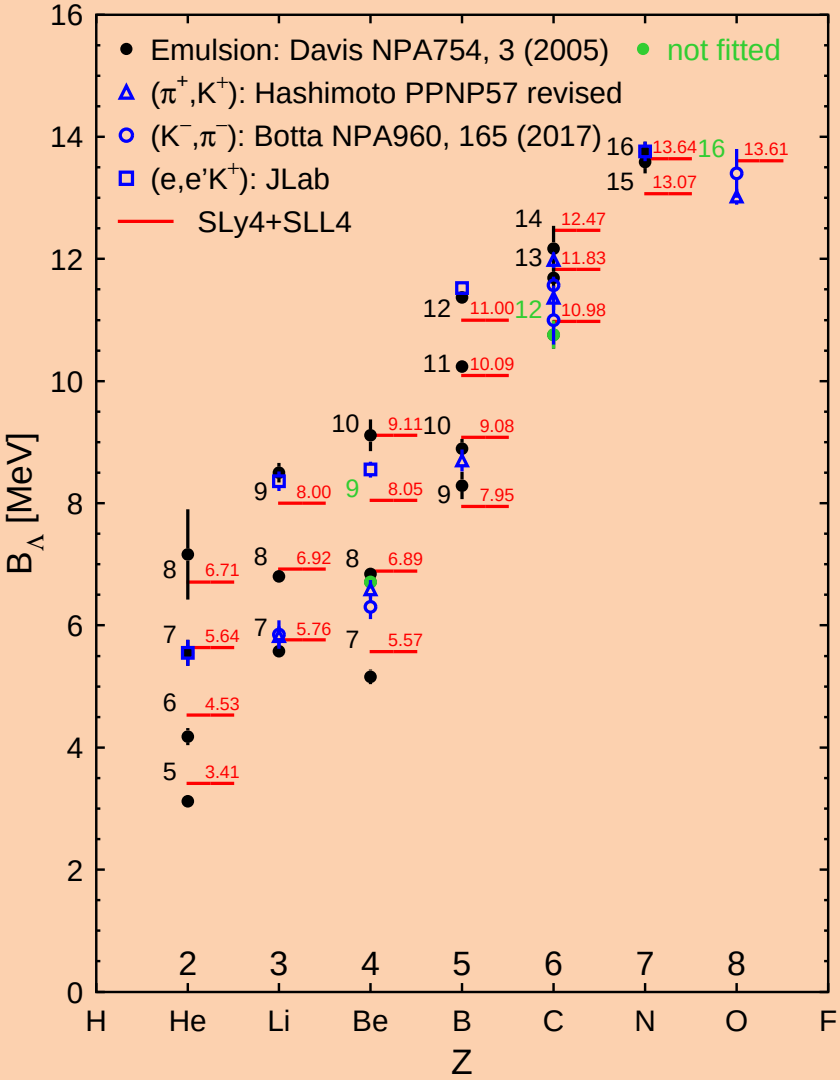
- Hypernuclear data set (19 + 19 data points):
  - $B_\Lambda^{(s)}$ for ${}^{5,6,8}_{\Lambda}\mathrm{He}$, ${}^{7,8,9}_{\Lambda}\mathrm{Li}$, ${}^{7,8,10}_{\Lambda}\mathrm{Be}$, ${}^{9,10,11,12}_{\Lambda}\mathrm{B}$, ${}^{12,13,14}_{\Lambda}\mathrm{C}$, ${}^{15,16}_{\Lambda}\mathrm{N}$

  (Emulsion) Davis, NPA 754, 3 (2005)
  - $B_\Lambda^{(s)}$ for ${}^{7}_{\Lambda}\mathrm{He}$, ${}^{9}_{\Lambda}\mathrm{Be}$, ${}^{11}_{\Lambda}\mathrm{B}$, ${}^{12,13}_{\Lambda}\mathrm{C}$, ${}^{15}_{\Lambda}\mathrm{N}$, ${}^{16}_{\Lambda}\mathrm{O}$

  (K^-, π^-) Botta et al., NPA 960, 165 (2017)
  - $B_\Lambda^{(s,p,d,f,g)}$ for ${}^{16}_{\Lambda}\mathrm{O}$, ${}^{28}_{\Lambda}\mathrm{Si}$, ${}^{51}_{\Lambda}\mathrm{V}$, ${}^{89}_{\Lambda}\mathrm{Y}$, ${}^{139}_{\Lambda}\mathrm{La}$, ${}^{208}_{\Lambda}\mathrm{Pb}$

