

# THERMAL ABUNDANCE OF HYPERONS FROM COUPLED-CHANNEL MODEL

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University of Wroclaw

EMMI WORKSHOP: BOUND STATES AND  
PARTICLE INTERACTIONS IN THE 21ST CENTURY  
03-06 JULY 2023

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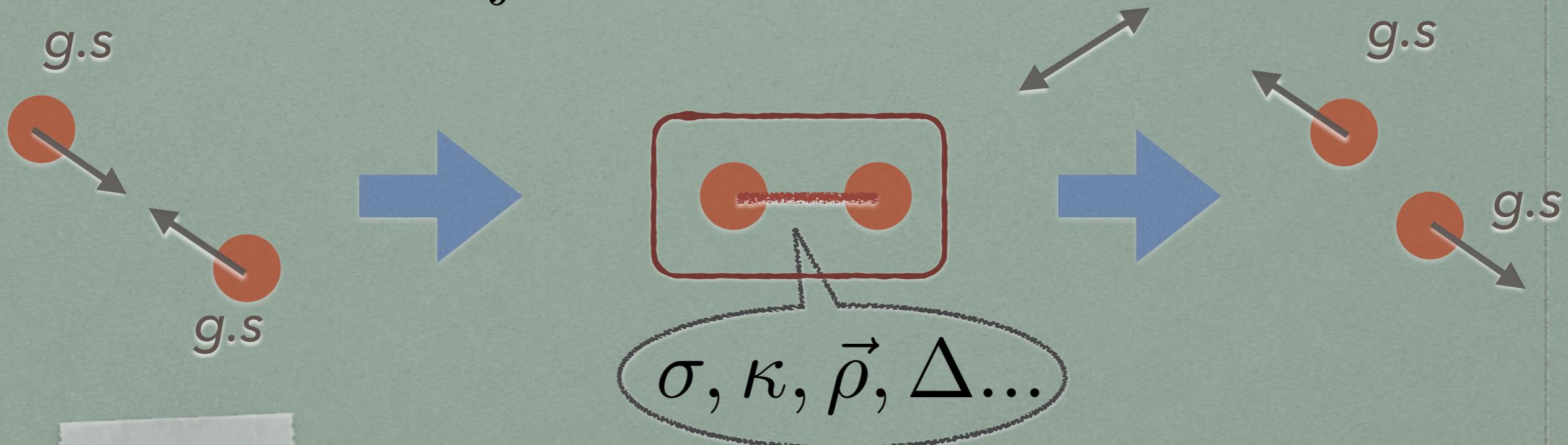
# S-MATRIX FORMULATION OF STAT. MECH.

R. Dashen, S. K. Ma and H. J. Bernstein,  
Phys. Rev. 187 (1969) 345.

R. Venugopalan and M. Prakash,  
Nucl. Phys. **A546**, 718 (1992).

# S-MATRIX FORMULATION OF STATISTICAL MECHANICS

$$\Delta \ln Z = \int dE e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \text{tr} (\delta_E) .$$



PWA

X

S-matrix thermo.

+ repulsions

$$\delta \longrightarrow Q(M) \equiv \frac{1}{2} \text{Im} (\text{tr} \ln S)$$

# HOW TO RELATE PHASE SHIFTS TO THERMODYNAMICS?

***thermo-statistical***

***dynamical***

$$\Delta \ln Z = \int dE e^{-\beta E} \frac{1}{4\pi i} \text{tr} \left\{ S_E^{-1} \frac{\partial}{\partial E} S_E \right\}_c$$

*single channel, elastic*

$$\frac{1}{\pi} \frac{d}{dE} \delta$$

*N-body &  
Coupled-Channel problem*  
*multi (coupled) channel*

$$\frac{1}{\pi} \frac{d}{dE} \mathcal{Q}$$

$$\begin{aligned} \mathcal{Q} &= \frac{1}{2} \text{ImTr} \ln S \\ &= \sum_{\text{channels}} \lambda_i \end{aligned}$$

# **S = -1 HYPERONS COUPLED CHANNEL SYSTEM**

JPAC, PRD **93**, 034029 (2016)

C. Fernandez-Ramirez, PML, and P. Petreczky,  
PRC **98**, 044910 (2018)

J. Cleymans, PML, K. Redlich, and N. Sharma  
PRC **103**, 014904 (2021)

# PHASE SHIFT FROM PWA

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Coupled Channels partial wave calculator for KN scattering  
by the Joint Physics Analysis Center (JPAC)  
Version: September 1, 2015

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Igor V. Danilkin (Jefferson Lab)  
Vincent Mathieu (Indiana University)  
Adam P. Szczepaniak (Indiana University and Jefferson Lab)

**Citation:** Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

**First version:** Cesar Fernandez-Ramirez (Jefferson Lab)  
**This version:** Cesar Fernandez-Ramirez (Jefferson Lab)

**Contact:** [cefera@gmail.com](mailto:cefera@gmail.com) (Cesar Fernandez-Ramirez)

**Disclaimers:**

- 1 - This code follows the 'garbage in, garbage out' philosophy. If your parameters do not make sense, the output will not make sense either.
  - 2 - You can use, share and modify this code under your own responsibility.
  - 3 - This code is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
  - 4 - No PhD students or postdocs were severely damaged during the development of this project.
-

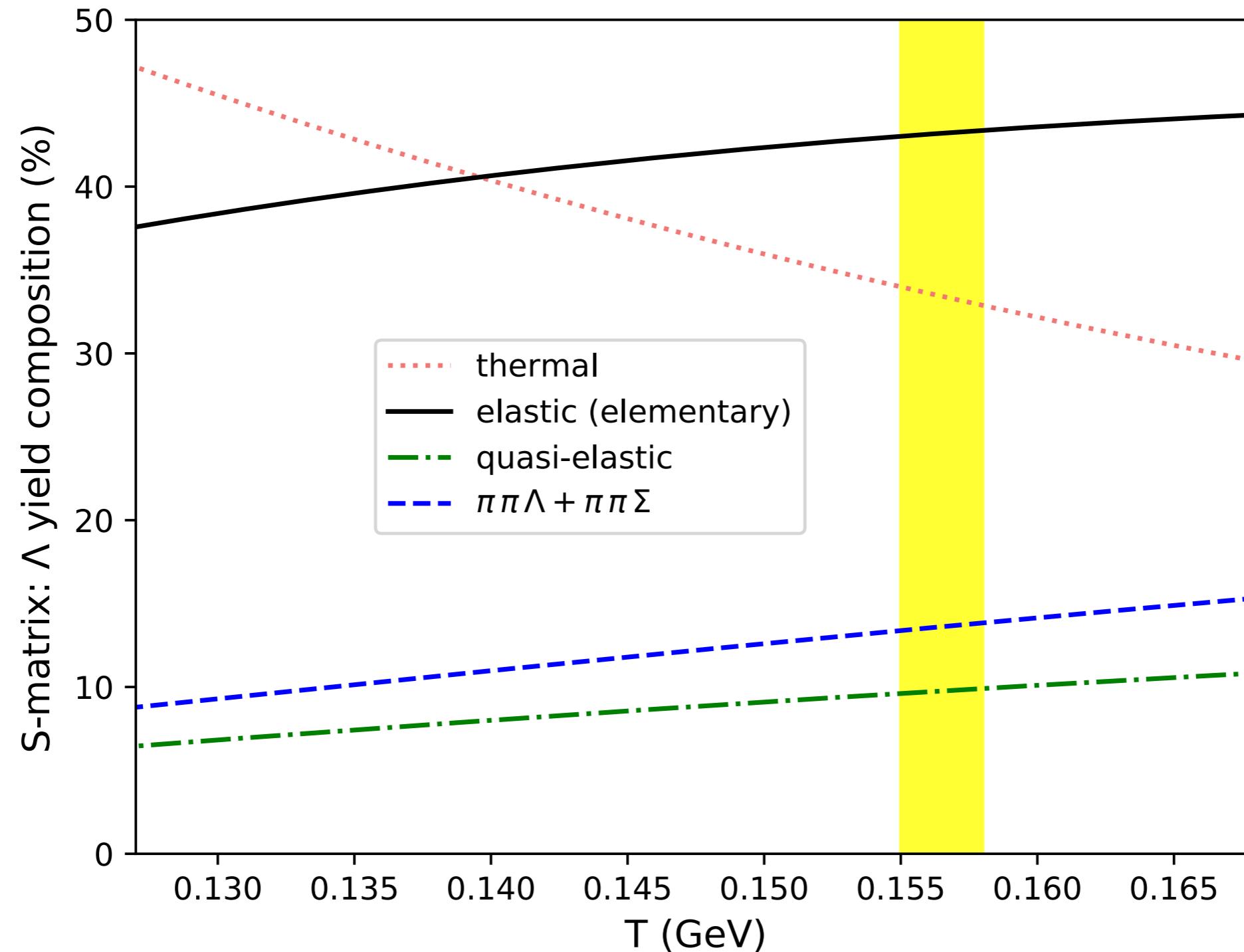
channel	elastic	channel	quasi-elastic	channel	unitarity
1	$\bar{K}N$	6	$\bar{K}_1^* N$	15	$\pi\pi\Lambda$
2	$\pi\Sigma$	7	$[\bar{K}_3^* N]_-$	16	$\pi\pi\Sigma$
3	$\pi\Lambda$	8	$[\bar{K}_3^* N]_+$		
4	$\eta\Lambda$	9	$[\pi\Sigma(1385)]_-$		
5	$\eta\Sigma$	10	$[\pi\Sigma(1385)]_+$		
		11	$[\bar{K}\Delta(1232)]_-$		
		12	$[\bar{K}\Delta(1232)]_+$		
		13	$[\pi\Lambda(1520)]_-$		
		14	$[\pi\Lambda(1520)]_+$		

elastic scatterings (elementary)

quasi elastic scatterings

unitarity background

channel	elastic	channel	quasi-elastic	channel	unitarity
1	$\bar{K}N$	6	$\bar{K}_1^* N$	-	15



# STRANGENESS CONTENT IN A HADRON GAS

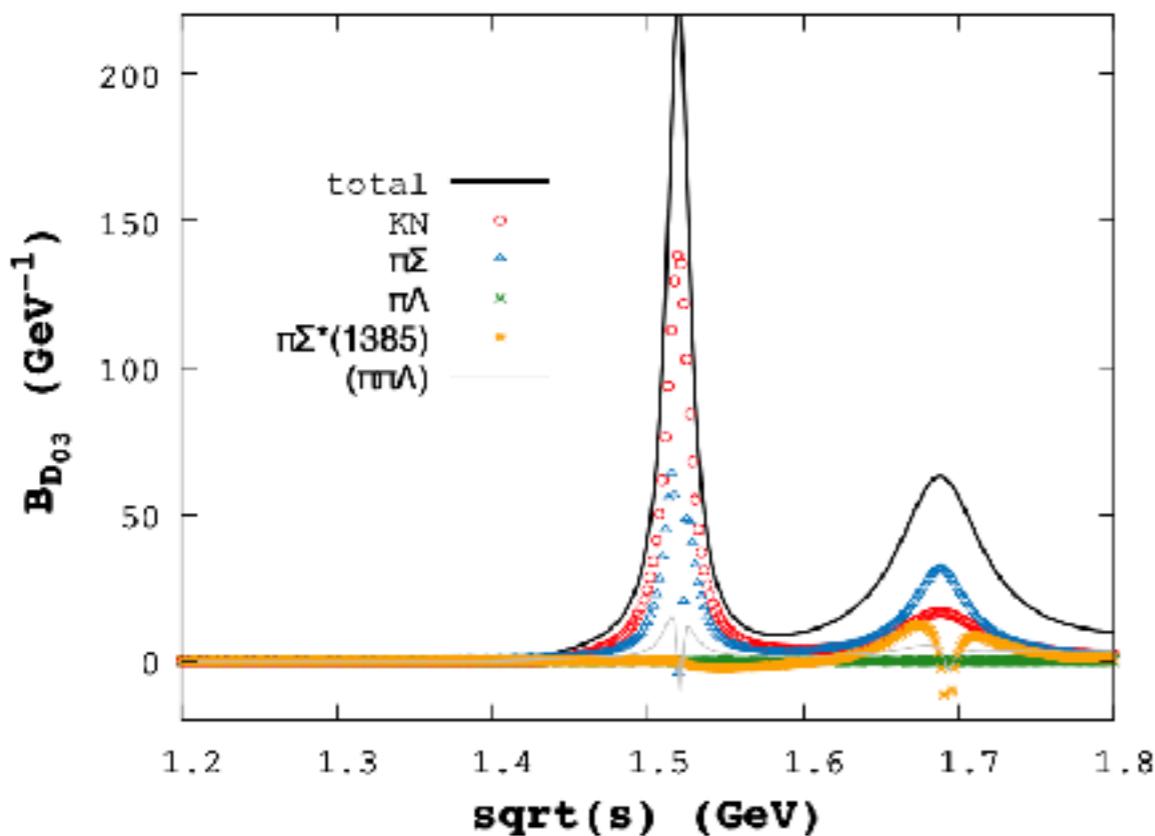
- K-N system requires a coupled channel analysis

$|\bar{K}N\rangle, |\pi\Sigma\rangle, |\pi\Lambda\rangle, |\eta\Lambda\rangle, \dots$  *16 basis states*

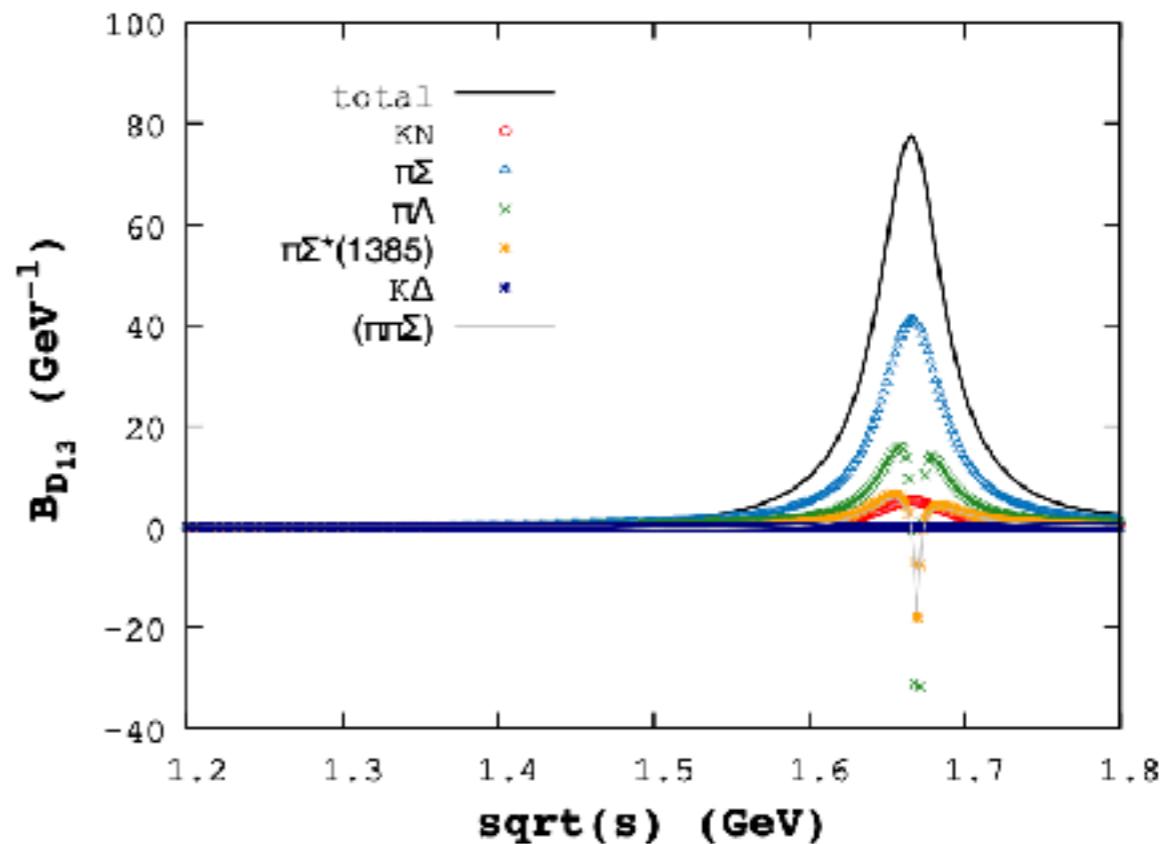
$$\begin{aligned} Q(M) &\equiv \frac{1}{2} \text{Im} (\text{tr} \ln S) \\ &= \frac{1}{2} \text{Im} (\ln \det [S]) \\ &= \delta_{\bar{K}N} + \delta_{\pi\Sigma} + \delta_{\pi\Lambda} + \dots \end{aligned}$$

Compute  $\det S$  for each  
 $\sqrt{s}$  for each channel  
isospin conserving

# 1520, 1690



# 1670



## $\Lambda(1520) 3/2^-$

$$I(J^P) = 0(\frac{3}{2}^-)$$

Mass  $m = 1519.5 \pm 1.0$  MeV [d]

Full width  $\Gamma = 15.6 \pm 1.0$  MeV [d]

$p_{beam} = 0.39$  GeV/c       $4\pi\chi^2 = 82.8$  mb

$\Lambda(1520)$ DECAY MODES	Fraction ( $\Gamma_i/\Gamma$ )	$p$ (MeV/c)
$N\bar{K}$	$45 \pm 1\%$	243
$\Sigma\pi$	$42 \pm 1\%$	268
$\Lambda\pi\pi$	$10 \pm 1\%$	259
$\Sigma\pi\pi$	$0.9 \pm 0.1\%$	169
$\Lambda\gamma$	$0.85 \pm 0.15\%$	350

## $\Sigma(1670) 3/2^-$

$$I(J^P) = 1(\frac{3}{2}^-)$$

Mass  $m = 1665$  to  $1685$  ( $\approx 1670$ ) MeV

Full width  $\Gamma = 40$  to  $80$  ( $\approx 60$ ) MeV

$p_{beam} = 0.74$  GeV/c       $4\pi\chi^2 = 28.5$  mb

$\Sigma(1670)$ DECAY MODES	Fraction ( $\Gamma_i/\Gamma$ )	$p$ (MeV/c)
$N\bar{K}$	7–13 %	414
$\Lambda\pi$	5–15 %	448
$\Sigma\pi$	30–60 %	394

## $\Lambda(1690) 3/2^-$

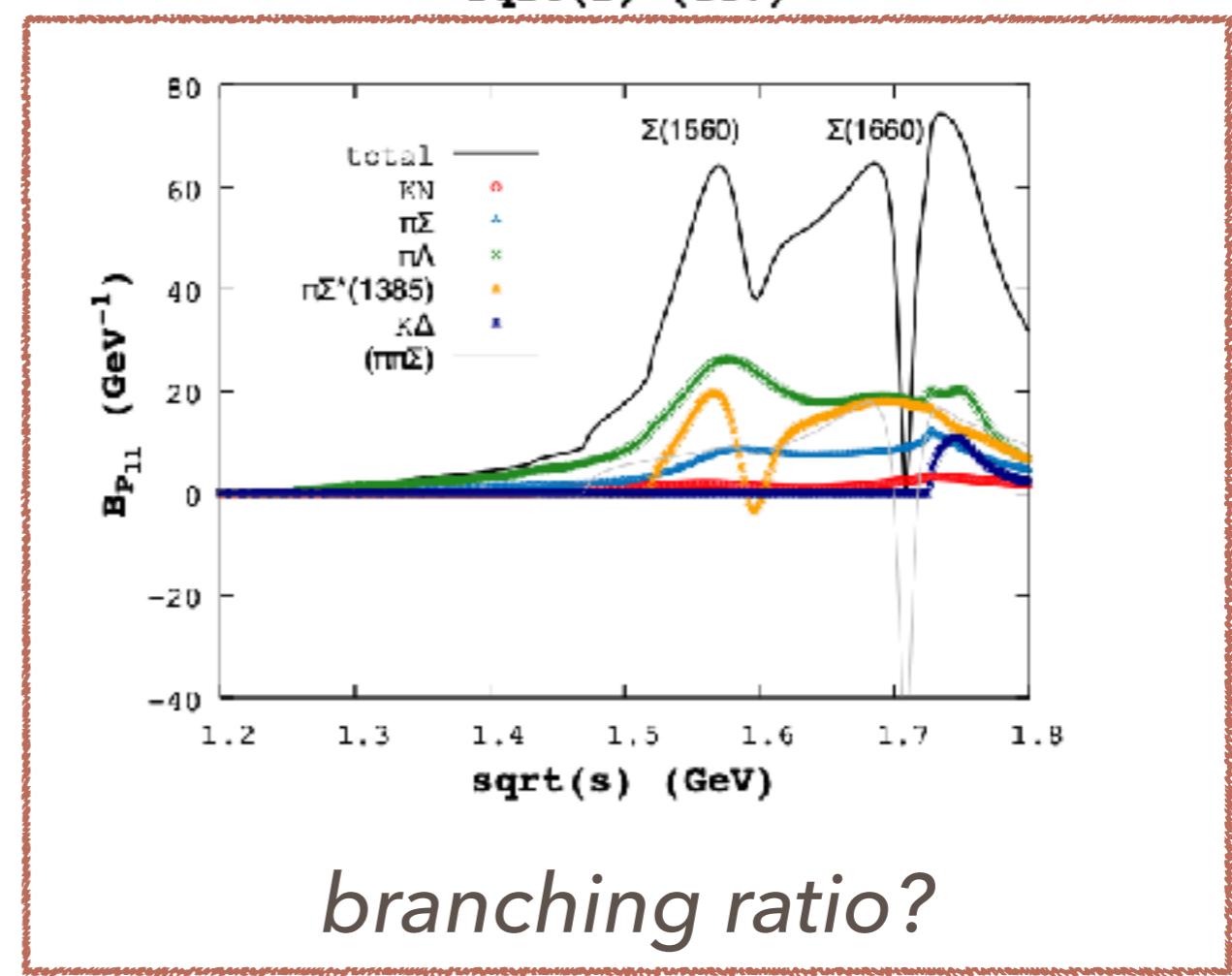
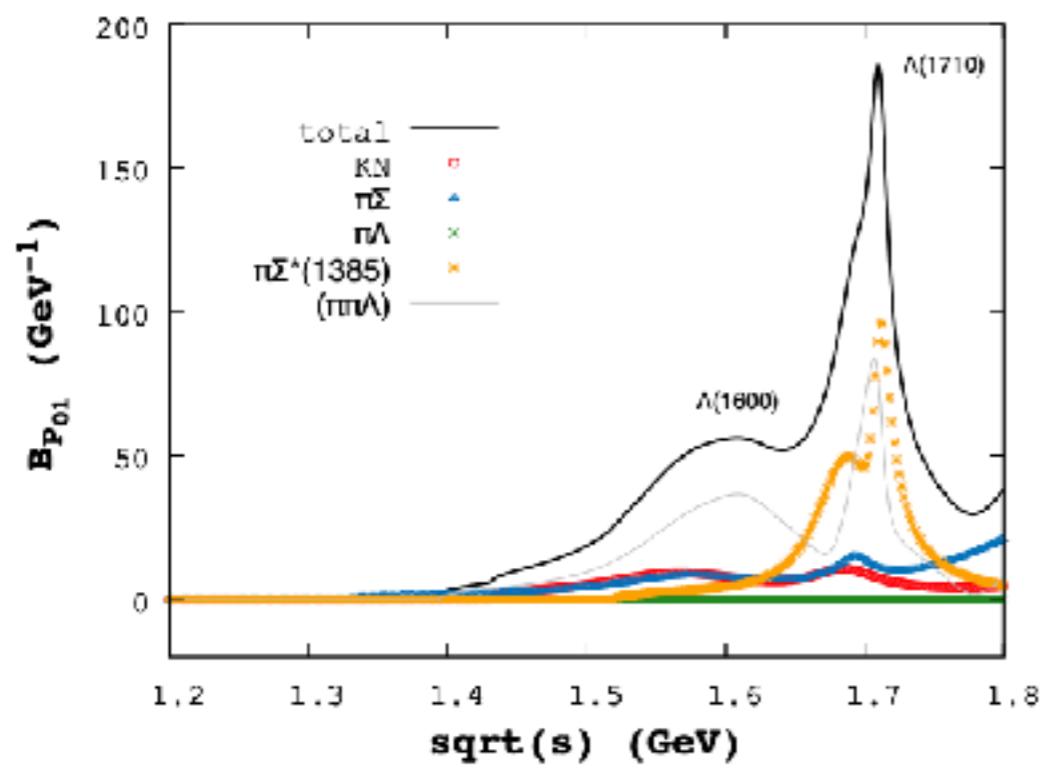
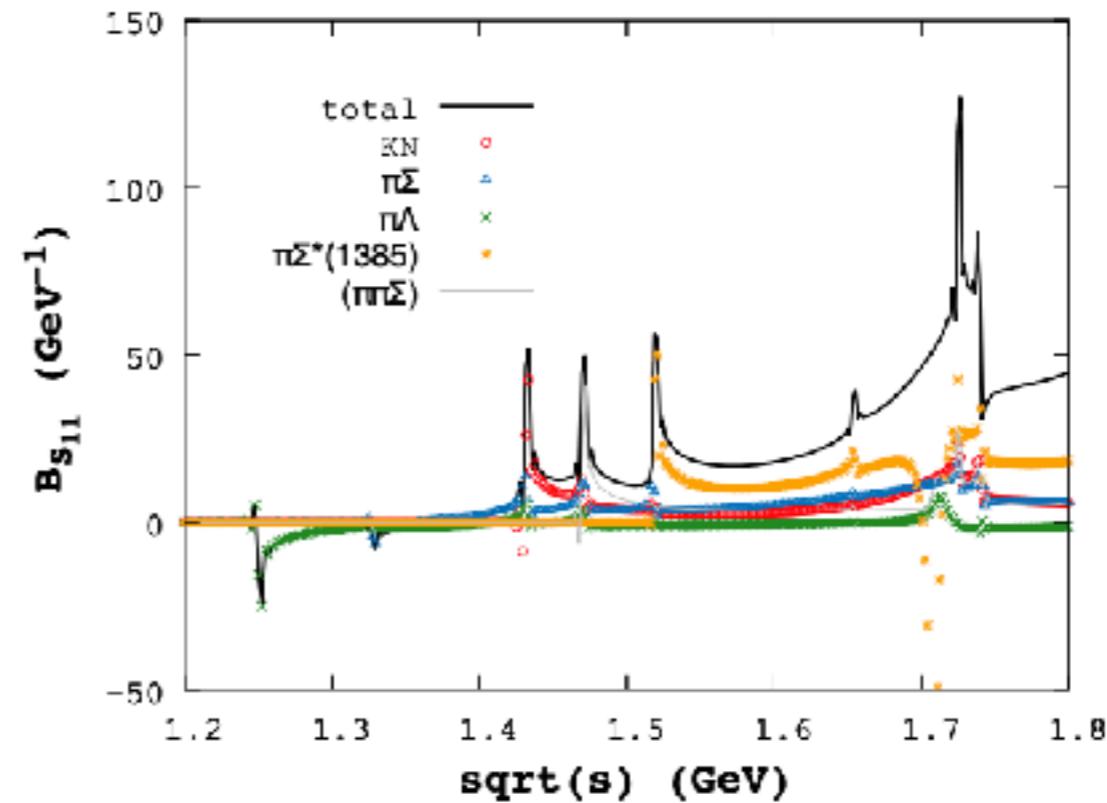
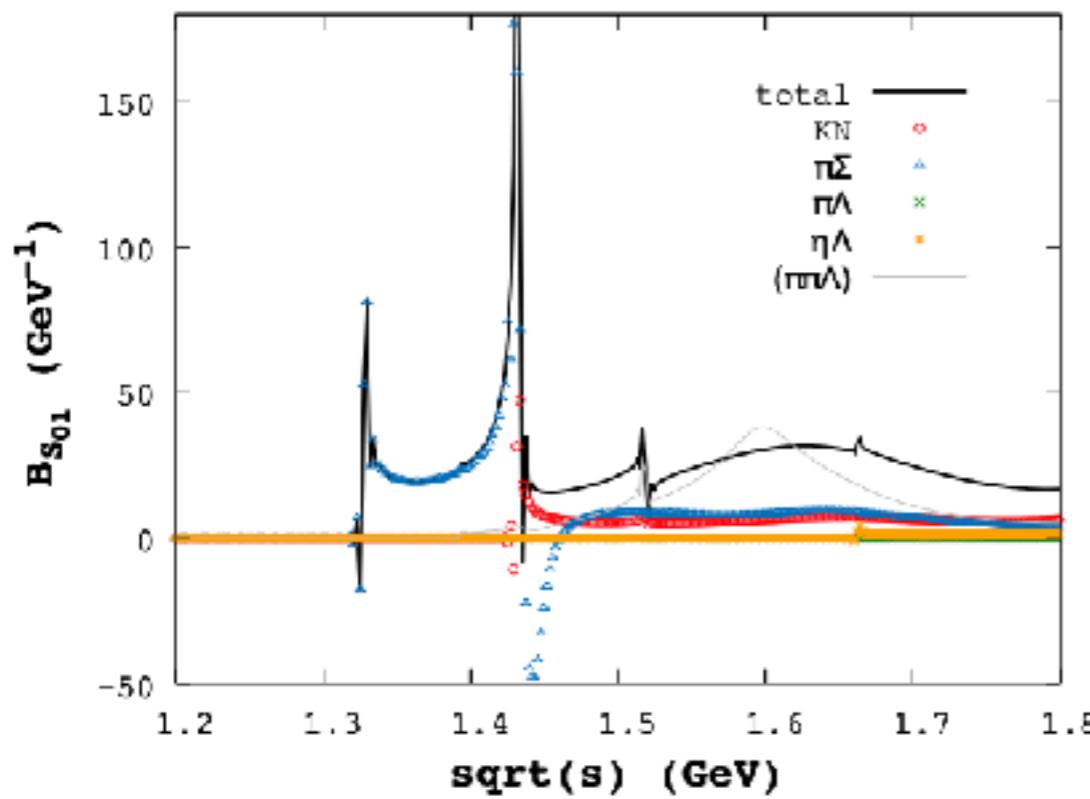
$$I(J^P) = 0(\frac{3}{2}^-)$$

