

Impact of chiral hyperonic three-body forces on neutron stars

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**3rd EMMI Workshop: Anti-matter, hyper-matter & exotica
production at the LHC**

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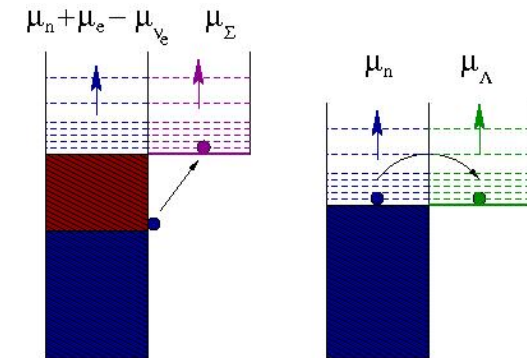
The Hyperon Puzzle: An Open Problem



Hyperons are expected to appear in the core of neutron stars at $\rho \sim (2-3)\rho_0$ when μ_N is large enough to make the conversion of N into Y **energetically favorable**.

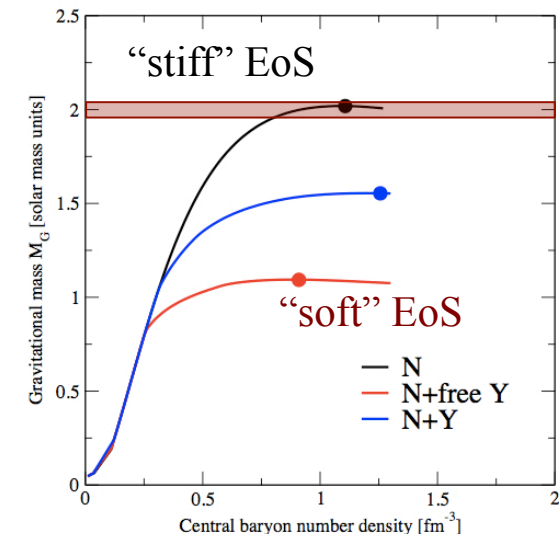
But

The relieve of Fermi pressure due to its appearance $\rightarrow$ **EoS softer** $\rightarrow$ **reduction of the mass** to values incompatible with observation



Observation of $\sim 2 M_{\odot}$ NS $\rightarrow$ Any reliable EoS of dense matter should predict $M_{\max}[EoS] > 2 M_{\odot}$

Can hyperons be present in the interior of neutron stars in view of this new constraint?



Possible Solutions to the Hyperon Puzzle

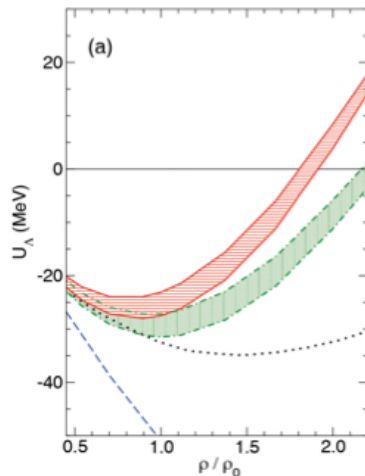
YN & YY

- YY vector meson repulsion

ϕ meson coupled only to hyperons yielding strong repulsion at high ρ

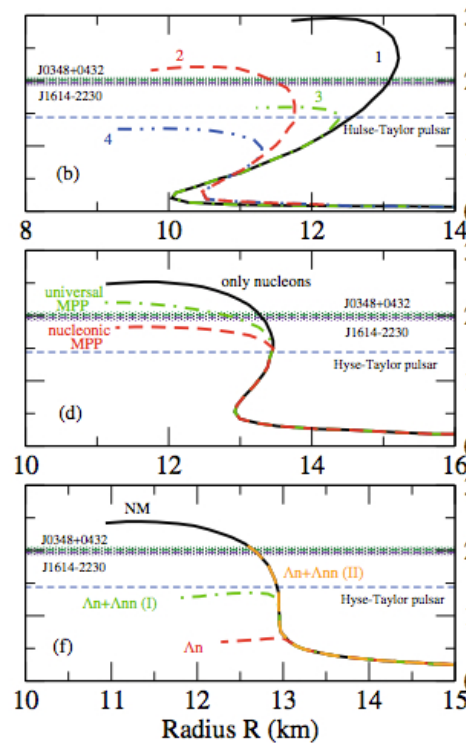
- Chiral forces

YN from χ EFT predicts Λ s.p. potential more repulsive than those from meson exchange



Hyperonic TBF

Natural solution based on the known importance of 3N forces in nuclear physics



Quark Matter

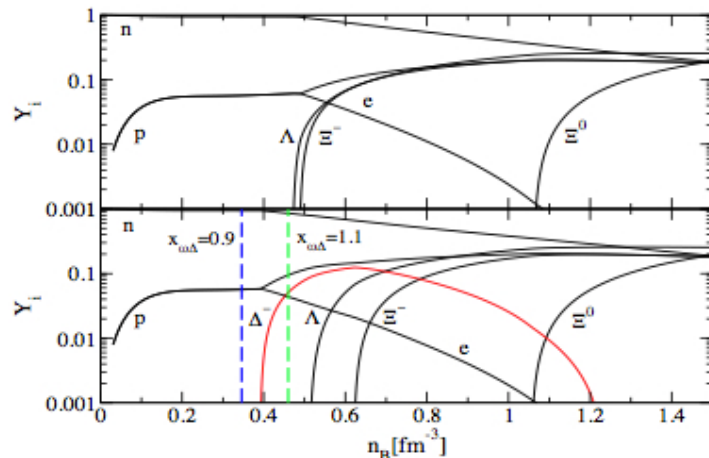
Phase transition to deconfined QM at densities lower than hyperon threshold

To yield $M_{\text{max}} > 2M_{\odot}$ QM should be

- significantly repulsive to guarantee a stiff EoS
- attractive enough to avoid reconfinement

Is there also a Δ isobar puzzle ?

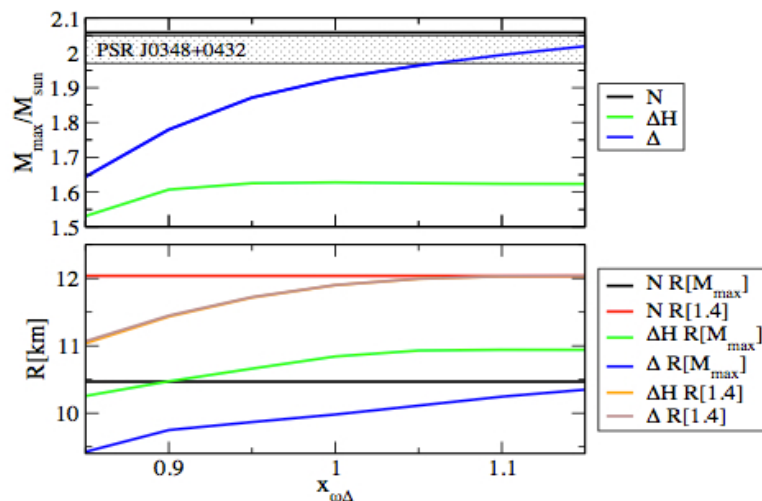
Drago et al. (2014) & other authors have studied in the context of RMF models the role of the Δ isobar in neutron star matter & its interplay with hyperons



❖ Constraints from L indicate an **early appearance of Δ isobars** in neutron stars matter at $\sim 2-3 \rho_0$ (same range as hyperons)

❖ Appearance of **Δ isobars** modify the composition & structure of hadronic stars

❖ $M_{\max}$ is dramatically affected by the presence of **Δ isobars**



If Δ potential is close to that indicated by π^- , e-nucleus or photoabsorption nuclear reactions then **EoS is too soft** $\rightarrow$ **Δ puzzle** similar to the hyperon one

