

Coalescence from correlation functions

Kfir Blum
CERN & Weizmann Institute

EMMI Wroclaw 2019

$$\boxed{\frac{\mathcal{B}_A}{m^{2(A-1)}} \approx \frac{2J_A + 1}{2^A \sqrt{A}} \left(\frac{m R}{\sqrt{2\pi}} \right)^{3(1-A)}}$$

O(1) prediction of coalescence (Hydro / Gaussian source model)

R. Scheibl, U. Heinz, Phys.Rev. C59 (1999) 1585-1602

KB et al, Phys.Rev. D96 (2017) no.10, 103021

Closely related, and implied by, e.g.

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P. Danielewicz, P. Schuck, Phys.Lett. B274 (1992)

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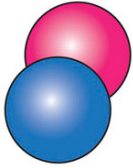
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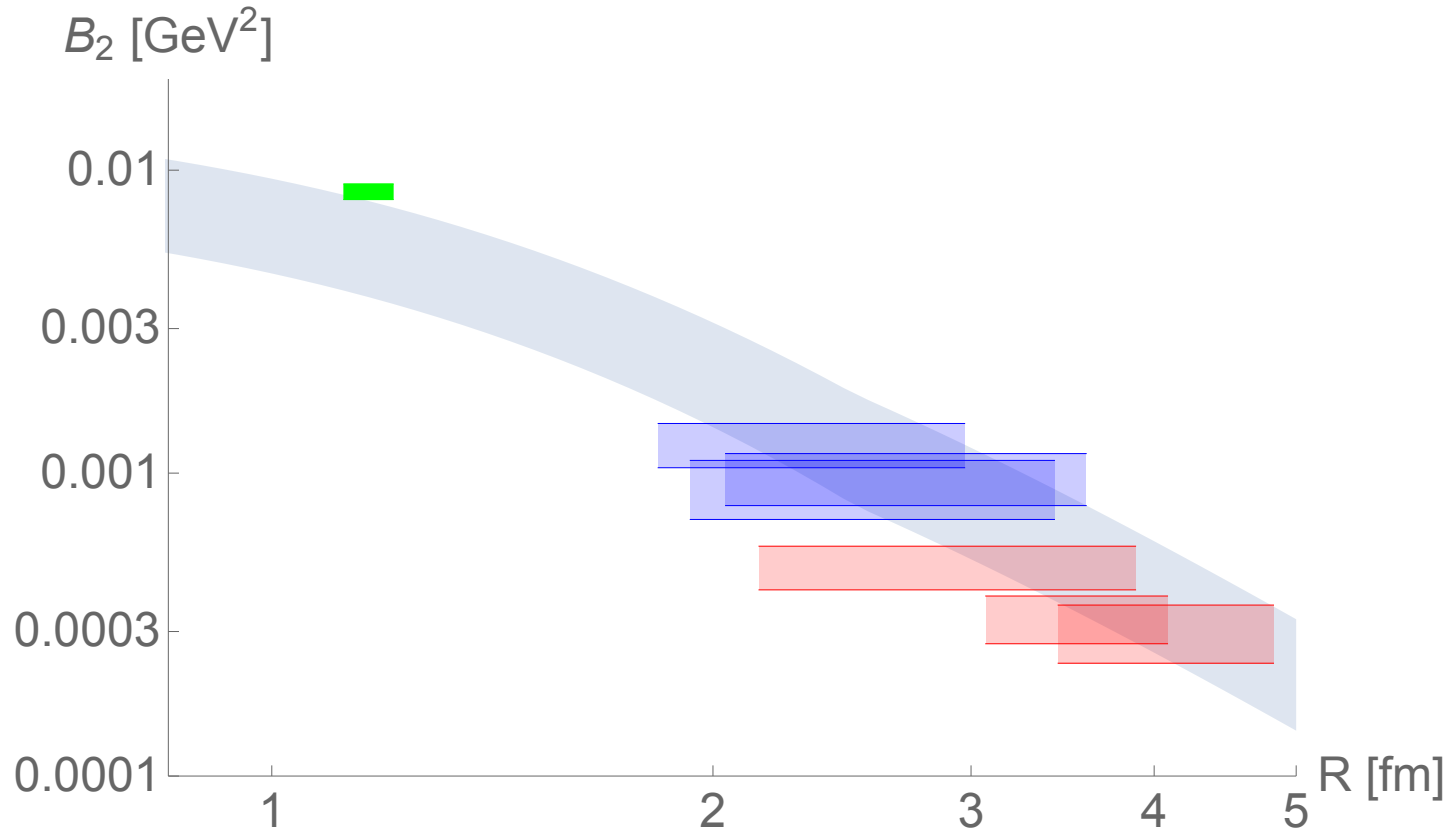
I was a little child when these papers were written.

Little did I know, that I was about to get in trouble in Wroclaw 2019 over them.



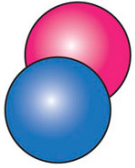
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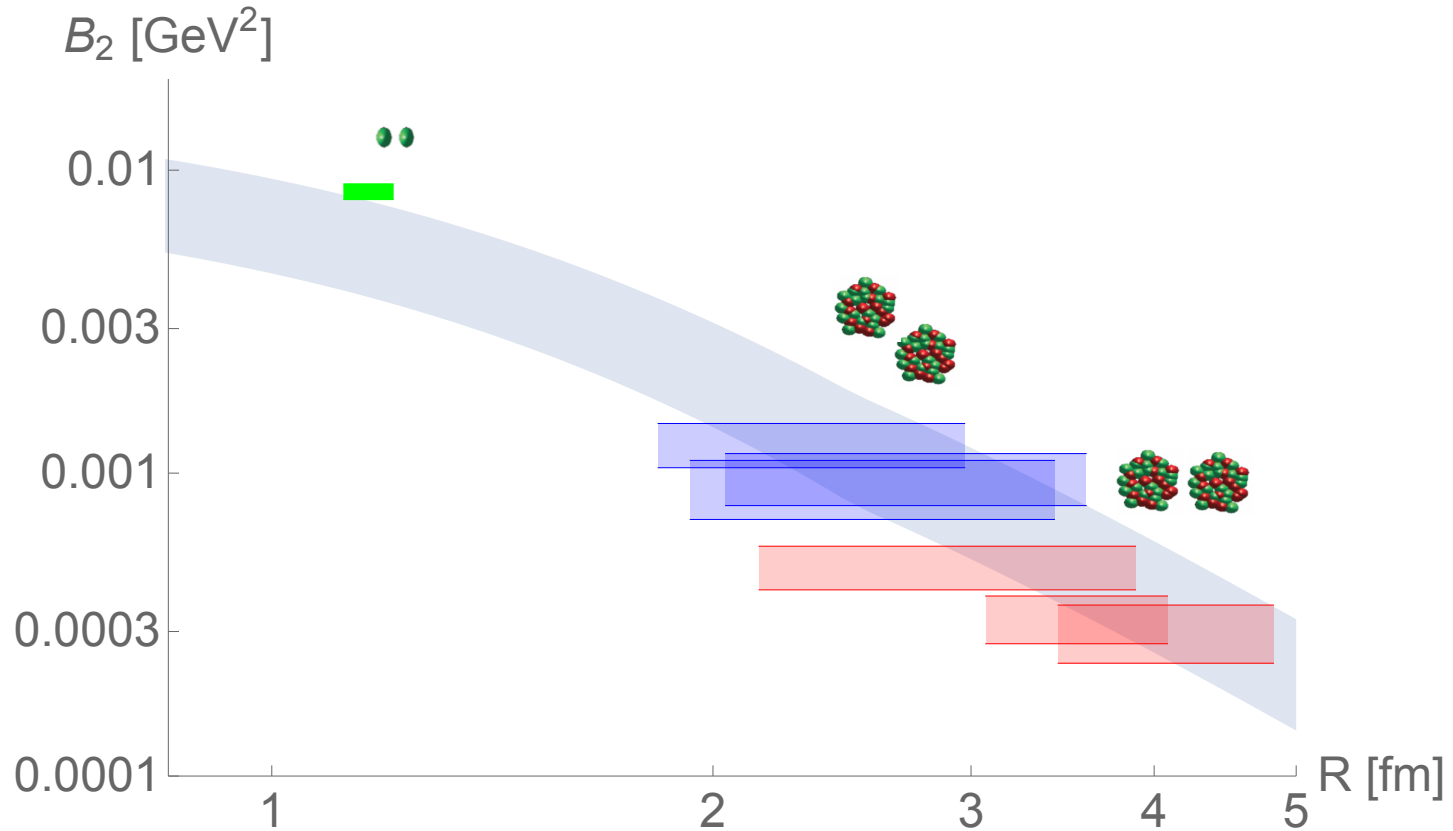
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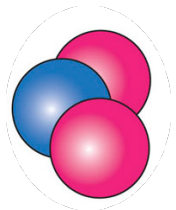
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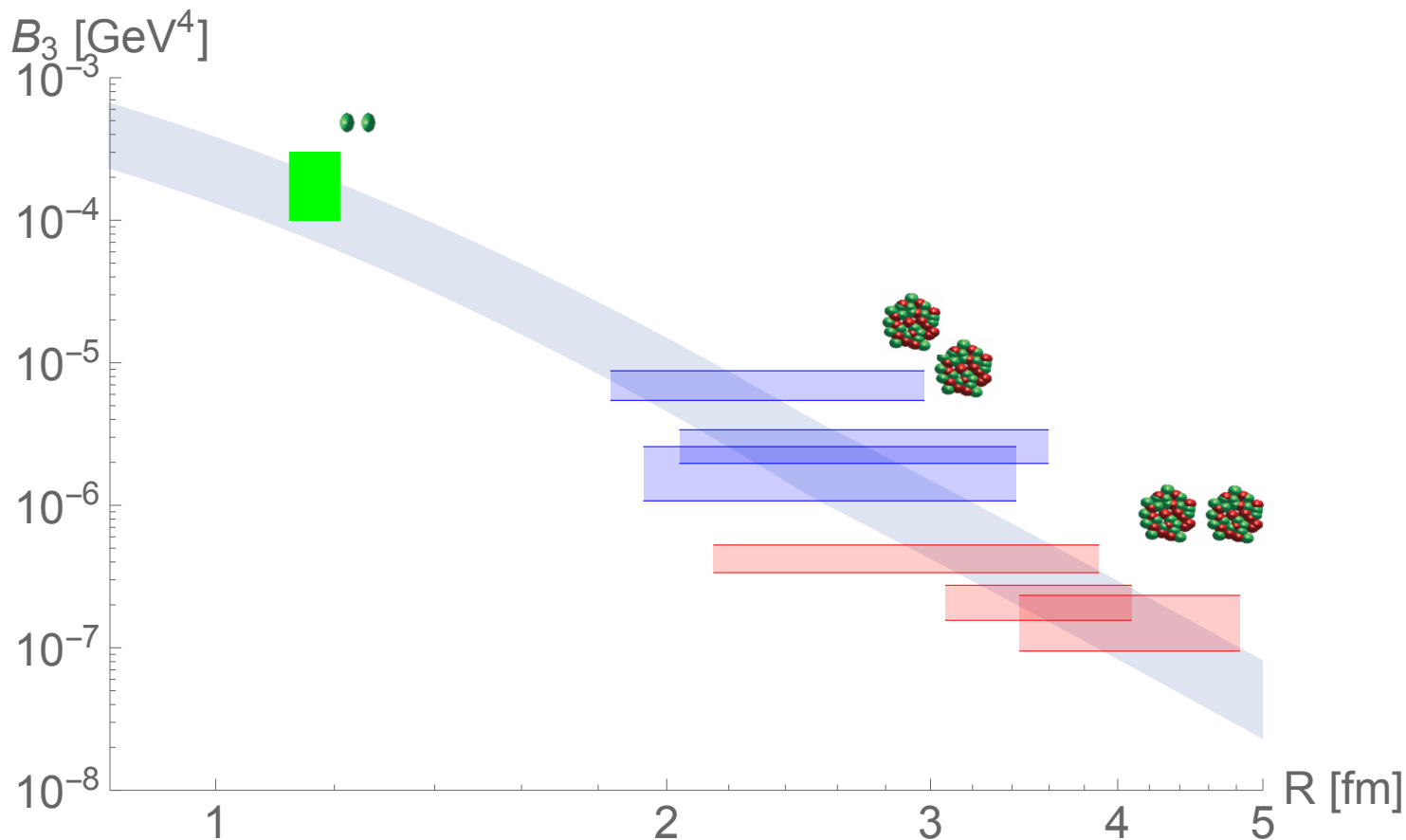


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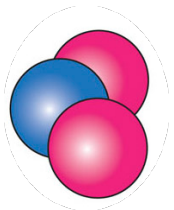


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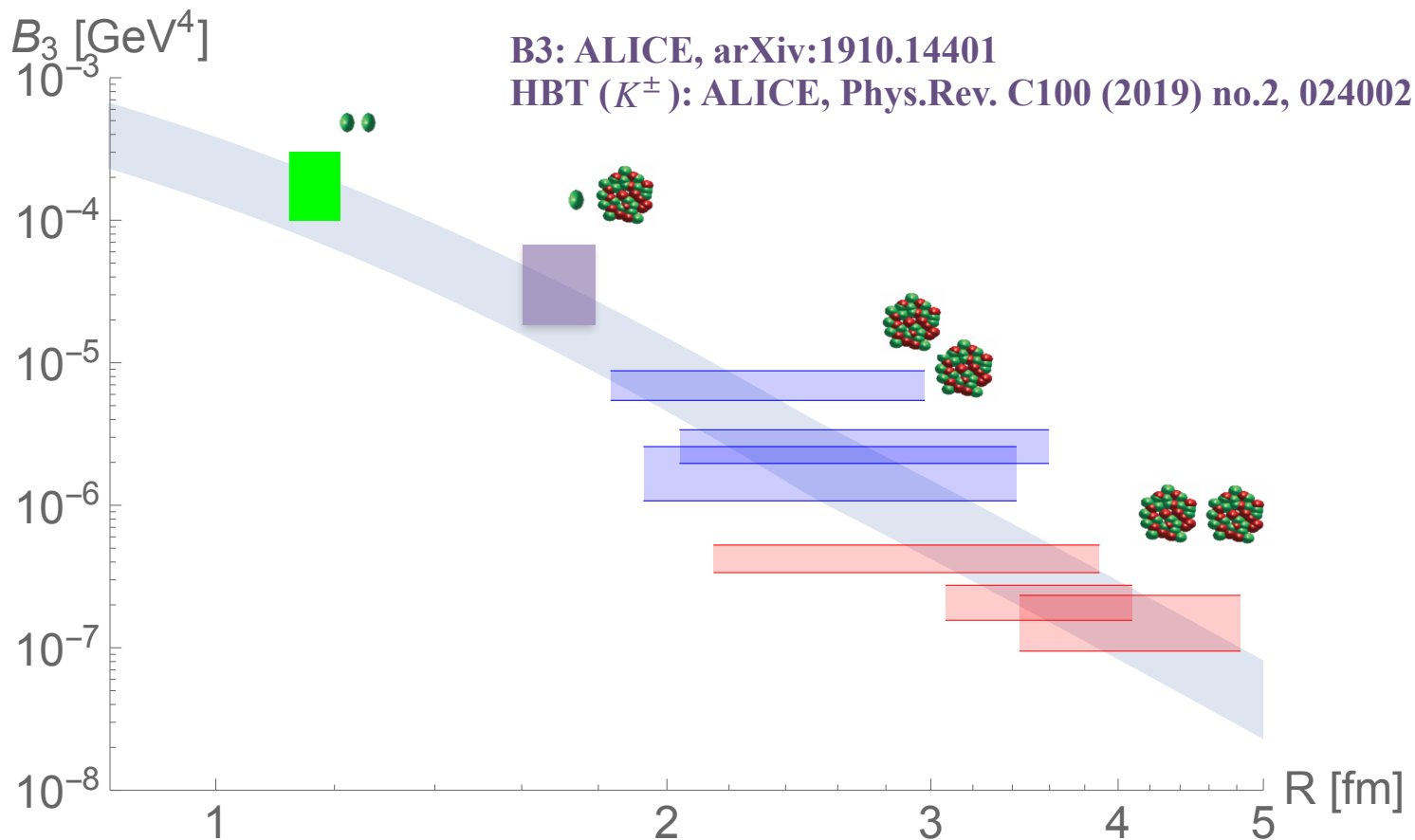


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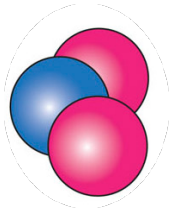


