



UNIVERSITÀ
DEGLI STUDI
DI TRIESTE

EMMI Workshop 



復旦大學
FUDAN UNIVERSITY

Bound states and particle interactions in the 21st century

Production of (Hyper)Nuclei within a Coalescence Approach

KaiJia Sun (孙开佳)

kjsun@fudan.edu.cn; 5/7/2023

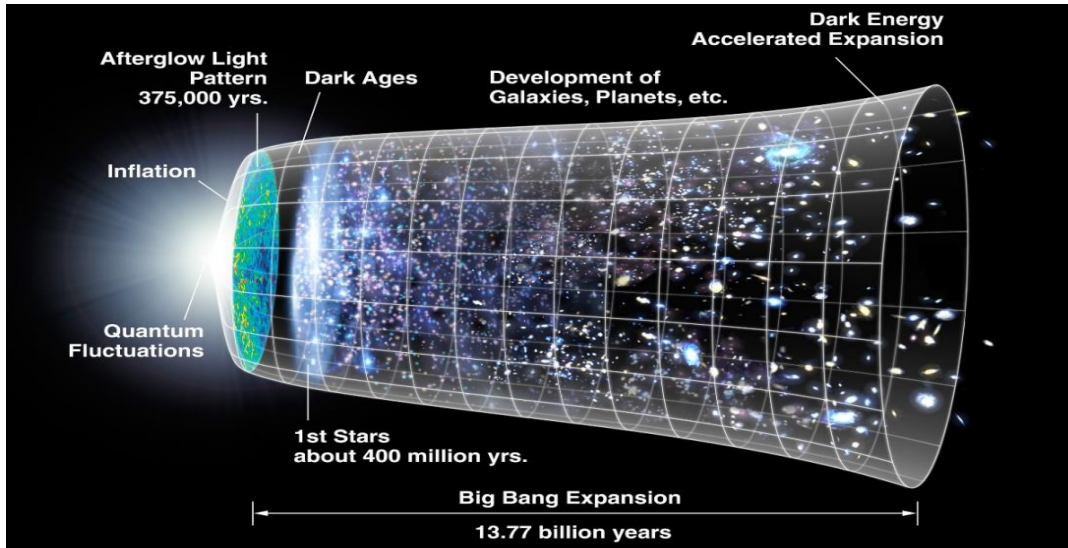
Institute of Modern Physics

Fudan University, China

I. Little Bang Nucleosynthesis

Little Bang Nucleosynthesis

(1)

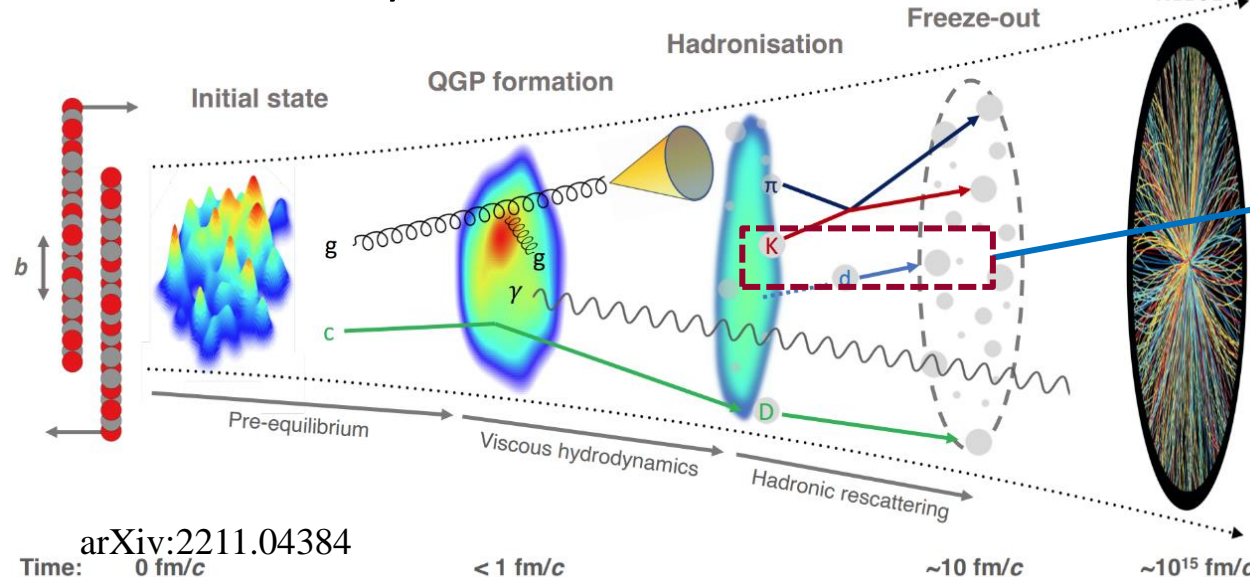


Big bang nucleosynthesis is responsible for the formation of light nuclei (e.g., d , ${}^3\text{He}$, ${}^4\text{He}$) in our Universe. $t \sim 100\text{ s}$, $T \sim \text{a few MeV}$

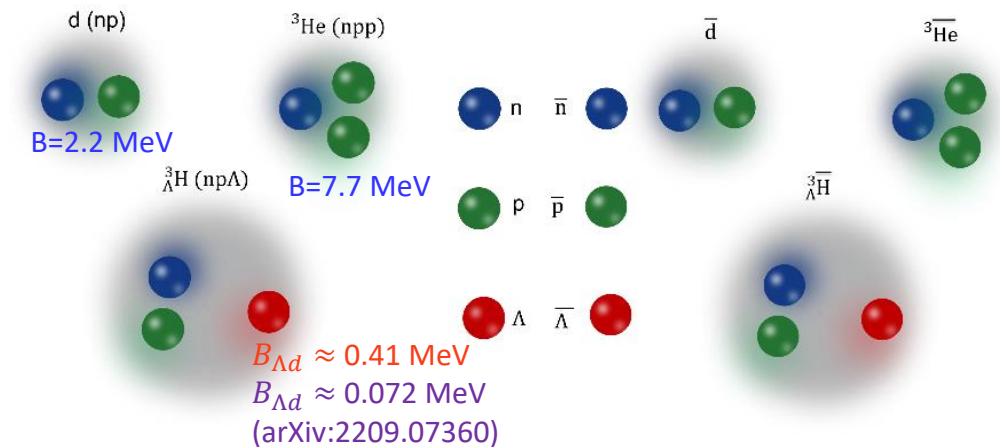
K. A. Olive et al., Phys. Rept. 333, 389–407 (2000); J. Chen et al., Phys. Rep. 760, 1 (2018); P. Braun-Munzinger and B. Donigus NPA987, 144 (2019)

ALICE (Nature Phys. 11,811(2015); Phys. Rev. Lett. 128, 252003 (2022); Nature Phys. 19, 61 (2023))

Relativistic Heavy-Ion collisions



Little bang nucleosynthesis produces both matter and antimatter nuclei; $t \sim 10^{-23}\text{ s}$, $T \sim 100\text{ MeV}$



STAR (Science 328, 58(2010); Nature 473,353(2011); Nat. Phys. 16 409(2020))

arXiv:2211.04384
Time: 0 fm/c

Multi-scale: Binding energies (E_B) $\ll T_c$ ($\sim 150\text{ MeV}$) $\ll m_N$ (938MeV); The size $r \sim \frac{1}{\sqrt{4\mu E_B}}$ ($r_d \sim 2\text{ fm}$, $r_{{}^3\text{He}} \sim 2\text{ fm}$, $r_{{}^3_{\Lambda}\text{H}} \sim 5\text{ fm}$); Huge cross sections.

Little bang nucleosynthesis

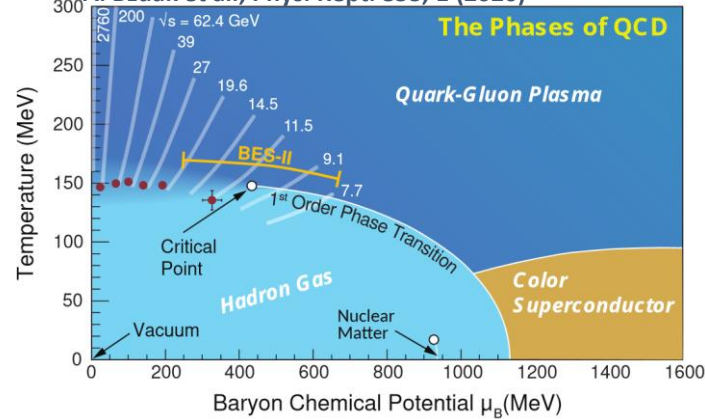
(2)

Related Physics:

QCD phase structure

X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017)

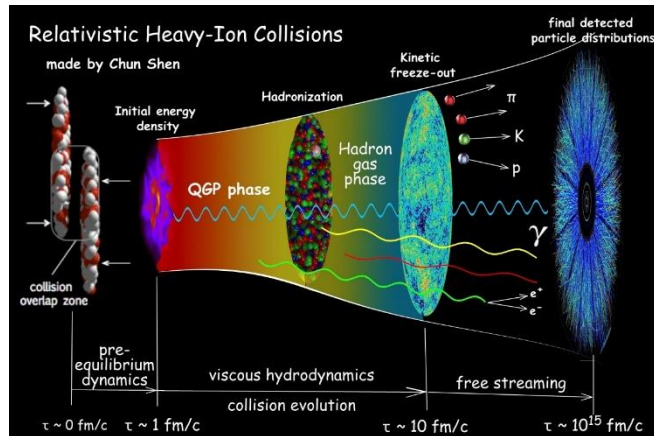
A. Bzdak et al., Phys. Rept. 853, 1 (2020)



K. J. Sun, L. W. Chen, C. M. Ko, and Z. Xu, Phys. Lett. B 774, 103 (2017);

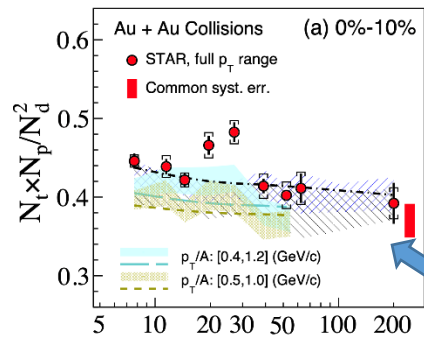
K. J. Sun, C. M. Ko, and F. Li, PLB 816, 136258 (2021)