Mass  $m = 1685$  to  $1695$  ( $\approx 1690$ ) MeV

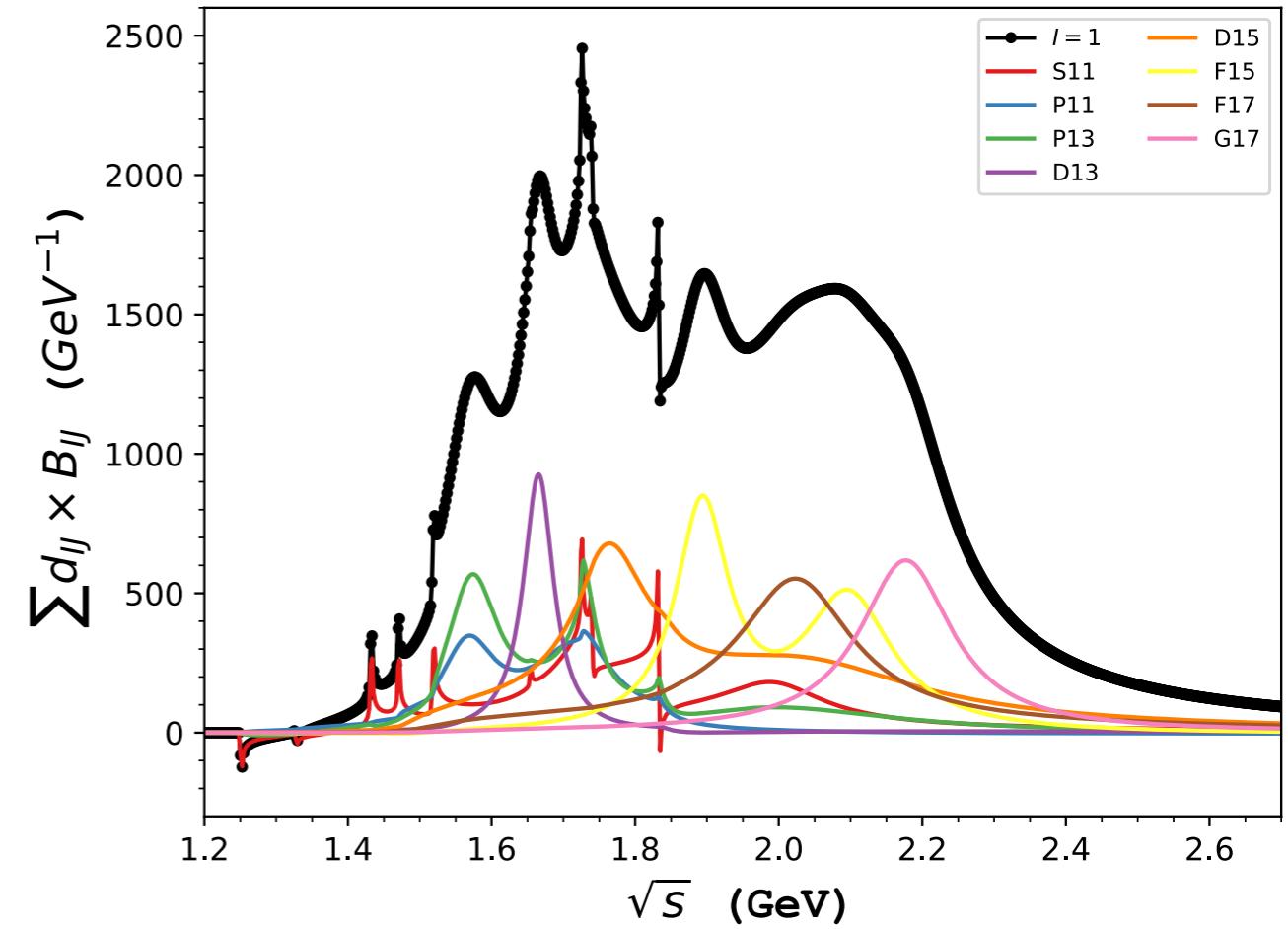
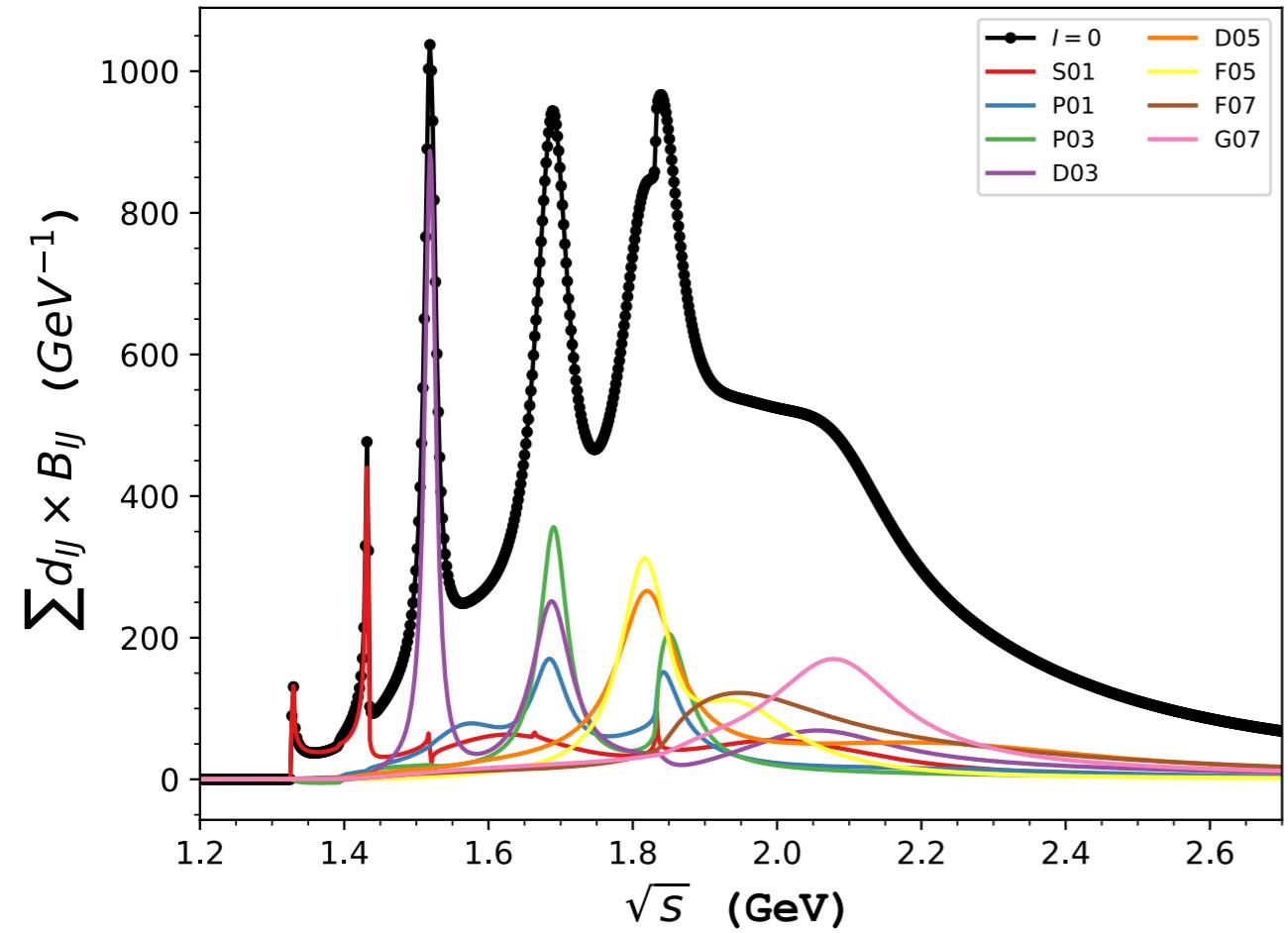
Full width  $\Gamma = 50$  to  $70$  ( $\approx 60$ ) MeV

$p_{beam} = 0.78$  GeV/c       $4\pi\chi^2 = 26.1$  mb

$\Lambda(1690)$ DECAY MODES	Fraction ( $\Gamma_i/\Gamma$ )	$p$ (MeV/c)
$N\bar{K}$	20–30 %	439
$\Sigma\pi$	20–40 %	410
$\Lambda\pi\pi$	$\sim 25$ %	419
$\Sigma\pi\pi$	$\sim 20$ %	358

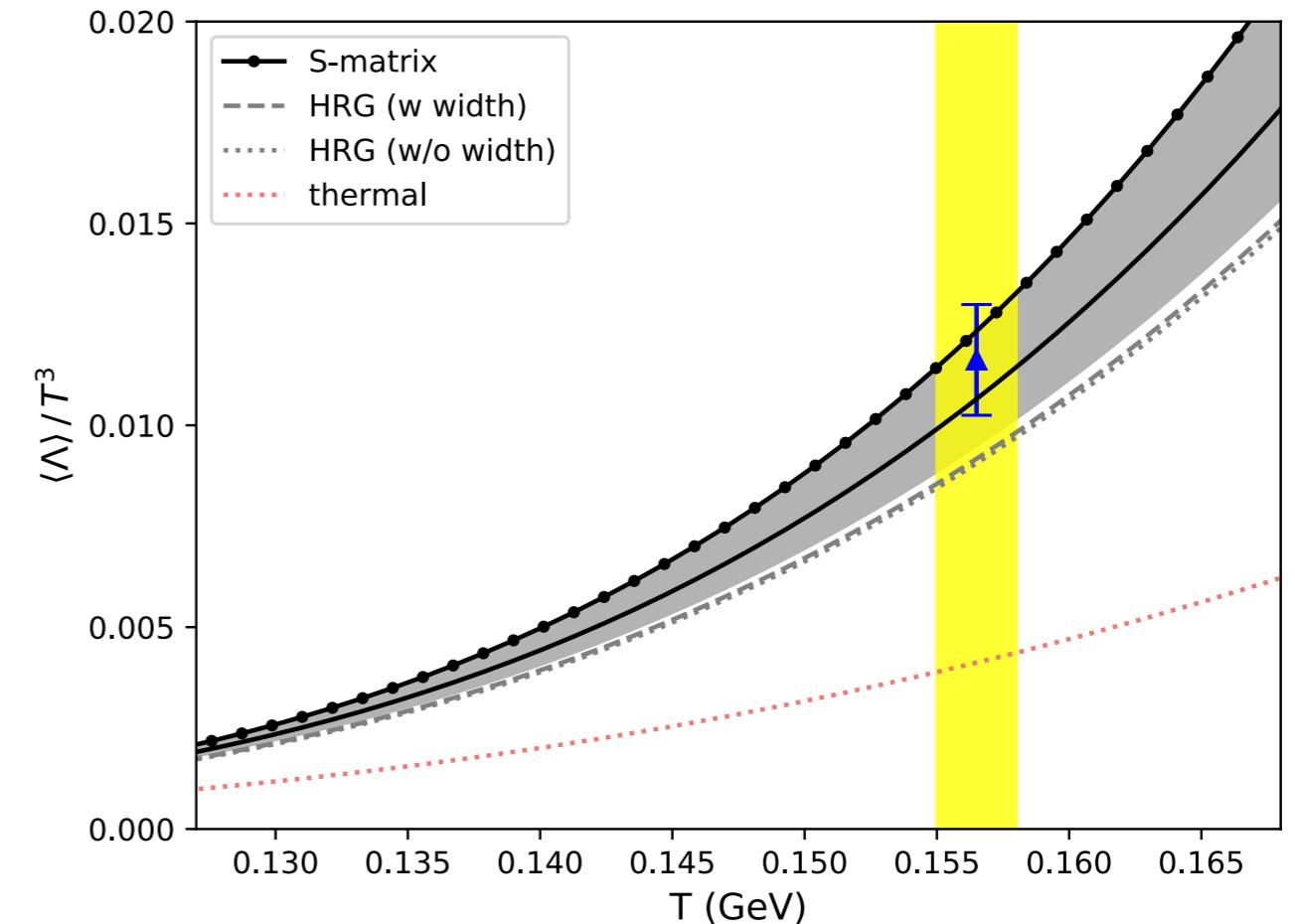
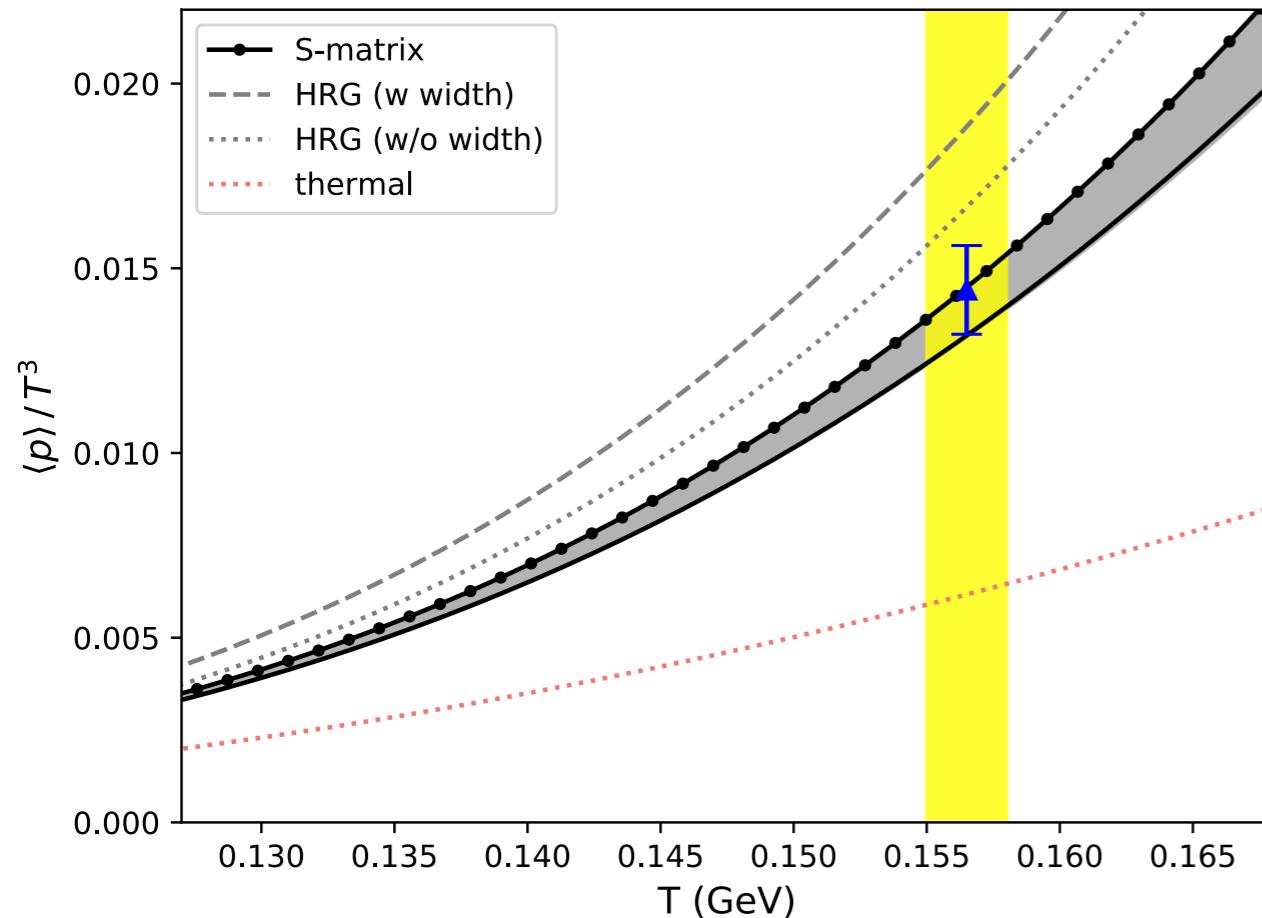


*branching ratio?*



# **FINAL RESULTS**

# *S-matrix VS HRG*

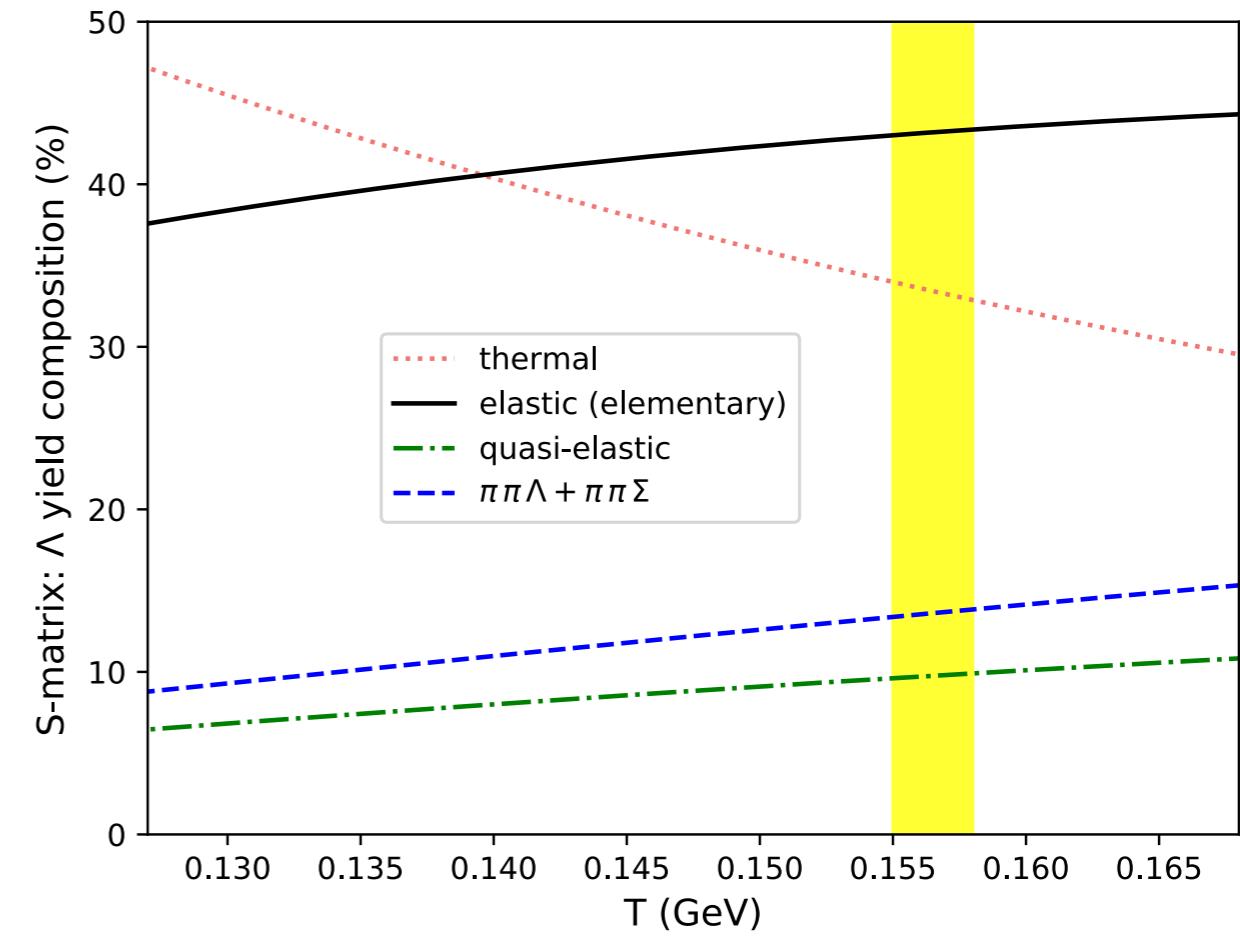
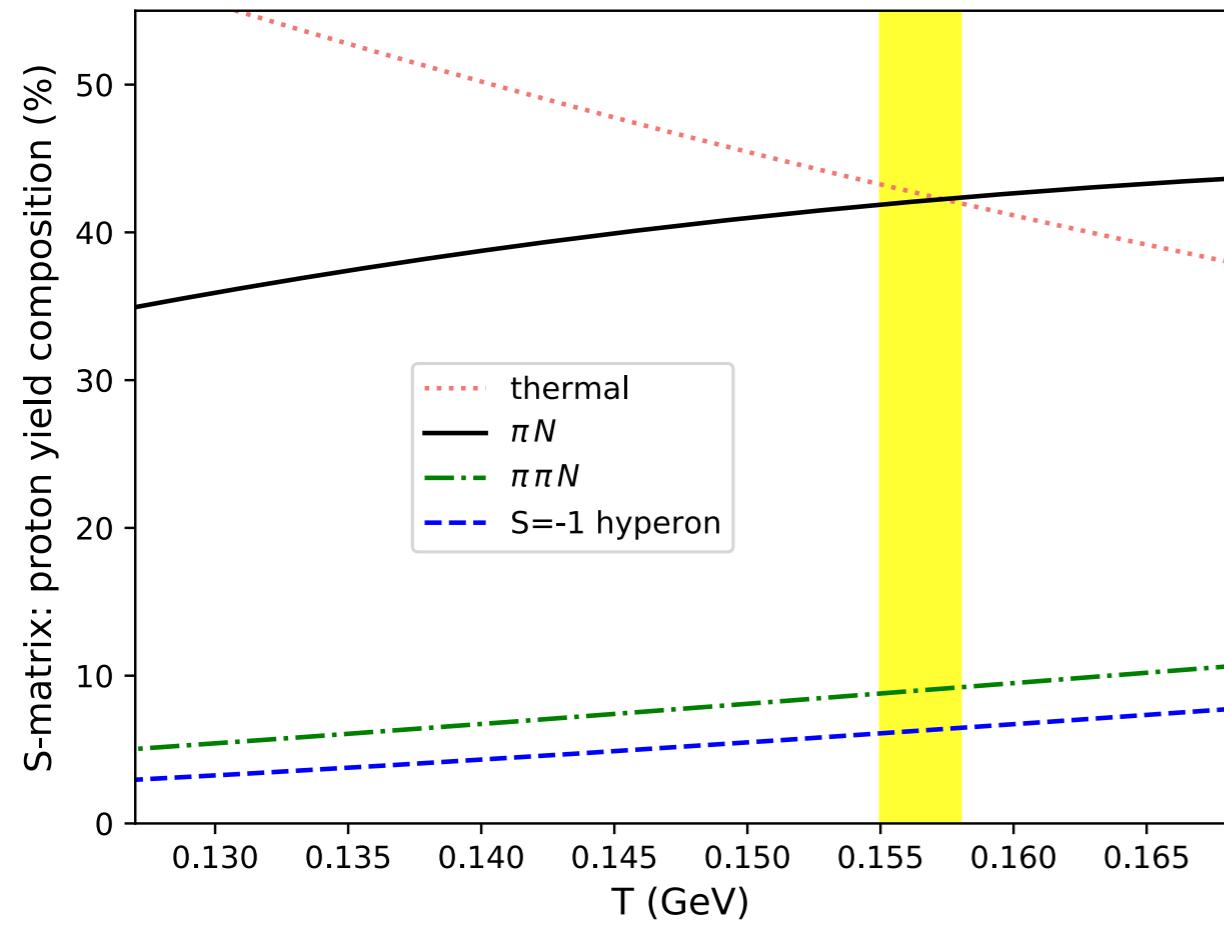


piN phase shifts  
pipiN BGs  
hyperons

*consistent treatment of res and non-res. int.*



*Coupled-Channel model:*  
 $\bar{k}N, \pi\Lambda, \pi\Sigma, \dots$   
*extra hyperon states*  
*beyond PDG*  
*unitarity BGs*