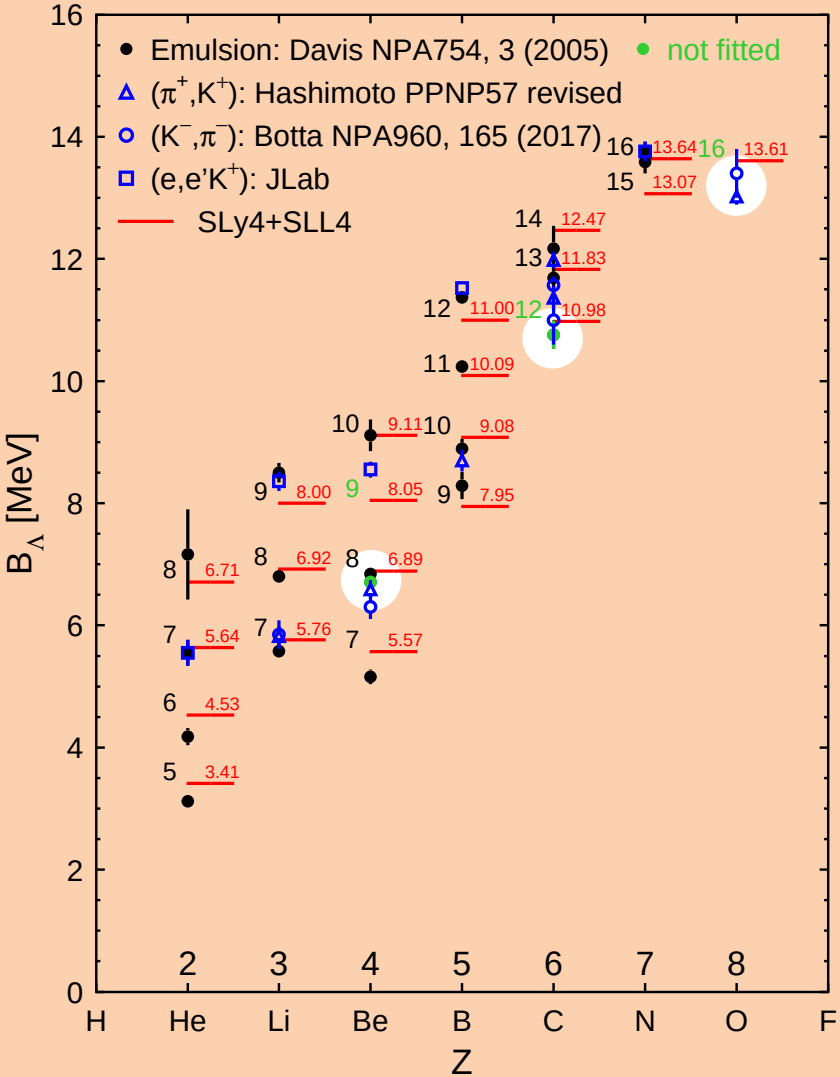
  (π^+, K^+) Hashimoto & Tamura, PPNP 57, 564 (2006)
  Gal & Hungerford & Millener, RMP 88, 035004 (2016)

- Fit parameters a_0, a_1, a_2, a_3 → 'SLL4' $N\Lambda$ force



Results: Light Single-Λ Hypernuclei:

- Reasonable fit with $\langle \Delta B_\Lambda \rangle_{\rm rms} \approx 0.32$ MeV



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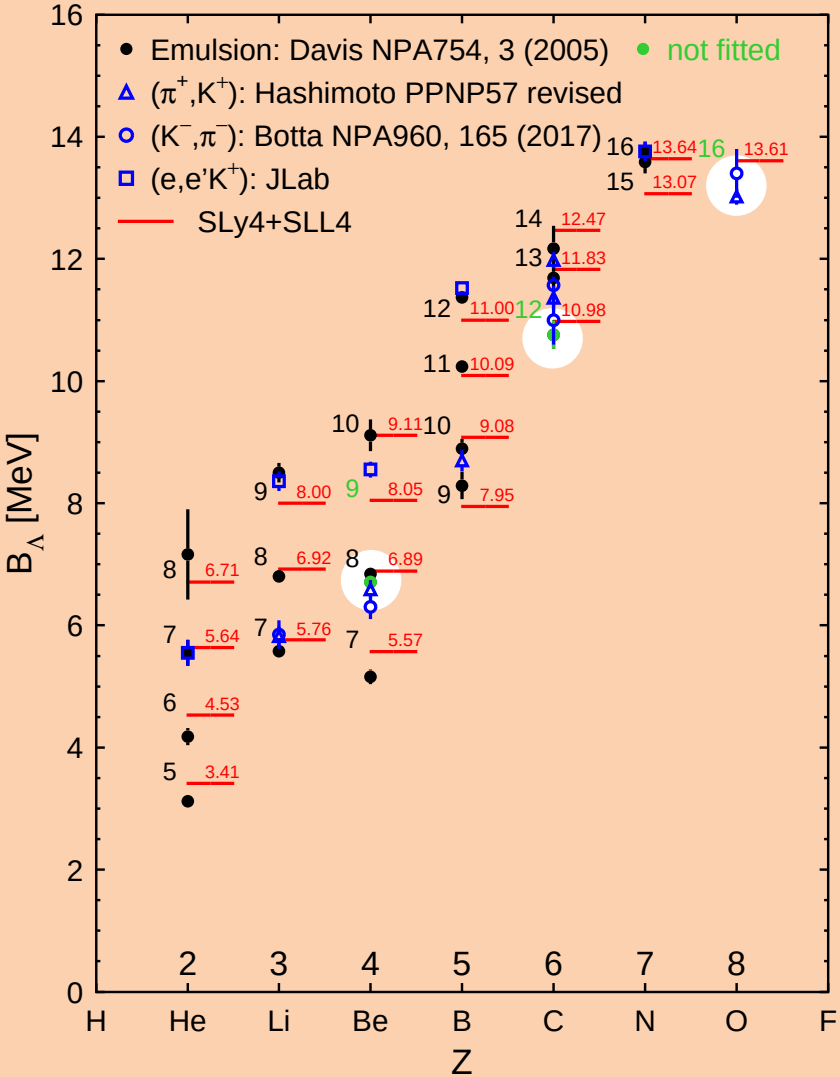
  corrected ${}^{12}_{\Lambda}$C ?
  cluster nucleus ${}^{9}_{\Lambda}$Be :



