Approaching a SU(3) Energy Density Functional

Horst Lenske
Institut für Theoretische Physik, JLU Giessen
and
Madhumita Dhar
Cooch Behar Engineering College

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Cooch Behar Government Engineering College

Agenda

- **SU(3) symmetry and octet BB-interactions**
- **Exploiting SU(3) relations for a covariant Octet-EDF**
- **Baryon mean-fields in asymmetric nuclear matter**
- **New effect: Λ-Σ mixing induced by the nuclear isovector mean-field**
- **Summary and outlook**

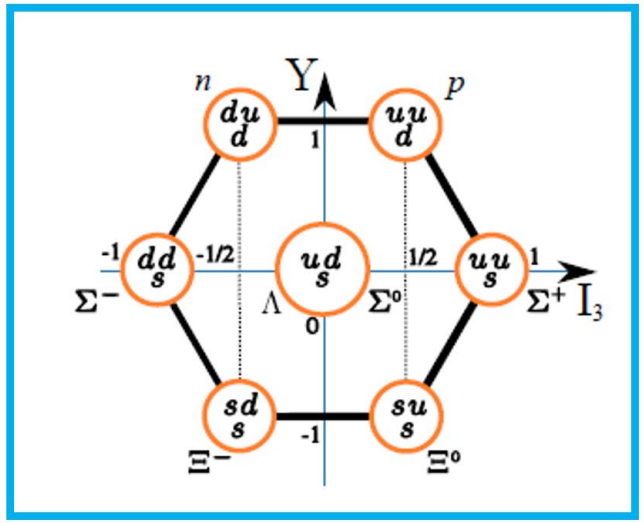
H. Lenske, M. Dhar, EPJ Web Conf. 271 (2022) 05003 (Hyp 2022, Prague), 2208.04916 [nucl-th]

Theoretical Background:

H. Lenske, M. Dhar, *Lect.Notes Phys. 948 (2018) 161*

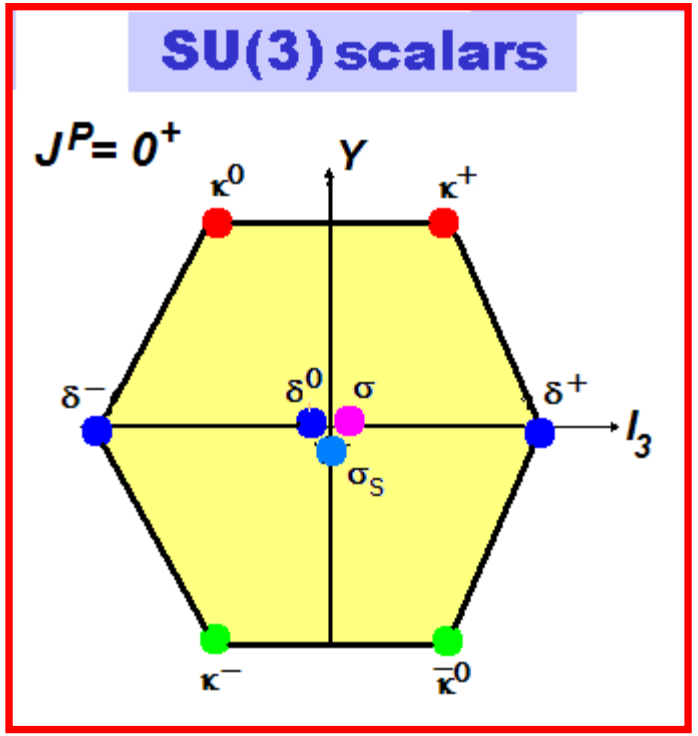
H. Lenske, M. Dhar, Th. Gaitanos, Xu Cao, *Prog.Part.Nucl.Phys.* 98 (2018) 119

SU(3) Scheme of Octet Baryon Interactions

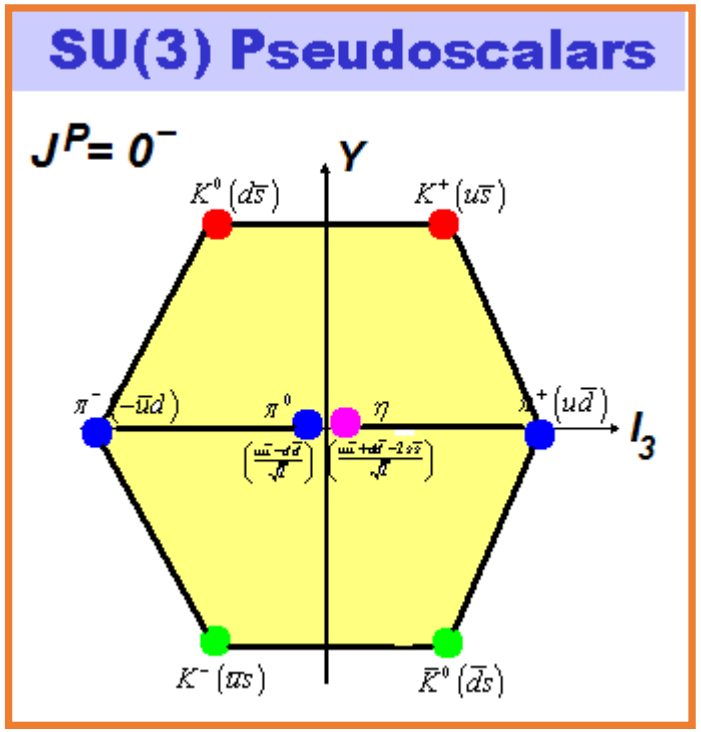


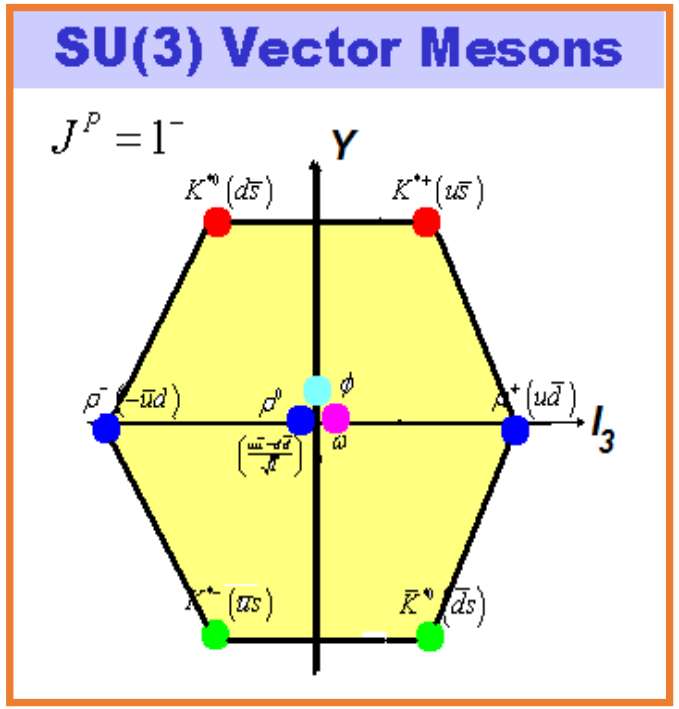
$$B = \frac{1}{\sqrt{2}} \sum_{a=1}^{8} \lambda^a B^a = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

The Octet/Nonet Mesons



Nuclear Mean-Fields





Lagrangian Density of BB-Octet Interactions

$$\mathcal{L}_{int}^{\mathcal{P}} = -\sqrt{2}\left\{g_D\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_8\right]_D + g_F\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_8\right]_F\right\} - g_S\frac{1}{\sqrt{3}}\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_1\right]_S$$

P=Pseudoscalar, Vector, and Scalar Meson Exchange

anti−symmetric $[\bar{B}, B] = \bar{B}B - B\bar{B}$ and symmetric $\{\bar{B}, B\} = \bar{B}B + B\bar{B}$ configurations

$$\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}\right]_D = \mathrm{Tr}\left(\left\{\overline{\mathcal{B}}, \mathcal{B}\right\}\mathcal{P}_8\right) \quad , \quad \left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}\right]_F = \mathrm{Tr}\left(\left[\overline{\mathcal{B}}, \mathcal{B}\right]\mathcal{P}_8\right) \quad , \quad \left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}\right]_S = \mathrm{Tr}(\overline{\mathcal{B}}\mathcal{B})\mathrm{Tr}(\mathcal{P}_1)$$

