Clusters and hypernuclei production within the PHQMD

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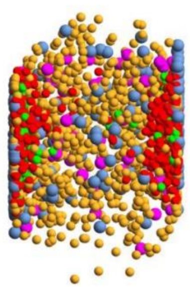
arXiv:1907.03860

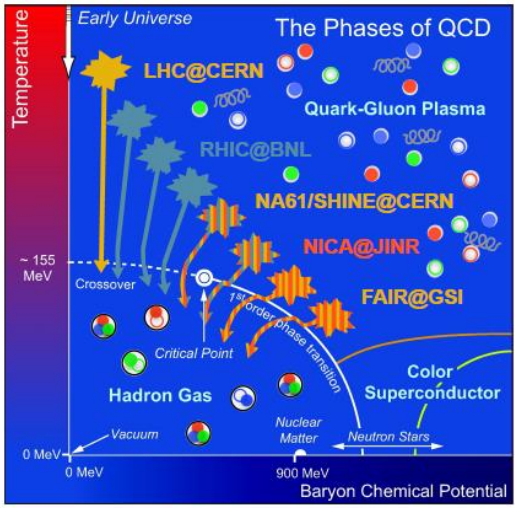
PHQMD

3rd EMMI Workshop: Anti-matter, hyper-matter and exotica production at the LHC
2-6 December 2019, University of Wroclaw, Poland



The ‚holy grail‘ of heavy-ion physics:

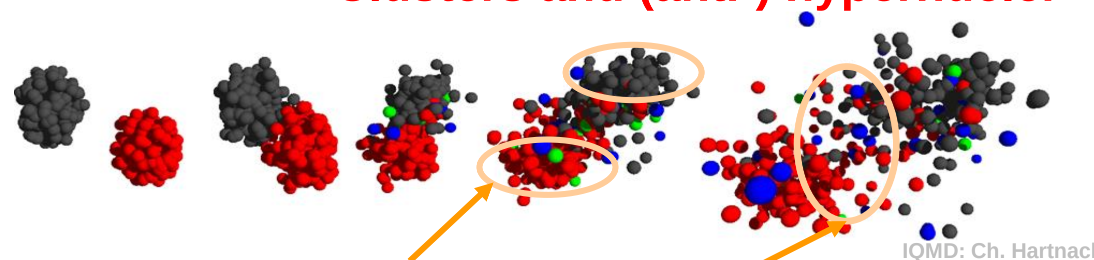
The phase diagram of QCD



Experimental observables:

...

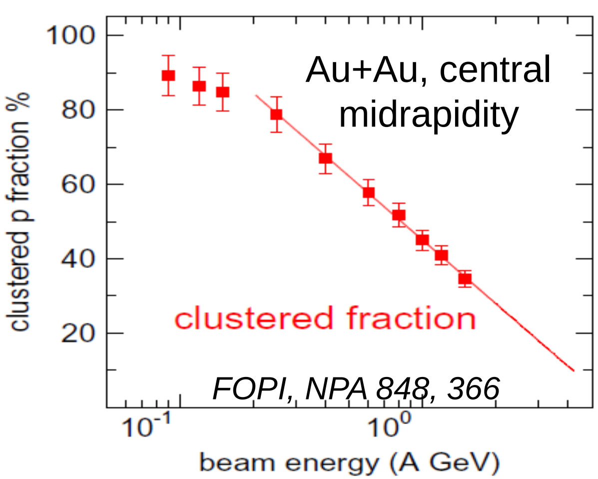
Clusters and (anti-) hypernuclei



- projectile/target spectators ➔ heavy cluster formation
- midrapidity ➔ light clusters

! Hyperons are created in participant zone

- Clusters are very **abundant at low energy**



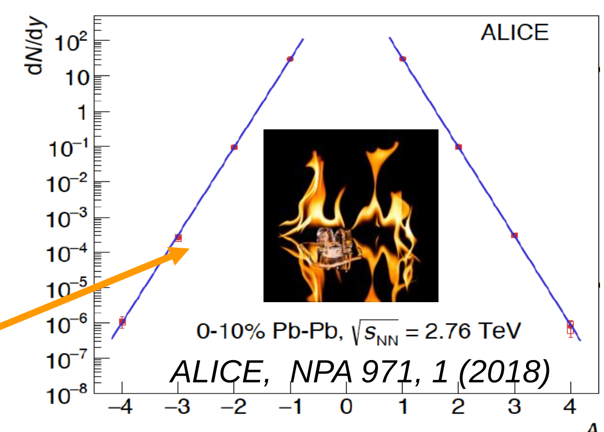
(Anti-) hypernuclei production:

- at mid-rapidity by Λ coalessance during expansion

- at projectile/target rapidity by rescattering/absorption of Λ by spectators

High energy HIC:

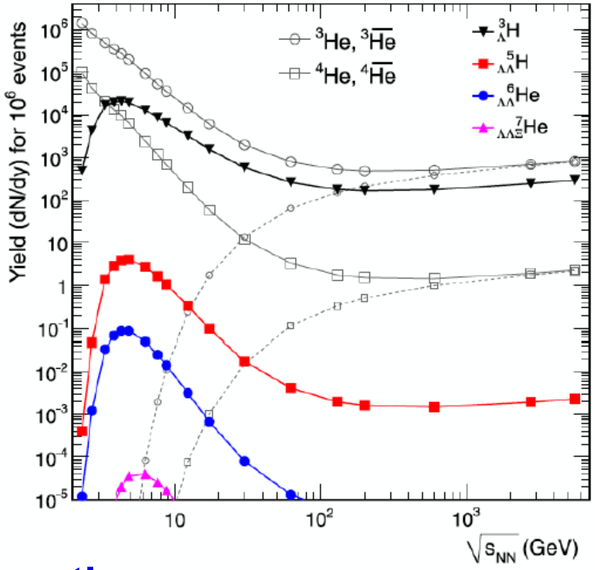
‚Ice in a fire‘ puzzle: how the weakly bound objects can be formed in a hot enviroment ?!



Modeling of cluster and hypernuclei formation

Existing models for clusters formation:

A. Andronic et al., PLB 697, 203 (2011)



- **statistical model:**
  - assumption of thermal equilibrium (difficult to justify at target and projectile rapidity)
  - strong sensitivity of nuclei yields to choice of T_{ch}
  - binding energies are small compared to T_{ch}

- **coalescence model:**
  - determination of clusters at a given point in time by coalescence radii in coordinate and momentum spaces

➔ **don‘t provide information on the dynamics of clusters formation**

In order to understand the microscopic origin of clusters formation one needs:
- **a realistic model for the dynamical time evolution of the HIC ➔ transport models**
- **dynamical modeling of cluster formation based on interactions**

- **Cluster formation is sensitive to nucleon dynamics**

➔ **One needs to keep the nucleon correlations (initial and final) by realistic nucleon-nucleon interactions in transport models:**
- **QMD (quantum-molecular dynamics) – allows to keep correlations**
- **MF (mean-field based models) – correlations are smeared out**

PHQMD: ‚price to pay‘ for ‚profits‘

QMD dynamics for baryons (in PHQMD):

Equation-of-motions: based on quantum-mechanical Hamiltonian formulation from the Ritz variational principle
Propagate: Gaussian wave functions (Ansatz) in 6+1 dimensions (r, p + time)

Consequences:
- ❑ Non-relativistic consideration
- ❑ Non-covariant formulation

➔ Relativistic extension of QMD is lacking for the assumptions (hopefully acceptable for the goals here)

❑ N-body dynamics is realized by 2 body forces between all baryons in the system
➔ allows to keep correlations

BUU dynamics for baryons (cf. PHSD):

Equation-of-Motions: based on field theoretical equations-of-motion - BUU equation is the on-shell limit of the Kadanoff-Baym many-body theory
Propagate: Green functions – in KB and f(x,p) – single-particle distribution functions in BUU; n=8 dimensions [(r,t) (E,p)]

Consequences:
- ❑ Relativistic consideration
- ❑ Covariant formulation

❑ N-body dynamics is reduced to mean-field dynamics in the self generated mean-field potential + off-shell interactions
➔ loosing higher order correlations (not very relevant for the "bulk" dynamics!)

N-body correlations are important for cluster formation !

PHQMD

<u>The goal:</u> to develop a **unified n-body microscopic transport approach** for the description of heavy-ion dynamics and dynamical cluster formation from low to ultra-relativistic energies

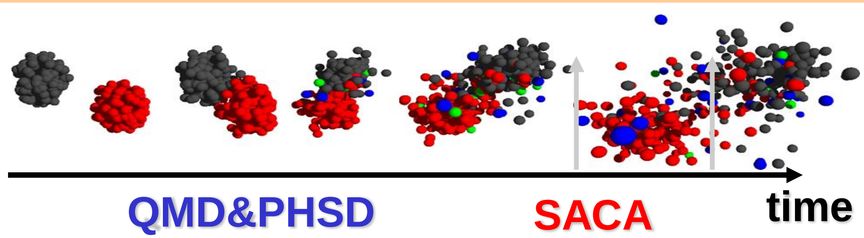
<u>Realization:</u> combined model **PHQMD = (PHSD & QMD) & SACA**

Parton-Hadron-Quantum-Molecular Dynamics

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

Clusters recognition:
SACA (Simulated Annealing Clusterization Algorithm)
vs. MST (Minimum Spanning Tree)



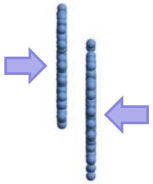
Parton-Hadron-String-Dynamics (PHSD)

collision integral → PHQMD

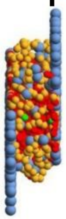
PHSD is a **non-equilibrium microscopic transport approach** for the description of **strongly-interacting hadronic and partonic matter** created in heavy-ion collisions

