



# Effective Field Theory for Light Hypernuclei

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Nir Barnea

EMMI Workshop, Trieste

July 3-6, 2023

## Jerusalem, Israel

A. Gal, B. Bazak, M. Bagnarol

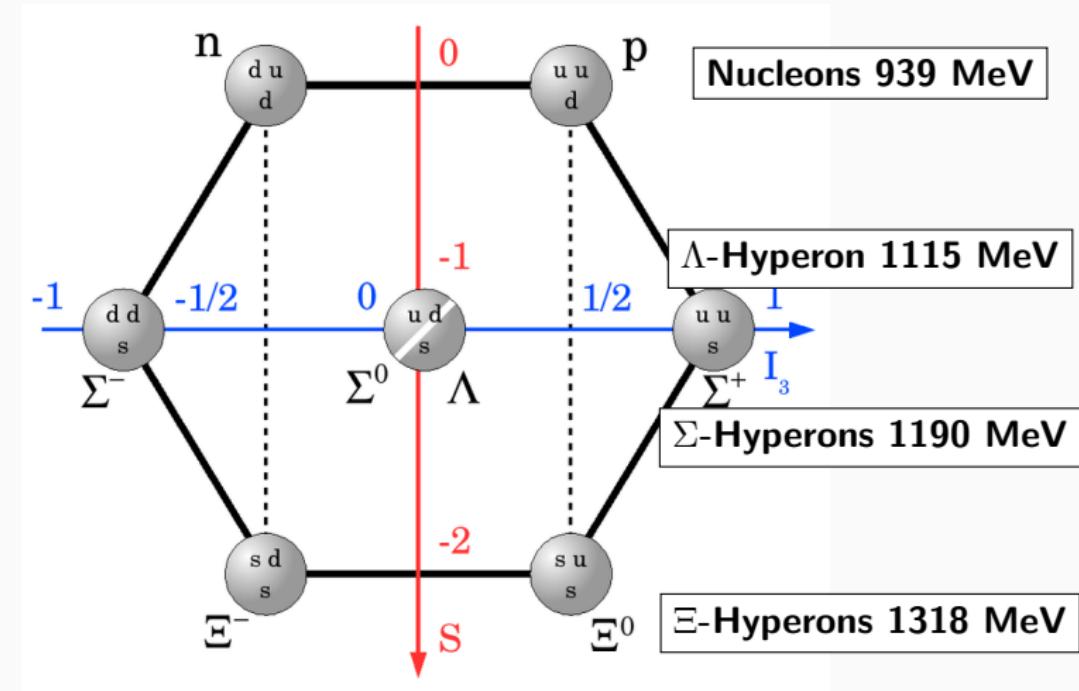
## CEA, Saclay, France

L. Contessi

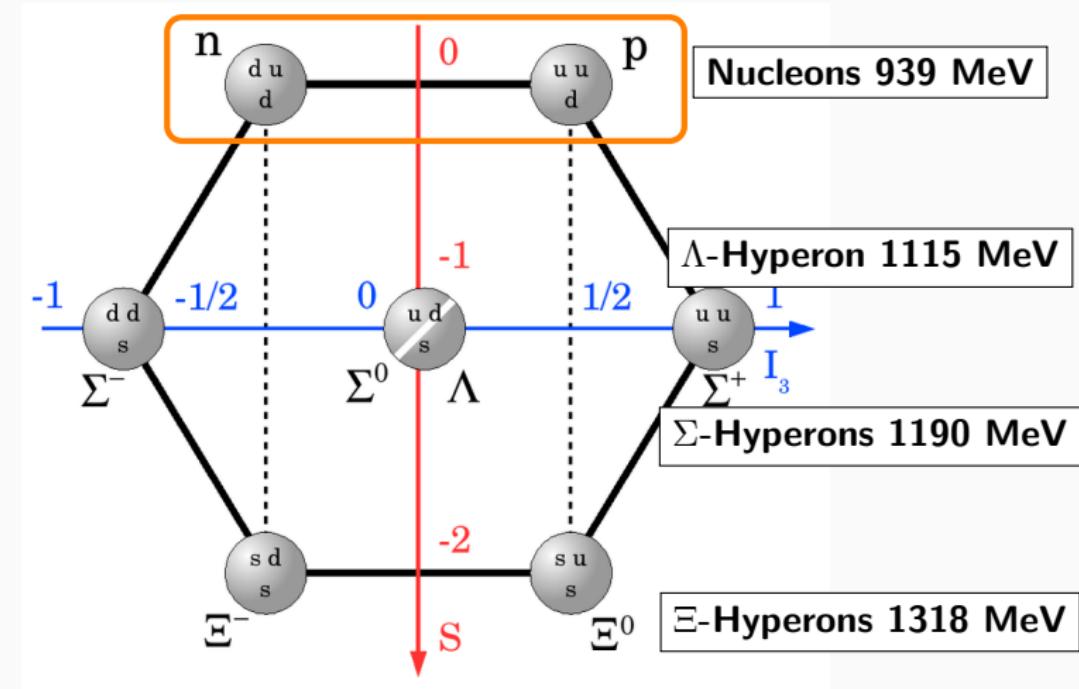
## Rez/Prague, Czech Republic

M. Schäfer, J. Mareš

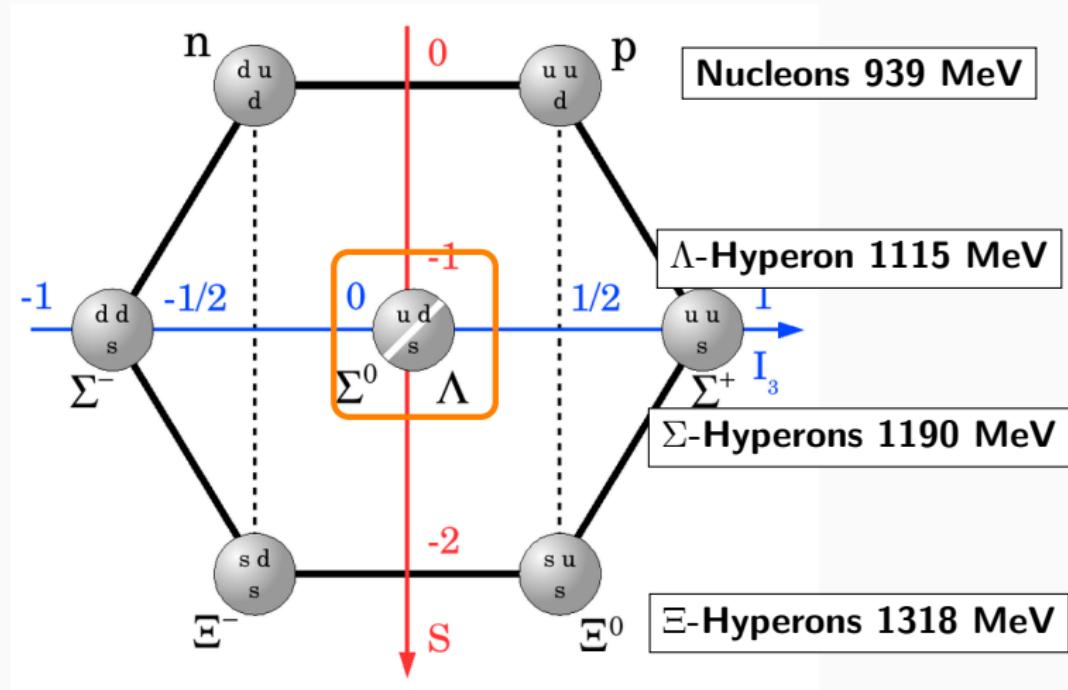
# Introduction - the baryon octet



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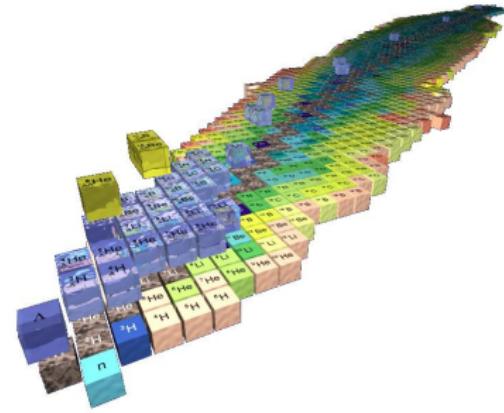


