

Calculations of $\Lambda\text{nn}(1/2^+)$, ${}^3_{\Lambda}\text{H}^*(3/2^+)$, ${}^4_{\Lambda\Lambda}\text{H}(1^+)$, and $\Lambda\Lambda\text{nn}(0^+)$ states within LO \neq EFT

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EMMI Workshop: Bound states and particle interactions in the 21st century

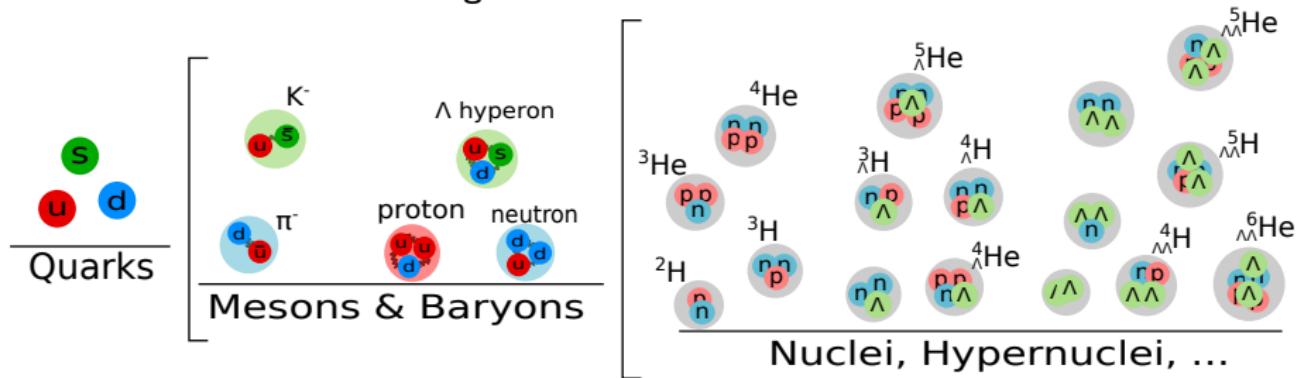
6th of July 2023

Why few-body hypernuclei ?

Interactions of hadrons :

- currently described by QCD

At low and intermediate energies ...



- **QCD is notoriously difficult to solve in this energy regime !**
→ lattice QCD and effective field theories (EFTs)

Observed properties of
few-body hypernuclei



Precise few-body
methods

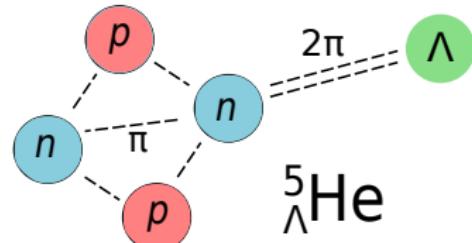


Underlying interaction
models

Hypernuclei

Where do we stand ?

- experimentaly observed more than 30 Λ -hypernuclei
- three well-established $\Lambda\Lambda$ -hypernuclei
- **femtoscopy data**
- scarce ΛN and no $\Lambda\Lambda$ scattering data



→ difficult to fix parameters of interaction models,
many parameters and few data points → large uncertainties

What do we do ?

→ we build low-energy EFT without π (π EFT) employing both scattering lengths and s -shell hypernuclear data (3-body NNN , ΛNN , and $\Lambda\Lambda N$ interaction)

Hyper(nuclear) \neq EFT (introduced in Nir's talk)

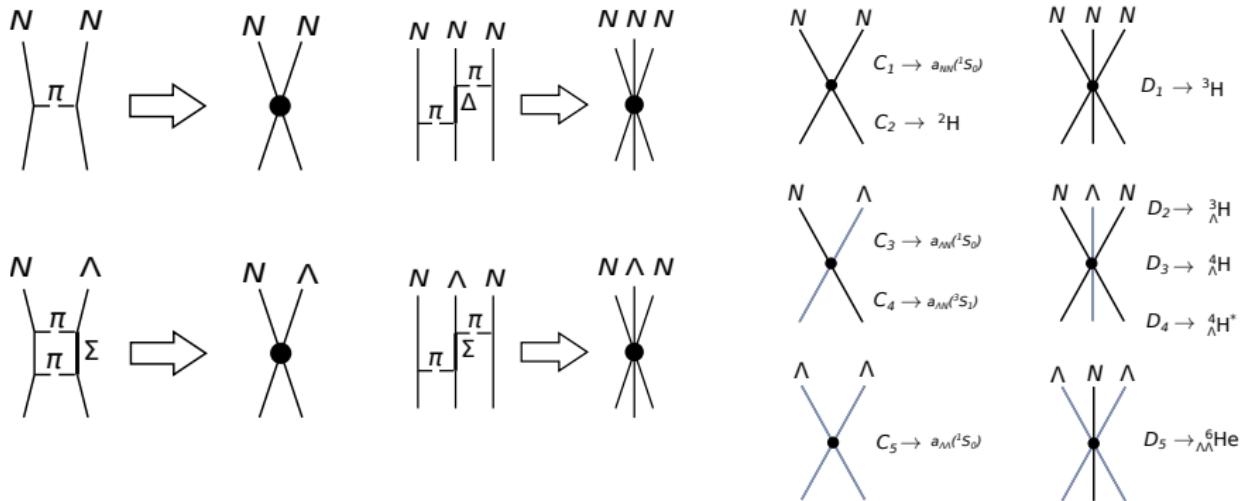
Hamiltonian :

$$H_{\lambda}^{(\text{LO})} = T_k + V_2 + V_3$$

$$V_2 = \sum_{I,S} C_{\lambda}^{I,S} \sum_{i < j} \mathcal{P}_{ij}^{I,S} \delta_{\lambda}(\mathbf{r}_{ij})$$