Short Summary of the Talk

✧ Study of the effects of $NN\Lambda$ force on neutron stars

To such end EoS & NS structure derived within the BHF approach using:

- NN at $N^3\text{LO}$ in χEFT including Δ isobar in intermediate states of NN scattering
- NNN at $N^2\text{LO}$ in χEFT
- $N\Lambda$ from meson-exchange (Nijmegen group). Weak point of the work
- $NN\Lambda$ derived by the Juelich-Munich-Bonn group in χEFT at $N^2\text{LO}$

✧ Inclusion of $NN\Lambda$ force improves description of heavy hypernuclei

✧ Inclusion of $NN\Lambda$ force leads to an EoS stiff enough such that the resulting NS maximum mass is compatible with current observations **but the model contains only N, leptons & Λ 's**

✧ We have **NOT SOLVED** the hyperon puzzle **but have taken an additional step towards its solution**

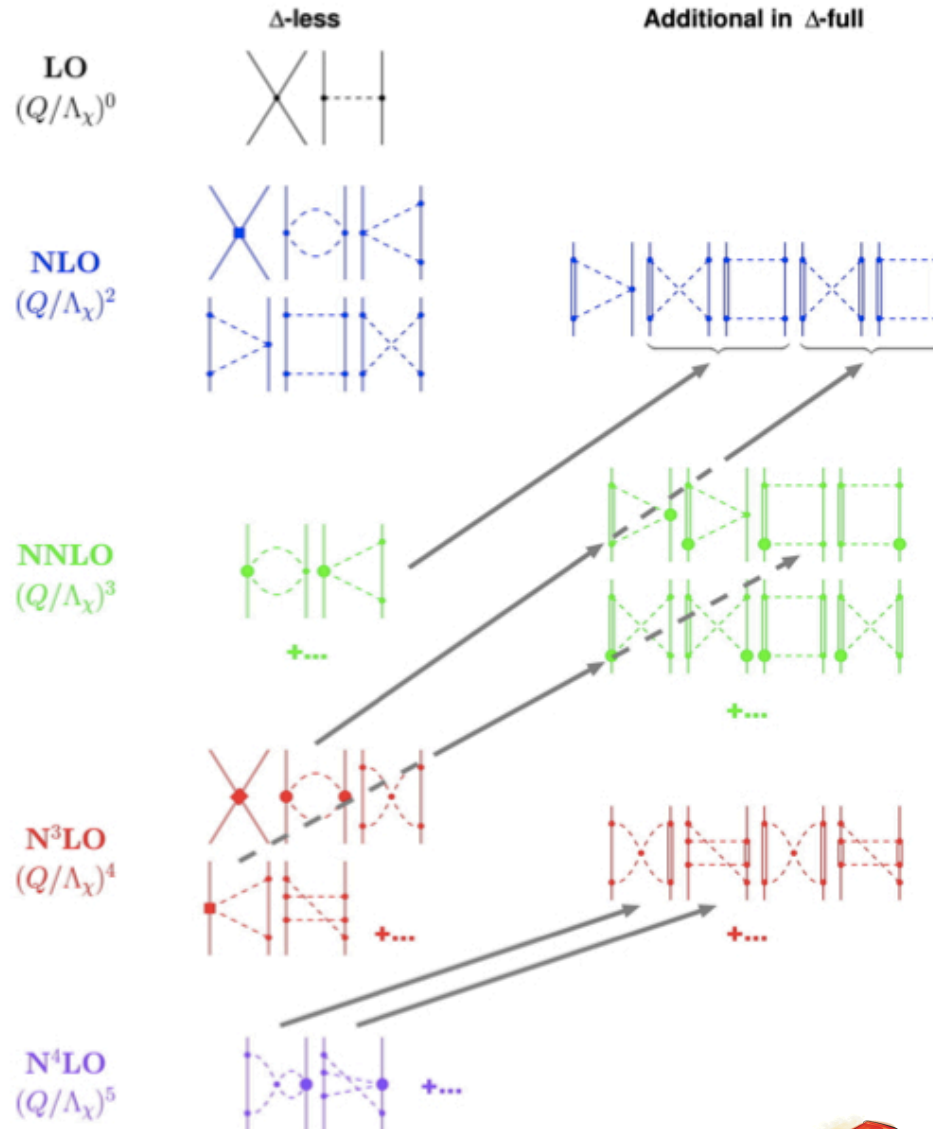
In collaboration with: D. Logoteta & I. Bombaci (University of Pisa)

For details see:



EPJA 55, 207 (2019)

NN interaction



- ✧ **Local chiral potential** derived in coordinate space
- ✧ **Δ isobar** included in intermediate NN scattering
- ✧ A **gaussian regulator** for short range contributions

$$\frac{1}{\pi^{3/2} R_s^3} \exp\left[-\frac{r^2}{R_s^2}\right], \quad R_s = 0.7 \text{ fm}$$

- ✧ **Long range** contributions regularized with

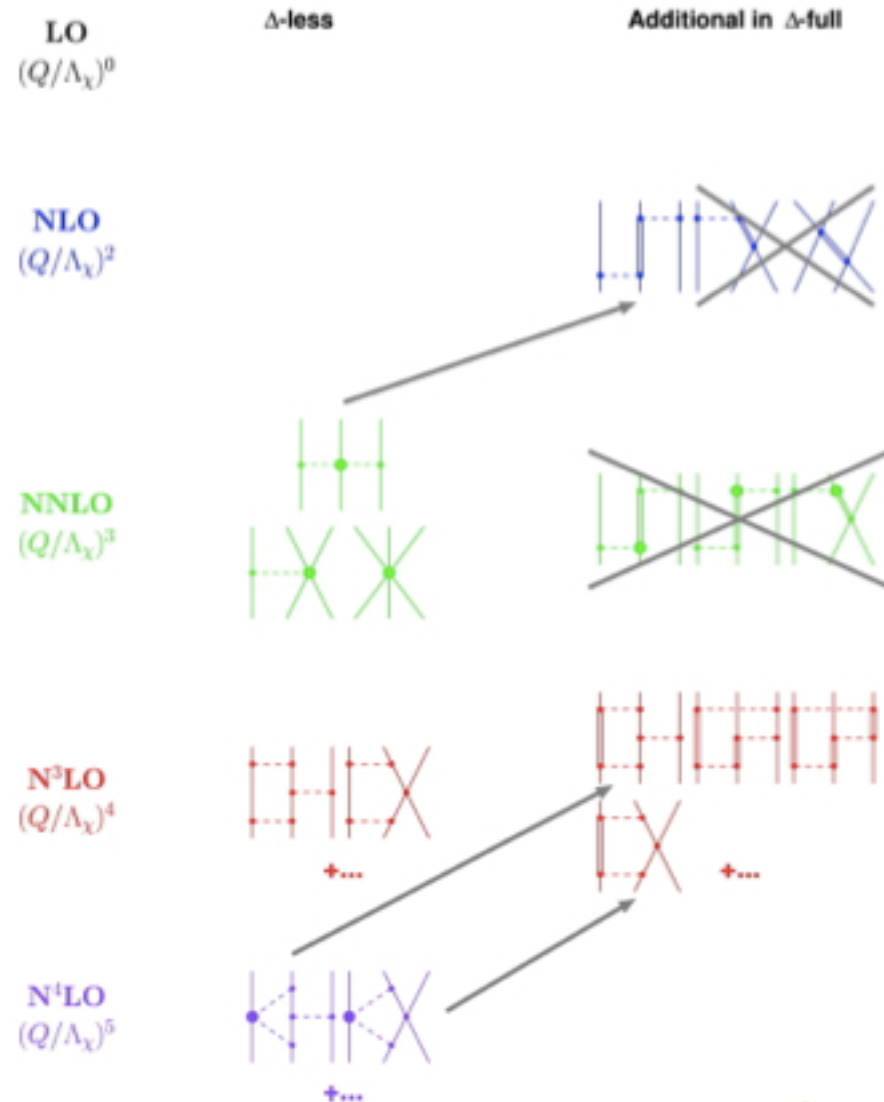
$$1 - \left(\left(\frac{r}{R_l} \right)^6 \exp\left(\frac{2r}{R_l} - 2 \right) + 1 \right)^{-1}, \quad R_l = 1 \text{ fm}$$