## Nuclei & Hypernuclei

≈3300 nuclear isotopes

≈40 single Lambda hypernuclei

3 double Lambda hypernuclei

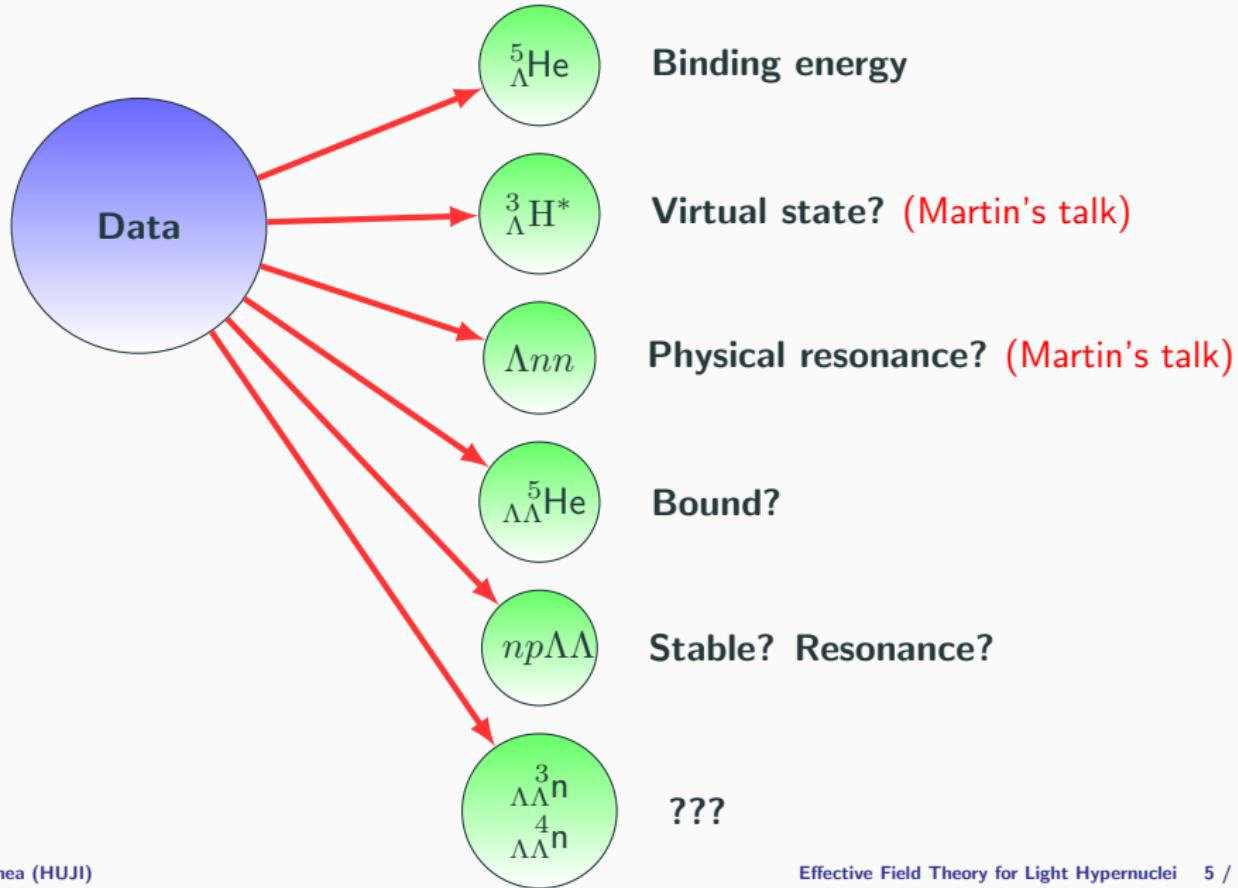


## QCD → EFT

The program:

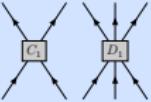
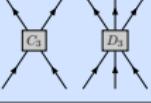
- Use observed hyperons properties
- Precise few-body methods
- Effective description of nature

# Hypernuclear BEFT/ $\pi$ EFT in a nut shell



# Baryonic EFT

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	2-body	3-body	4-body	5-body
LO			-	-
NLO		-		-
$N^2\text{LO}$			?	?

# The Nuclear Interaction - $\chi$ EFT

Weinberg, van Kolck, Epelbaum, Machleidt, Meissner, ...

$$\mathcal{L}_{QCD}(q, G) \longrightarrow \mathcal{L}_{\chi EFT}(B, \pi, K)$$

	Two-nucleon force	Three-nucleon force	Four-nucleon force
$Q^0$	X H	—	—
$Q^2$	X P K N H	—	—
$Q^3$	H K	H H X *	—
$Q^4$	X P K K ...	H H K X ...	H K H H ...

work in progress...

$$V_{LO} = \underbrace{V_\pi(r)}_{1-\text{pion exchange}} + \underbrace{(c_S + c_T \sigma_1 \cdot \sigma_2) \delta(r)}_{\delta \text{ interactions}}$$

# The Nuclear Interaction - $\chi$ EFT

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$$\mathcal{L}_{QCD}(q, G) \longrightarrow \mathcal{L}_{\chi EFT}(B, \pi, K)$$

	Two-nucleon force	Three-nucleon force	Four-nucleon force
$Q^0$		$V(\mathbf{r}) = c\delta(\mathbf{r})$	—
$Q^2$		—	—
$Q^3$			—
$Q^4$		... work in progress...	...

$$V_{LO} = \underbrace{V_\pi(\mathbf{r})}_{1-\text{pion exchange}} + \underbrace{(c_S + c_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \delta(\mathbf{r})}_{\delta \text{ interactions}}$$

# The Nuclear Interaction - $\chi$ EFT

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$$\mathcal{L}_{QCD}(q, G) \longrightarrow \mathcal{L}_{\chi EFT}(B, \pi, K)$$

	Two-nucleon force	Three-nucleon force	Four-nucleon force
$Q^0$	X H	One Pion Exchange $\approx \exp(-\mu_\pi r)/r$	
$Q^2$	X P K N	-	-
$Q^3$	P K	H X *	-
$Q^4$	X P K N ...	K H N K ...	H K N H ...

work in progress...

$$V_{LO} = \underbrace{V_\pi(\mathbf{r})}_{1-\text{pion exchange}} + \underbrace{(c_S + c_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \delta(\mathbf{r})}_{\delta \text{ interactions}}$$

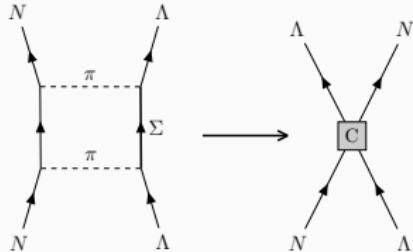
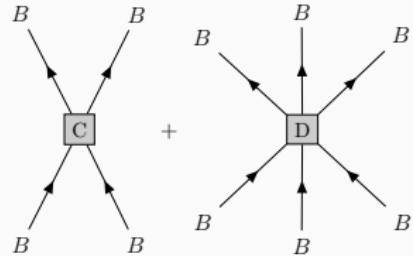
# Baryonic EFT aka $\neq$ EFT

- $B = n, p, \Lambda$  are the only DOF.

