Four body force in Pionless Effective Field Theory

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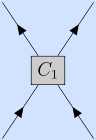
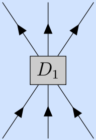
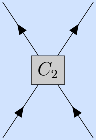
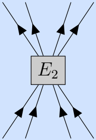
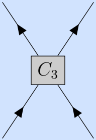
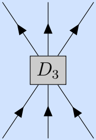
- Quantum Chromodynamics (QCD) is the established fundamental theory of the strong interaction. At low energies ($E \lesssim 200$ MeV) it is not perturbative.
- Two approaches:
  - Solve the Lagrangian by brute force, regardless of the cost $\Rightarrow$ Lattice QCD (LQCD)
  - Work with more appropriate low-energy degrees of freedom $\Rightarrow$ Effective Field Theory (EFT)
- We employ Pionless EFT ($\not\pi$EFT), where the the degrees of freedom are the nucleons and the pions are integrated out

$\not\pi$EFT interaction

	2-body	3-body	4-body	5-body
LO	 C_1	 D_1	–	–
NLO	 C_2	–	 E_2	–
N^{2}LO	 C_3	 D_3	?	?

$\not\pi$EFT interaction, LO

- Without pions, our Leading Order (LO) interaction is a contact interaction:

$$V_{\mathrm{LO,\ 2B}}(\vec{r}) = C_{S,I}\,\delta(\vec{r})$$
$$V_{\mathrm{LO,\ 3B}}(\vec{r}_{ij},\vec{r}_{jk}) = D_{S,I}\,\delta(\vec{r}_{ij})\,\delta(\vec{r}_{jk})$$

- The promotion of a repulsive three body force at LO prevents the Thomas collapse
- In order to numerically solve Schrödinger's equation, we have to smear the Dirac delta, introducing the cutoff Λ

$$\delta_\Lambda(\vec{r}) = e^{-\frac{\Lambda^2 r^2}{4}}$$

$\not\pi$EFT interaction, NLO

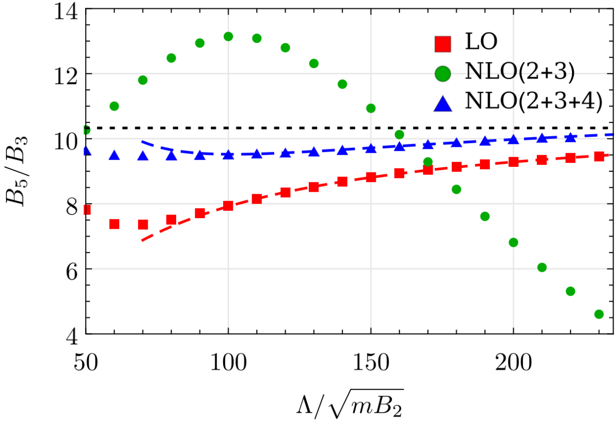
- Our Next to LO (NLO) interaction has momentum dependent two-body terms, three counterterms and a four-body force:

$$
\begin{aligned}
V_{\text{NLO, 2B}} &= C_{S,I} \nabla^2 \delta(\vec{r}) \\
V_{\text{NLO, counter}} &= C_{S,I} \delta_\Lambda(\vec{r}) + D_{S,I} \delta(\vec{r}_{ij}) \delta(\vec{r}_{jk}) \\
V_{\text{NLO, 4 Body}} &= E_{S,I} \prod_{ab \in \text{pairs}} \delta_\Lambda(\vec{r}_{ab})
\end{aligned}
$$

- The momentum dependent terms introduce an effective range to the interaction
- The counterterms have the same form of the LO terms and serve to keep the LO observables reproduced at NLO

Inclusion of the four body force

- The inclusion of the four body force at NLO is necessary and sufficient for the renormalization at this order.
- It was found by *B. Bazak et al.* in 2018 studying 4 to 6 boson systems. They conjectured the necessity of an $A-3$ force at N^{A-3}LO!



Final remarks on the model

- Between LO and NLO, out model has six parameters fixed to few body observables:

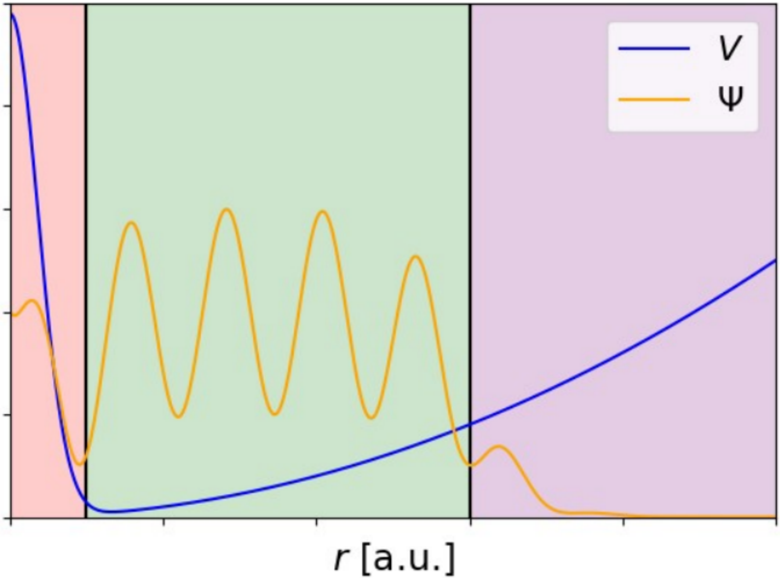
$$
\begin{array}{lll}
\text{LO:} & a^0_{nn} & = -18.95 \text{ fm} \\
 & B(^2\text{H}) & = 2.2246 \text{ MeV} \\
 & B(^3\text{H}) & = 8.482 \text{ MeV} \\
\text{NLO:} & r^0_{nn} & = 2.75 \text{ fm} \\
 & r^1_{np} & = 1.753 \text{ fm} \\
 & B(^4\text{He}) & = 28.3 \text{ MeV}
\end{array}
$$