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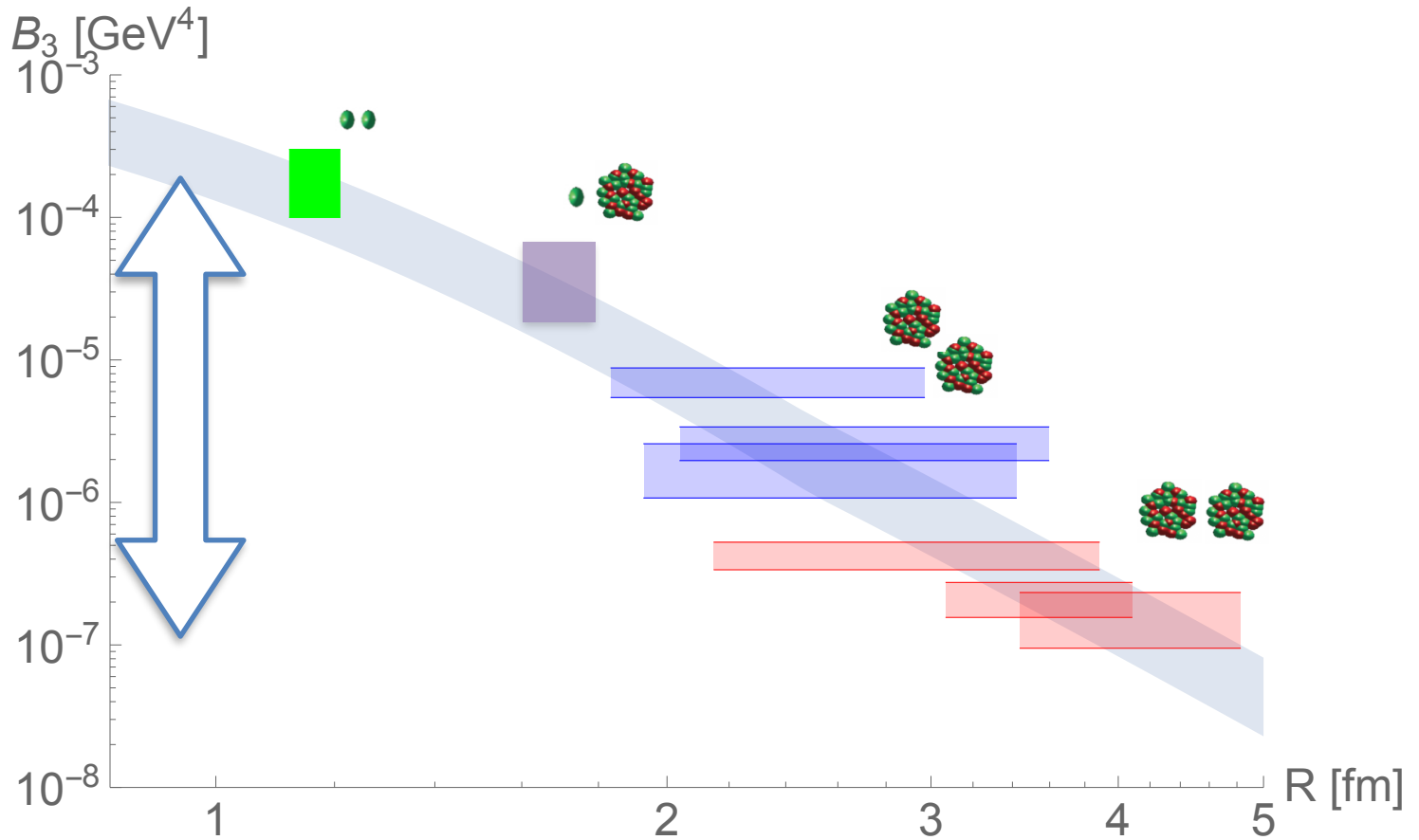


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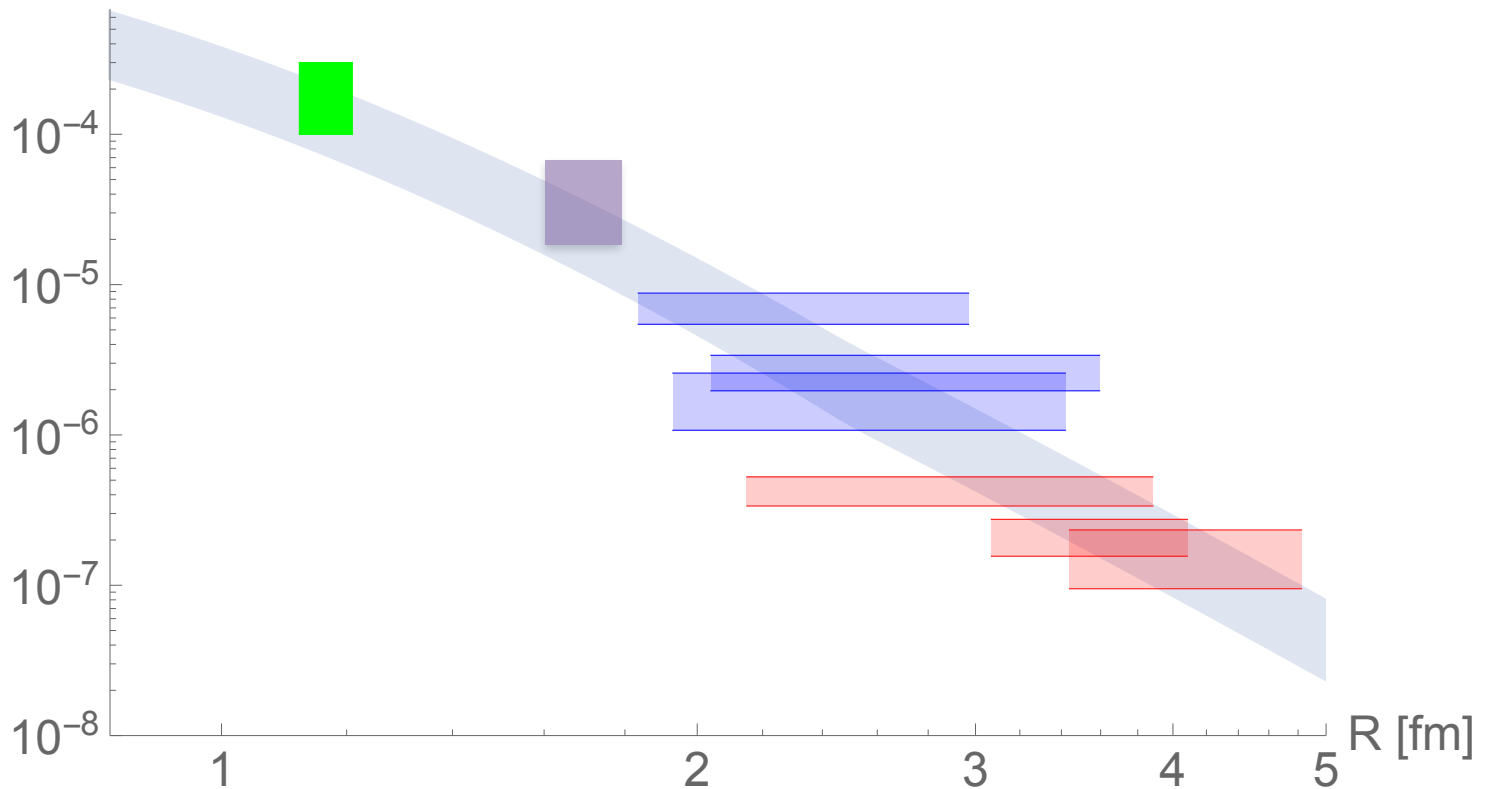
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Questions:

1. Where are the hydro model parameters?



$$\frac{\mathcal{B}_A}{m^{2(A-1)}} \approx \frac{2J_A + 1}{2^A \sqrt{A}} \left(\frac{m R}{\sqrt{2\pi}} \right)^{3(1-A)}$$

Questions:

1. Where are the hydro model parameters?
- 2.

arXiv:1904.06592

Hydrodynamic flow in small systems

or: “How the heck is it possible that a system emitting only a dozen particles can be described by fluid dynamics?”

Ulrich Heinz^{1a}, in collaboration with J. Scott Moreland^b

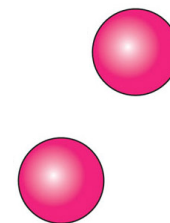
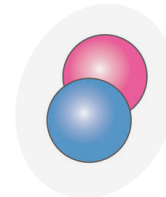
^aDepartment of Physics, The Ohio State University, Columbus, OH 43210-1117, USA

^bDepartment of Physics, Duke University, Durham, NC 27708-0305, USA

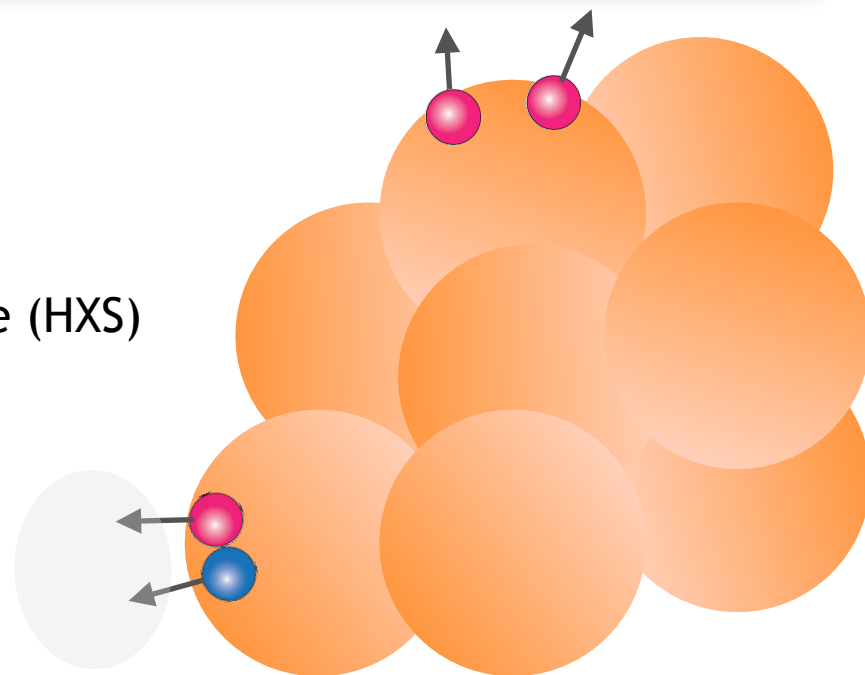
E-mail: heinz.9@osu.edu

Abstract. The “unreasonable effectiveness” of relativistic fluid dynamics in describing high energy heavy-ion and even proton-proton collisions will be demonstrated and discussed. Several recent ideas of optimizing relativistic fluid dynamics for the specific challenges posed by such collisions will be presented, and some thoughts will be offered why the framework works better than originally expected. I will also address the unresolved question where exactly hydrodynamics breaks down, and why.

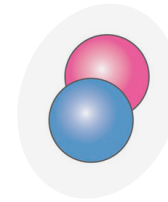
$$\frac{\mathcal{B}_A}{m^{2(A-1)}} \approx \frac{2J_A + 1}{2^A \sqrt{A}} \left(\frac{m R}{\sqrt{2\pi}} \right)^{3(1-A)}$$



High excitation state (HXS)

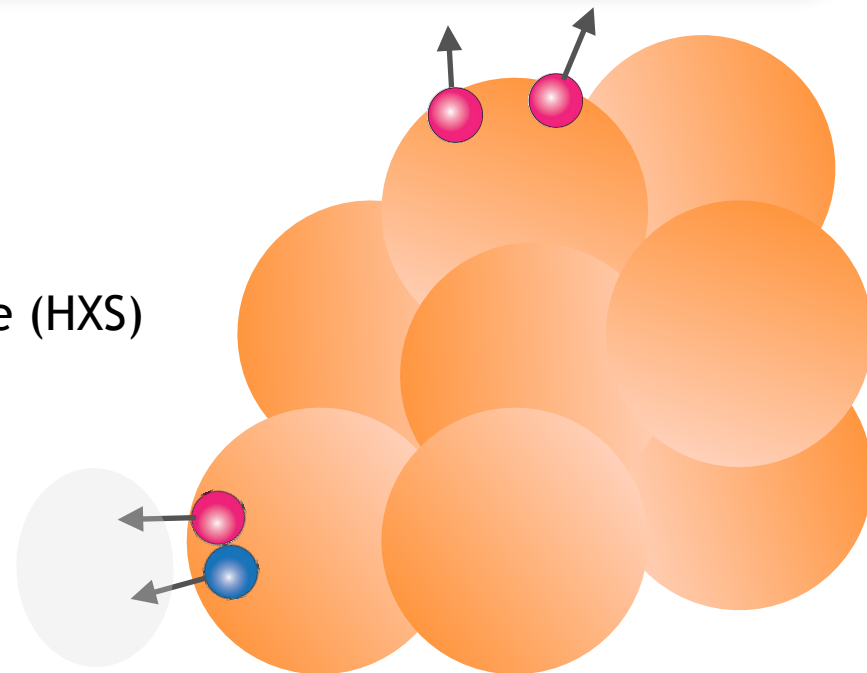


$$\psi_{P_d}(x_1, x_2) = e^{i\vec{P}_d \vec{X}} \phi_d(\vec{r})$$

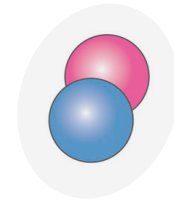


$$\frac{dN_d}{d^3P_d} = \langle \psi_{P_d} | \hat{\rho}_{\text{HX}} | \psi_{P_d} \rangle$$

High excitation state (HXS)

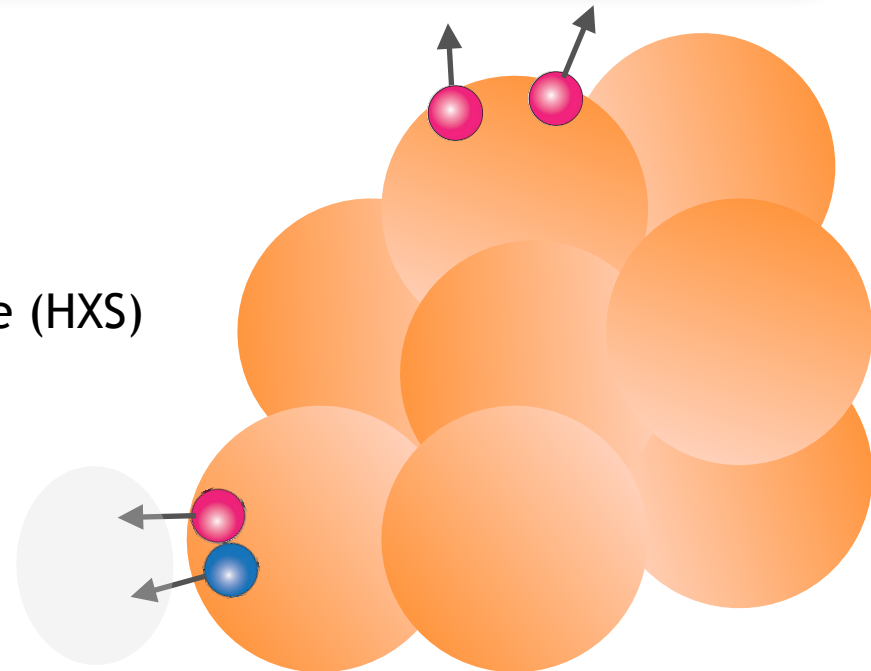


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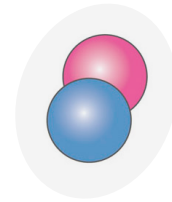


$$\begin{aligned} \frac{dN_d}{d^3 P_d} &= \langle \psi_{P_d} | \hat{\rho}_{\text{HX}} | \psi_{P_d} \rangle \\ &= G_d \int d^3 x_1 \int d^3 x_2 \int d^3 x'_1 \int d^3 x'_2 \times \\ &\quad \psi_{P_d}^*(x'_1, x'_2) \psi_{P_d}(x_1, x_2) \rho_2(x'_1, x'_2; x_1, x_2; t_f) \end{aligned}$$

High excitation state (HXS)



$$\psi_{P_d}(x_1, x_2) = e^{i\vec{P}_d \vec{X}} \phi_d(\vec{r})$$

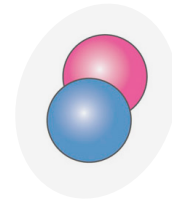


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$$\rho_2(x'_1, x'_2; x_1, x_2; t) \approx \rho_1(x'_1, x_1; t) \rho_1(x'_2, x_2; t)$$

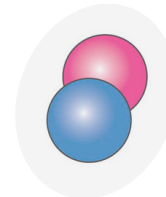
$$\rho_1(x, x'; t) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k}(\vec{x}' - \vec{x})} f_1^W \left(\vec{k}, \frac{\vec{x} + \vec{x}'}{2}; t \right)$$

$$\mathcal{D}_d(\vec{q}, \vec{r}) = \int d^3\zeta e^{-i\vec{q}\vec{\zeta}} \phi_d\left(\vec{r} + \frac{\vec{\zeta}}{2}\right) \phi_d^*\left(\vec{r} - \frac{\vec{\zeta}}{2}\right)$$



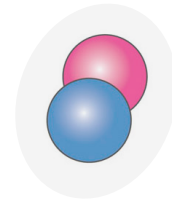
$$\frac{dN_d}{d^3P_d} = G_d \int d^3R \int \frac{d^3q}{(2\pi)^3} \int d^3r \mathcal{D}_d(\vec{q}, \vec{r}) \times \\ f_1^W\left(\frac{\vec{P}_d}{2} + \vec{q}, \vec{R} + \frac{\vec{r}}{2}; t_f\right) f_1^W\left(\frac{\vec{P}_d}{2} - \vec{q}, \vec{R} - \frac{\vec{r}}{2}; t_f\right)$$

$$|\phi_d(\vec{r})|^2 = \int d^3k e^{i\vec{k}\vec{r}} \mathcal{D}(\vec{k})$$



$$\frac{dN_d}{d^3P_d} \approx G_d \int d^3q \mathcal{D}(\vec{q}) \int d^3R \int d^3r e^{i\vec{q}\vec{r}} \times \\ f_1^W\left(\frac{\vec{P}_d}{2}, \vec{R} + \frac{\vec{r}}{2}; t_f\right) f_1^W\left(\frac{\vec{P}_d}{2}, \vec{R} - \frac{\vec{r}}{2}; t_f\right)$$