Octet Baryon Physics:
3 sets of 3 Fundamental Coupling Constants $\{g_D, g_F, g_S\}$ fix the in total 48 BB'M Vertices

BBM Vertices under Singlet-Octet Mixing: (mean-field producing) Vector Couplings

Vertex	Coupling constant
$NN\omega$	$g_N^\omega = g_S \cos(\theta) + \sqrt{\frac{3}{2}} g_F \sin(\theta) - \frac{1}{\sqrt{6}} g_D \sin(\theta)$
$NN\phi$	$g_N^\phi = g_S \sin(\theta) - \sqrt{\frac{3}{2}} g_F \cos(\theta) + \frac{1}{\sqrt{6}} g_D \cos(\theta)$
$NN\rho$	$g_N^\rho = \sqrt{2}(g_F + g_D)$
$\Lambda\Lambda\omega$	$g_\Lambda^\omega = g_S \cos(\theta) - \sqrt{\frac{2}{3}} g_D \sin(\theta)$
$\Lambda\Lambda\phi$	$g_\Lambda^\phi = g_S \sin(\theta) + \sqrt{\frac{2}{3}} g_D \cos(\theta)$
$\Sigma\Sigma\omega$	$g_\Sigma^\omega = g_S \cos(\theta) + \sqrt{\frac{2}{3}} g_D \sin(\theta)$
$\Sigma\Sigma\phi$	$g_\Sigma^\phi = g_S \sin(\theta) - \sqrt{\frac{2}{3}} g_D \cos(\theta)$
$\Sigma\Sigma\rho$	$g_\Sigma^\rho = \sqrt{2} g_F$
$\Lambda\Sigma\rho$	$g_{\Lambda\Sigma}^\rho = \sqrt{\frac{2}{3}} g_D$
$\Xi\Xi\omega$	$g_\Xi^\omega = g_S \cos(\theta) - \sqrt{\frac{3}{2}} g_F \sin(\theta) - \frac{1}{\sqrt{6}} g_D \sin(\theta)$
$\Xi\Xi\phi$	$g_\Xi^\phi = g_S \sin(\theta) + \sqrt{\frac{3}{2}} g_F \cos(\theta) + \frac{1}{\sqrt{6}} g_D \cos(\theta)$
$\Xi\Xi\rho$	$g_\Xi^\rho = \sqrt{2}(g_F - g_D)$

Guiding Principles for a SU(3) DFT Mean-Field Dynamics

- **Interactions inherit SU(3) symmetry from NN (BB) scattering data**
- **Incorporate SU(3)-breaking by use of physical masses and empirical coupling constants**
- **Free space, in-medium Bethe-Salpeter, and vertex equations conserve the fundamental symmetries**

The SU(3) DFT/EDF Program:

- **Three in-medium couplings are needed to fix the scalar and vector sets of $\{g_D, g_F, g_S\}$**
- **For known $g_{NN\omega}(\rho)$, $g_{NN\rho}(\rho)$ and imposing $g_{NN\phi}(\rho)=0$ → vector $g_D(\rho)$, $g_F(\rho)$, $g_S(\rho)$**
- **For known $g_{NN\sigma}(\rho)$, $g_{NN\delta}(\rho)$ and imposing $g_{NN\sigma'}(\rho)= 0$ → scalar $g_D(\rho)$, $g_F(\rho)$, $g_S(\rho)$**
- **Fix the mixing angle – ideal mixing $\tan(\theta)=1/\sqrt{2}$ ($\theta\sim35°$)**
- **→ SU(3) BBM vector and scalar mean-field couplings**

Covariant SU(3) Density Functional

$$\mathcal{L}_{int}^{DF} = -\sqrt{2} \sum_{\mathcal{M}\in\{\mathcal{P},\mathcal{S},\mathcal{V}\}} \left\{ g_D^{*(\mathcal{M})}(\hat{\rho})\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_8\right]_D + g_F^{*(\mathcal{M})}(\hat{\rho})\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_8\right]_F - g_S^{*(\mathcal{M})}(\hat{\rho})\frac{1}{\sqrt{6}}\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_1\right]_S \right\}$$

$$\hat{\rho} = F(j_\mu j^\mu)$$

Meson and Baryon Field Equations:

$$\left(\partial_\mu\partial^\mu + m_{\mathcal{M}}^2\right)\Phi_{\mathcal{M}}^s = \sum_{BB'} g_{BB'\mathcal{M}}^*(\hat{\rho})\rho^{BB's}, \qquad \left(\partial_\mu\partial^\mu + m_{\mathcal{M}}^2\right)V_{\mathcal{M}}^\lambda = \sum_{BB'} g_{BB'\mathcal{M}}^*(\hat{\rho})\rho^{BB'\lambda}$$

$$\left(\gamma_\mu\left(p^\mu - \Sigma_{\mathcal{B}}^\mu(\hat{\rho})\right) - M_{\mathcal{B}} + \Sigma_{\mathcal{B}}^{(s)}(\hat{\rho})\right)\Psi_{\mathcal{B}} = 0\,.$$

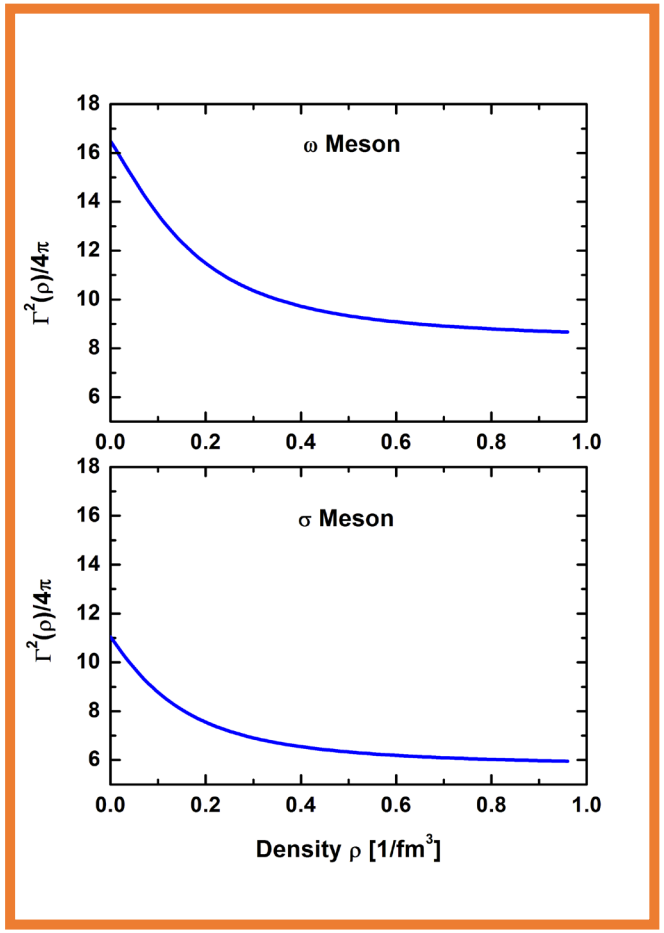
Evaluated in mean-field approximation

$$\Phi = \langle 0|\Phi|0\rangle + \delta\Phi \;;\; V^\mu = \langle 0|V^\mu|0\rangle + \delta V^\mu \;;\; \hat{\rho} = \langle 0|\hat{\rho}|0\rangle + \delta\rho \;;\; g_{BB'M}^*(\hat{\rho}) = g_{BB'M}(\rho_B) + \delta\rho\frac{\partial g_{BB'M}(\rho_B)}{\partial\rho_B}$$