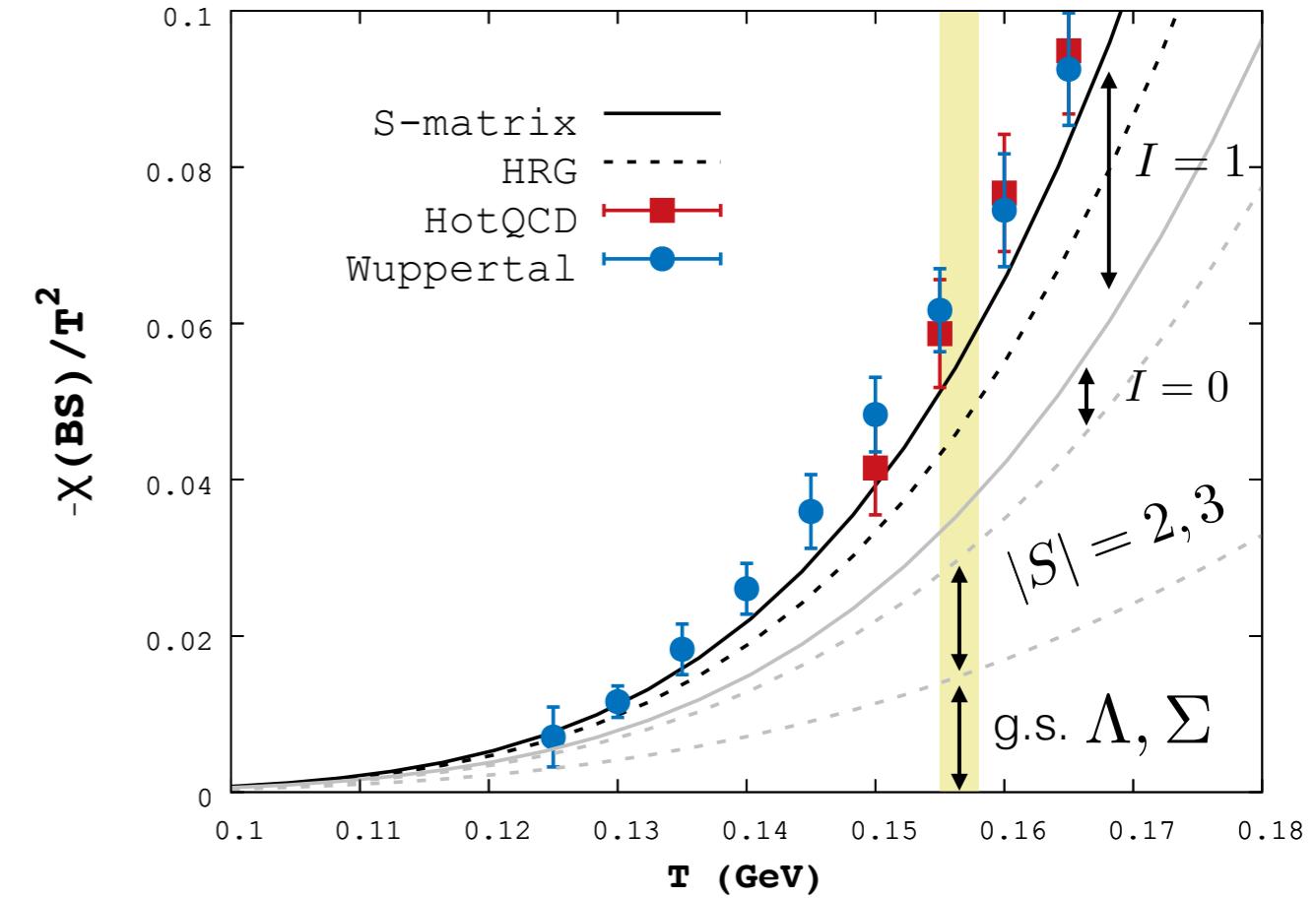
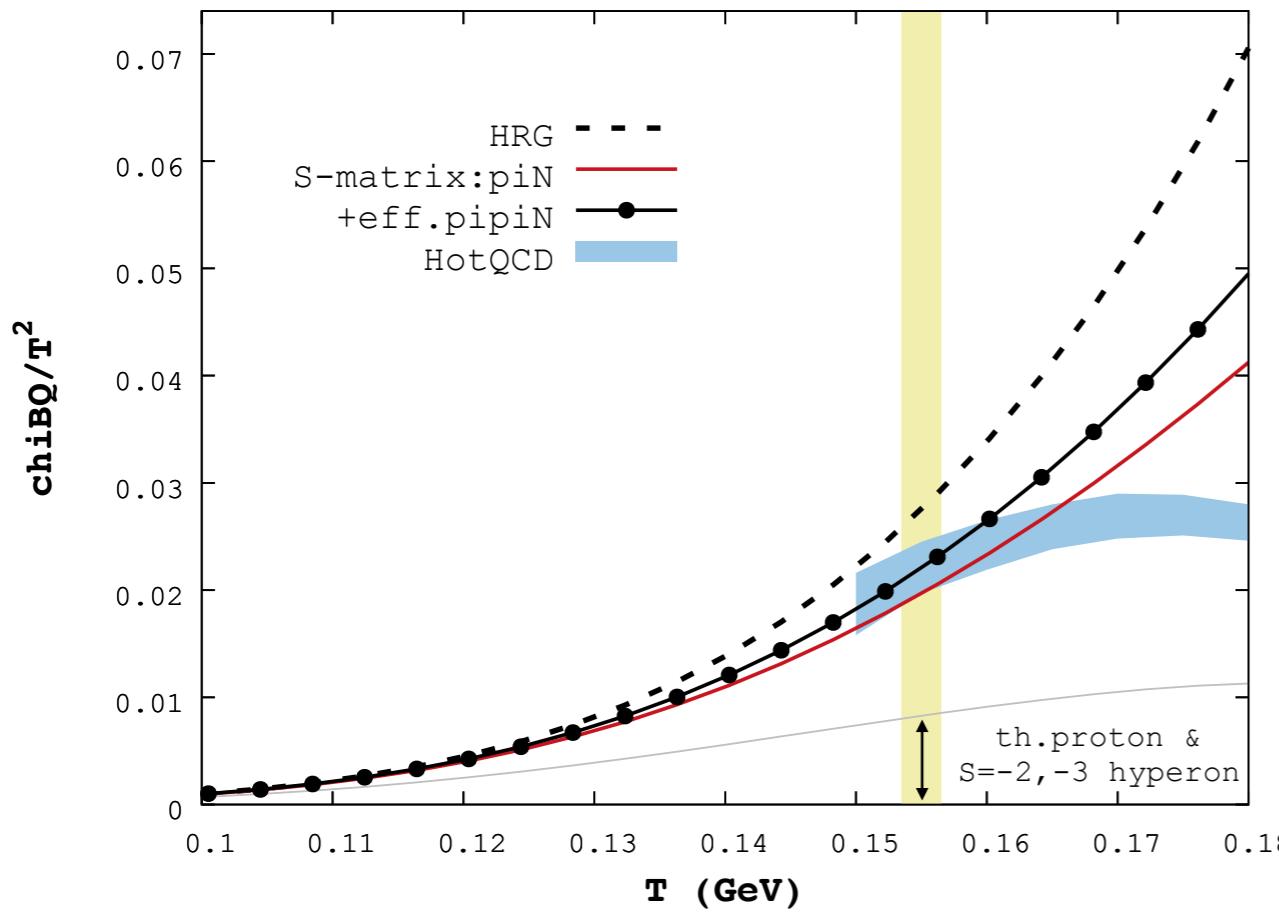
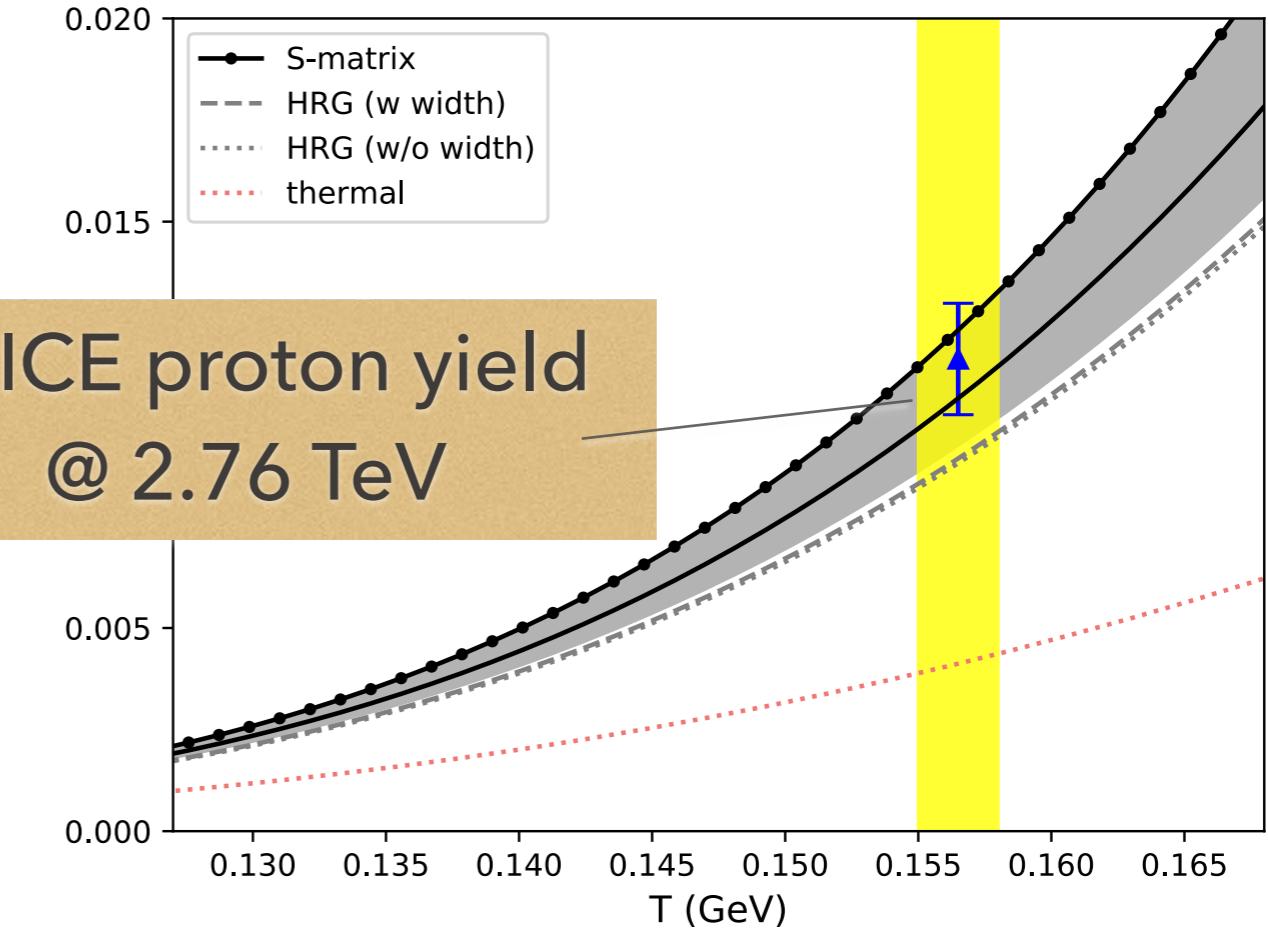
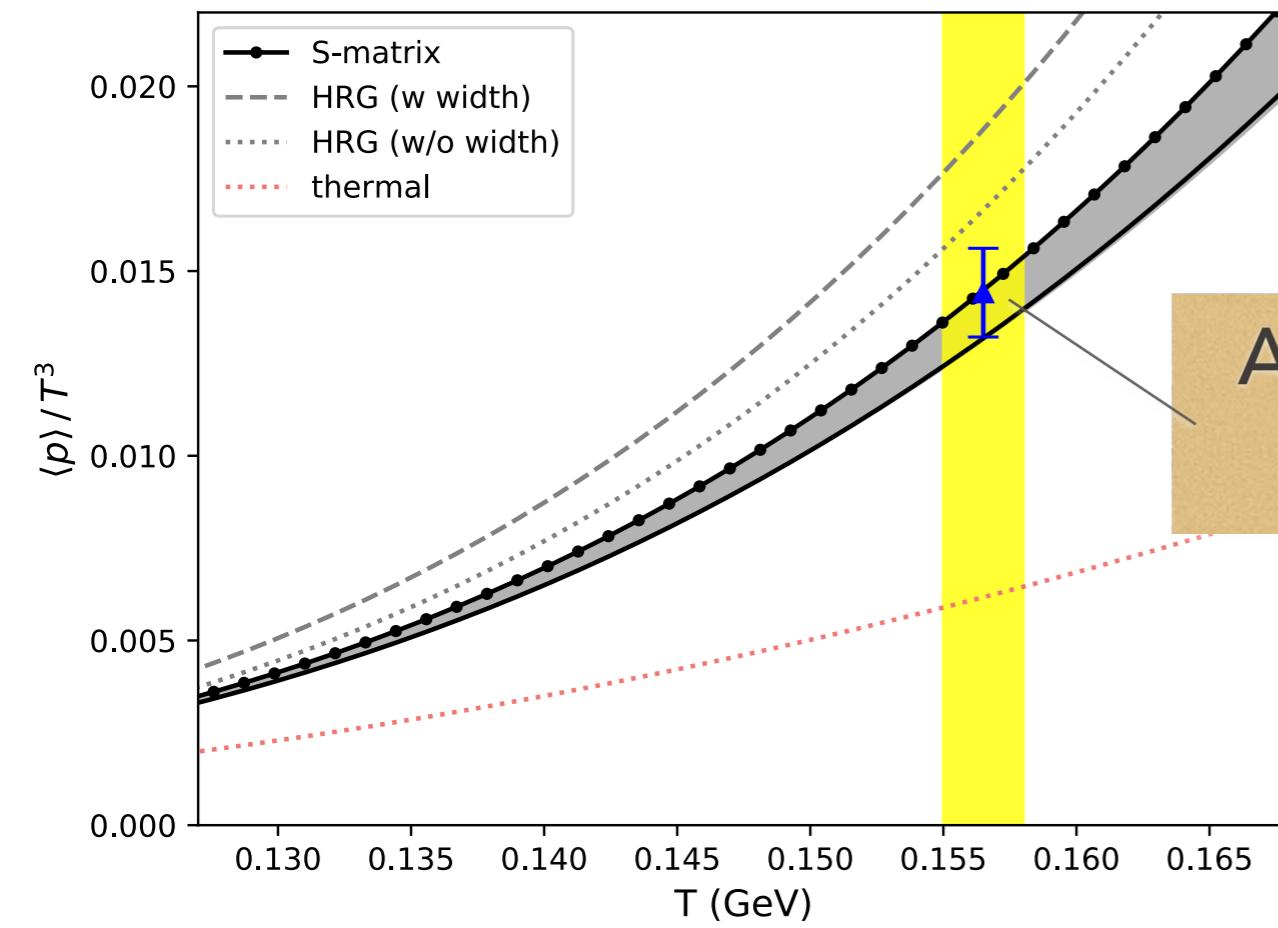
SAID GWU

$p\bar{N}$  phase shifts  
 $\pi\pi\bar{N}$  BGs  
hyperons

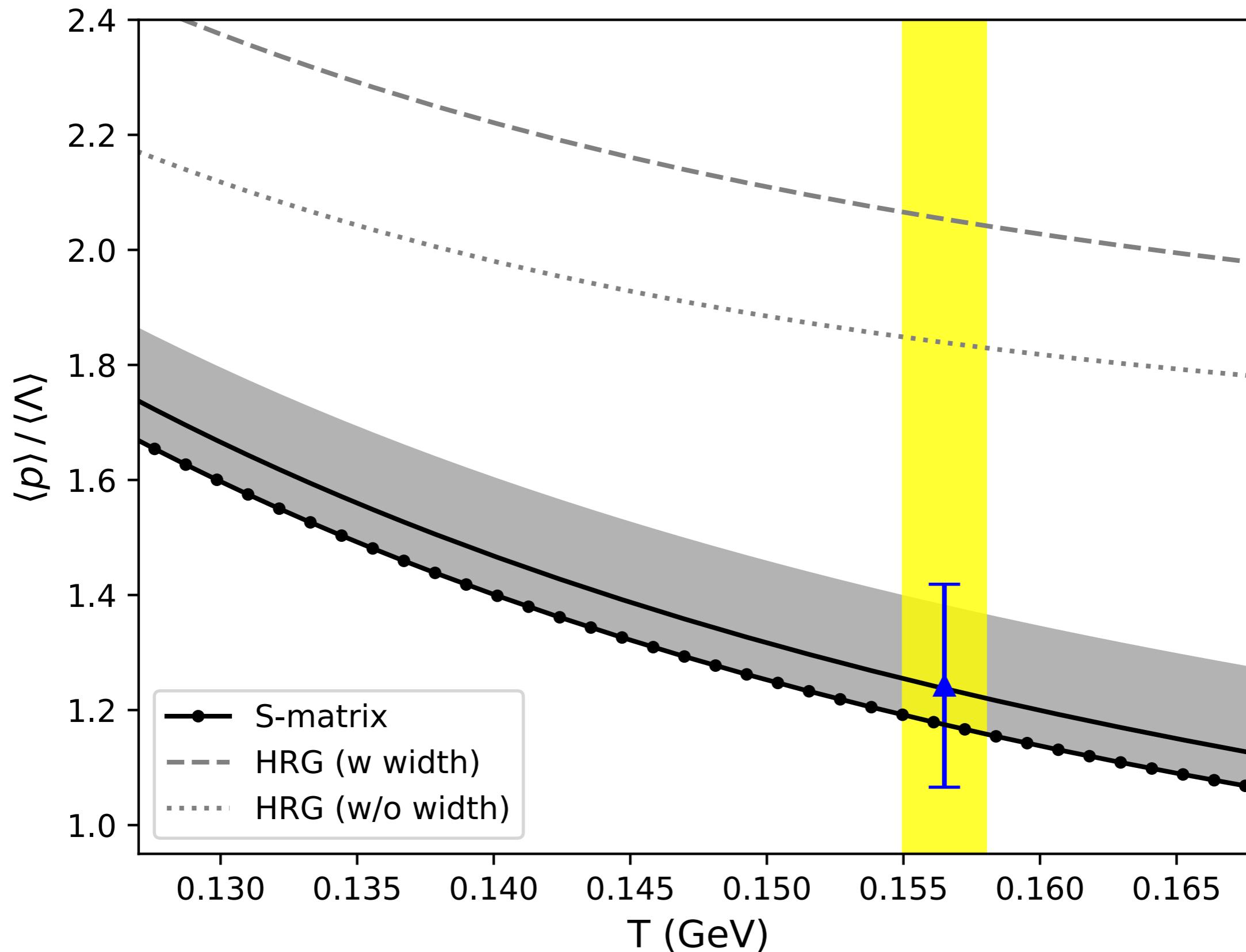
JPAC

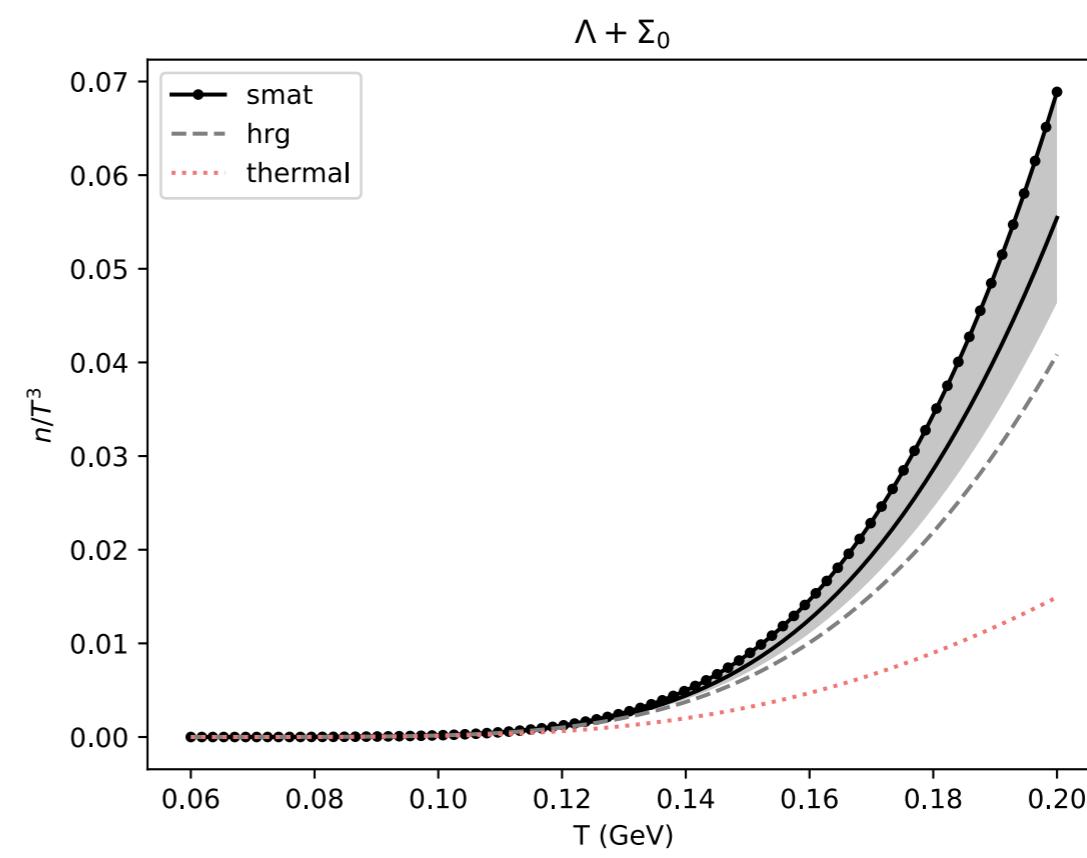
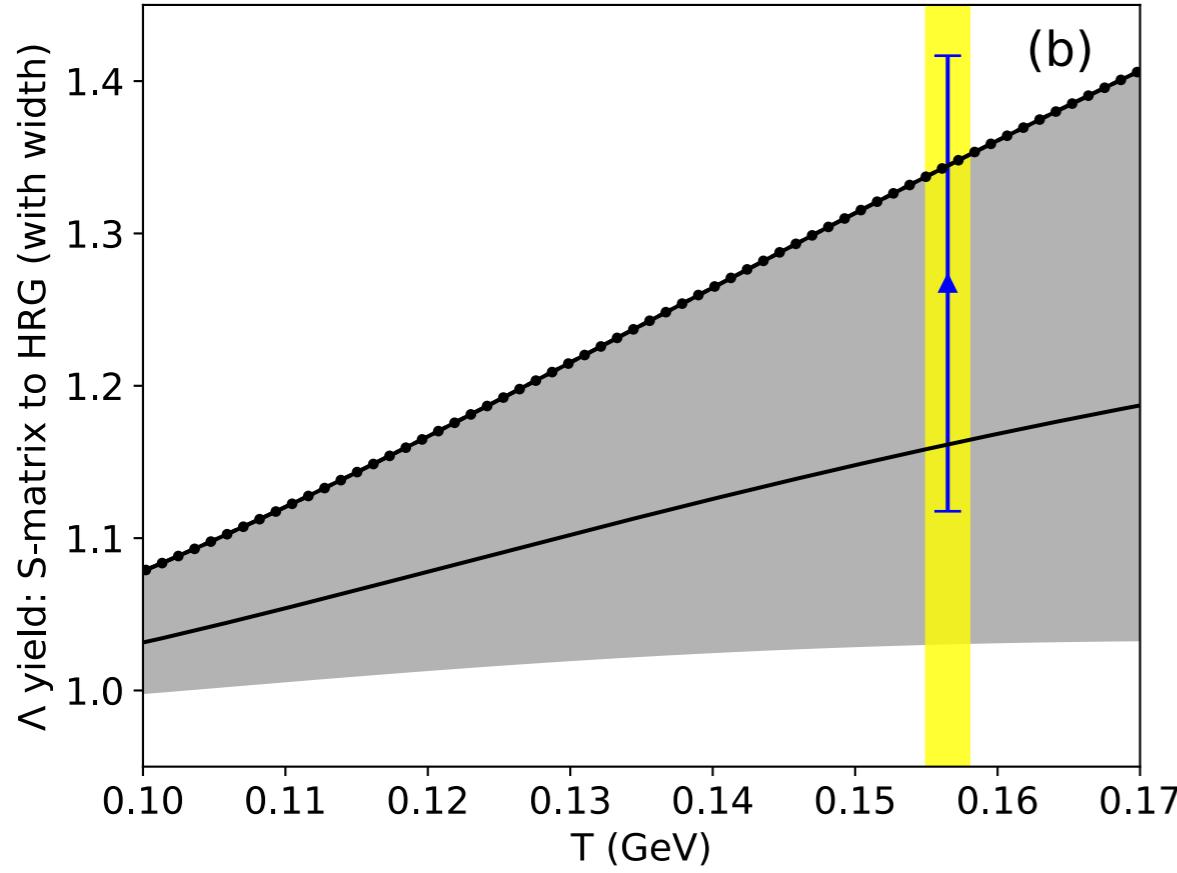
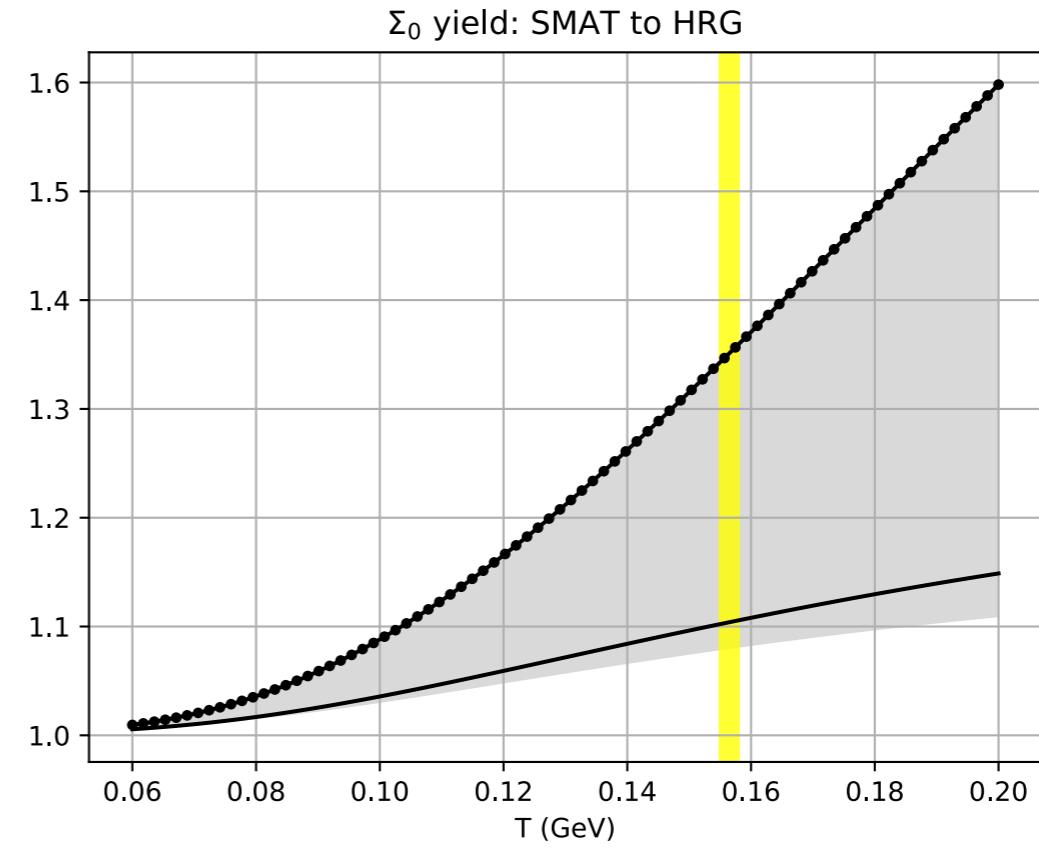
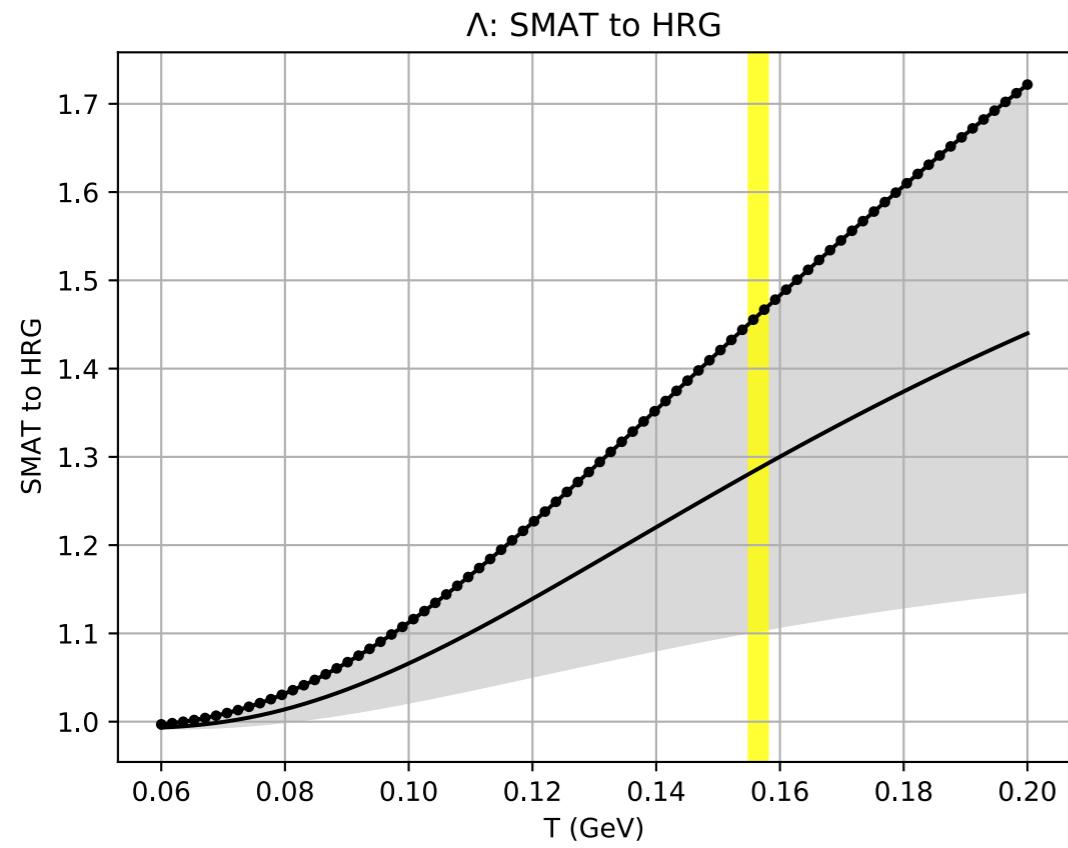
*Coupled-Channel system:  
 $\bar{k}N, \pi\Lambda, \pi\Sigma, \dots$*   
*extra hyperon states*  
*beyond PDG*  
*unitarity BGs*

*consistent treatment of res and non-res. int.*



*ratio of yields*



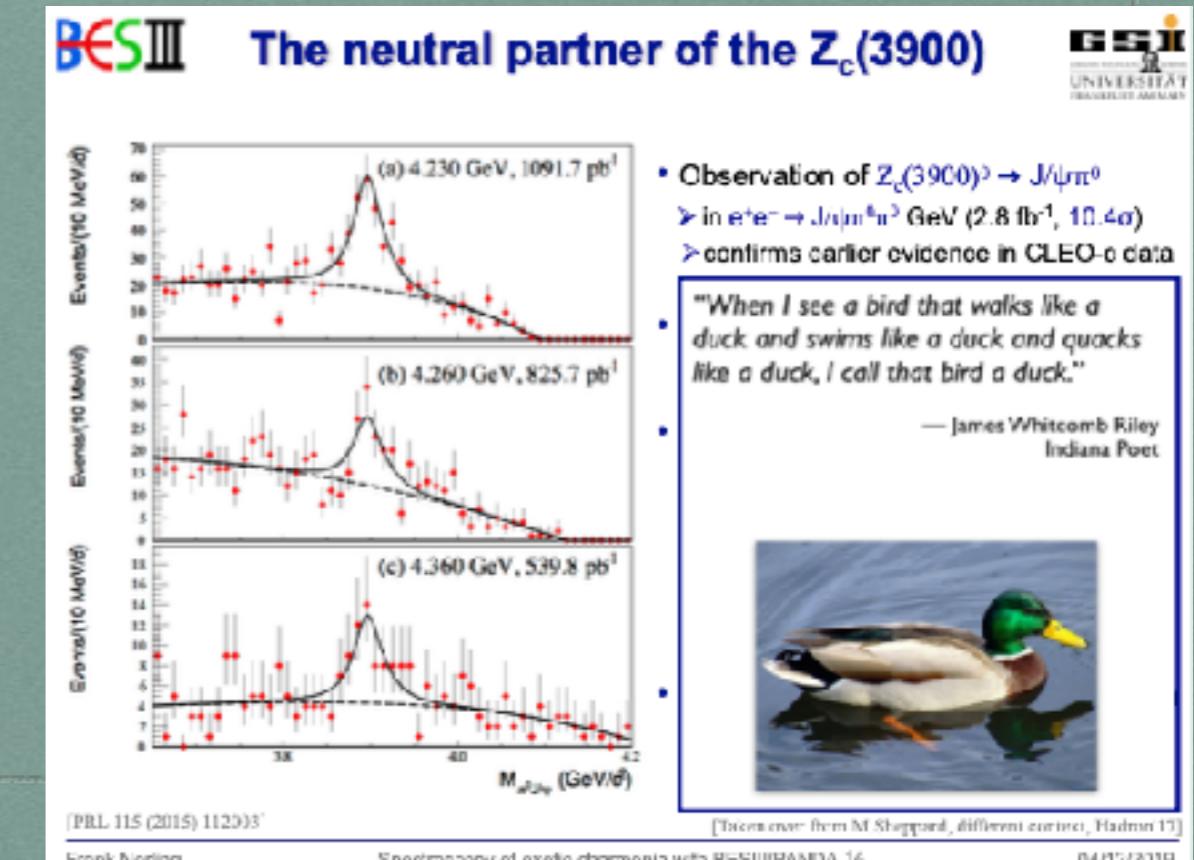
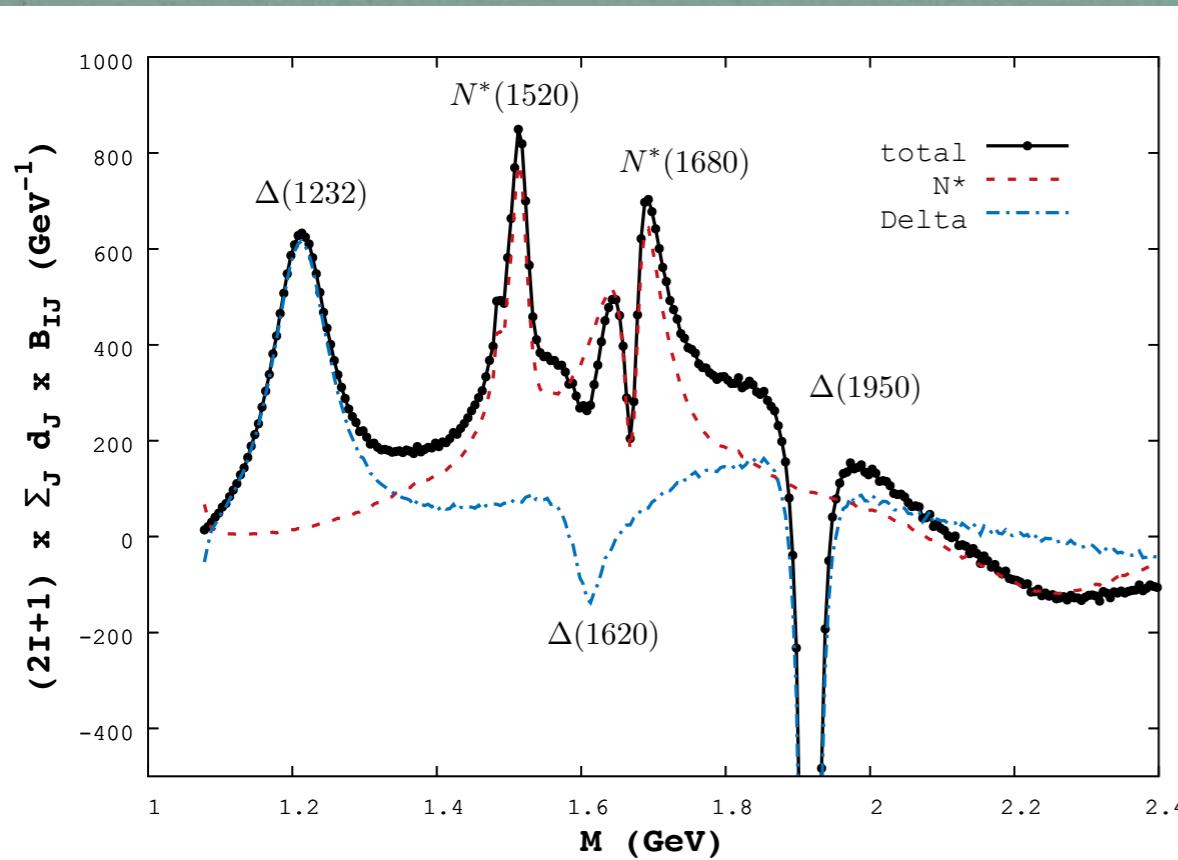