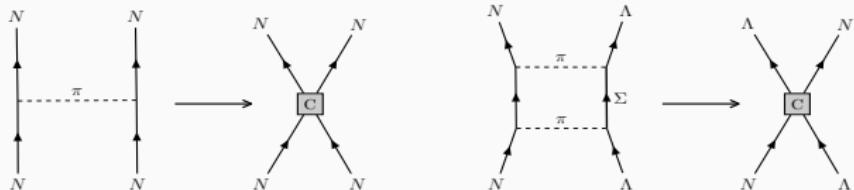
$$\mathcal{L}_{QCD}(q, G) \longrightarrow \mathcal{L}_{\chi EFT}(B, \pi, K) \longrightarrow \mathcal{L}(B)$$

- $\mathcal{L}$  is expanded in powers of  $Q/M_h$ .
- Include contact terms and derivatives.
- Not too many parameters

$$\begin{aligned} \mathcal{L} = & N^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m} \right) N + \Lambda^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m} \right) \Lambda \\ & + \mathcal{L}_{2B} + \mathcal{L}_{3B} + \dots \end{aligned}$$



# The expansion parameter



## Accuracy for light nuclei

**Nuclei** The pion mass is our breaking scale  $M_h$

$$\left(\frac{Q}{M_h}\right) = \frac{\sqrt{2B_N M_N}}{m_\pi} \approx 0.5 - 0.8$$

Seems to work better in practice as  $\Delta B(^4\text{He}) \approx 10\%$

**Hypernuclei** No OPE therefore breaking scale is  $2m_\pi$

$$\left(\frac{Q}{M_h}\right) = \frac{\sqrt{2B_\Lambda M_\Lambda}}{2m_\pi} \approx 0.3$$

At LO accuracy goes as  $(Q/M_h)^2$



## ① Universality

- At LO, **2-body** inputs are **scattering lengths**
- BEFT is well suited for studying universality

## ② The Wigner Bound Phillips, Beane and Cohen (1997-1998)

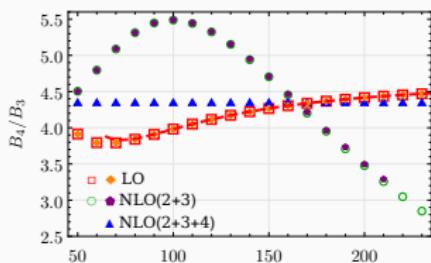
- The **effective range** is bounded by the cutoff  $r_{\text{eff}} \leq W/\lambda$
- All orders but LO are perturbation (Kaplan, van Kolck, ...).

## ③ The Thomas collapse Bedaque, Hammer, and van Kolck (1999)

- With LO 2-body interaction  $B_3 \propto \hbar \lambda^2 / m.$
- A **3-body** counter term must be introduced **at LO**.

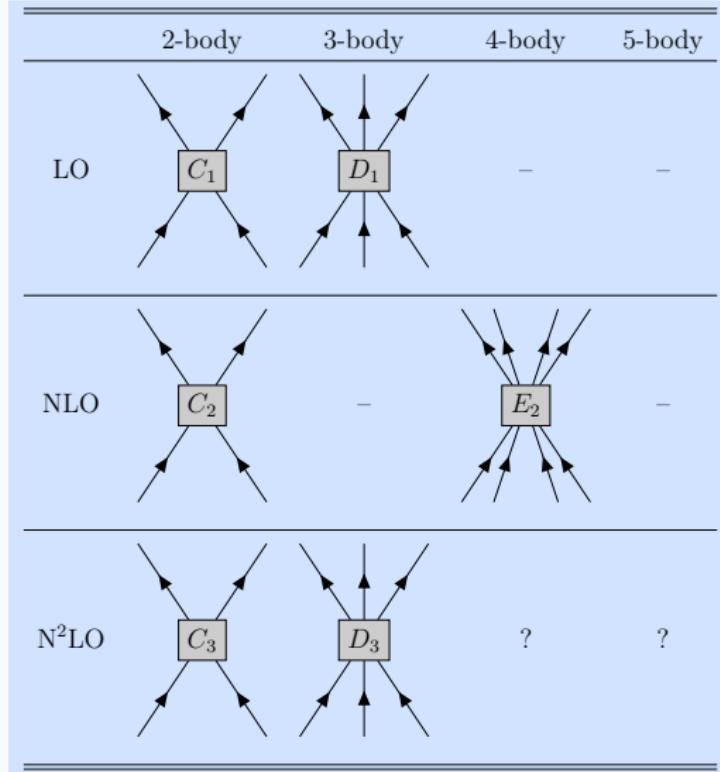
## ④ NLO - 4-body force Bazak et al. (2019)

- @NLO the 4-body system is unstable.
- A **4-body** force promoted to **NLO**.



# The Nuclear Interaction - BEFT/ $\not$ EFT

Kaplan, van Kolck, Bedaque, Hammer, Bazak, ...

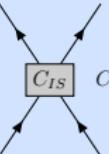
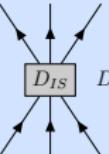
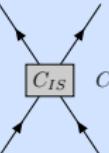
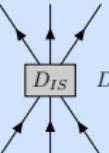
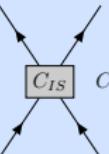
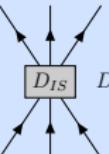


Bazak, Kirscher, König,  
Pavón Valderrama,  
Barnea, and van Kolck,  
*PRL* **122**, 143001 (2019)

Hammer, König and van  
Kolck, *Rev. Mod. Phys.*  
**92**, 025004 (2020)

# BEFT of $\Lambda np$ @ Leading Order

## 2-body & 3-body diagrams:

	2-body	3-body	#LECS
Strange = 0	 $C_{01}, C_{10}$	 $D_{\frac{1}{2}\frac{1}{2}}$	3
Strange = -1	 $C_{\frac{1}{2}0}, C_{\frac{1}{2}1}$	 $D_{0\frac{1}{2}}, D_{0\frac{3}{2}}, D_{1\frac{1}{2}}$	5
Strange = -2	 $C_{00}$	 $D_{\frac{1}{2}\frac{1}{2}}$	2

Contact terms

minimal amount of parameters

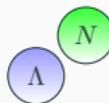
LECs

constrained by exp. data

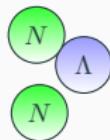
L. Contessi, M. Schafer, N. Barnea, A. Gal, J. Mareš, PLB 797 (2019) 134893



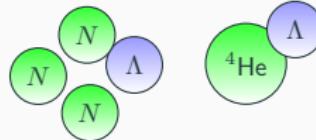
## What do we have?



**Not bound**  
**scarce scattering data**



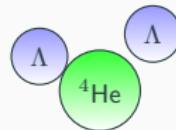
$^3_{\Lambda}\text{H}$ ,  $B_\Lambda \approx 0.1$  MeV



$^4_{\Lambda}\text{H}^{0,1}, ^4_{\Lambda}\text{He}^{0,1}$   $B_\Lambda \approx 3$  MeV  
 $^5_{\Lambda}\text{He}$ ,  $B_\Lambda \approx 3$  MeV



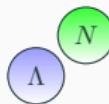
**Not bound**  
**scarce scattering data**



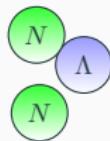
$^6_{\Lambda\Lambda}\text{He}$ ,  $B_\Lambda \approx 3$  MeV



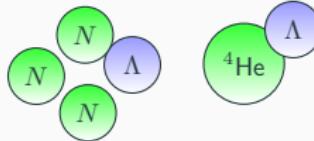
## What do we have?



$$C_{\frac{1}{2}0}, C_{\frac{1}{2}1}$$



$$^3_{\Lambda}\mathbf{H}, B_{\Lambda} \approx 0.1 \text{ MeV}$$

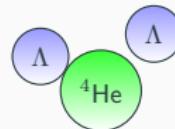


$$^4_{\Lambda}\mathbf{H}^{0,1}, ^4_{\Lambda}\mathbf{He}^{0,1} \quad B_{\Lambda} \approx 3 \text{ MeV}$$

$$^5_{\Lambda}\mathbf{He}, B_{\Lambda} \approx 3 \text{ MeV}$$



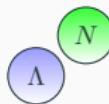
$$C_{00}$$



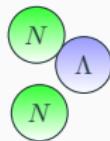
$$^6_{\Lambda\Lambda}\mathbf{He}, B_{\Lambda} \approx 3 \text{ MeV}$$



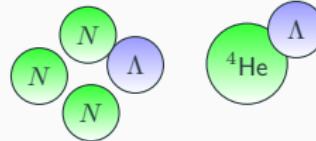
## What do we have?