QGP transport properties

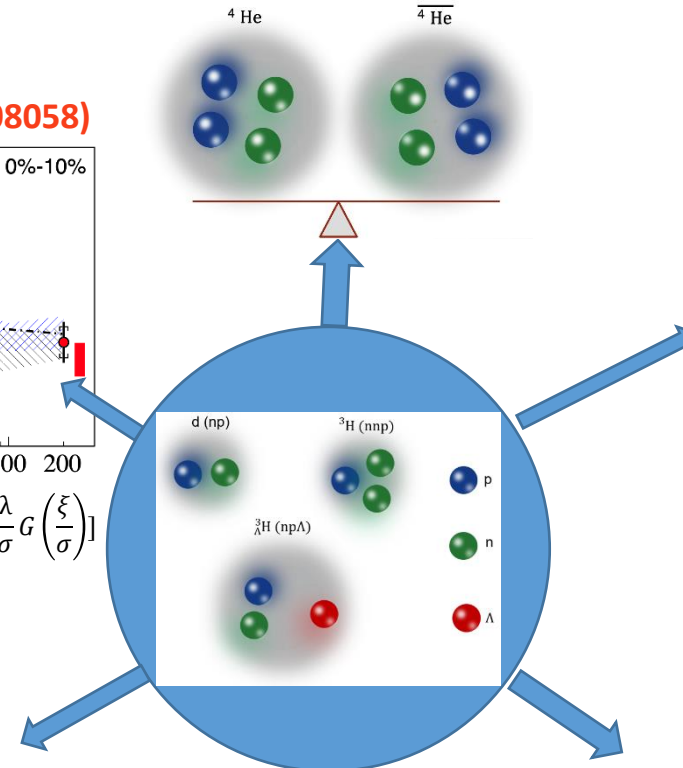


Matter-antimatter asymmetry

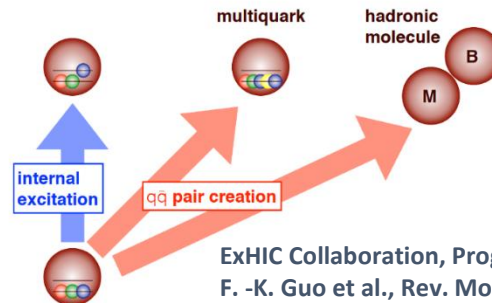
STAR, PRL (2209.08058)



$$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}} \left[1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right) \right]$$



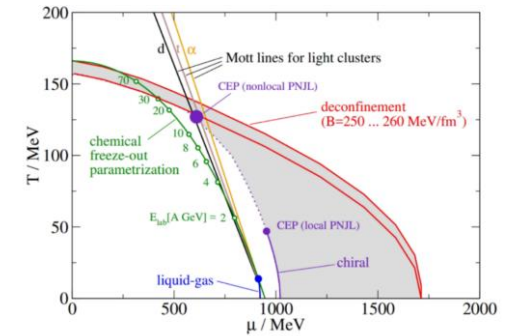
Extreme states



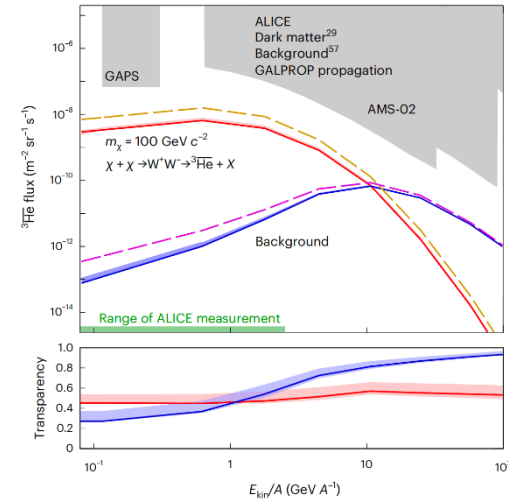
ExHIC Collaboration, Prog. Part. Nucl. Phys. 95, 279 (2017)

F. -K. Guo et al., Rev. Mod. Phys. 90, 015004 (2018)

Warm nuclear matter EOS



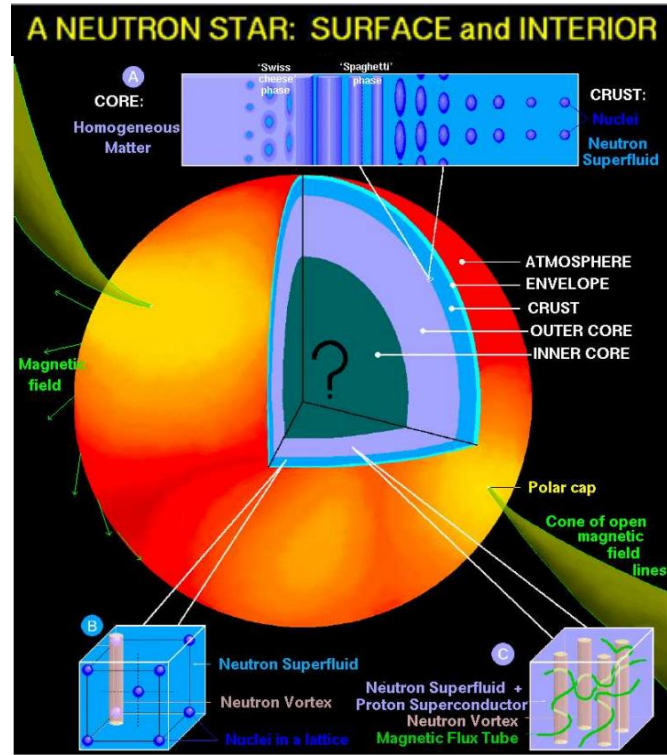
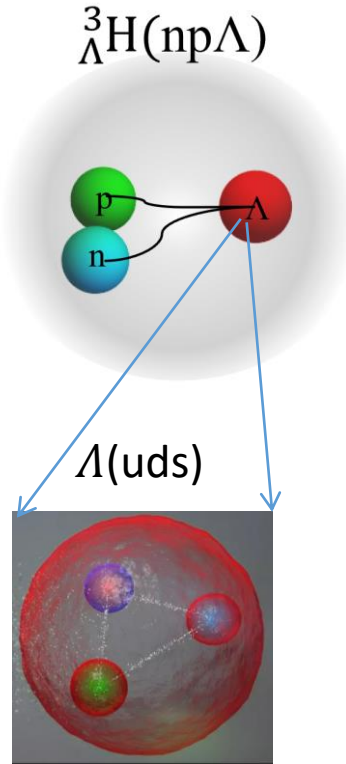
Dark matter searches



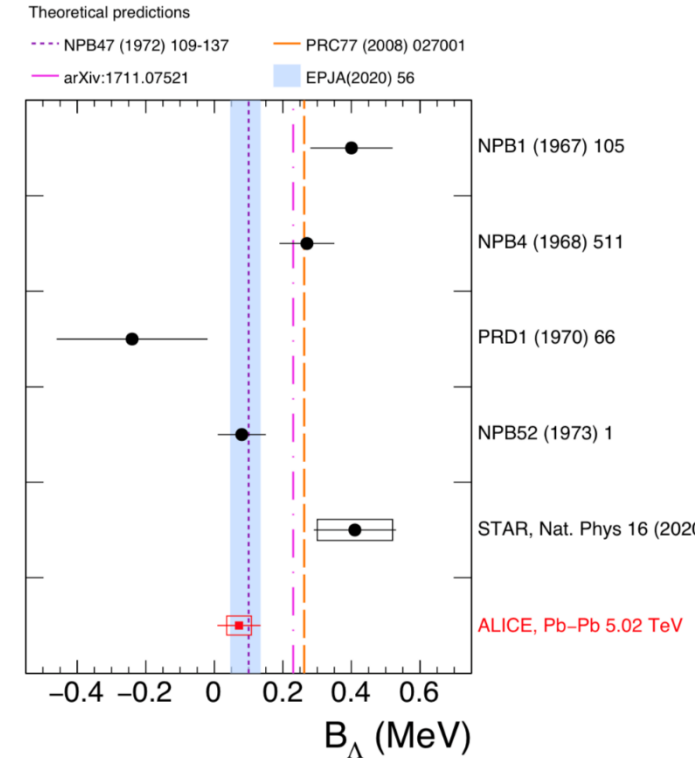
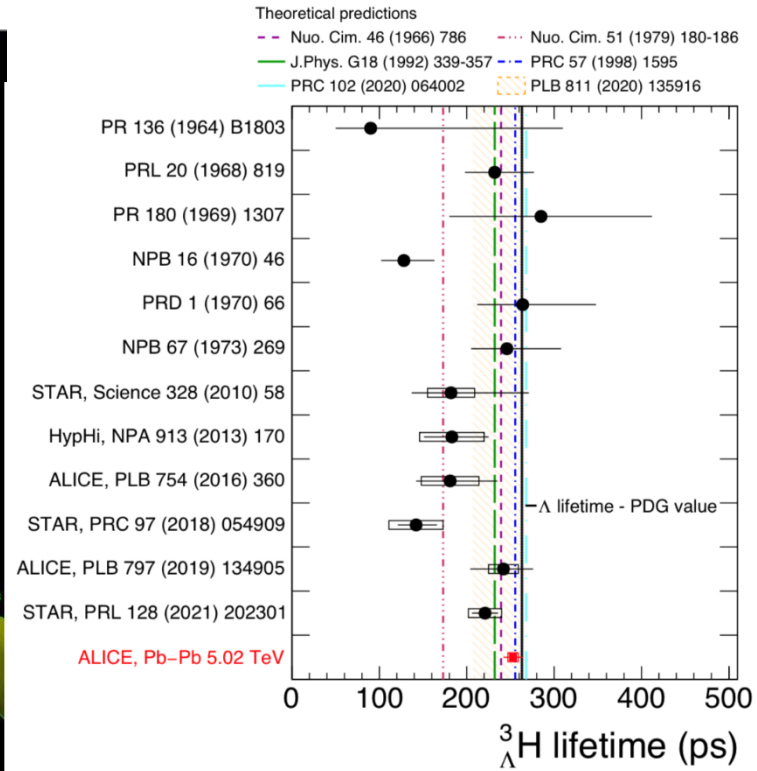
ALICE, Nature Phys. 19, 61-71 (2023)

Hypernuclei as a laboratory to understand the nature of hyperon-nucleon (YN) interactions

Y. G. Ma, Nucl. Sci. Tech. 3497 (2023)



J. M. Lattimer and M. Prakash, Science 304, 536-42 (2004).



$B_{\Lambda d} \approx 0.41\text{MeV}$
 STAR(Nat. Phys. 16 409(2020))

$B_{\Lambda d} \approx 0.072\text{MeV}$
 ALICE(2209.07360)