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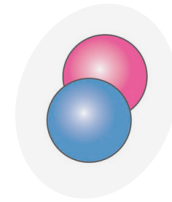
$$\psi_{p_1, p_2}^s(x_1, x_2) = \frac{1}{\sqrt{2}} e^{2i\vec{P}\vec{X}} \left(e^{i\vec{q}\vec{r}/2} - e^{-i\vec{q}\vec{r}/2} \right)$$



$$\frac{dN^s}{d^3p_1 d^3p_2} = \langle \psi_{p_1, p_2}^s | \hat{\rho}_{\text{HX}} | \psi_{p_1, p_2}^s \rangle \\ = G_2^s \int d^3x_1 \int d^3x_2 \int d^3x'_1 \int d^3x'_2 \\ \psi_{p_1, p_2}^{s*}(x'_1, x'_2) \psi_{p_1, p_2}^s(x_1, x_2) \rho_2(x'_1, x'_2; x_1, x_2; t_f)$$



$$|\phi_d(\vec{r})|^2 = \int d^3k e^{i\vec{k}\vec{r}} \mathcal{D}(\vec{k})$$



$$\frac{dN_d}{d^3P_d} \approx G_d \int d^3q \mathcal{D}(\vec{q}) \int d^3R \int d^3r e^{i\vec{q}\vec{r}} \times \\ f_1^W\left(\frac{\vec{P}_d}{2}, \vec{R} + \frac{\vec{r}}{2}; t_f\right) f_1^W\left(\frac{\vec{P}_d}{2}, \vec{R} - \frac{\vec{r}}{2}; t_f\right)$$

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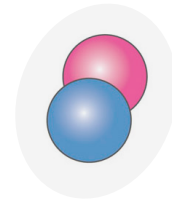


$$\frac{dN^s}{d^3p_1 d^3p_2} = G_2^s (\mathcal{A}_2(p_1, p_2) - \mathcal{F}_2(P, q)),$$



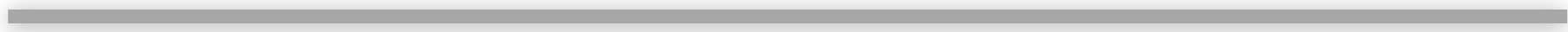
$$\mathcal{F}_2(P, q) = \int d^3R \int d^3r e^{i\vec{q}\vec{r}} \times \\ f_1^W\left(\vec{P}, \vec{R} + \frac{\vec{r}}{2}; t_f\right) f_1^W\left(\vec{P}, \vec{R} - \frac{\vec{r}}{2}; t_f\right)$$

$$\mathcal{A}_2(p_1, p_2) = \int d^3x f_1^W(\vec{p}_1, \vec{x}; t_f) \int d^3x f_1^W(\vec{p}_2, \vec{x}; t_f)$$



$$\frac{dN_d}{d^3P_d} \approx G_d \int d^3q \mathcal{D}(\vec{q}) \int d^3R \int d^3r e^{i\vec{q}\vec{r}} \times$$

$$f_1^W \left(\frac{\vec{P}_d}{2}, \vec{R} + \frac{\vec{r}}{2}; t_f \right) f_1^W \left(\frac{\vec{P}_d}{2}, \vec{R} - \frac{\vec{r}}{2}; t_f \right)$$

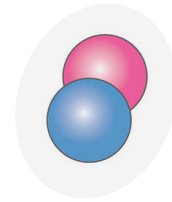


$$\frac{dN^s}{d^3p_1 d^3p_2} = G_2^s (\mathcal{A}_2(p_1, p_2) - \mathcal{F}_2(P, q)),$$

$$\mathcal{F}_2(P, q) = \int d^3R \int d^3r e^{i\vec{q}\vec{r}} \times$$

$$f_1^W \left(\vec{P}, \vec{R} + \frac{\vec{r}}{2}; t_f \right) f_1^W \left(\vec{P}, \vec{R} - \frac{\vec{r}}{2}; t_f \right)$$





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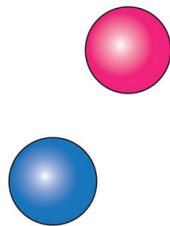
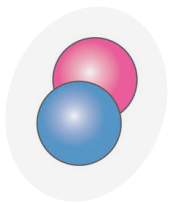
$$f_1^W \left(\frac{\vec{P}_d}{2}, \vec{R} + \frac{\vec{r}}{2}; t_f \right) f_1^W \left(\frac{\vec{P}_d}{2}, \vec{R} - \frac{\vec{r}}{2}; t_f \right)$$

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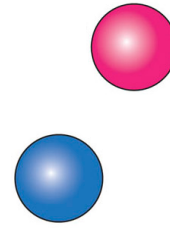
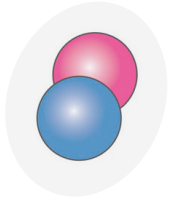
$$\mathcal{F}_2(P, q) = \int d^3R \int d^3r e^{i\vec{q}\vec{r}} \times$$

$$f_1^W \left(\vec{P}, \vec{R} + \frac{\vec{r}}{2}; t_f \right) f_1^W \left(\vec{P}, \vec{R} - \frac{\vec{r}}{2}; t_f \right)$$





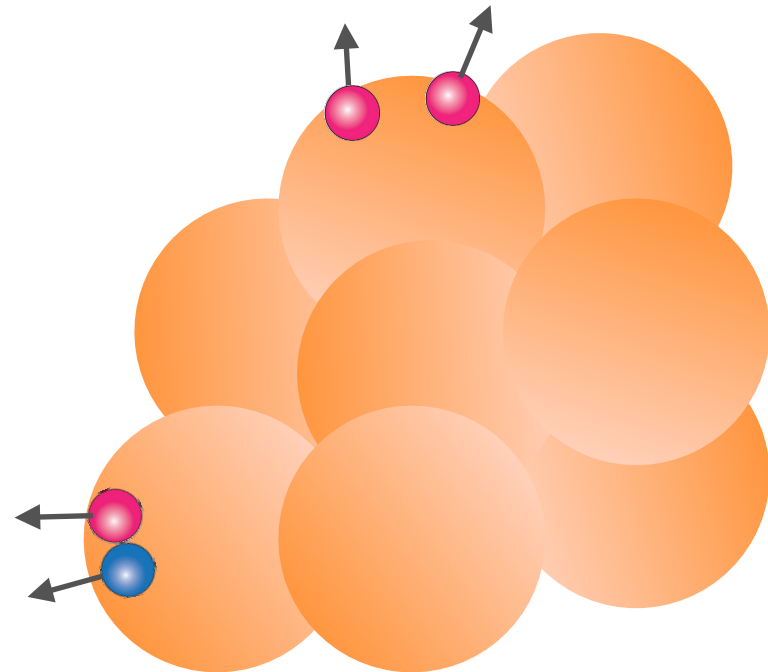
$$\frac{d}{d^3R} \left(\frac{dN_d}{d^3P_d} \right) \approx G_d \frac{d}{d^3R} \int d^3q \mathcal{D}(\vec{q}) \mathcal{F}_2 \left(\frac{\vec{P}_d}{2}, \vec{q} \right)$$

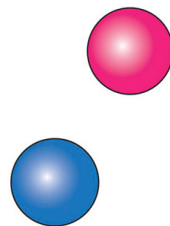
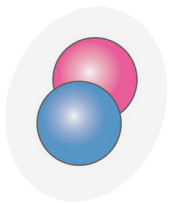


$$\underline{\frac{d}{d^3 R}} \left(\frac{dN_d}{d^3 P_d} \right) \approx G_d \underline{\frac{d}{d^3 R}} \int d^3 q \mathcal{D}(\vec{q}) \mathcal{F}_2 \left(\frac{\vec{P}_d}{2}, \vec{q} \right)$$

relativistic flow:

$$\gamma_d \int d^3 R f_d \rightarrow (1/2m) \int \left[d^3 \sigma_\mu P_d^\mu \right] f_d$$

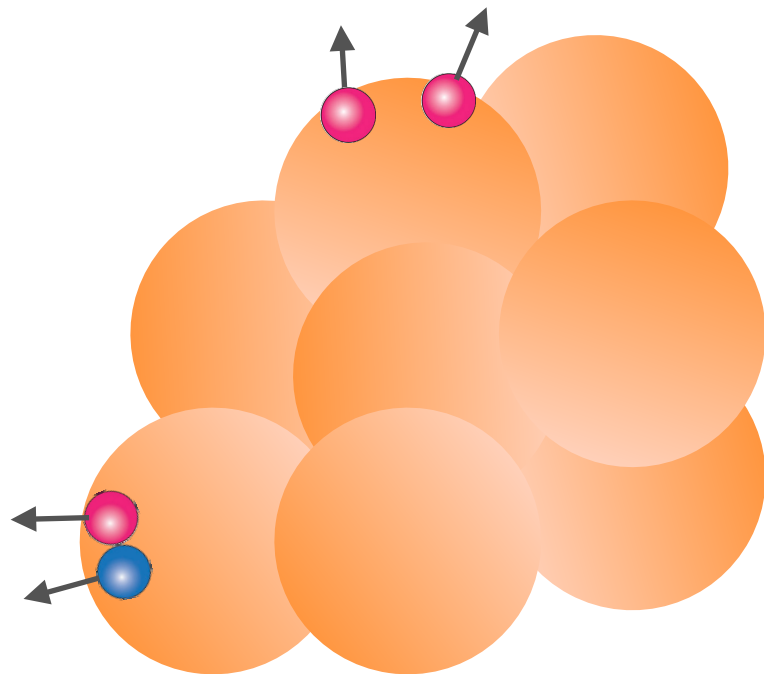




$$\left(\frac{dN_d}{d^3 P_d} \right) \approx G_d \int d^3 q \mathcal{D}(\vec{q}) \mathcal{F}_2 \left(\frac{\vec{P}_d}{2}, \vec{q} \right)$$

relativistic flow:

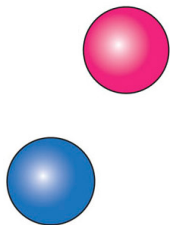
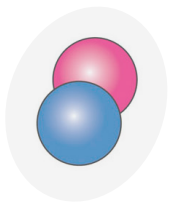
$$\gamma_d \int d^3 R f_d \rightarrow (1/2m) \int \left[d^3 \sigma_\mu P_d^\mu \right] f_d$$





$$\left(\frac{dN_d}{d^3P_d}\right) \approx G_d \int d^3q \, \mathcal{D}(\vec{q}) \, \mathcal{F}_2\left(\frac{\vec{P}_d}{2}, \vec{q}\right)$$

$$\mathcal{B}_2(p) = \frac{P_d^0 \frac{dN_d}{d^3P_d}}{\left(p^0 \frac{dN}{d^3p}\right)^2} \qquad C_2(P,q) = \frac{p_1^0 p_2^0 \frac{dN}{d^3p_1 d^3p_2}}{\left(p_1^0 \frac{dN}{d^3p_1}\right) \left(p_2^0 \frac{dN}{d^3p_2}\right)}$$



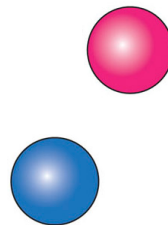
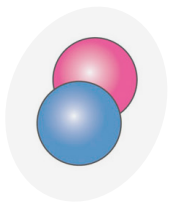
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$$\mathcal{B}_2(p) \; = \; \frac{P_d^0 \; \frac{dN_d}{d^3P_d}}{\left(p^0 \; \frac{dN}{d^3p}\right)^2}$$

$$C_2(P,q) \; = \; 1 - \frac{G_2^s - G_2^a}{G_2^s + G_2^a} \mathcal{C}_2(P,q)$$

$$\frac{G_d}{G_2^s+G_2^a} \; = \; \frac{3}{3+1}$$

$$\mathcal{C}_2^{\text{PRF}}\left(|\vec{q}|\ll m\right) \; = \; \frac{\mathcal{F}_2}{\mathcal{A}_2}$$



$$\mathcal{B}_2(p) \approx \frac{3}{2m} \int d^3q \mathcal{D}(\vec{q}) \mathcal{C}_2^{\text{PRF}}(\vec{p}, \vec{q})$$

$$\mathcal{B}_2(p) = \frac{P_d^0 \frac{dN_d}{d^3P_d}}{\left(p^0 \frac{dN}{d^3p}\right)^2}$$

$$C_2(P, q) = 1 - \frac{G_2^s - G_2^a}{G_2^s + G_2^a} C_2(P, q)$$

$$\frac{G_d}{G_2^s + G_2^a} = \frac{3}{3 + 1}$$

$$\mathcal{C}_2^{\text{PRF}}(|\vec{q}| \ll m) = \frac{\mathcal{F}_2}{\mathcal{A}_2}$$

Coalescence from correlation functions:

$$\mathcal{B}_2(p) \approx \frac{3}{2m} \int d^3q \mathcal{D}(\vec{q}) \mathcal{C}_2^{\text{PRF}}(\vec{p}, \vec{q})$$

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R. Scheibl, U. Heinz, Phys.Rev. C59 (1999) 1585-1602

$$\frac{\mathcal{B}_A}{m^{2(A-1)}} \approx \frac{2J_A + 1}{2^A \sqrt{A}} \left(\frac{m R}{\sqrt{2\pi}} \right)^{3(1-A)}$$

KB et al, Phys.Rev. D96 (2017) no.10, 103021

KB, M. Takimoto, Phys.Rev. C99 (2019) no.4, 044913

Coalescence from correlation functions:

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R. Scheibl, U. Heinz, Phys.Rev. C59 (1999) 1585-1602

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R. Scheibl, U. Heinz, Phys.Rev. C59 (1999) 1585-1602

assumed: hydro model

$$\begin{aligned} \eta_l(\tau, \eta, \rho) &= \eta & \eta_t(\tau, \eta, \rho) &= \eta_f \left(\frac{\rho}{\Delta\rho} \right)^\alpha \\ f_i(R, P) &= e^{\mu_i/T} e^{-P \cdot u(R)/T} H(R), \quad i = \text{p, n} \\ H(R) &= H(\eta, \rho) = \exp \left(-\frac{\rho^2}{2(\Delta\rho)^2} - \frac{\eta^2}{2(\Delta\eta)^2} \right) \end{aligned}$$

Hydrodynamic flow in small systems

or: “How the heck is it possible that a system emitting only a dozen particles can be described by fluid dynamics?”

Ulrich Heinz^{1a}, in collaboration with J. Scott Moreland^b

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^bDepartment of Physics, Duke University, Durham, NC 27708-0305, USA

E-mail: heinz.9@osu.edu

Abstract. The “unreasonable effectiveness” of relativistic fluid dynamics in describing high energy heavy-ion and even proton-proton collisions will be demonstrated and discussed. Several recent ideas of optimizing relativistic fluid dynamics for the specific challenges posed by such collisions will be presented, and some thoughts will be offered why the framework works better than originally expected. I will also address the unresolved question where exactly hydrodynamics breaks down, and why.

R. Scheibl, U. Heinz, Phys.Rev. C59 (1999) 1585-1602

assumed: hydro model

$$\eta_l(\tau, \eta, \rho) = \eta \quad \eta_t(\tau, \eta, \rho) = \eta_f \left(\frac{\rho}{\Delta\rho} \right)^\alpha$$

$$f_i(R, P) = e^{\mu_i/T} e^{-P \cdot u(R)/T} H(R), \quad i = p, n$$

$$H(R) = H(\eta, \rho) = \exp \left(-\frac{\rho^2}{2(\Delta\rho)^2} - \frac{\eta^2}{2(\Delta\eta)^2} \right)$$

Coalescence from correlation functions:

$$\mathcal{B}_2(p) \approx \frac{3}{2m} \int d^3q \mathcal{D}(\vec{q}) \mathcal{C}_2^{\text{PRF}}(\vec{p}, \vec{q})$$

R. Scheibl, U. Heinz, Phys.Rev. C59 (1999) 1585-1602

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$$H(R) = H(\eta, \rho) = \exp \left(-\frac{\rho^2}{2(\Delta\rho)^2} - \frac{\eta^2}{2(\Delta\eta)^2} \right)$$

hydro model gives Gaussian source:

Chapman, Nix, Heinz, Phys.Rev. C52 (1995) 2694-2703

$$\mathcal{C}_2^{\text{PRF}} = e^{-R_\perp^2 \vec{q}_\perp^2 - R_\parallel^2 \vec{q}_\parallel^2}$$


$$\mathcal{R}_\perp(m_t) = \frac{\Delta\rho}{\sqrt{1 + \frac{m_t}{T} \eta_f^2}},$$

$$\mathcal{R}_\parallel(m_t) = \frac{\tau_0 \Delta\eta}{\sqrt{1 + \frac{m_t}{T} (\Delta\eta)^2}}.$$

Coalescence from correlation functions:

$$\mathcal{B}_2(p) \approx \frac{3}{2m} \int d^3q \mathcal{D}(\vec{q}) \mathcal{C}_2^{\text{PRF}}(\vec{p}, \vec{q})$$


R. Scheibl, U. Heinz, Phys.Rev. C59 (1999) 1585-1602


assumed: Gaussian Source Model (GSM)  $\mathcal{C}_2^{\text{PRF}} = e^{-R_\perp^2 \vec{q}_\perp^2 - R_\parallel^2 \vec{q}_\parallel^2}$

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R. Scheibl, U. Heinz, Phys.Rev. C59 (1999) 1585-1602

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assumed: $\phi_d(\vec{r}) = \frac{e^{-\frac{\vec{r}^2}{2d^2}}}{(\pi d^2)^{\frac{3}{4}}}$  $\mathcal{D}(\vec{k}) = e^{-\frac{\vec{k}^2 d^2}{4}}$

Coalescence from correlation functions:

With these
assumptions:

$$\mathcal{B}_2 = \frac{3\pi^{\frac{3}{2}}}{2m \left(R_{\perp}^2 + \left(\frac{d}{2}\right)^2 \right) \sqrt{R_{\parallel}^2 + \left(\frac{d}{2}\right)^2}}$$

R. Scheibl, U. Heinz, Phys.Rev. C59 (1999) 1585-1602

assumed: Gaussian Source Model (GSM)



$$\mathcal{C}_2^{\text{PRF}} = e^{-R_{\perp}^2 \vec{q}_{\perp}^2 - R_{\parallel}^2 \vec{q}_{\parallel}^2}$$

assumed:

$$\phi_d(\vec{r}) = \frac{e^{-\frac{\vec{r}^2}{2d^2}}}{(\pi d^2)^{\frac{3}{4}}}$$



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R. Scheibl, U. Heinz, Phys.Rev. C59 (1999) 1585-1602



$$B_2 = \frac{3\pi^{3/2} \langle \mathcal{C}_d \rangle}{2m_t \mathcal{R}_{\perp}^2(m_t) \mathcal{R}_{\parallel}(m_t)} . \quad (6.3)$$

$$\langle \mathcal{C}_d \rangle \approx \frac{1}{\left(1 + \left(\frac{d}{2\mathcal{R}_{\perp}(m)} \right)^2 \right) \sqrt{1 + \left(\frac{d}{2\mathcal{R}_{\parallel}(m)} \right)^2}} . \quad (4.12)$$

Coalescence from correlation functions:

With these assumptions:

$$\mathcal{B}_2 = \frac{3\pi^{\frac{3}{2}}}{2m \left(R_{\perp}^2 + \left(\frac{d}{2} \right)^2 \right) \sqrt{R_{\parallel}^2 + \left(\frac{d}{2} \right)^2}}$$

Pair rest frame

R. Scheibl, U. Heinz, Phys.Rev. C59 (1999) 1585-1602



$$B_2 = \frac{3\pi^{3/2} \langle \mathcal{C}_d \rangle}{2m_t \mathcal{R}_{\perp}^2(m_t) \mathcal{R}_{\parallel}(m_t)} \quad (6.3)$$

YKP frame

$$\langle \mathcal{C}_d \rangle \approx \frac{1}{\left(1 + \left(\frac{d}{2\mathcal{R}_{\perp}(m)} \right)^2 \right) \sqrt{1 + \left(\frac{d}{2\mathcal{R}_{\parallel}(m)} \right)^2}} \quad (4.12)$$

Coalescence from correlation functions:

$$\mathcal{B}_2 = \frac{3\pi^{\frac{3}{2}}}{2m \left(R_{\perp}^2 + \left(\frac{d}{2} \right)^2 \right) \sqrt{R_{\parallel}^2 + \left(\frac{d}{2} \right)^2}}$$

S. Mrowczynski, Acta Phys.Polon. B48 (2017) 707

Coalescence from correlation functions:

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S. Mrowczynski, Acta Phys.Polon. B48 (2017) 707

assumed: 1D GSM

$$D(\mathbf{r}) = \frac{e^{-\frac{\mathbf{r}^2}{4R_{\text{kin}}^2}}}{(4\pi R_{\text{kin}}^2)^{3/2}} \quad \Rightarrow \quad C_2^{\text{PRF}} = e^{-R_{\text{kin}}^2 q^2}$$

obtained:

$$\frac{dN_d}{d^3\mathbf{p}} = \mathcal{A} \frac{dN_p}{d^3\left(\frac{1}{2}\mathbf{p}\right)} \frac{dN_n}{d^3\left(\frac{1}{2}\mathbf{p}\right)}, \quad \mathcal{A} = \frac{3}{4} \frac{\pi^{3/2}}{(R_{\text{kin}}^2 + R_d^2)^{3/2}}$$

Coalescence from correlation functions:

$$\mathcal{B}_2 = \frac{3\pi^{\frac{3}{2}}}{2m \left(R_{\perp}^2 + \left(\frac{d}{2} \right)^2 \right) \sqrt{R_{\parallel}^2 + \left(\frac{d}{2} \right)^2}}$$

S. Mrowczynski, Acta Phys.Polon. B48 (2017) 707



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Coalescence from correlation functions:

$$\mathcal{B}_2 = \frac{3\pi^{\frac{3}{2}}}{2m \left(R_{\perp}^2 + \left(\frac{d}{2}\right)^2 \right) \sqrt{R_{\parallel}^2 + \left(\frac{d}{2}\right)^2}}$$

S. Mrowczynski, Acta Phys.Polon. B48 (2017) 707



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obtained: $\frac{dN_d}{d^3\mathbf{p}} = \mathcal{A} \frac{dN_p}{d^3(\frac{1}{2}\mathbf{p})} \frac{dN_n}{d^3(\frac{1}{2}\mathbf{p})}, \quad \mathcal{A} = \frac{3}{4} \frac{\pi^{3/2}}{(R_{\text{kin}}^2 + R_d^2)^{3/2}} = \boxed{\gamma_d} \frac{m}{2} B_2$

$\gamma_d \rightarrow 1$



Coalescence from correlation functions:

$$\mathcal{B}_2(p) \approx \frac{3}{2m} \int d^3q \mathcal{D}(\vec{q}) \mathcal{C}_2^{\text{PRF}}(\vec{p}, \vec{q})$$

This formula is naive, of course.

Coalescence from correlation functions:

$$\mathcal{B}_2(p) \approx \frac{3}{2m} \int d^3q \mathcal{D}(\vec{q}) \mathcal{C}_2^{\text{PRF}}(\vec{p}, \vec{q})$$

$$C_2(P, q) = 1 - \frac{G_2^s - G_2^a}{G_2^s + G_2^a} C_2(P, q)$$

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$$C_2(P, q) = 1 - \frac{G_2^s - G_2^a}{G_2^s + G_2^a} C_2(P, q)$$

Final-state interactions (Coulomb and strong) distort the correlation function.

They distort it differently in pp state and in pn state,
and they distort it differently in spin-symmetric/antisymmetric states.

Coalescence from correlation functions:

$$\mathcal{B}_2(p) \approx \frac{3}{2m} \int d^3q \mathcal{D}(\vec{q}) \mathcal{C}_2^{\text{PRF}}(\vec{p}, \vec{q})$$

$$C_2(P, q) = 1 - \frac{G_2^s - G_2^a}{G_2^s + G_2^a} C_2(P, q)$$

We don't have any better idea, than to keep the experimental analysis that mods-out the final-state interactions to reconstruct underlying correlation.

Coalescence from correlation functions:

$$\mathcal{B}_2(p) \approx \frac{3}{2m} \int d^3q \mathcal{D}(\vec{q}) \mathcal{C}_2^{\text{PRF}}(\vec{p}, \vec{q})$$

Gaussian source; chaoticity λ

$$\mathcal{C}_2^{\text{PRF}} = \lambda e^{-R_\perp^2 \vec{q}_\perp^2 - R_\parallel \vec{q}_l^2}$$

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Gaussian source; chaoticity λ

$$\mathcal{C}_2^{\text{PRF}} = \lambda e^{-R_\perp^2 \vec{q}_\perp^2 - R_\parallel \vec{q}_l^2}$$

$$\mathcal{B}_2 = \frac{3\pi^{\frac{3}{2}} \lambda}{2m \left(R_\perp^2 + \left(\frac{d}{2} \right)^2 \right) \sqrt{R_\parallel^2 + \left(\frac{d}{2} \right)^2}}$$

Coalescence from correlation functions:

$$\mathcal{B}_2(p) \approx \frac{3}{2m} \int d^3q \mathcal{D}(\vec{q}) \mathcal{C}_2^{\text{PRF}}(\vec{p}, \vec{q})$$

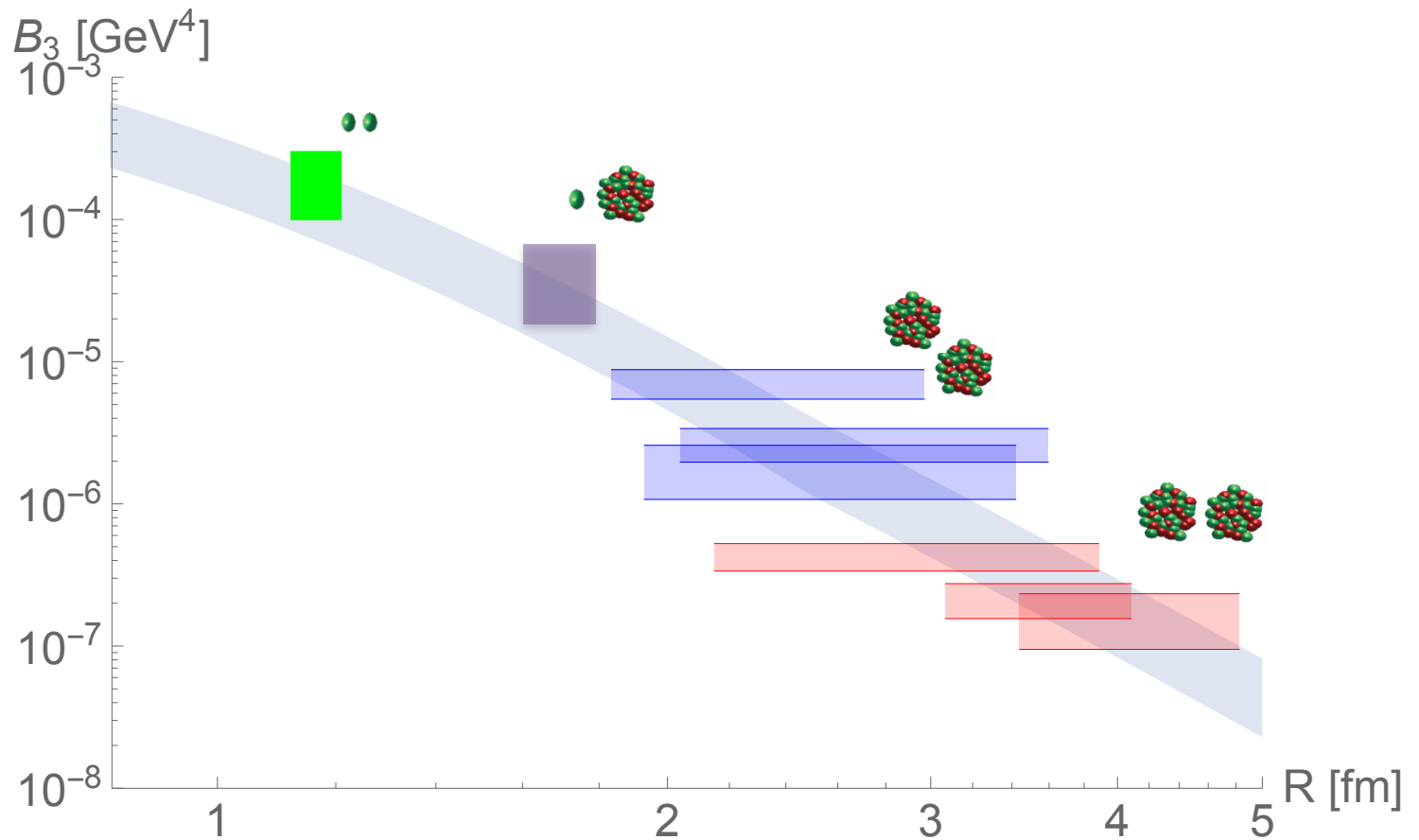
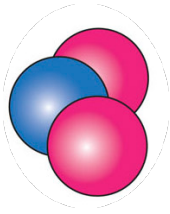
Gaussian source; chaoticity λ

$$\mathcal{C}_2^{\text{PRF}} = \lambda e^{-R_\perp^2 \vec{q}_\perp^2 - R_\parallel \vec{q}_l^2}$$

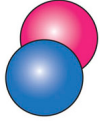
$$\frac{\mathcal{B}_A}{m^{2(A-1)}} = \lambda^{\frac{A}{2}} \frac{2J_A + 1}{2^A \sqrt{A}} \left[\frac{(2\pi)^{\frac{3}{2}}}{m^3 \left(R_\perp^2 + \left(\frac{d_A}{2} \right)^2 \right) \sqrt{R_\parallel^2 + \left(\frac{d_A}{2} \right)^2}} \right]^{A-1}$$

Nuclear coalescence from correlation functions

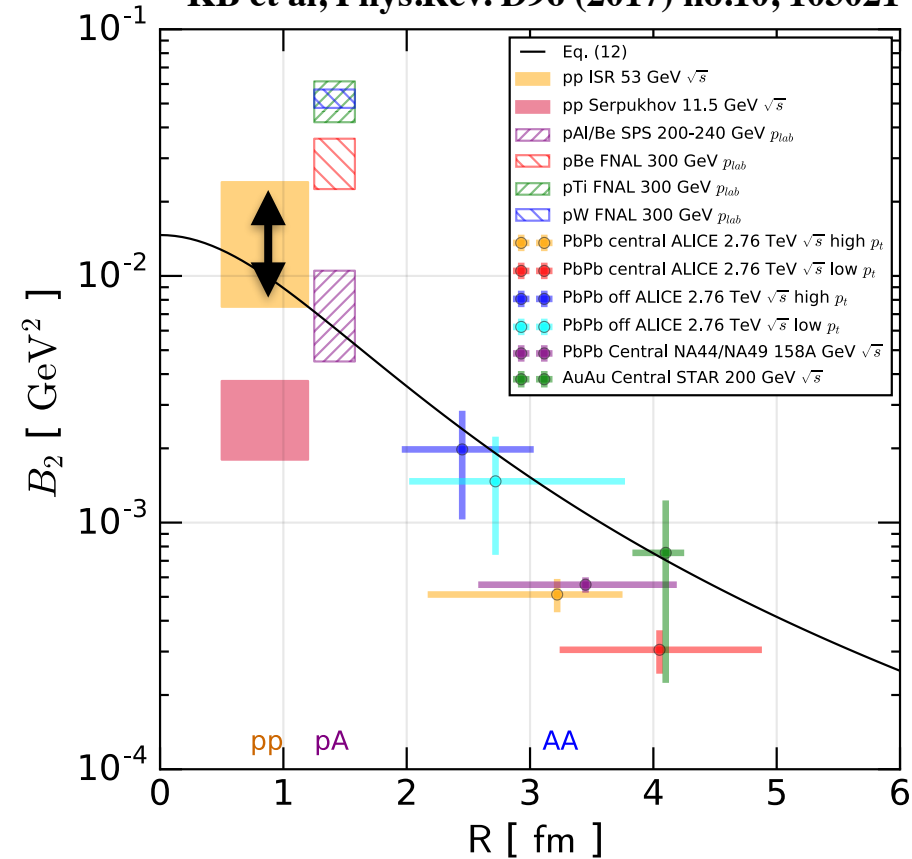
KB, M. Takimoto, Phys.Rev. C99 (2019) no.4, 044913



Xtra



KB et al, Phys.Rev. D96 (2017) no.10, 103021



ALICE, PRC97, 024615 (2018)

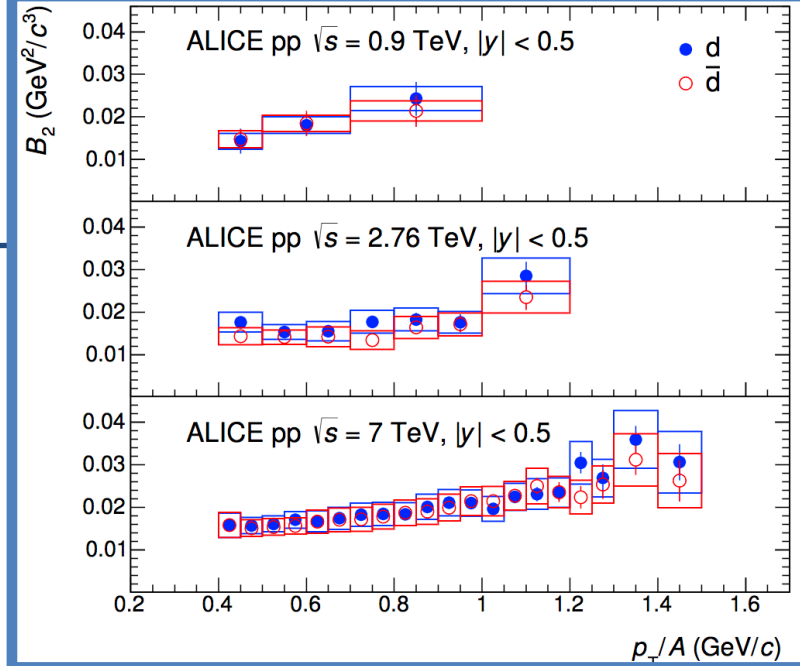


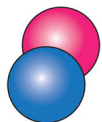
CERN-EP-2017-255
September 26, 2017

Production of deuterons, tritons, ³He nuclei and their anti-nuclei in pp collisions at $\sqrt{s} = 0.9, 2.76$ and 7 TeV