$C_{\frac{1}{2}0}, C_{\frac{1}{2}1}$



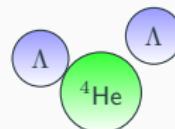
$D_{0\frac{1}{2}}$



$D_{0\frac{3}{2}}, D_{1\frac{1}{2}}$   
 ${}^5_\Lambda\text{He}, B_\Lambda \approx 3 \text{ MeV}$



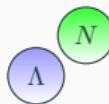
$C_{00}$



${}^6_{\Lambda\Lambda}\text{He}, B_\Lambda \approx 3 \text{ MeV}$



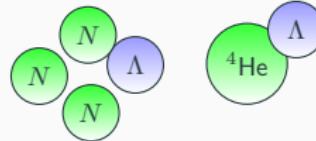
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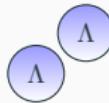
$C_{\frac{1}{2}0}, C_{\frac{1}{2}1}$



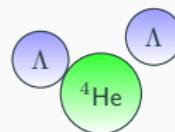
$D_{0\frac{1}{2}}$



$D_{0\frac{3}{2}}, D_{1\frac{1}{2}}$   
 ${}^5_\Lambda\text{He}, B_\Lambda \approx 3 \text{ MeV}$



$C_{00}$



$D_{\frac{1}{2}\frac{1}{2}}$

# $\Lambda N$ scattering data

- Cross-sections for  $p_{lab} \geq 100$  MeV/c
- Spin dependence not resolved

- **Alexander et al.** PR173, 1452 (1968)

$$a_{\Lambda N}^0 = -1.8^{+2.3}_{-4.2} \text{ fm}$$

$$a_{\Lambda N}^1 = -1.6^{+1.1}_{-0.8} \text{ fm}$$

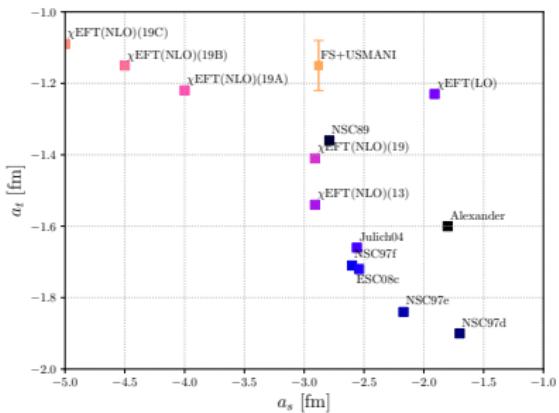
- **Sechi-Zorn et al.** PR175, 1735 (1968)

$$0 > a_{\Lambda N}^0 > -9.0 \text{ fm}$$

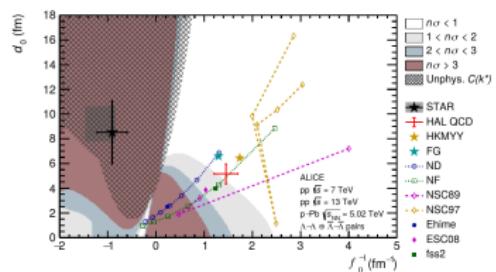
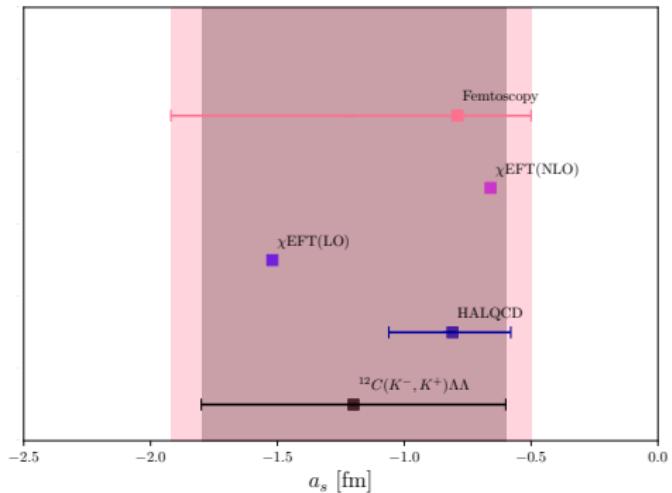
$$-0.8 > a_{\Lambda N}^1 > -3.2 \text{ fm}$$

- **Femtoscopy** (2023)

Tight constraint on  $a_s, a_t$   
inconsistent with existing models



# $\Lambda\Lambda$ scattering lengths



ALICE Collaboration, PLB 797,  
134822 (2019)

## $\Lambda\Lambda$ scattering length

Exp./Model	$a_{\Lambda\Lambda}^0$ [fm]	
$^{12}\text{C}(K^-, K^+)\Lambda\Lambda X$	-1.2(6)	PRC 85, 015204 (2012)
HALQCD	$-0.81 \pm 0.23^{0.0}_{-0.1}$	NPA 998, 121737 (2020)
$\chi\text{EFT(LO;600)}$	-1.52	PLB 653, 29 (2007)
$\chi\text{EFT(NLO;600)}$	-0.66	NPA 954, 273 (2016)
Femtoscopy	$-0.79^{+0.29}_{-1.13}$	PRC 91, 024916 (2016)

## What do we have?

- LO and NLO  $\not\!\! EFT$  fitted to low-energy experimental constraints
- The Schrödinger equation

## What do we want to know?

- Bound state spectrum
- Resonances and virtual states
- Scattering cross-sections

## How do we get there?

- Gaussian basis functions
- Bound states  $\Rightarrow$  SVM (Suzuki and Varga)
- Scattering  $\Rightarrow$  Busch formula
- Resonances  $\Rightarrow$  Complex rotation, analytic continuation



- **Nuclear scattering**

Elastic  $s$ -wave scattering @NLO for  $A \leq 5$  (Mirko's talk)

- **$\Lambda$  hypernuclei ( ${}^A_\Lambda Z$ )**

$s$ -shell hypernuclei - overbinding of  ${}^5_\Lambda$ He

Hypernuclear resonances

- **$\Lambda\Lambda$  hypernuclei ( ${}^A_{\Lambda\Lambda} Z$ )**

Onset of binding,  $A=4$  or  $5$ ?

- **Charge symmetry breaking**

The Dalitz von Hippel parameters from SU(3) symmetry.

- **Few nucleons in a box**

EFT matching of LQCD calcs.

## The nuclear sector

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## Universal fermionic relations (STM, Petrov, Deltuva, ...)