Dynamics: based on the solution of **generalized off-shell transport equations** derived from Kadanoff-Baym many-body theory

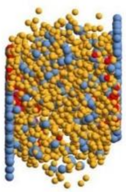
Initial A+A collision



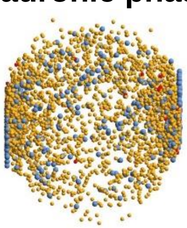
Partonic phase



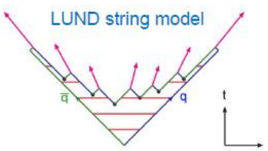
Hadronization



Hadronic phase

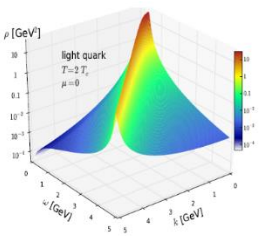


- **Initial A+A collisions** :
  N+N → string formation → decay to pre-hadrons + leading hadrons



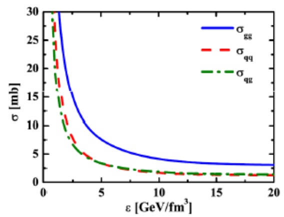
- **Formation of QGP stage** if local $\varepsilon > \varepsilon_{critical}$:
  dissolution of pre-hadrons → partons

- **Partonic phase - QGP:**
  QGP is described by the Dynamical QuasiParticle Model (DQPM) matched to reproduce lattice QCD EoS for finite T and μ_B (crossover)

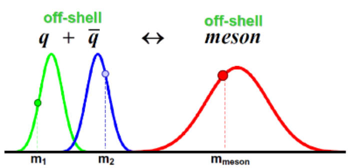


- **Degrees-of-freedom:** strongly interacting quasiparticles: massive quarks and gluons (g,q,q_{bar}) with sizeable collisional widths in a self-generated mean-field potential

- **Interactions:** (quasi-)elastic and inelastic collisions of partons



- **Hadronization** to colorless off-shell mesons and baryons:
  Strict 4-momentum and quantum number conservation



- **Hadronic phase:** hadron-hadron interactions – off-shell HSD

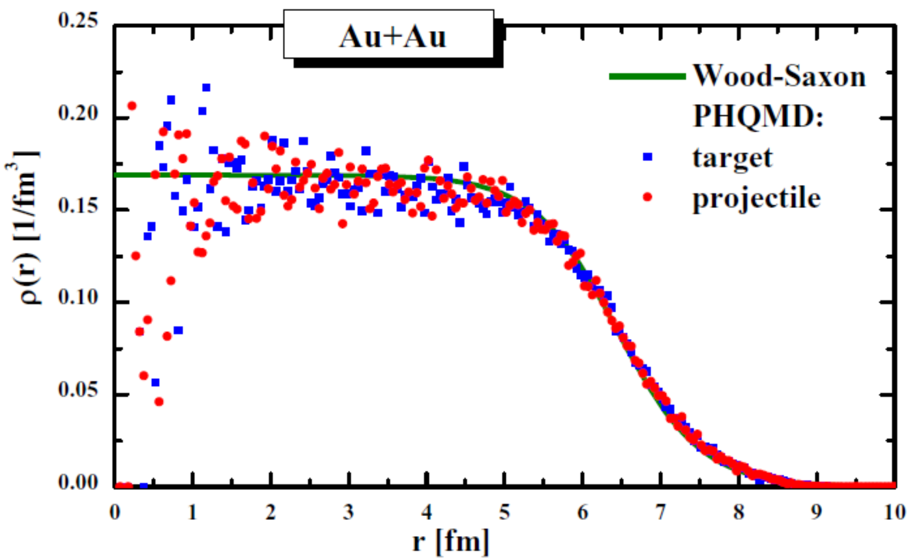
W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W. Cassing, EPJ ST 168 (2009) 3

Initial conditions of PHQMD

Cluster formation is sensitive to the initial correlations of nucleons ➔
Initialization of a nucleus:
the initial distributions of nucleons in projectile and target has to be carefully modelled:

- **Right density distribution**
- **Right binding energy to guarantee the stability of nuclei**

❑ **Initialization in coordinate space: Wood-Saxon distribution:**



$$\rho^{WS}(r) = \frac{\rho_0}{1 + e^{\frac{r - R_A}{a}}}$$

$$R_A = r_0 A^{1/3}$$

$$\rho_0 = 0.1695 \text{ fm}^{-2}, \ a = 0.535 \text{ fm}$$

❑ **Initialization in momentum space: Thomas-Fermi distribution $p < p_F$ with the additional requirement that nucleons are bound:**

$$0 \leq \sqrt{m^2 + \mathbf{p_{io}}^2(t=0)} - m \leq - < V(\mathbf{r_{io}}) >$$

QMD propagation

- **Generalized Ritz variational principle**: $\delta \int_{t_1}^{t_2} dt < \psi(t)| i\frac{d}{dt} - H|\psi(t) >= 0.$

Assume that $\psi_N = \prod_{i=1}^{N} f(r_i, p_i, r_{i0}, p_{i0}, t)$ **for N particles** **(neglecting antisymmetrization !)**

single-particle Wigner density of the nucleon "*i*"

Trial wave function for one particle "*i*" : Gaussian with width *L* centered at r_{i0}, p_{i0}

L=4.33 fm²

$$\boxed{f(\mathbf{r_i}, \mathbf{p_i}, \mathbf{r_{i0}}, \mathbf{p_{i0}}, t) = \frac{1}{\pi^3 \hbar^3} e^{-\frac{2}{L}(\mathbf{r_i}-\mathbf{r_{i0}}(t))^2} e^{-\frac{L}{2\hbar^2}(\mathbf{p_i}-\mathbf{p_{i0}}(t))^2}}$$

- **Equations-of-motion (EoM) for Gaussian centers in coordinate and momentum space:**

$$\boxed{\dot{r}_{i0} = \frac{\partial \langle H \rangle}{\partial p_{i0}} \qquad \dot{p}_{i0} = -\frac{\partial \langle H \rangle}{\partial r_{i0}}}$$

Hamiltonian: $H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i (T_i + \sum_{j \neq i} V_{i,j})$

$$V_{i,j} = V(\mathbf{r_i}, \mathbf{r_j}, \mathbf{r_{i0}}, \mathbf{r_{j0}}, t) = V_{\text{Skyrme}} + V_{\text{Coul}}$$

QMD interaction potential and EoS

The expectation value of the Hamiltonian:

$$\langle H\rangle = \langle T\rangle + \langle V\rangle = \sum_i (\sqrt{p_{i0}^2 + m^2} - m) + \sum_i \langle V_{Skyrme}(\mathbf{r_{i0}}, t)\rangle$$

- **Skyrme potential - scalar ('static') * :**

$$\langle V_{Skyrme}(\mathbf{r_{i0}}, t)\rangle = \alpha\left(\frac{\rho_{int}(\mathbf{r_{i0}}, t)}{\rho_0}\right) + \beta\left(\frac{\rho_{int}(\mathbf{r_{i0}}, t)}{\rho_0}\right)^\gamma$$

	α (MeV)	β (MeV)	γ	K [MeV]
S	-390	320	1.14	200
H	-130	59	2.09	380

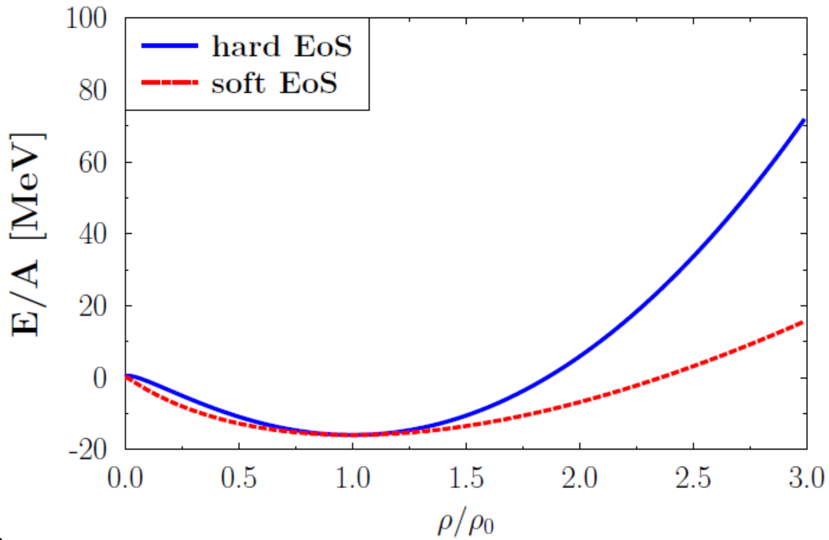
- **modifed interaction density (with relativistic extension):**

$$\rho_{int}(\mathbf{r_{i0}}, t) \rightarrow C\sum_j \left(\frac{4}{\pi L}\right)^{3/2} \mathrm{e}^{-\frac{4}{L}(\mathbf{r}_{i0}^{\mathrm{T}}(t) - \mathbf{r}_{j0}^{\mathrm{T}}(t))^2} \times \mathrm{e}^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r}_{i0}^{\mathrm{L}}(t) - \mathbf{r}_{j0}^{\mathrm{L}}(t))^2},$$

- **HIC ←→ EoS for infinite matter at rest**
- **compression modulus K of nuclear matter:**

$$K = -V\frac{dP}{dV} = 9\rho^2 \frac{\partial^2 (E/A(\rho))}{(\partial\rho)^2}\Big|_{\rho=\rho_0}.$$