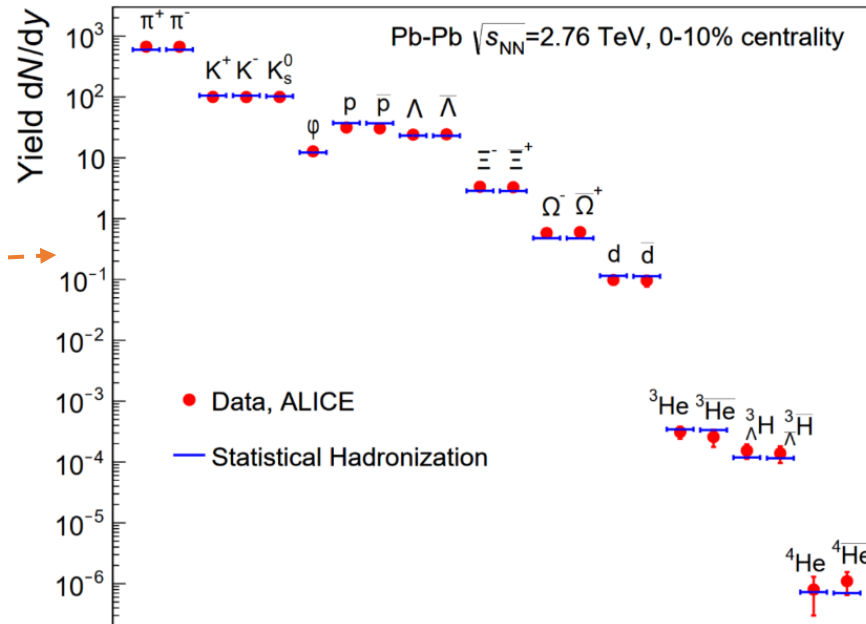
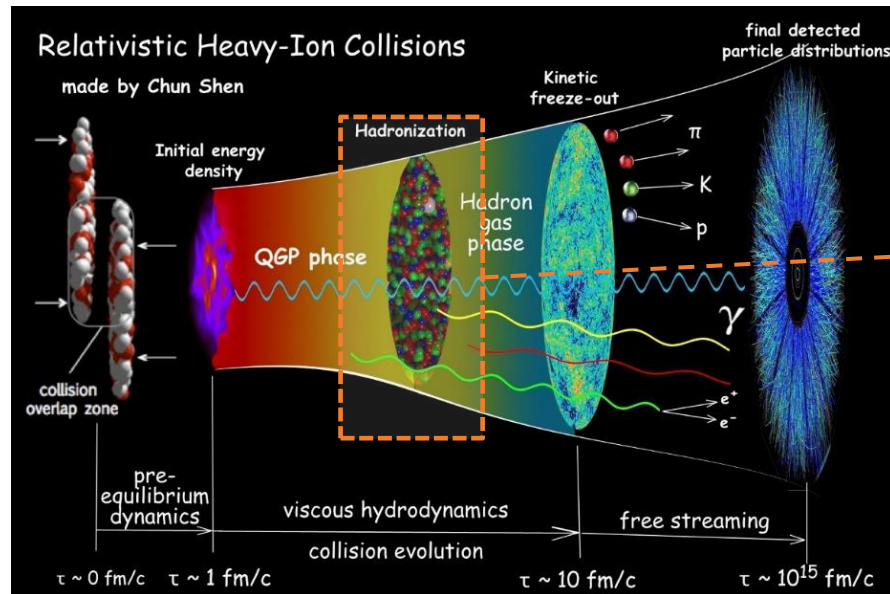
II. Production mechanisms

Statistical hadronization

(4)

Andronic, Braun-Munzinger, Redlich, Stachel, Nature 561, 321 (2018)

See talk by Röpke



$$N_h \approx \frac{\gamma_h g_h V_C}{2\pi^2} m_h^2 T_C K_2\left(\frac{m_h}{T_C}\right)$$

$$\approx \gamma_h g_h V_C \left(\frac{m_h T_C}{2\pi}\right)^{3/2} e^{-m_h/T_C}$$

T_C : Chemical freeze-out temperature, which is close to the chiral transition temperature (LQCD)

γ_h : Fugacity

Light (hyper)nuclei and ordinary hadrons share the same high chemical freezeout temperature $T_c = 156.6 \pm 1.7$ MeV, which almost coincides with the pseudo transition temperature from the QGP phase to the hadron phase.

Any effect of hadronic dynamics?

Effects of Hadronic Dynamics?

(5)

K. J. Sun, R. Wang, C. M. Ko, Y. G. Ma, and C. Shen, 2207.12532(2022)

Theoretically, the dynamics of deuteron production is governed by the evolution of 2-nucleon Green's function in the vicinity of a bound state (quasi-particle).

Relativistic kinetic equation for $\pi NN \leftrightarrow \pi d$

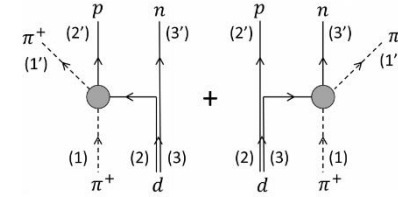
$$\frac{\partial f_d}{\partial t} + \frac{\mathbf{P}}{E_d} \cdot \frac{\partial f_d}{\partial \mathbf{R}} = -\mathcal{K}^> f_d + \mathcal{K}^<(1 + f_d)$$

with collision integral:

$$\begin{aligned} \text{R.H.S.} = & \frac{1}{2g_d E_d} \int \prod_{i=1'}^{3'} \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} \frac{d^3 \mathbf{p}_\pi}{(2\pi)^3 2E_\pi} \frac{E_d d^3 \mathbf{r}}{m_d} \\ & \times 2m_d W_d(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}) (|\mathcal{M}_{\pi+n \rightarrow \pi+n}|^2 + n \leftrightarrow p) \\ & \times \left[- \left(\prod_{i=1'}^{3'} (1 \pm f_i) \right) g_\pi f_\pi g_d f_d + \frac{3}{4} \left(\prod_{i=1'}^{3'} g_i f_i \right) \right. \\ & \left. \times (1 + f_\pi)(1 + f_d) \right] \times (2\pi)^4 \delta^4(p_{\text{in}} - p_{\text{out}}) \end{aligned}$$

Nonlocal collision integral to take into account the effects of **finite nuclei sizes**.
 W_d denotes deuteron Wigner function.

Impulse approximation (IA): Length/energy scale:



$$\lambda_{\text{thermal}} \sim 0.5 \text{ fm} \ll r_{np} \sim 4 \text{ fm}$$

FIG. 1. Diagrams for the reaction $\pi^+ d \leftrightarrow \pi^+ np$ in the impulse approximation. The filled bubble indicates the intermediate states such as a Δ resonance.

Solving kinetic equations with the stochastic method using test particles

Probability for reaction $\pi d \leftrightarrow \pi NN$ to take place in volume ΔV and time interval Δt are given by

$$\begin{aligned} \rightarrow P_{23}|_{\text{IA}} & \approx F_d v_{\pi+p} \sigma_{\pi+p \rightarrow \pi+p} \frac{\Delta t}{N_{\text{test}} \Delta V} + (p \leftrightarrow n). \\ P_{32}|_{\text{IA}} & \approx \frac{3}{4} F_d v_{\pi+p} \sigma_{\pi+p \rightarrow \pi+p} \frac{\Delta t W_d}{N_{\text{test}}^2 \Delta V} + (p \leftrightarrow n) \end{aligned}$$

For triton or helium-3:

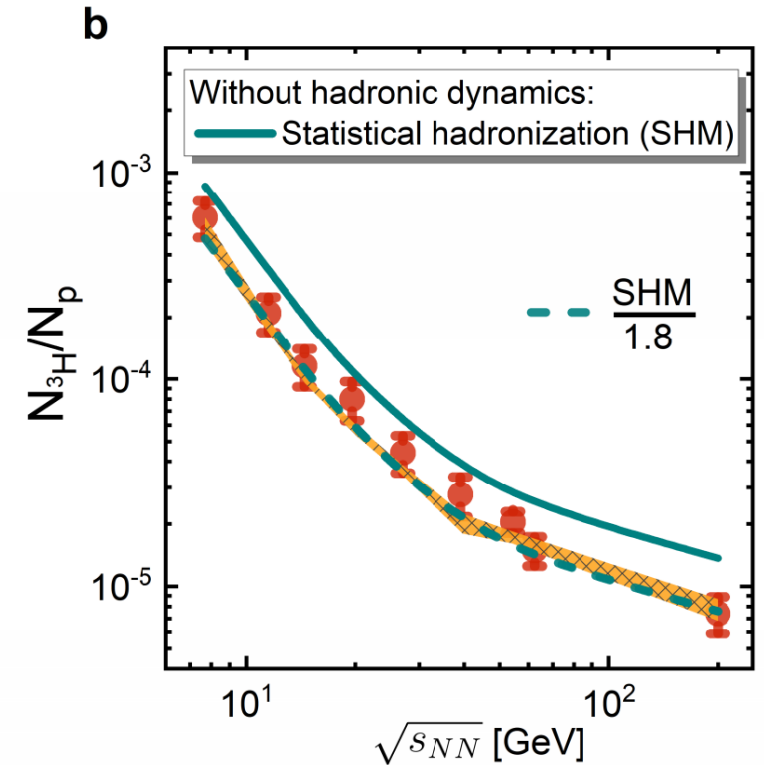
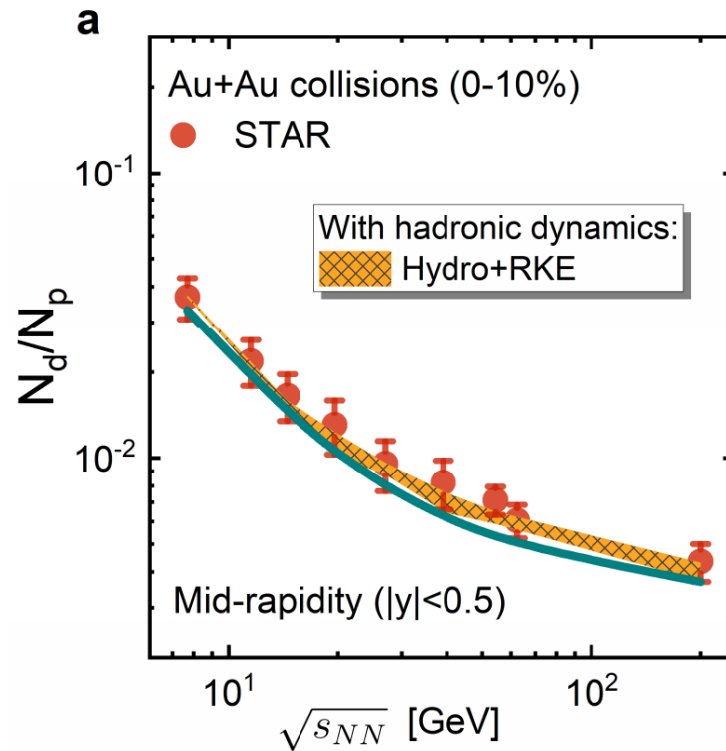
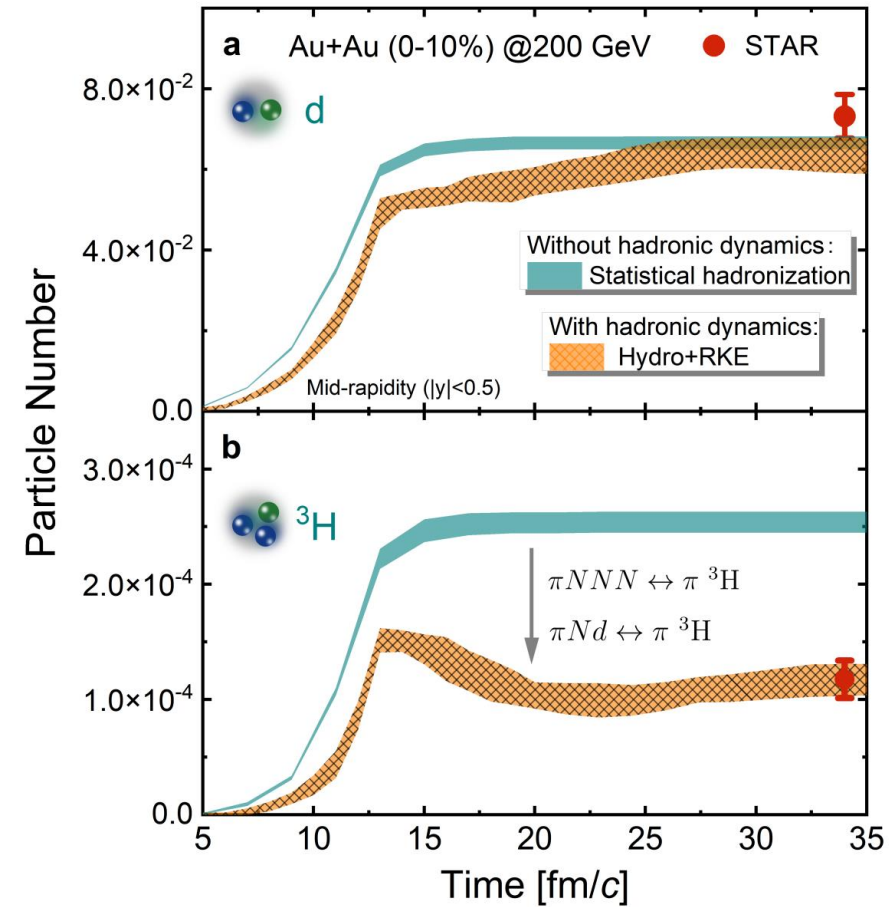
$$P_{42}|_{\text{IA}} \approx \frac{1}{4} F_t \frac{v_{\pi N} \sigma_{\pi N \rightarrow \pi N} \Delta t}{N_{\text{test}}^3 \Delta V} W_t$$