Atom-Dimer scattering

$$\frac{a_{ad}}{a_{aa}} = 1.1791 + 0.553 \frac{r_{aa}}{a_{aa}} \quad ; \quad \frac{r_{ad}}{a_{aa}} = -0.038 + 1.04 \frac{r_{aa}}{a_{aa}}$$

Dimer-Dimer scattering

$$\frac{a_{dd}}{a_{aa}} = 0.5986 + 0.105 \frac{r_{aa}}{a_{aa}} \quad ; \quad \frac{r_{dd}}{a_{aa}} = 0.133 + 0.51 \frac{r_{aa}}{a_{aa}}$$

These results are reproduced for spin saturated system:

- Neutron-Deuteron  $S = \frac{3}{2}$  scattering
- Deuteron-Deuteron  $S = 2$  scattering.

① Near-threshold  $^3\text{H}^*$

virtual state

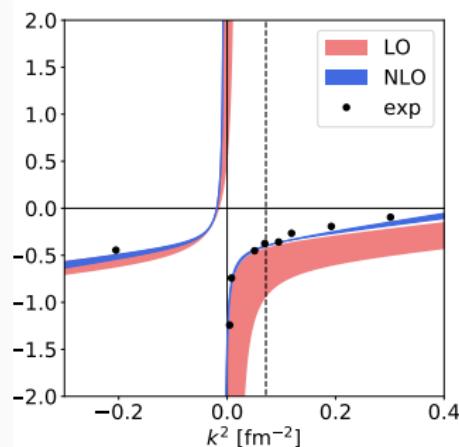
$\Rightarrow$  pole of S-matrix

② Near-threshold zero in S-matrix

$$\frac{1}{k \cotg(\delta) - ik} = 0$$

$$\lim_{k \rightarrow k_0} k \cotg(\delta) = \pm \infty$$

$\Rightarrow$  modified ERE



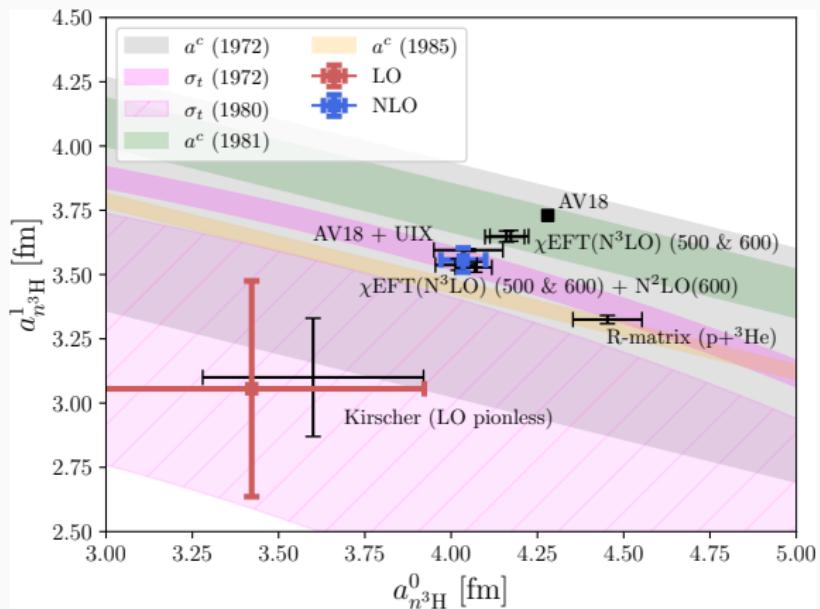
Oers and Seagrave, PLB 24, 11 (1967)

$$a_{n^2\text{H}}^{1/2} = 0.29 \text{ fm}$$

$$r_{n^2\text{H}}^{1/2} = 1.70 \text{ fm}$$

$$k \cotg(\delta) = A + B k^2 + \frac{C}{(1 + D k^2)} \quad ; \quad a = -\frac{1}{A + C} \quad \text{and} \quad r = 2B$$

# Experiment & Theory : $n + {}^3\text{H}$ scattering lengths



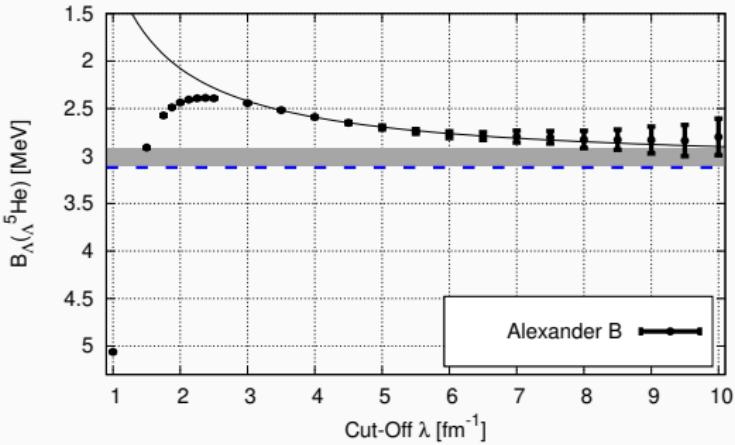
M. Schäfer and B. Bazak, Phys. Rev. C 107, 064001 (2023)

## Light Hypernuclei @LO

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# The ${}^5_{\Lambda}\text{He}$ binding energy

$B_{\Lambda}({}^5_{\Lambda}\text{He})$  vs. cut-off  $\lambda$  in LO BEFT



L.Contessi N.Barnea A.Gal, PRL 121 (2018) 102502

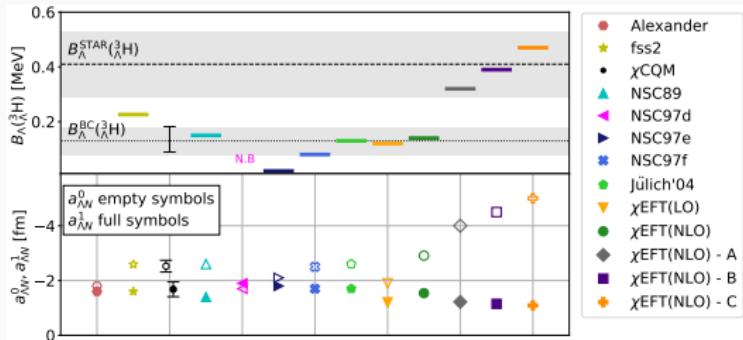
With Alexander &  $\chi$ EFT(NLO) scattering lengths  $a_s, a_t$   
 $B_{\Lambda}({}^5_{\Lambda}\text{He})$  is reproduced within theoretical error

Cut-off dependence

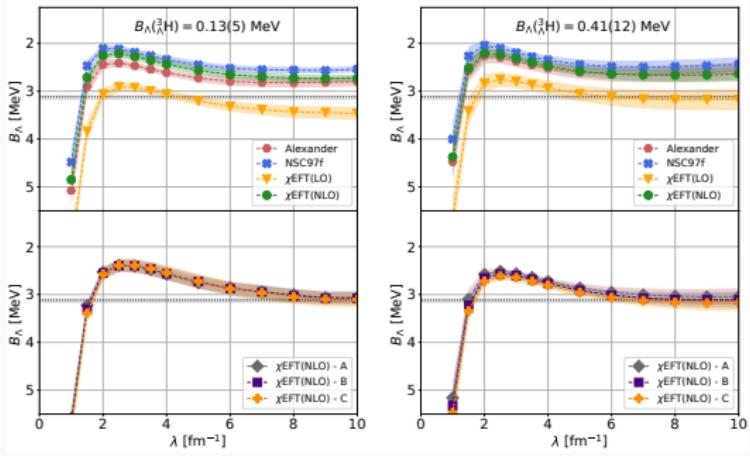
$$\frac{B_{\Lambda}(\lambda)}{B_{\Lambda}(\infty)} = 1 + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \dots$$