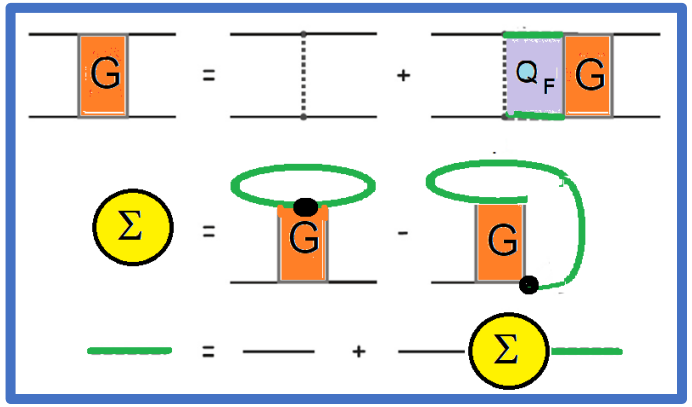
$$\phi = \langle 0|\Phi|0\rangle \;;\; V^0 = \langle 0|V^\mu|0\rangle\delta^{\mu 0} \;;\; \rho_B \equiv \langle 0|\hat{\rho}|0\rangle \;;\; \delta\rho = \hat{\rho} - \rho_B$$

The Practicioneer‘s Approach

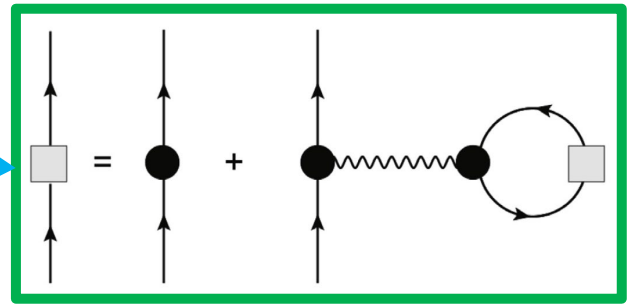
Covariant Nuclear EDF from DBHF Interactions

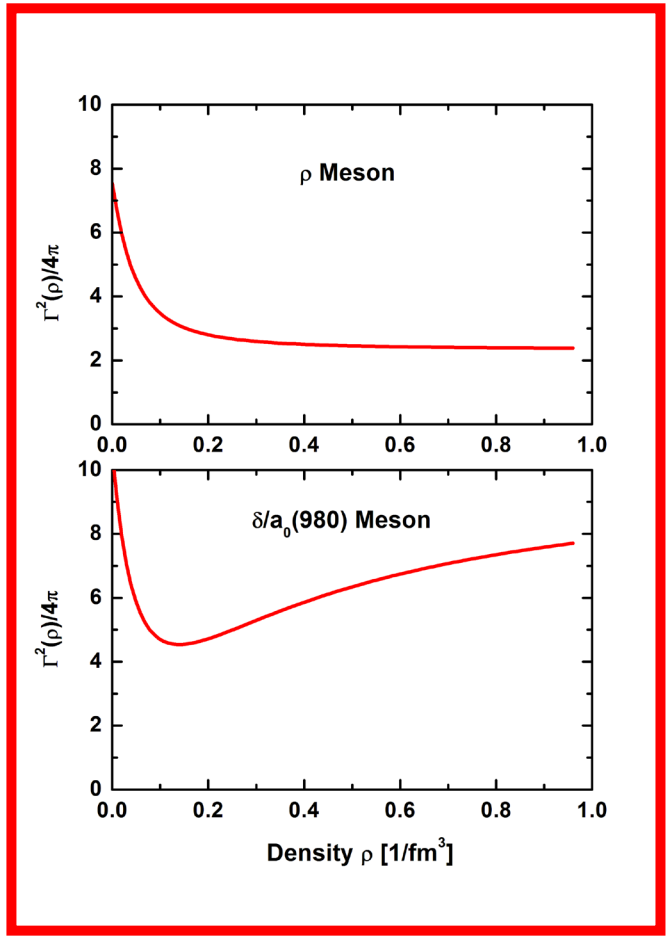


Isoscalar Vertices



$$\Gamma_{\mathcal{B}\mathcal{B}'a}(q_s, k_F) \simeq \frac{1}{1 - \int dq' V_a \mathcal{G}^* Q_F \,|_{\mathcal{B}\mathcal{B}'}} \; g_{\mathcal{B}\mathcal{B}'a}.$$