EoS for infinite matter at rest



*Work in progress: implementation of momentum dependent potentials

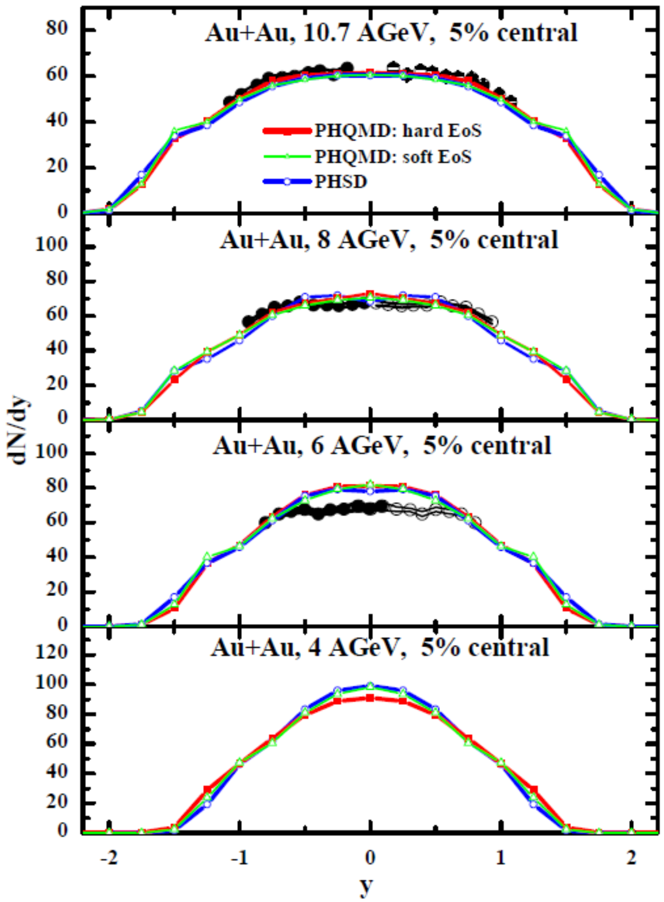
First PHQMD results on ‚bulk‘ observables

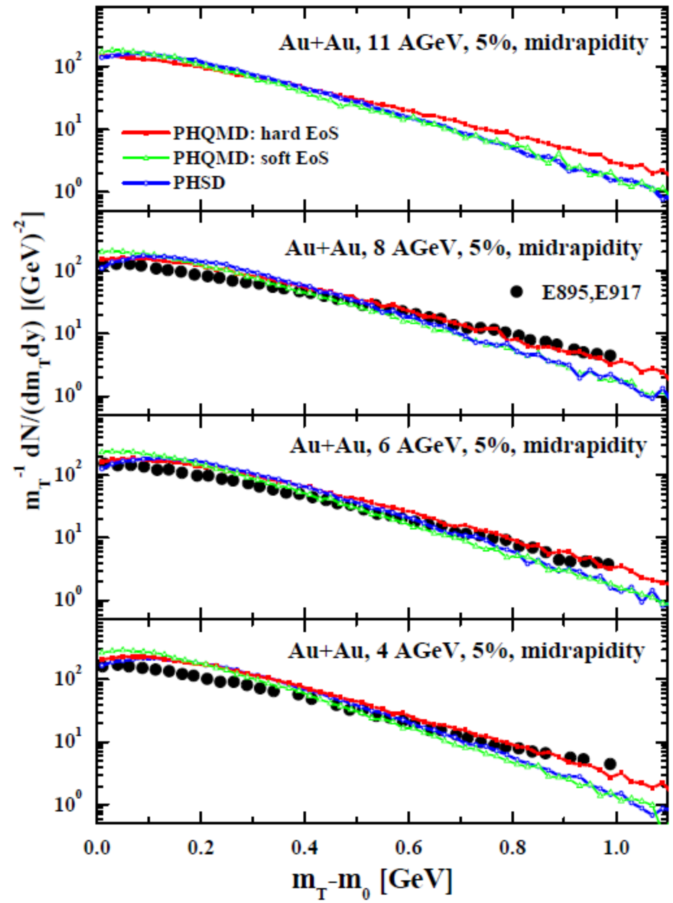
arXiv:1907.03860

PHQMD

PHQMD: ‚bulk' dynamics at AGS/FAIR/NICA energies

The rapidity and m_T distributions for protons from 5% central Au+Au collisions at 4, 6, 8, 10.7 A GeV



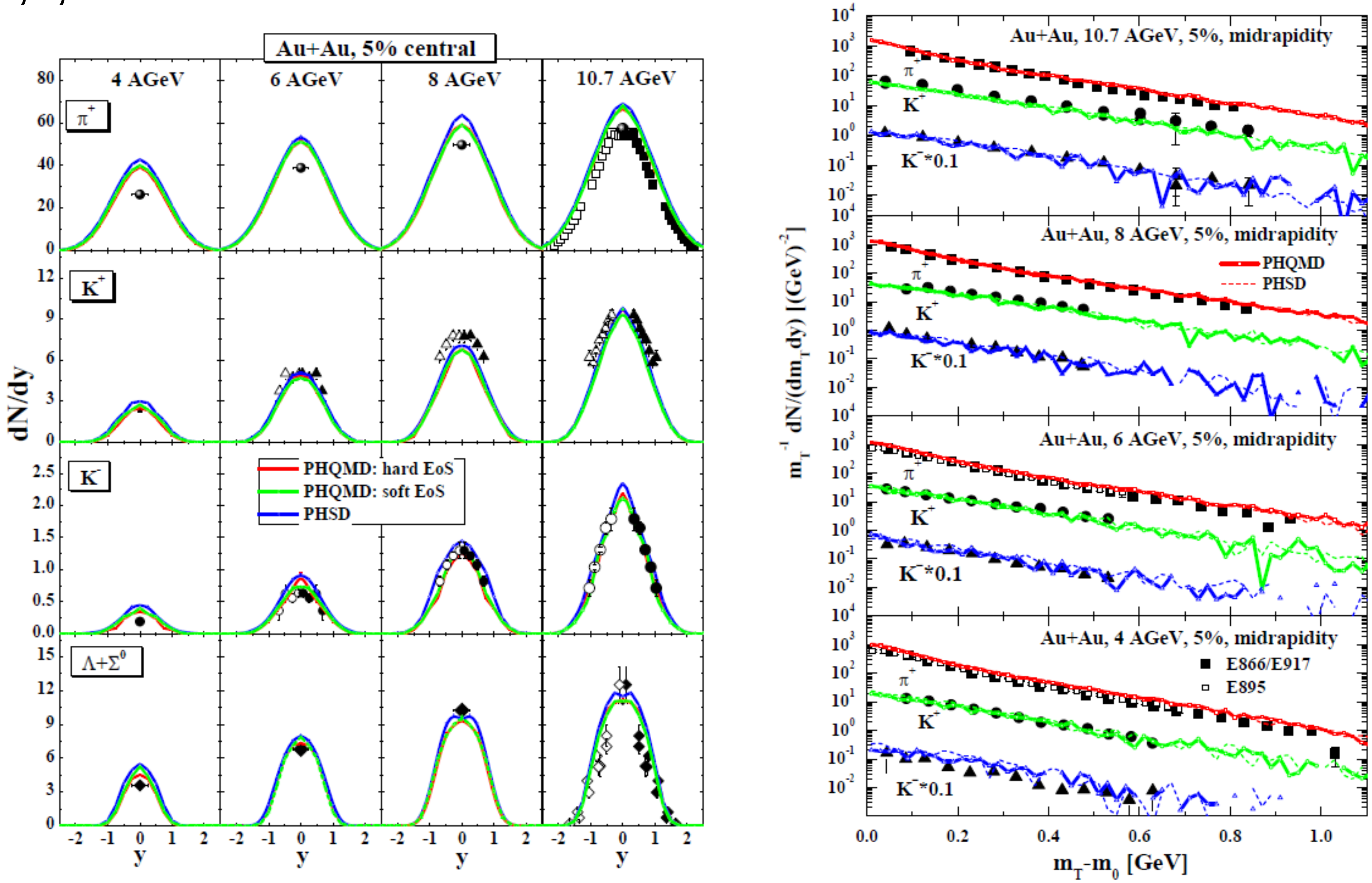


- the influence of EoS is slightly visible in rapidity spectra of protons
- m_T spectra of protons from PHQMD with a 'hard' EoS are harder then with 'soft' EoS
- PHQMD results for the m_T spectra with 'soft' EoS are in a good agreement with the PHSD spectra (using 'soft' EoS in default PHSD4.0 version)