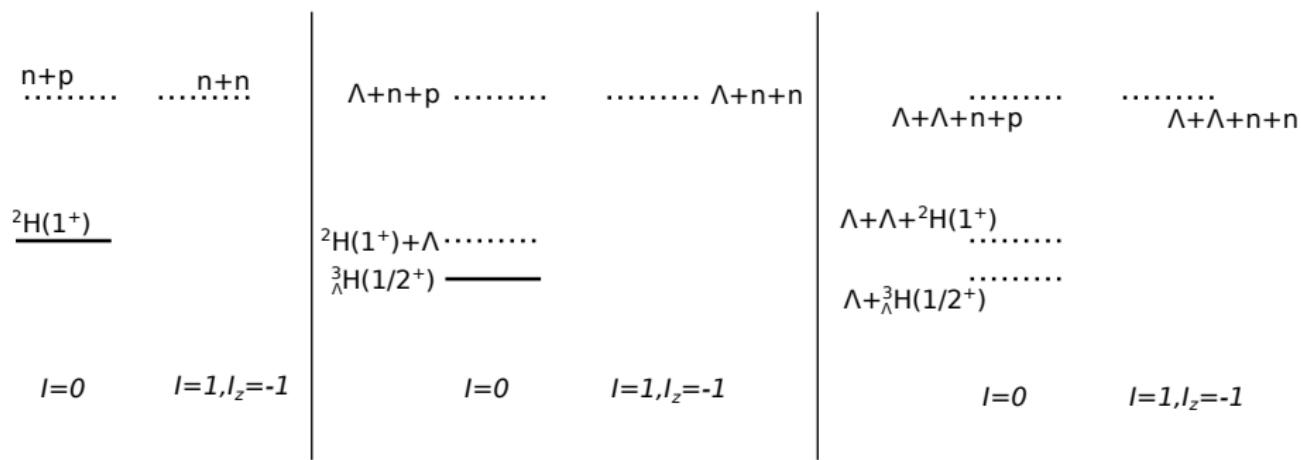
$$V_3 = \sum_{I,S,\alpha} D_{\lambda,\alpha}^{I,S} \sum_{i < j < k} \mathcal{Q}_{ijk}^{I,S,\alpha} \sum_{\text{cyc}} \delta_{\lambda}(\mathbf{r}_{ij}) \delta_{\lambda}(\mathbf{r}_{jk})$$

Contact terms (minimal amount of parameters) \rightarrow constrained by exp. data



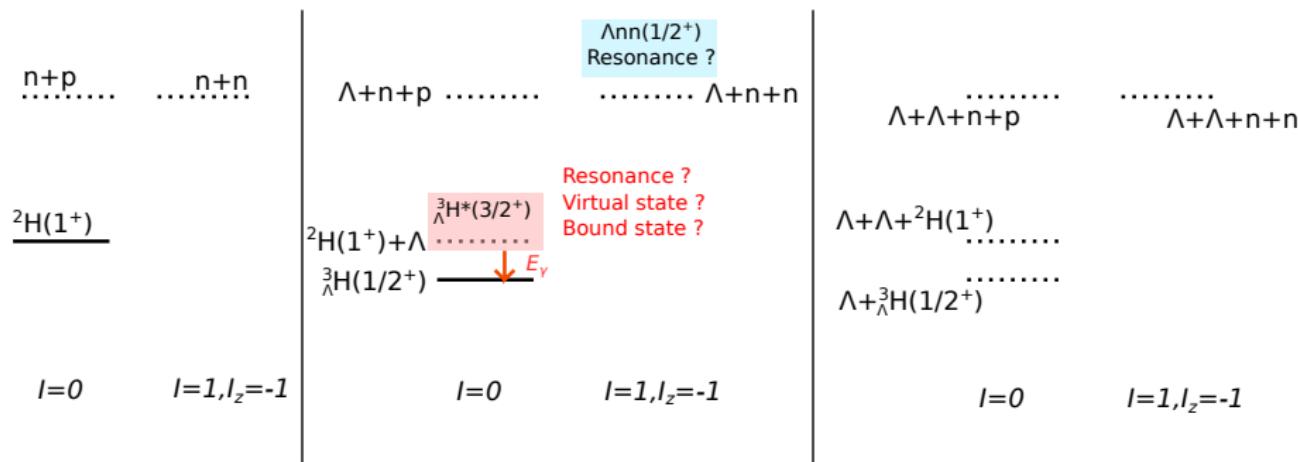
\rightarrow prediction of Λnn , $\Lambda \Lambda n$, $\Lambda \Lambda nn$, ${}^3\Lambda \text{H}^*$, ${}^5\Lambda \text{He}$, ${}^4\Lambda \text{H}$, ${}^5\Lambda \text{H}$, ${}^5\Lambda \text{He}$