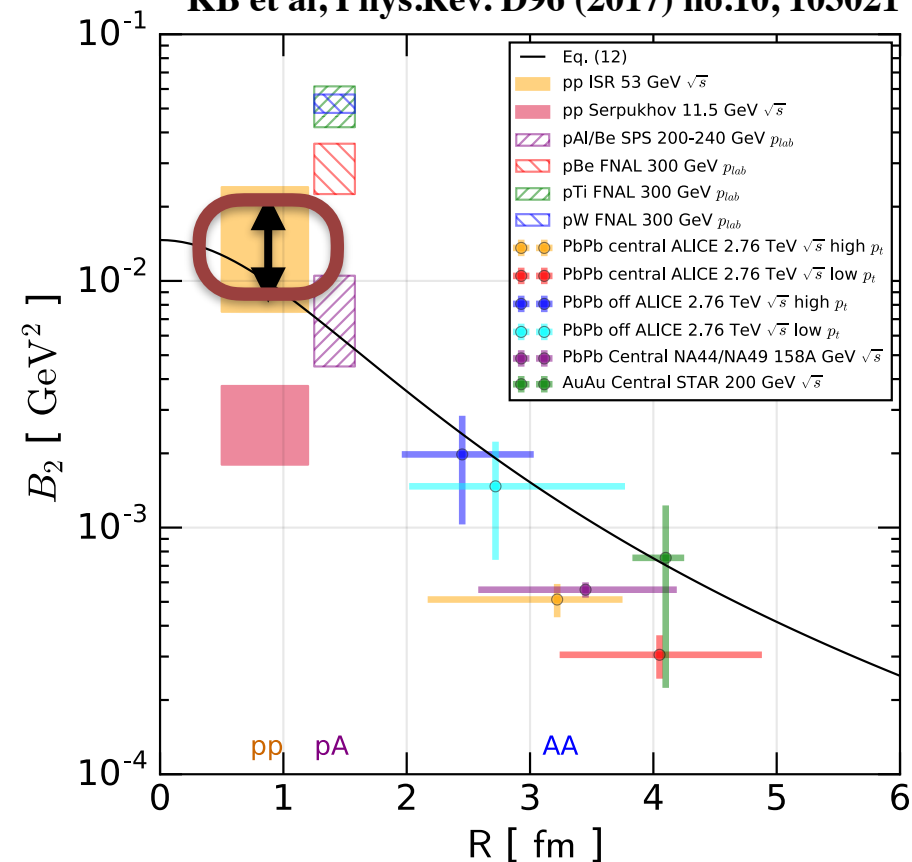
ALICE Collaboration

[nucl-ex] 25 Sep 2017





KB et al, Phys.Rev. D96 (2017) no.10, 103021



ALICE, PRC97, 024615 (2018)

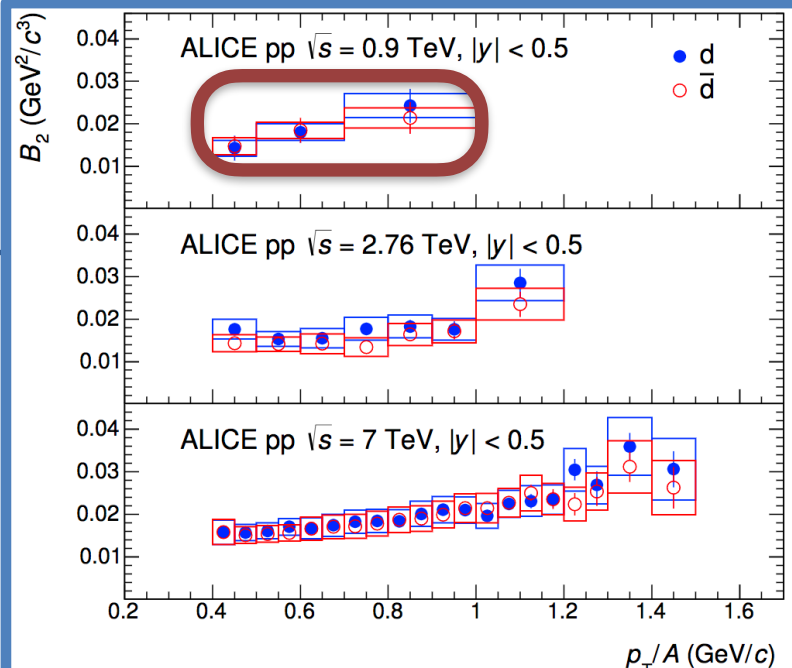


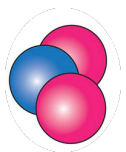
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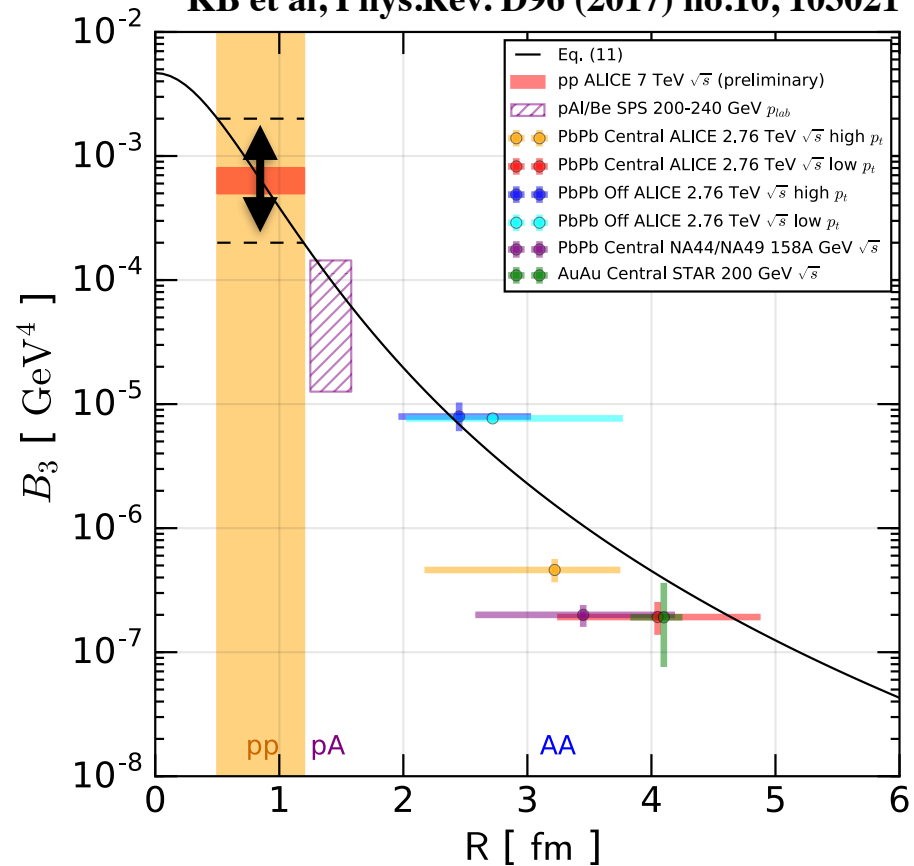
ALICE Collaboration

[nucl-ex] 25 Sep 2017





KB et al, Phys.Rev. D96 (2017) no.10, 103021



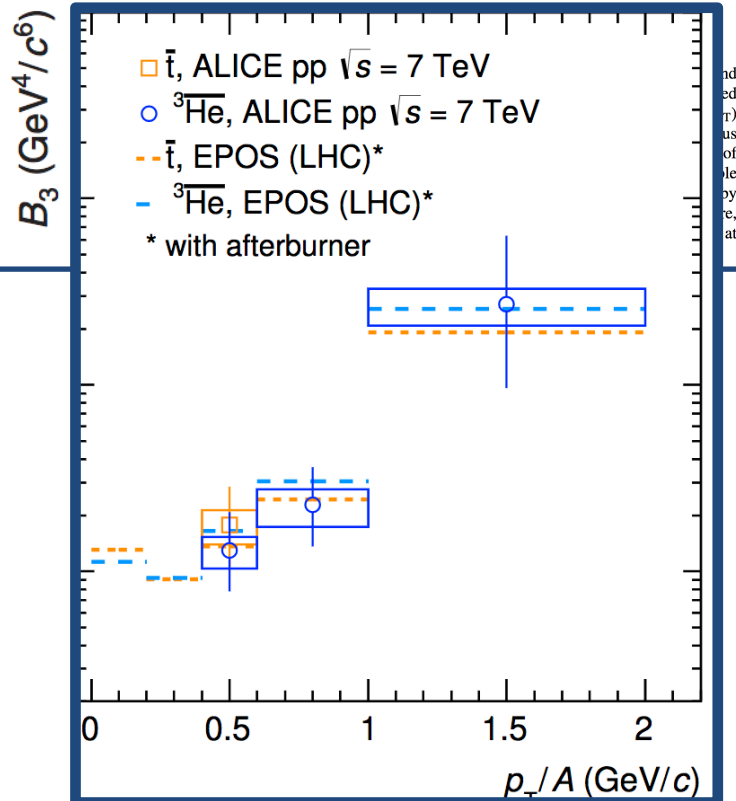
ALICE, PRC97, 024615 (2018)

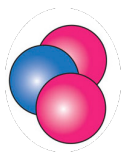


CERN-EP-2017-255
September 26, 2017

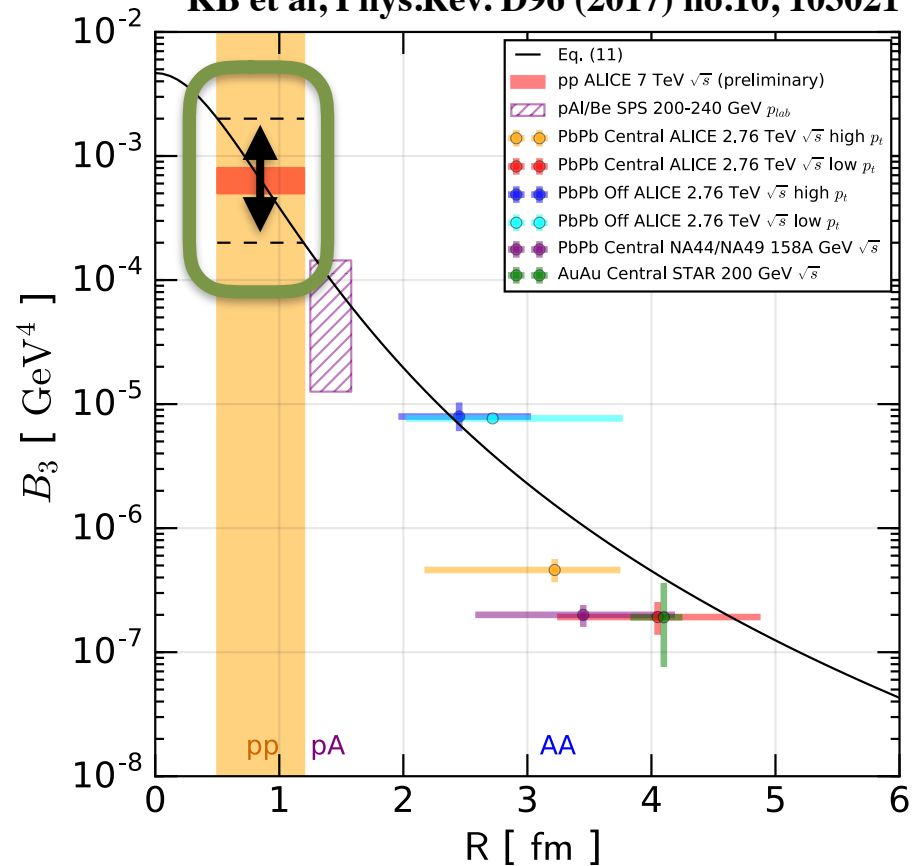
Production of deuterons, tritons, ³He nuclei and their anti-nuclei in pp collisions at $\sqrt{s} = 0.9, 2.76$ and 7 TeV

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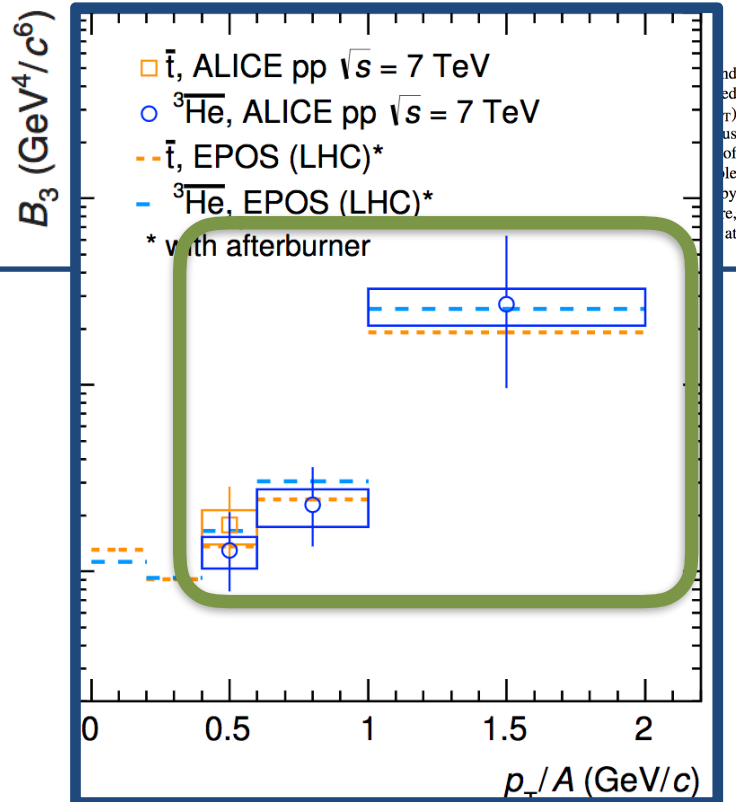
ALICE, PRC97, 024615 (2018)



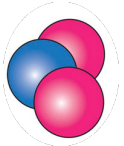
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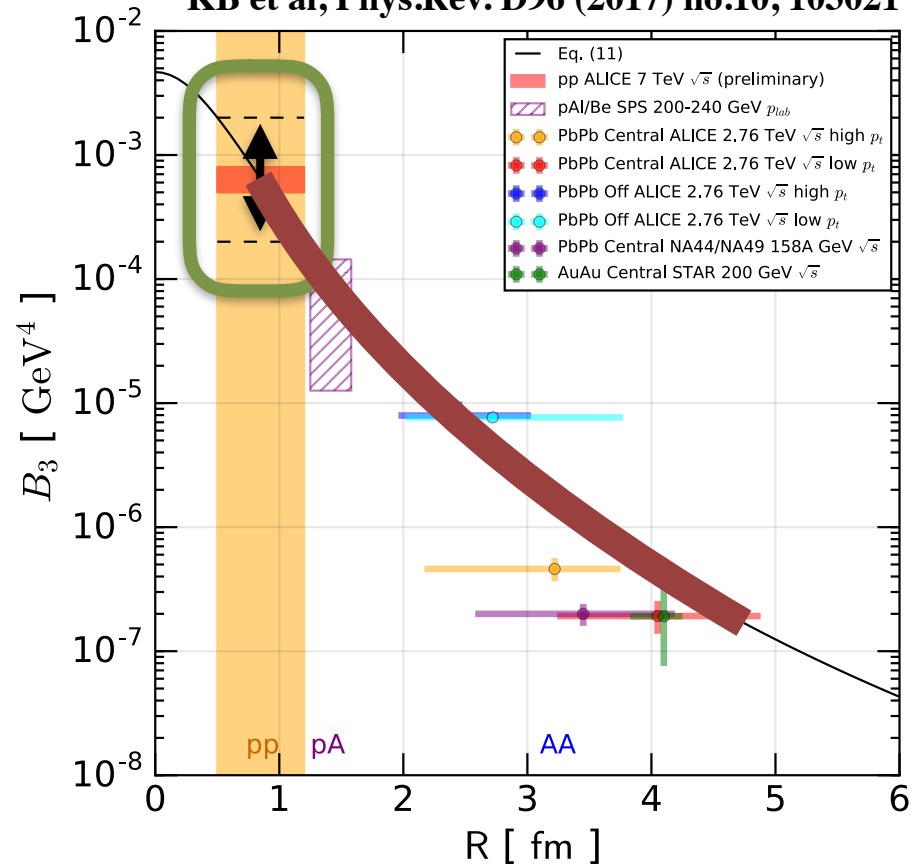
ALICE Collaboration



$$\frac{\mathcal{B}_A}{m^{2(A-1)}} \approx \frac{2J_A + 1}{2^A \sqrt{A}} \left(\frac{m R}{\sqrt{2\pi}} \right)^{3(1-A)}$$



KB et al, Phys.Rev. D96 (2017) no.10, 103021



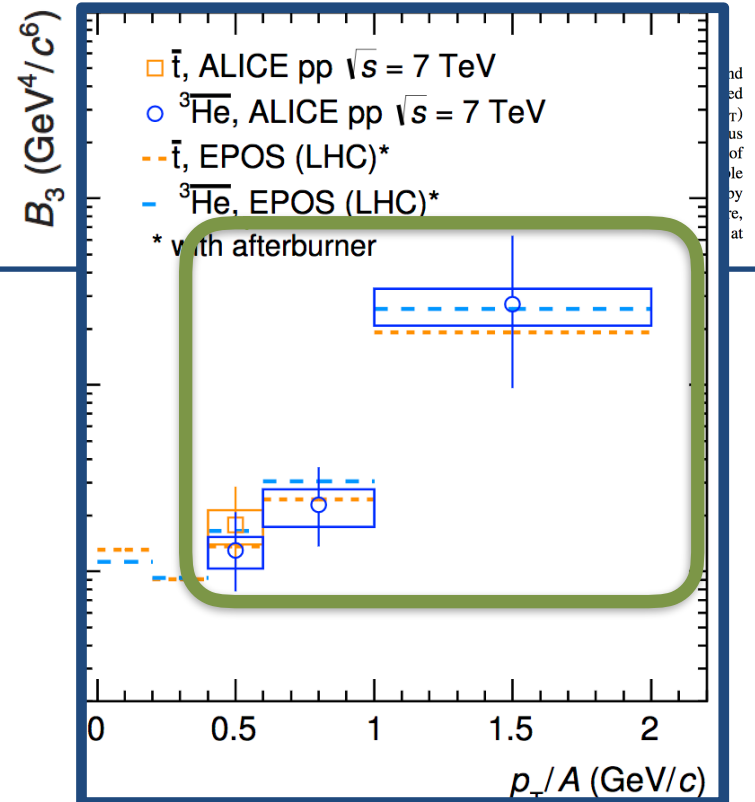
ALICE, PRC97, 024615 (2018)