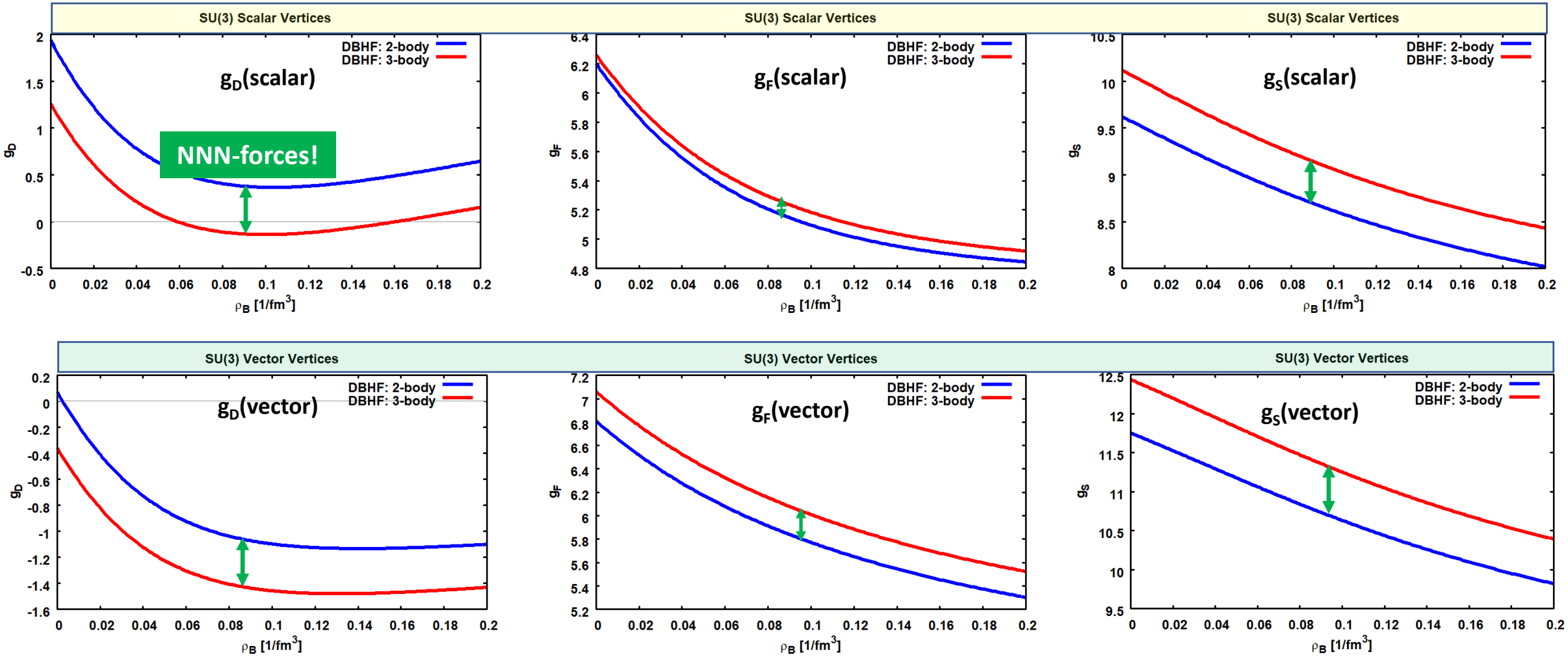


Isovector Vertices

H.L., Lect.Notes Phys. 641 (2004) 147; PPNP 98 (2018) 119

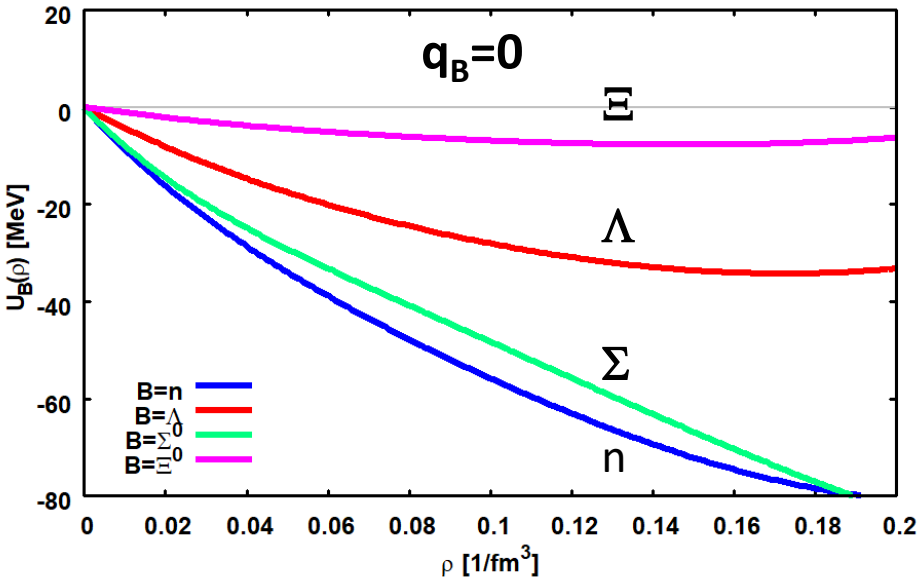
GI-DBHF SU(3) Vertices in Infinite Nuclear Matter

Ideal Mixing: θ=35.26°

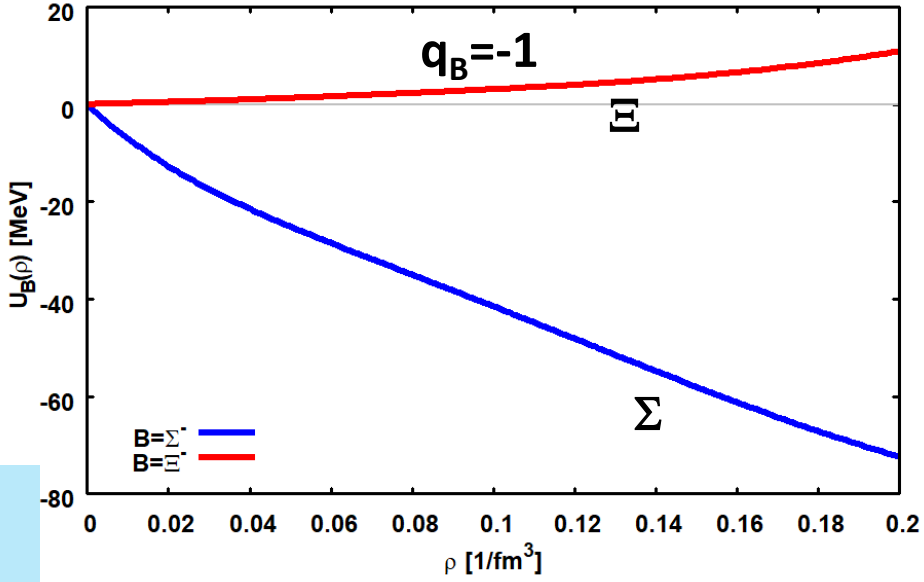


g_D g_F g_S

SU(3) Baryon Mean-Fields



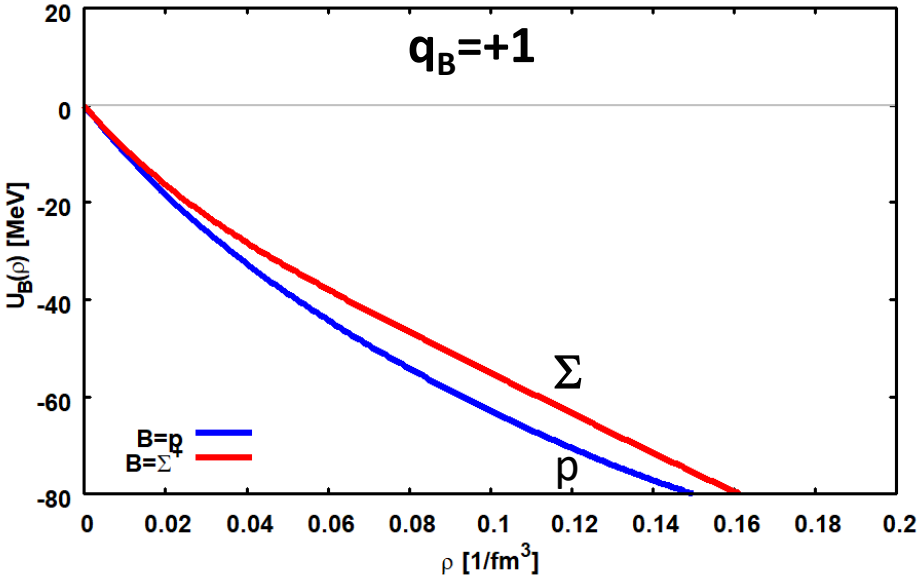
$U_B = U^S_B + U^V_B$



Z/A=0.4

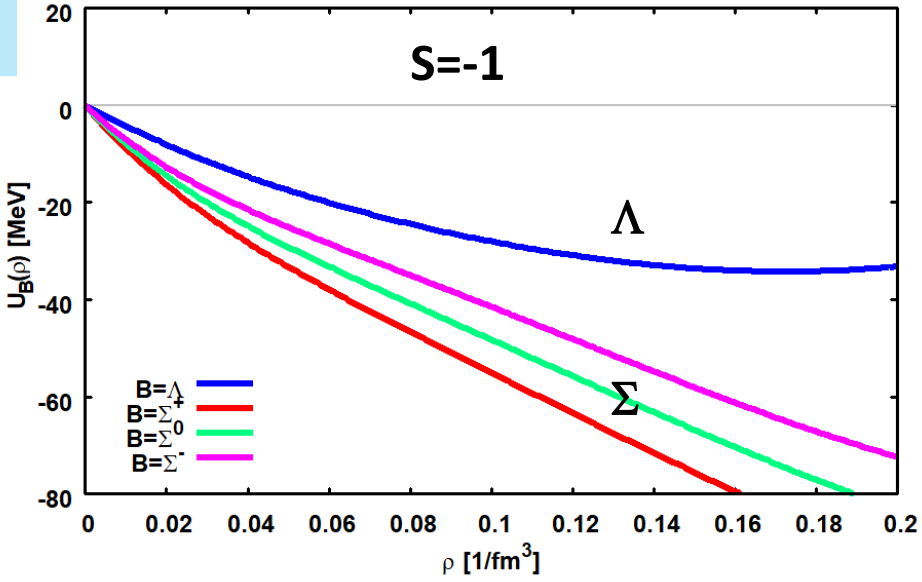
as e.g.