The onset of Λ and $\Lambda\Lambda$ -hypernuclear binding



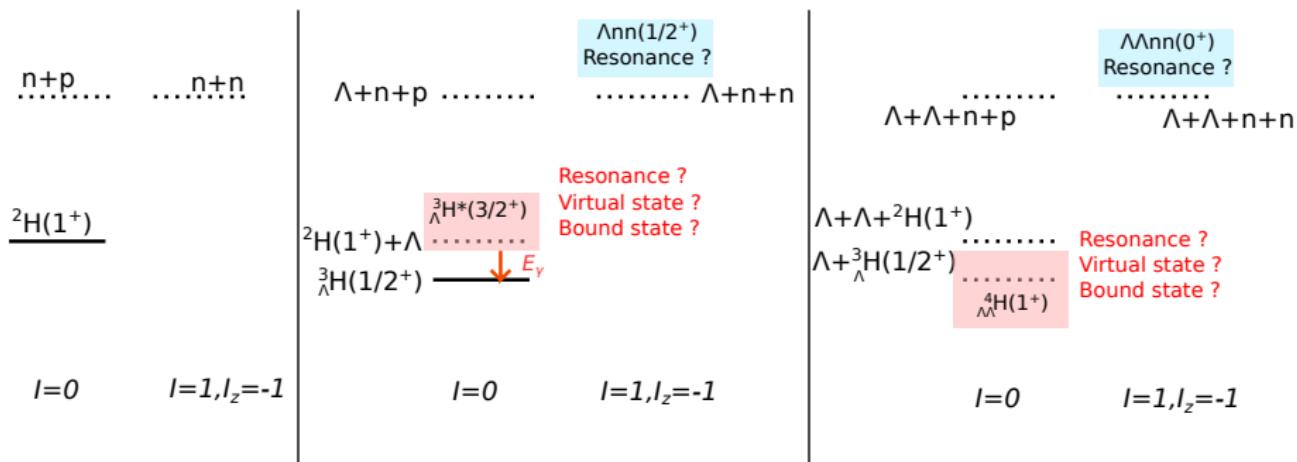
Glue-like role of the Λ hyperon.

The onset of Λ and $\Lambda\Lambda$ -hypernuclear binding



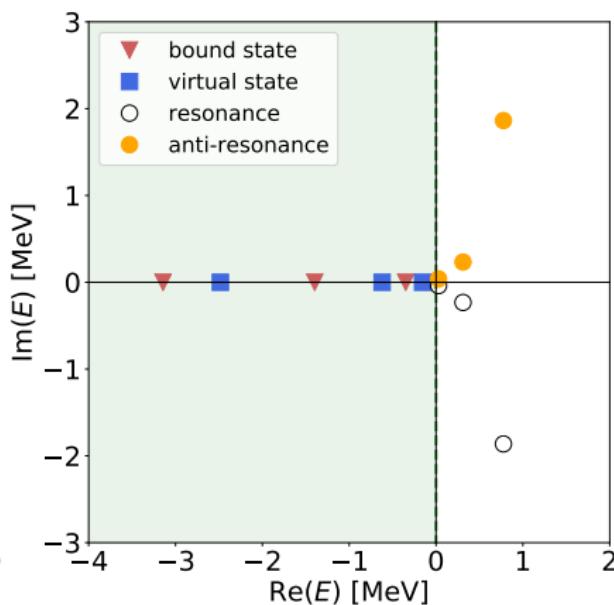
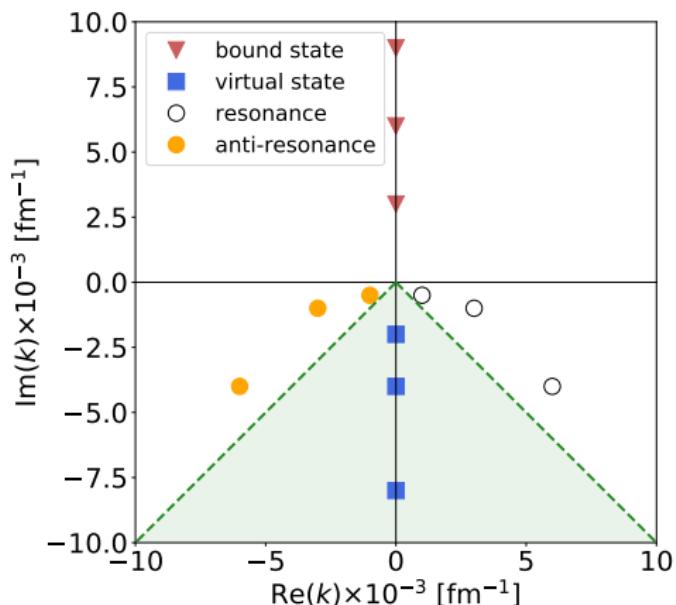
Glue-like role of the Λ hyperon.

The onset of Λ and $\Lambda\Lambda$ -hypernuclear binding



Glue-like role of the Λ hyperon.

Bound states, resonances, virtual states



Bound state	$a > 0, r < a/2$
Virtual state	$a < 0, r > 0$ ($a < 0, r < 0, a < 2r$)
Resonance	$a < 0, r < 0, r < a$
Subthreshold resonance	$a < 0, r < 0, 2r < a < r$

Hypernucler trios ${}^3_{\Lambda}\text{H}$, ${}^3_{\Lambda}\text{H}^*$, Ann - physical motivation

${}^3_{\Lambda}\text{H}$

- lightest bound hypernucleus with $1/2^+$ spin-parity g.s.
- established from hypertriton weak-decay measurements

$$R_3 = \frac{\Gamma({}^3_{\Lambda}\text{H} \rightarrow \pi^- + {}^3\text{He})}{\Gamma_{\pi^-}({}^3_{\Lambda}\text{H})} = 0.35 \pm 0.04$$

(G. Keyes et al., NPB67, 269, 1973)