# RESONANCES / EXCITATIONS VIA SCATTERING STATES

- broad /overlapping resonances
- molecular states
- threshold effects /cusps

*non-resonant interactions: +/-*

$$\pi N \rightarrow \Delta \rightarrow \pi N$$



# THEORETICAL ISSUES

$$muB = 0 \quad @ T = 155 \text{ MeV}$$

- LHC conditions = pion rich:  $p = p\bar{p}$ ;  $\langle \pi \rangle / \langle p \rangle \approx 15$   
Need to Take Pions Seriously!  
NN is heavily (Boltzmann) suppressed compared to  
 $\pi N$
- How to include a resonance?
- Why it is NOT a Breit-Wigner?
- In-medium Effects from S-matrix

# S-MATRIX FORMULATION OF THERMODYNAMICS

*thermo-statistical*

*dynamical*

$$\Delta \ln Z = \int dE e^{-\beta E} \frac{1}{4\pi i} \text{tr} \left\{ S_E^{-1} \frac{\partial}{\partial E} S_E \right\}_c$$



$$b_{\pi\pi}\xi_\pi^2 + b_{\pi K}\xi_\pi\xi_K + b_{\pi N}\xi_\pi\xi_N + b_{\pi\eta}\xi_\pi\xi_\eta + b_{K\bar{K}}\xi_K\xi_{\bar{K}} + \dots$$

$$b_{\pi N} = 2 \times b_{\pi N}^{I=1/2} + 4 \times b_{\pi N}^{I=3/2}$$

*orbital L:  
S, P, D, F, etc..*

# S-MATRIX FORMULATION OF THERMODYNAMICS

***thermo-statistical***

$$\Delta \ln Z = \int dE e^{-\beta E} \frac{1}{4} \cdot \text{tr} \left\{ S_E^{-1} \frac{\partial}{\partial E} S_E \right\}_c$$

NN effects

$$a_S = 20 \text{ fm}$$

convergence?

$$r \approx 0.0727$$

LHC

$$r \approx 0.36$$

$$T = 60 \text{ MeV}$$

$$r \approx 1.92$$

$$\mu_B = 700, 800 \text{ MeV}$$

***dynamical***

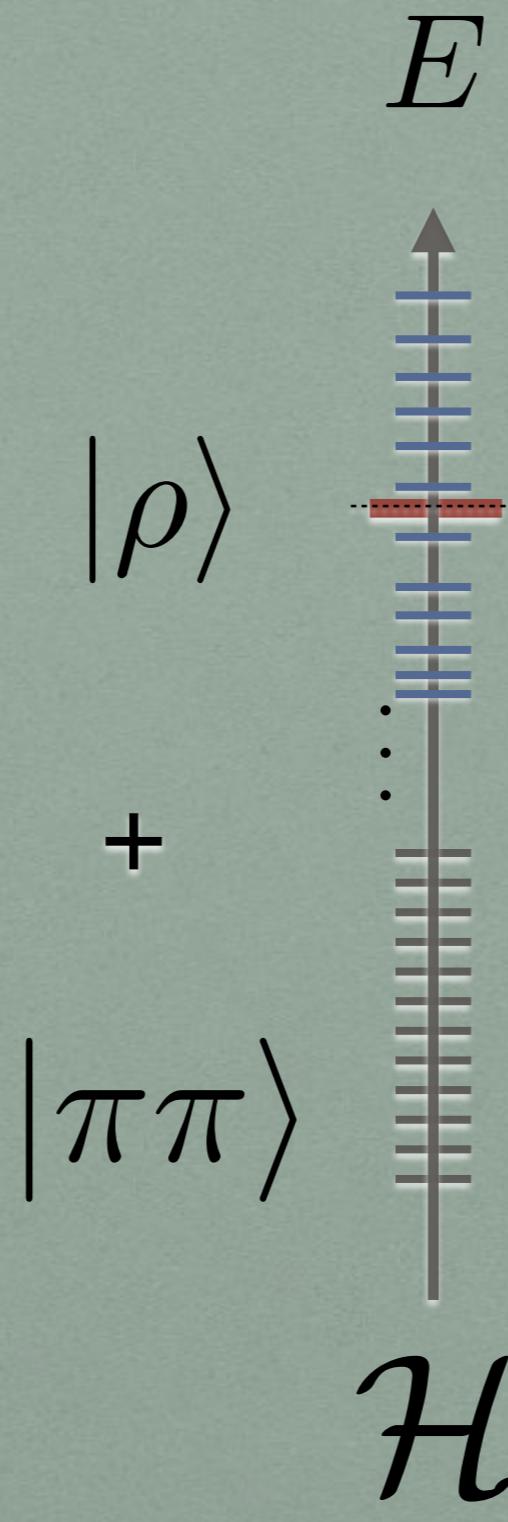
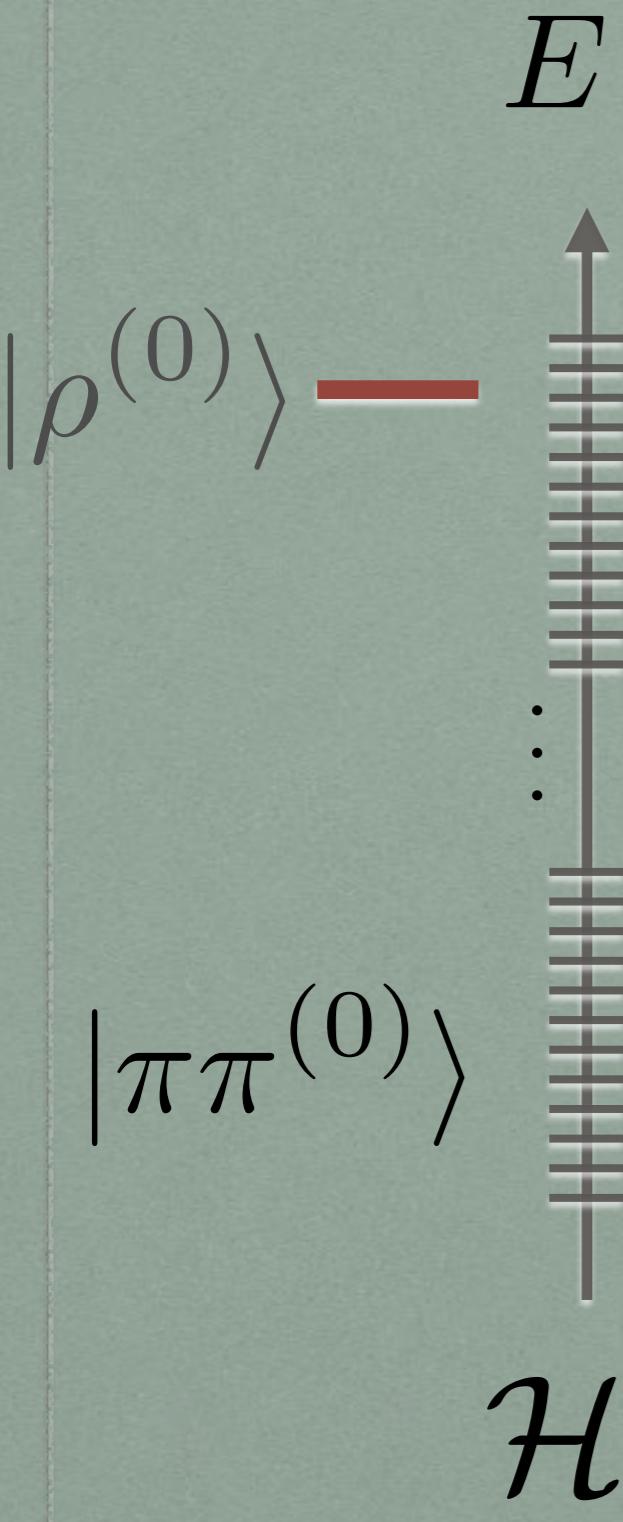
$$\pi \xi_\eta + b_{K\bar{K}} \xi_K \xi_{\bar{K}} + \dots$$

$\times b_{\pi N}^{I=3/2}$  orbital L:  
 $S, P, D, F$ , etc..

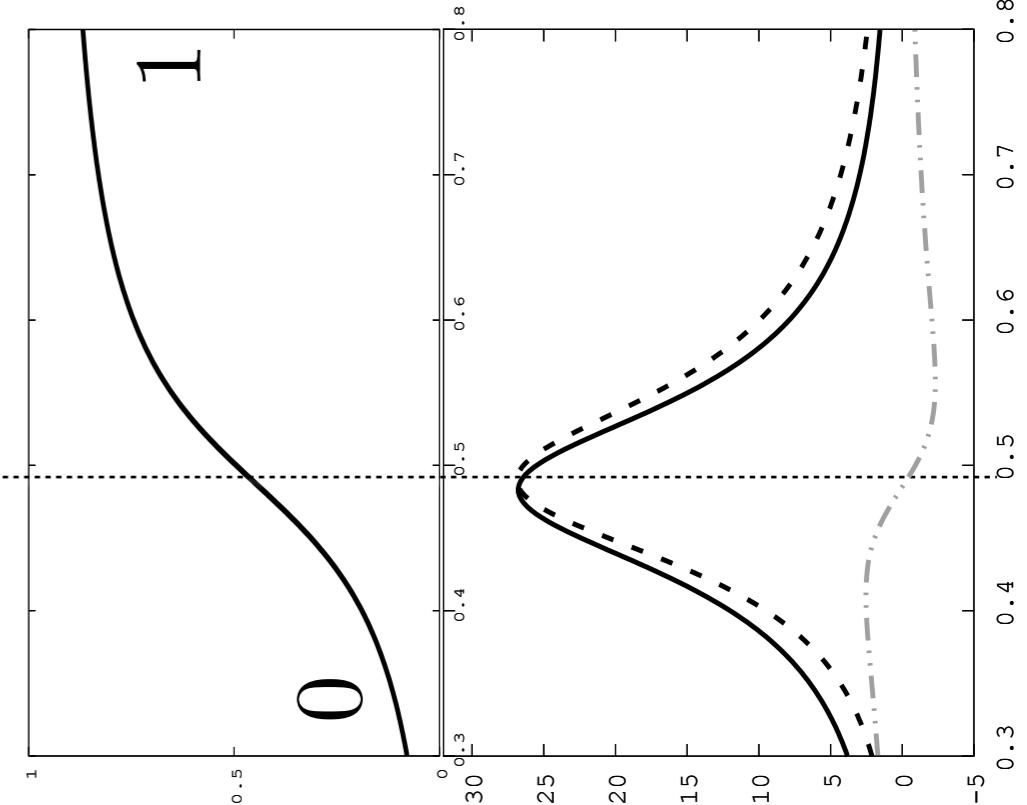
and H. J. Bernstein,  
Rev. 187 (1969) 345.

**HOW TO INCLUDE  
RESONANCES?**

$\text{Tr } e^{-\beta \mathcal{H}_0}$     *vs*     $\text{Tr } e^{-\beta \mathcal{H}}$



$$\Delta g(E, \epsilon) \quad B(E)$$



$$g(E, \epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E, \epsilon)$$

$$= A_\rho + \boxed{\Delta A_{\pi\pi}}$$

# PHYSICS OF B

$$\delta = -\text{Im} \text{Tr} \ln G_\rho^{-1}$$

$$B = 2 \frac{\partial}{\partial E} \delta$$

$$= -2 \text{Im} \frac{\partial}{\partial E} \ln G_\rho^{-1}$$

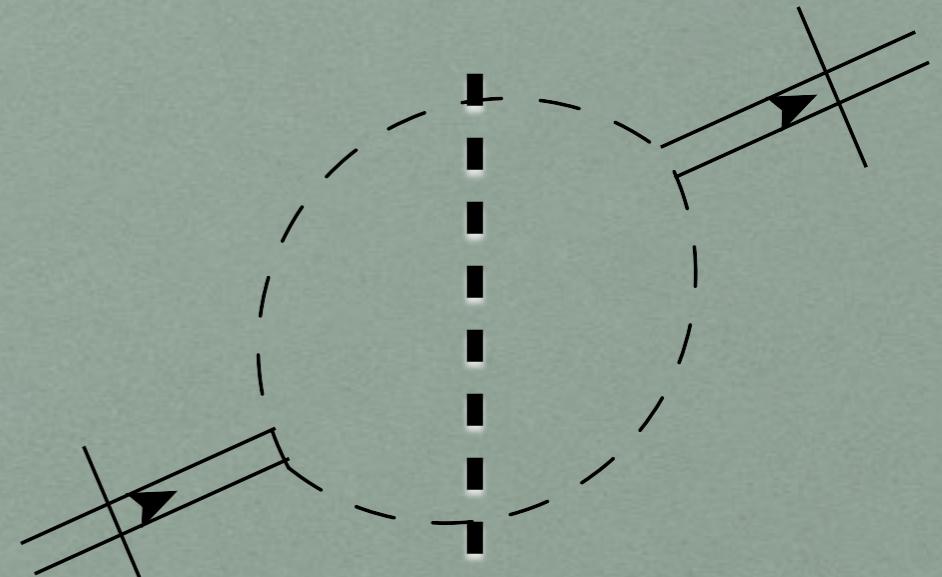
$$= -2 \text{Im}[G_\rho](2E) + 2 \text{Im}\left[\frac{\partial \Sigma_\rho}{\partial E} G_\rho\right]$$

$$= A_\rho(E) + \Delta A_{\pi\pi}$$

$$\downarrow \quad \quad \quad \downarrow$$

$$-\frac{\partial}{\partial E} \int d\phi_E T_{\text{re}}$$

pipi -> pipi



$$\frac{\partial \Sigma_\rho}{\partial E}$$

*physical interpretation:*

*contribution from  
correlated pi pi pair*

# PHYSICS OF B

to rho or not to rho?  
that's out of the question!

$$\delta = -\text{Im } T$$

$$B = 2 \frac{\partial}{\partial E} \delta$$

$$= -2 \text{Im} \frac{\partial}{\partial E}$$

$$= -2 \text{Im}[G]$$

$$= A_\rho(E) + \Delta A_{\pi\pi}$$

*resonance's picture:*

$$B(E) = A_\rho(E) + \Delta A_{\pi\pi}$$

rho

*scattering picture:*

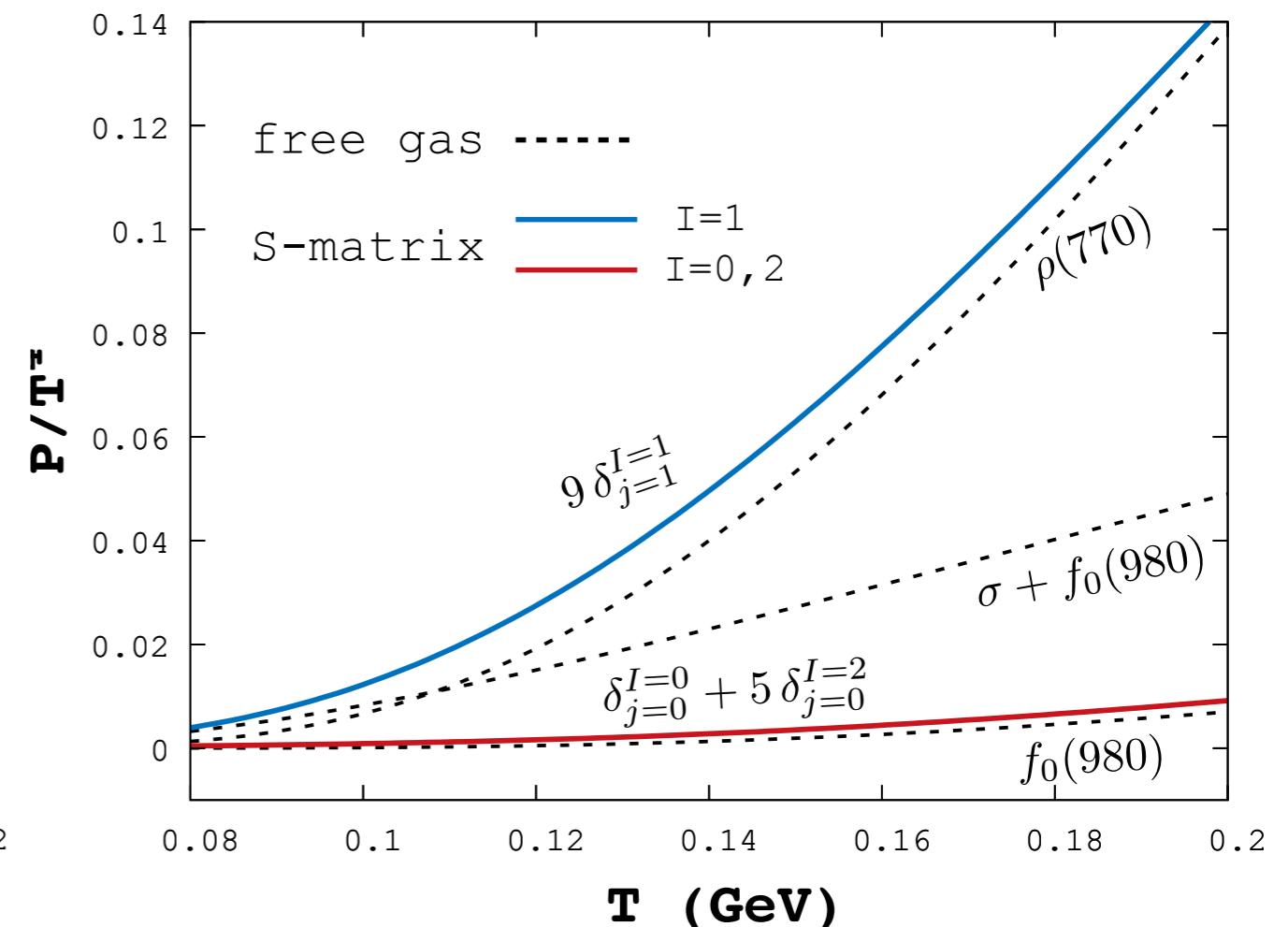
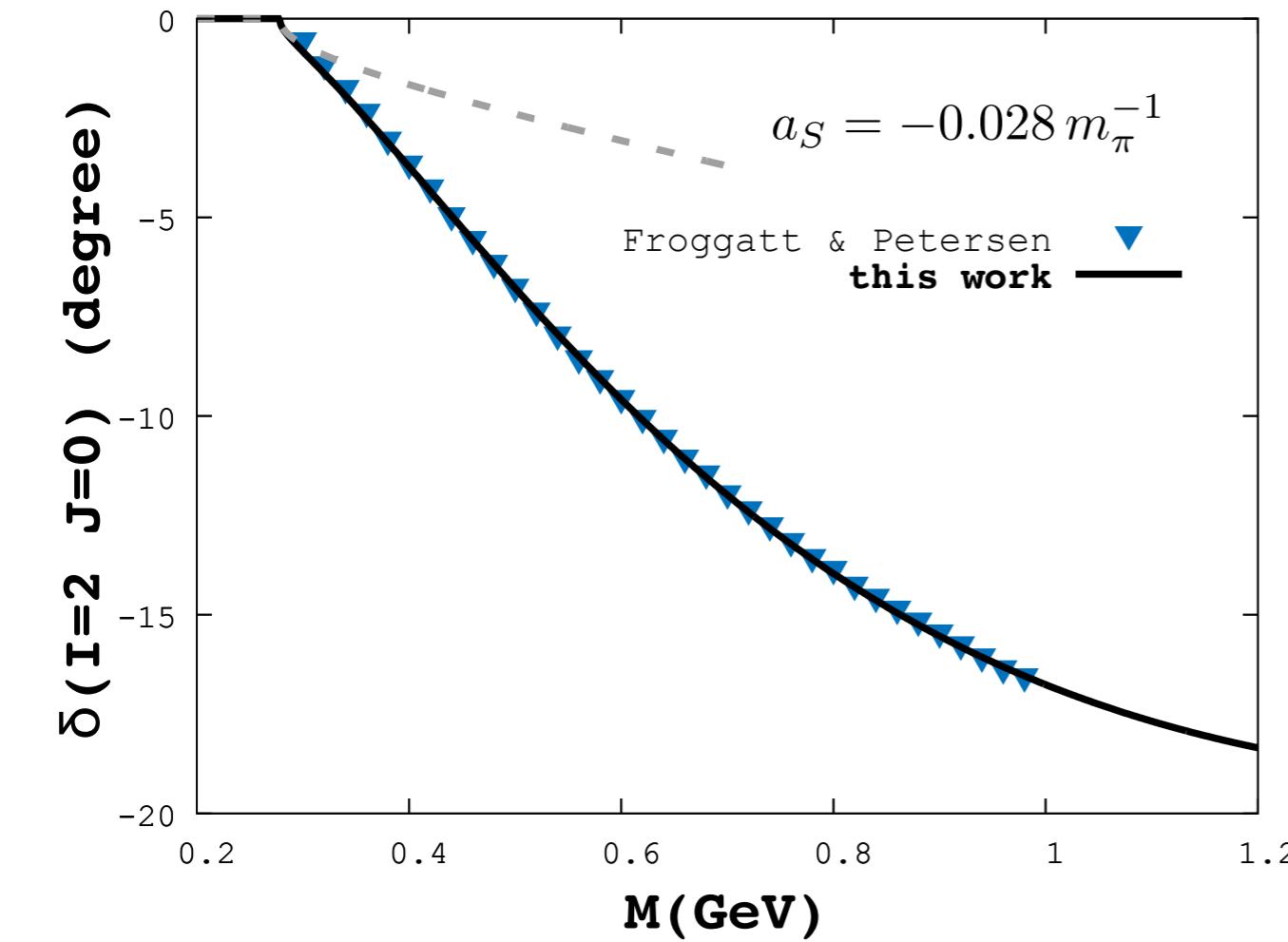
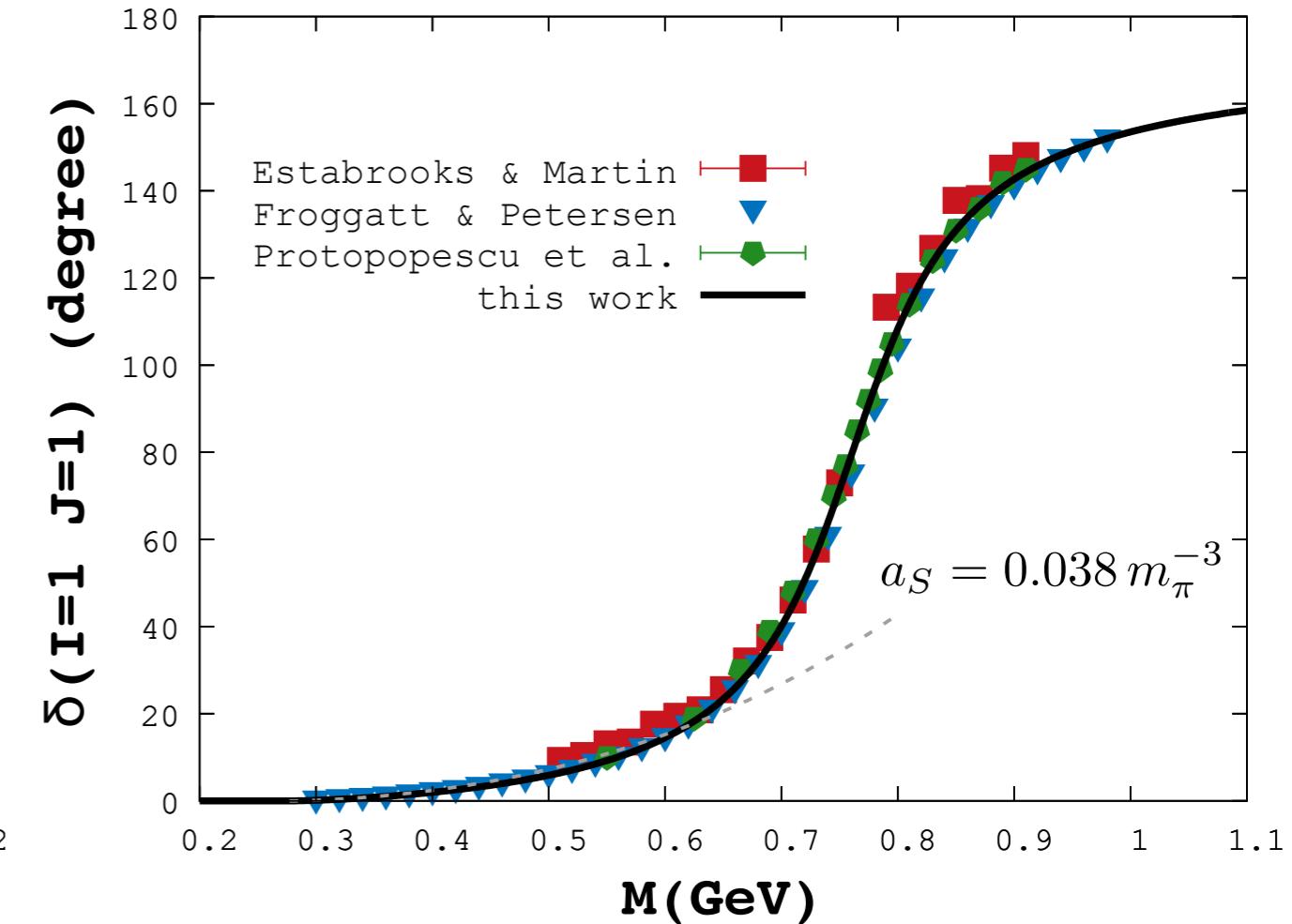
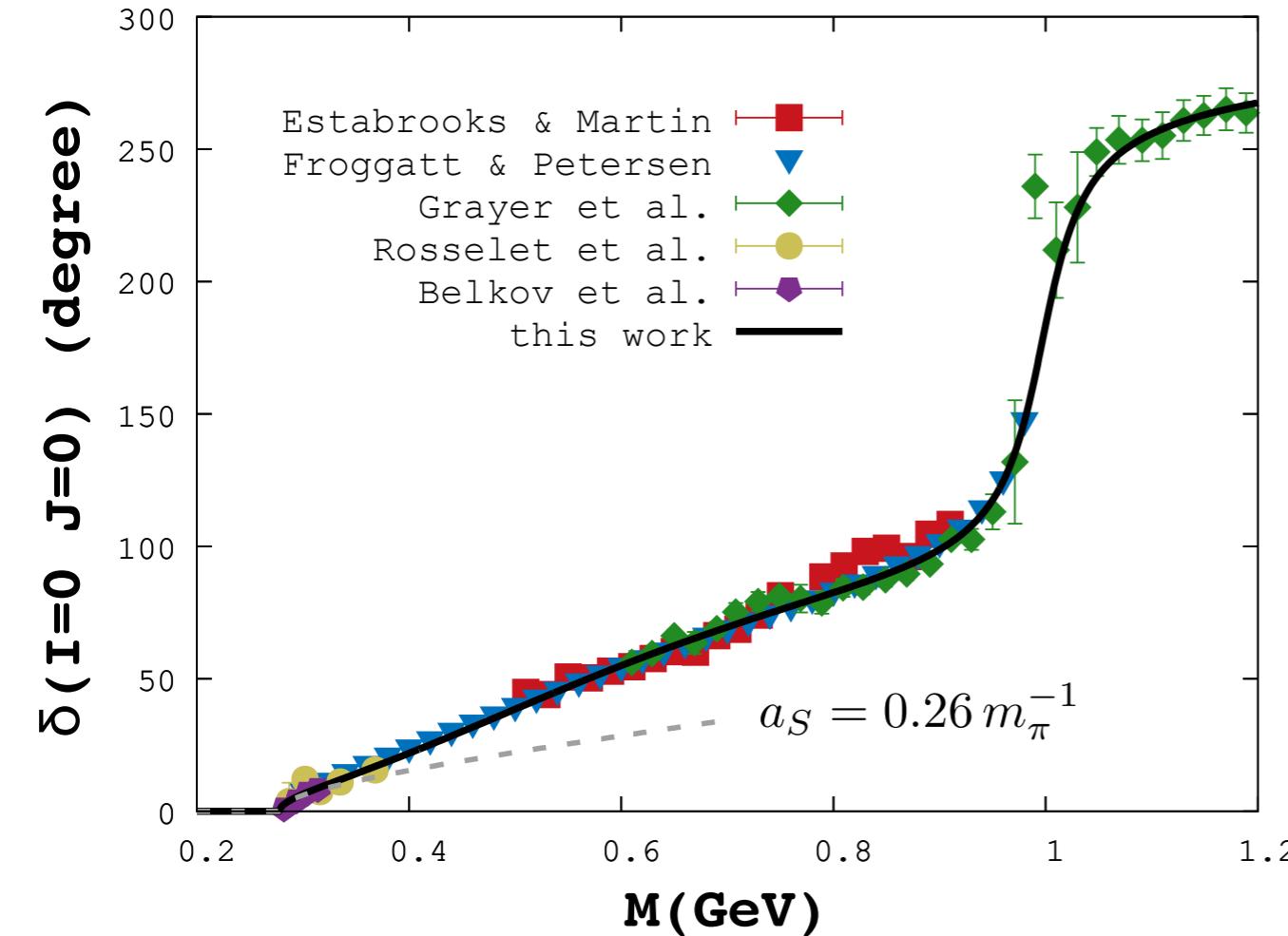
$$B_1 = \frac{\partial}{\partial E} \text{Tr } \hat{t}_{\text{re}}$$