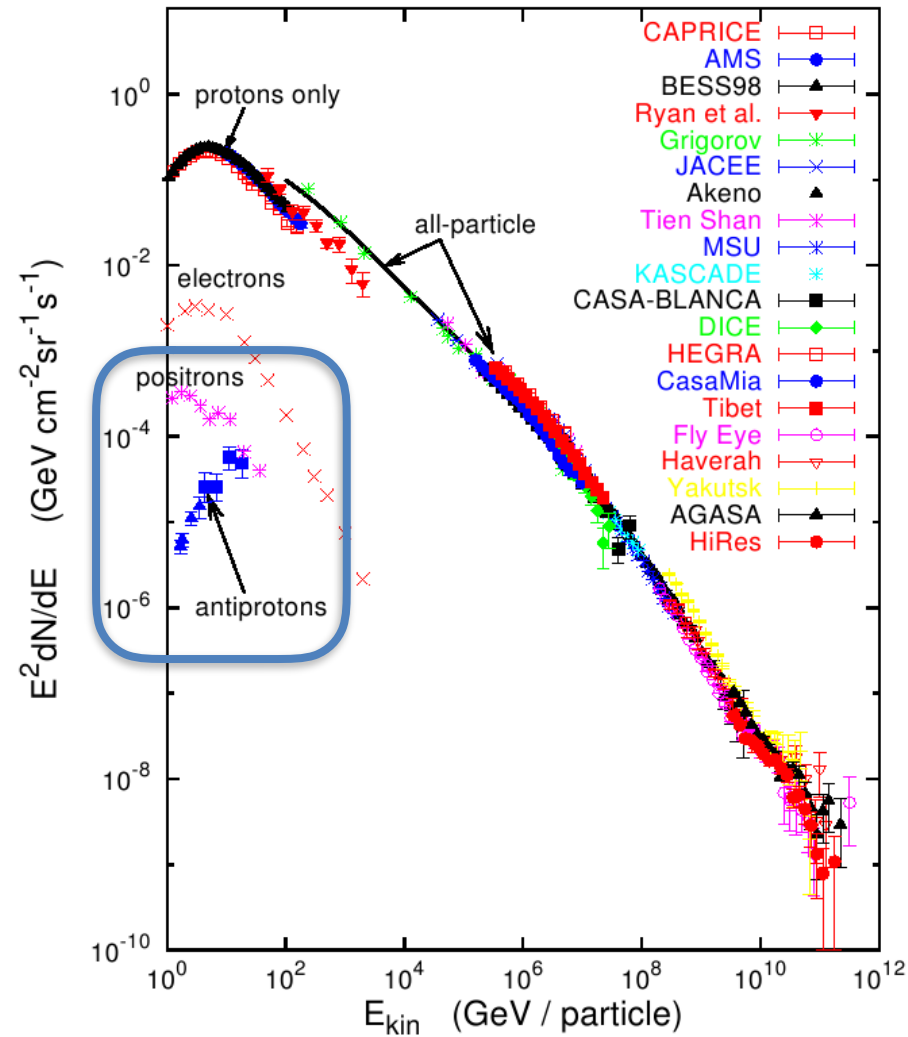
CERN-EP-2017-255
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Production of deuterons, tritons, ^3He nuclei and their anti-nuclei in pp collisions at $\sqrt{s} = 0.9, 2.76$ and 7 TeV

ALICE Collaboration



Cosmic Ray antimatter – \bar{p} , e^+ , \bar{d} , and $\overline{{}^3\text{He}}$ – long thought a smoking gun of exotic high-energy physics like dark matter annihilation



Hillas, astro-ph/0607109

anti He3

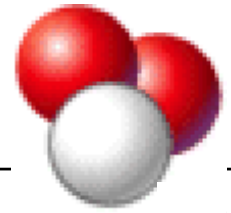
Handful of events?

AMS reports (unpublished):
2 anti-He4 candidates,
6 anti-He3 candidates.

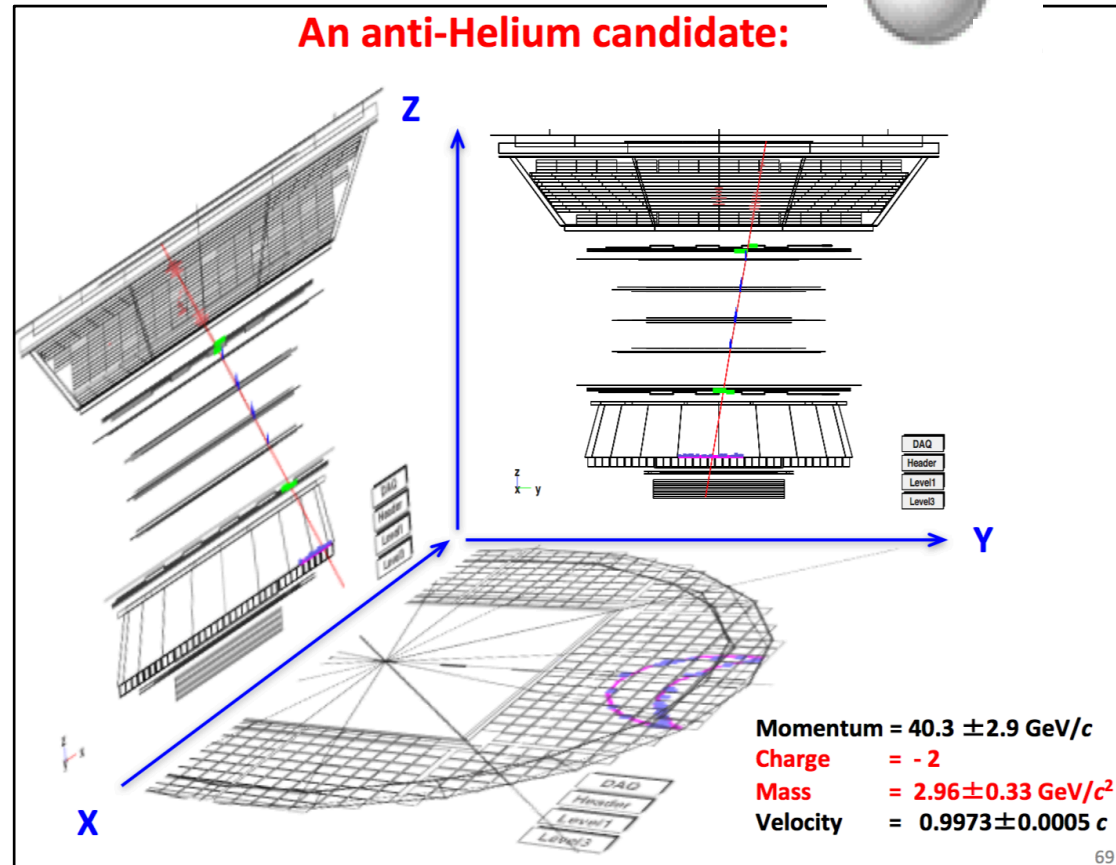
...cosmic rays, or background?
Need to reject background
at a level of $\sim 1:100\text{M}$...

Take it as motivation for theory
examination of astro flux.

AMS02, Dec 2016



An anti-Helium candidate:

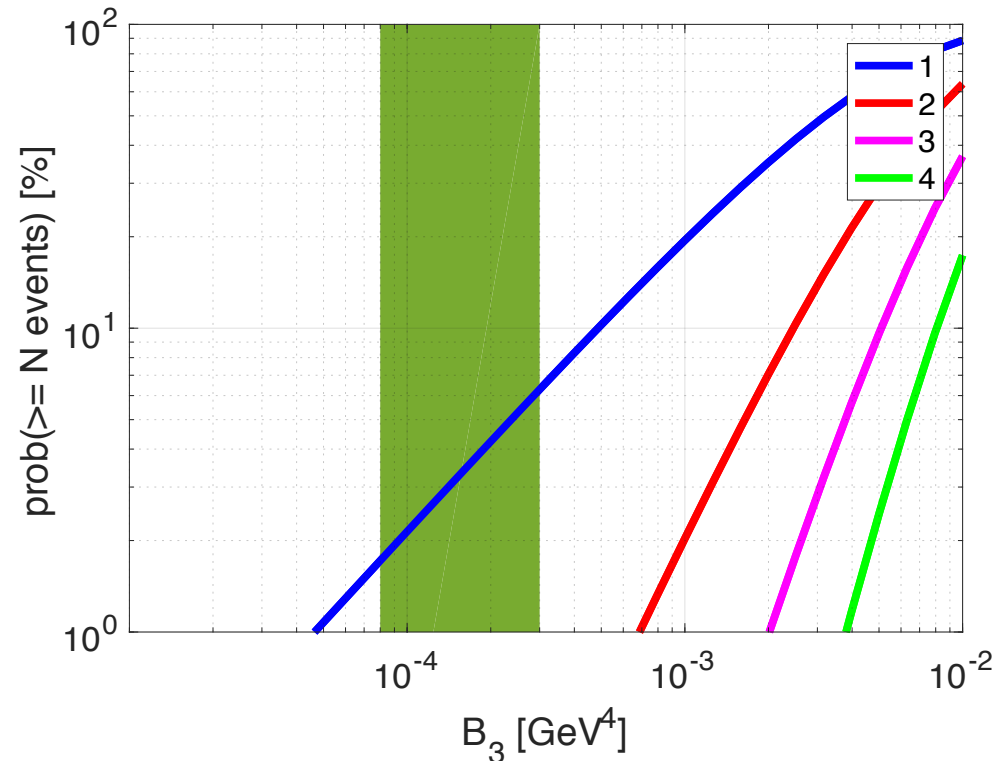


anti He3

The difficult part is to get the cross section right.

Coalescence ansatz:
$$E_A \frac{dN_A}{d^3p_A} = B_A R(x) \left(E_p \frac{dN_p}{d^3p_p} \right)^A$$

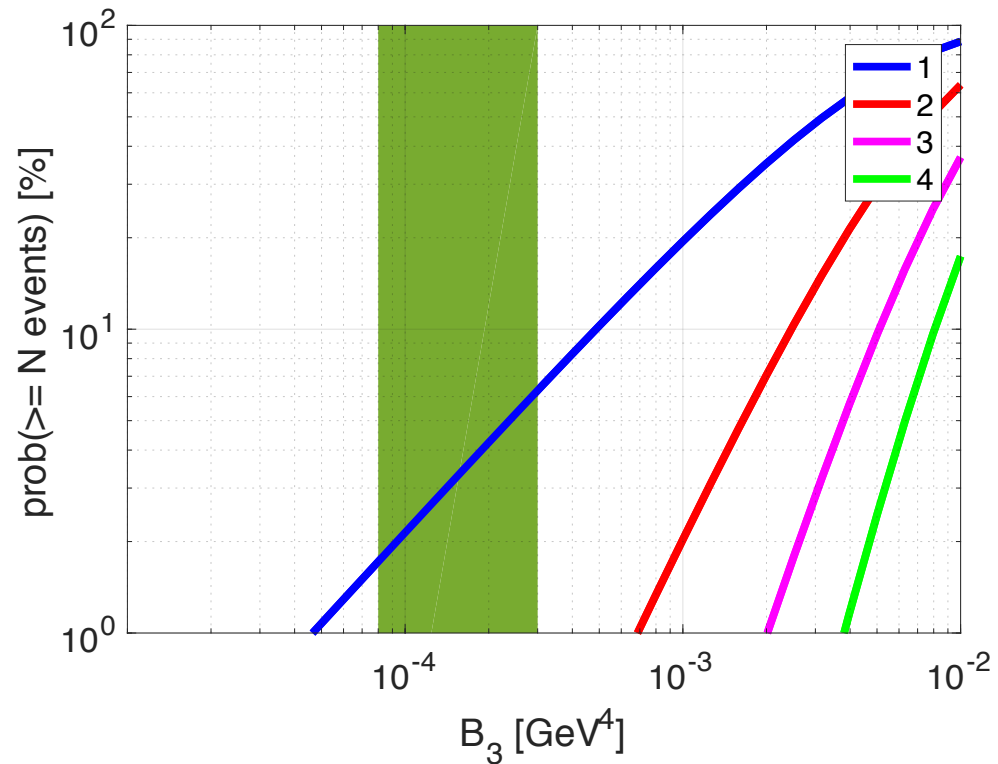
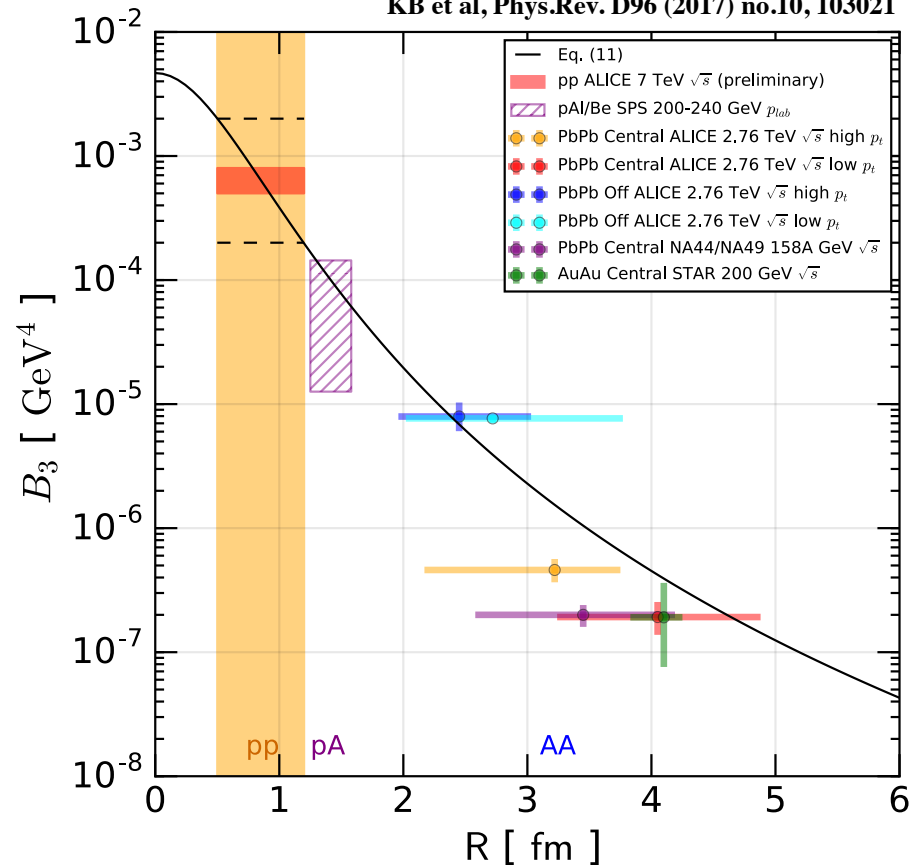
We need B_3