$J^\pi = 1/2^+$ requires R_3 about 0.4
 $J^\pi = 3/2^+$ requires R_3 about 0.1
(Bertrand et al., NPB16, 77, 1970)



Hypernucler trios $^3\Lambda$ H, $^3\Lambda$ H*, Λ nn - physical motivation

$^3\Lambda$ H($1/2^+$)

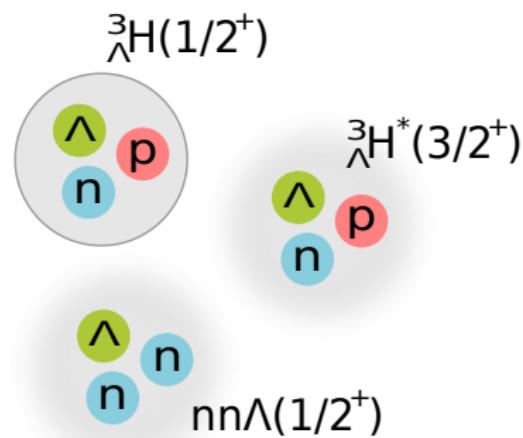
- lightest bound hypernucleus
- currently no experimental consensus on its B_Λ
- constraint in ΛN interaction models

$^3\Lambda$ H*($3/2^+$)

- no experimental evidence
- strict constraint on $\Lambda N S = 1$ interaction

Λ nn($1/2^+$)

- experiment (HypHI)
- JLab E12-17-003 experiment
- valuable source of Λn interaction
- structure of neutron-rich Λ -hypernuclei
- talk of L. Tang (Jlab) and C. Rappold (HypHI)



Λ nn and $^3\Lambda$ H* - early work

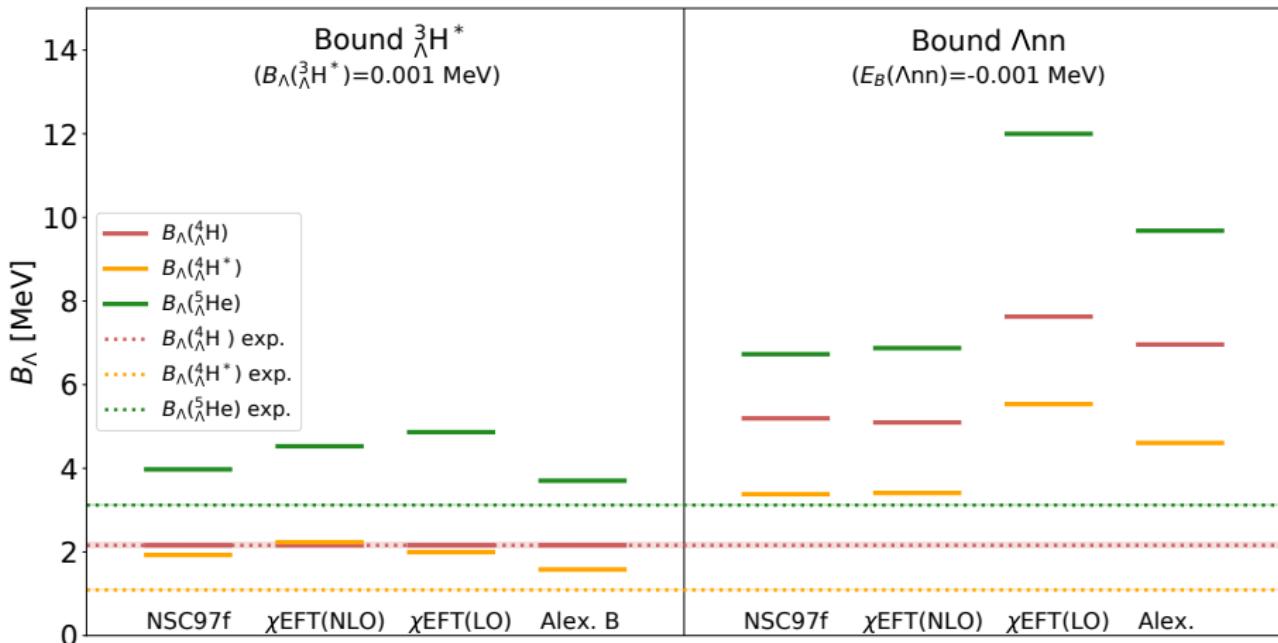
- **R. H. Dalitz, B. W. Downs** (PR110, 958, 1958; PR111, 967, 1958; PR114, 593, 1959)
→ first calculation, variational approach, unbound Ann
- **H. Garcilazo** (J. Phys. G: Nucl. Phys. 13, 63, 1987)
→ Faddeev equations, separable potentials, unbound Ann
- **K. Miyagawa et al.** (PRC51, 2905, 1995)
→ Faddeev equations, realistic Nijmegen interaction, unbound Ann and $^3\Lambda$ H*
- **H. Garcilazo et al.** (PRC75, 034002, 2007; PRC76, 034001, 2007)
→ Faddeev equations, Chiral Quark Model ($N\Lambda - N\Sigma$ coupling, tensor force)
→ unbound Ann
→ constraints on $a_{\Lambda N}^{S=0}$, $a_{\Lambda N}^{S=1}$ from $^3\Lambda$ H, unbound $^3\Lambda$ H*, and Λp data
- **V. B. Belyaev et al.** (NPA803, 210, 2008)
→ first resonance calculation, 3-body Jost function, phenomenological potential
→ Ann pole just above/below the threshold, large widths