# $^5_{\Lambda}\text{He}$ - The impact of “Deep” hypertriton

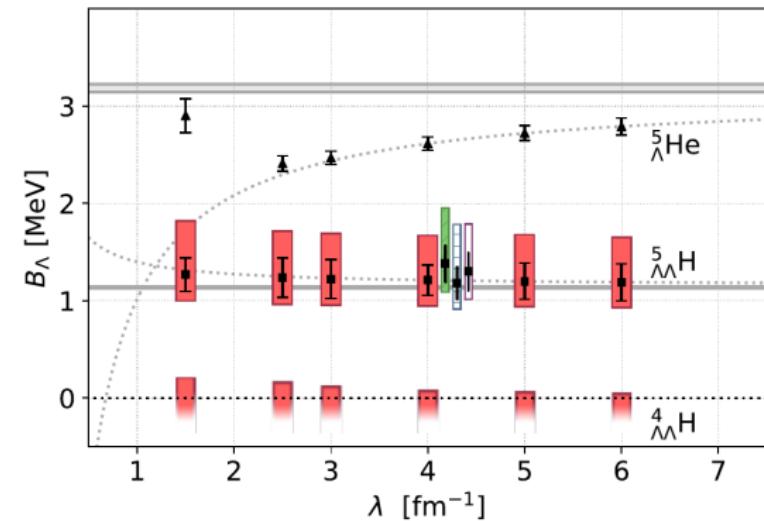
## Scattering lengths



## $^5_{\Lambda}\text{He}$ Binding energy



# Onset of $\Lambda\Lambda$ hypernuclear binding



Contessi-Schafer-Barnea-Gal-Mareš, PLB 797 (2019) 134893.

## Double- $\Lambda$ systems:

- The neutral systems  $\Lambda\Lambda n$ ,  $\Lambda\Lambda nn$  are far from threshold
- $\Lambda\Lambda^4 H$  on verge of binding. Better data is needed for clarification.
- In our theory  $\Lambda\Lambda^5 H$  is comfortably bound

# Search for excited trios (Martin's talk)

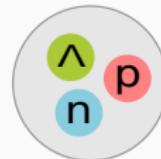
$^3\Lambda\text{H}^*(3/2^+)$

- no experimental evidence
- JLab C12-19-002 proposal

$\Lambda\text{nn}(1/2^+)$

- experiment (HypHI)
- JLab E12-17-003 experiment
- structure of neutron-rich  $\Lambda$ -hypernuclei

$^3\Lambda\text{H}(1/2^+)$



$^3\Lambda\text{H}^*(3/2^+)$



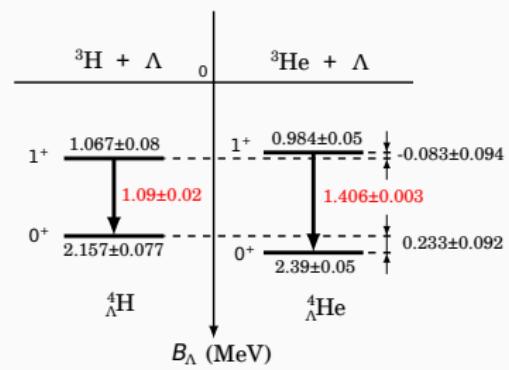
$\text{nn}\Lambda(1/2^+)$

**Calculating resonance states is a non-trivial task**

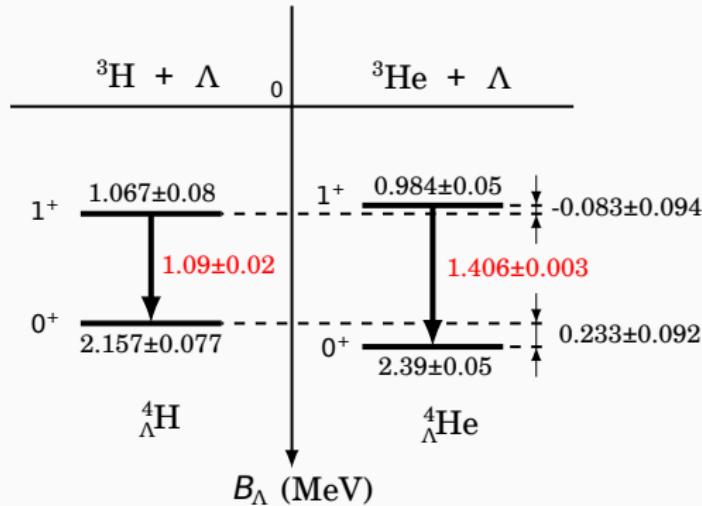
We have used two techniques:

- Complex scaling method (CSM)
- Inverse analysis continuation in coupling constraint (IACCC)

## Charge symmetry breaking



# $A = 4$ hypernuclear level scheme



- $B_{\Lambda}(^4_{\Lambda}\text{He}; 0^+)$  - **Emulsion measurement**

Nucl. Phys. A 754, 3c (2005)

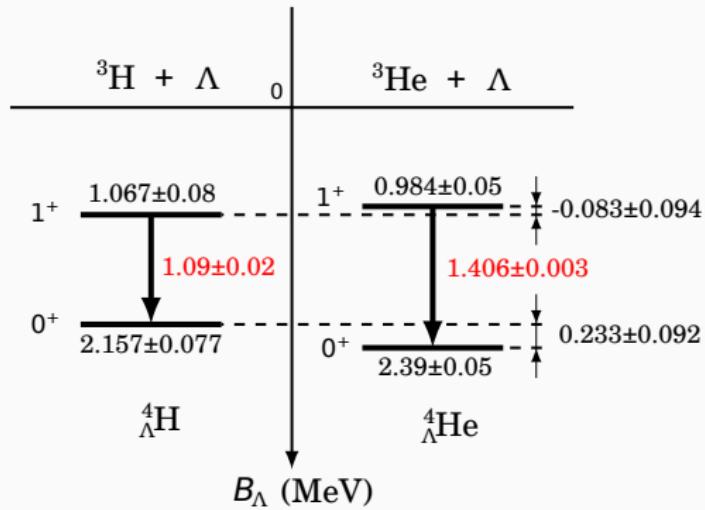
- $B_{\Lambda}(^4_{\Lambda}\text{H}; 0^+)$  - **MAMI experiment**

Nucl. Phys. A 954, 149 (2016)

- $E_{\gamma}(^4_{\Lambda}\text{H}; 1^+ \rightarrow 0^+), E_{\gamma}(^4_{\Lambda}\text{He}; 1^+ \rightarrow 0^+)$  -  **$\gamma$ -rays at J-PARC**

Phys. Rev. Lett. 115, 222501 (2015)

# $A = 4$ hypernuclear level scheme



- Charge symmetry: **invarince** under  $n \leftrightarrow p$ , e.g.  ${}^3\text{H} \leftrightarrow {}^3\text{He}$
- Nuclei: for  ${}^3\text{He} - {}^3\text{H}$ ,  $\Delta E_{CSB}$  without Coulomb is about 70 keV
- For  ${}^3\text{He} - {}^3\text{H}$ :  $\Delta E_{CSB}/\Delta E \approx 0.01$
- Hypernuclei: CSB in  ${}^4_\Lambda\text{He}-{}^4_\Lambda\text{H}$ :  $\Delta E_{CSB}/\Delta E \approx 0.22$

# Theoretical considerations

**Dalitz, von Hippel** Phys. Lett. 10, 153 (1964)

$\Lambda - \Sigma^0$  mixing in  $SU(3)_f$  (following Coleman & Glashow)

$$\mathcal{A}_{I=1}^{(0)} = -\frac{\langle \Sigma^0 | \delta M | \Lambda \rangle}{M_{\Sigma^0} - M_\Lambda} = -0.0148(6)$$

CSB OPE contribution by  $g_{\Lambda\Lambda\pi} = 2\mathcal{A}_{I=1}^{(0)} g_{\Lambda\Sigma\pi}$