  ➔ QMD and MF dynamics gives similar results with similar EoS

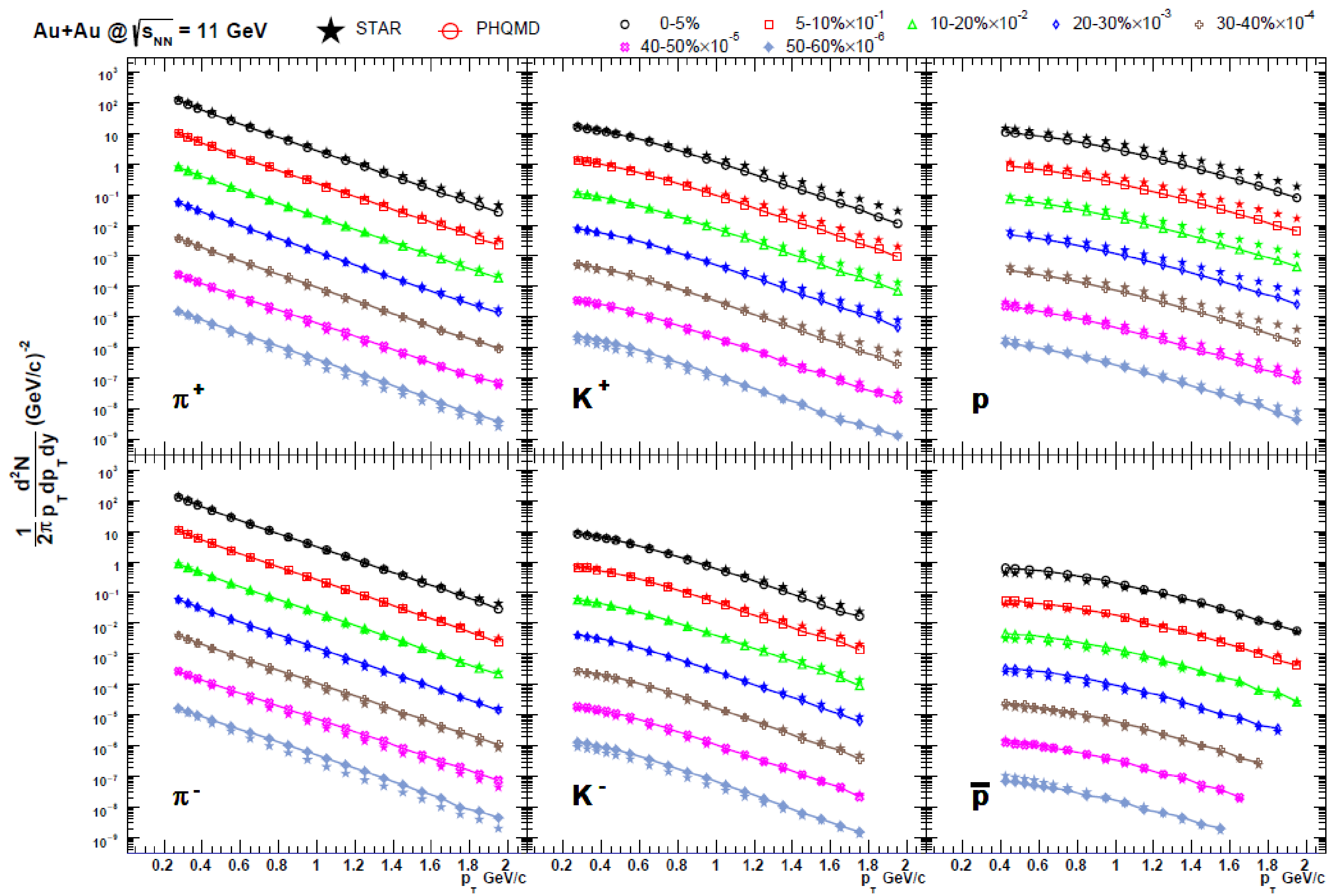
PHQMD: ‚bulk' dynamics at AGS/FAIR/NICA energies

The rapidity and m_T distributions for π^+, K^+, K^-, $\Lambda+\Sigma^0$ from 5% central Au+Au collisions at 4, 6, 8, 10.7 A GeV



- the influence of EoS is slightly visible in rapidity and m_T spectra of newly produced hadrons

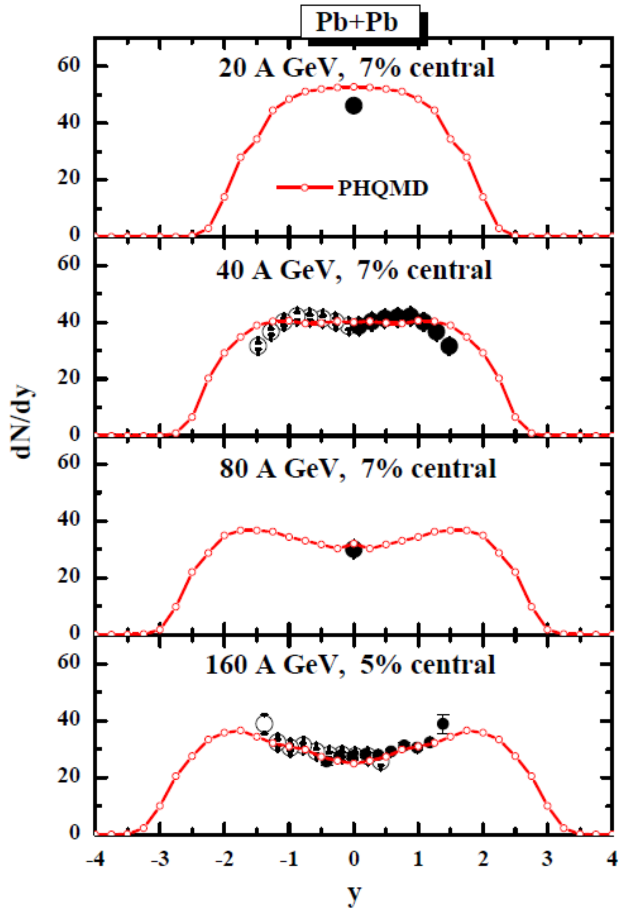
PHQMD: ‚bulk‘ dynamics at RHIC BES

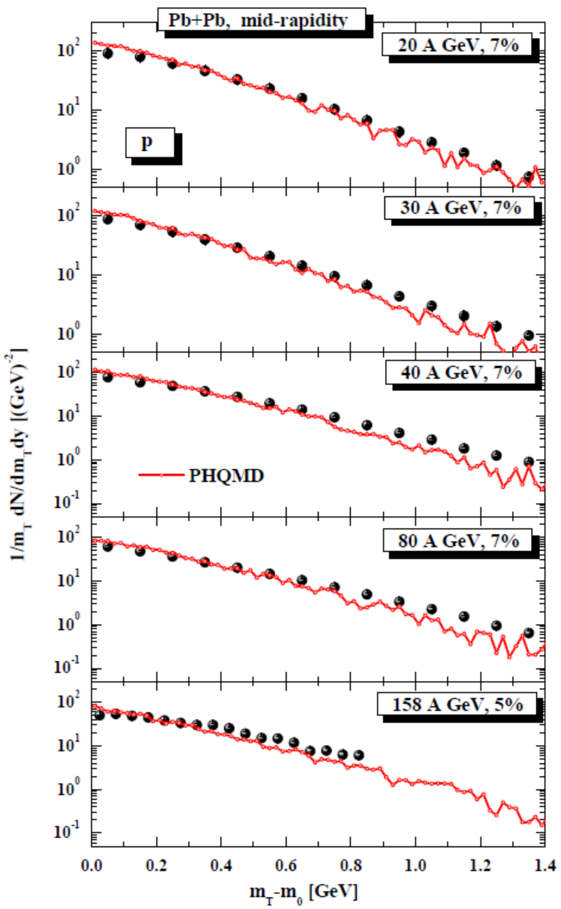


PHQMD: **good agreement with RHIC BES exp. data**

PHQMD: ‚bulk' dynamics at SPS

The rapidity and m_T distributions for protons from 7% central Pb+Pb collisions at 20, 40, 80, 158 A GeV

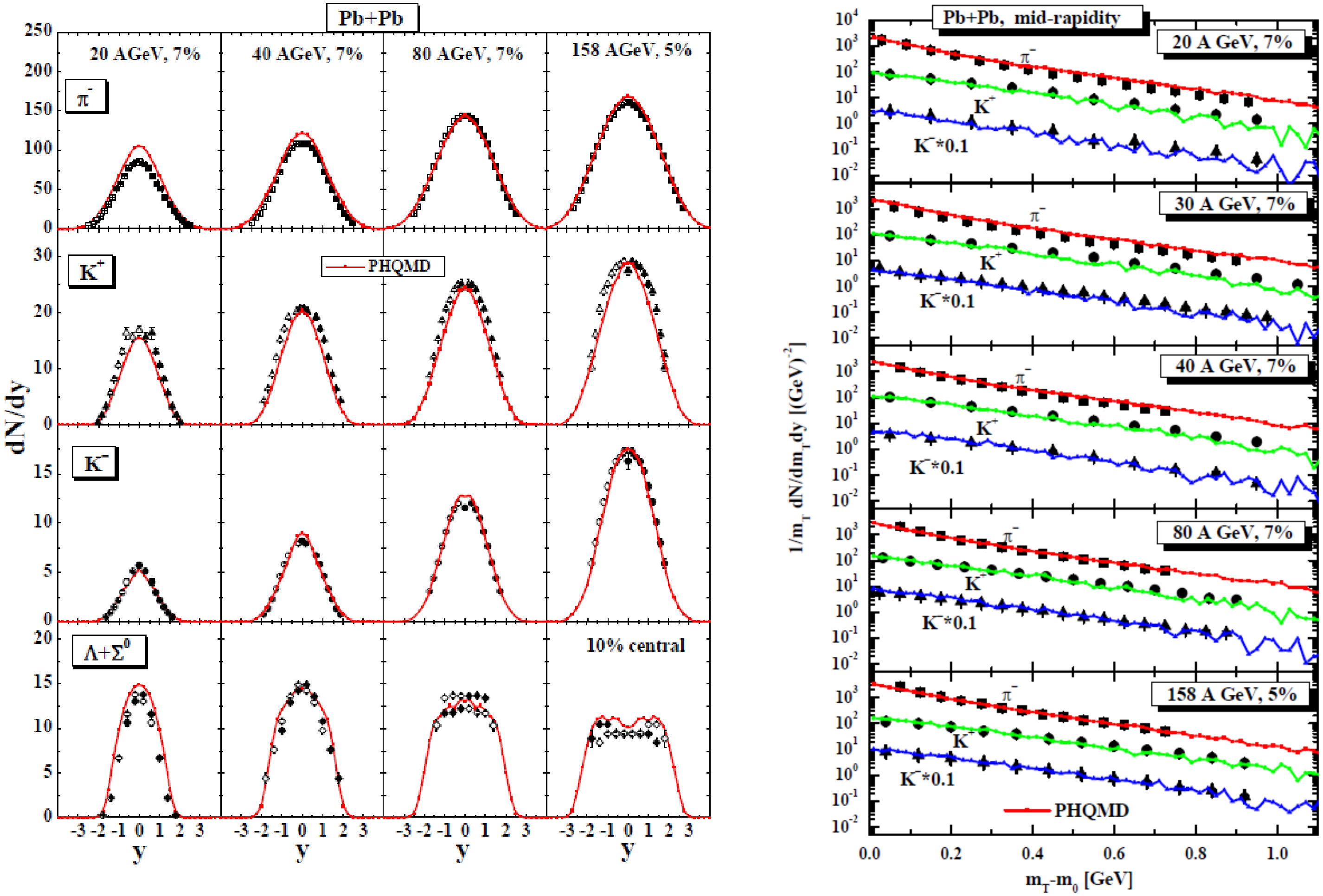




PHQMD : stopping is controlled by exp. data
very weak dependence on baryonic potential