- NLO interaction is included perturbatively to circumvent the Wigner bound

Application to ^{4}He$+n$ scattering

- We applied our interaction to ^{4}He$+n$ scattering in the $^2S_{\frac{1}{2}^+}$ channel
- We confined our system in an harmonic potential and used the Busch formula to extract the scattering parameters, a_0 and r_{eff}
- We solved the Schrödinger equation with the Stochastic Variational Method (SVM)

Busch formula's idea



- We apply the Busch formula in order to extract the free space scattering parameters (scattering length a_0 and effective range r_{eff})

$$k \cot \delta_0 = -2\sqrt{\mu\omega}\frac{\Gamma\left(\frac{3}{4} - \frac{E}{2\hbar c\omega}\right)}{\Gamma\left(\frac{1}{4} - \frac{E}{2\hbar c\omega}\right)}$$

- The effective range expansion (ERE) gives us the scattering parameters

$$k \cot \delta_0 \approx \frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2$$

- The Busch formula relates trapped energies (solvable with bound state methods like SVM) with free space, untrapped scattering parameters

Busch formula convergence

- The Busch formula includes a Gamma ratio: it needs very high energy accuracy to give reliable results, below 10^{-2} MeV
- The harmonic constant ω has to be as low as possible, in order to well separate the scales of the system
- The typical scale of ^{4}He+n is estimated as the scattering length $a \approx 2.5$ fm

$$\hbar\omega = \frac{\hbar^2}{\mu L^2} < 8 \text{ MeV}$$

- In practice, it has to be at most 2 MeV in order to have negligible trap effects

Stochastic Variational Method

- Method to solve the Schrödinger equation standing on the variational principle, proposed by Suzuki and Varga in 1996
- The wave function is expanded as

$$|\Psi\rangle = \sum_{k=1}^{M} \alpha_k |\Phi_k\rangle$$

- Each $|\Phi_k\rangle$ depends on some parameters, which are chosen randomly
- More and more states are added until convergence

SVM single state

- The single basis state is expressed as a correlated Gaussian and an orbital, spin and isospin part

$$|\Phi\rangle = \mathcal{A}(G(A)|c\rangle)$$
$$\langle\vec{x}|G(A)\rangle = G(\vec{x},A) = e^{-\frac{1}{2}\vec{x}^{\mathrm{T}}A\vec{x}}$$
$$\langle\vec{x},\vec{s},\vec{I}|c\rangle = \langle\vec{x},\vec{s},\vec{I}|(LS)JM_JIM_I\rangle = [\varphi_L\otimes\varphi_S]_{J,M_J}\varphi_{I,M_I}$$

- The Gaussian form of the wave function allows analytical calculations of matrix elements
- The spin and isospin parts are just coupling of the single spins

$$\varphi_{S,M_S} = |[\ldots[[s_1\otimes s_2]_{s_{12}}\otimes s_3]\cdots\otimes s_N]_{S,M_S}\rangle$$

In presence of multiple configurations, they are chosen randomly as well!

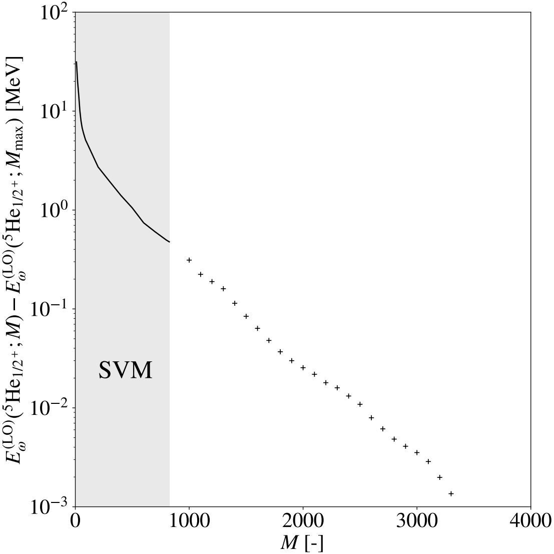
- The stochastic selection process eventually becomes too slow when the basis is big enough
- In order to reach the desired accuracy ad-hoc desgined states can be generated
- We generated states that capture the ^{4}He core - n dynamic as follows

$$A = \begin{pmatrix} (3\times 3) & 0 \\ 0 & \frac{1}{(n\beta)^2} \end{pmatrix}$$