^{10}Be, ^{48}Ca, ^{124}Sn, ^{208}Pb



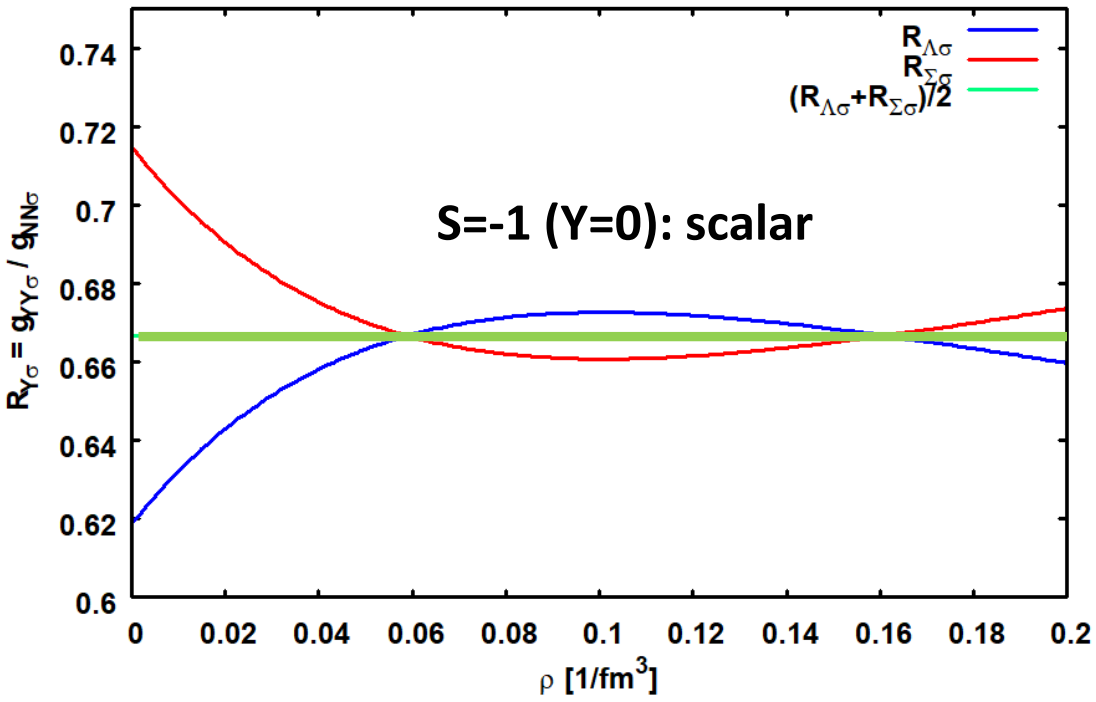
Similarity:

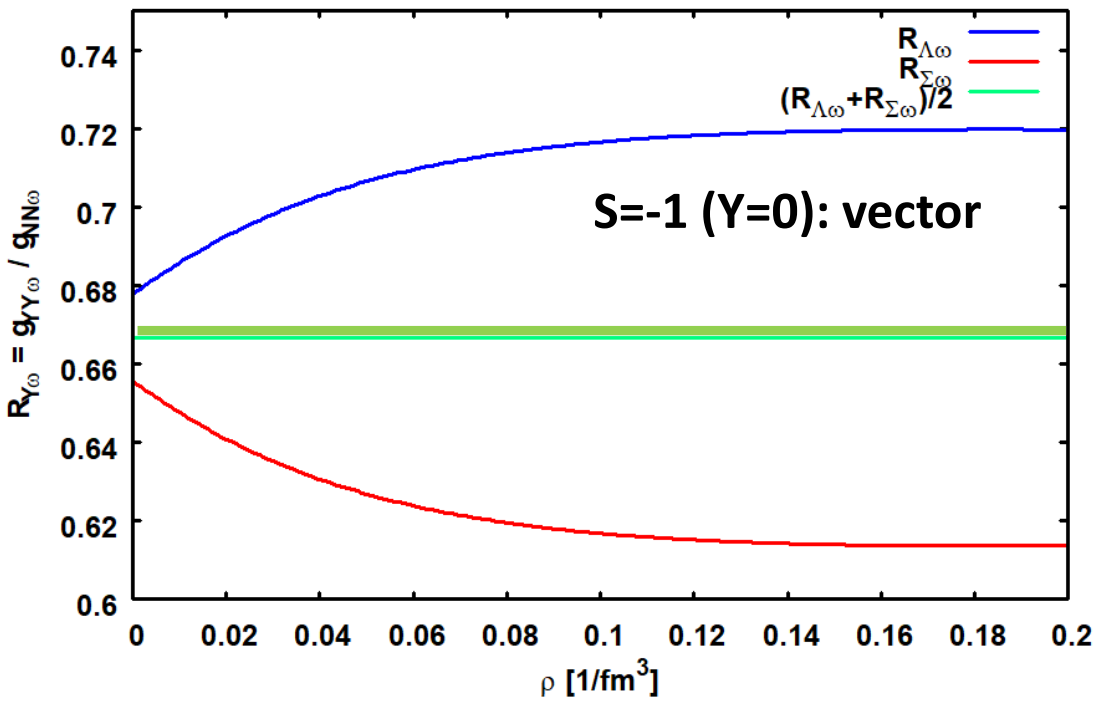
$\{\Sigma^+,\Sigma^0\} \sim \{p,n\}$



Hyperon Couplings and „Quark Scaling“

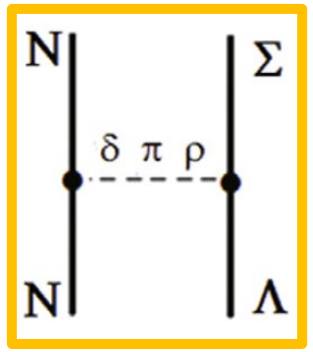
$$R_{YM} = \frac{g_{YYm}}{g_{NNm}} \approx \frac{N_Y(u,d-\text{quarks})}{N_Y(\text{quarks})} \underset{S=-1}{=} \frac{2}{3}$$





In-Medium Quark Scaling Hypothesis recovered for
Y($\Lambda\Sigma$)=0 and Y(Ξ)=-1 Hypercharge-Multiplets

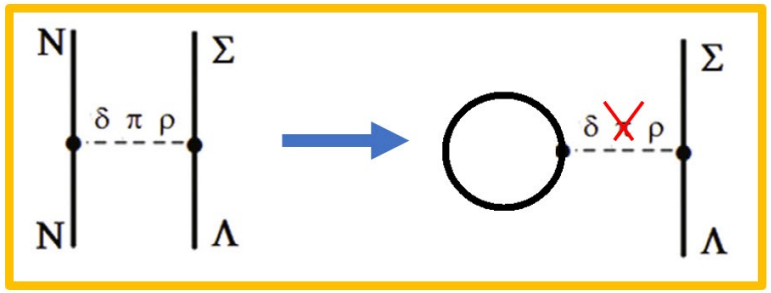
Λ-Σ Mixing by Isovector Interactions



$$\mathcal{L} = \sum_{b=\Lambda,\Sigma} \bar{\psi}_b \left(\gamma_\mu \left(p^\mu + V_b^\mu \right) - M_b - \phi_b \right) \psi_b + \bar{\psi}_\Lambda U_{\Lambda\Sigma} \psi_\Sigma + h.c.$$

$$U_{\Lambda\Sigma} = \left(g_{\Lambda\Sigma\delta} \boldsymbol{\phi}_\delta + g_{\Lambda\Sigma\rho}\, \boldsymbol{\gamma}_\mu \mathbf{V}_\rho^\mu \right) \cdot \boldsymbol{\tau}_{\Lambda\Sigma}$$

Λ-Σ Mixing inAsymmetric Nuclear Matter Induced by the Static Isovector Mean-Field