For pp we had no B_3



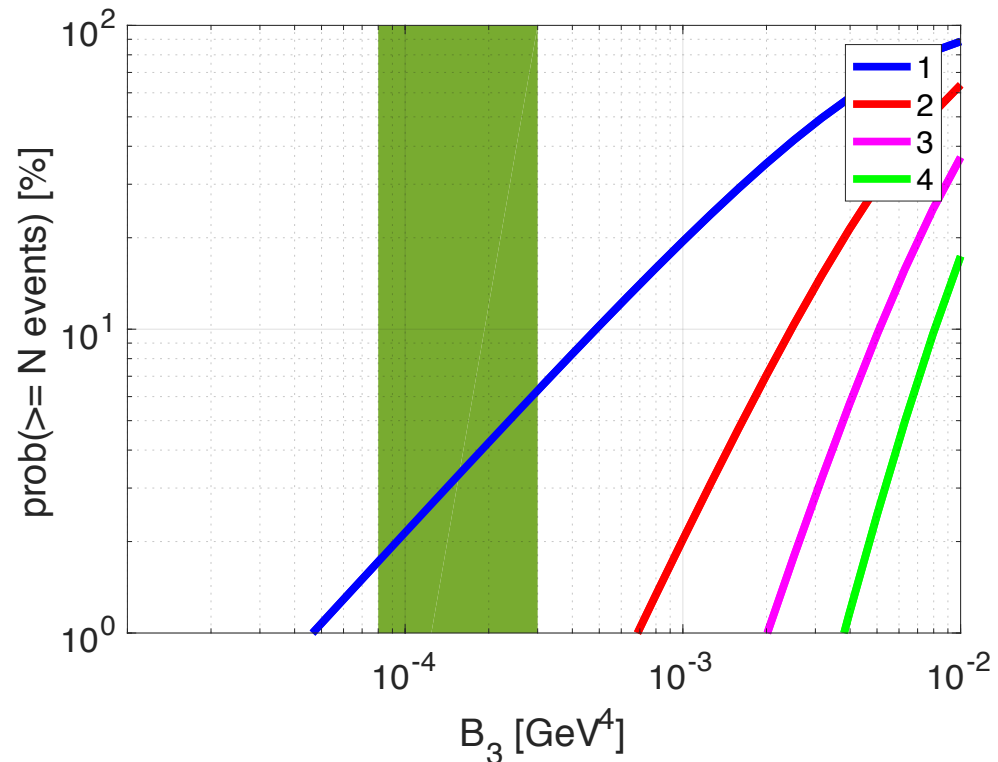
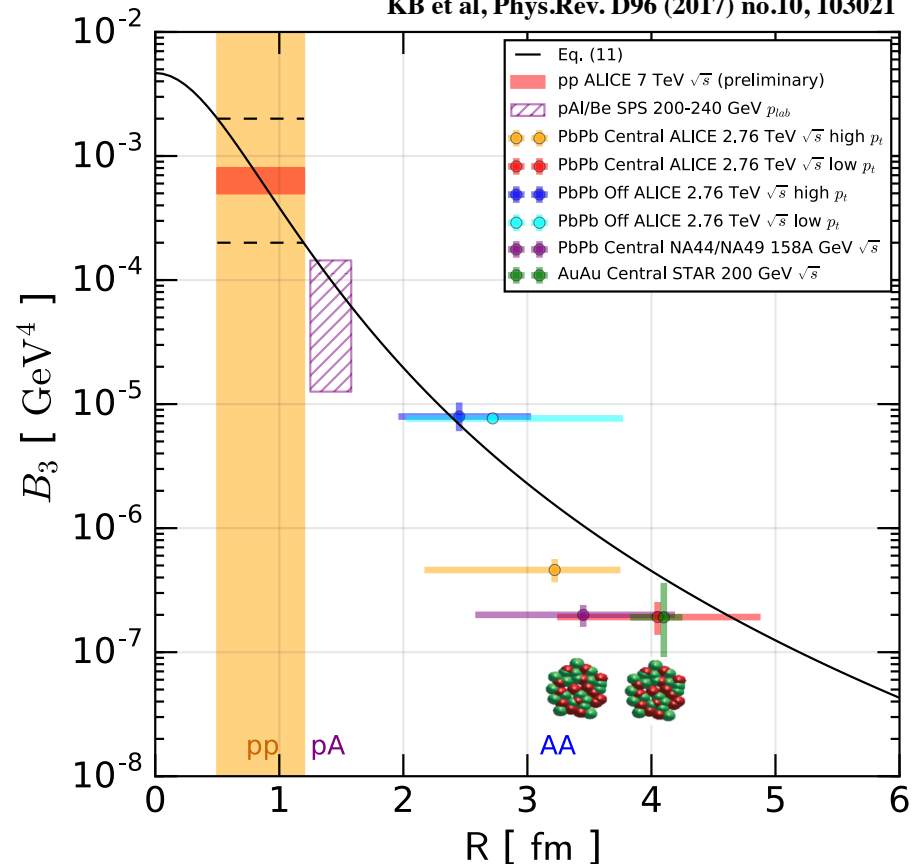
KB et al, Phys.Rev. D96 (2017) no.10, 103021



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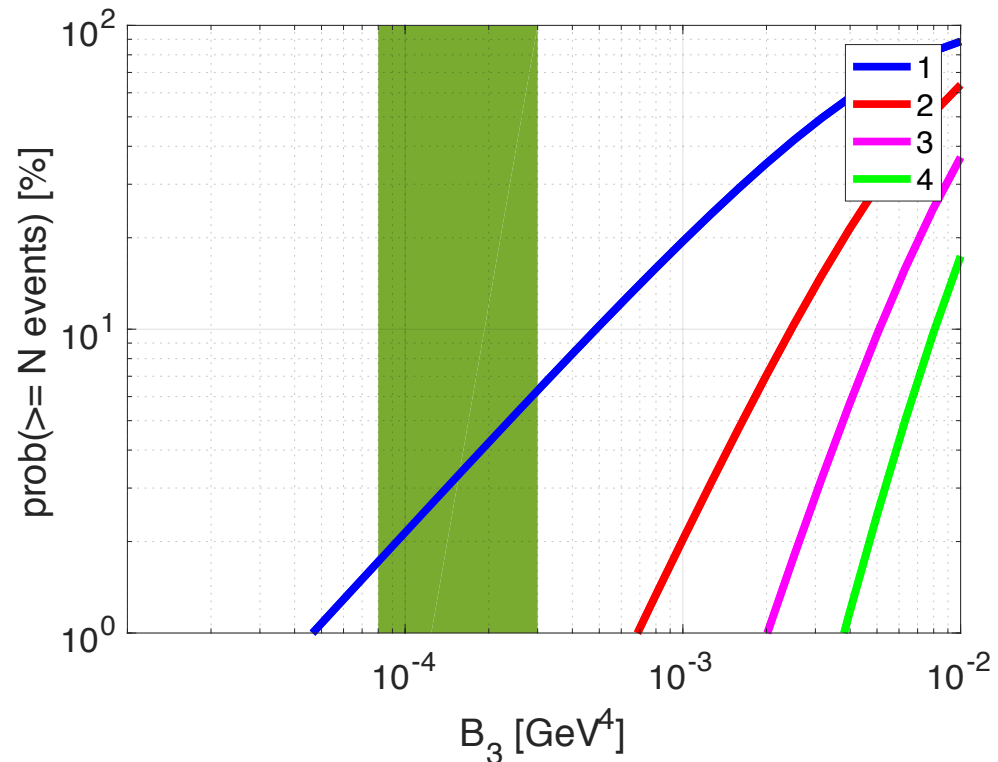
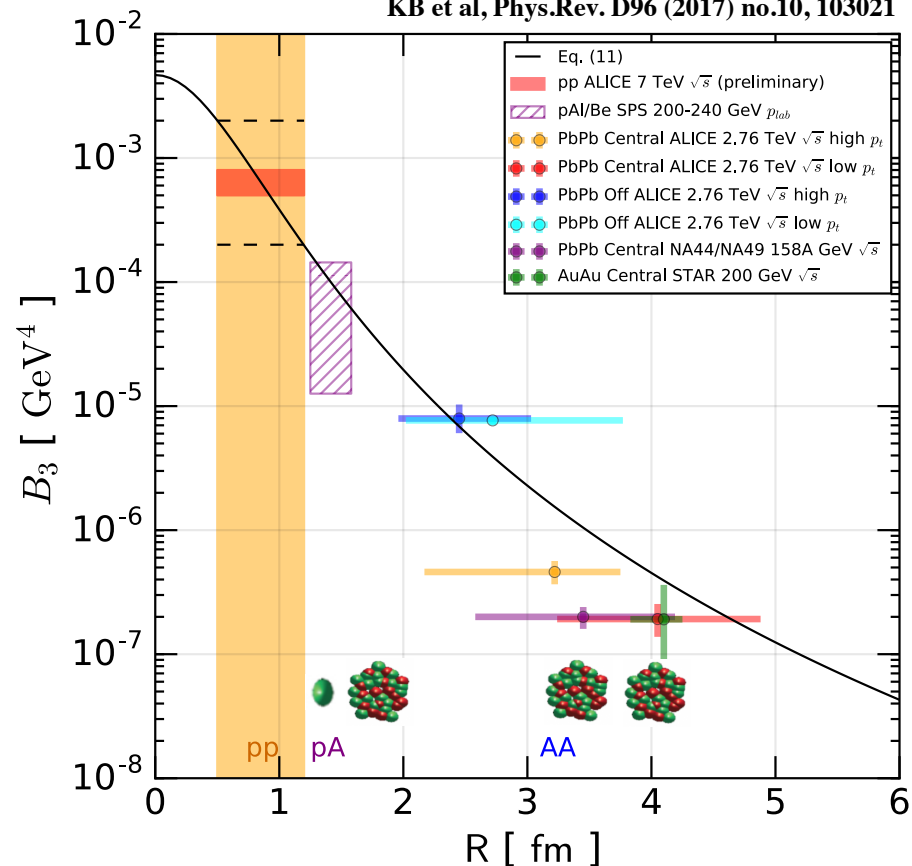
KB et al, Phys.Rev. D96 (2017) no.10, 103021



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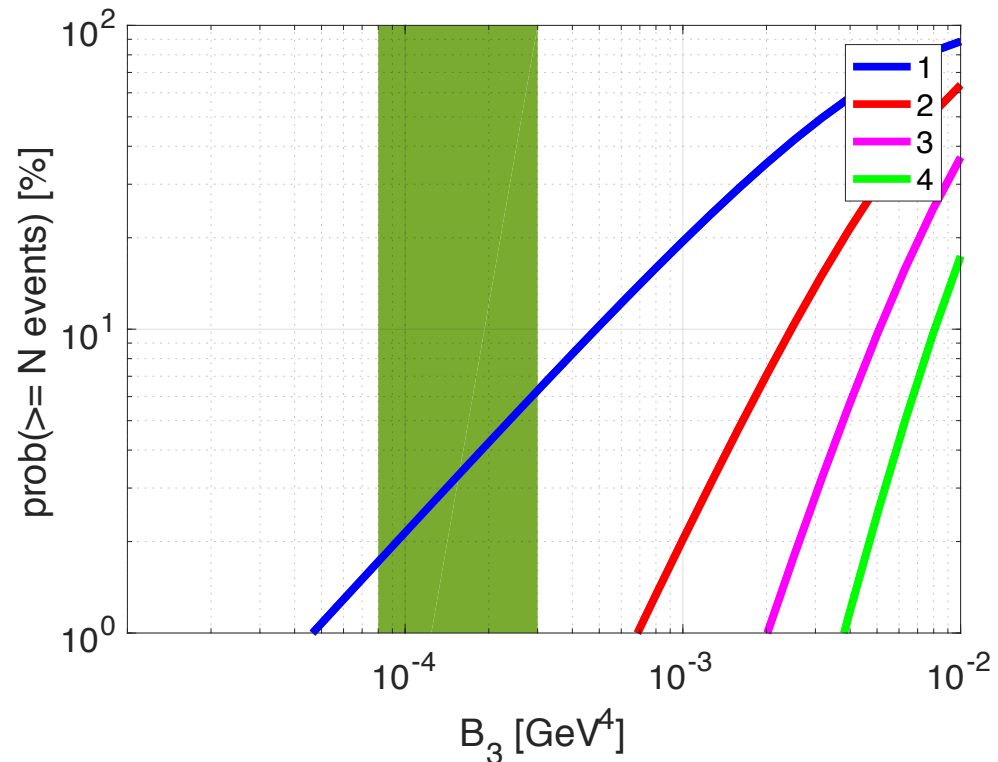
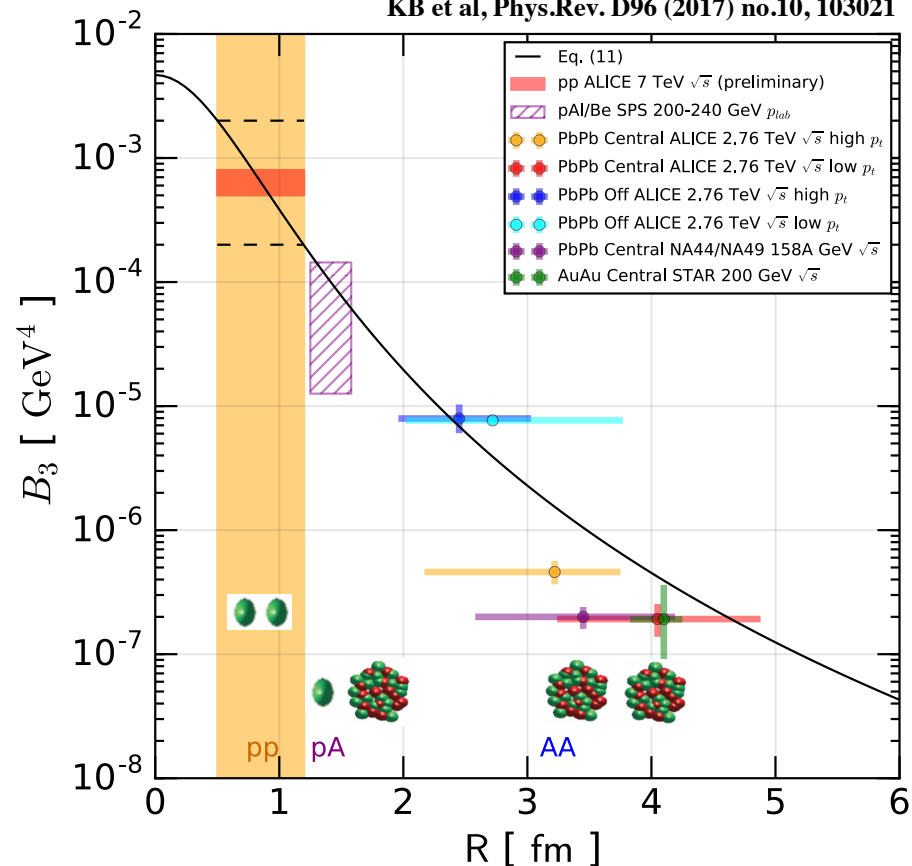
KB et al, Phys.Rev. D96 (2017) no.10, 103021



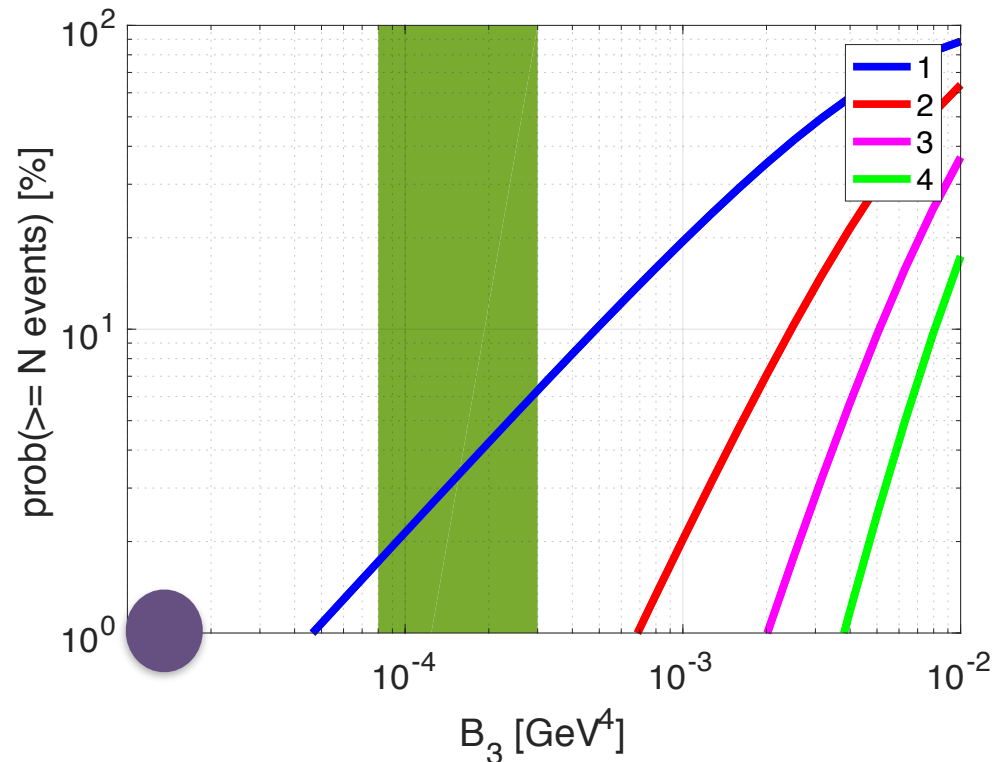
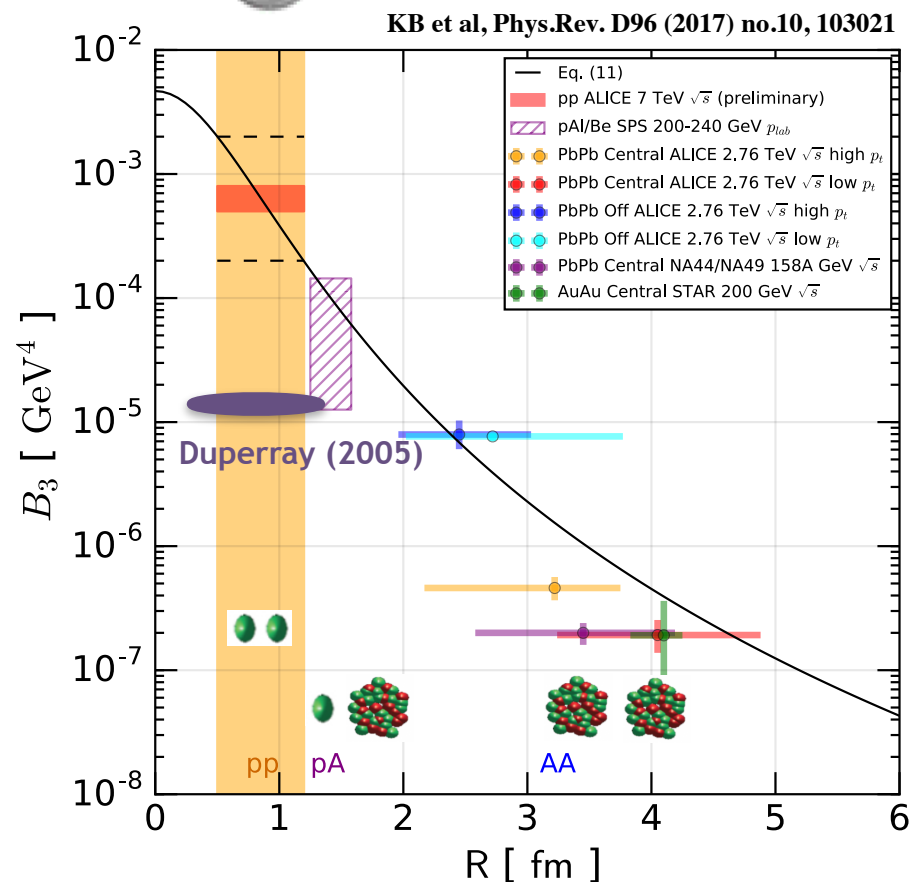
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KB et al, Phys.Rev. D96 (2017) no.10, 103021