Λ nn and $^3\Lambda$ H* - current status

- **HypHI Collaboration** (PRC88, 041001(R), 2013)
→ suggestion of bound Λ nn, $^6\text{Li} + ^{12}\text{C}$ @ 2A GeV
- **E. Hiyama et al.** (PRC89, 061302(R), 2014)
→ YN model equivalent to NSC97f; changing $^3V_{N\Lambda-N\Sigma}^T$, $^0V_{NN}$ to bind Λ nn
→ nonexistence of bound Λ nn ($^3_\Lambda\text{H}$, $^3\Lambda\text{H}^*$, $^4_\Lambda\text{H}$, ^3H)
- **A. Gal, H. Garcilazo** (PLB736, 93, 2014)
→ Faddeev equations, separable potentials
→ nonexistence of bound Λ nn ($\sigma_{\Lambda p}$, $^3_\Lambda\text{H}$, and $^4_\Lambda\text{H}$ exc. energy)
- **I. R. Afnan, B. F. Gibson** (PRC92, 054608, 2015)
→ Faddeev equations, Λ nn resonance calculations, separable potentials
→ subthreshold (non-physical) Λ nn resonance
- **JLab E12-17-003 Experiment** (PTEP92 2022, 013D01, 2022)
→ $^3\text{H}(e, e' K^+) \Lambda$ nn
→ No significant structures observed

Implications of just bound Λ nn and $^3\Lambda$ H* ($\lambda = 6 \text{ fm}^{-1}$)

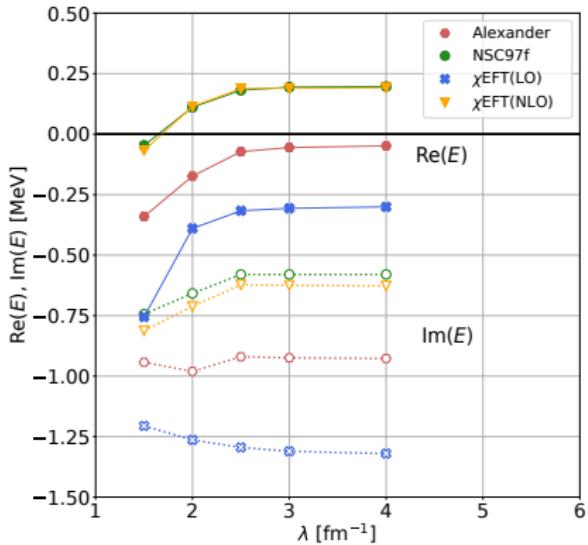
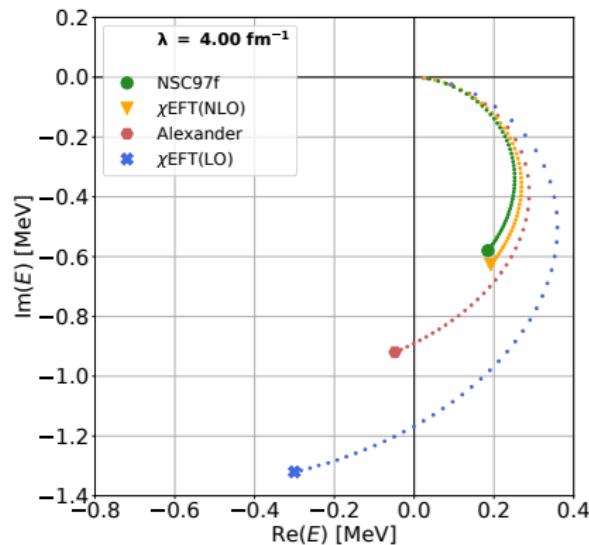
(PLB 808, 2020, 135614; PRC 103, 2021, 025204)



- $B_\Lambda(^3\Lambda H)$ is used to fix three-body force in $I, S = 0, 1/2$ channel and remains unaffected

Resonance in Λ nn system

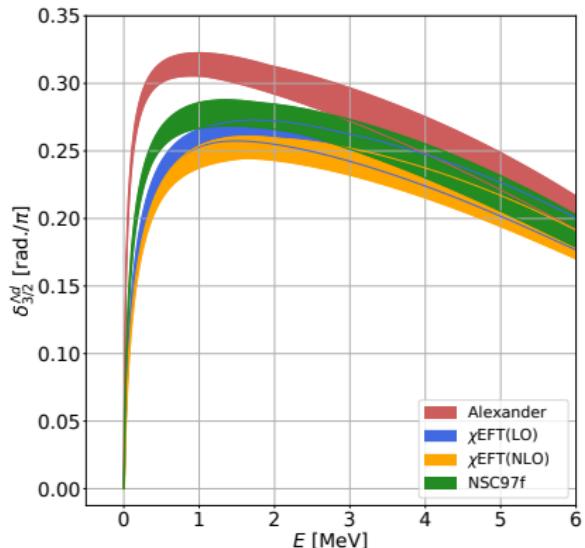
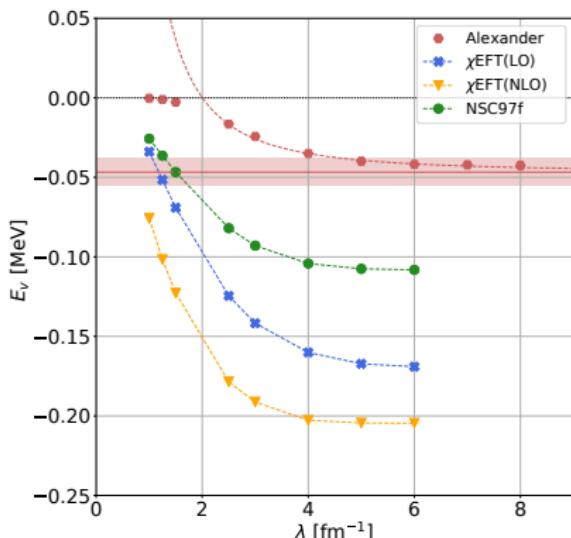
(PLB 808, 2020, 135614; PRC 103, 2021, 025204)