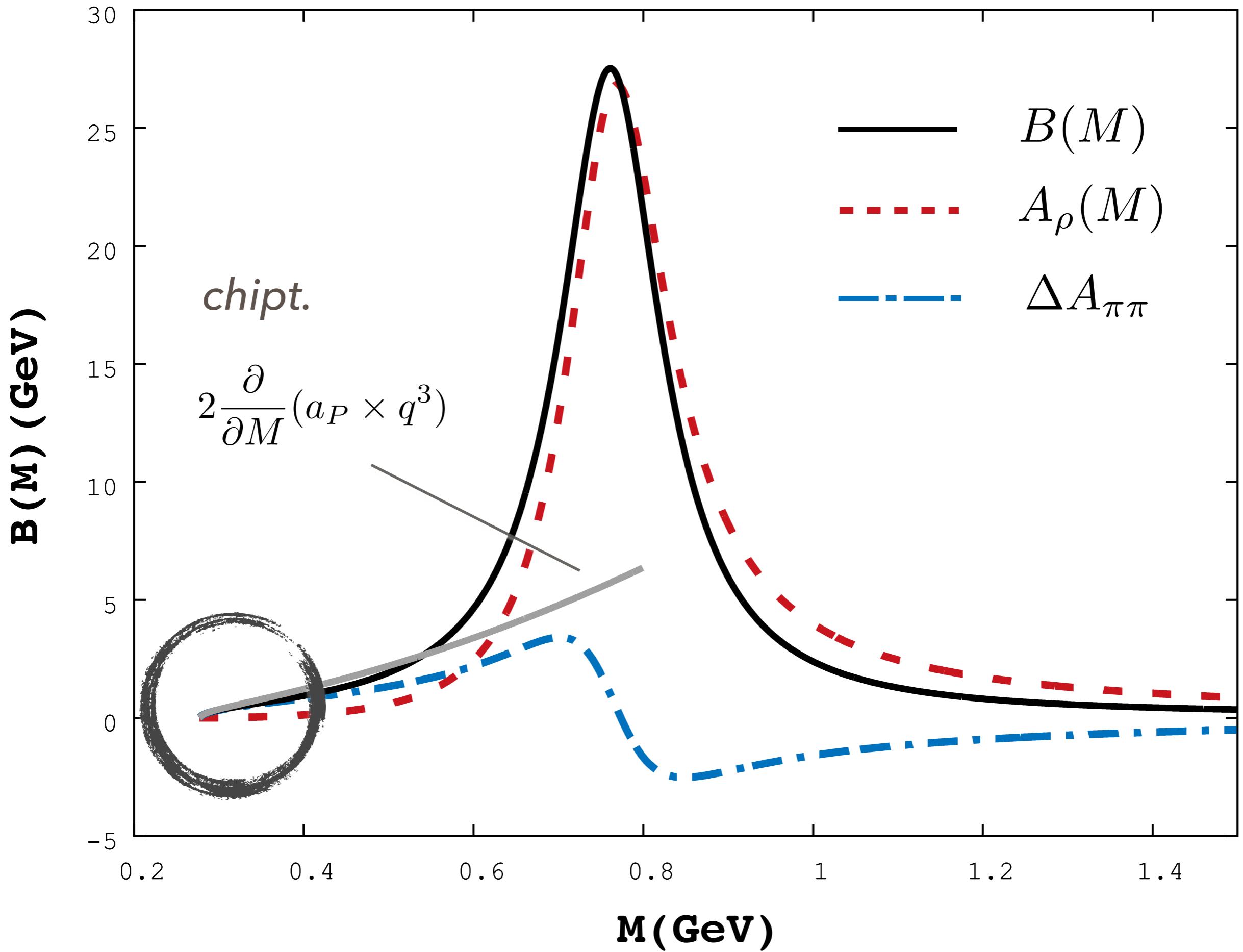
pipi -> pipi

$$B_2 = \frac{1}{2} \text{Im} \text{Tr } \hat{t}^\dagger \overleftrightarrow{\partial}_E \hat{t}$$

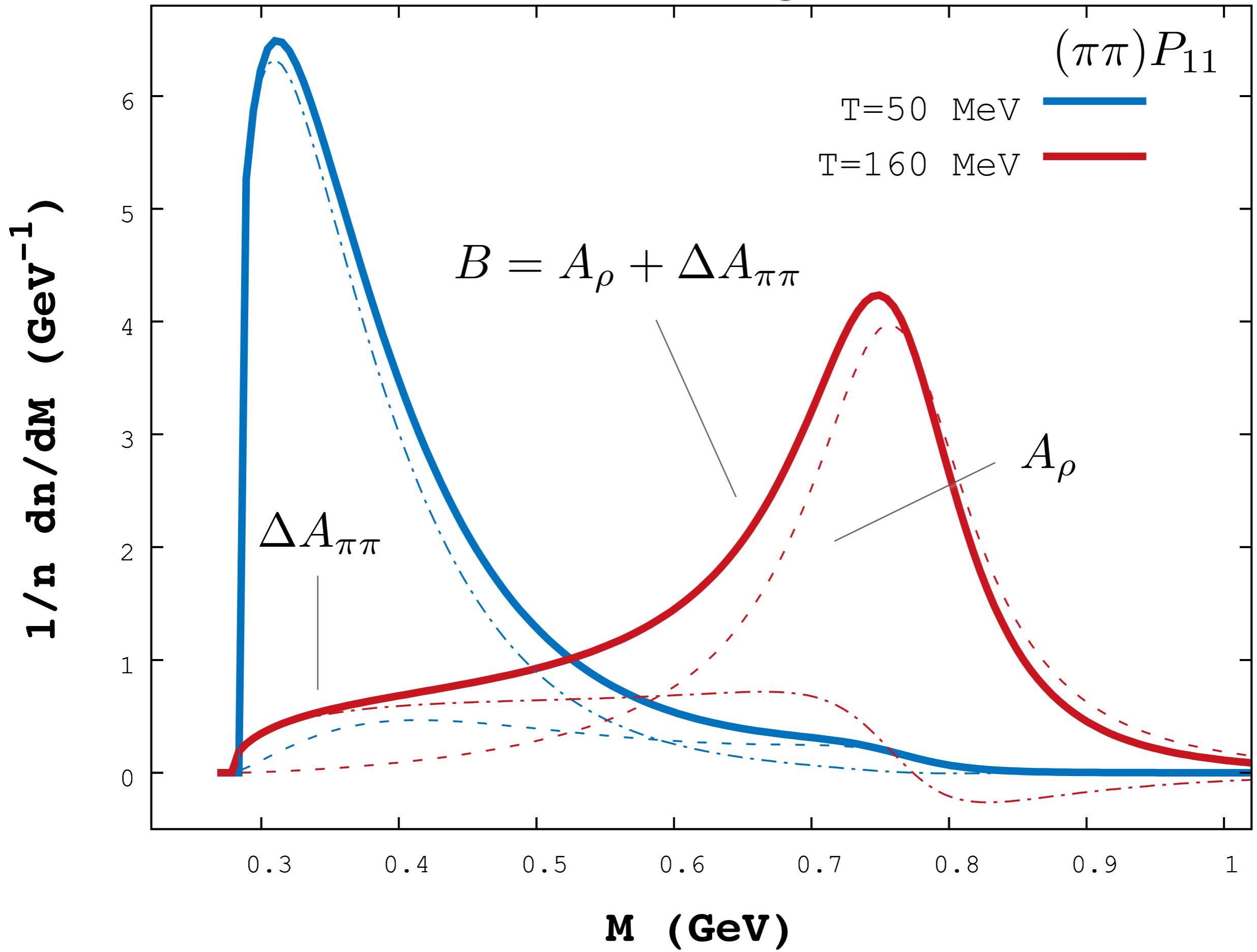
$$-\frac{\partial}{\partial E} \int d\phi_E T_{\text{re}} \quad \text{pipi -> pipi}$$

$$\frac{\partial \Sigma_\rho}{\partial E}$$





# *What the medium see at low $T$ and high $T$*



# WHY IT IS NOT A SUM OF BREIT-WIGNERS

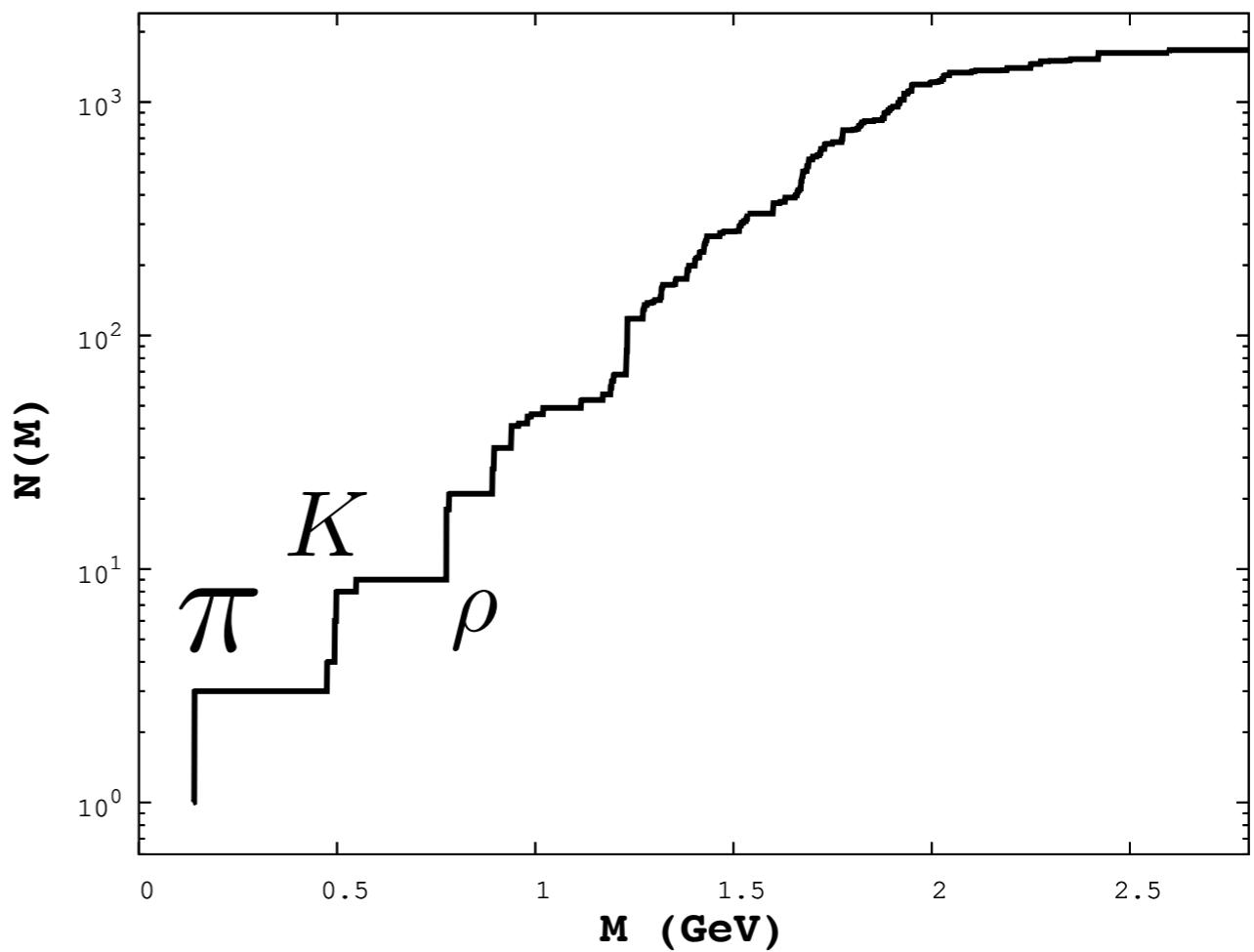
# “IMPROVING HRG”

- Not (just) about adding width
- Not (just) about choosing the right table of resonances
- To bind or not to bind, that's NOT the question!

# HRG AS AN S-MATRIX SCHEME

$$\det S(E) = \prod_{\{\text{res}\}} \frac{z_{\text{res}}^* - E}{z_{\text{res}} - E}, \quad z_{\text{res}} \approx m_{\text{res}} - i 0^+.$$

$$Q_{\text{HRG}}(E) = \sum_{\text{res}} d_{IJ} \times \pi \theta(E - m_{\text{res}}),$$

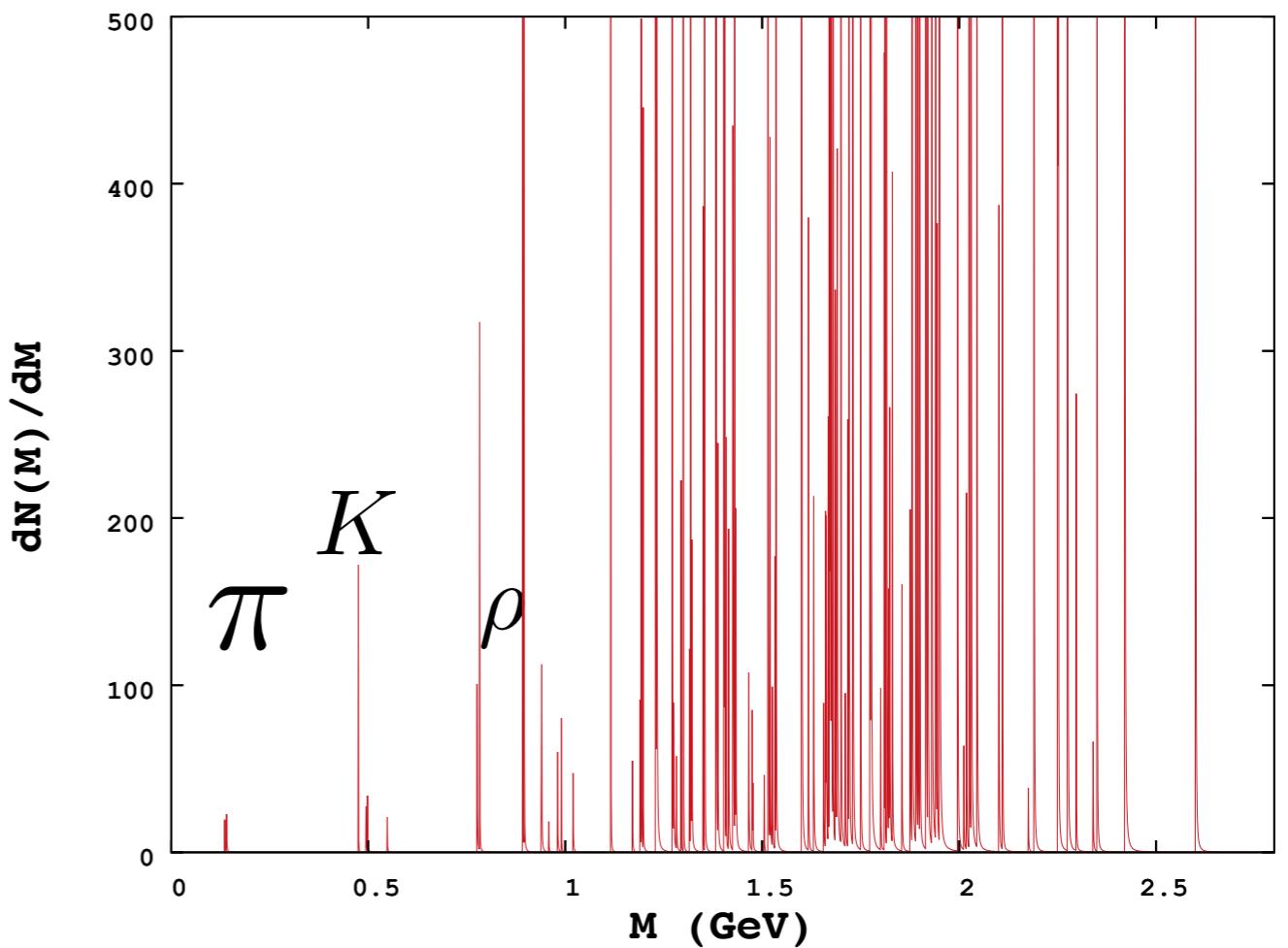


# HRG AS AN S-MATRIX SCHEME

$$\det S(E) = \prod_{\{\text{res}\}} \frac{z_{\text{res}}^* - E}{z_{\text{res}} - E}, \quad z_{\text{res}} \approx m_{\text{res}} - i 0^+.$$

$$Q_{\text{HRG}}(E) = \sum_{\text{res}} d_{IJ} \times \pi \theta(E - m_{\text{res}}),$$

→  $\frac{\partial}{\partial E}$

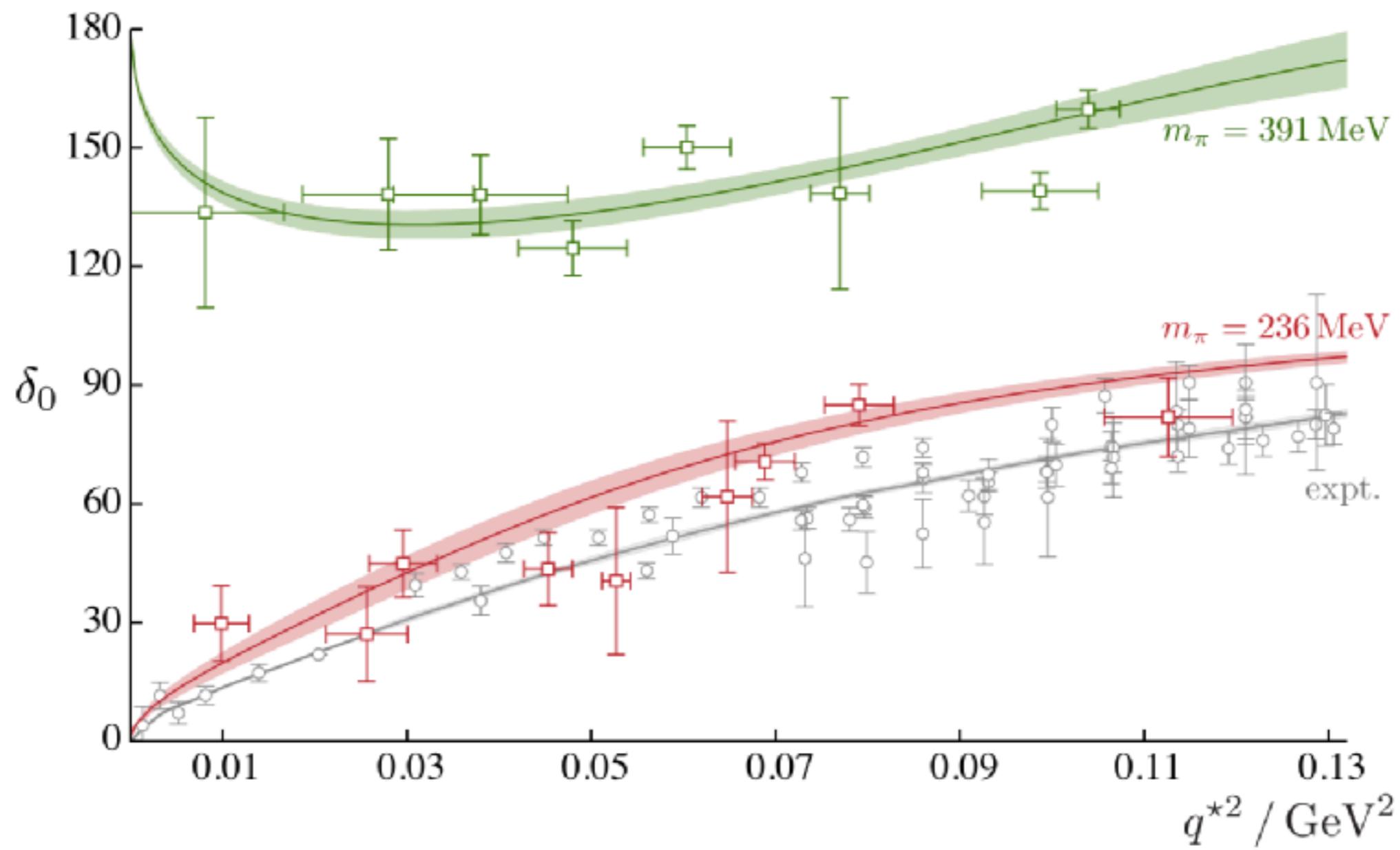


# DYNAMICAL GENERATION OF BS / RESONANCES

- dynamical generation of bound states / resonances:  
 $f(980)$  close to  $K\bar{K}$  threshold  
 $f(500)$  dynamically generated
- coupling of open channels:  $\pi\pi$ ,  $KK\bar{K}$   
with a  $|q\bar{q}\rangle$  state

# LATTICE COMPUTATIONS ON PHASE SHIFT

deuteron physics?



*what you give*  $\neq$  *what you get*

*1 in 5 out!*

$$\frac{1}{E - \mathcal{H}_0} = |\pi\pi\rangle + |K\bar{K}\rangle + |R^0\rangle + |q\bar{q}\rangle$$

$$\left[ \begin{array}{c} \Pi_{\pi\pi}(E) \\ \Pi_{K\bar{K}}(E) \\ \frac{1}{E - m_{res}^0} \end{array} \right]$$

$$V_{int} = \begin{bmatrix} g_{\pi\pi} & g_{\pi K} & g_{\pi R} \\ g_{\pi K} & g_{KK} & g_{KR} \\ g_{\pi R} & g_{KR} & \end{bmatrix}$$

$$G = G_0 + G_0 V_{int} G$$

# *From Hamiltonian to Scattering Matrix*

$$\begin{aligned}\tilde{S} &= (I - G_-^0 V) (I + G_+^0 T) \\ &= I - G_-^0 V + G_+^0 T - G_-^0 V G_+^0 T \\ &= I - G_-^0 V + G_+^0 V + G_+^0 V G_+^0 T - G_-^0 V G_+^0 T \\ &= I + (G_+^0 - G_-^0) V + (G_+^0 - G_-^0) V G_+^0 T \\ &= I + (G_+^0 - G_-^0) T \\ &\rightarrow I + 2 i \operatorname{Im} (G_+^0) \times T. \quad \textit{on-shell limit}\end{aligned}$$

A diagram consisting of two parallel grey arrows originating from the left side of the equation and pointing towards the fraction  $\frac{1}{E - \mathcal{H}_0 \pm i\delta}$ .