- ✧ Operatorial structure **similar to the Av18** (but not the same)



Pirauilli et al., PRC 94, 054007 (2016)

NNN interaction



✧ First TNF contribution at NLO

✧ Δ isobar included

✧ Gaussian regulator used

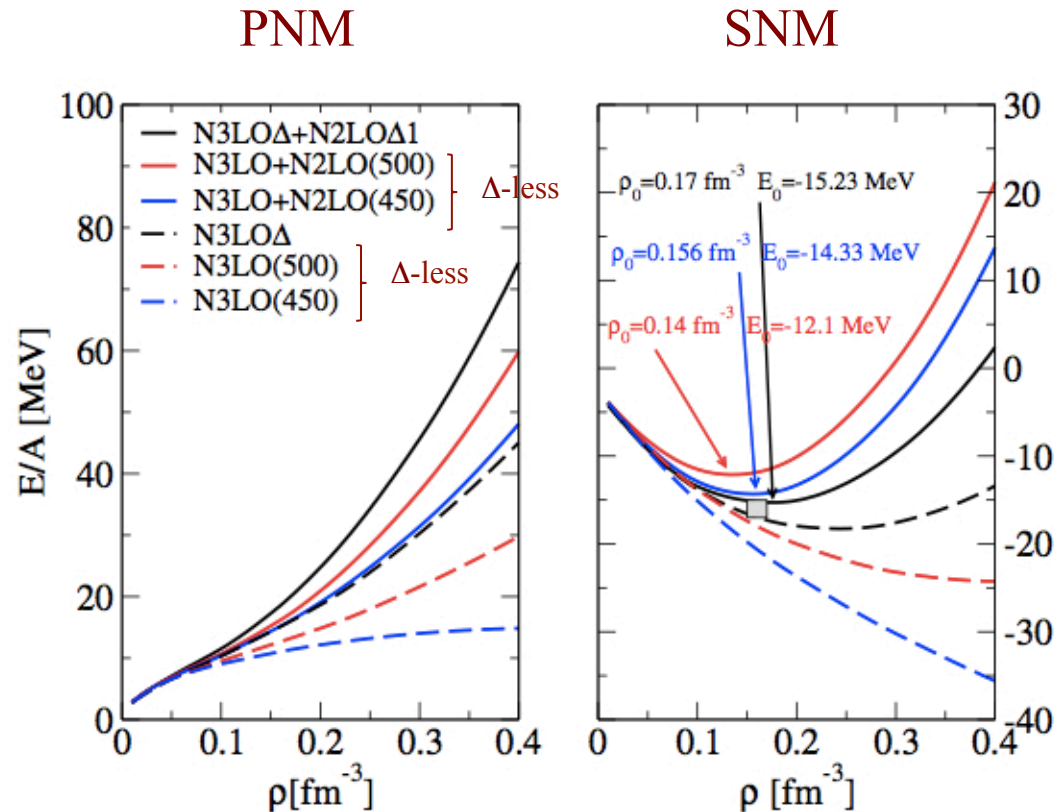
$$\frac{1}{\pi^{3/2} R_s^3} \exp\left[-\frac{r^2}{R_s^2}\right], \quad R_s = 0.7 \text{ fm}$$

✧ 5 LEC:

- c_1, c_3, c_4 fixed from the two-body interaction
- c_E & c_D fixed from observables of few body nuclear systems (^3H) or to reproduce the saturation point (this work)



Nuclear Matter EoS



✧ Significant **improvement** of SNM **saturation point** due to TNF

✧ Explicit **inclusion** of the **Δ isobar** diminishes the **strength** of the TNF needed to obtain a good saturation point

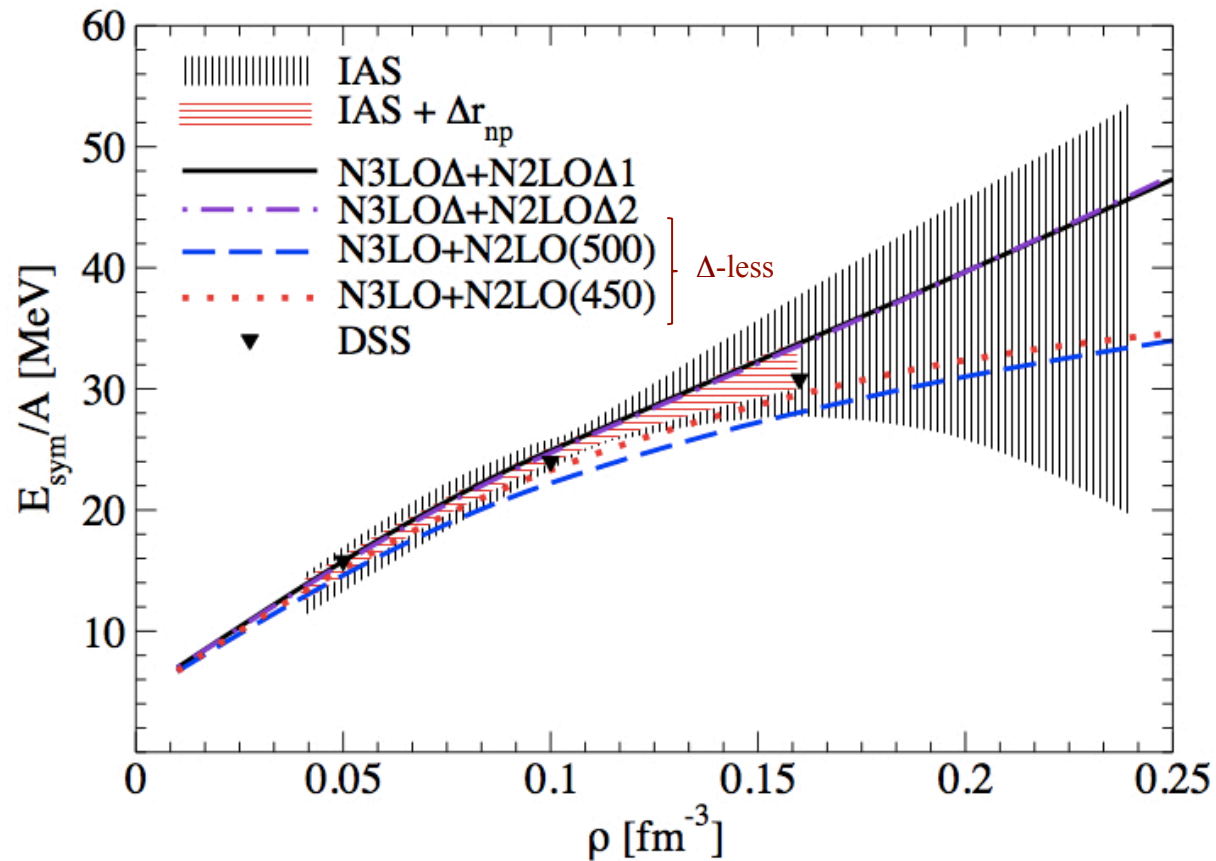
Nuclear Matter Properties at Saturation

	Model	$\rho_0(\text{fm}^{-3})$	E/A (MeV)	E_{sym} (MeV)	L (MeV)	K_{∞} (MeV)
Δ -less	N3LOΔ+N2LOΔ1	0.171	-15.23	35.39	76.0	190
	N3LOΔ+N2LOΔ2	0.176	-15.09	36.00	79.8	176
	N3LO+N2LO(500)	0.135	-12.12	25.89	38.3	153
	N3LO+N2LO(450)	0.156	-14.32	29.20	39.8	205