- Λ nn resonance pole moves with increasing cut-off towards physical Riemann sheet

Excited state of hypertriton $^3\Lambda$ H* as a virtual state

(PLB 808, 2020, 135614; PRC 103, 2021, 025204)



- $^3\Lambda$ H* virtual state solution for all considered cut-offs and scattering lengths
- convergence of $^3\Lambda$ H* virtual state pole with increasing cut-off
- at LO χ EFT there is a virtual state lying from 0.02 up to 0.25 MeV near the $^2\text{H} + \Lambda$ threshold

Possibility of a shallow ${}^3_{\Lambda}\text{H}^*$ bound state (PRC 105, 2022, 015202)

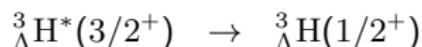
Weak decay of ${}^3_{\Lambda}\text{H}^*$

(closure approximation)

$$\Gamma_{WD}(1/2^+)/\Gamma_{\Lambda} = 1.154$$

$$\Gamma_{WD}(3/2^+)/\Gamma_{\Lambda} = 0.986$$

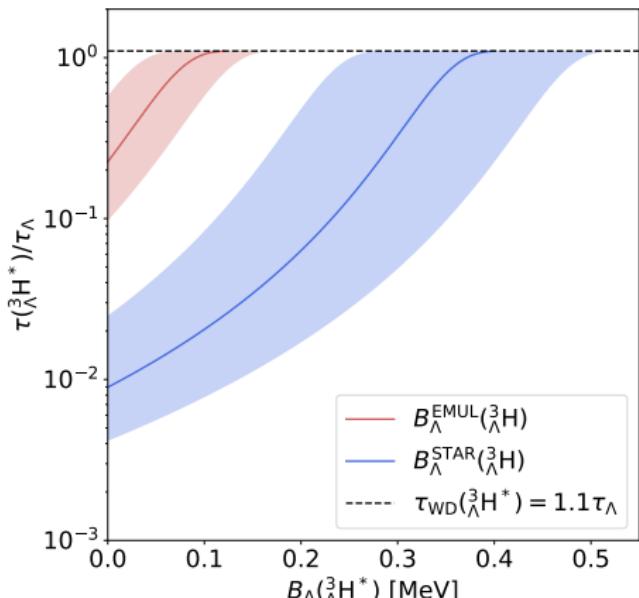
Electromagnetic M1 transition



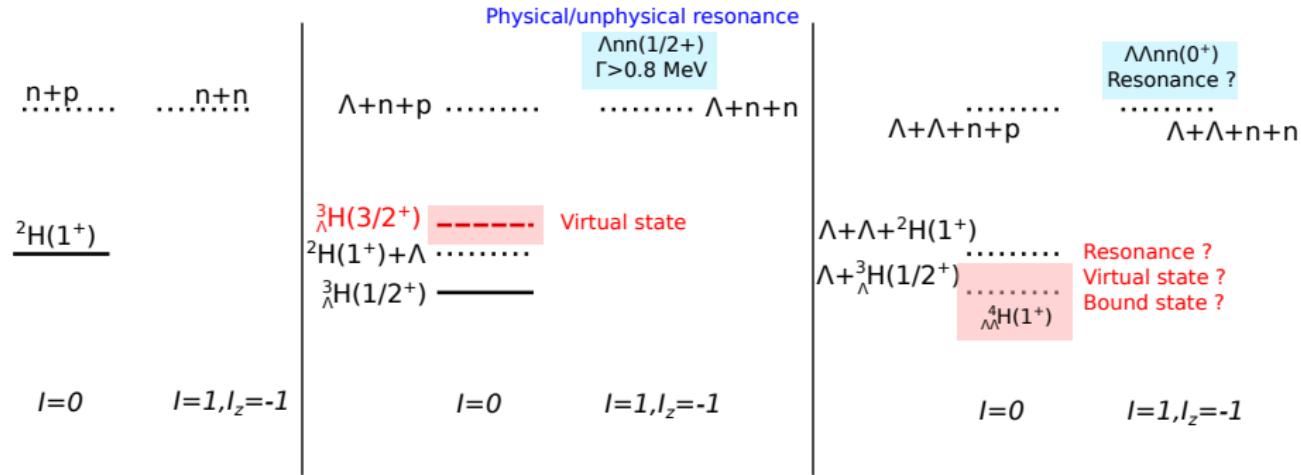
$$\Gamma_{M1} = \alpha(\Delta E)^{3/2} (g_c - g_{\Lambda}) s^{-1}$$

Overall lifetime of ${}^3_{\Lambda}\text{H}^*(3/2^+)$

$$\tau(3/2^+) = \left(\frac{1}{\tau_{M1}} + \frac{1}{\tau_{WD}} \right)^{-1}$$



The onset of Λ and $\Lambda\Lambda$ -hypernuclear binding



Glue-like role of the Λ hyperon.