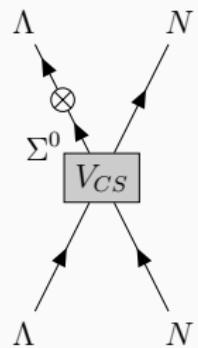
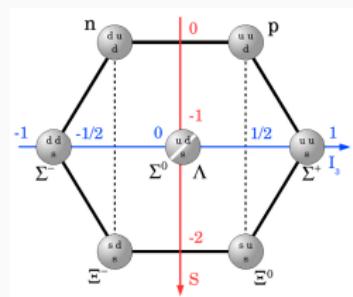
- **A. Gal** Phys. Lett. B 744, 352, (2015)  
Generalization of DvH

$$\langle \Lambda N | V_{\text{CSB}} | \Lambda N \rangle = -\frac{2}{\sqrt{3}} \mathcal{A}_{I=1}^{(0)} \langle \Sigma N | V_{\text{CS}} | \Lambda N \rangle \tau_z.$$

$$\Delta B_\Lambda(0^+) \approx 240 \text{ keV} \quad \Delta B_\Lambda(1^+) \approx 35 \text{ keV}$$

- **Gazda, Gal** PRL 116, 122501 (2016)  
generalized DvH; LO  $\chi$ EFT  $YN$  interaction; NSCM

$$\Delta B_\Lambda(0^+) \approx 180 \pm 130 \text{ keV} \quad \Delta B_\Lambda(1^+) \approx -200 \pm 30 \text{ keV}$$



# Charge symmetry breaking - $\chi$ EFT

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- 2 d.p. and 2 parameters  
 $C_s^{CSB}, C_t^{CSB}$
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## Question:

Can BEFT explain these last 2 observations?

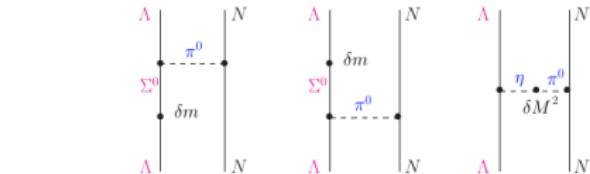


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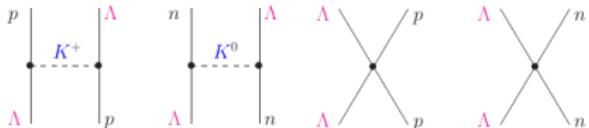


Fig. 2 CSB contributions from  $K^\pm/K^0$  exchange (left) and from contact terms (right).

$A$	NLO13		NLO19	
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500	$4.691 \times 10^{-3}$	$-9.294 \times 10^{-4}$	$5.590 \times 10^{-3}$	$-9.505 \times 10^{-4}$
550	$6.724 \times 10^{-3}$	$-8.625 \times 10^{-4}$	$6.863 \times 10^{-3}$	$-1.260 \times 10^{-3}$
600	$9.960 \times 10^{-3}$	$-9.870 \times 10^{-4}$	$9.217 \times 10^{-3}$	$-1.305 \times 10^{-3}$
650	$1.500 \times 10^{-2}$	$-1.142 \times 10^{-3}$	$1.240 \times 10^{-2}$	$-1.395 \times 10^{-3}$

Haidenbauer et al., Few-body sys.

(2019)

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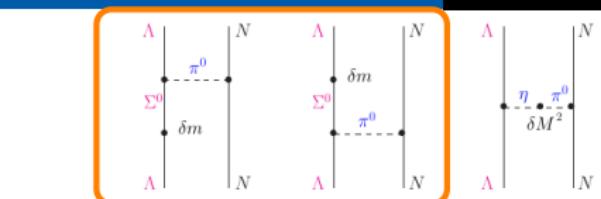


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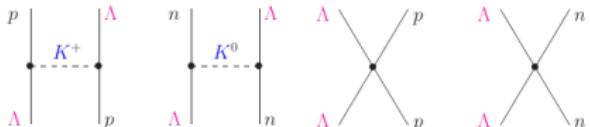


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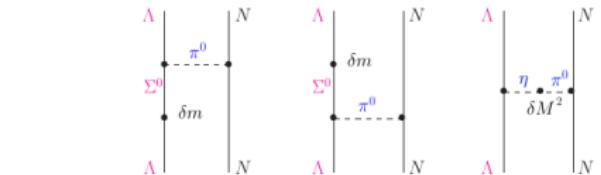


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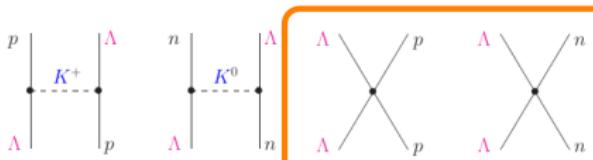


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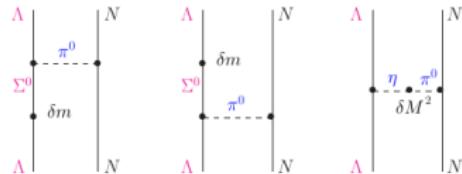


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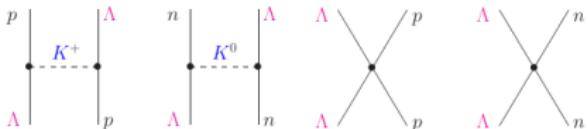


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Haidenbauer et al., Few-body sys.

(2019)

## Charge Symmetric $\Lambda N$

$$V_{\Lambda N} = \sum_S C_{\Lambda N}^S(\lambda) \mathcal{P}_S \delta_\lambda(\mathbf{r})$$

## CSB $\Lambda N$ interaction

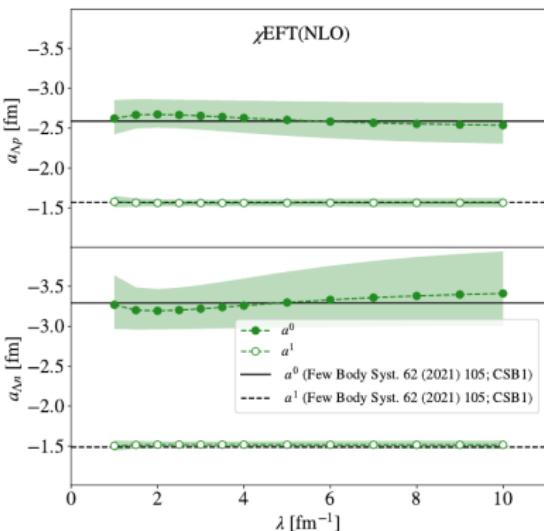
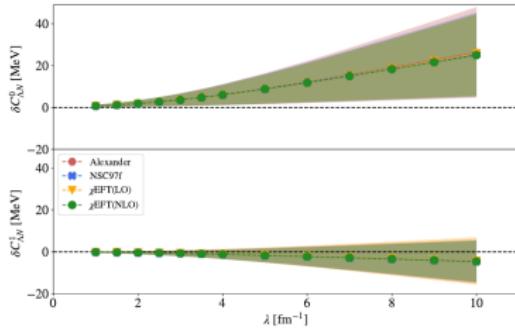
$$C_{\Lambda N}^S \rightarrow \left[ C_{\Lambda p}^S \frac{1 + \tau_z}{2} + C_{\Lambda n}^S \frac{1 - \tau_z}{2} \right]$$

Resulting LECs

$$C_{\Lambda N}^S = \frac{1}{2} (C_{\Lambda p}^S + C_{\Lambda n}^S)$$

$$\Delta C_{\Lambda N}^S = \frac{1}{2} (C_{\Lambda p}^S - C_{\Lambda n}^S)$$

Same observations as in  $\chi$ EFT



# The DvH mechanism in BEFT

Dalitz, von Hippel for BEFT

$$\langle \Lambda N | C_{\text{CSB}} | \Lambda N \rangle = -\frac{2}{\sqrt{3}} \mathcal{A}_{I=1}^{(0)} \langle \Sigma N | C_{\text{CS}} | \Lambda N \rangle \tau_z.$$

Assuming  $SU(3)_f$  symmetry we can relate  $C_{\Lambda N, \Sigma N}^S$  to the  $NN$  and  $\Lambda N$  LECs:

$$C_{\Lambda N, \Sigma N}^0 = -3(C_{NN}^0 - C_{\Lambda N}^0),$$

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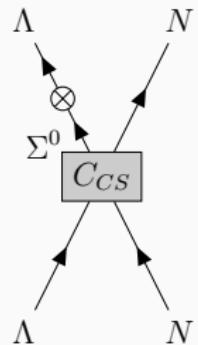
Dover, Feshbach, Ann. Phys. (NY) 198, 321 (1990)

The resulting CSB LECs are opposite in sign and

$$|C_s^{\text{CSB}}| \gg |C_t^{\text{CSB}}|$$

Having  $\mathcal{A}_{I=1}^{(0)}$  we have no free parameters.