Logoteta et al., PRC 94, 064001 (2016)

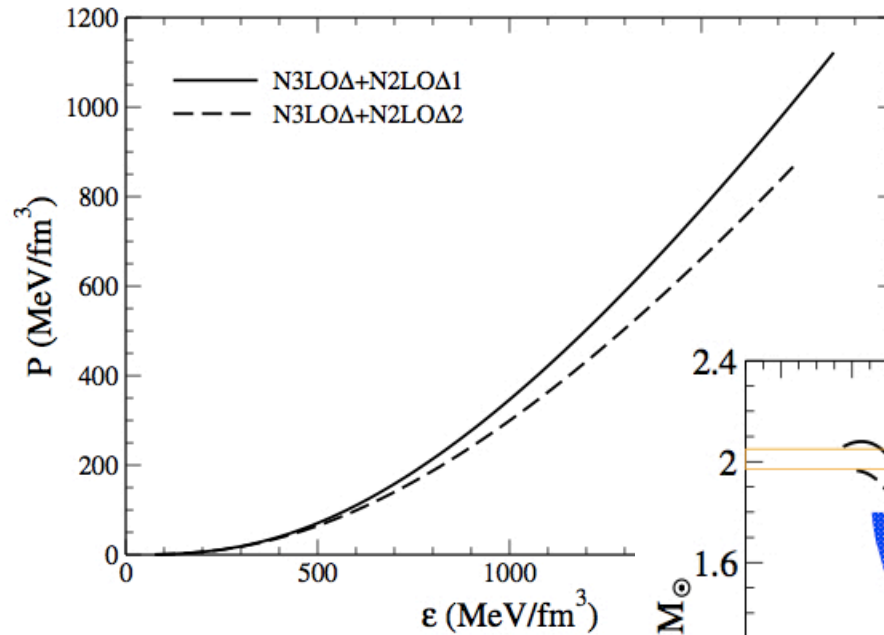
Nuclear Symmetry Energy



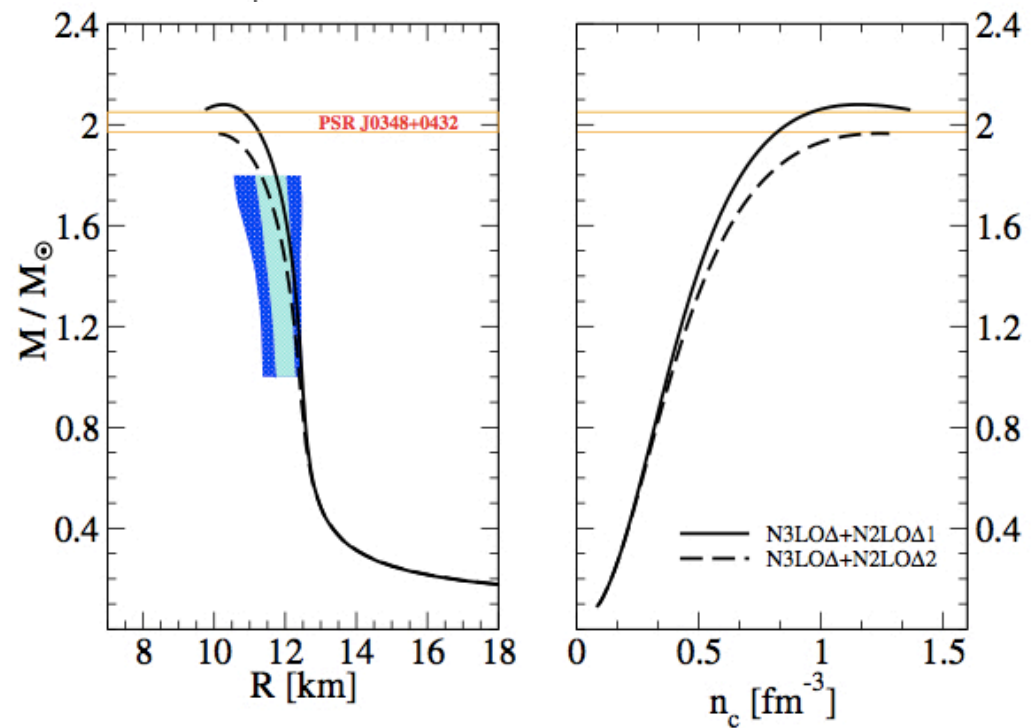
Good agreement with
experimental
constraints from
Isobaric Analog
States & neutron skin
thickness



NS EoS & Structure at $N^3\text{LO}\Delta+N^2\text{LO}\Delta$



Good agreement with
with recent observations
of $2M_{\odot}$ NS



Logoteta et al., A&A 609, A128 (2018)

$N\Lambda$ interaction

✧ NY & YY interactions in χ EFT derived by the Juelich-Bonn-Munich group

- NY at

LO: Polinder et al., NPA 779, 244 (2006)

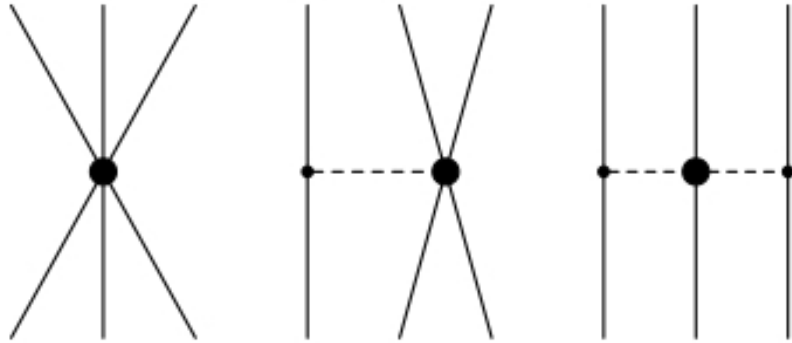
NLO: Haidenbauer et al., NPA 915, 24 (2013)

Haidenbauer et al., arXiv:1906.11681

- YY at NLO: Haidenbauer et al., NPA 954, 273 (2016)

WEAK POINT OF THE PRESENT WORK: unfortunately at the time this work was done **we did not have at our disposal this interaction** and, therefore, instead we used a NY meson-exchange interaction from the Nijmegen group (NSC97a & NSC97f)

NN Λ interaction



(π exchange expected to be dominant, heavy meson (K, η) exchange absorbed into contact terms)

✧ First contributions to NNY appear in χ EFT at **N²LO**

✧ Leading terms at **N²LO**

- ✓ Three-baryon contact terms
- ✓ One & two-meson exchange

✧ LEC estimated through **decouplet saturation**. Only one LEC H' remains a free parameter. A value of $H' = \pm 1/f_\pi$ where $f_\pi = 93$ MeV has been considered by Petschauer et al.