'renormalization' factor F_d, F_t which can be fixed by πd and πt cross sections.

Effects of Hadronic Dynamics

(6)

K. J. Sun, R. Wang, C. M. Ko, Y. G. Ma, and C. Shen, 2207.12532(2022) Data from STAR, PRL 130, 202301 (2023)



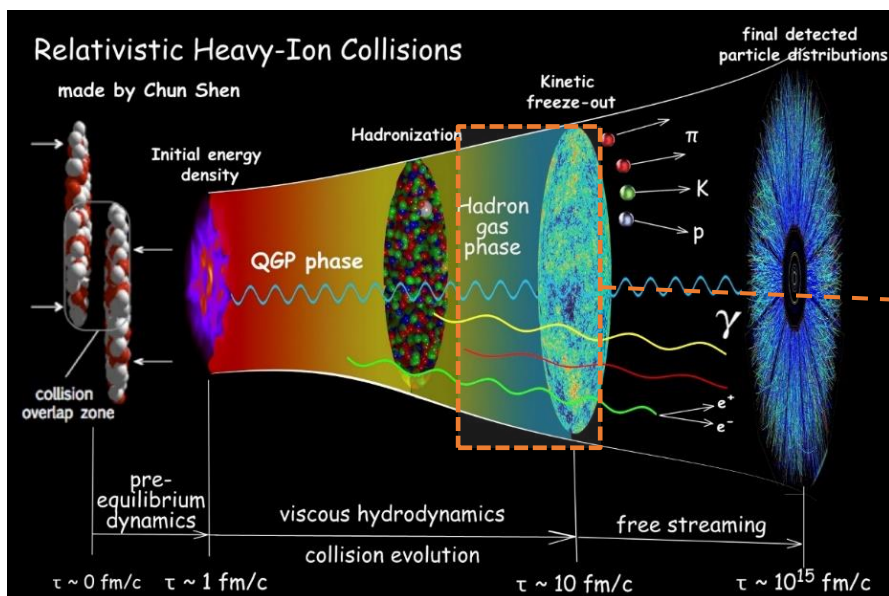
Hadronic re-scatterings have small effects on the final deuteron yield (see also D. Oliinychenko et al. PRC 99, 044907 (2019)), but they reduce the triton yield by about a factor of 2

Final-State Coalescence

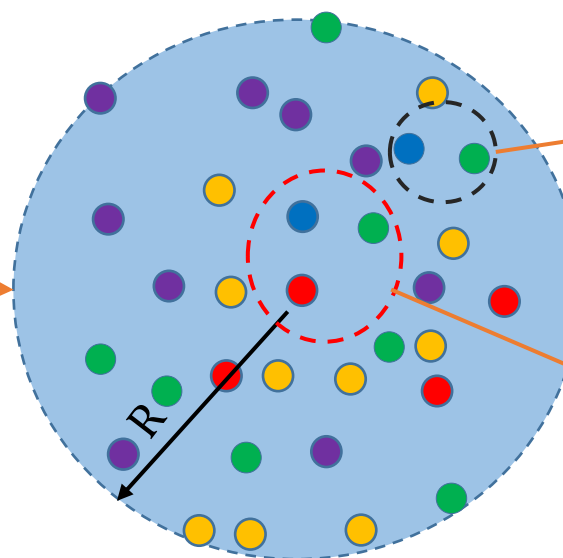
(7)

The strong hadronic re-scatterings push the chemical freeze-out of light (hyper)nuclei to later hadronic stage. The final-state coalescence model can be derived under from the kinetic approach certain approximations (quasi-particle, small binding energy and etc.).

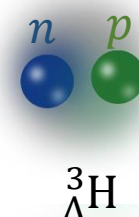
R. Scheibl and U. W. Heinz, PRC59. 1585(1999);
P. Danielewicz et al., NPA533, 712 (1991).



Coalescence Model



Deuteron



Density Matrix Formulation

$$N_A = Tr(\hat{\rho}_S \hat{\rho}_A)$$

$$= g_c \int d\Gamma \rho_s(\{x_i, p_i\}) \times W_A(\{x_i, p_i\})$$

Wigner function of light cluster

Overlap between source distribution function and Wigner function of light nuclei

F. Bellini et al., PRC 99, 044913 (2019)

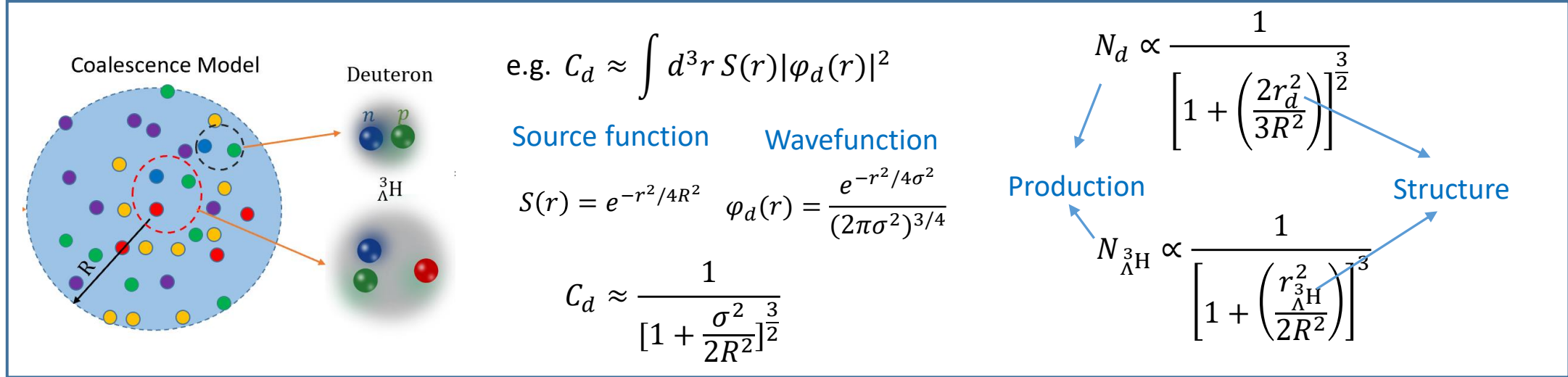
Internal wavefunction plays an important role

III. Hypertriton Production within a Coalescence Model

Quantum Mechanical Correction on Yields

(8)

K. J. Sun, C. M. Ko, and B. Dönigus, Phys. Lett. B792, 132-137(2019)



LHC energies:

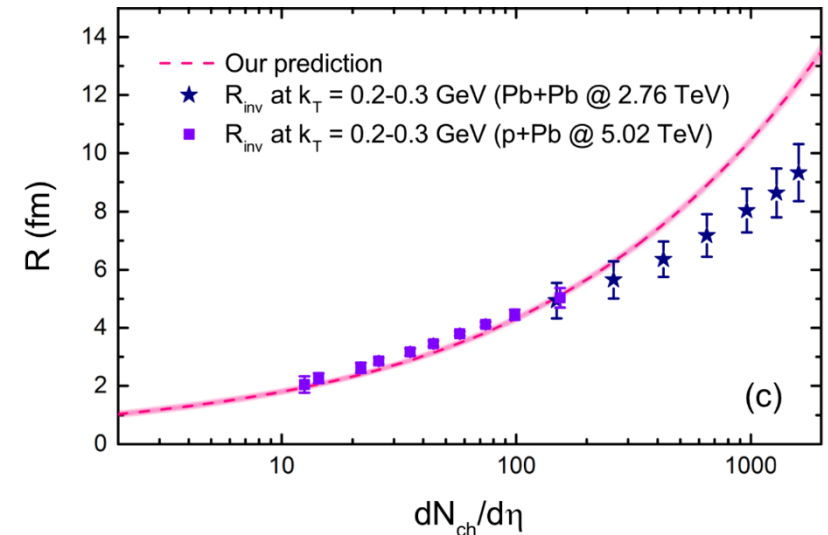
$$\frac{N_d}{N_p} \approx \frac{4.0 \times 10^{-3}}{\left[1 + \frac{2r_d^2}{3R^2}\right]^{3/2}}$$

$$\frac{N_{^3\text{He}}}{N_p} \approx \frac{7.1 \times 10^{-6}}{\left[1 + \frac{r_{^3\text{He}}^2}{2R^2}\right]^3}$$

$$\frac{N_{^3\text{He}}}{N_\Lambda} \approx \frac{7.1 \times 10^{-6}}{\left[1 + \frac{r_{^3\text{He}}^2}{2R^2}\right]^3}$$