$^4_{\Lambda\Lambda}\text{H}(J^\pi = 1^+)$ and $\Lambda\Lambda nn(J^\pi = 0^+)$ systems

- **J. K. Ahn et al.** (PRL87, 132504, 2001)

AGS-E906 counter experiment; production of bound $^4_{\Lambda\Lambda}\text{H}(1^+)$

- **Filikhin and Gal** (PRL89,172502, 2002)

$^4_{\Lambda\Lambda}\text{H}(1^+)$, Faddeev-Yakubovski calculations, $\Lambda\Lambda$ fitted to $\Delta B_{\Lambda\Lambda}(^6_{\Lambda\Lambda}\text{He})$, no bound state

- **Nemura et al.** (PRC67, 051001(R), 2003)

$^4_{\Lambda\Lambda}\text{H}(1^+)$, Stochastic Variational calculations, interactions same as above, bound state

- **Nemura et al.** (PRL94, 202502, 2005)

all s-shell $\Lambda\Lambda$ hypernuclei, Stochastic Variational full coupled-channel calculations, simulated form of YN and YY Nijmegen potentials, onset at $B_\Lambda(^4_{\Lambda\Lambda}\text{H}(1^+))=2$ keV

- **S. D. Randeniya and E. V. Hungerford** (PRC76, 064308, 2007)

AGS-906 counter experiment; reevaluation of the reported observation of the $^4_{\Lambda\Lambda}\text{H}(1^+)$

$^4_{\Lambda\Lambda}H(J^\pi = 1^+)$ and $\Lambda\Lambda nn(J^\pi = 0^+)$ systems

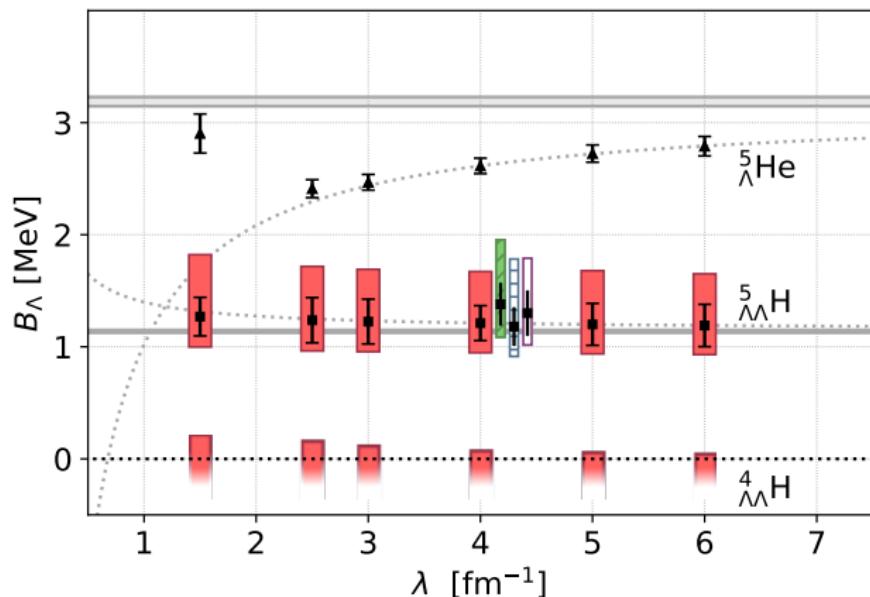
- **J.-M. Richard et al.** (PRC91, 014003, 2015)
 $\Lambda\Lambda nn(0^+)$, claim that a stability of this system is within uncertainties of the baryon-baryon interaction
- **H. Garcilazo et al.** (Chin. Phys. C41, 0741102, 2017)
 $\Lambda\Lambda nn(0^+)$, NN Yukawa type Malfliet-Tjon interaction and $N\Lambda$, $\Lambda\Lambda$ ESC08c Nijmegen potential, unbound - just above the threshold
- **S. Bleser et al.** (PLB790, 502, 2019)
AGS-E906 counter experiment; neutral $\Lambda\Lambda nn(0^+)$ system assigned to the main yet unexplained signal
- **L. Contessi et al.** (PLB797, 134893, 2019)
LO χ EFT ; fit to $^6_{\Lambda\Lambda}\text{He}$; $\Lambda\Lambda nn(0^+)$ unbound and $^4_{\Lambda\Lambda}H(1^+)$ on the verge of binding
- **H. Le et al.** (EPJA57, 217, 2021)
LO & NLO χ EFT; sign of unbound $^4_{\Lambda\Lambda}H(1^+)$; definite conclusion would require more precise numerical results

Implications of just bound ${}^4_{\Lambda\Lambda}\text{H}(1^+)$ and $\Lambda\Lambda\text{nn}(0^+)$ systems

(PLB 797, 2019 134893)

Table: Λ separation energies $B_\Lambda({}_{\Lambda\Lambda}^A\text{Z})$ for $A=3-6$, calculated using $a_{\Lambda\Lambda}=-0.8$ fm, cutoff $\lambda=4$ fm $^{-1}$ and the Alexander[B] ΛN interaction model. In each row a $\Lambda\Lambda N$ LEC was fitted to the underlined binding energy constraint.