✧ In this work $H' = \beta/f_\pi^2$ with β a parameter fixed to reproduce $U_\Lambda(k=0) \sim -28 - 30$ MeV in SNM at saturation $\longrightarrow$ Models NN Λ_1 & NN Λ_2

✧ A non-local regulator $\exp(-(p^4 + p'^4)/\Lambda^4)$ with $\Lambda = 500$ MeV is used



Petschauer et al., PRC93, 014001 (2016)

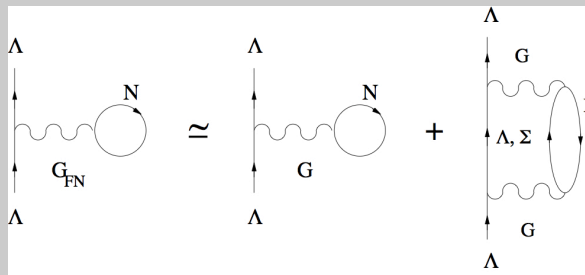
Effect of $NN\Lambda$ interaction on hypernuclei

Before analyzing the **effect of $NN\Lambda$** interaction on NS we consider its role on hypernuclear structure. To such end

A perturbative many-body approach is employed to obtain the Λ self-energy in finite nuclei from which the Λ s.p. bound states can be obtained

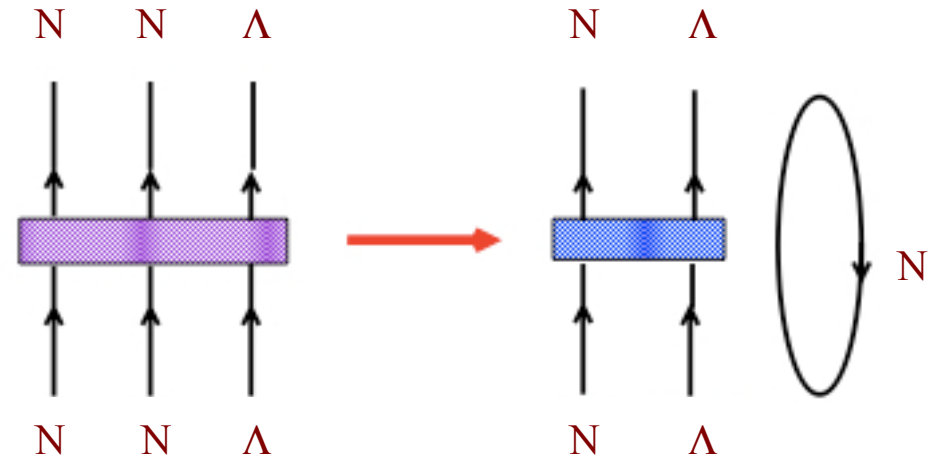
$$G_{NM} = V + V \left(\frac{Q}{E} \right)_{NM} G_{NM} \quad \longrightarrow \quad G_{FN} = G_{NM} + G_{NM} \left[\left(\frac{Q}{E} \right)_{FN} - \left(\frac{Q}{E} \right)_{NM} \right] G_{FN}$$

BHF approximation to the Λ irreducible self-energy in finite nuclei



NN Λ interaction within the BHF approach

The **NN Λ interaction** can be introduced in our BHF approach by adding an **effective density-dependent N Λ force** to the N Λ two-body one when solving the Bethe-Goldstone equation



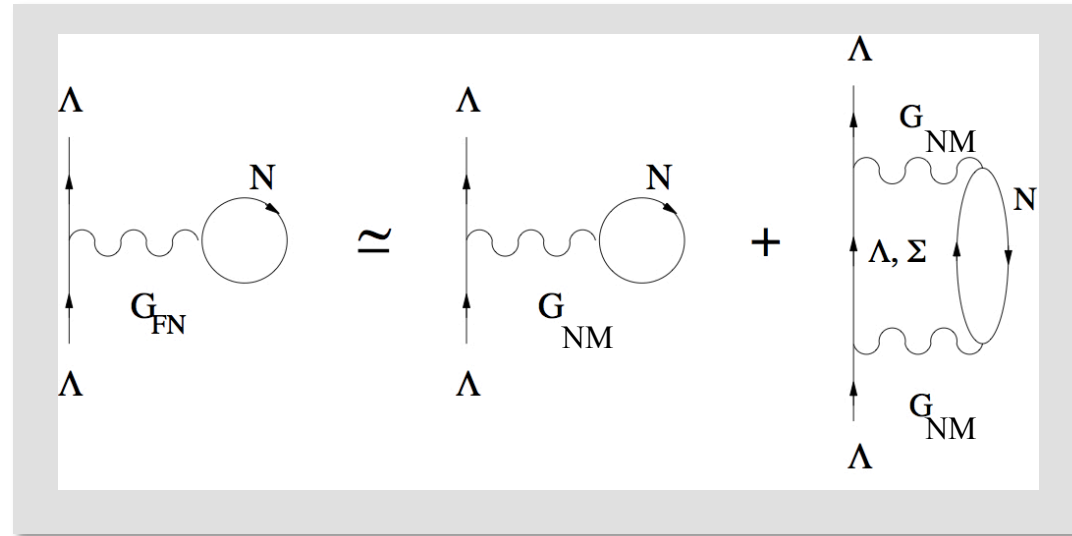
$$W_{B_i B_j}(\vec{r}_{ij}) = Tr_{(\tau_k, \sigma_k)} \int W_3(\vec{r}_i, \vec{r}_j, \vec{r}_k) n(\vec{r}_i, \vec{r}_j, \vec{r}_k) d^3 \vec{r}_k$$

$$W_3(\vec{r}_i, \vec{r}_j, \vec{r}_k): \text{genuine TBF} \quad n(\vec{r}_i, \vec{r}_j, \vec{r}_k): \text{three-body correlation function}$$

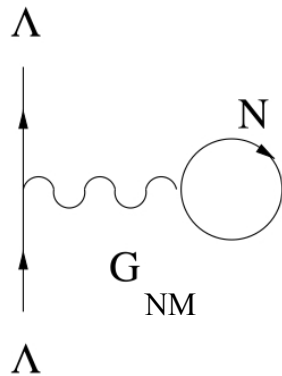
N.B.: Petschauer et al., have derived this effective density-dependent N Λ force allowing a straightforward inclusion of the NNL force in our BHF approach

Finite nucleus Λ self-energy in the BHF approximation

Using G_{FN} as an effective YN interaction, the finite nucleus Λ self-energy is given as sum of a **1st order term** & a **2p1h correction**



✧ 1st order term

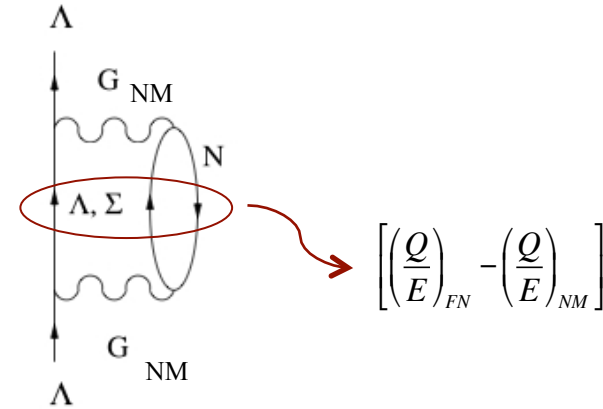


$$\mathcal{V}_1(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda) = \frac{1}{2j_\Lambda + 1} \sum_{\mathcal{J}} \sum_{n_h l_h j_h t_{z_h}} (2\mathcal{J} + 1) \times \langle (k'_\Lambda l_\Lambda j_\Lambda) (n_h l_h j_h t_{z_h}) \mathcal{J} | G | (k_\Lambda l_\Lambda j_\Lambda) (n_h l_h j_h t_{z_h}) \mathcal{J} \rangle$$

This contribution is **real** & **energy-independent**

N.B.: most of the effort is on the basis transformation $|(k_\Lambda l_\Lambda j_\Lambda) (n_h l_h j_h t_{z_h}) J\rangle \rightarrow |KLqLSJTM_T\rangle$

✧ 2p1h correction



This contribution is the **sum of two terms**:

- The first, due to the piece $G_{NM}(Q/E)_{FN}G_{NM}$, gives rise to an **imaginary energy-dependent** part in the Λ self-energy

$$\begin{aligned} \mathcal{W}_{2p1h}(k_{\Lambda}, k'_{\Lambda}, l_{\Lambda}, j_{\Lambda}, \omega) &= -\frac{\pi}{2j_{\Lambda} + 1} \sum_{n_h l_h j_h t_{z_h}} \sum_{\mathcal{L} L S J} \sum_{\mathcal{J} Y' = \Lambda \Sigma} \int dq q^2 \int dK K^2 (2\mathcal{J} + 1) \\ &\times \langle (k'_{\Lambda} l_{\Lambda} j_{\Lambda}) (n_h l_h j_h t_{z_h}) \mathcal{J} | G | K \mathcal{L} q L S J \mathcal{J} T M_T \rangle \\ &\times \langle K \mathcal{L} q L S J \mathcal{J} T M_T | G | (k_{\Lambda} l_{\Lambda} j_{\Lambda}) (n_h l_h j_h t_{z_h}) \mathcal{J} \rangle \\ &\times \delta \left(\omega + \varepsilon_h - \frac{\hbar^2 K^2}{2(m_N + m_{Y'})} - \frac{\hbar^2 q^2 (m_N + m_{Y'})}{2m_N m_{Y'}} - m_{Y'} + m_{\Lambda} \right) \end{aligned}$$

From which can be obtained the **contribution to the real part of the self-energy** through a **dispersion relation**

$$\mathcal{V}_{2p1h}^{(1)}(k_{\Lambda}, k'_{\Lambda}, l_{\Lambda}, j_{\Lambda}, \omega) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} d\omega' \frac{\mathcal{W}_{2p1h}(k_{\Lambda}, k'_{\Lambda}, l_{\Lambda}, j_{\Lambda}, \omega')}{\omega' - \omega}$$

- The second, due to the piece $G_{\text{NM}}(Q/E)_{\text{NM}}G_{\text{NM}}$, gives also a **real & energy-independent** contribution to the Λ self-energy and avoids double counting of $Y'N$ states

$$\begin{aligned} & \mathcal{V}_{2p1h}^{(2)}(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda) \\ &= \frac{1}{2j_\Lambda + 1} \sum_{n_h l_h j_h t_{z_h}} \sum_{\mathcal{L} L S J} \sum_{\mathcal{J} Y' = \Lambda \Sigma} \int dq q^2 \int dK K^2 (2\mathcal{J} + 1) \\ & \times \langle (k'_\Lambda l_\Lambda j_\Lambda) (n_h l_h j_h t_{z_h}) \mathcal{J} | G | K \mathcal{L} q L S J \mathcal{J} T M_T \rangle \\ & \times \langle K \mathcal{L} q L S J \mathcal{J} T M_T | G | (k_\Lambda l_\Lambda j_\Lambda) (n_h l_h j_h t_{z_h}) \mathcal{J} \rangle \\ & \times Q_{Y'N} \left(\Omega - \frac{\hbar^2 K^2}{2(m_N + m_{Y'})} - \frac{\hbar^2 q^2 (m_N + m_{Y'})}{2m_N m_{Y'}} - m_{Y'} + m_\Lambda \right)^{-1} \end{aligned}$$

Summarizing, in the **BHF** approximation the finite nucleus Λ self-energy is given by:

$$\Sigma_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) = \mathcal{V}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) + i\mathcal{W}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega)$$

with

$$\mathcal{V}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) = \mathcal{V}_1(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda) + \mathcal{V}_{2p1h}^{(1)}(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda, \omega) - \mathcal{V}_{2p1h}^{(2)}(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda)$$

$$\mathcal{W}_{l_\Lambda j_\Lambda}(k_\Lambda, k'_\Lambda, \omega) = \mathcal{W}_{2p1h}(k_\Lambda, k'_\Lambda, l_\Lambda, j_\Lambda, \omega)$$

Λ single-particle bound states

Λ s.p. bound states can be obtained using the **real part of the Λ self-energy** as an **effective hyperon-nucleus** potential in the Schoedinger equation

$$\sum_{i=1}^{N_{\max}} \left[\frac{\hbar^2 k_i^2}{2m_{\Lambda}} + \mathcal{V}_{l_{\Lambda} j_{\Lambda}}(k_n, k_i, \omega = \varepsilon_{l_{\Lambda} j_{\Lambda}}) \right] \Psi_{il_{\Lambda} j_{\Lambda} m_{j_{\Lambda}}} = \varepsilon_{l_{\Lambda} j_{\Lambda}} \Psi_{nl_{\Lambda} j_{\Lambda} m_{j_{\Lambda}}}$$

solved by diagonalizing the Hamiltonian in a complete & orthonormal set of regular basis functions within a spherical box of radius R_{box}

$$\Phi_{nl_{\Lambda} j_{\Lambda} m_{j_{\Lambda}}}(\vec{r}) = \langle \vec{r} | k_n l_{\Lambda} j_{\Lambda} m_{j_{\Lambda}} \rangle = N_{nl_{\Lambda}} j_{l_{\Lambda}}(k_n r) \psi_{l_{\Lambda} j_{\Lambda} m_{j_{\Lambda}}}(\theta, \phi)$$

- $N_{nl_{\Lambda}}$ $\longrightarrow$ normalization constant
- $N_{\max}$ $\longrightarrow$ maximum number of basis states in the box
- $j_{j_{\Lambda}}(k_n r)$ $\longrightarrow$ Bessel functions for discrete momenta ($j_{j_{\Lambda}}(k_n R_{\text{box}}) = 0$)
- $\psi_{l_{\Lambda} j_{\Lambda} m_{j_{\Lambda}}}(\theta, \phi)$ $\longrightarrow$ spherical harmonics including spin d.o.f.
- $\Psi_{nl_{\Lambda} j_{\Lambda} m_{j_{\Lambda}}} = \langle k_n l_{\Lambda} j_{\Lambda} m_{j_{\Lambda}} | \Psi \rangle$ $\longrightarrow$ projection of the state $|\Psi\rangle$ on the basis $|k_n l_{\Lambda} j_{\Lambda} m_{j_{\Lambda}}\rangle$

N.B.: a self-consistent procedure is required for each eigenvalue

Effect of $NN\Lambda$ interaction on hypernuclei

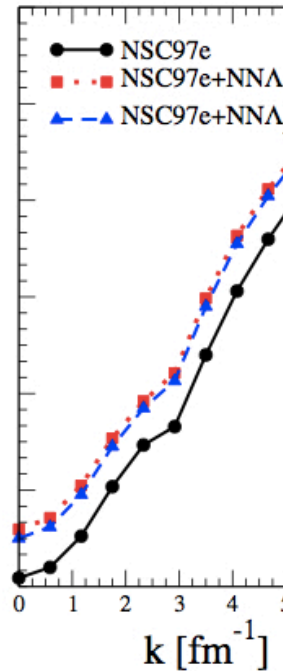
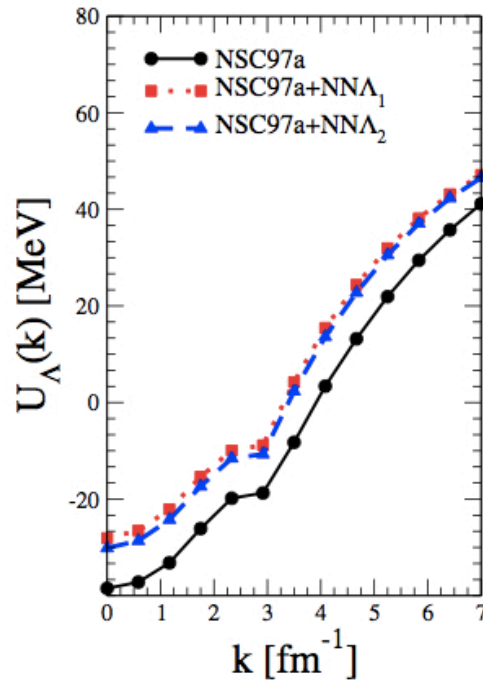
Λ separation energy in $^{41}_{\Lambda}\text{Ca}$, $^{91}_{\Lambda}\text{Zr}$ & $^{209}_{\Lambda}\text{Pb}$