PHQMD: ‚bulk‘ dynamics at SPS

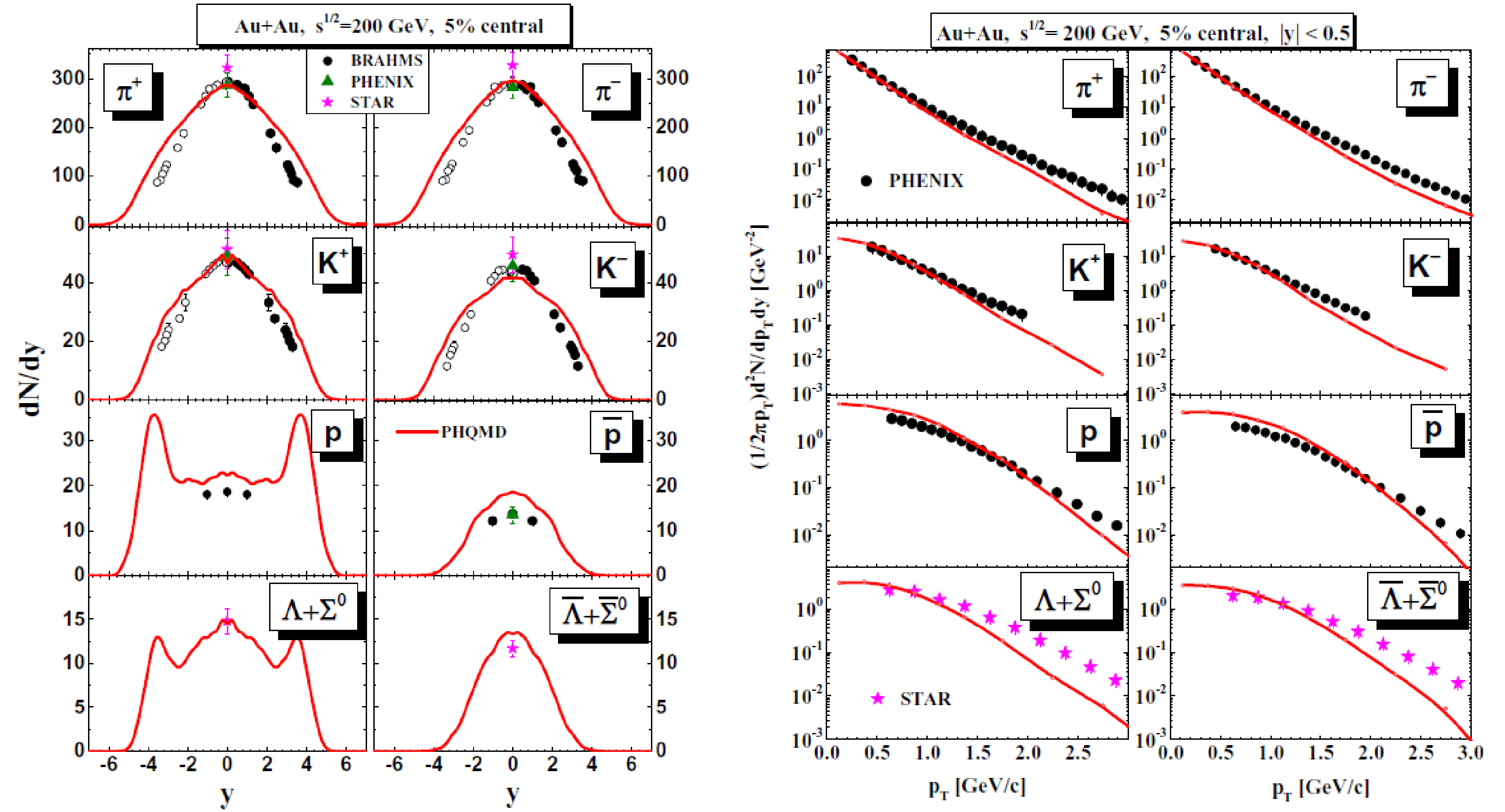
The rapidity and m_T distributions for π^-, K^+, K^-, $\Lambda+\Sigma^0$ from 7% central Pb+Pb collisions at 20, 40, 80, 158 A GeV



PHQMD: good agreement with exp. data - similar to PHSD - since at high energies the dynamics is dominated by collisions rather than potential interactions!

PHQMD: ‚bulk‘ dynamics at RHIC

The rapidity and p_T distributions for p, anti-p, π^-, K^+, K^-, $\Lambda+\Sigma^0$, anti($\Lambda+\Sigma^0$) from 5% central Au+Au collisions at $s^{1/2}$ = 200 GeV



PHQMD: results are similar to PHSD - since at RHIC energies the dynamics is dominated by collisions of partons/hadrons rather than nuclear potential interactions!

Clusters in PHQMD:
MST & SACA

arXiv:1907.03860

PHQMD

Cluster recognition: Minimum Spanning Tree (MST)

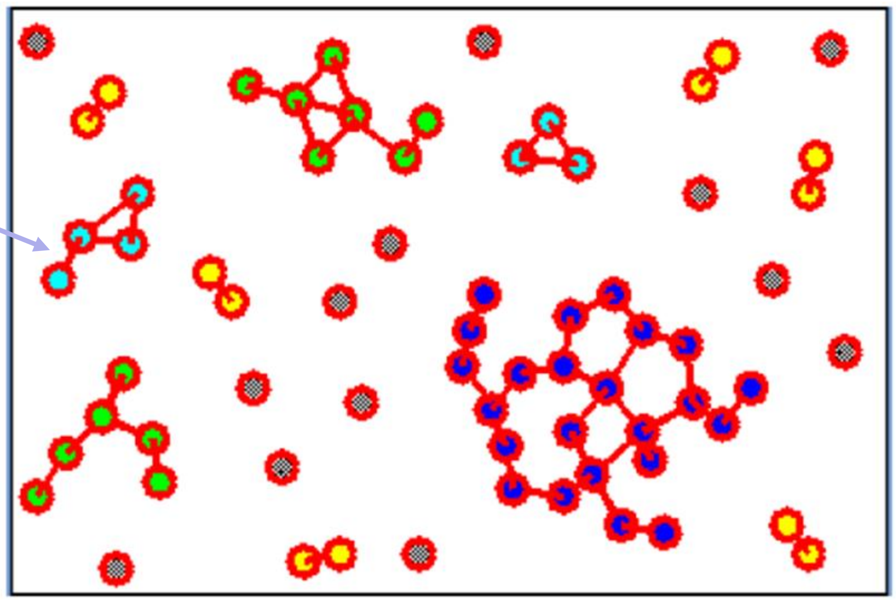
The Minimum Spanning Tree (MST) is a cluster recognition method applicable for the (asymptotic) final states where coordinate space correlations may only survive for bound states.

The MST algorithm searches for accumulations of particles in coordinate space:

1. **Two particles are 'bound' if their distance in coordinate space fulfills**

$$\left|\vec{r}_i - \vec{r}_j\right| \leq 2.5\, fm$$

2. **Particle is bound to a cluster if it bounds with at least one particle of the cluster.**



*** Remark:**
inclusion of an additional momentum cuts (coalescence) lead to a small changes: particles with large relative momentum are mostly not at the same position

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266

Simulated Annealing Clusterization Algorithm (SACA)

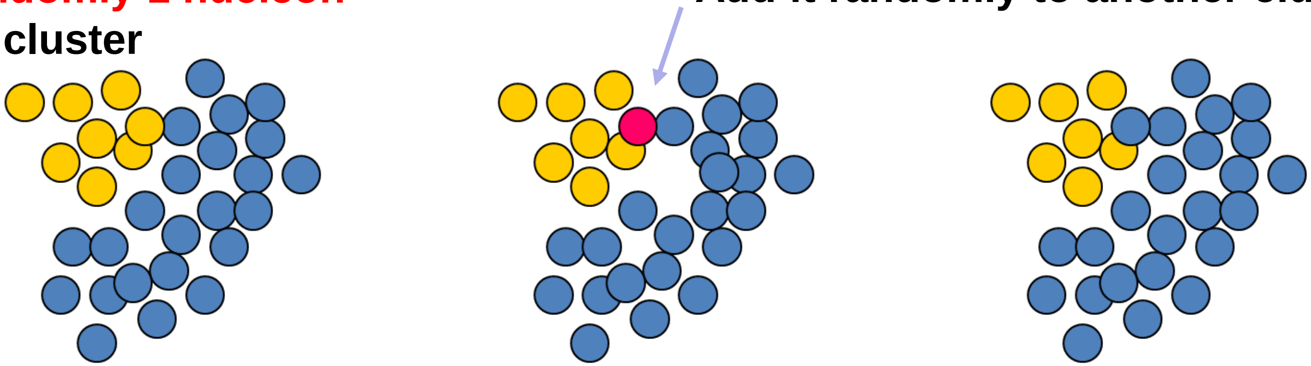
Basic ideas of clusters recognition by SACA:

Based on idea by Dorso and Randrup (Phys.Lett. B301 (1993) 328)

- **Take the positions and momenta of all nucleons at time t**
- **Combine them in all possible ways into all kinds of clusters or leave them as single nucleons**
- **Neglect the interaction among clusters**
- **Choose that configuration which has the highest binding energy:**

Take randomly 1 nucleon out of a cluster

Add it randomly to another cluster



$E=E^{1}_{kin}+E^{2}_{kin}+V^{1}+V^{2}$

$E'=E^{1'}_{kin}+E^{2'}_{kin}+V^{1'}+V^{2'}$

If E' < E take a new configuration
If E' > E take the old configuration with a probability depending on E'-E
Repeat this procedure many times

→ **Leads automatically to finding of the most bound configurations**

R. K. Puri, J. Aichelin, PLB301 (1993) 328, J.Comput.Phys. 162 (2000) 245-266;
P.B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nuclear Physics A 619 (1997) 379-390

Cluster recognition by SACA

- **SACA searches for the most bound configurations ➔ clusters**
  Clusters are bound by potential interactions between nucleons V_i
  V_i – Skyrme potential (as in PHQMD!)