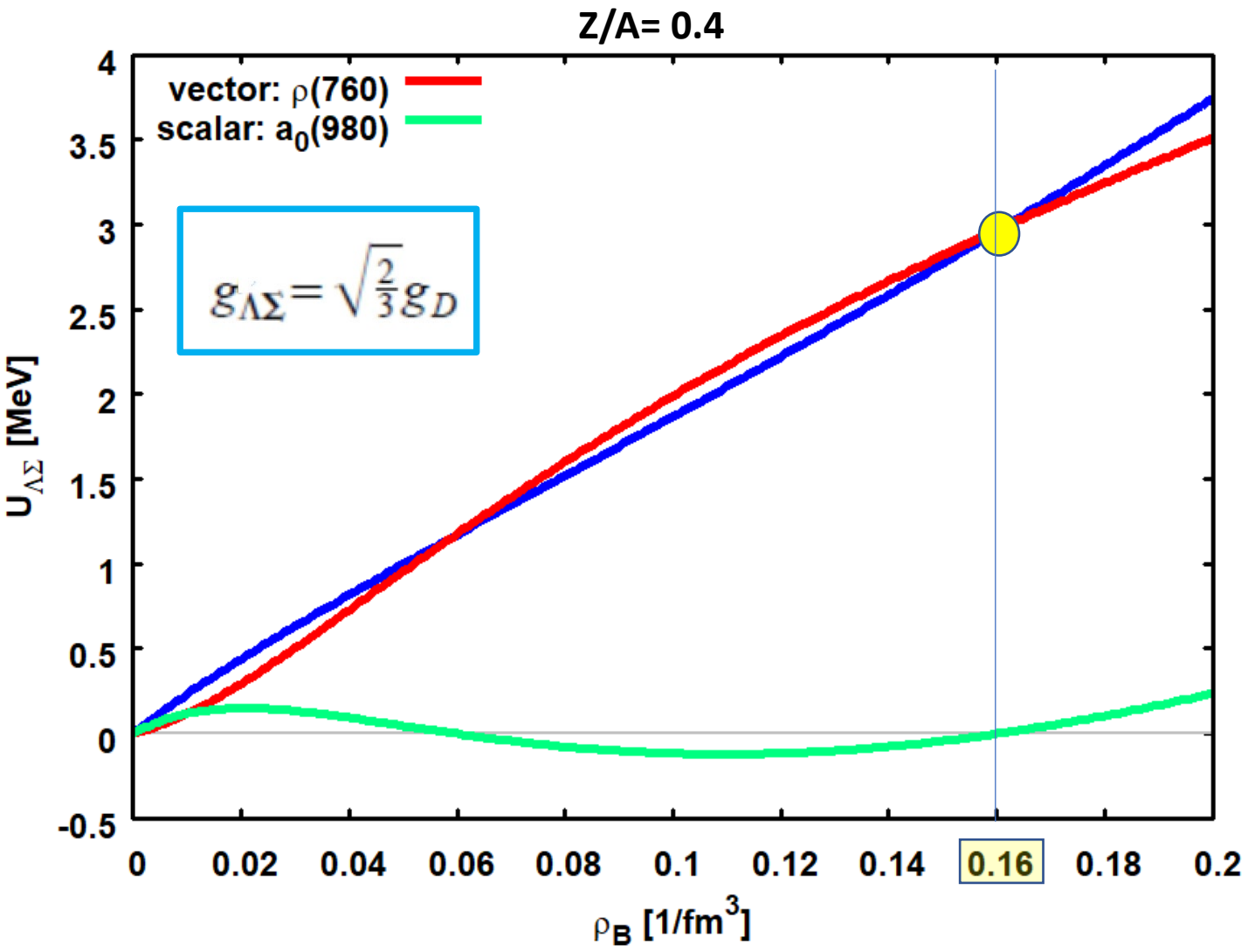


$$U_{\Lambda\Sigma}(\rho_B) = U_\delta^{(NN)}(\rho_B)\left(\frac{g_{\Lambda\Sigma\delta}}{g_{NN\delta}}\right) + U_\rho^{(NN)}(\rho_B)\left(\frac{g_{\Lambda\Sigma\rho}}{g_{NN\rho}}\right)$$

Mean-Field Induced Mixing

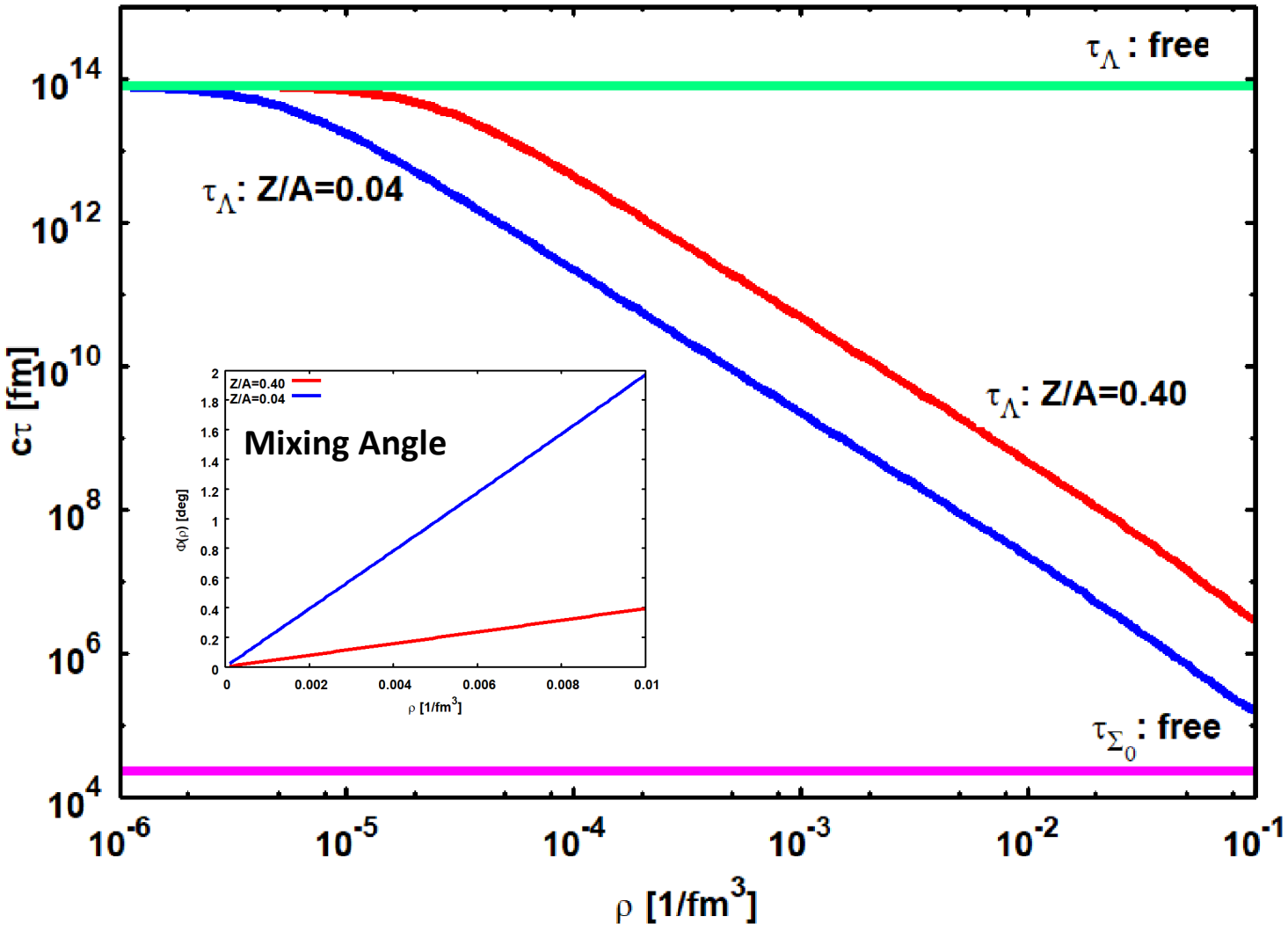
$$\begin{pmatrix} H_{\Lambda A} - E & U_{\Lambda\Sigma} \\ U_{\Lambda\Sigma}^\dagger & H_{\Lambda A} + m_{\Sigma\Lambda} - E \end{pmatrix} \begin{pmatrix} \left[\phi_\Lambda \otimes |A\rangle\right]_{I_A N_A} \\ \left[\phi_\Sigma \otimes |A\rangle\right]_{I_A N_A} \end{pmatrix} = 0$$