	$^{41}_{\Lambda}\text{Ca}$	$^{91}_{\Lambda}\text{Zr}$	$^{209}_{\Lambda}\text{Pb}$
NSC97a	23.0	31.3	38.8
NSC97a+ $NN\Lambda_1$	14.9	21.1	26.8
NSC97a+ $NN\Lambda_2$	13.3	19.3	24.7
NSC97e	24.2	32.3	39.5
NSC97e+ $NN\Lambda_1$	16.1	22.3	27.9
NSC97e+ $NN\Lambda_2$	14.7	20.7	26.1
Exp.	18.7(1.1)*	23.6(5)	26.9(8)

- ✧ We consider only hypernuclei described as a **closed shell nuclear core + a Λ sitting in a s.p. state**. Comparison with the **closest hypernucleus for which exp. data is available**
- ✧ **Inclusion of $NN\Lambda$ improves** the agreement with data for $^{91}_{\Lambda}\text{Zr}$ & $^{209}_{\Lambda}\text{Pb}$.
- ✧ **$NN\Lambda$** predict too much repulsion in the case $^{41}_{\Lambda}\text{Ca}$ & lighter hypernuclei
- ✧ **No refit** of any parameter **has been done** to perform these calculations

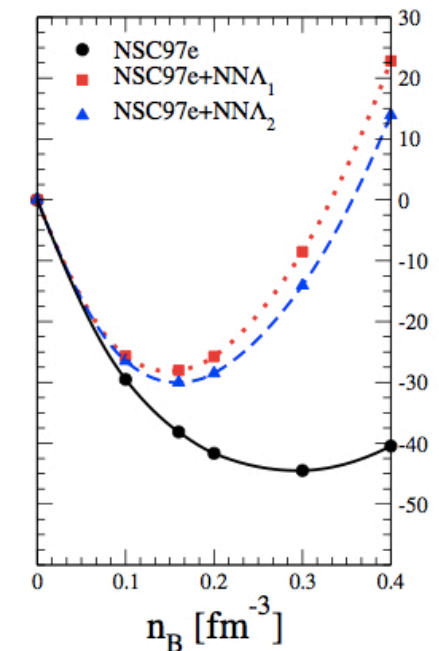
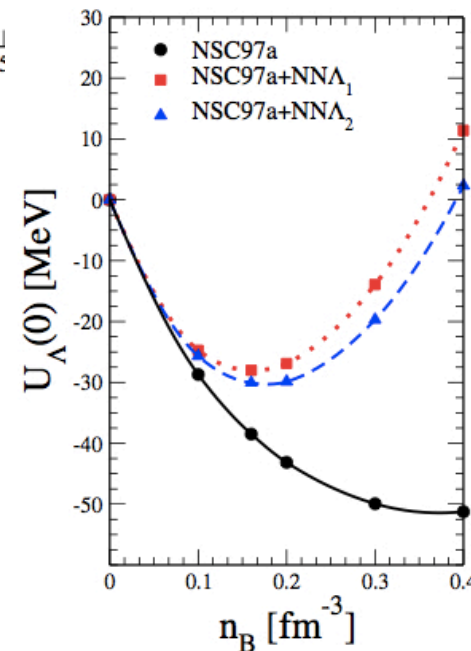
* Taken from Pile et al., PRL 66, 2585 (1991). Not included in the recent Gal et al., RMP 88, 035004 (2016)

Λ BHF single-particle potential in SNM at ρ_0

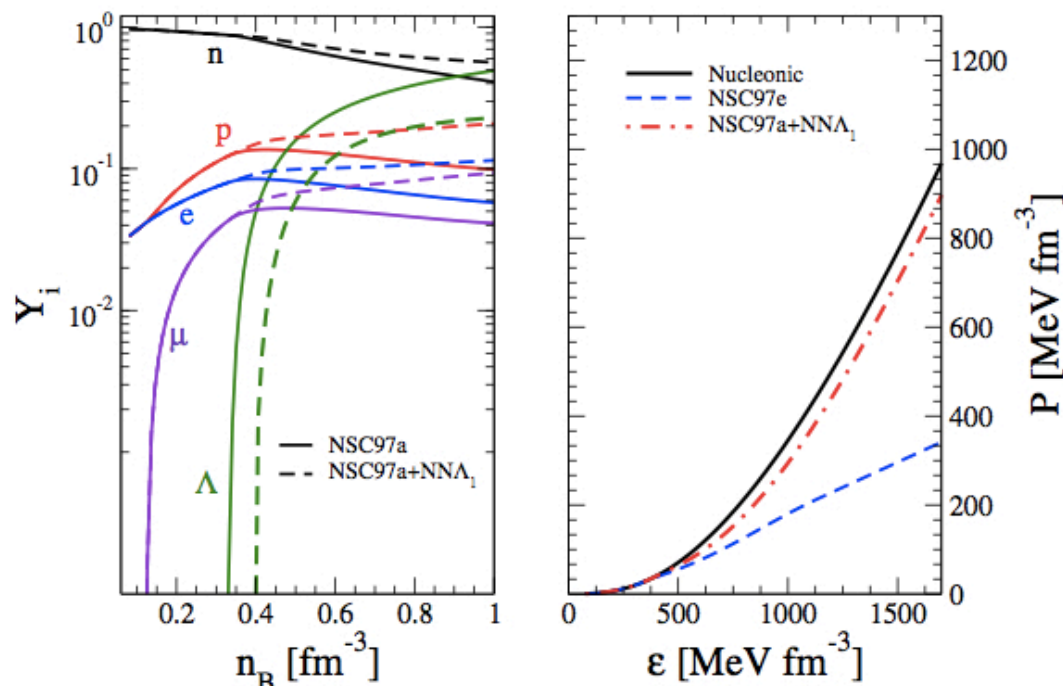


- ✧ NSC97a & NSC97e (with no NNA) predict $U_\Lambda(0) \sim -40$ MeV.
- ✧ NSC97a predict more attraction than NSC97e even when NNA is included
- ✧ NNA induces as expected repulsion

- ✧ $U_\Lambda(0)$ very deep when only two-body forces are considered
- ✧ NNA induces repulsion for densities larger than $\sim 0.1 \text{ fm}^{-3}$ & shift the minimum to $\sim 0.16 \text{ fm}^{-3}$
- ✧ NNA negligible in the low density region