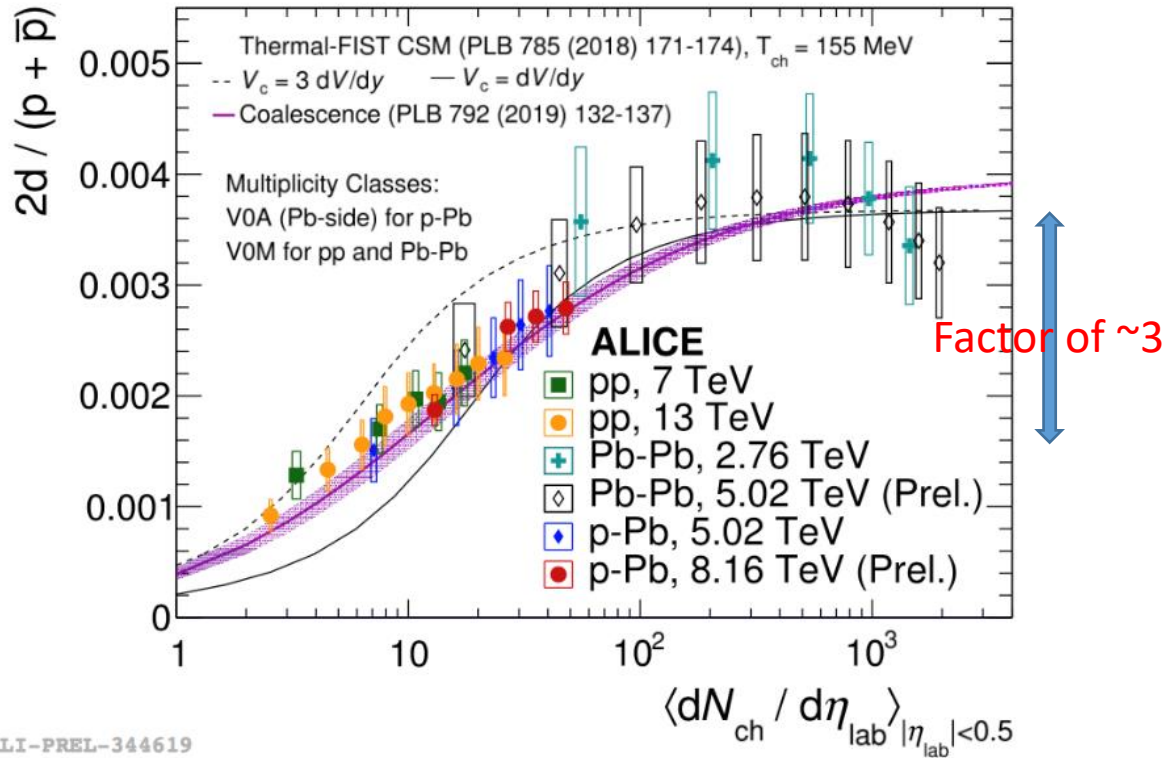
:3-body coal. (equilateral triangle)

HBT measurements:

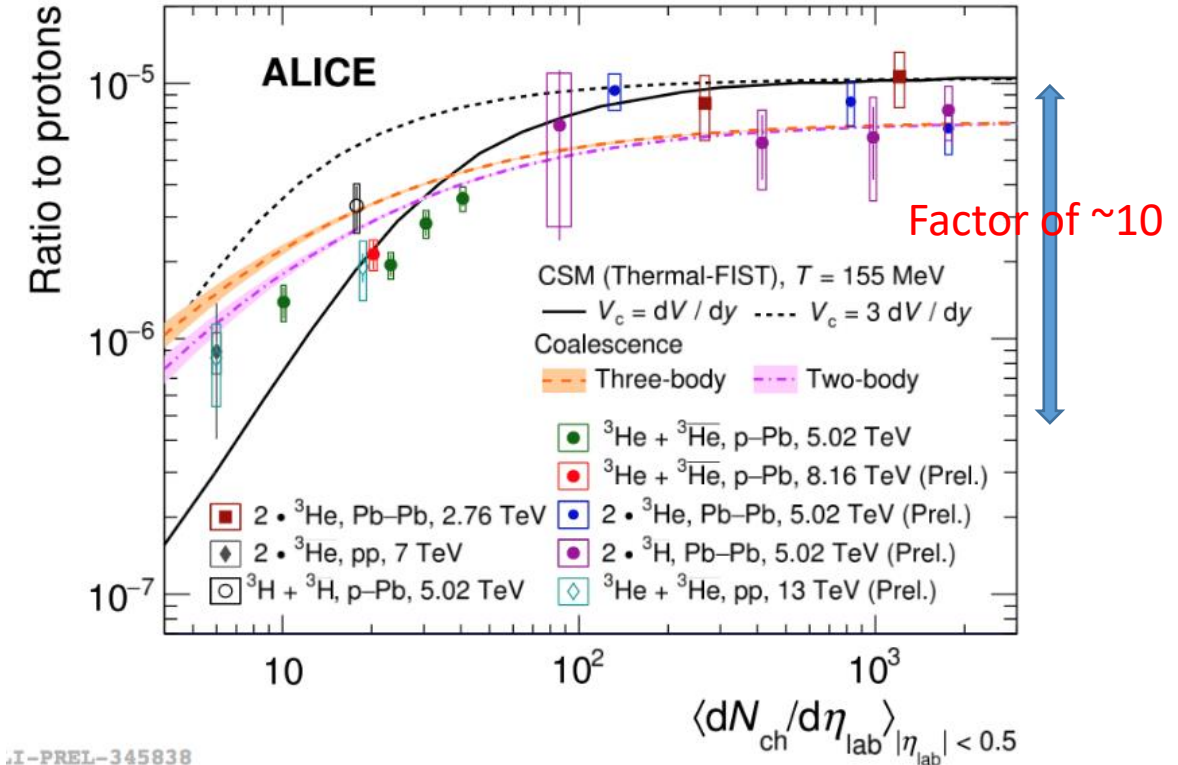


Suppression of light nuclei production in small systems (9)

L. Barioglio for ALICE Collaboration. PoS LHCP2021 (2021) 056;



CSM: V. Vovchenko et al., PLB 785, 171 (2019), PRC 100,054906 (2019)



Coalescence: K. J. Sun, C. M. Ko and B. D. Ö nigus, Phys. Lett. B 792, 132 (2019)

Large suppression factors in collisions of small systems

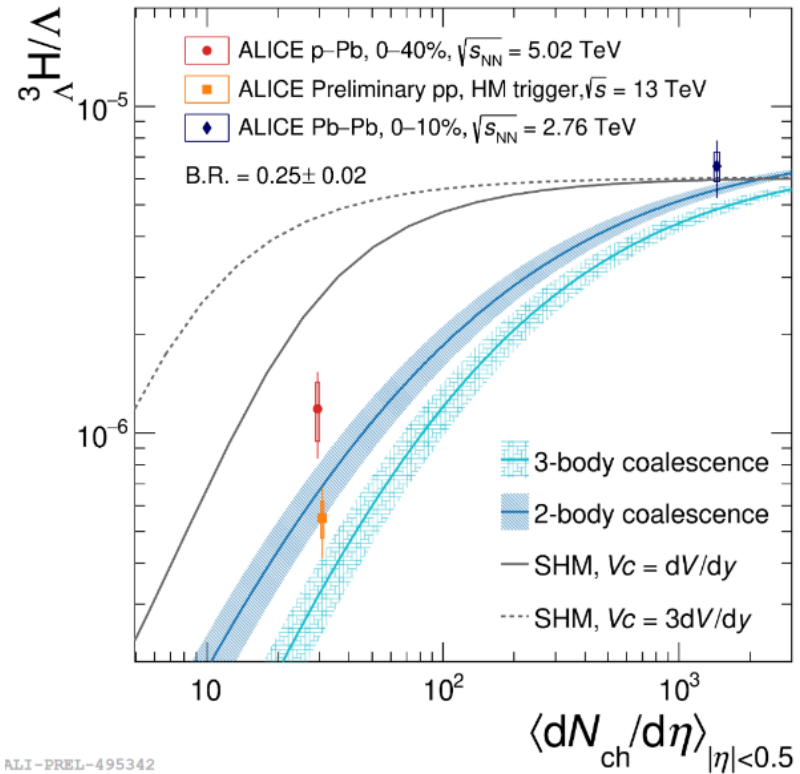
Coal: Finite nuclei sizes (better description on hypertriton production in p+p and p+Pb collisions)

CSM: Baryon number conservation

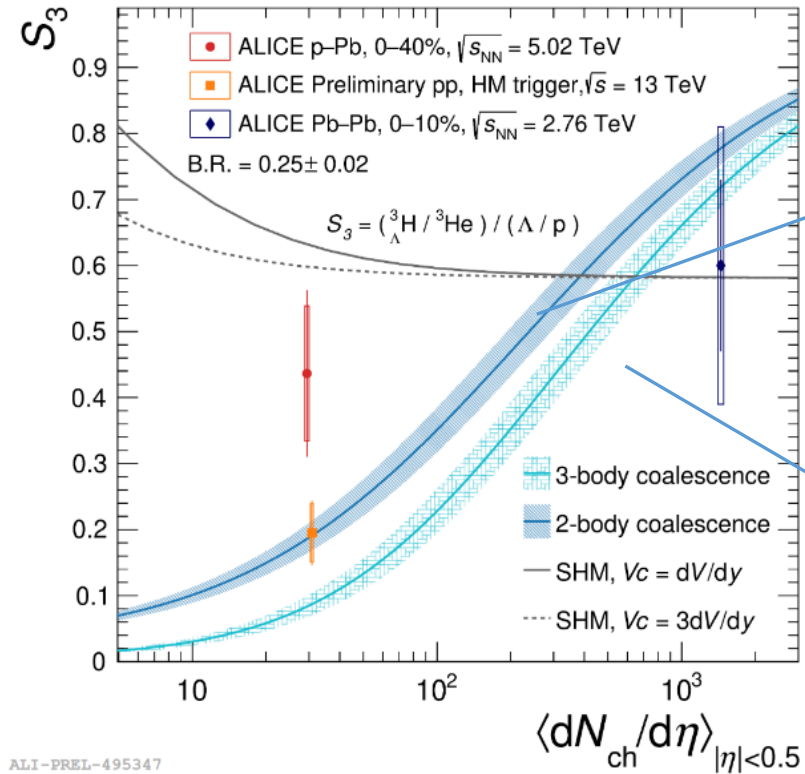
Suppression of hypernuclei production in small systems (10)