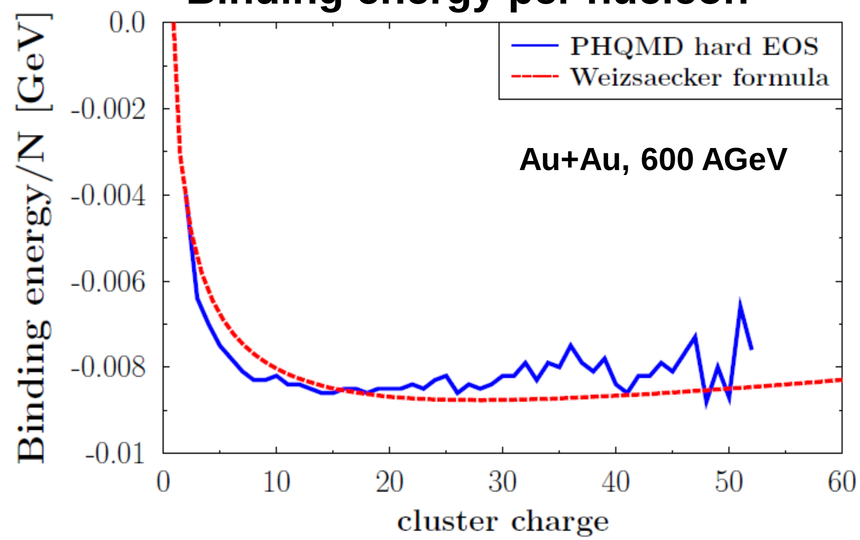
- **Binding energy (SACA) vs. Weizsaecker formula**

There are two kinds of clusters:

I) Heavy clusters formed from spectator matter close to beam and target rapidity:
- **initial-final state correlations**
- **HIC makes spectator matter unstable**

II) Light clusters formed from participant matter created during the expansion of the fireball ➔
"ice" (E_{bind} ≈-8 MeV/N) in "fire"(T≥ 100 MeV)
- **origin is not well understood**
- **seen from SIS to RHIC**
- **quantum effects may be important**

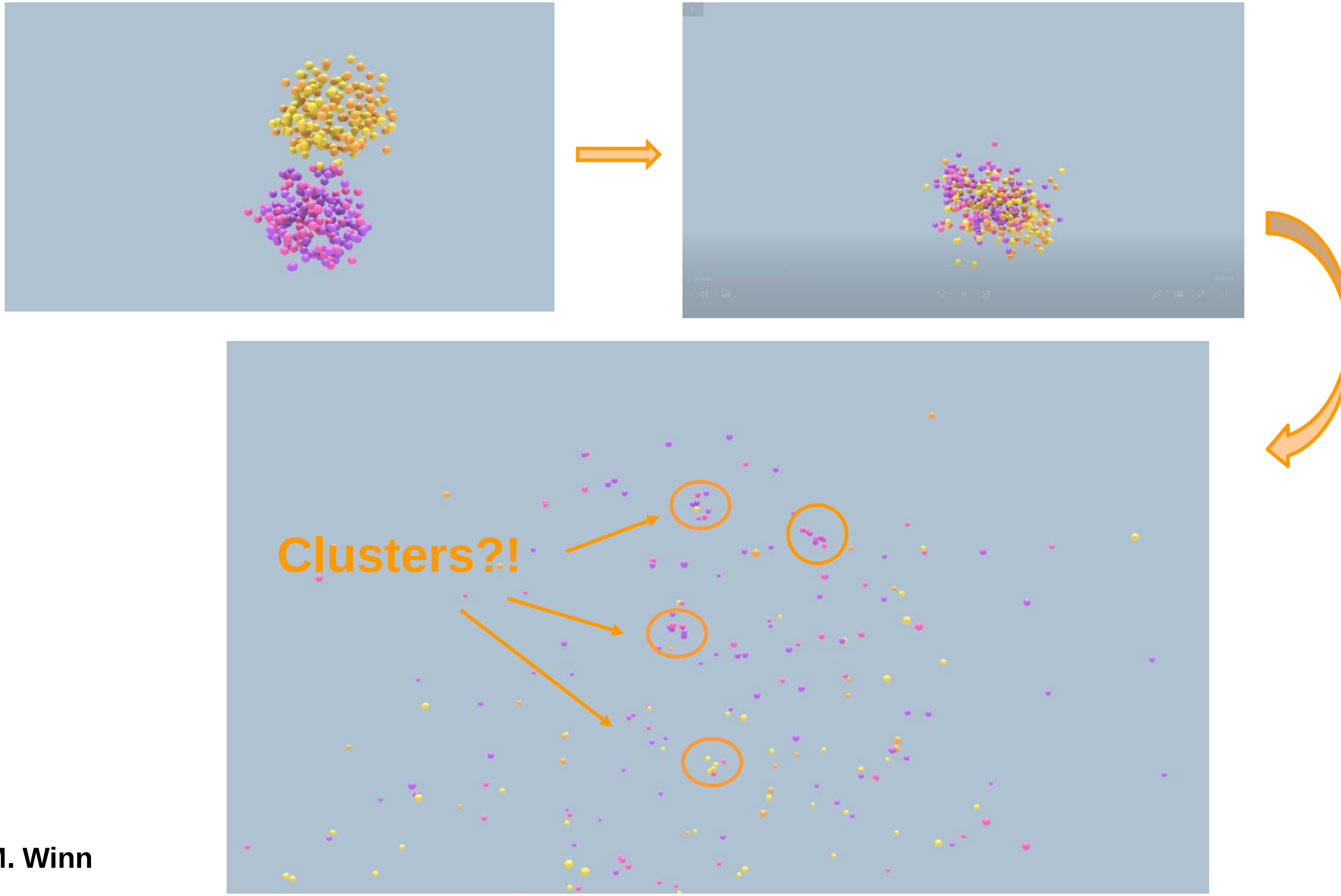
Binding energy per nucleon



➔ average binding energy of the clusters identified by SACA at late times (>100 fm/c) is in agreement with Weizsaecker formula

PHQMD

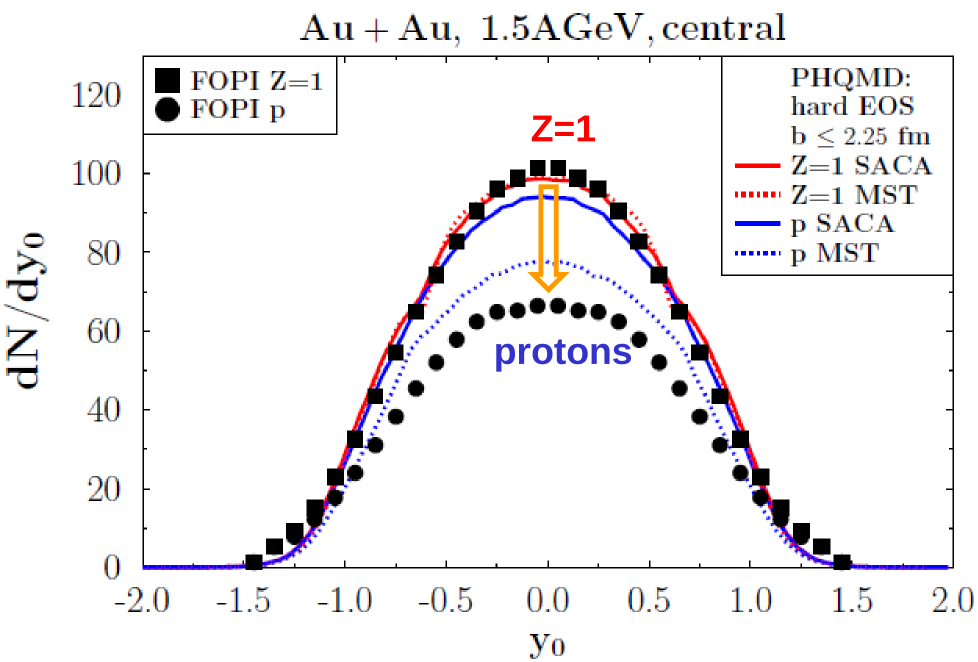
Time evolution: Au+Au, b=2 fm, 600 AGeV

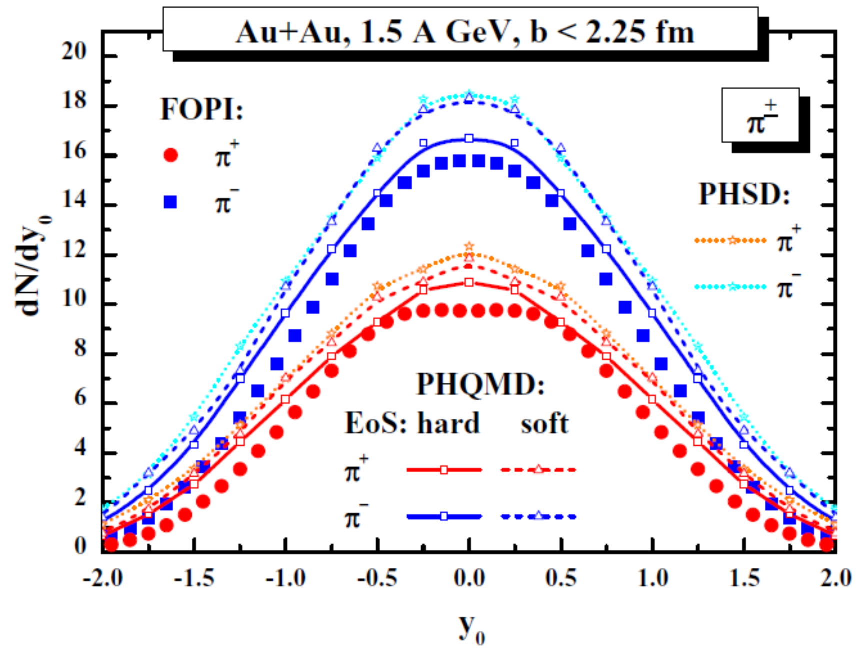


By M. Winn

PHQMD: light clusters and ‚bulk' dynamics at SIS

Scaled rapidity distribution $y_0 = y/y_{proj}$ in central Au+Au reactions at 1.5 AGeV