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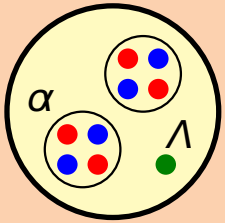
  large $\langle R_N^2 \rangle \rightarrow$ low ρ_N
  $\rightarrow$ low $V_\Lambda \rightarrow$ low B_Λ



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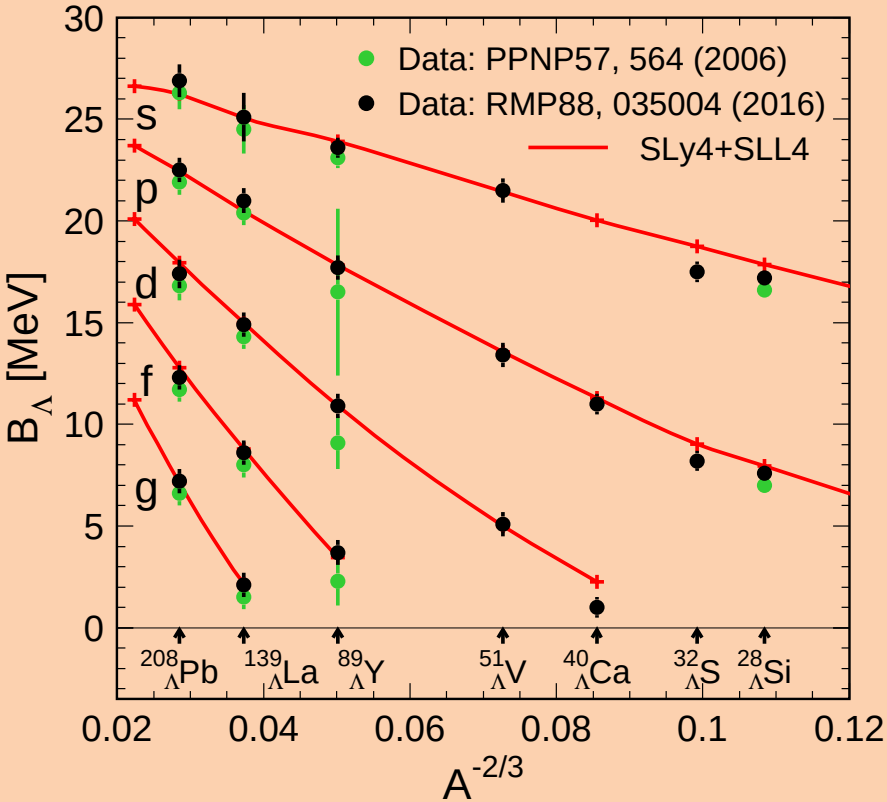


- Exp. data too low by ~ 0.6 MeV ? (J. Millener, H. Tamura): Emulsion $^{12}_{\Lambda}$C used for normalization is not accurate

  see Gal & Hungerford & Millener, RMP 88, 035004 (2016)

Results: $N\Lambda$ Skyrme Parameters:

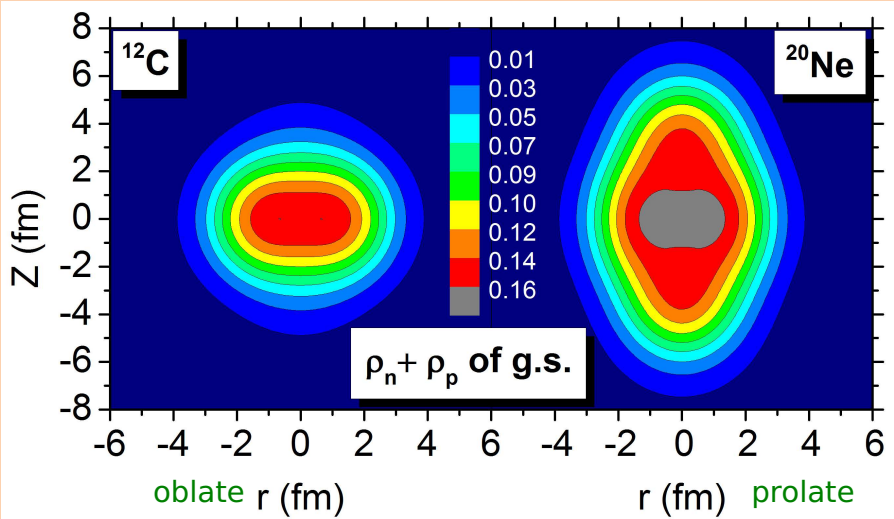
Force		a_0	a_1	a_2	a_3	α	ΔB_Λ[MeV]
SLL4		-322.0	15.75	19.63	715.0	1	0.32
SLL4'		-316.0	23.25	13.88	650.0	1	0.40
RAY12	[1]	-237.4	0	-6.85	375.00	1	0.72
MDG3	[2]	-456.0	25.86	0	864.75	2/3	1.87
YBZ5	[3]	-315.3	0	11.57	750.00	1	0.57
SKSH2	[4]	-290.0	0.35	10.68	693.75	1	0.66
YMR	[5]	-1056.2	26.25	35.00	1054.20	1/8	0.51
HPΛ2	[6]	-302.8	23.72	29.85	514.25	1	1.02

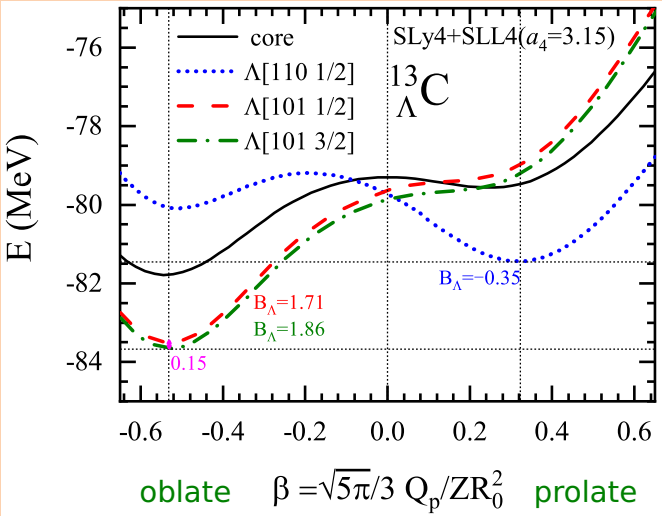
SLL4' : $B_\Lambda\big|_{\text{PPNP57}}$ not corrected by 0.6 MeV

[1] Rayet, Ann. Phys. 102, 226 (1976); Nucl. Phys. A367, 381 (1981)
[2] Millener & Dover & Gal, Phys. Rev. C38, 2700 (1988)
[3] Y. Yamamoto & Bandō & Žofka, Prog. Theor. Phys. 80, 757 (1988)
[4] Fernández & López-Arias & Prieto, Z. Phys. A334, 349 (1989)
[5] Yamamoto & Motoba & Rijken, Prog. Theor. Phys. Suppl. 185, 72 (2010)
[6] Guleria & Dhiman & Shyam, Nucl. Phys. A886, 71 (2012)

↪ Substantial improvement for corrected data

More Sophistication: Deformation and Spin-Orbit Splitting:

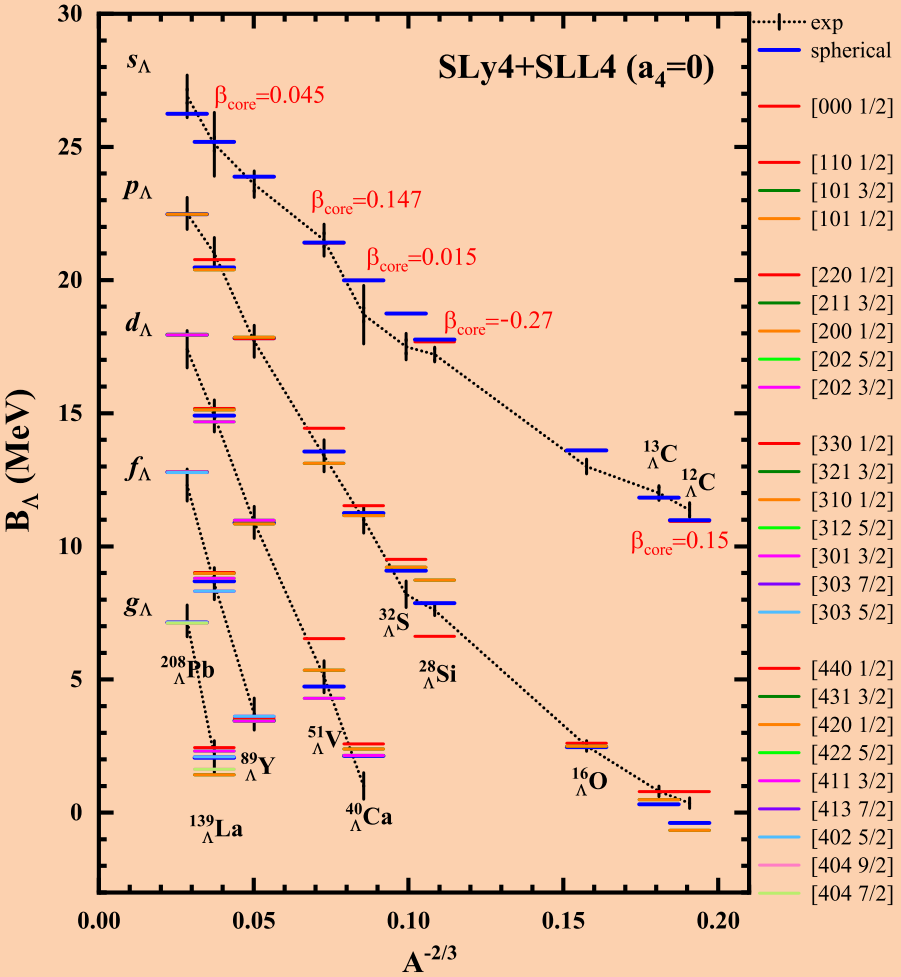




- Core deformation lowers g.s. energy and splits the Λ s.p. levels, in addition to an explicit *ΛN* spin-orbit force
- Exp. s.o. splitting in $^{13}_{\Lambda}$C: Kohri et al., PRC 65, 034607 (2002)

$$E(1p_{1/2}) - E(1p_{3/2}) = 0.152 \pm 0.054 \pm 0.036\,\text{MeV}$$

- Deformation effect on Λ s.p. levels:



- Hyperon binding varies by $\mathcal{O}(1\,\mathrm{MeV})$ due to core deformation: similar shapes of core and hyperon w.f. are energetically preferred
- Precise calculations and precise data are required: $\mathcal{O}(0.1\,\mathrm{MeV})$!
- But:
  - Only few nuclear deformations are known
  - Skyrme forces are not predictive for deformation...

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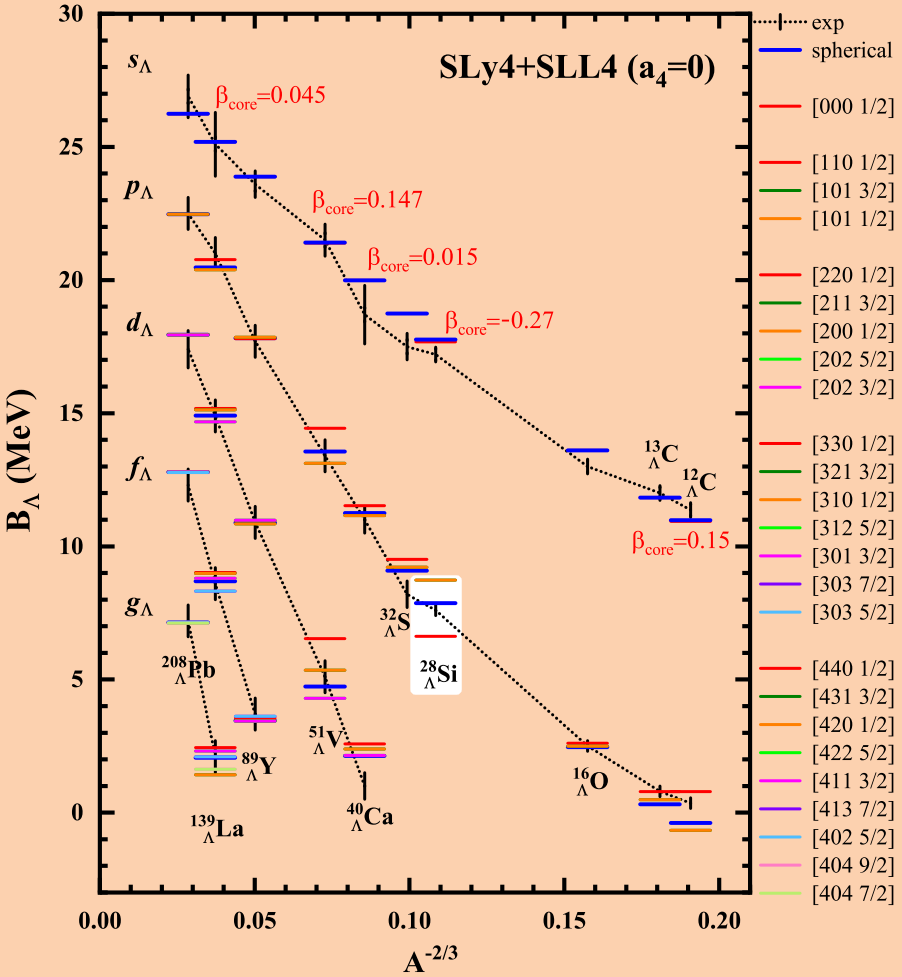