Constraint (MeV)	${}^3_{\Lambda\Lambda}\text{n}$	${}^4_{\Lambda\Lambda}\text{n}$	${}^4_{\Lambda\Lambda}\text{H}$	${}^5_{\Lambda\Lambda}\text{H}$	${}^6_{\Lambda\Lambda}\text{He}$
$\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^6\text{He})=\underline{0.67}$	—	—	—	1.21	3.28
$B_\Lambda({}_{\Lambda\Lambda}^4\text{H})=\underline{0.05}$	—	—	0.05	2.28	4.76
$B({}_{\Lambda\Lambda}^4\text{n})=\underline{0.10}$	—	0.10	0.86	4.89	7.89
$B({}_{\Lambda\Lambda}^3\text{n})=\underline{0.10}$	0.10	15.15	18.40	22.13	25.66

$^4\Lambda\Lambda H(1^+)$ hypernucleus (PLB 797, 2019 134893)

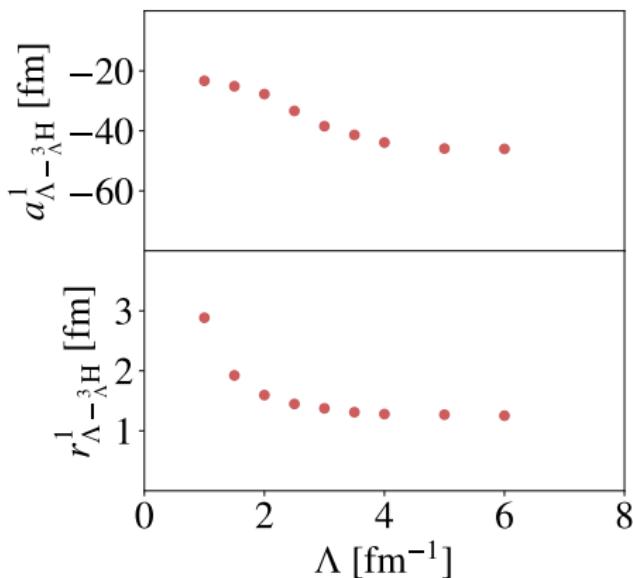
$^4\Lambda\Lambda H(1^+)$ bound only for $a_{\Lambda\Lambda}^0 \leq -1.9$ fm

$^4_{\Lambda\Lambda}\text{H}(1^+)$ hypernucleus in terms of $\Lambda + ^3_{\Lambda}\text{H}$ scattering (preliminary)

s-wave $\Lambda + ^3_{\Lambda}\text{H}$ scattering

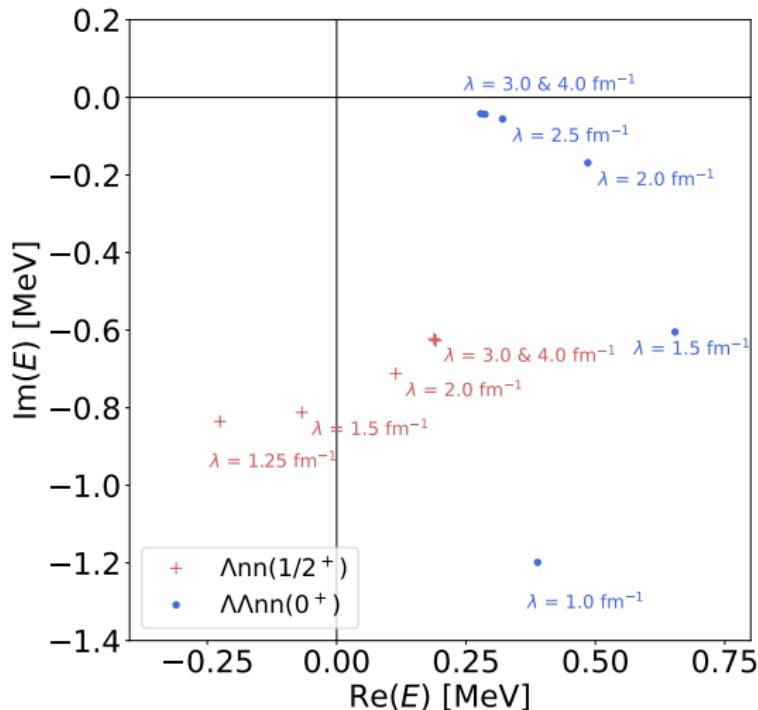
- Bush formula
(introduced in Mirko's talk)
- negative $a_{\Lambda - ^3_{\Lambda}\text{H}}^1$ scattering length
- positive $r_{\Lambda - ^3_{\Lambda}\text{H}}^1$ effective range

$^4_{\Lambda\Lambda}\text{H}(1^+)$ enters continuum as
a virtual state !



$\Lambda\Lambda$ nn(0^+) resonance (preliminary)

Possibility of the narrow
 $\Lambda\Lambda$ nn(0^+) resonance !



Summary

- comprehensive study of the $\Lambda\text{nn}(\frac{1}{2}^+)$, $^3_{\Lambda}\text{H}^*(\frac{3}{2}^+)$, $\Lambda\Lambda\text{nn}(0^+)$, and $^4_{\Lambda\Lambda}\text{H}(1^+)$ systems in LO $\not\!\Lambda\text{EFT}$
→ ΛN , $\Lambda\Lambda$ scattering lengths, three-body forces, connection to $^4_{\Lambda}\text{H}$, $^4_{\Lambda}\text{H}^*$, $^6_{\Lambda\Lambda}\text{He}$

$\Lambda\text{nn}(\frac{1}{2}^+)$ - **resonant state**

$^3_{\Lambda}\text{H}^*(\frac{3}{2}^+)$ - **virtual state**

$^4_{\Lambda\Lambda}\text{H}(1^+)$ - **weakly bound or virtual state** (preliminary)

$\Lambda\Lambda\text{nn}(0^+)$ - **narrow resonant state** (preliminary)