Neutron star matter EoS & Composition



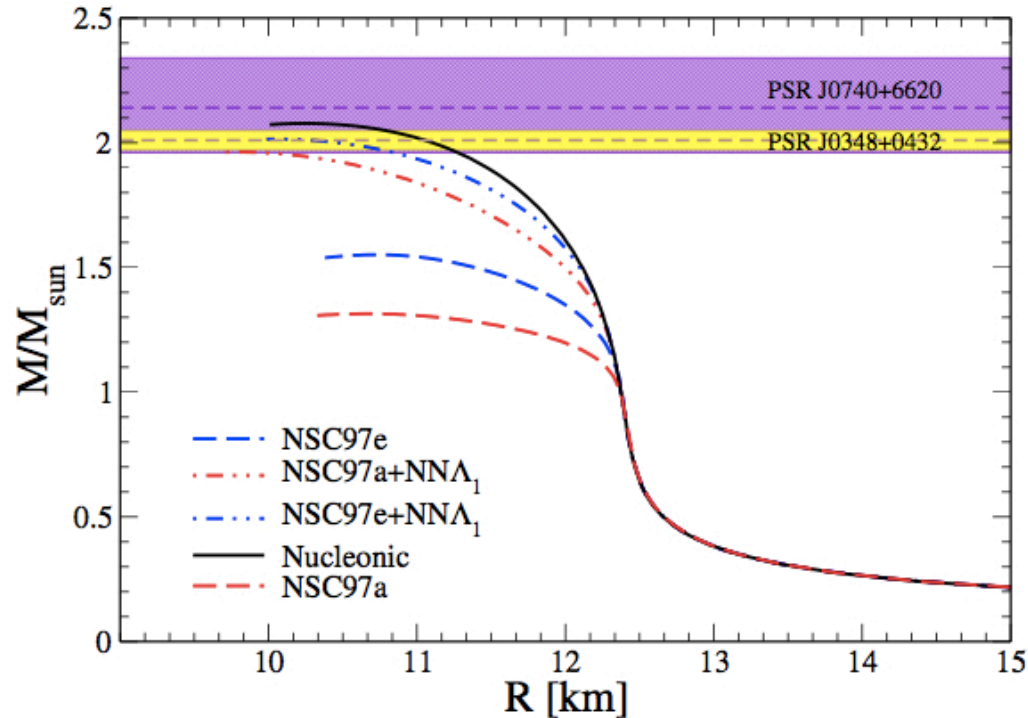
Only n , p , e^- , μ^- & Λ included

- ✓ First exploratory work. We are interested on the role of NNA
- ✓ More complete study requires the inclusion of other hyperons (work on progress)

✧ The effect of NNA interaction is twofold:

- ✓ Shift the onset of the Λ to slightly larger baryon density
- ✓ Strong reduction of the amount of Λ 's at large baryon densities with the consequent stiffening of the EoS compared to the case in which the NNA is not included $\longrightarrow$ important consequences for NS mass (M_{max} increases)

Neutron star Properties



	$M_{max}(M_{\odot})$	R (km)	n_c (fm $^{-3}$)
Nucleonic	2.08	10.26	1.15
NSC97a	1.31	10.60	1.40
NSC97a+NNA $_1$	1.96	9.80	1.30
NSC97a+NNA $_2$	1.97	9.87	1.28
NSC97e	1.54	10.81	1.18
NSC97e+NNA $_1$	2.01	10.10	1.20
NSC97e+NNA $_2$	2.02	10.15	1.19

NS $M_{\max}$ compatible with the largest NS observed

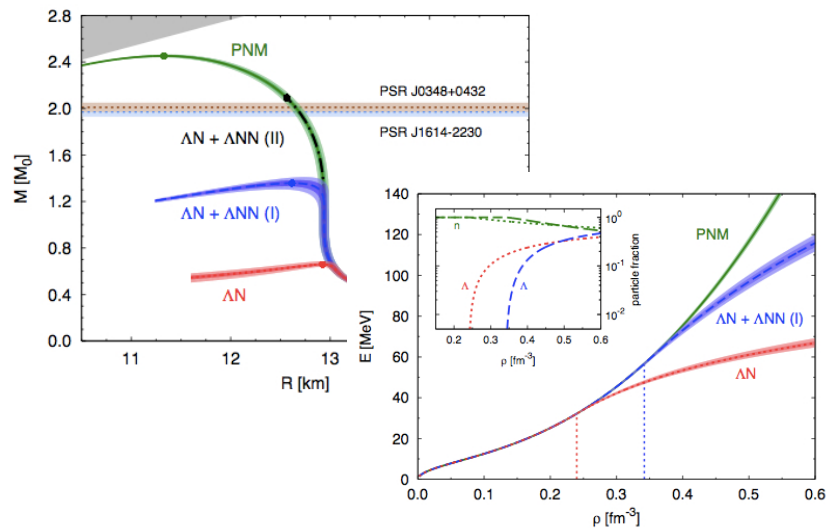
But

- ✓ We have ignored the presence of other hyperons in the NS interior that could change this conclusion
- ✓ Hypothetical repulsive NNY, NYY & YYY forces could lead to a similar conclusion

In view of this we CANNOT say that we have solved the hyperon puzzle

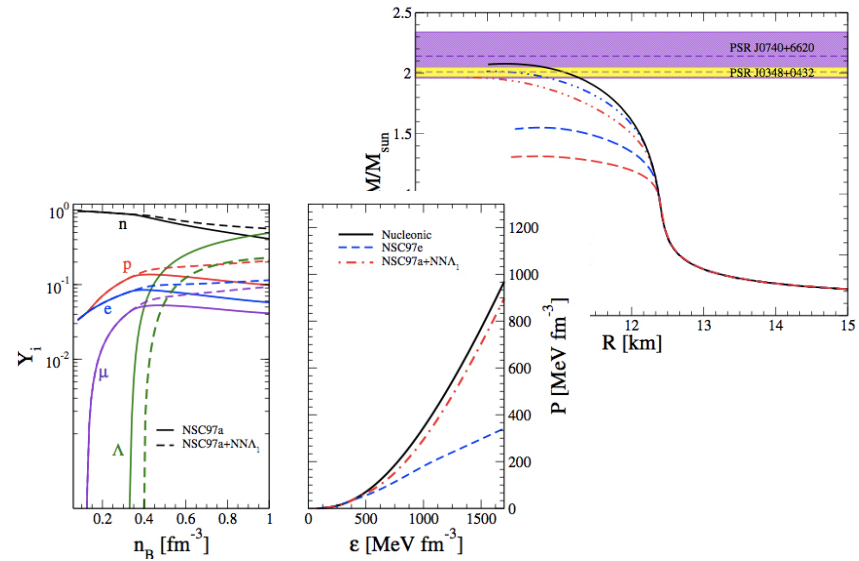
A comparison with QMC calculations

Lonardoni et al., PRL 114, 092301 (2016)



- ✧ NS matter described as mixture of neutrons & Λ 's in β -equilibrium
- ✧ Simple interaction models: $Av8'$ +UIX (nn,nnn) & Bodmer-Usmani (N Λ) potential, NNA (2π exchange + phenomenological repulsive term)
- ✧ The only NNA able to give $2M_{\odot}$ lead to the total disappearance of Λ in NS, but this in fact just pure neutron matter

This work



- ✧ NS matter described as a mixture of n , p , e^- , μ^- & Λ 's in β -equilibrium
- ✧ χ EFT (NN, NNN, NNA) + meson-exchange (NY)
- ✧ Even if the concentration of Λ 's is strongly reduced they are still present in the interior of a $2M_{\odot}$ NS

Summary & Conclusions (again)

✧ Study of the effects of $NN\Lambda$ force on neutron stars

To such end EoS & NS structure derived within the BHF approach using:

- NN at $N^3\text{LO}$ in χEFT including Δ isobar in intermediate states of NN scattering
- NNN at $N^2\text{LO}$ in χEFT
- $N\Lambda$ from meson-exchange (Nijmegen group). Weak point of the work
- $NN\Lambda$ derived by the Juelich-Munich-Bonn group in χEFT at $N^2\text{LO}$

✧ Inclusion of $NN\Lambda$ force improves description of heavy hypernuclei

✧ Inclusion of $NN\Lambda$ force leads to an EoS stiff enough such that the resulting NS maximum mass is compatible with current observations **but the model contains only N, leptons & Λ 's**

✧ We have **NOT SOLVED** the hyperon puzzle **but have taken an additional step towards its solution**

- ✧ You for your time & attention
- ✧ Benjamin, Stefania, Krzysztof, Juergen & Ludwik for their invitation