ALICE, Phys. Rev. Lett. 128 (2022) 25, 252003

See talk by C. Pinto



ALI-PREL-495342



ALI-PREL-495347

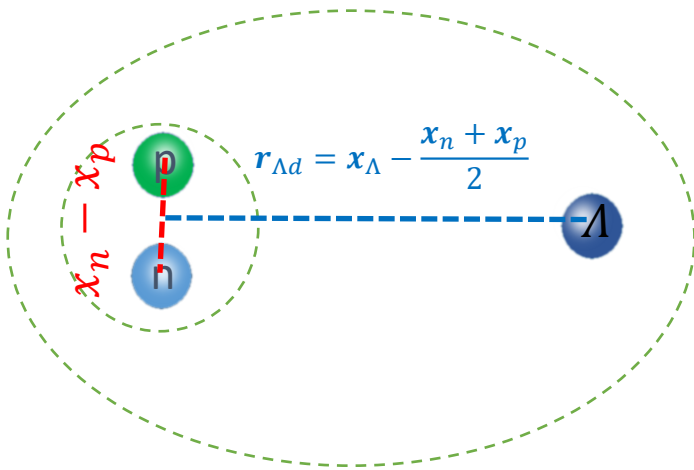
Momentum spectrum encodes much more detailed information

Quantum Mechanical Correction on Spectrum

(11)

MUSIC + UrQMD + Coalescence

Due to small lambda separation energy, the hypertriton can be well approximated as a bound state of deuteron and lambda hyperon.



$$W_{ht} = 8^2 e^{-\frac{x_1^2}{\sigma_1^2} - k_1^2 \sigma_1^2 - \frac{x_2^2}{\sigma_2^2} - k_2^2 \sigma_2^2}$$

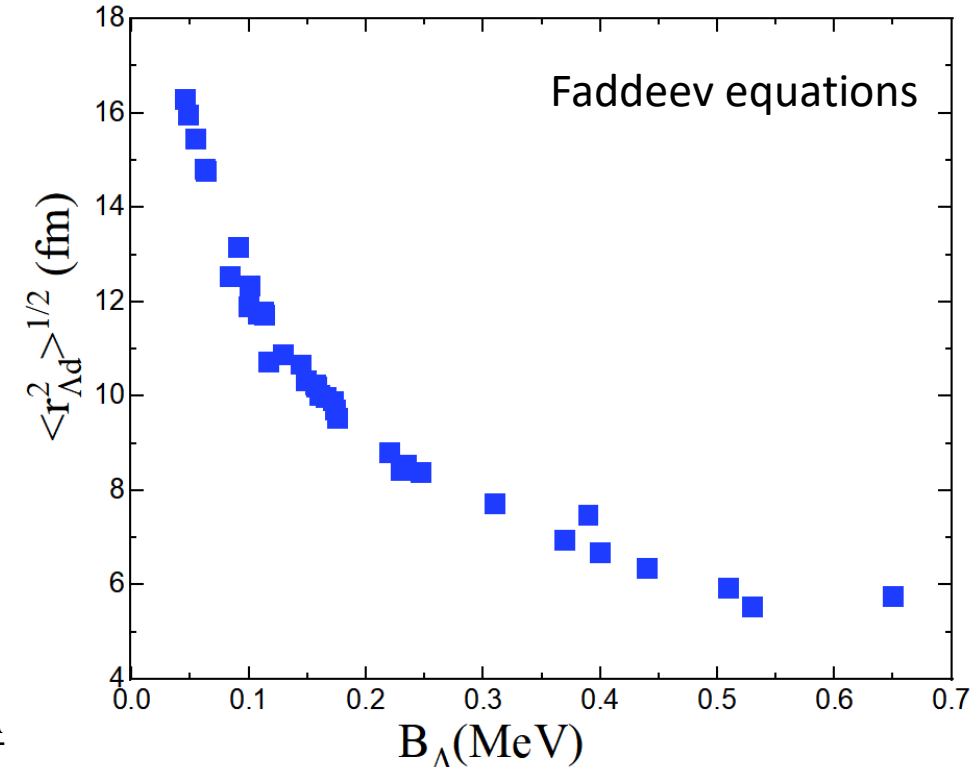
$$x_1 = \frac{x_n - x_p}{\sqrt{2}}$$

$$k_1 = \sqrt{2} \frac{m_p k_n - m_n k_p}{m_n + m_p}$$

$$x_2 = \sqrt{\frac{2}{3}} \left(\frac{m_n x_n - m_p x_p}{m_n + m_p} - x_\Lambda \right)$$

$$k_2 = \sqrt{\frac{2}{3}} \frac{3 m_\Lambda (k_n + k_p) - (m_n + m_p) k_\Lambda}{m_n + m_p + m_\Lambda}$$

$$\sqrt{\langle r_{\Lambda d}^2 \rangle} = \frac{3}{2} \sigma_2$$



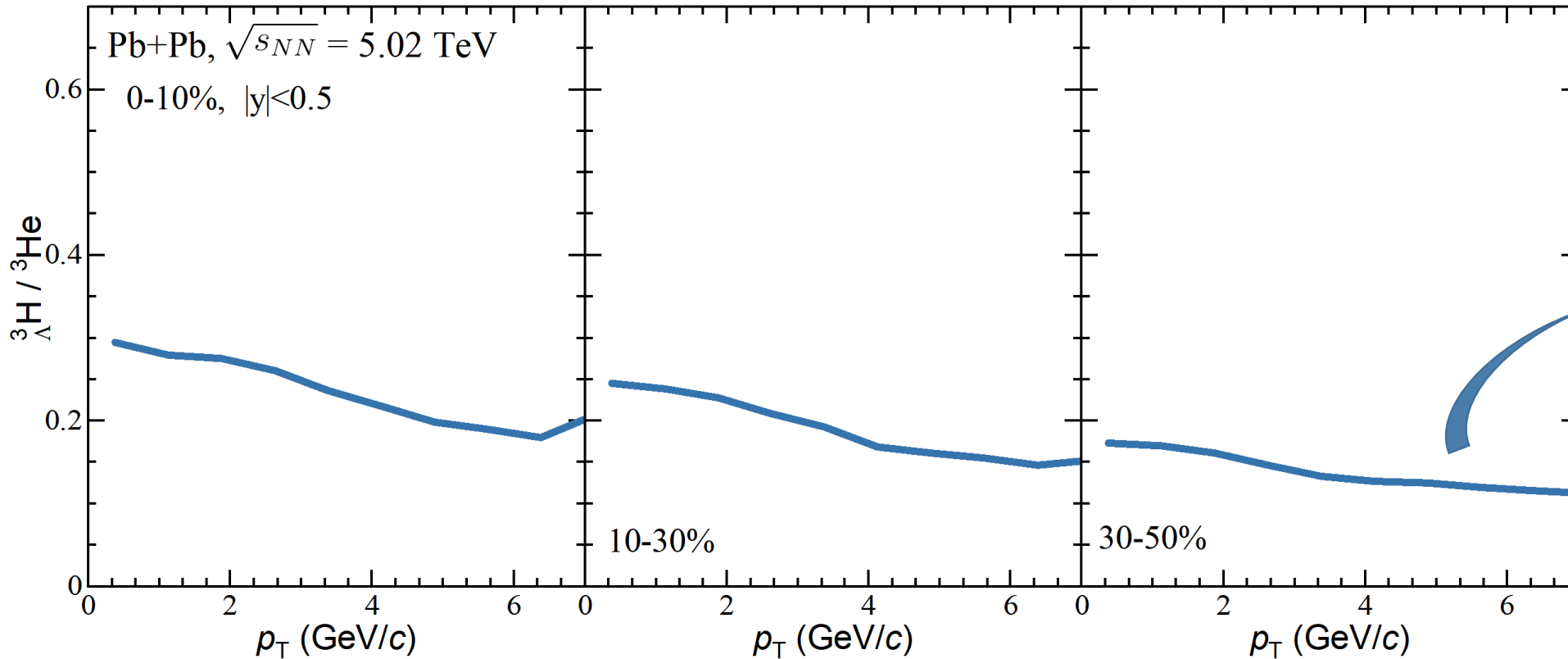
Taking $B_\Lambda = 0.13$ MeV

$$\sqrt{\langle r_{\Lambda d}^2 \rangle} \approx 10 \text{ fm} \quad \sigma_2 \approx 6.7 \text{ fm}$$

Quantum Mechanical Correction on Spectrum

(12)

Preliminary

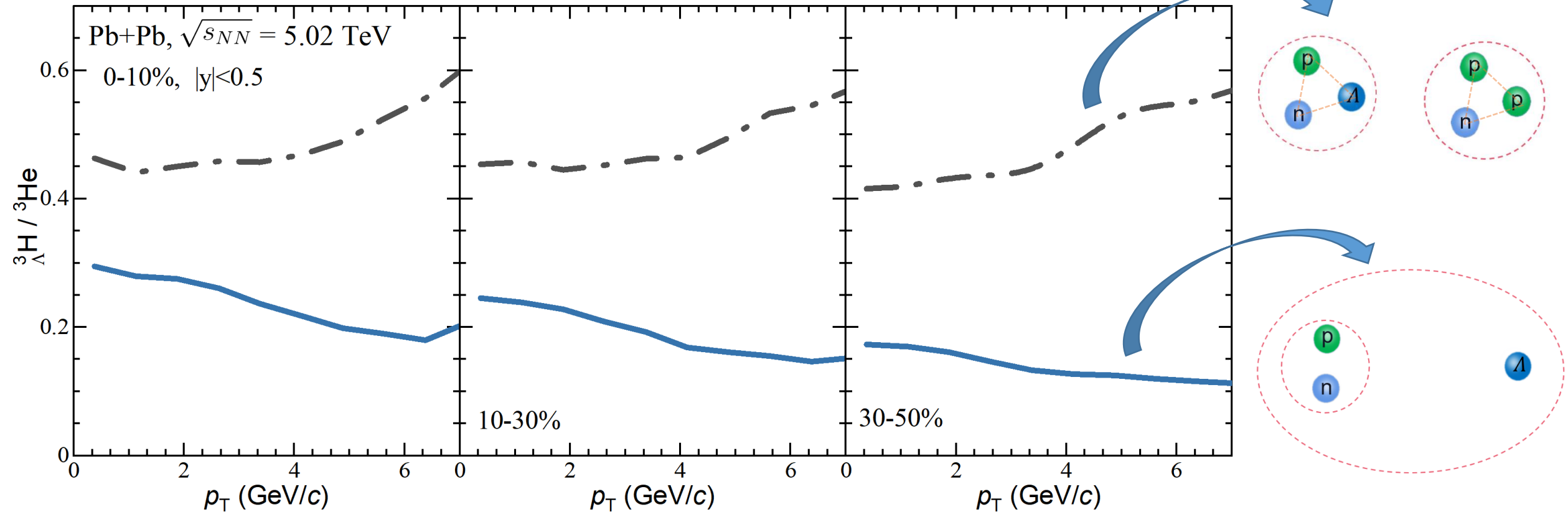


$\frac{{}^3\Lambda\text{H}}{{}^3\text{He}}$ decreases for all centralities

Quantum Mechanical Correction on Spectrum

(13)