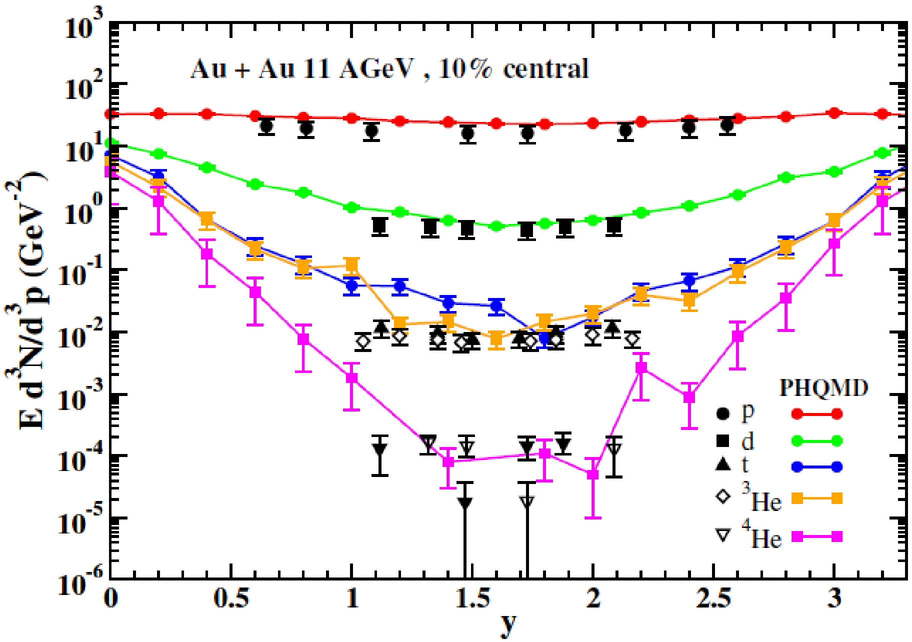
- ➢ **30% of protons are bound in clusters at 1.5 A GeV**
- ➢ **Presently MST is better identifying light clusters than SACA**
  ➔ To improve in SACA: more realistic potentials for small clusters, quantum effects

- ❑ **Pion spectra are sensitive to EoS: better reproduced by PHQMD with a 'hard' EoS**
- ❑ **PHQMD with soft EoS is consistent with PHSD (default – soft EoS)**
  *** To improve in PHQMD: momentum dependent potentials**

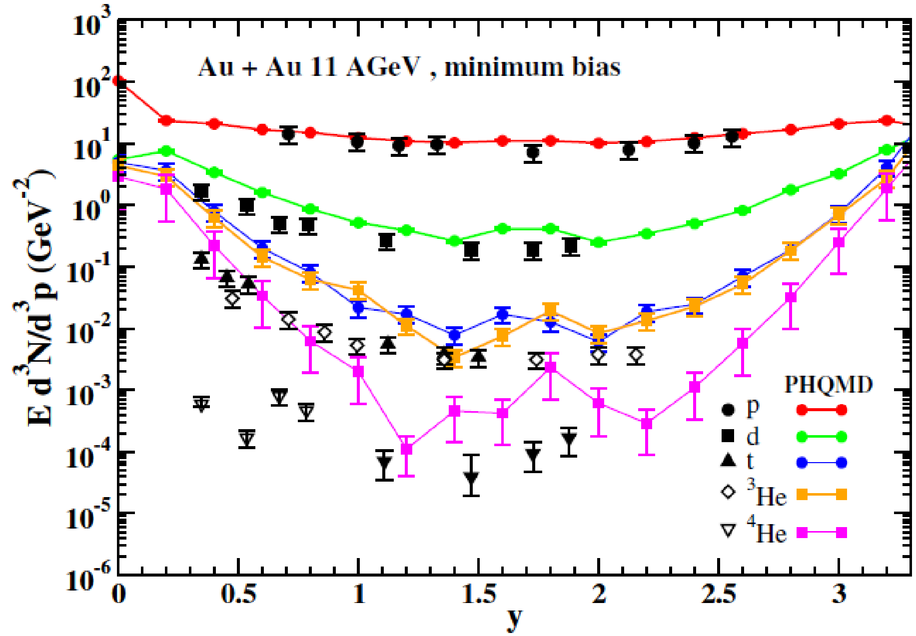
PHQMD: light clusters at AGS energies

The invariant multiplicities for p, d, t, ^{3}He, ^{4}He at p_T <0.1 GeV versus rapidity

Au+Au, 11 AGeV, 10% central



Au+Au, 11 AGeV, minimal bias



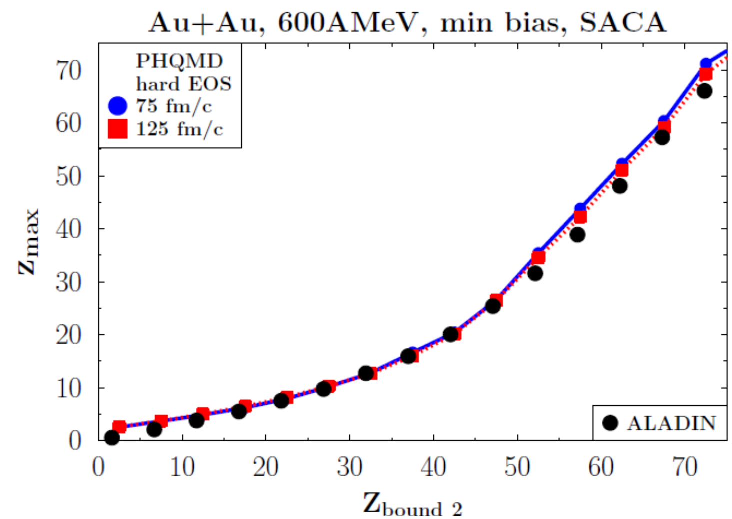
PHQMD: clusters recognition by MST provides a reasonable description of exp. data on light clusters at AGS energies

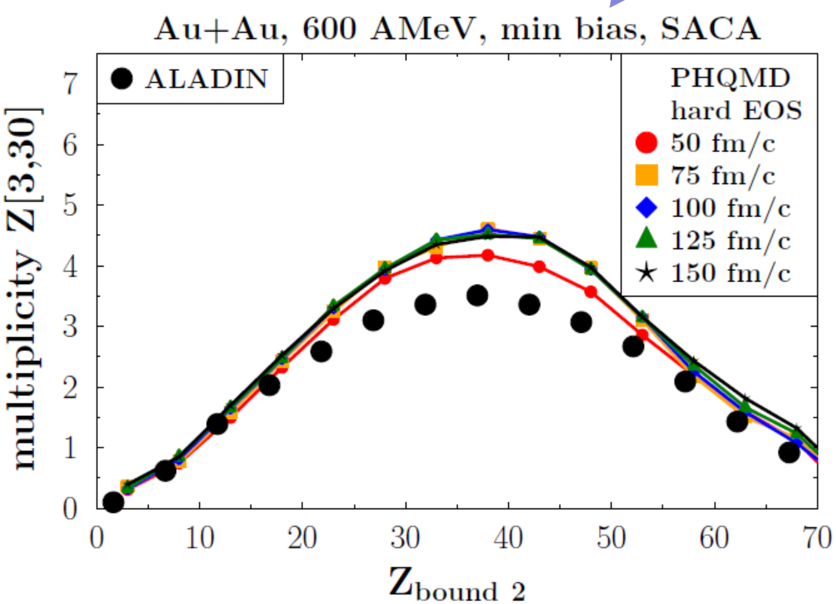
PHQMD: heavy clusters

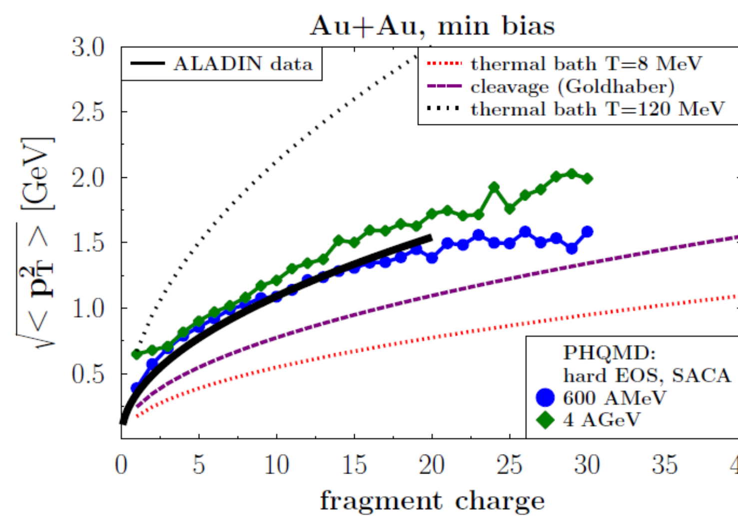
Heavy clusters (spectator fragments): experim. measured up to E_{beam} =1 AGeV (ALADIN Collab.)