# TESTING THE ROBUSTNESS

$$\mathcal{Q}(E) = \frac{1}{2} \text{ImTr}\{\ln S_E\}$$

*effective DOS*

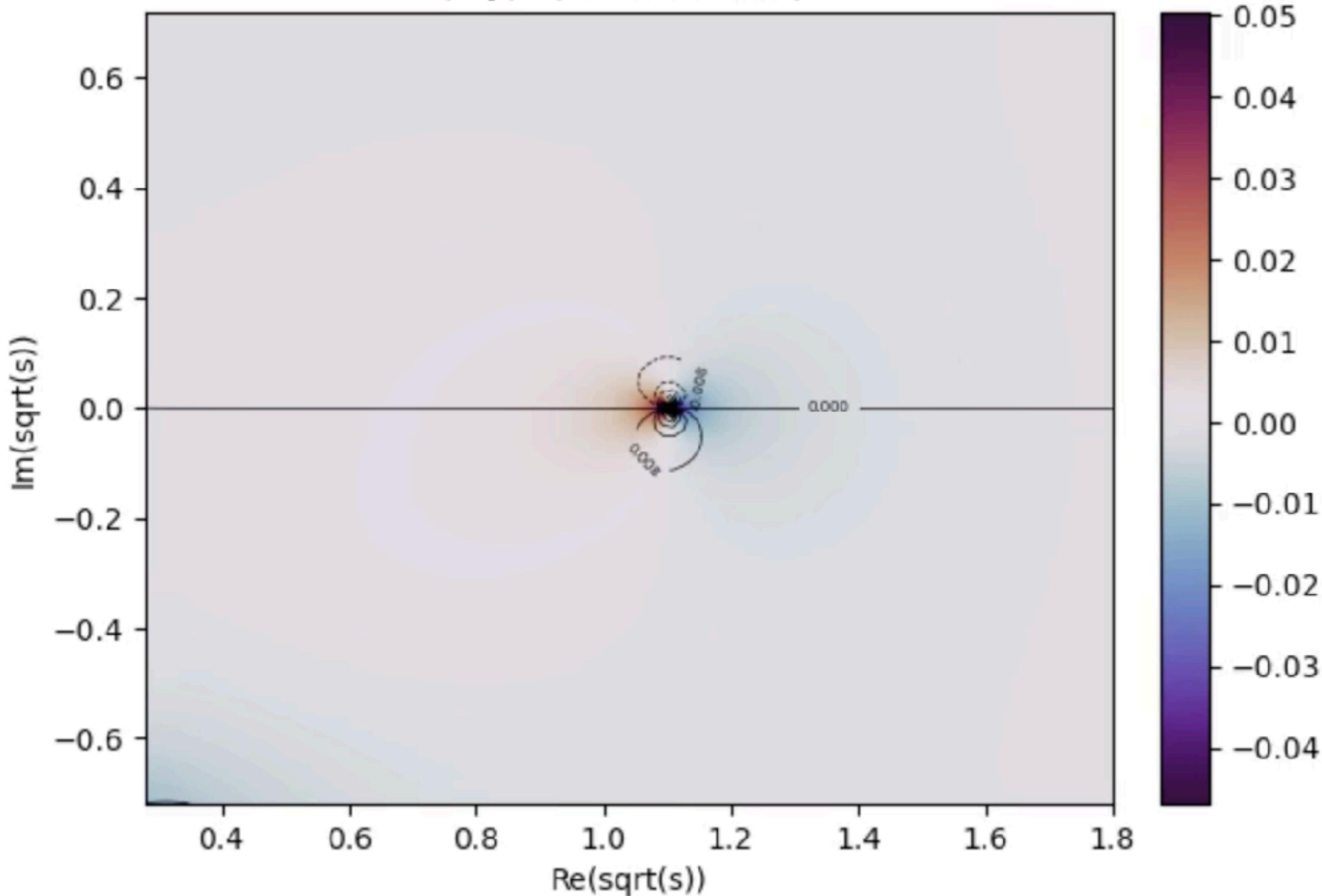
$$B = 2 \frac{d}{dE} \mathcal{Q}$$

*Getting  
Effective DOS  
on  
REAL Energy*

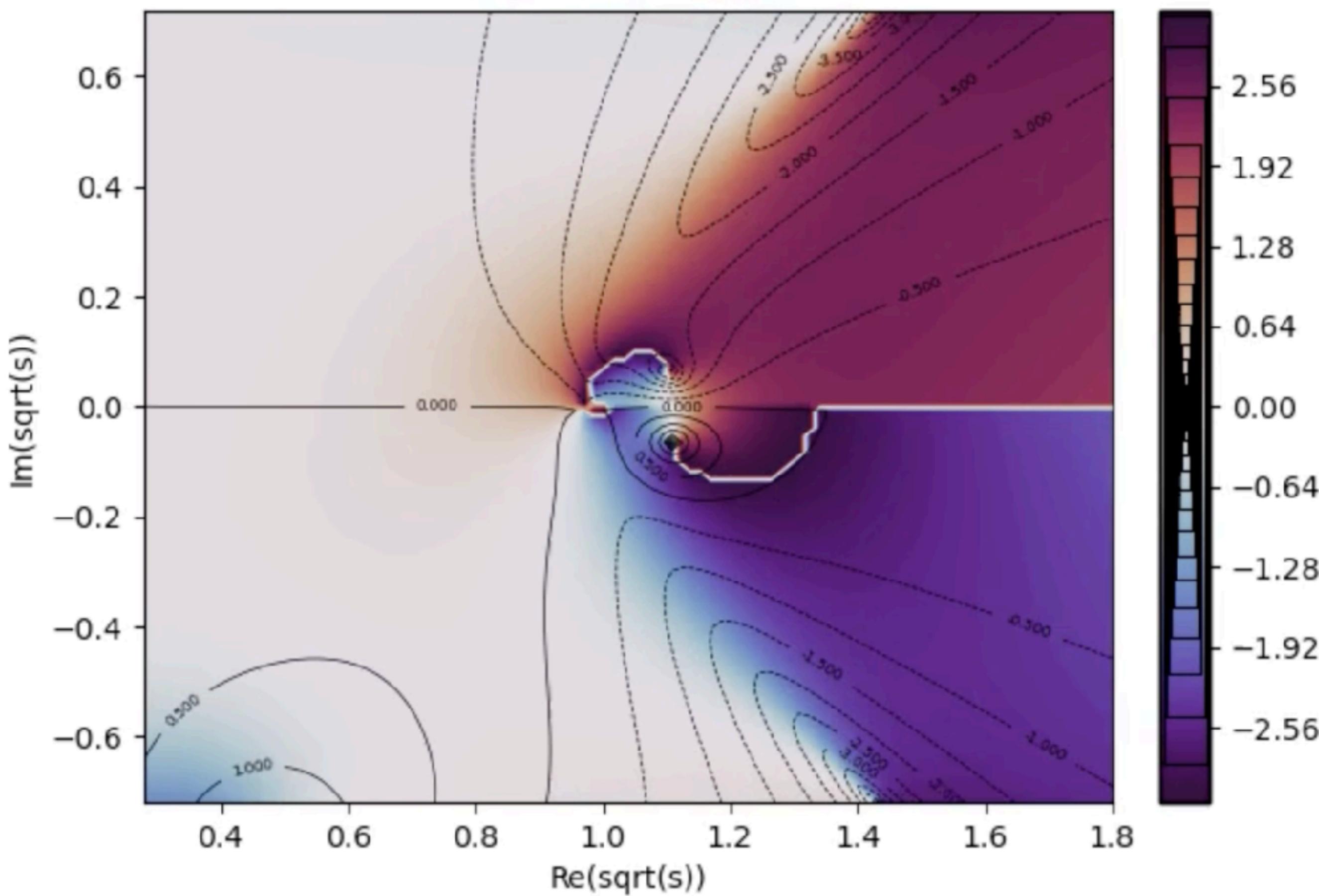
*what is being counted?*

*can it handle dynamically generated states?*

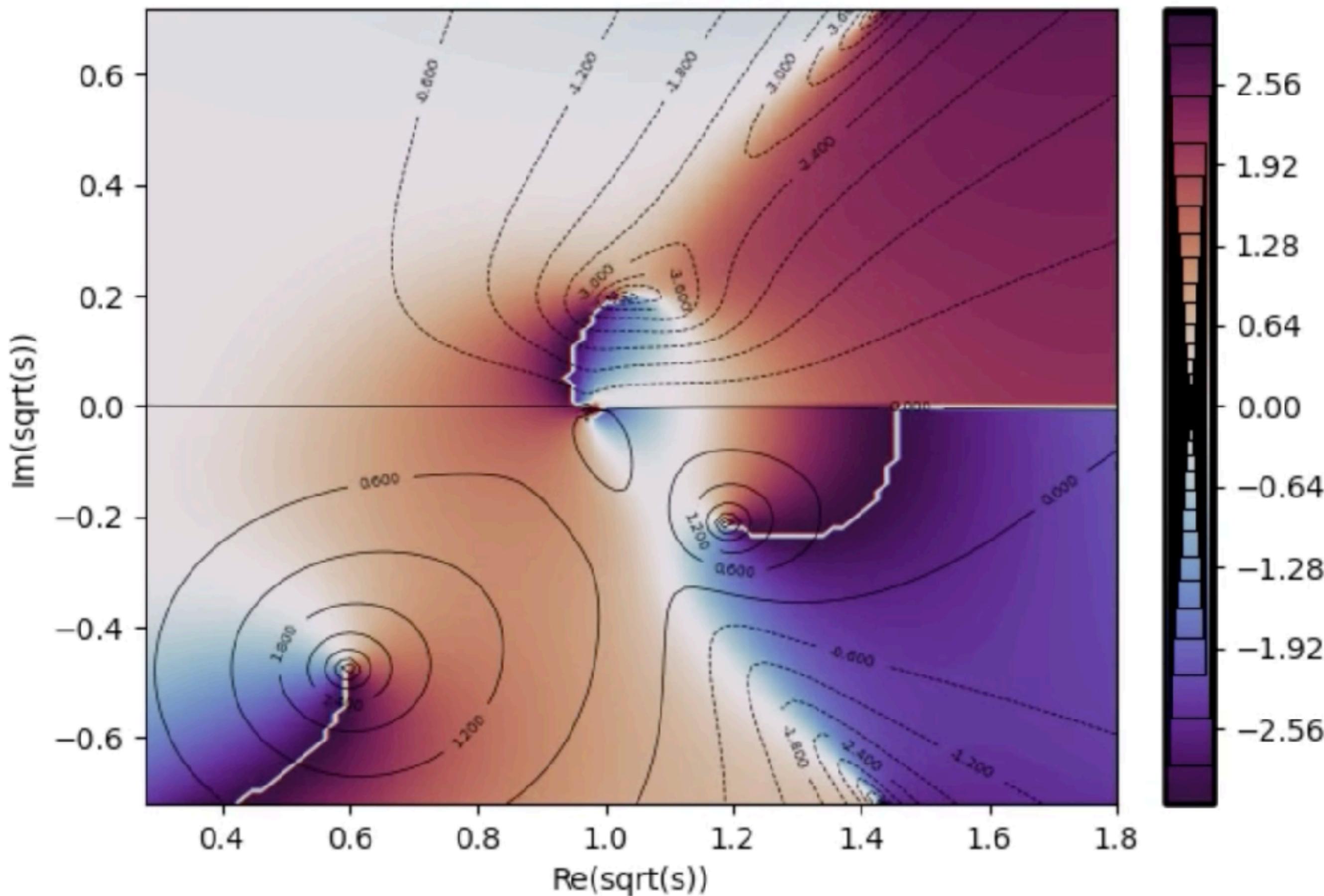
$(x, y) = (0.001, 0.001)$



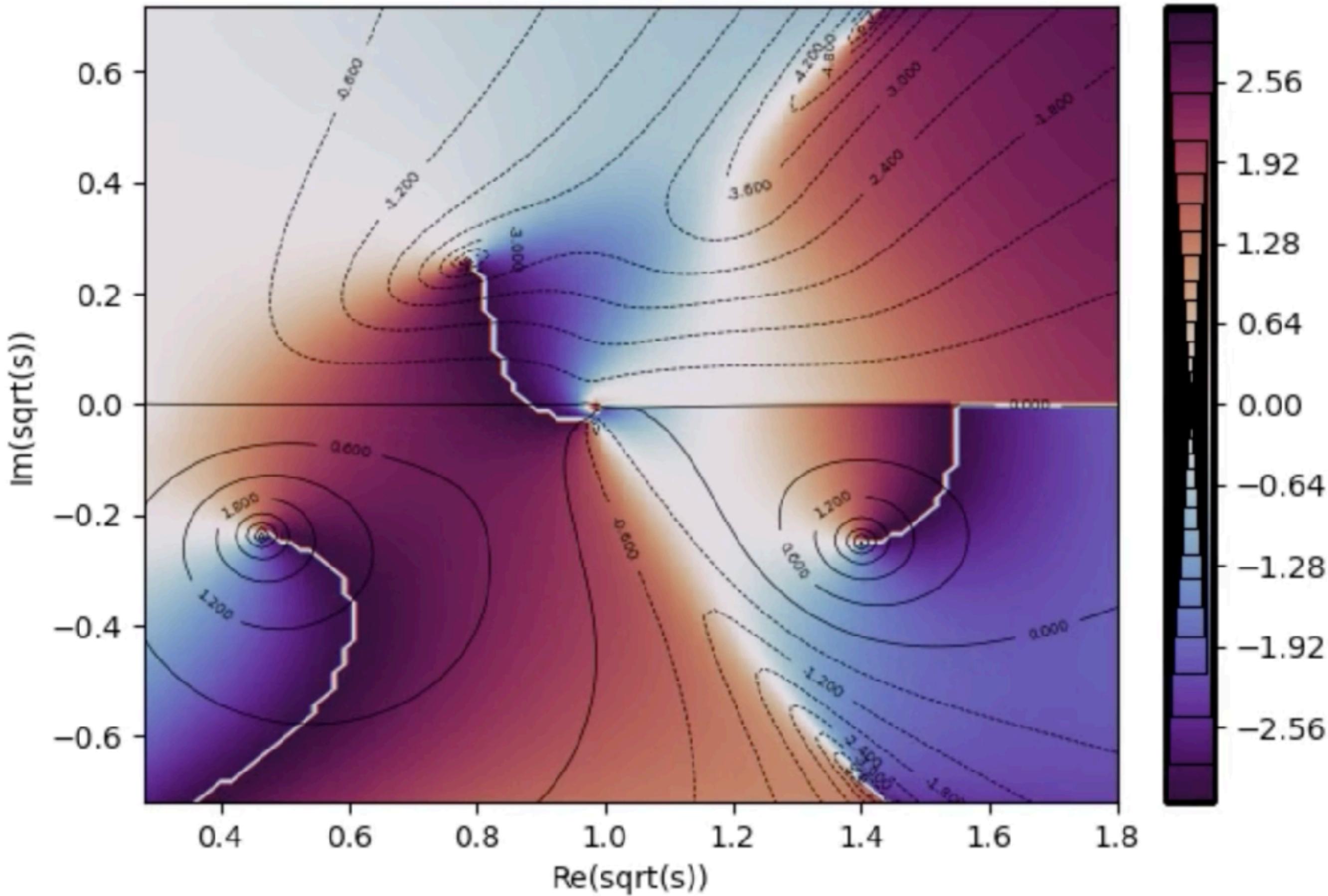
$(x,y)=(0.155, 1.0)$



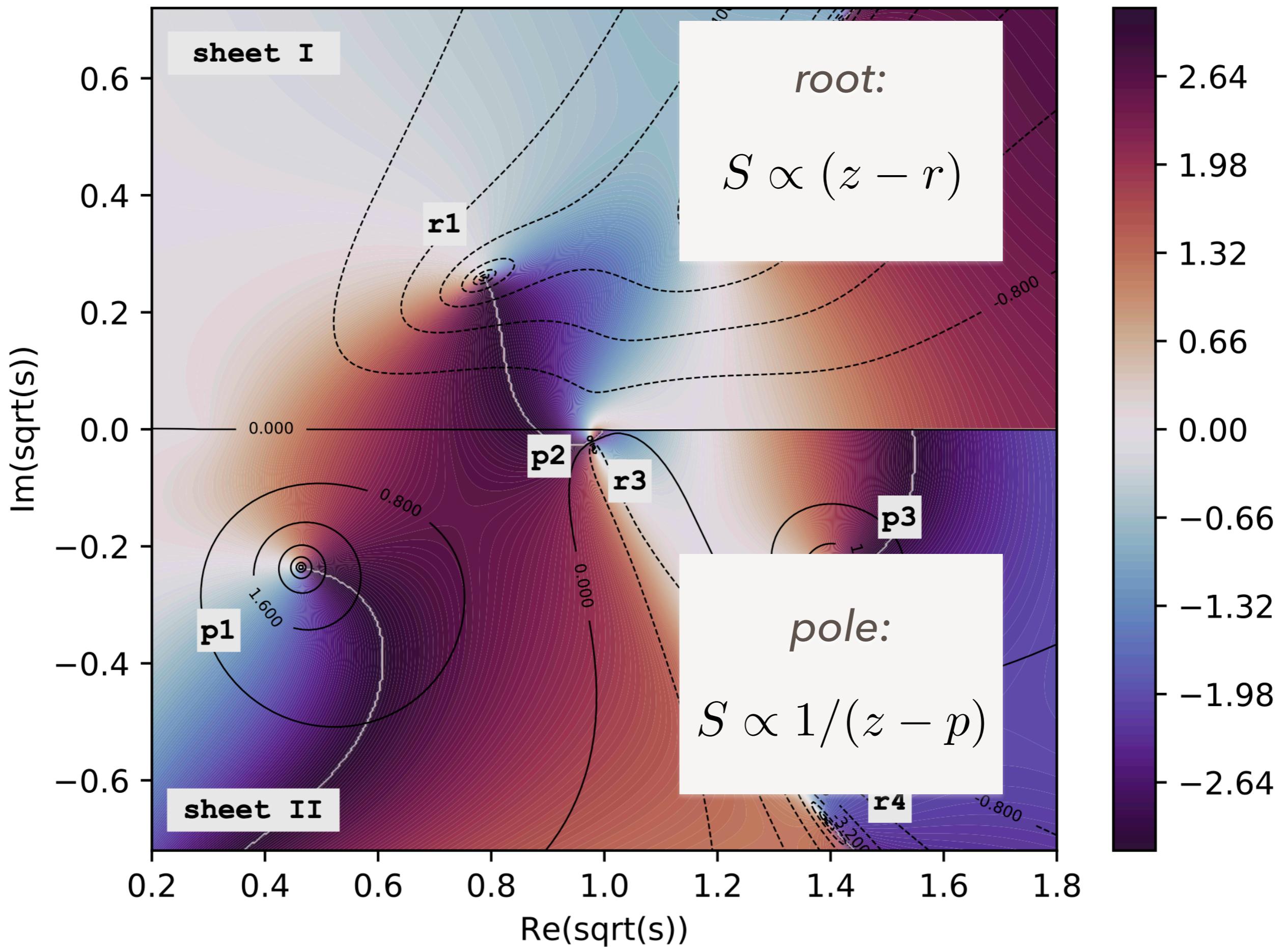
$(x,y)=(0.488, 1.0)$

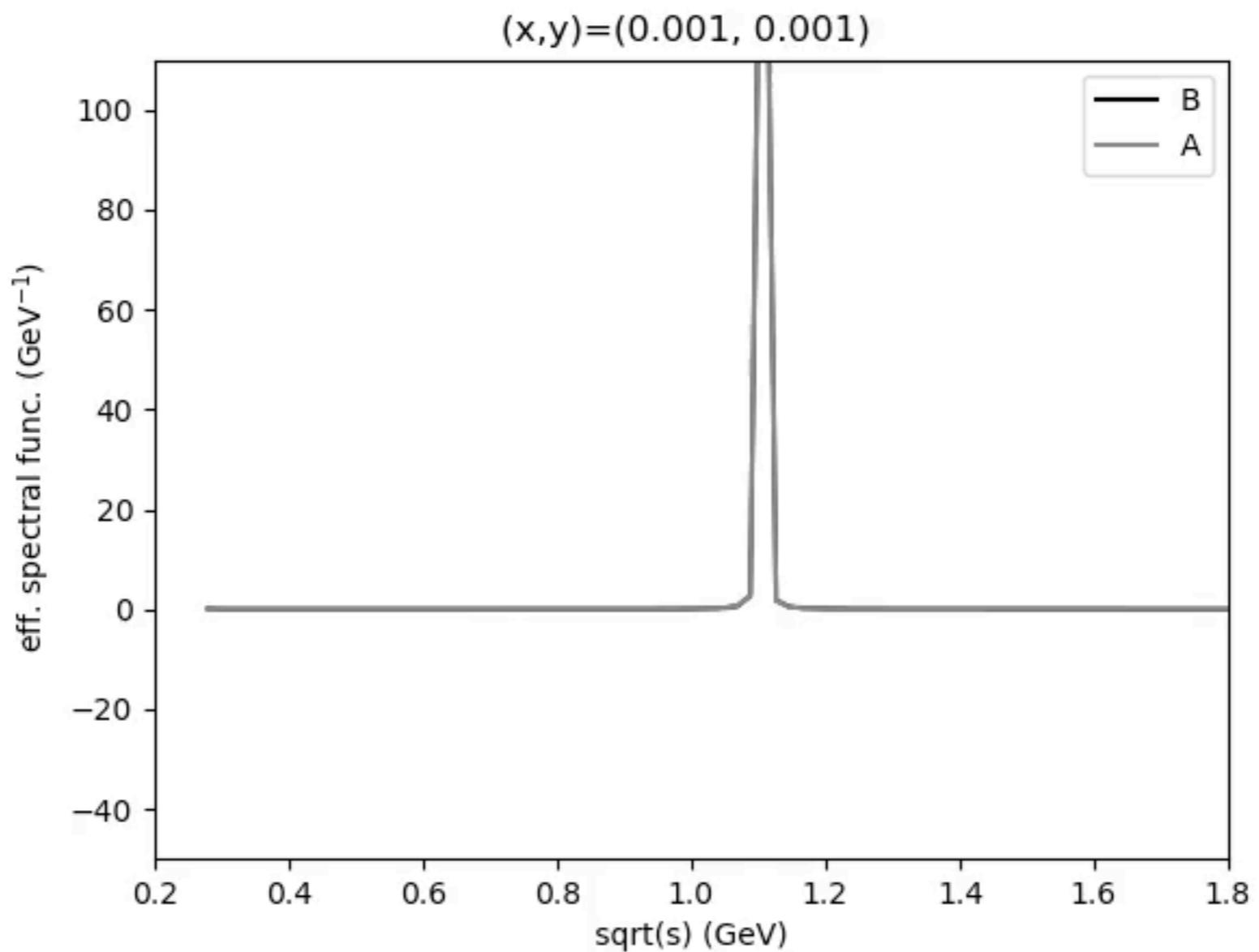


$(x,y)=(1.0, 1.0)$



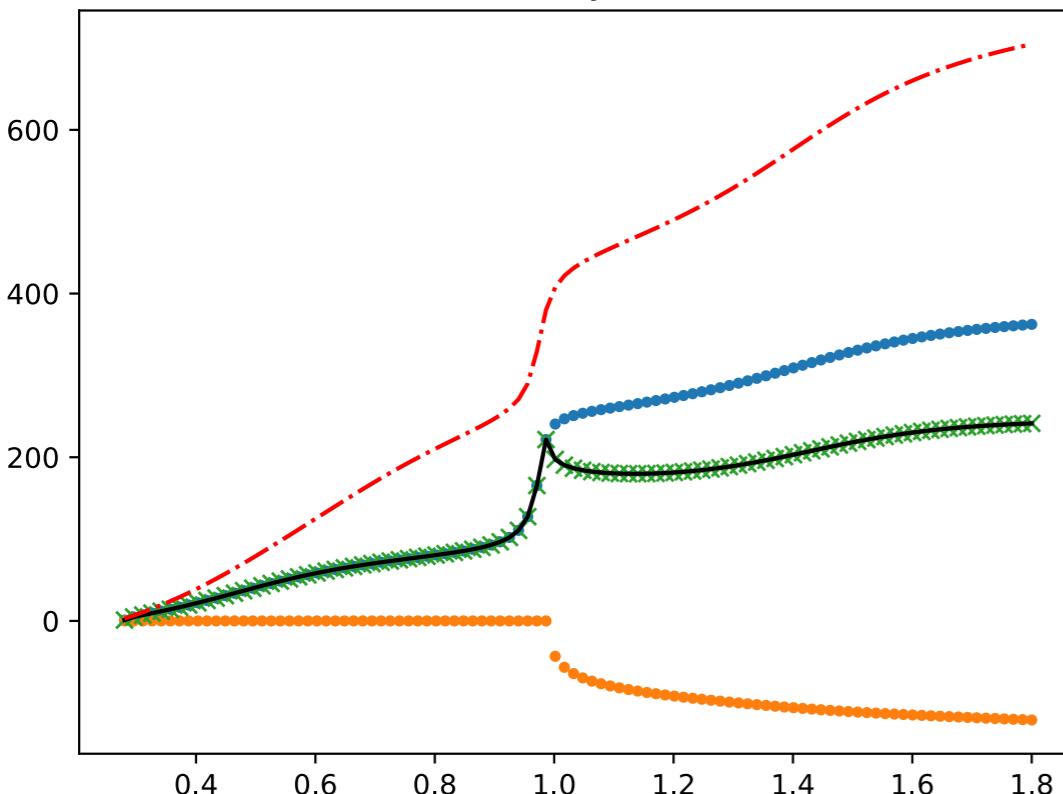
$\det S(\sqrt{s})$



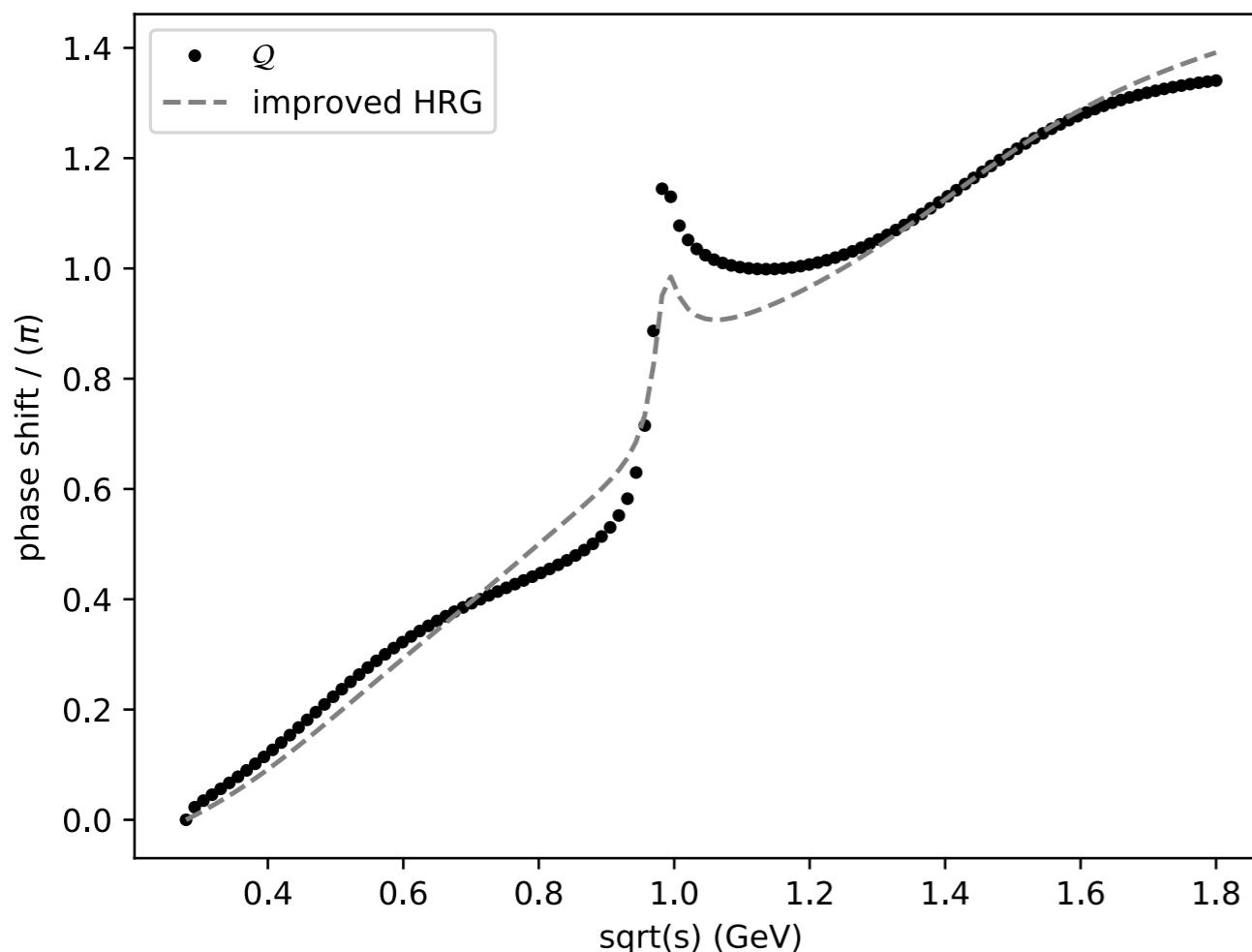


$x = 1.0, y = 1.0$

IEEE I. Definition of Riemann sheets. Convention follows  
f. [54, 55]



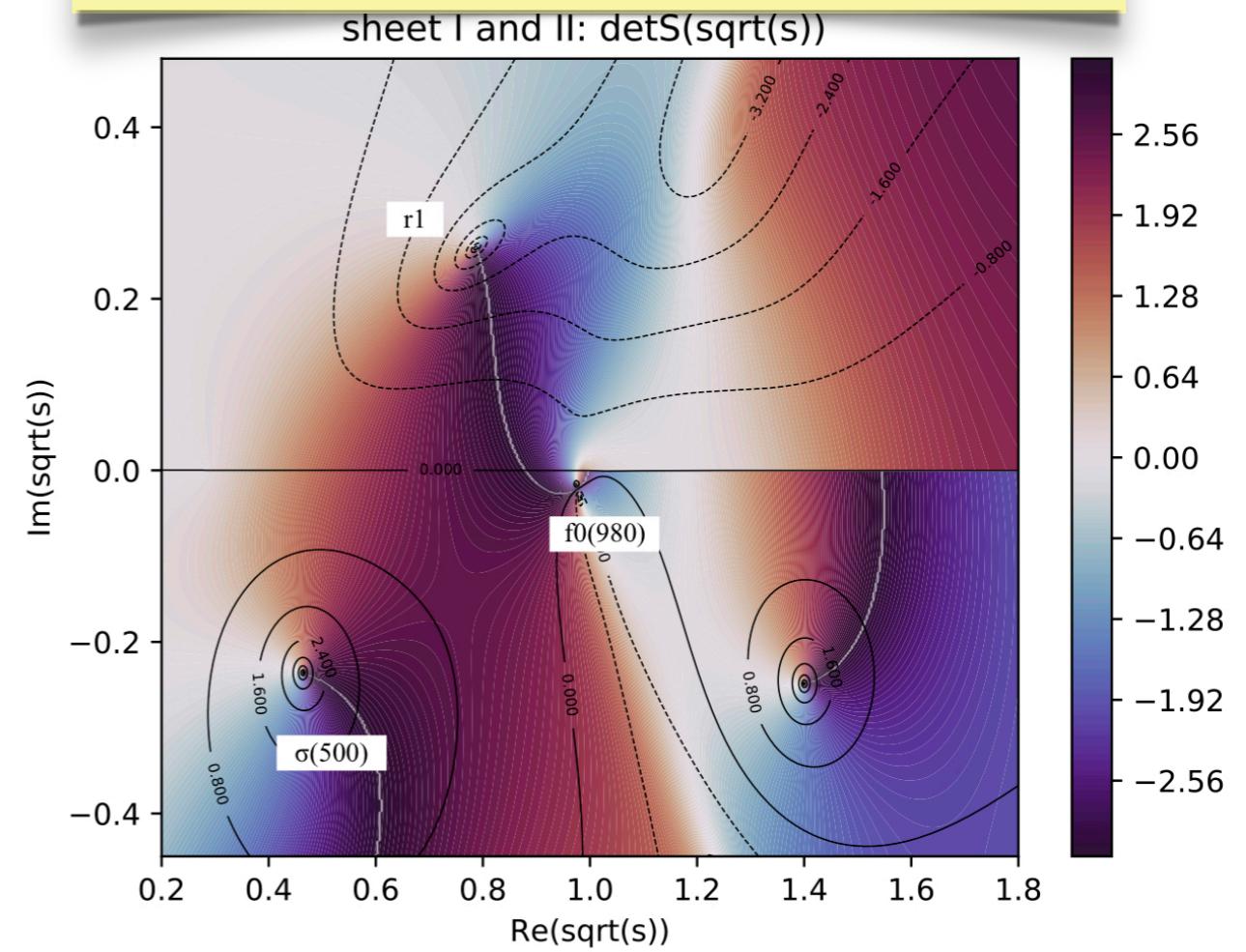
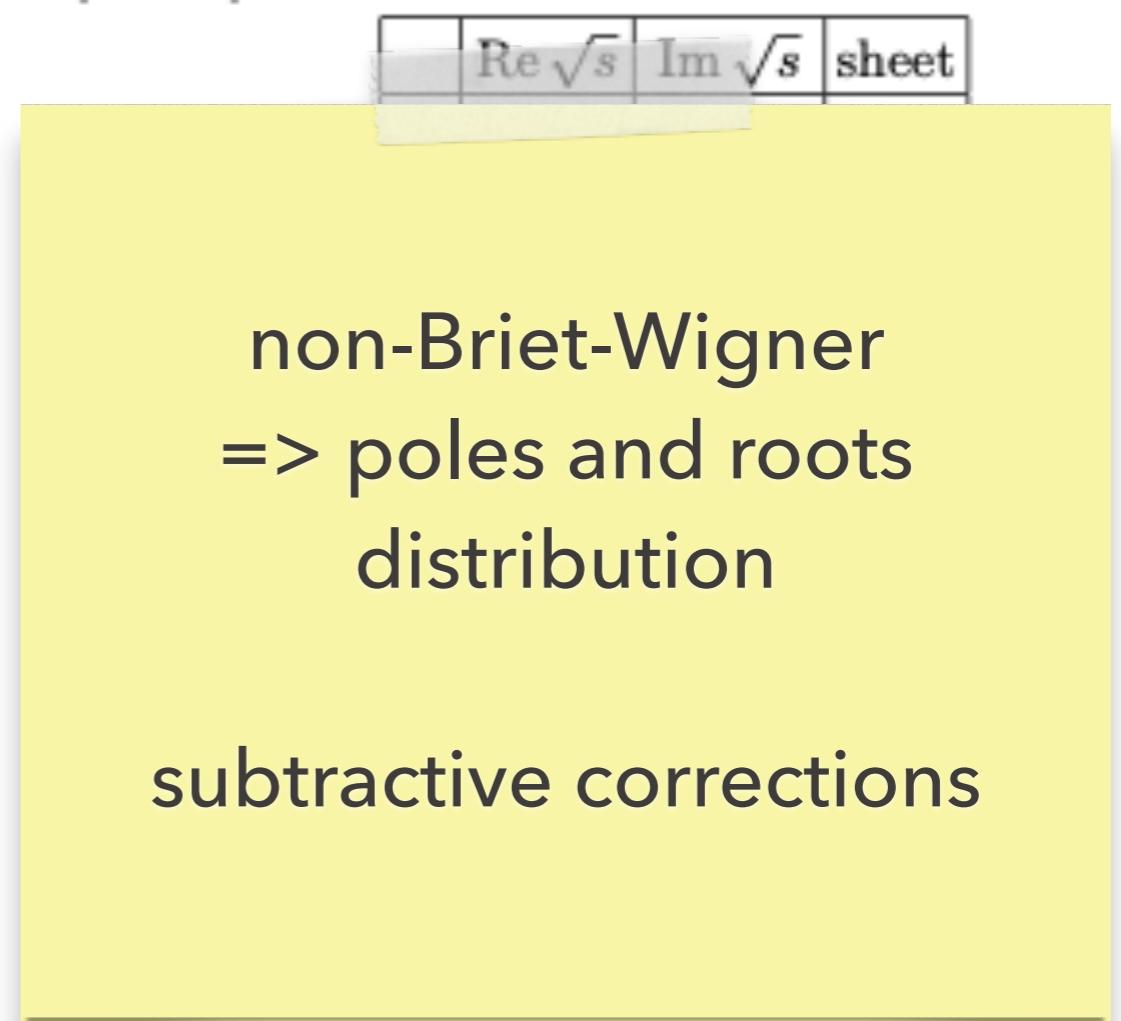
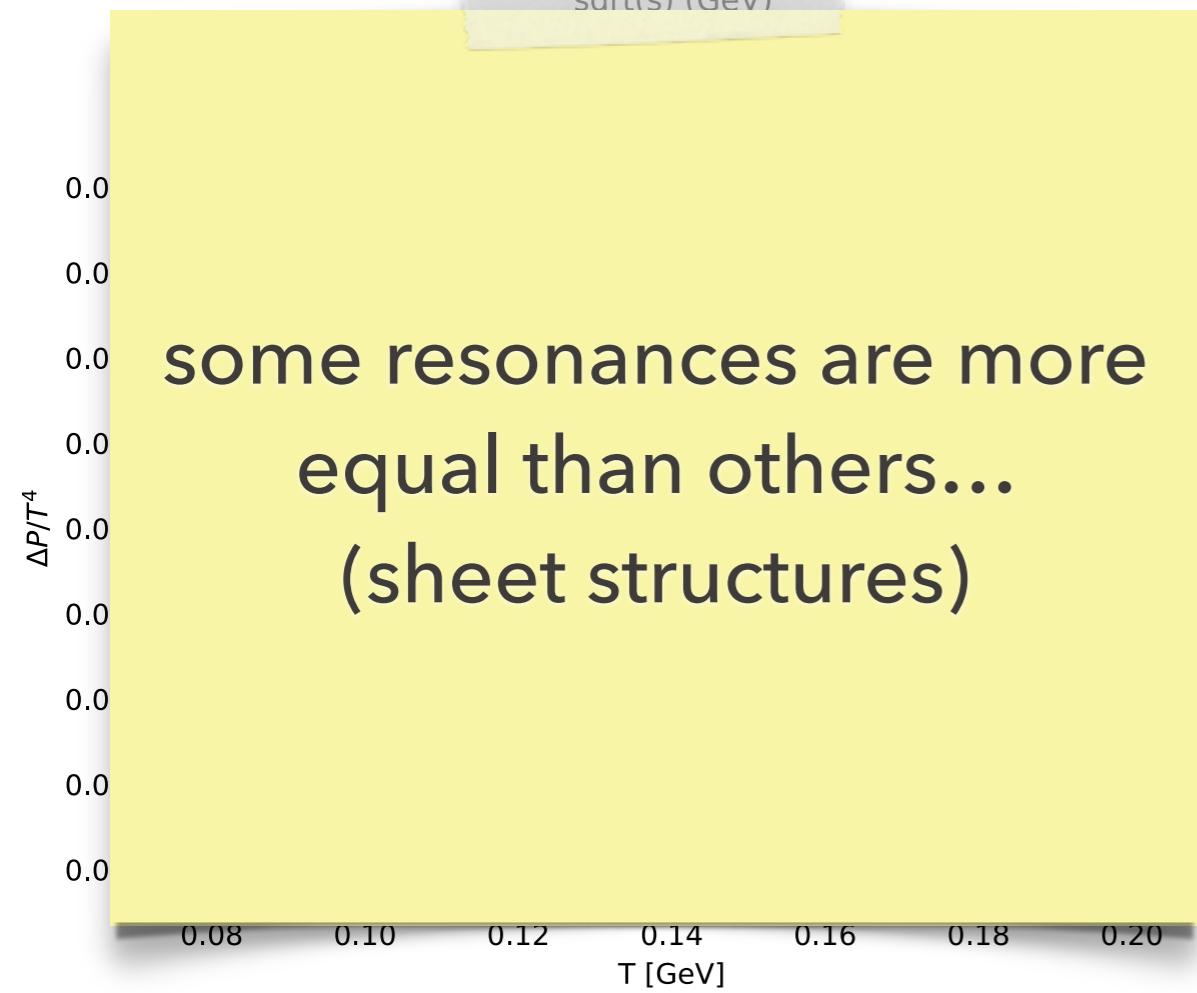
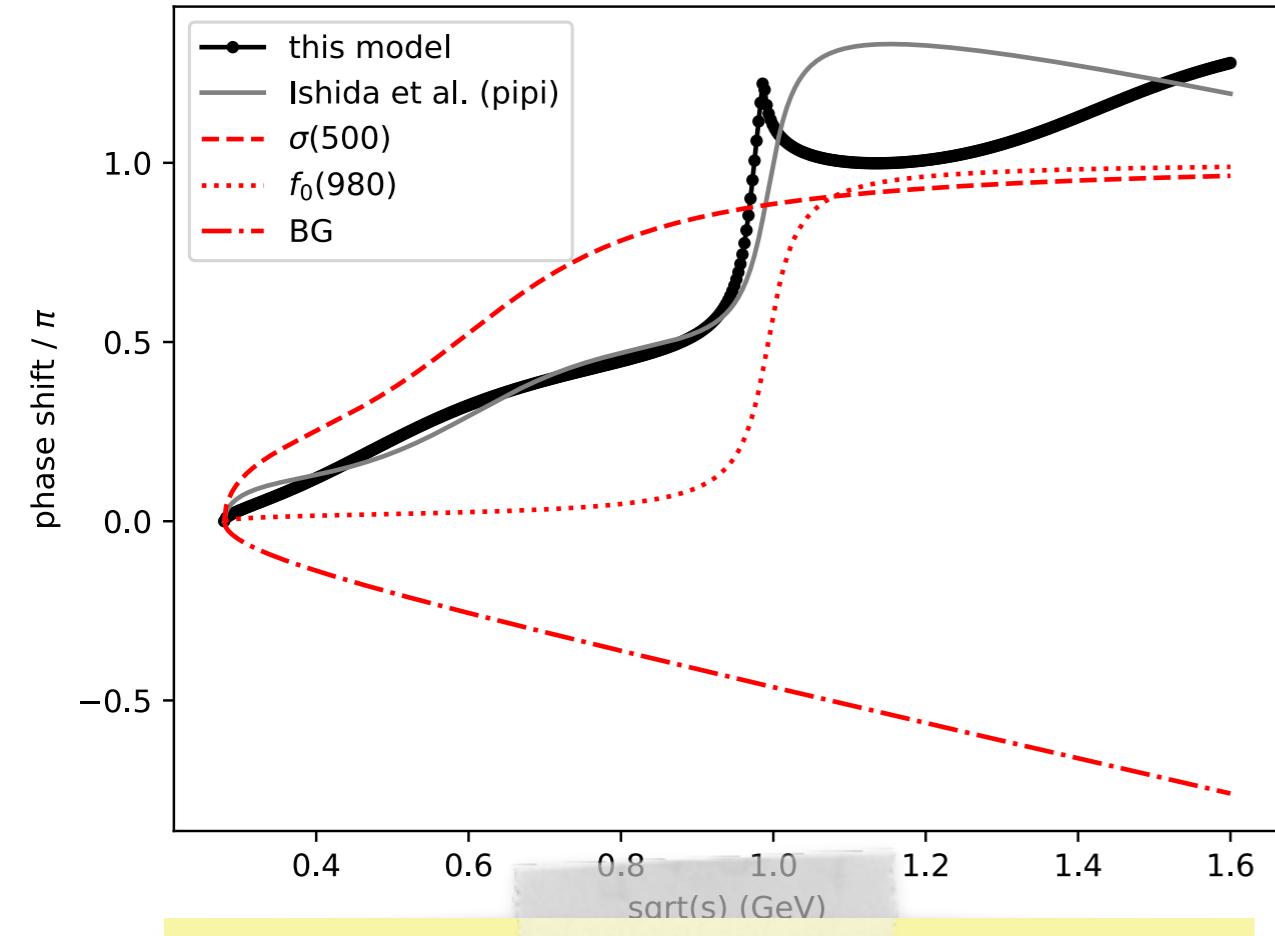
	$\text{Re } \sqrt{s}$	$\text{Im } \sqrt{s}$	sheet
p1	0.4637	-0.2357	II
p2	0.975	-0.0164	II
p3	1.401	-0.249	II
p4	0.6654	-0.2263	III
p5	1.4176	-0.2640	III
r1	0.787	+0.259	I
r2	1.410	+0.691	I
r3	0.981	-0.032	II
r4	1.393	-0.669	II
r5	0.918	+0.248	IV