Preliminary

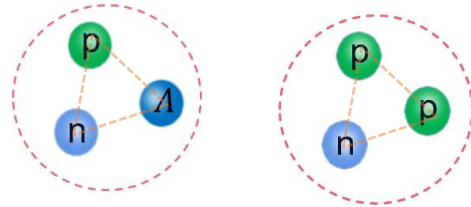
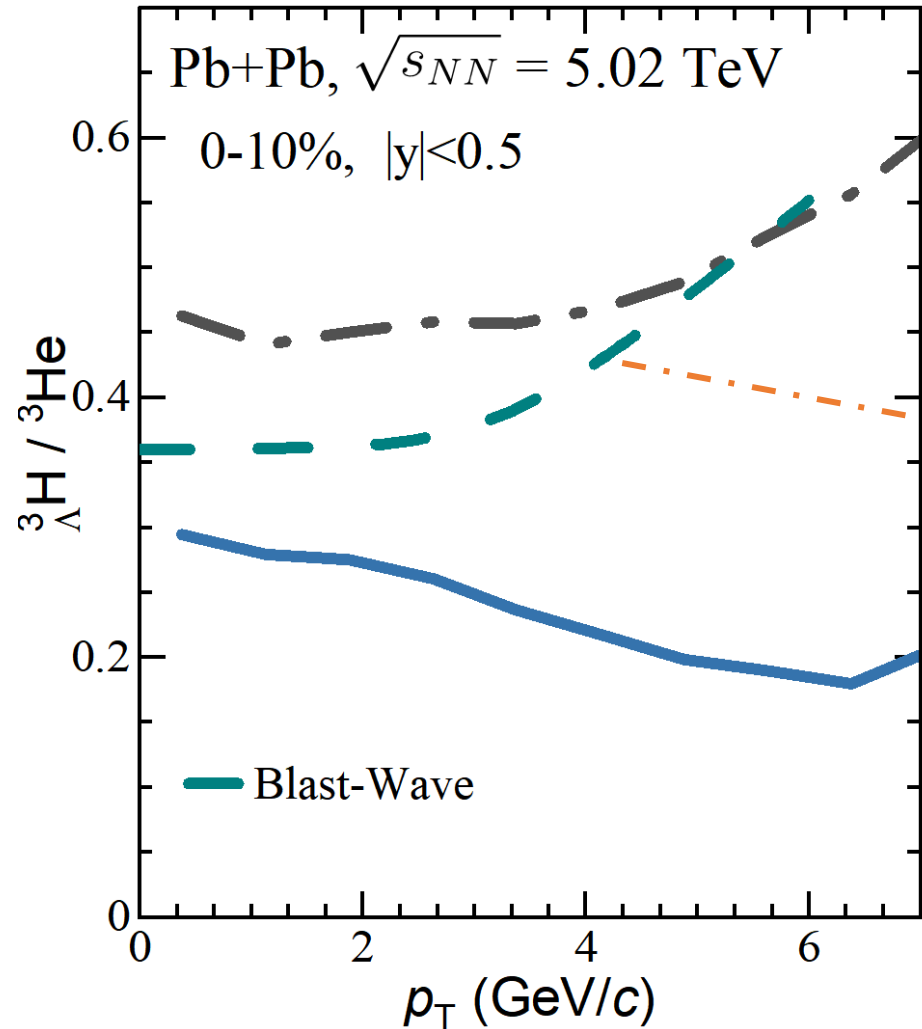


Sensitive to its internal wavefunction

Quantum Mechanical Correction on Spectrum

(14)

Preliminary

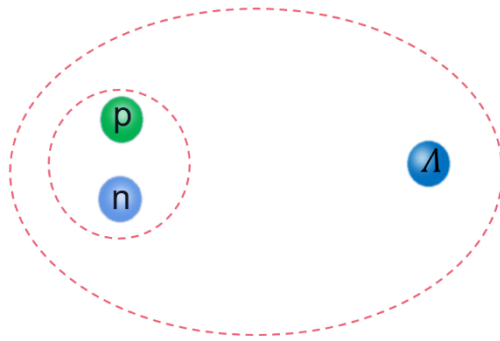


Coalescence model:

- $\frac{{}^3\Lambda\text{H}}{{}^3\text{He}}$ increases for small $\Lambda - d$ distance.
- It decreases for large $\Lambda - d$ distance (~ 10 fm)

Blast-wave model:

- $\frac{{}^3\Lambda\text{H}}{{}^3\text{He}}$ increases as p_T

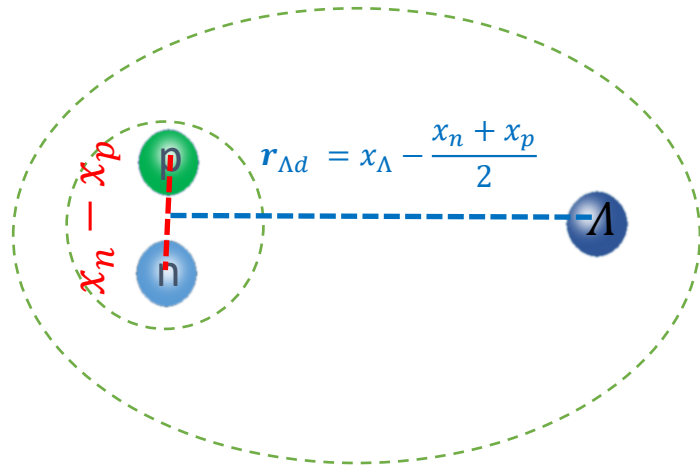


Why?

Two models yield opposite p_T dependence even in the **most central collisions!**

Quantum Mechanical Correction on Spectrum

(15)



$$W_{ht} = 8^2 e^{-\frac{x_1^2}{\sigma_1^2} - \mathbf{k}_1^2 \sigma_1^2 - \frac{x_2^2}{\sigma_2^2} - \mathbf{k}_2^2 \sigma_2^2}$$

$$x_1 = \frac{x_n - x_p}{\sqrt{2}}$$

$$x_2 = \sqrt{\frac{2}{3}} \left(\frac{m_n x_n - m_p x_p}{m_n + m_p} - x_\Lambda \right)$$

$$\mathbf{k}_1 = \sqrt{2} \frac{m_p \mathbf{k}_n - m_n \mathbf{k}_p}{m_n + m_p}$$

$$\mathbf{k}_2 = \sqrt{\frac{3}{2}} \frac{m_\Lambda (\mathbf{k}_n + \mathbf{k}_p) - (m_n + m_p) \mathbf{k}_\Lambda}{m_n + m_p + m_\Lambda}$$

- Static Gaussian source

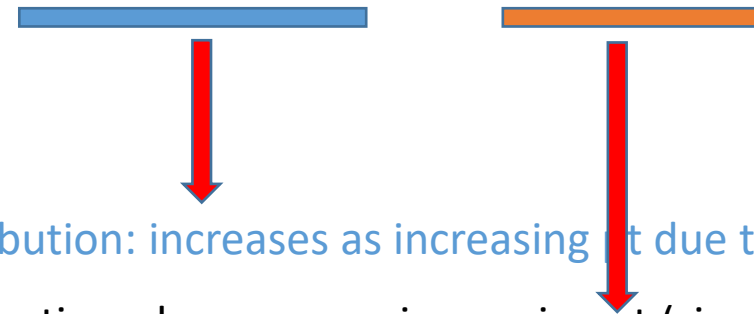
$$C_{ht}(R) \approx \frac{1}{\left[1 + \frac{\sigma_1^2}{2R^2}\right]^{\frac{3}{2}} \left[1 + \frac{\sigma_2^2}{2R^2}\right]^{\frac{3}{2}}} \quad \begin{array}{l} \sigma_1 \approx 2.26 \text{ fm} \\ \sigma_2 \approx 6.7 \text{ fm} \end{array}$$

$$C_{he3}(R) \approx \frac{1}{\left[1 + \frac{\sigma^2}{2R^2}\right]^3} \quad \sigma \approx 2.26 \text{ fm}$$

The quantum correction factor on ${}^3_\Lambda\text{H}/{}^3\text{He} \propto C_{ht}/C_{he3}$ decreases as increasing R.

- Expanding source

$${}^3_\Lambda\text{H}/{}^3\text{He} \propto \frac{\int p^\mu \cdot d\sigma_\mu f_{BL}(m_{ht}, p_T, \beta_s) C_{ht}(R(p_T))}{\int p^\mu \cdot d\sigma_\mu f_{BL}(m_{he3}, p_T, \beta_s) C_{he3}(R(p_T))}$$



Thermal contribution: increases as increasing pt due to the flow effect

Quantum correction: decreases as increasing pt (since the inhomogeneity length R decreases)

Competition between Flow effects and Quantum effects

- I. Post-hadronization dynamics plays a vital role in the little-bang nucleosynthesis. It suppresses triton yields by about a factor of two. (observed in STAR measurements [[PRL, 130, 202301 \(2023\)](#)]).
K. J. Sun, R. Wang, C. M. Ko, Y. G. Ma, and C. Shen, 2207.12532(2022)
- II. Large radii of light (hyper)nuclei suppress their yields in collisions of small system. (observed in ALICE measurements [[PRL 128 \(2022\) 25, 252003](#)]).
K. J. Sun, C. M. Ko and B. Dönigus, Phys. Lett. B 792, 132 (2019)
- III. It also leads to a decreasing $\frac{{}^3\text{H}}{{}^3\text{He}}$ as a function of p_T (*event in the most central collisions*), which is different from that predicted by the blast-wave model. **Such an effect can be readily tested at the LHC and RHIC.**