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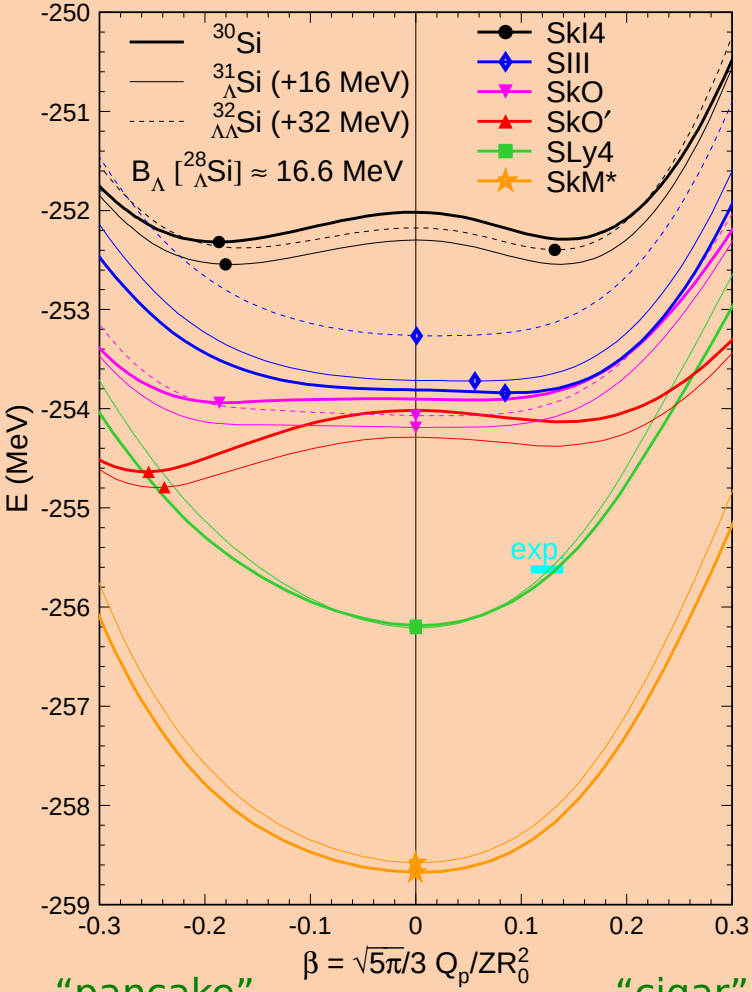
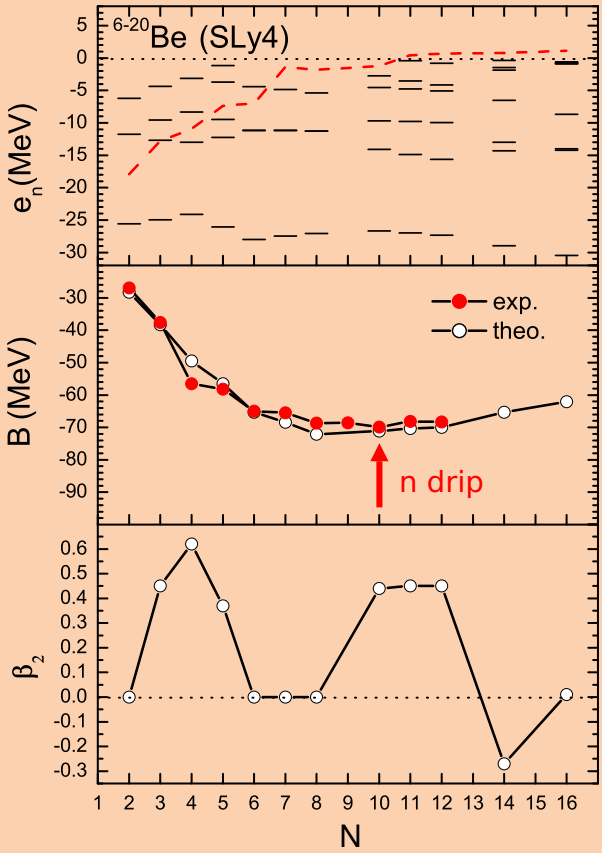
- Deformed (hyper)nuclei, e.g., ^{30}Si:



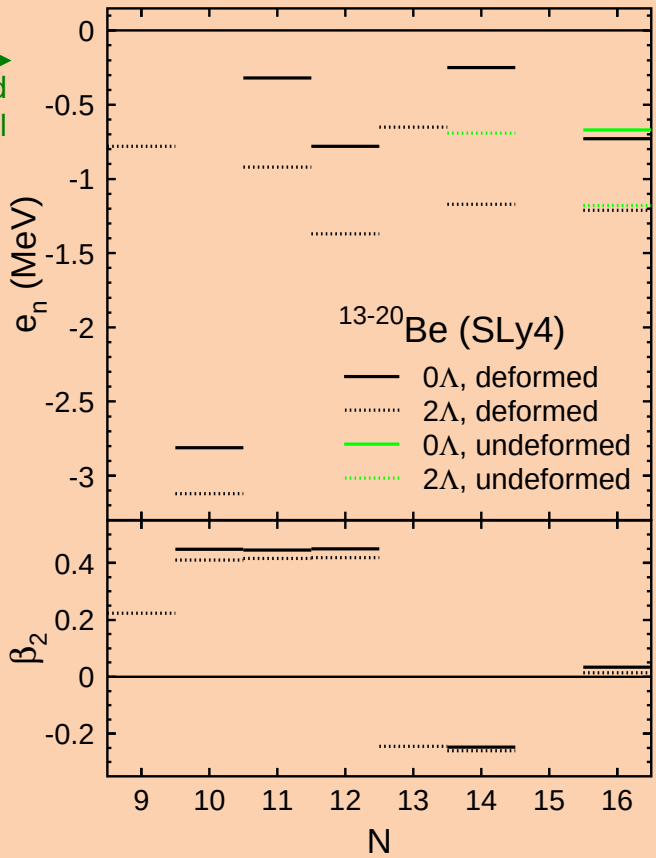
➥ Strong dependence on the *NN* Skyrme force, not predictive

➥ The Λ’s might ‘pull’ together a nucleus with a weak deformation minimum

- Neutron-rich (halo) hypernuclei, e.g., Be isotopes:



highest occupied neutron s.p. level (1d5/2)



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(SHF+BCS, better approach required for halo states)

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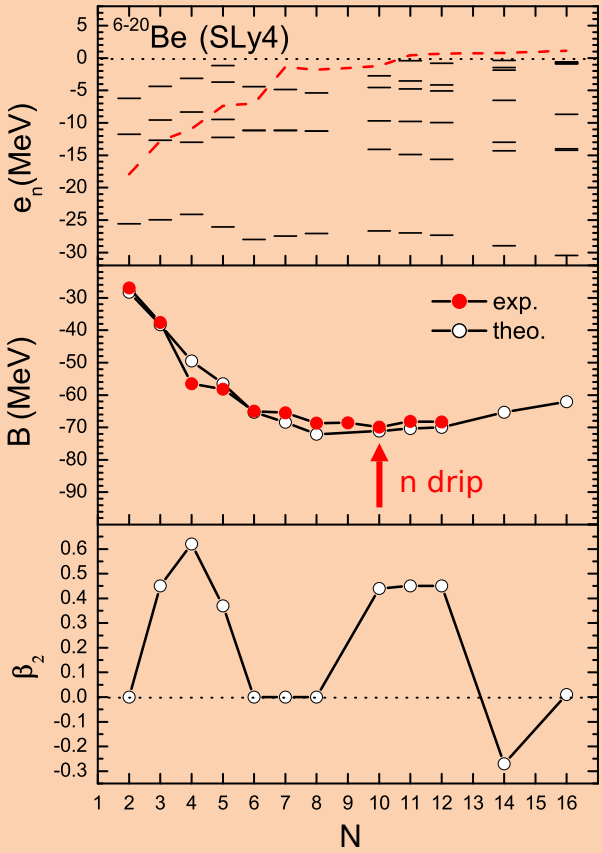