PHQMD with SACA shows an agreement with ALADIN data for very complex cluster observables as

- **Largest clusters (Z_{bound})**
- **Energy independent 'rize and fall'**
- **Rms p^2_T**





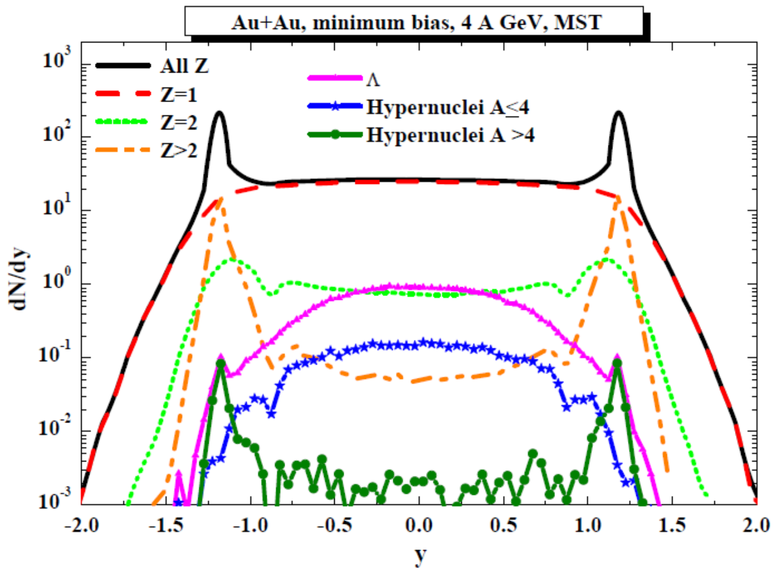


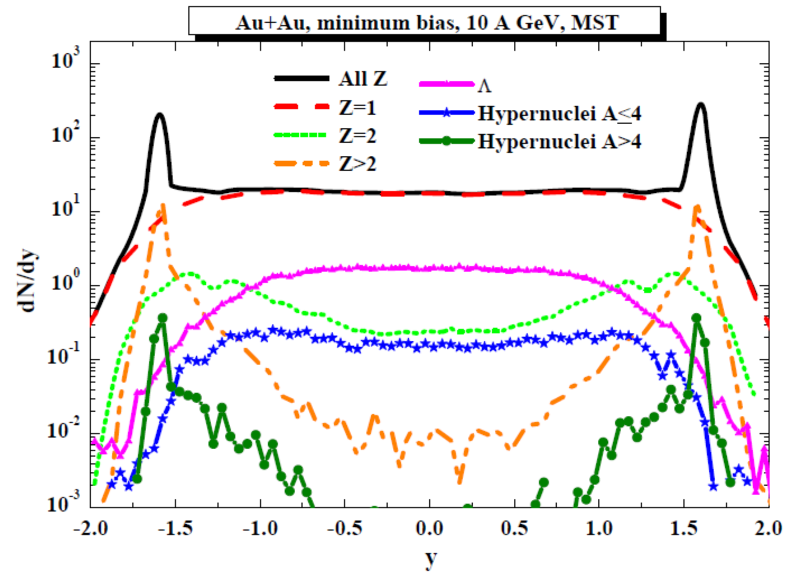
$$Z_{bound\ 2} = \sum_i Z_i\ \Theta(Z_i - (1+\epsilon)) \quad (\epsilon < 1)$$

PHQMD shows $\sqrt{p_T^2(Z)} \propto \sqrt{Z}$ dependence as exp. data

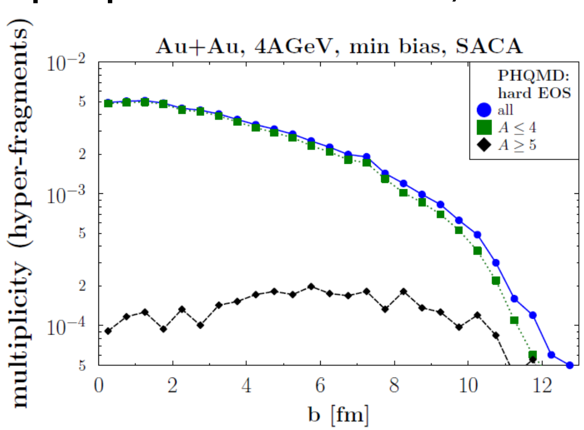
PHQMD: hypernuclei

PHQMD results (with a hard EoS and MST algorithm) for the rapidity distributions of all charges, Z = 1 particles, Z=2, Z>2, as well as Λ's, hypernuclei A≤4 and A>4 for Au+Au at 4 and 10AGeV





The multiplicity of light hypercluster vs. impact parameter b for Au+Au, 4 AGeV

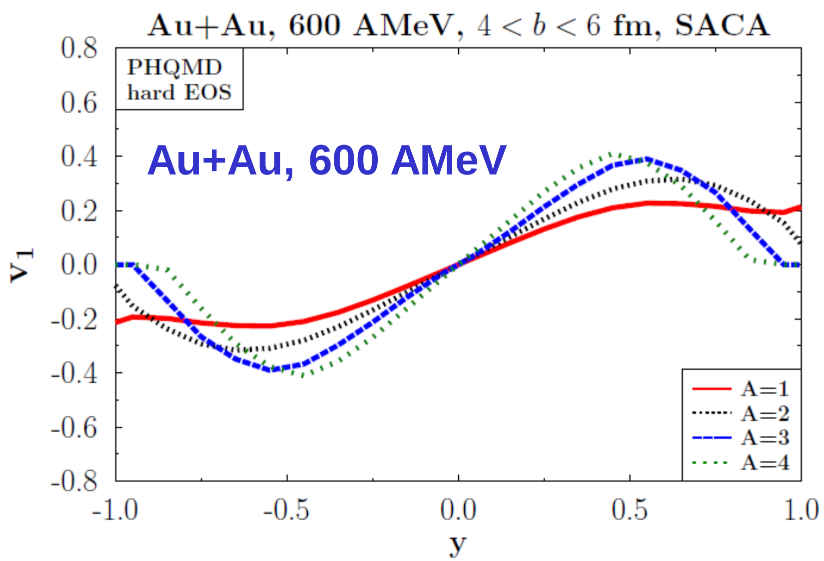


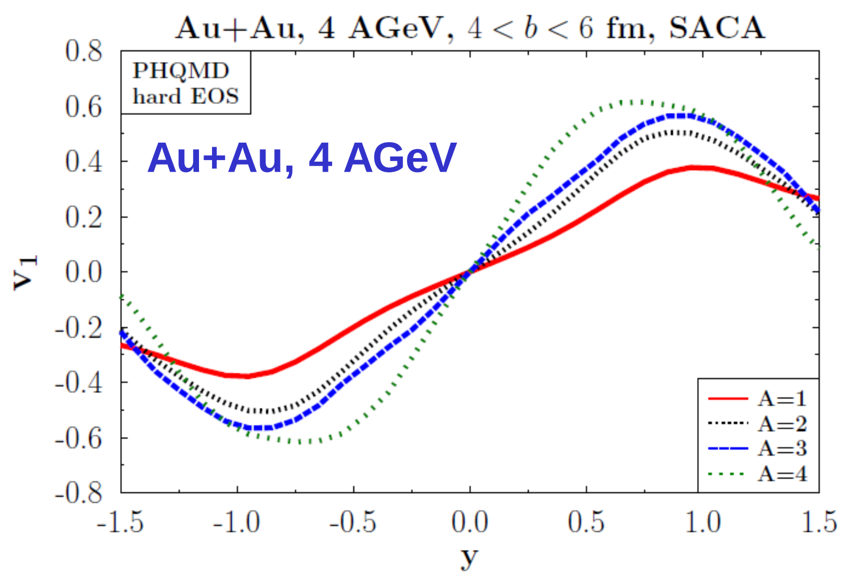
- ❑ **Central collisions → light hypernuclei**
- ❑ **Peripheral collisions → heavy hypernuclei**

Penetration of Λ's, produced at midrapidity, to target/projectile region due to rescattering

→ Possibility to study ΛN interaction

PHQMD: collectivity of clusters





PHQMD with hard EoS, with SACA:
v_1 of light clusters (A=1,2,3,4) vs rapidity
for mid-central Au+Au at 600 AMeV, 4AGeV

- **v_1 : quite different for nucleons and clusters (as seen in experiments)**
- **Nucleons come from participant regions (→ small density gradient) while clusters from interface spectator-participant (strong density gradient)**
- **v_1 increases with E_{beam}**
- **→ larger density gradient**

Summary

The PHQMD is a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster formation

combined model PHQMD = (PHSD & QMD) & SACA

PHQMD

- **provides the good description of hadronic ‘bulk’ observables from SIS to RHIC energies**
- **shows sensitivity to EoS: m_T spectra of baryons**
- **predicts the dynamical formation of clusters from low to ultra-relativistic energies**
- **allows to understand the proton spectra and the properties of clusters ($dn/dp_T dy$, v_1,v_2, fluctuations)**
- **allows to understand clusters formation in the participant and spectator region (consistent with available cluster data by ALADIN collaboration)**
- **allows to understand the formation of hypernuclei**

❖ ***PHQMD : under development***

Thank you for your attention !

Thanks to the Organizers !