Now we can go in the other direction and predict  $\mathcal{A}_{I=1}^{(0)}$  from the hypernuclear spectrum.



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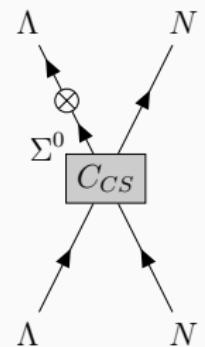
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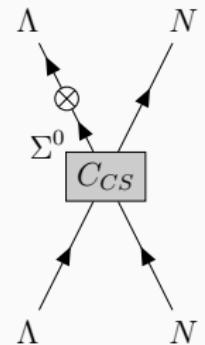
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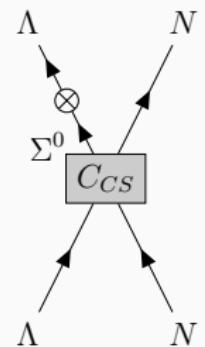
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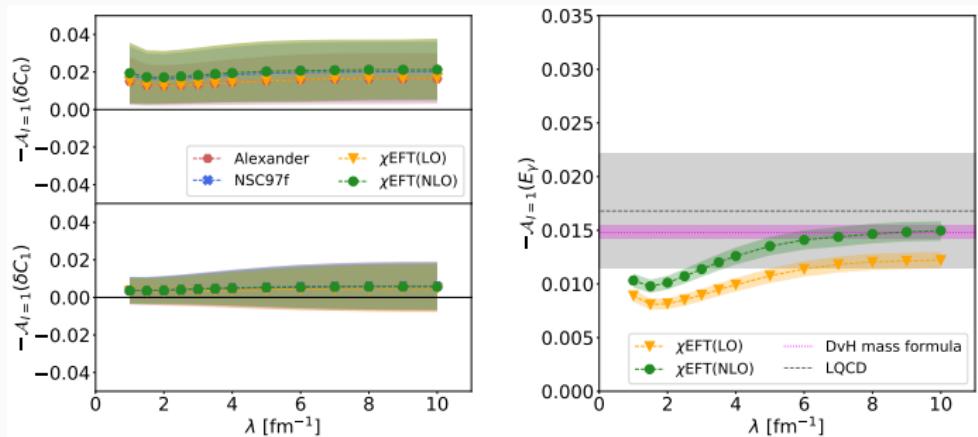
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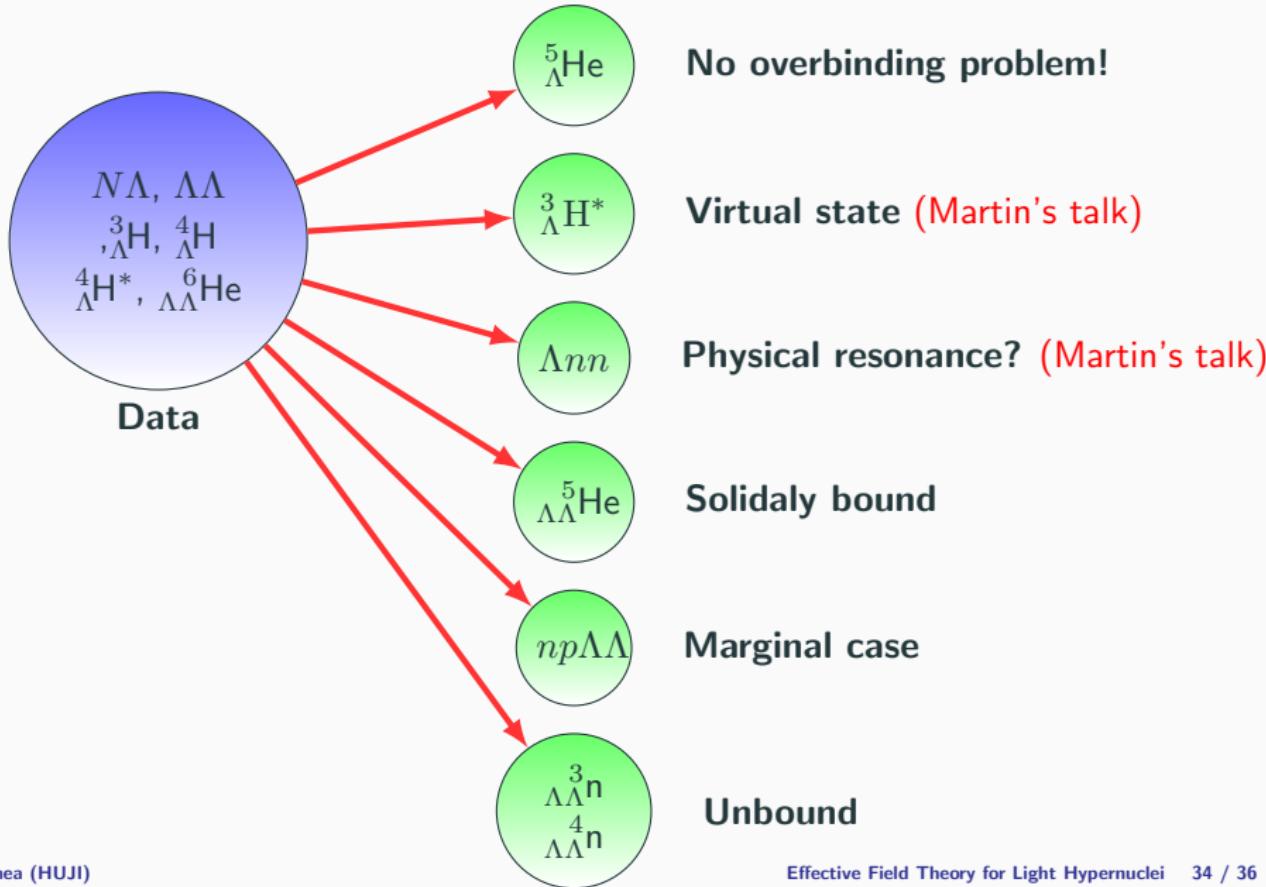


# Extracting the DvH parameter



Method/Input	$-\mathcal{A}_{I=1}$
SU(3) <sub>f</sub> [DvH64]	$0.0148 \pm 0.0006$
LQCD [LQCD20]	$0.0168 \pm 0.0054$
$\chi$ EFT(LO)/ $\chi$ EFT(LO) [Polinder06]	$0.0139 \pm 0.0013$
$\chi$ EFT(LO)/ $\chi$ EFT(NLO) [Haidenbauer13]	$0.0168 \pm 0.0014$

# Hypernuclear BEFT/ $\frac{1}{\Lambda}$ EFT in a nut shell







Thank you !