II. Location of resonance poles ( $p_i$ ) and roots ( $r_i$ )  
in the model.

repulsive corrections in  
HRG-like scheme:  
via roots

M (GeV)



# IN-MEDIUM EFFECTS

# VACUUM PHYSICS?

**Quantum statistical mechanics of gases in terms of dynamical filling fractions and scattering amplitudes**

André LeClair

Newman Laboratory, Cornell University, Ithaca, NY, USA

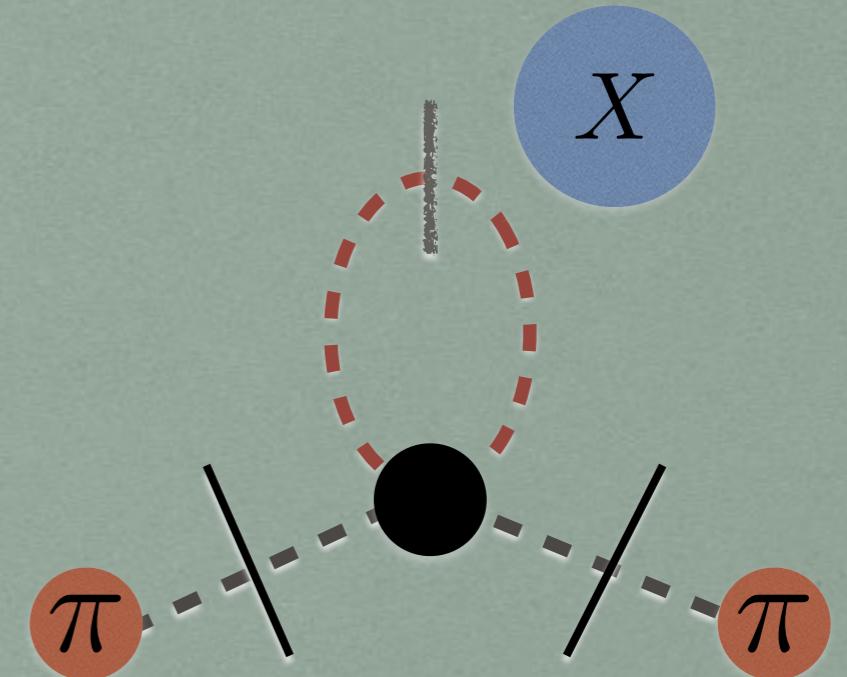
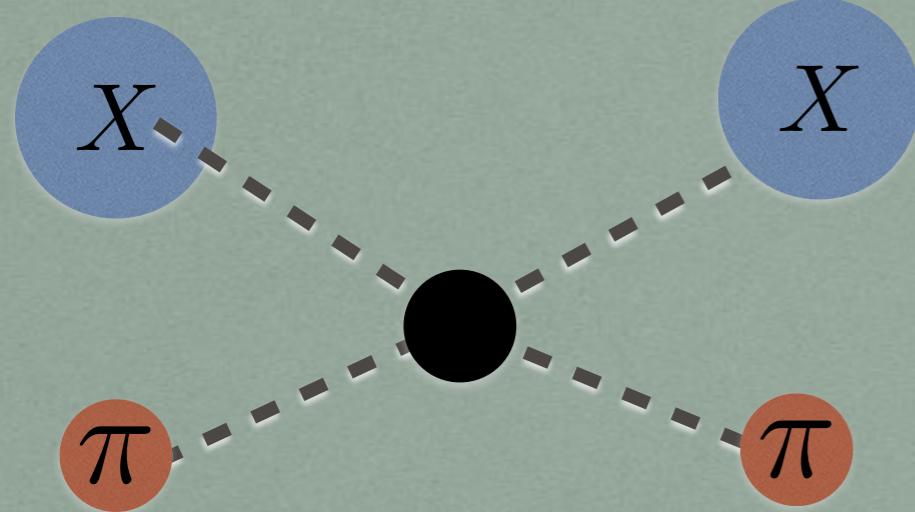
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Online at [stacks.iop.org/JPhysA/40/9655](https://stacks.iop.org/JPhysA/40/9655)

It helps to realize that at least in principle it is possible to decouple the zero temperature dynamics and the quantum statistical sums. The argument is simple: the computation of the partition function  $Z = \text{Tr}(e^{-\beta H})$  is in principle possible from the complete knowledge of the zero temperature eigenstates of the Hamiltonian  $H$ . In practice this is rather difficult and one resorts to perturbative methods such as the Matsubara method, which unfortunately entangles the zero temperature dynamics from the quantum statistical mechanics. However,

# IN-MEDIUM EFFECTS FROM S-MATRIX



$$\Sigma_A(E_A) = \int \frac{d^3 k_B}{(2\pi)^3} \frac{1}{2E_B} n_{\text{th}}(E_B) T(AB \rightarrow AB).$$

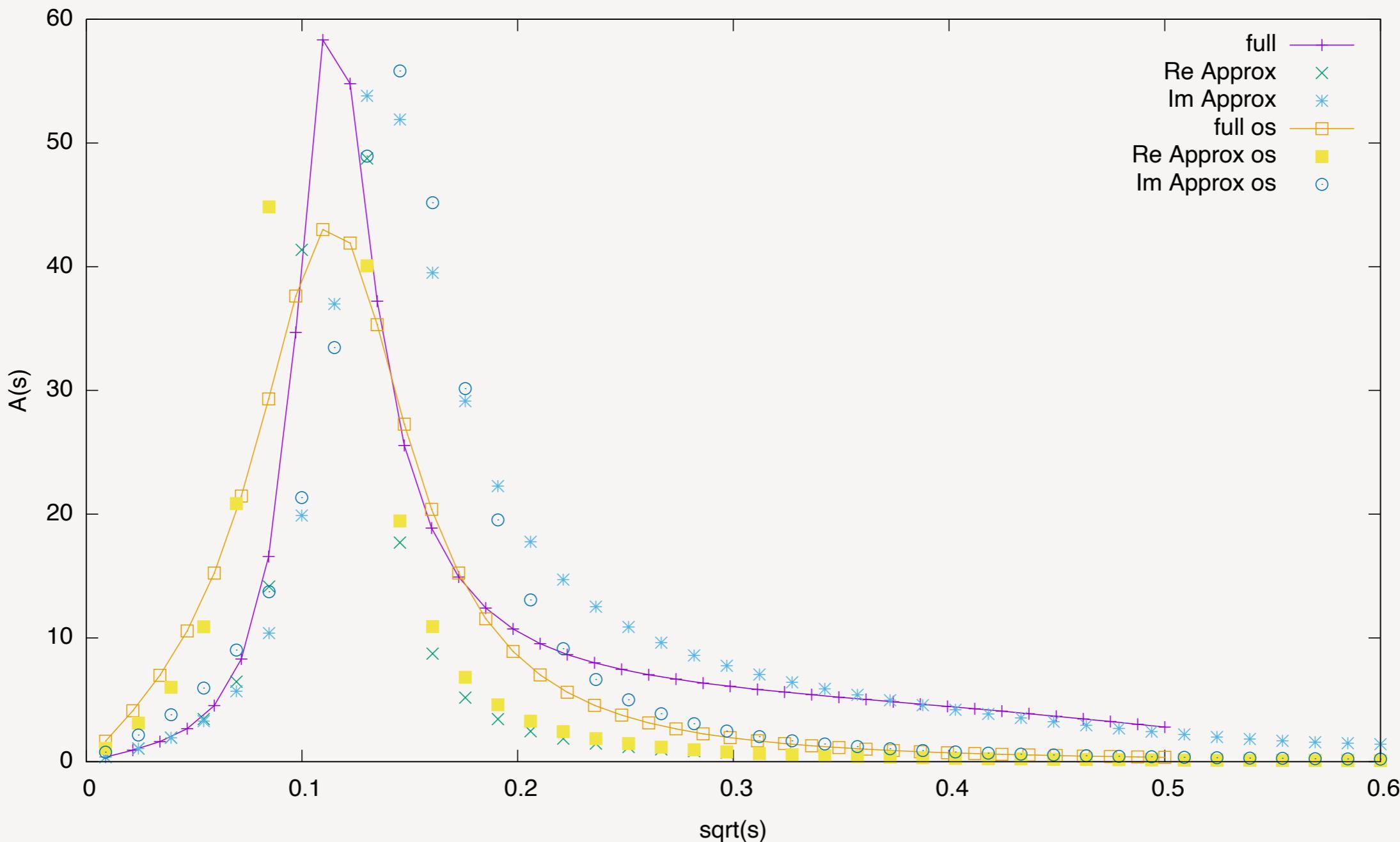
$$\Delta P = N_{\text{th}}^A N_{\text{th}}^B \times \frac{4\pi f}{2m_{\text{red}}}.$$

$$\begin{aligned}\Delta m_A &= \frac{1}{2E_A} \operatorname{Re} \Sigma_A(p) \\ &\approx N_{\text{th}}^B \times \frac{-4\pi f}{2m_{\text{red}}}.\end{aligned}$$

A. Schenk NPB 363 (1991)

S. Jeon and P. J. Ellis PRD 58 045013 (1998)

# IN-MEDIUM EFFECTS FROM S-MATRIX

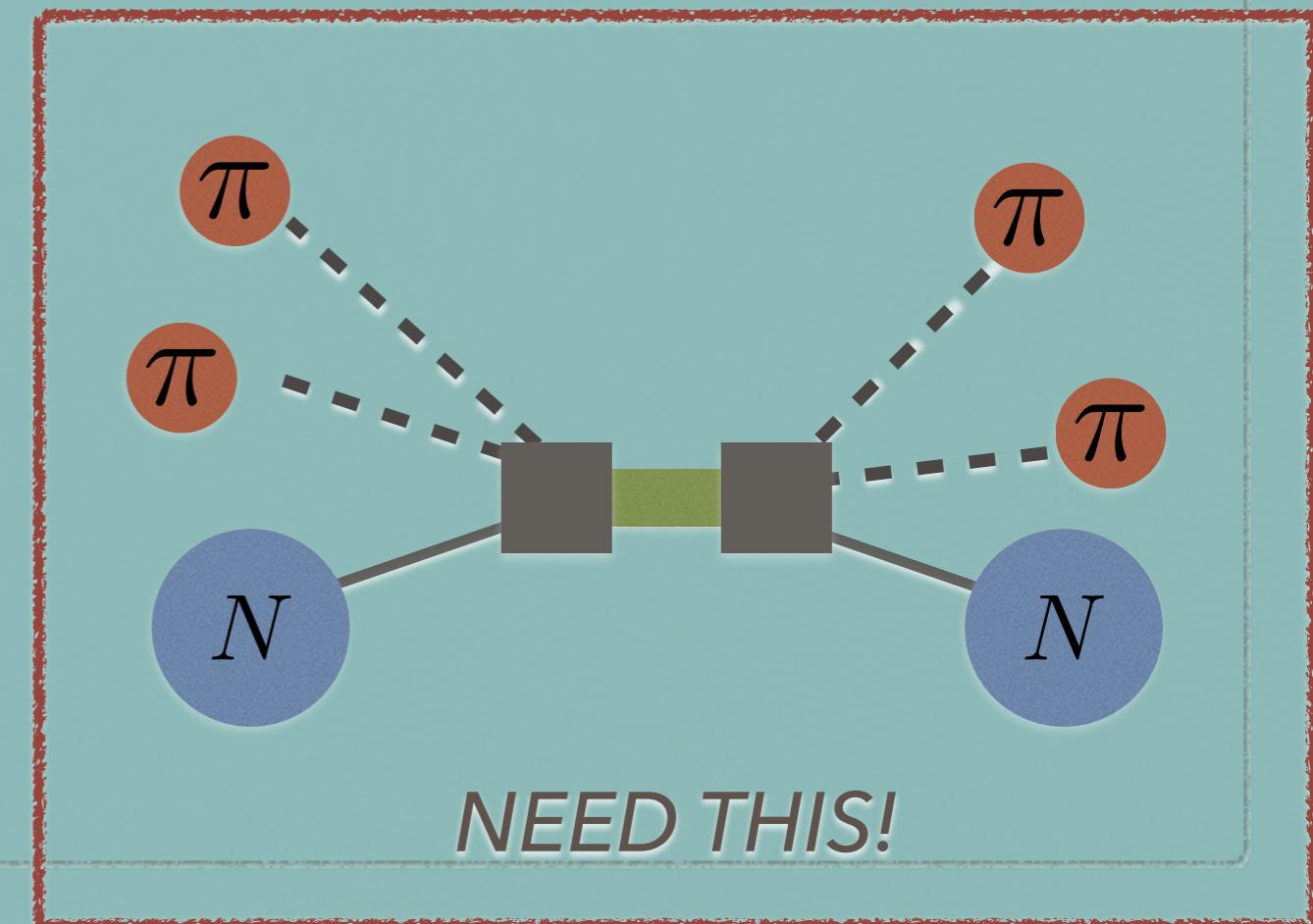
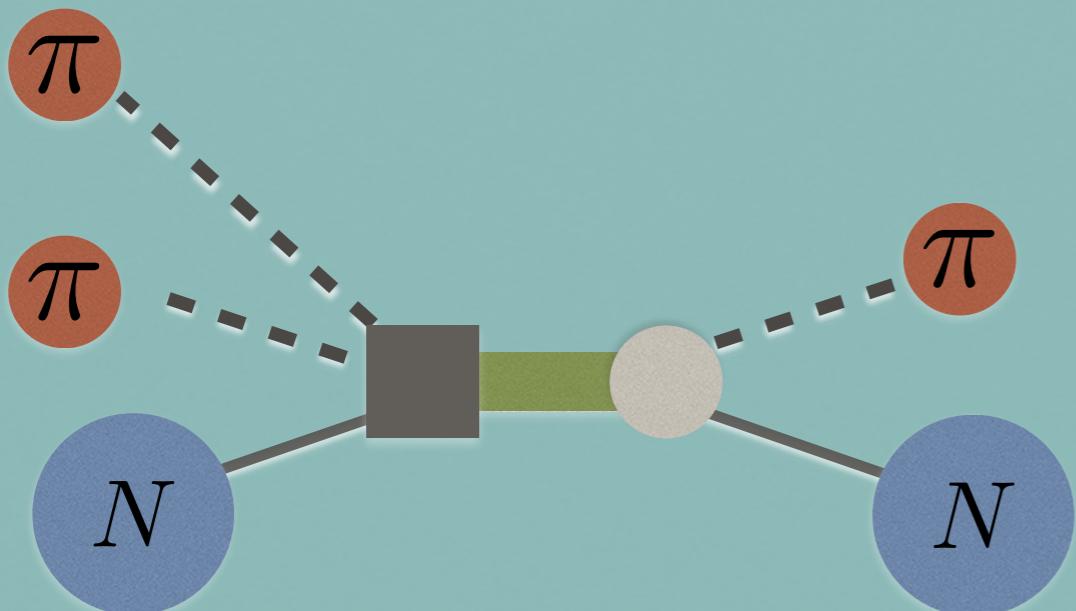
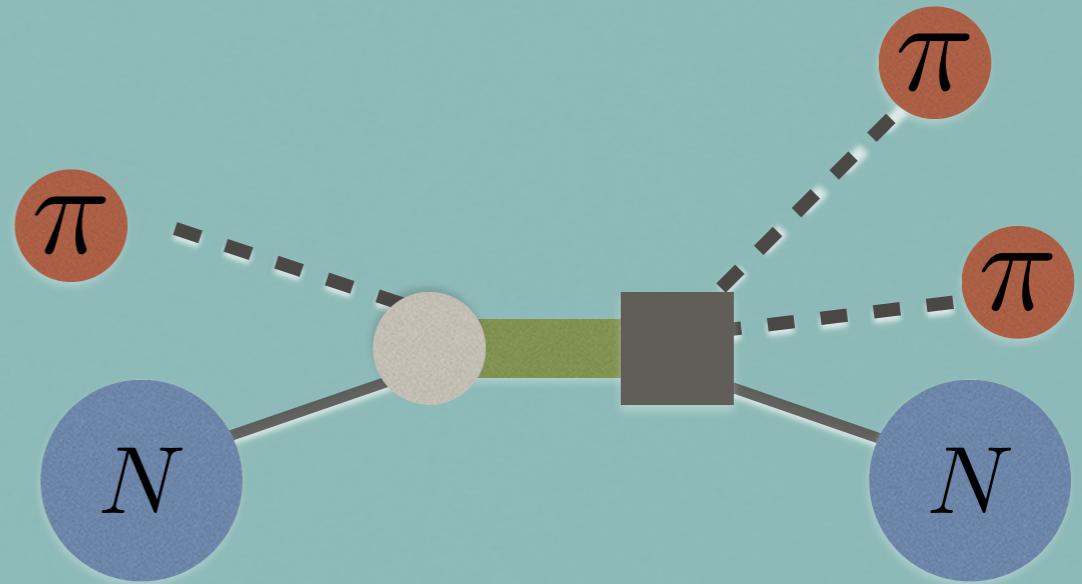
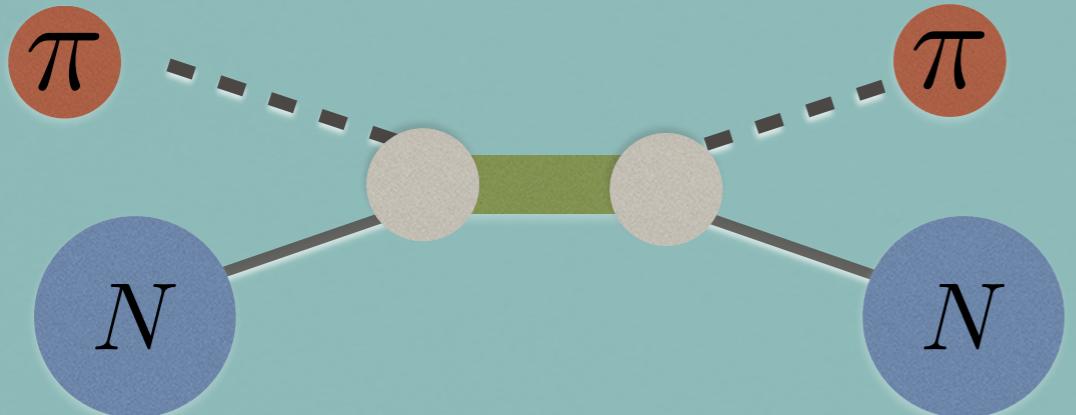


A. Schenk NPB 363 (1991)

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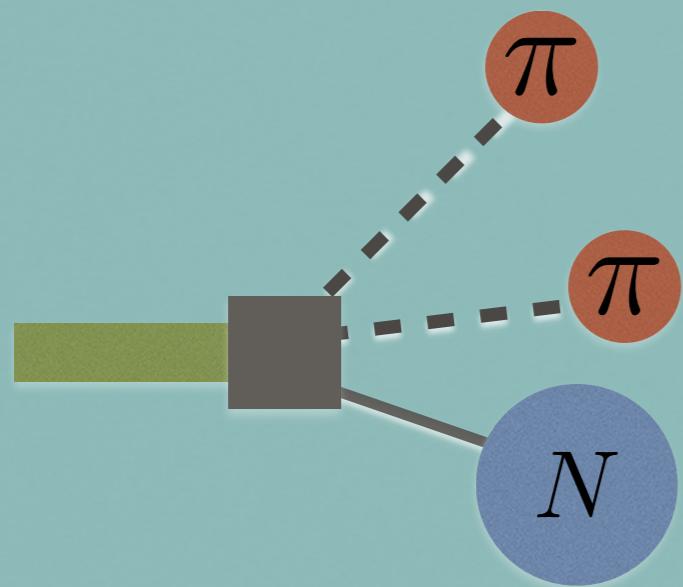
$B)$   
 $)$

# ISOBAR MODEL

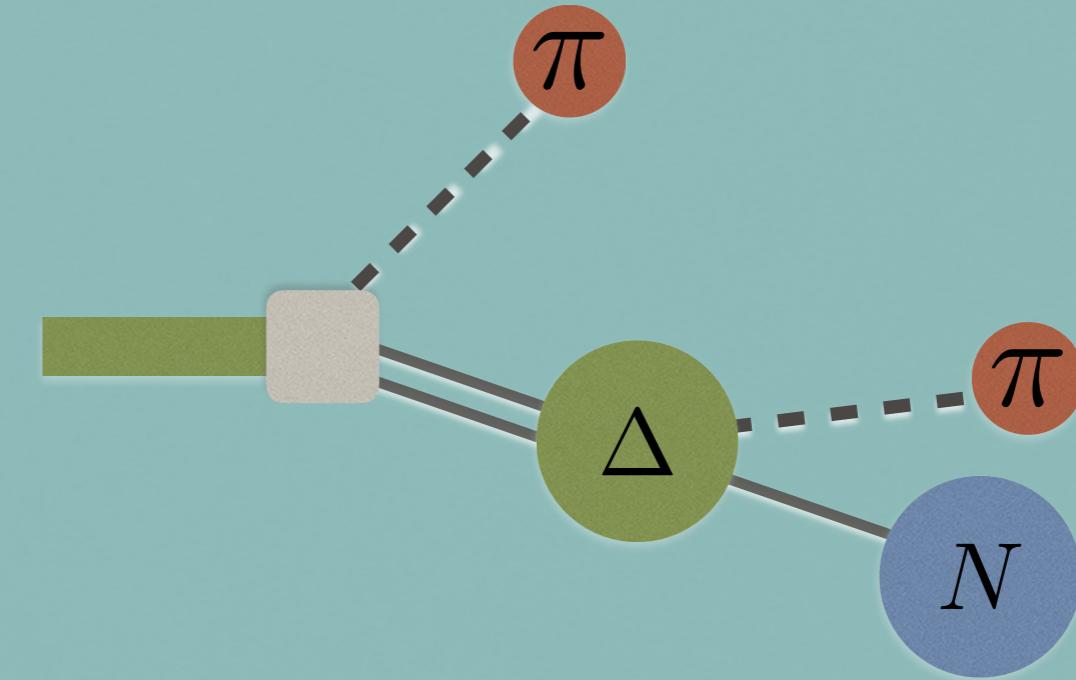


# ISOBAR MODEL

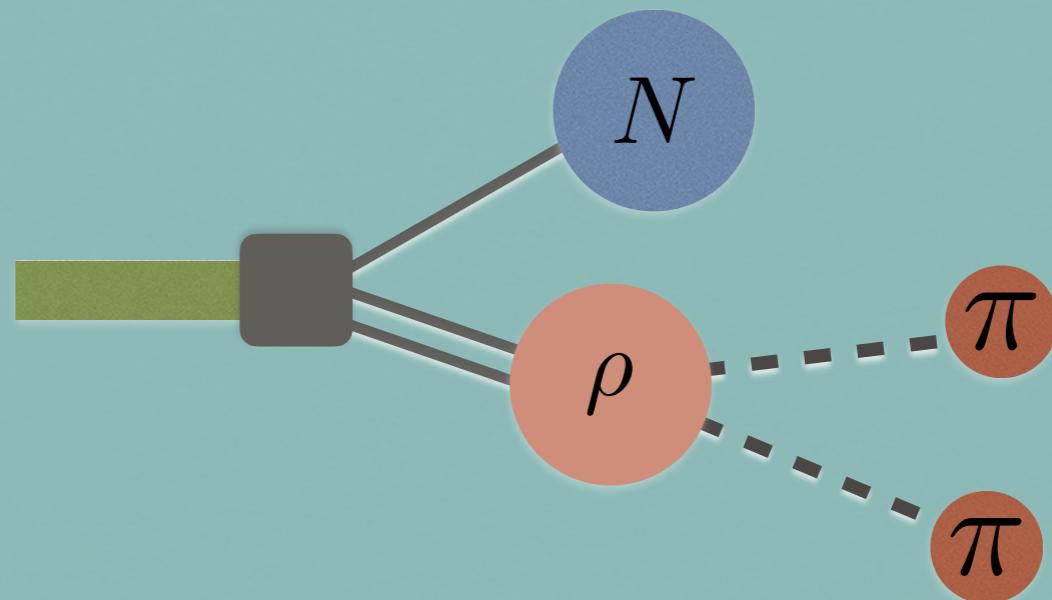
*sequential decay model*



$\approx$



*and / or*



# COMING SOON

- Guestimate of  $\pi\pi N$  from LQCD & invariant mass spectrum
- Towards femto
- Virial expansion approach to dense(r) matter