Λ–Σ Mean-Field Mixing SU(3) Potential in Asymmetric Nuclear Matter



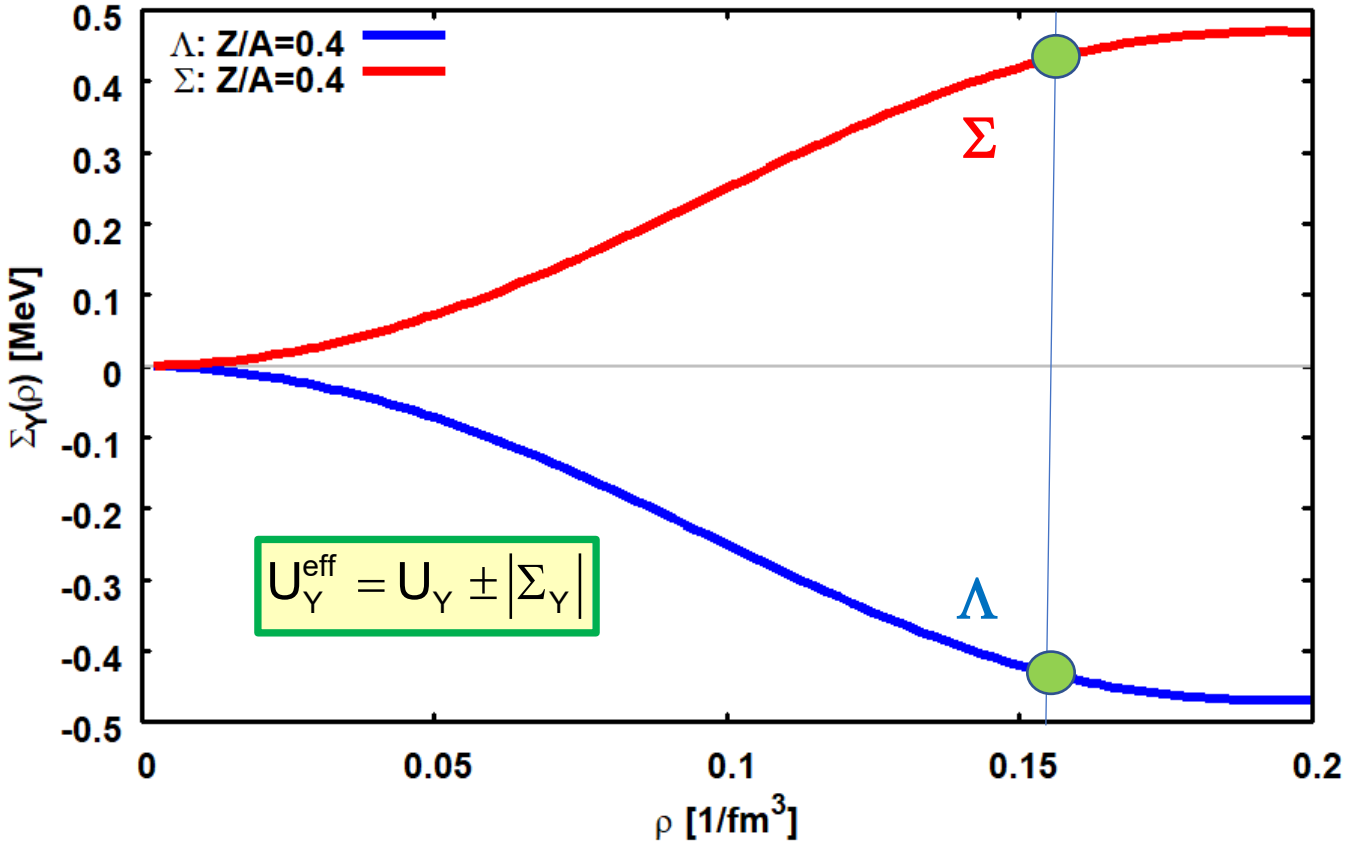
In-Medium Λ–Σ Mixing and the Λ Lifetime τ_Λ

$$\left|\Lambda^*(\rho)\right\rangle = \cos(\Phi)\left|\Lambda\right\rangle - \sin(\Phi)\left|\Sigma\right\rangle \qquad \left|\Sigma^*(\rho)\right\rangle = \cos(\Phi)\left|\Sigma\right\rangle + \sin(\Phi)\left|\Lambda\right\rangle$$



Λ and Σ Self-Energies Induced by the Isovector Mean-Field

$$\Sigma_\Lambda(1,2) = \left[\tau_{A3}(2)\ T_{Y3}(2)\right] U_{\Lambda\Sigma}(2) G_\Sigma(2,1) U_{\Sigma\Lambda}(1) \left[\tau_{A3}(1)\ T_{Y3}(1)\right]$$



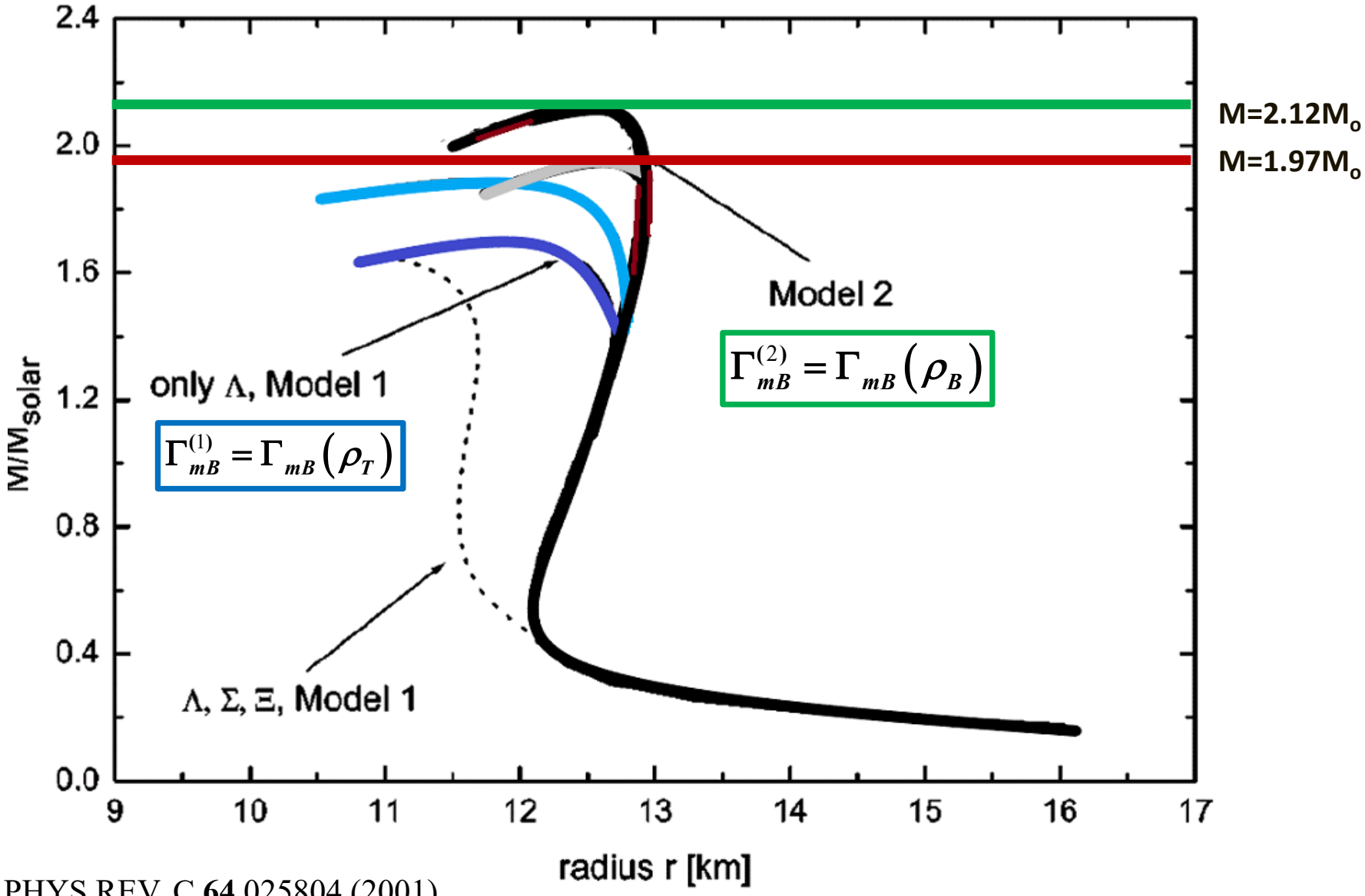
- Depth of effective Λ potential increases
- Depth of effective Σ potential decreases

Neutron Stars

GiEDF Results for Neutron Stars – Fit to Hypernuclei and SU(6)