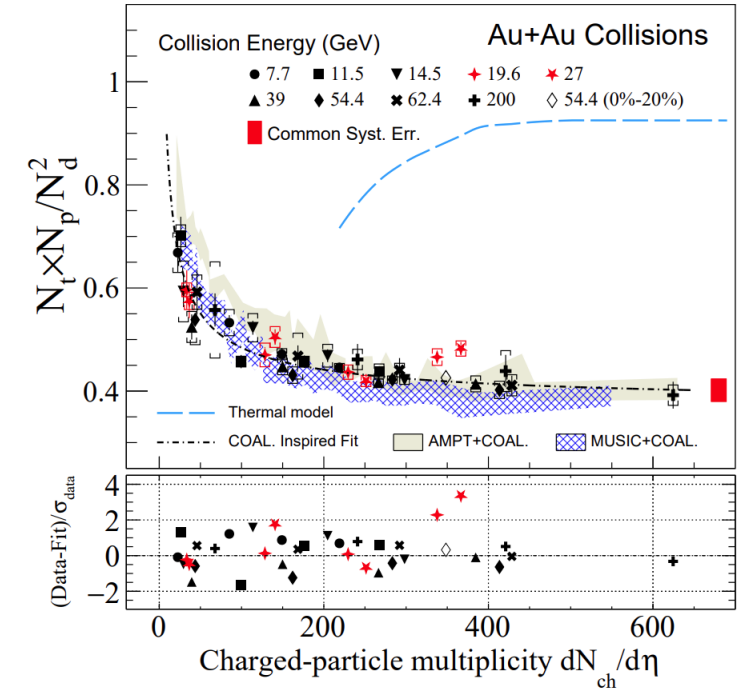
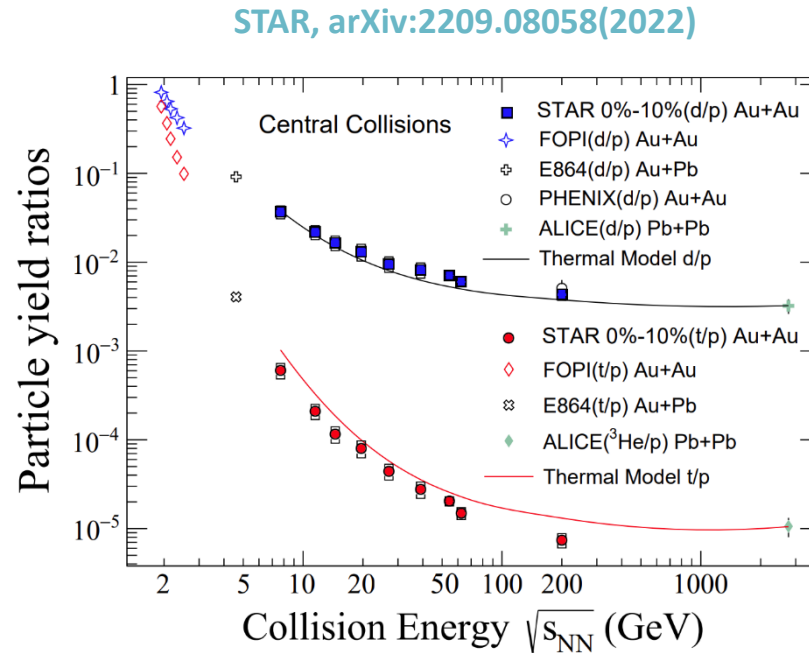
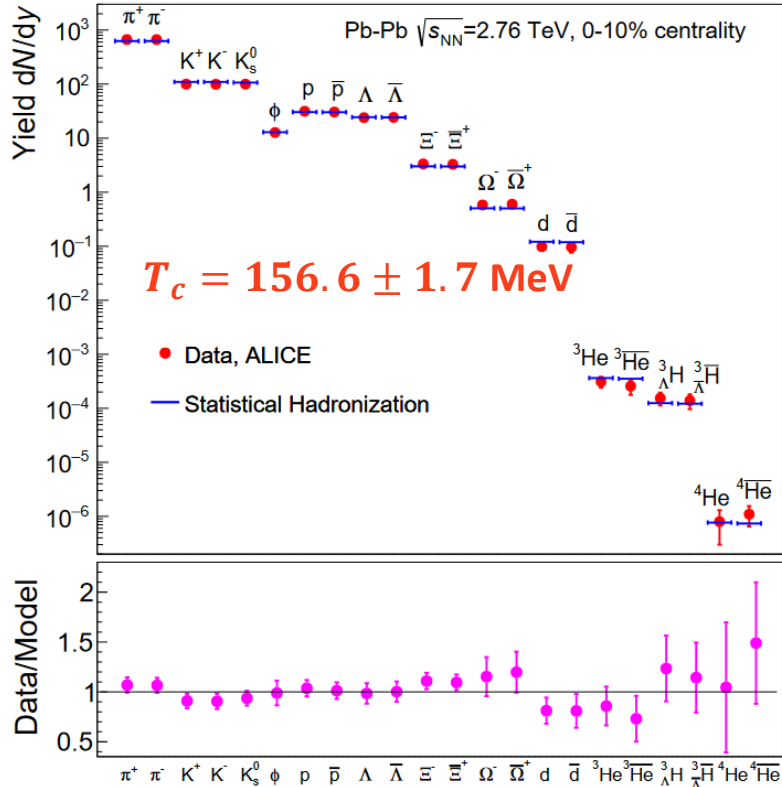
Open questions:

- A. A unified description from low to high energies and small to large collision systems?
- B. Medium corrections? (Hot pion gas, high baryon density [[R. Wang et al., arXiv: 2305.02988](#)])
- C. Dynamics of nucleus $A > 4$ and hypernucleus ?
- D. How does QCD phase transitions affect light nuclei production?
Indirect dark matter searches

Backup

The Triton Puzzle

(8)



**Triton yields at RHIC are overestimated by the statistical hadronization model!
The effects of hadronic re-scatterings need to be re-examined.**

- $A = 2$ $\pi NN \leftrightarrow \pi d, NNN \leftrightarrow Nd$
- $A = 3$ $\pi NNN \leftrightarrow \pi t(h), \pi Nd \leftrightarrow \pi t(h), NNNN \leftrightarrow Nt(h), NNd \leftrightarrow Nt(h)$

A. Andronic, P. Braun-Munzinger, J. Stachel, H. Stöcker, PLB 697, 203 (2011)

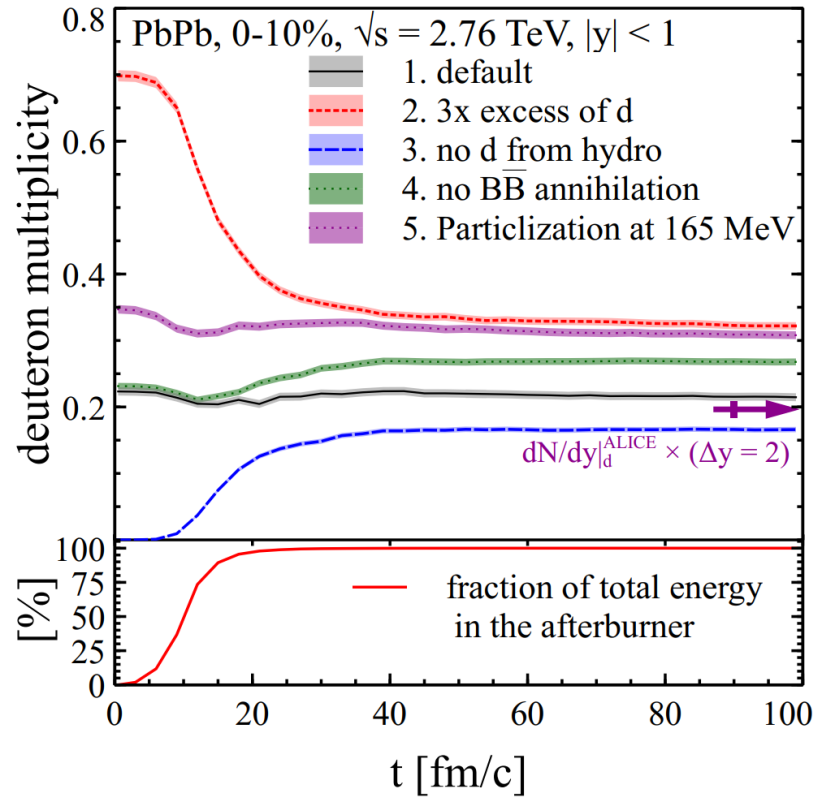
A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561, 321 (2018)

The Triton Puzzle

(9)

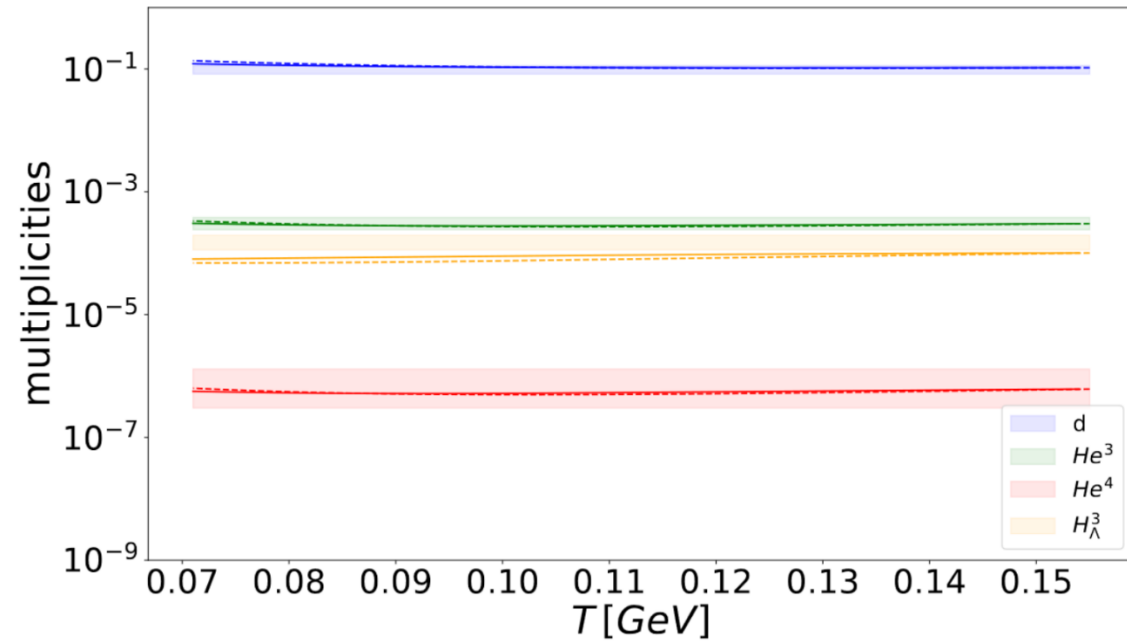
$\pi NN \leftrightarrow \pi d$

D. Oliinychenko, et al., PRC99, 044907 (2019)



V. Vovchenko, et al., PLB800, 135131 (2020)

T. Neidig, et al., PLB827, 136891 (2022)



The obtained hadronic effects on light nuclei production are small

Box calculation