**THANK YOU**

**LQCD**

# FLUCTUATIONS

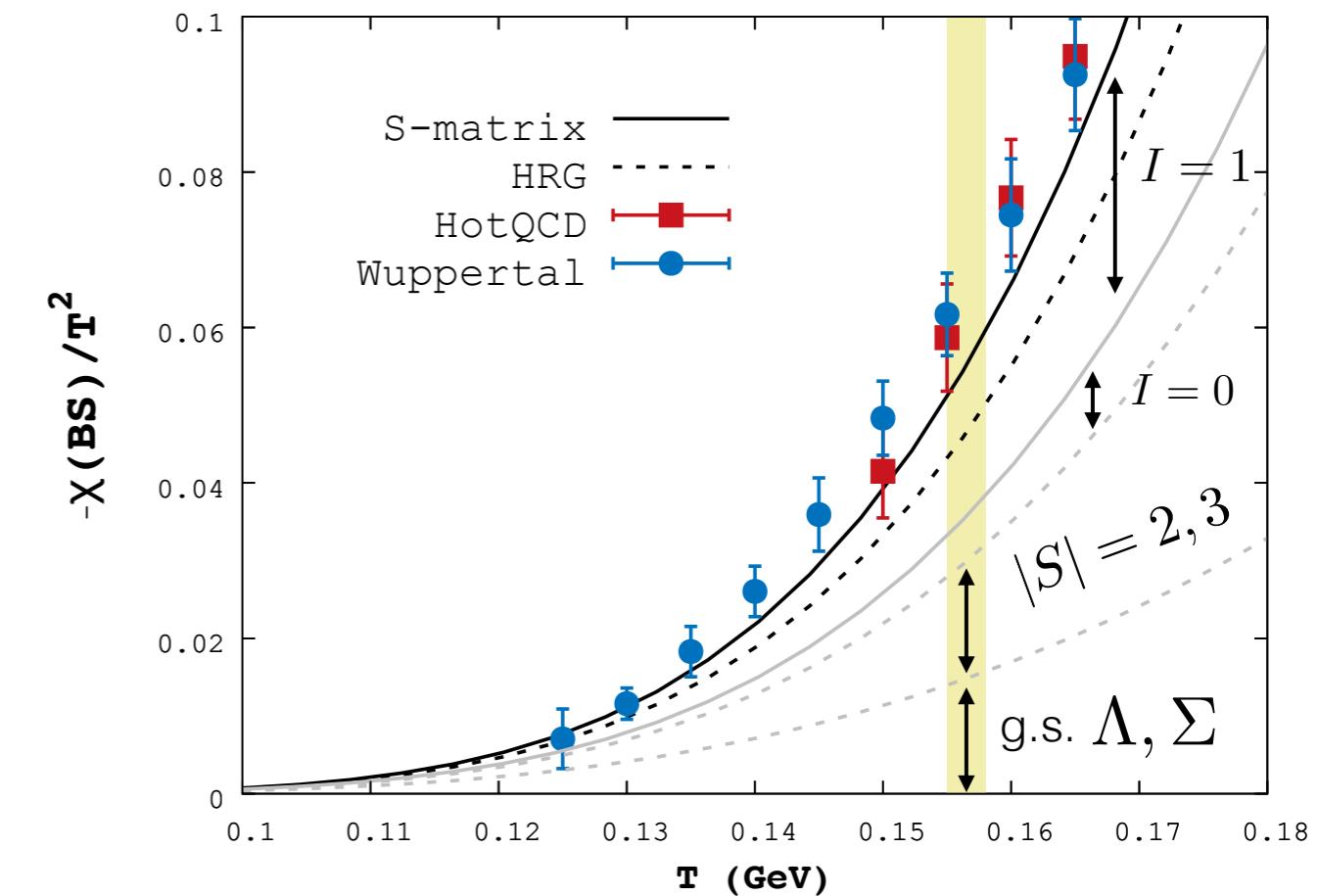
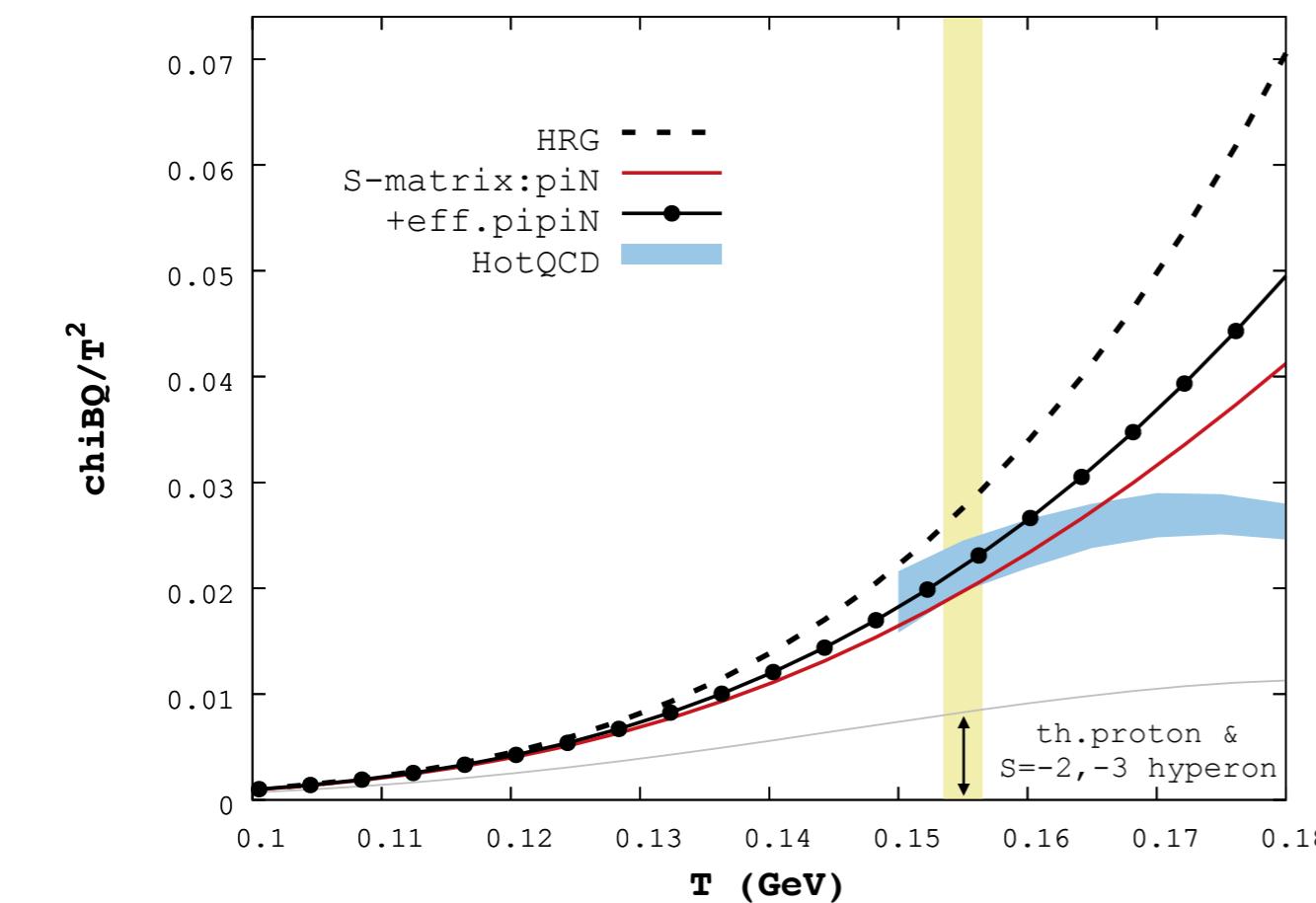
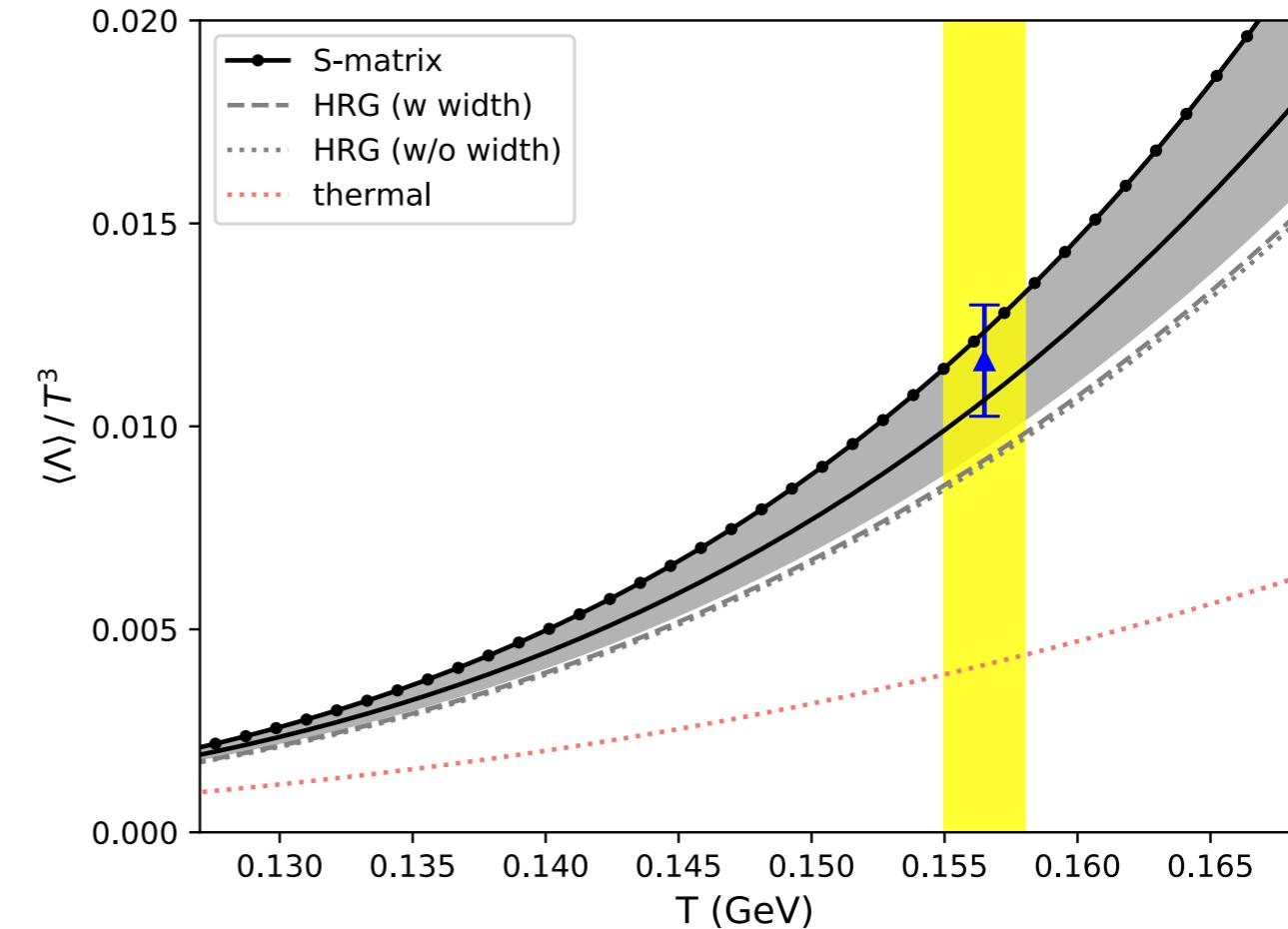
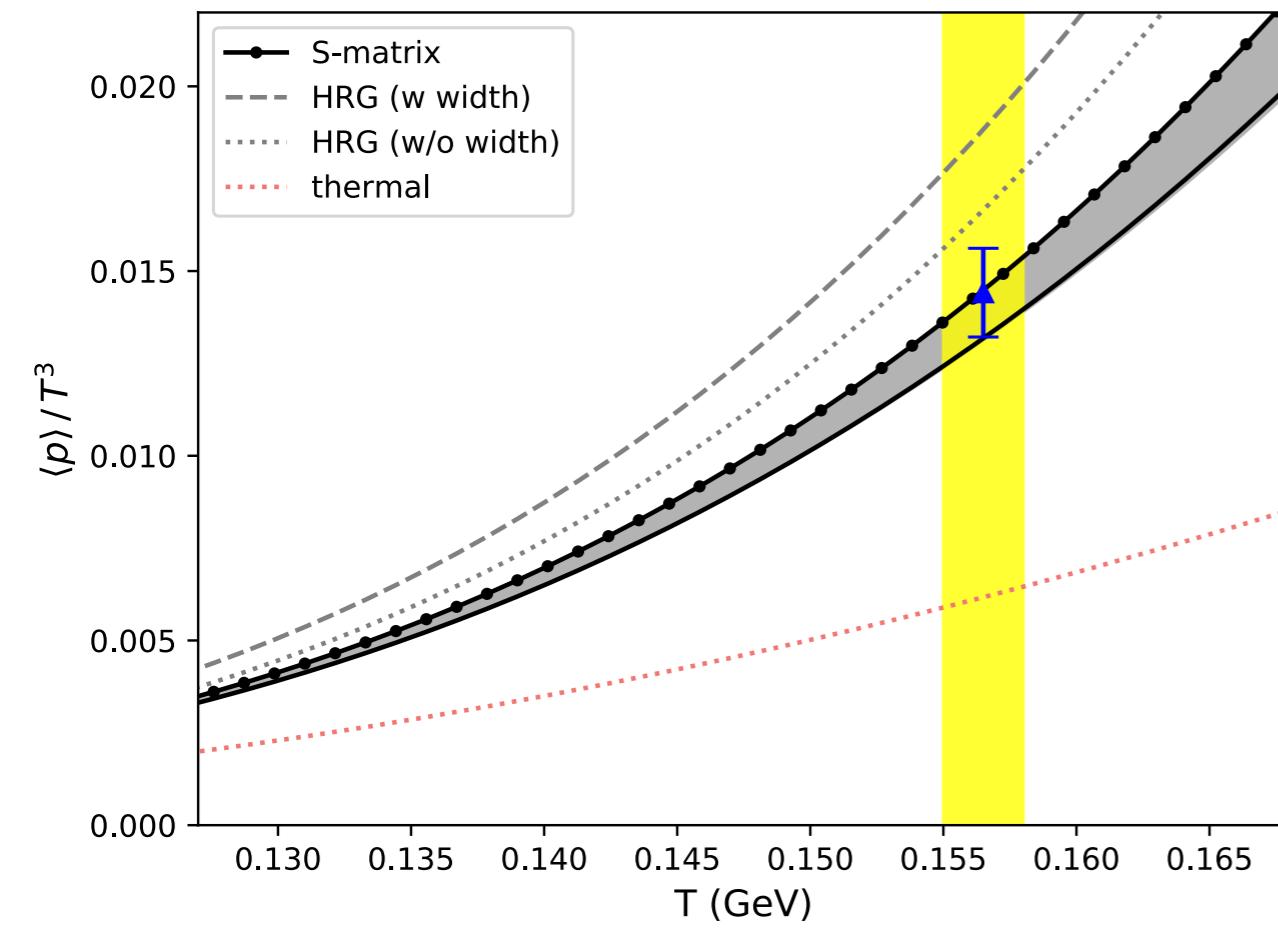
- studying the system by linear response

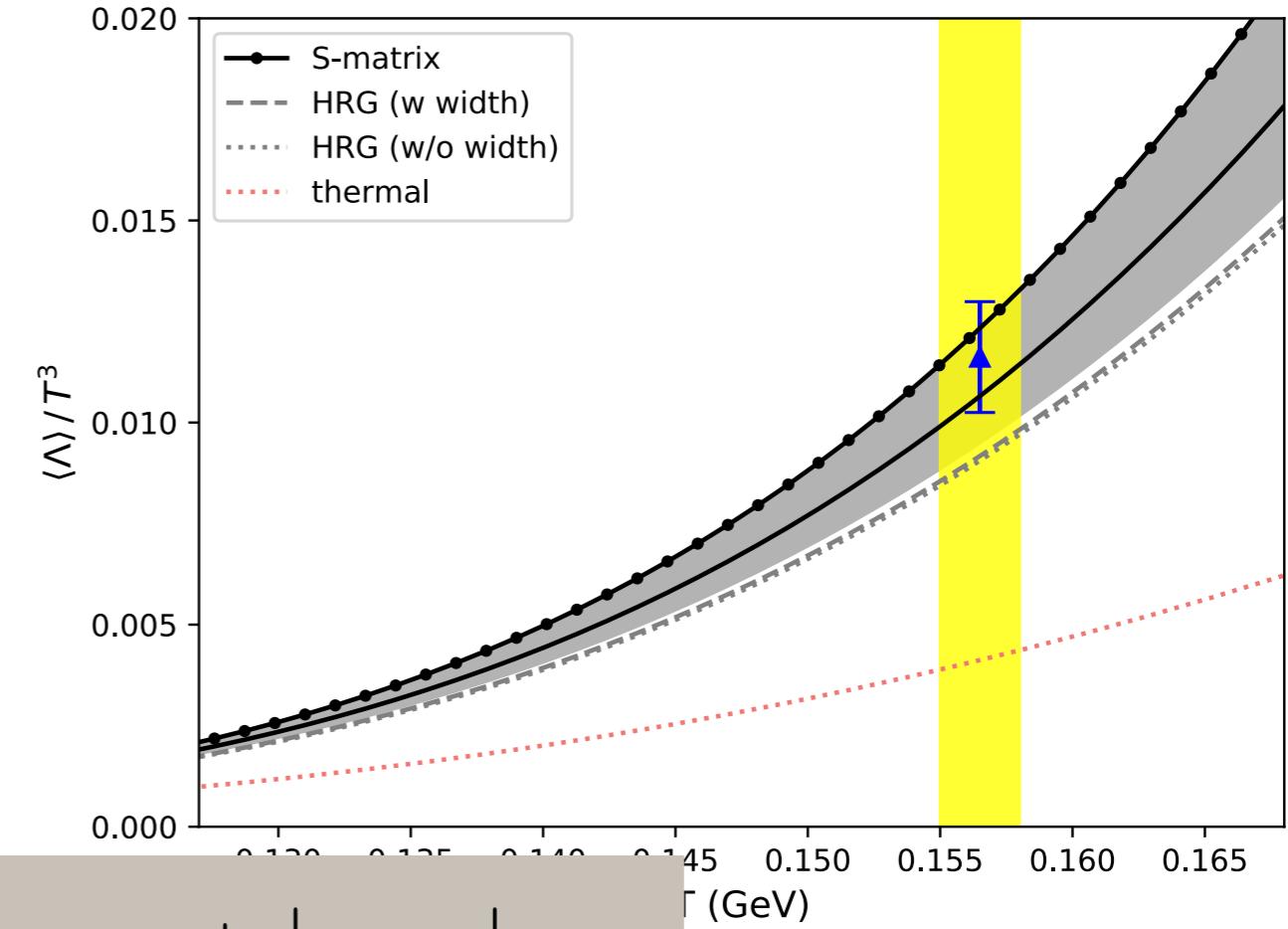
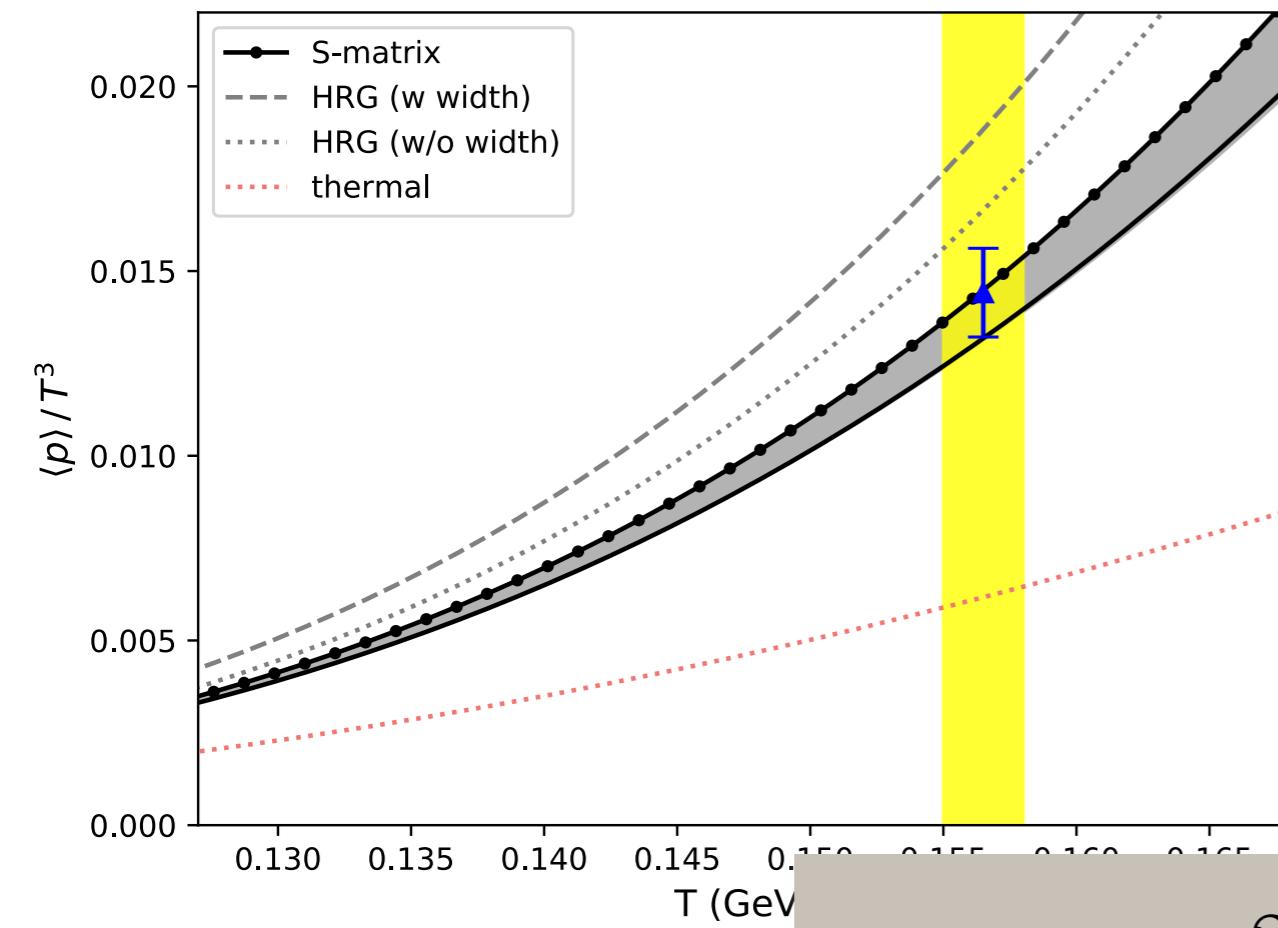


$$\mu = \mu_B B + \mu_Q Q + \mu_S S$$

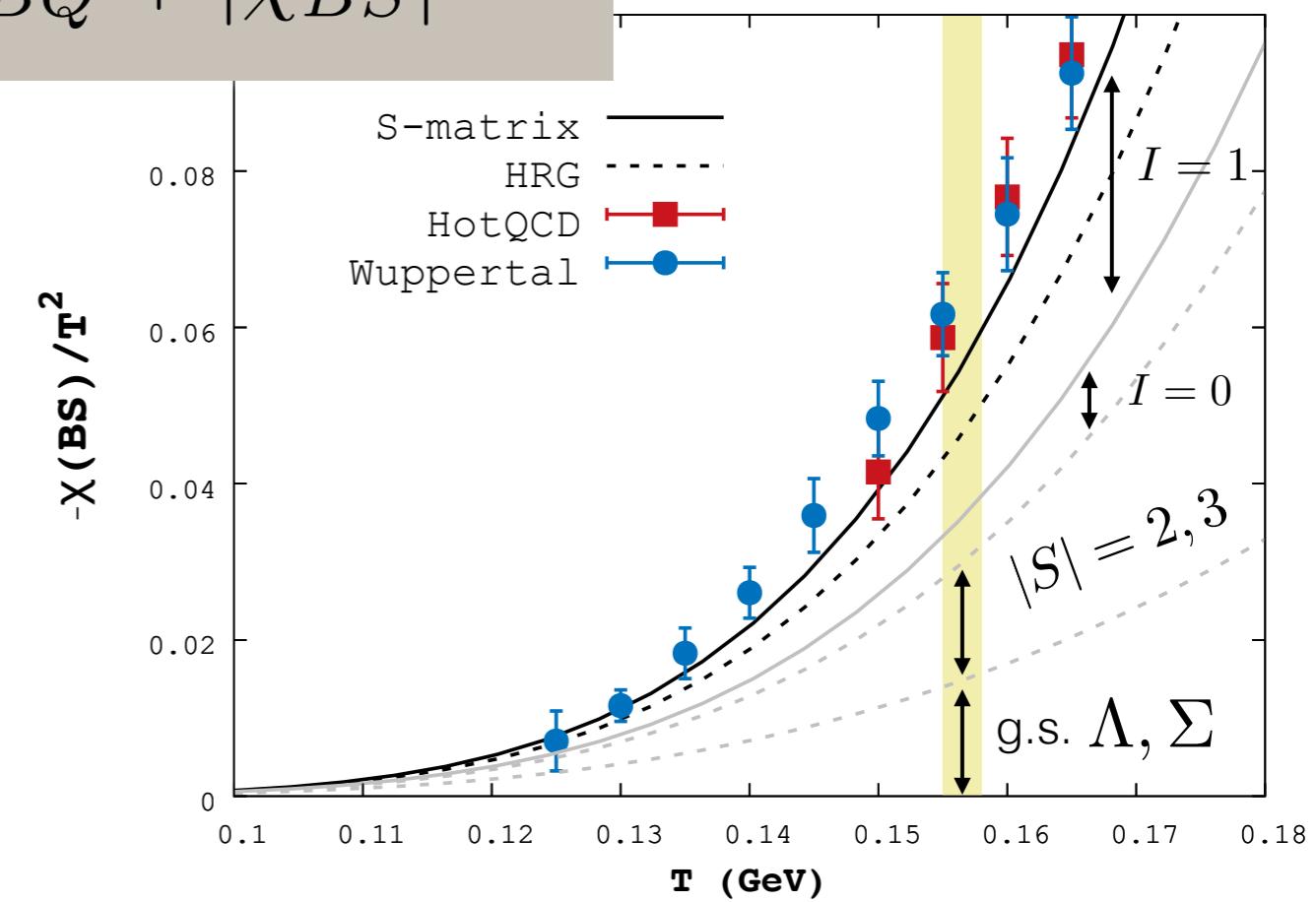
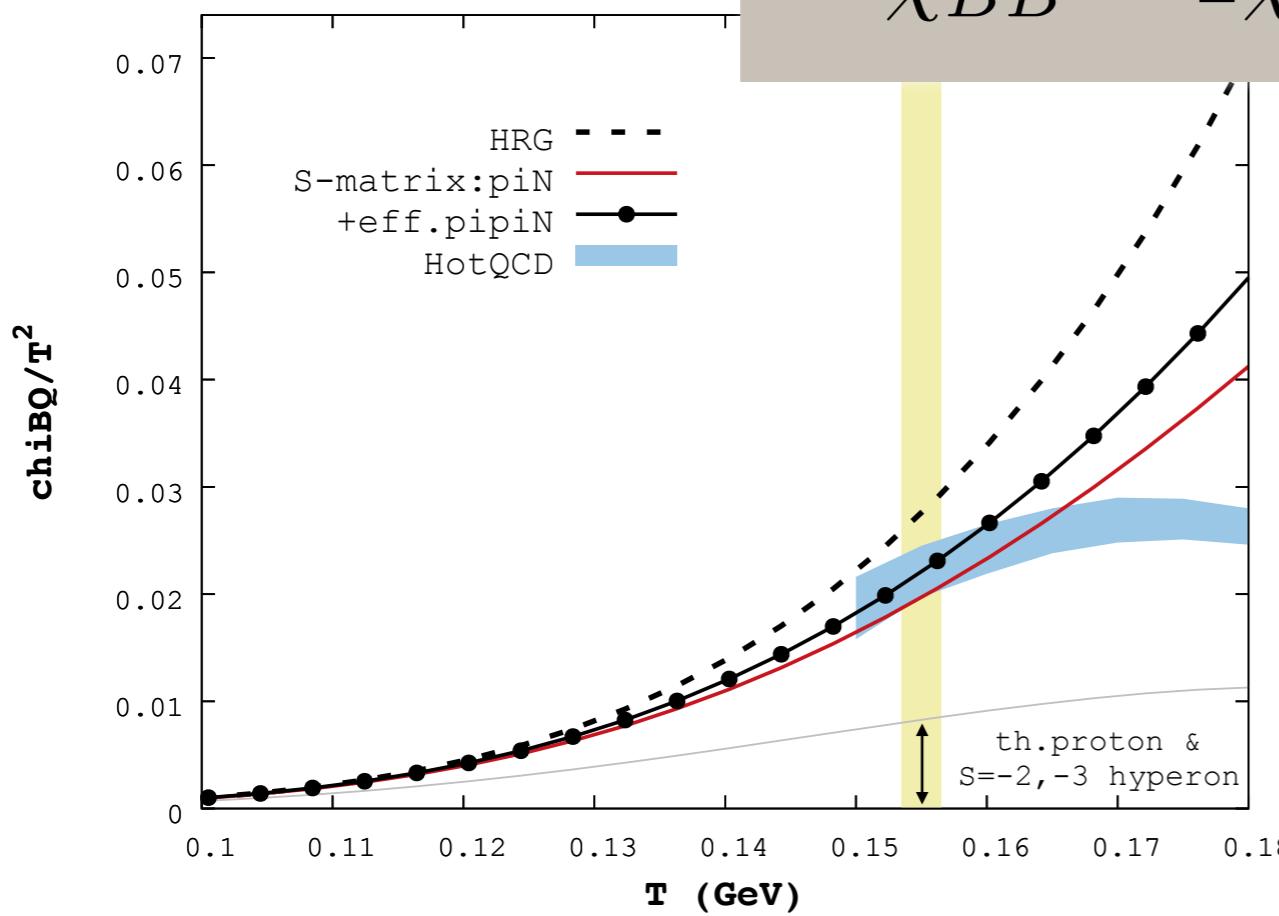
$$\chi_{B,S,\dots} = \frac{1}{\beta V} \frac{\partial^2}{\partial \bar{\mu}_B \partial \bar{\mu}_S \dots} \ln Z$$

 $\mu_B$  $\mu_Q$  $\mu_S$  $m_q$





$$\chi_{BB} = 2\chi_{BQ} + |\chi_{BS}|$$



# WIGNER, EISENBUD, SMITH, ...

$$S \rightarrow U^\dagger S_d U$$

$$S_d = \begin{pmatrix} e^{2i\delta_{\text{res}}(s)} & 0 \\ 0 & 1 \end{pmatrix},$$
$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}.$$

$$\text{BR}_a = \cos^2 \theta = \frac{g_a^2 \phi_a}{g_a^2 \phi_a + g_b^2 \phi_b},$$

$$\text{BR}_b = \sin^2 \theta = \frac{g_b^2 \phi_b}{g_a^2 \phi_a + g_b^2 \phi_b}.$$