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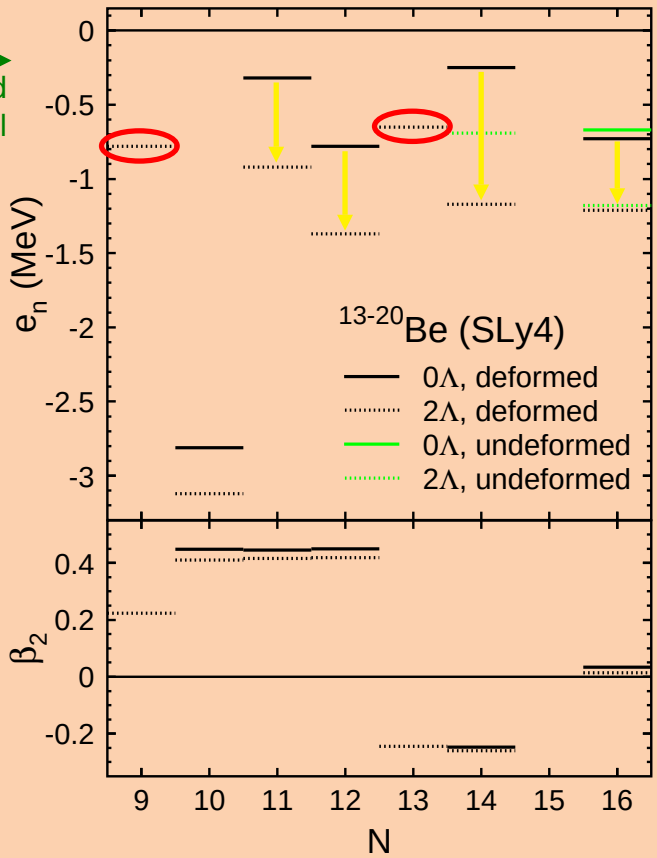

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Summary:

- Optimized SHF parameters for currently known light and heavy hypernuclei: $\Delta B_\Lambda \approx 0.32$ MeV
- Experimental open problems: ${}^{12,13}_{\Lambda}\mathrm{C}$, ${}^{16}_{\Lambda}\mathrm{O}$
- Nuclear core deformation affects $B_\Lambda(p, d, \ldots)$ to $\mathcal{O}(1\,\mathrm{MeV})$

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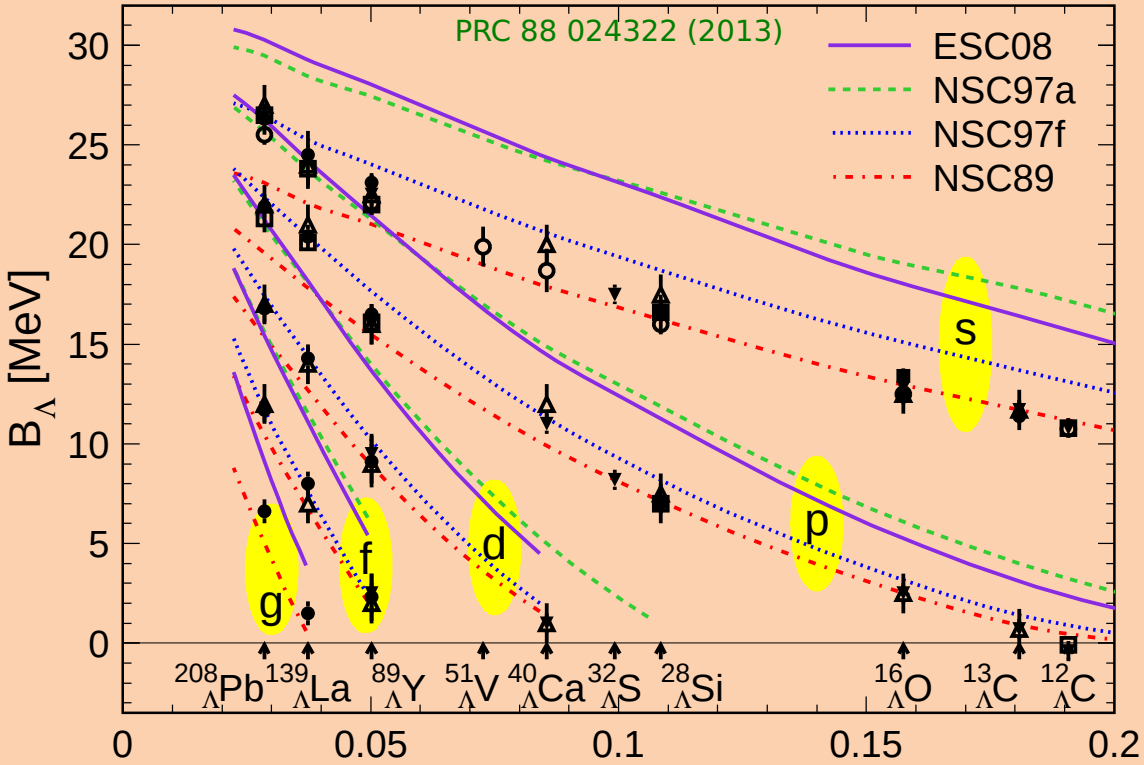
Outlook:

- More sophisticated SHF model: pairing, deformation, s.o. splitting, ...
- Beyond mean field: AMP, GCM, ...
- Extend comparison with cluster etc. models
- Extension to double-Λ and Ξ^- hypernuclei ...

Results: Single-Λ Hypernuclei:

- BHF G-matrix results with Nijmegen YN potentials (no YNN):



↪ Best agreement with NSC89 and NSC97f potentials

No indication of strong hyperon TBF