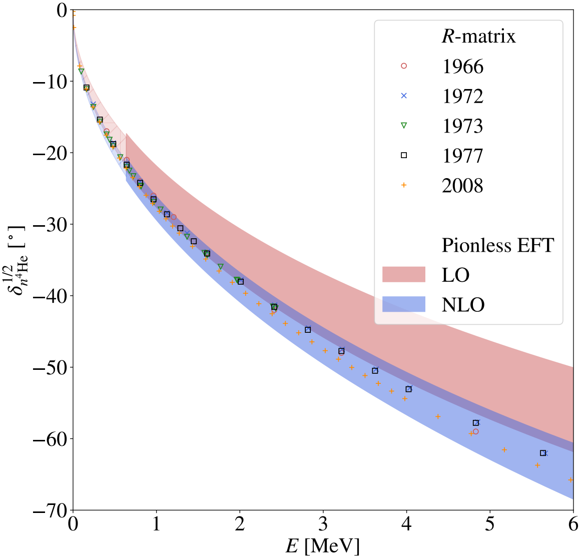
$$\exp\left(-\frac{1}{2}\vec{x}^{\mathsf{T}} A \vec{x}\right) = \exp\left(^4\text{He core}\right)\exp\left(-\frac{1}{2}\frac{{x_4}^2}{(n\beta)^2}\right)$$

- The 3×3 matrices are generated for ^{4}He with SVM, β is an optimized parameter and n runs from 1 to 10

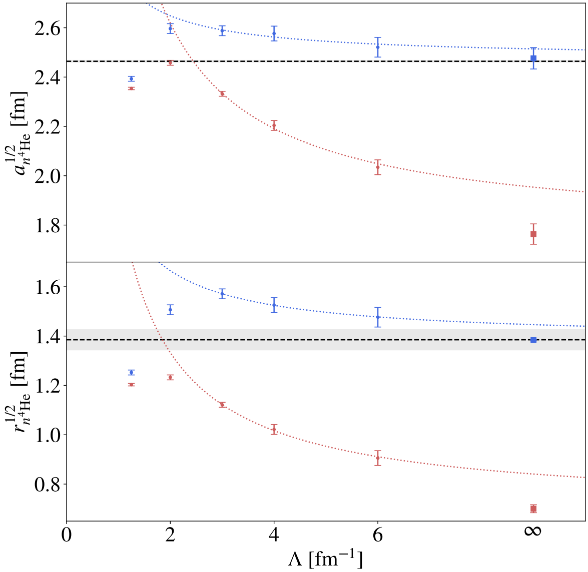
Convergence example



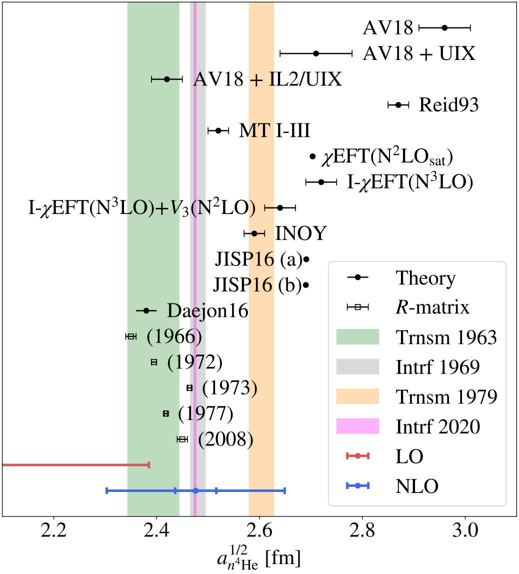
Phase shifts



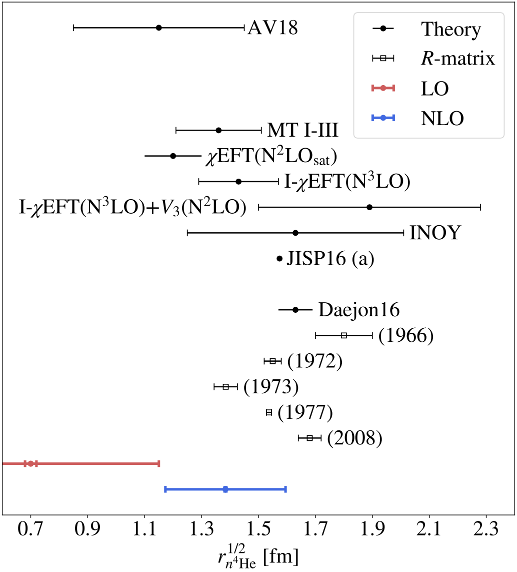
Scattering parameters



a_0 in the literature



r_0 in the literature



Conclusions & summary

- We presented the pionless Effective Field Theory potential up to NLO for the $L = 0$ case
- We emphasized the importance of the inclusion of a four-body force at NLO for the renormalization of the theory
- We extracted the scattering parameters a_0 and r_{eff} with the Busch formula
- We got amazing results compared to the literature and to other more sophisticated models!

Thank you for your attention!