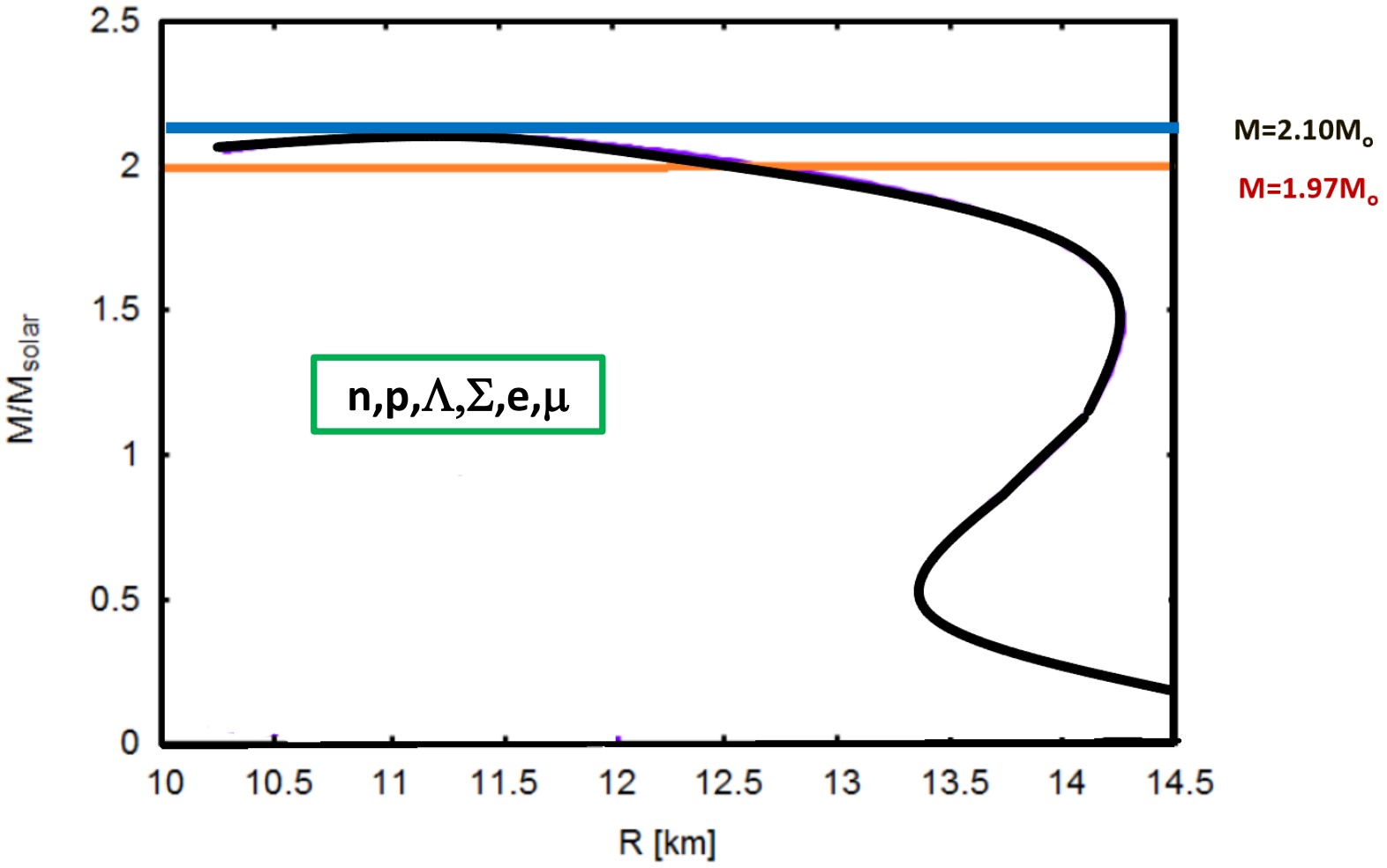
Quark-model SU(6):
upper curves

Fit to Hypernuclei:
Lower curve

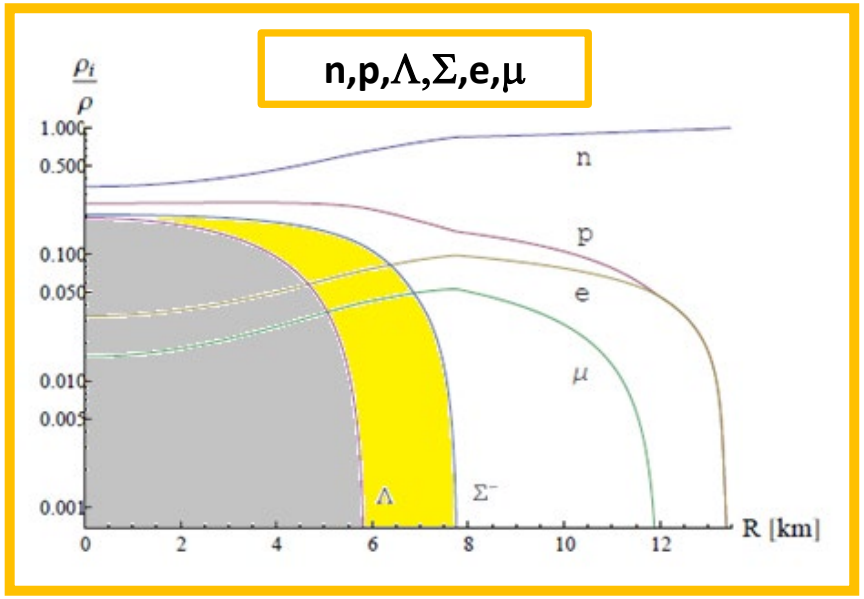


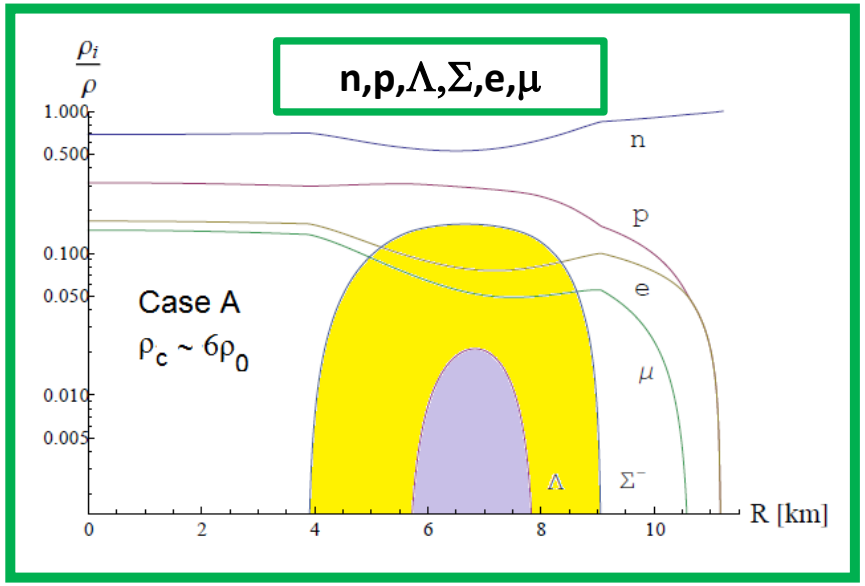
F. Hofman, C. Keil, H.L., PHYS.REV, C **64** 025804 (2001)

Simulating YYN/YYY Repulsion
Exploratory SU(3)-EDF Study of a Neutron Star



Looking inside a Neutron Star





Gi-DHBF EDF Standard Scenario:
Hyperon core in a neutron star

***Tomography* by Gravitational Waves?**

Hang Yu, N.N. Weinberg (MIT)
***Mon.Not.Roy.Astron.Soc.* 470 (2017) 1, 350:**
„Hyperonic Gradients lead to small Phase shifts ~10^{-3} rad“

SU(3)-EDF with repulsive YYN/YYY TBI:
Hyperon shell in a neutron star

Summary and Outlook

- **Octet BB interactions in the SU(3) scheme**
- **Derivation of a SU(3)-based EDF directly from NN interactions**
- **„Quark Scaling“ in Y=0,-1 (S=-1,-2) hypercharge multiplets**
- **Λ–Σ mixing induced by the isovector mean-field**
- **to come: Λ–Σ mixing induced by dynamical self-energies**
- **to come: neutron star EoS and mass-radius relation**

H. Lenske, M. Dhar, EPJ Web Conf. 271 (2022) 05003, 2208.04916 [nucl-th]

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