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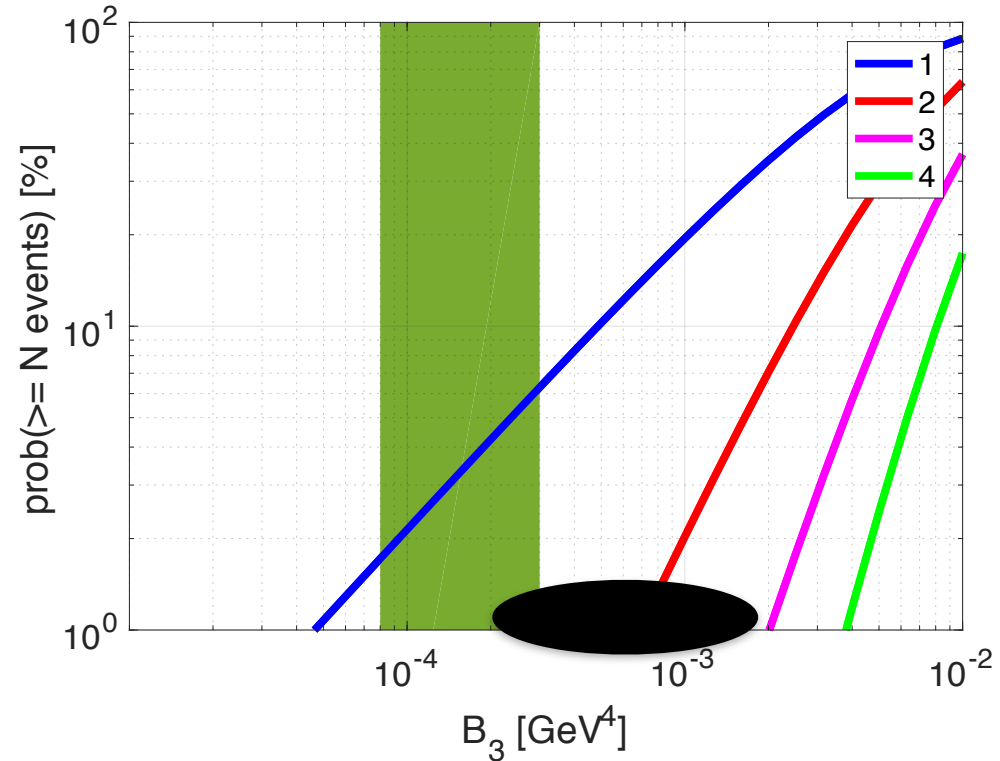
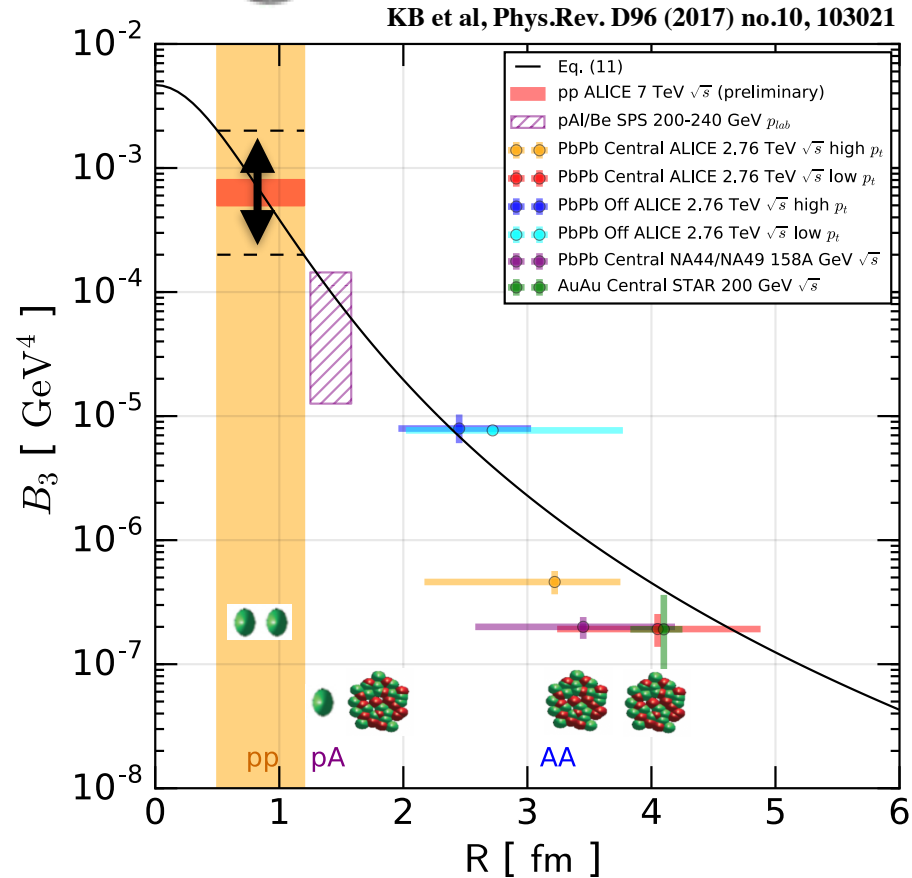
For pp we had no B_3 , but we *did* have *HBT*

$$\frac{\mathcal{B}_A}{m^{2(A-1)}} \approx \frac{2J_A + 1}{2^A \sqrt{A}} \left(\frac{m R}{\sqrt{2\pi}} \right)^{3(1-A)}$$

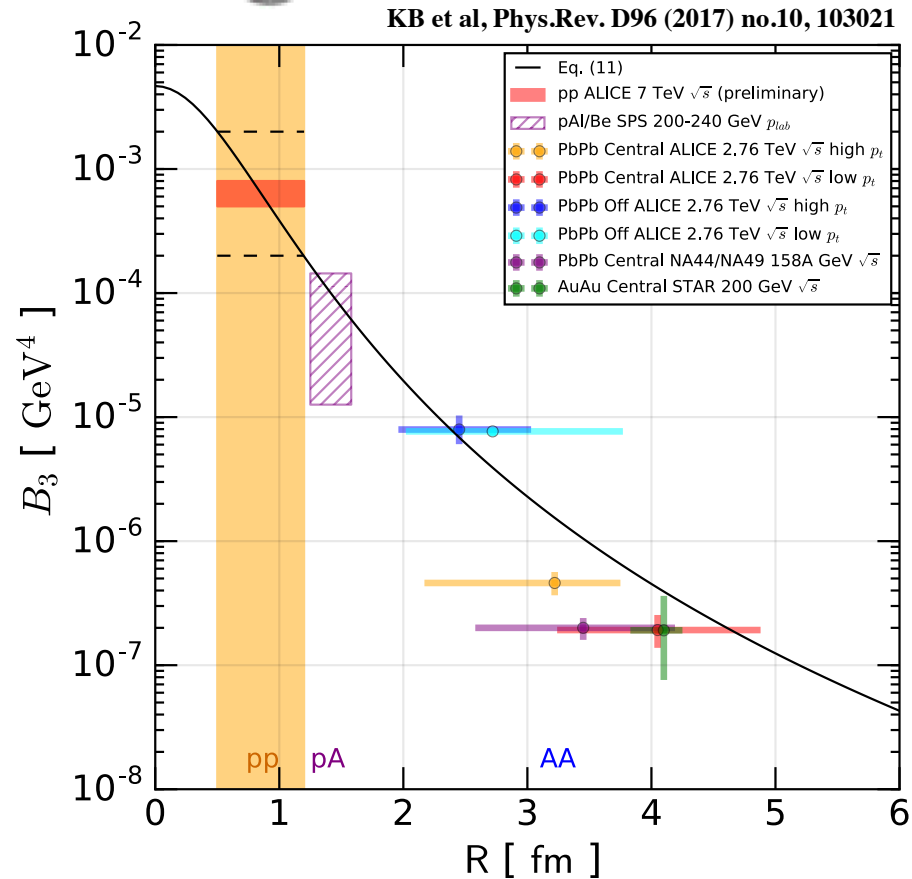
Scheibl & Heinz, PRC59, 1585 (1999)

KB et al, Phys.Rev. D96 (2017) no.10, 103021

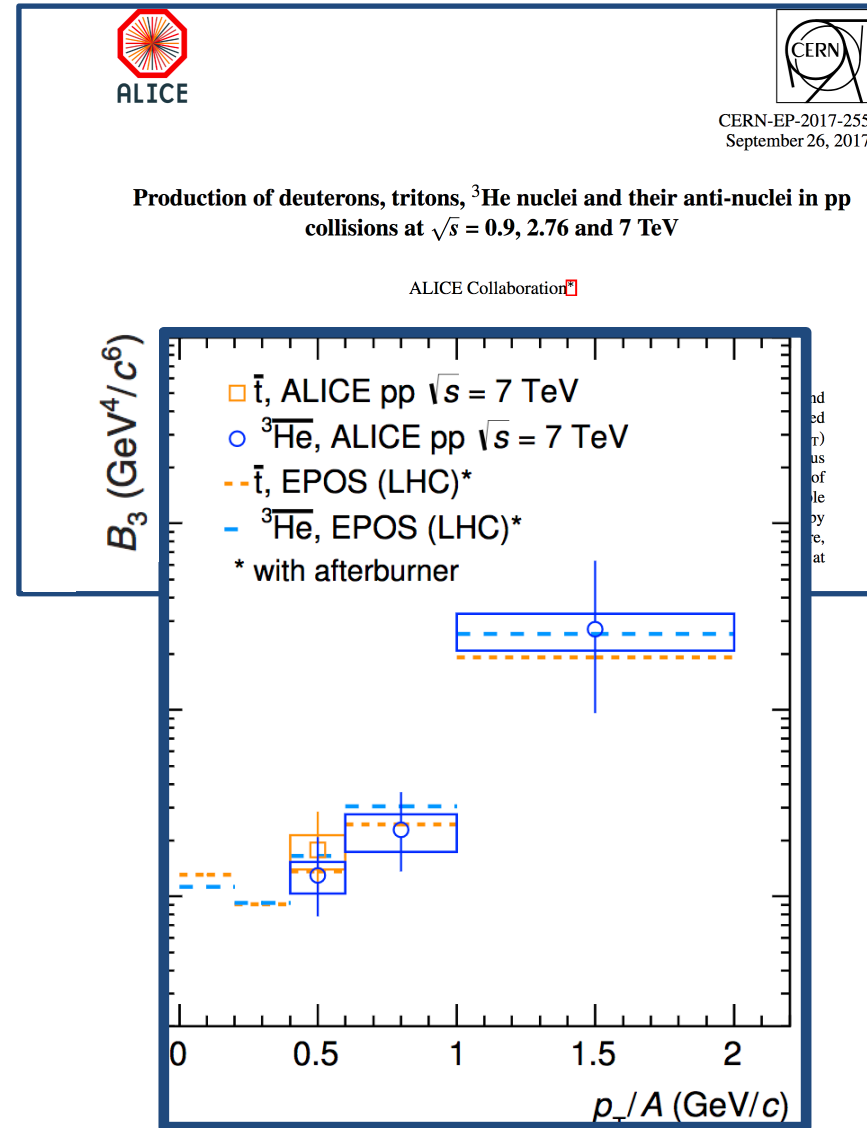
KB & Takimoto, 1901.07088



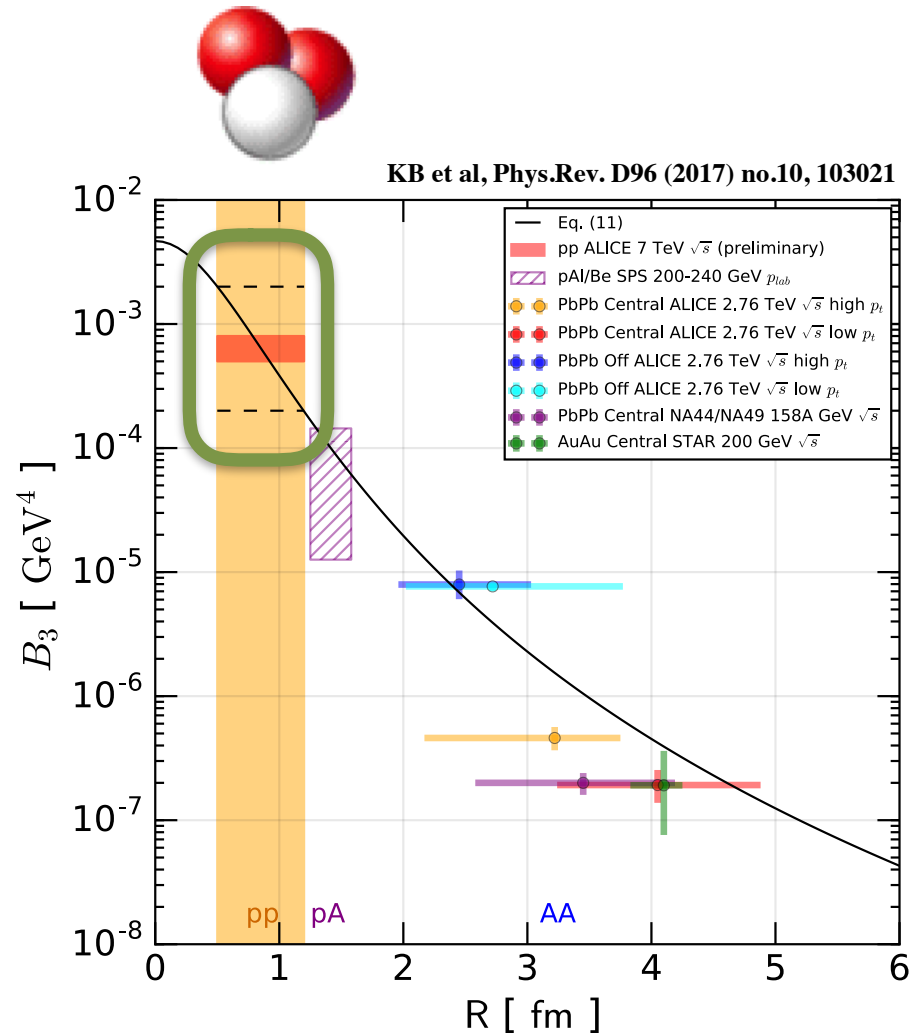
For pp we had no B_3 until Sep 26, 2017



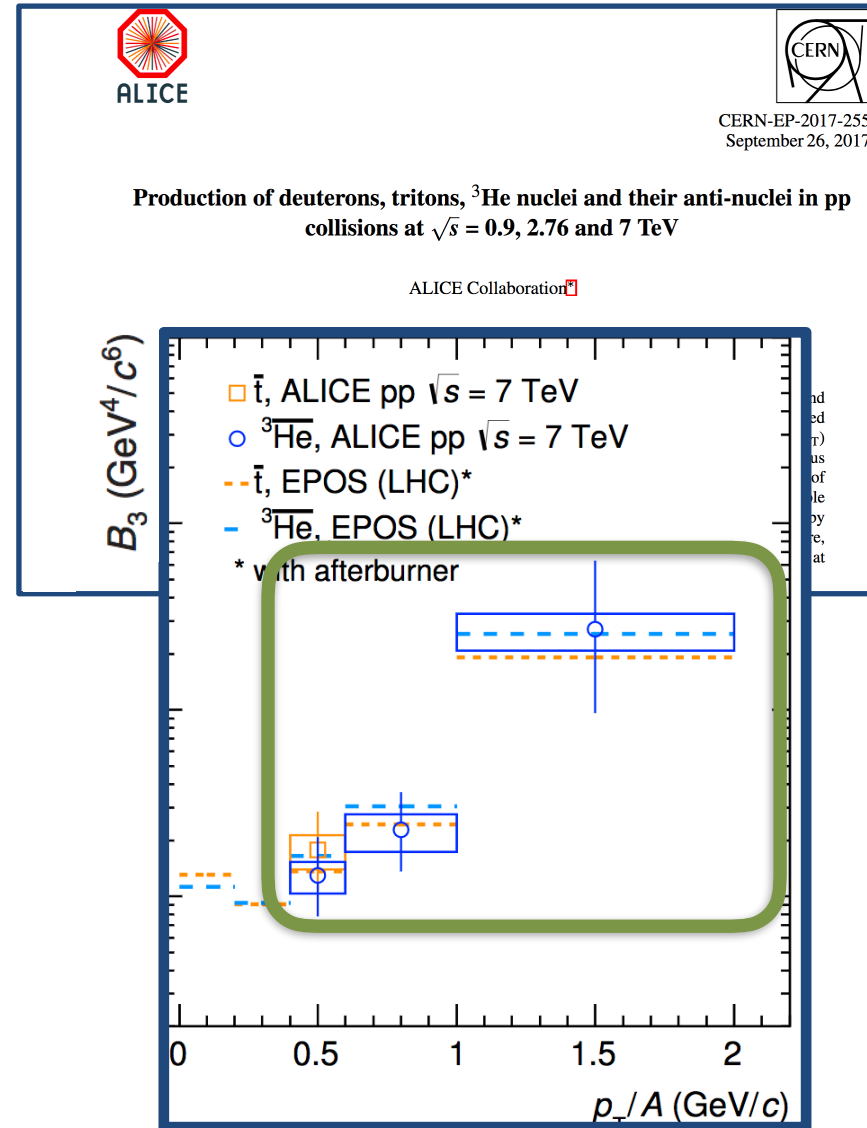
ALICE, PRC97, 024615 (2018)



For pp we had no B_3 until Sep 26, 2017



ALICE, PRC97, 024615 (2018)

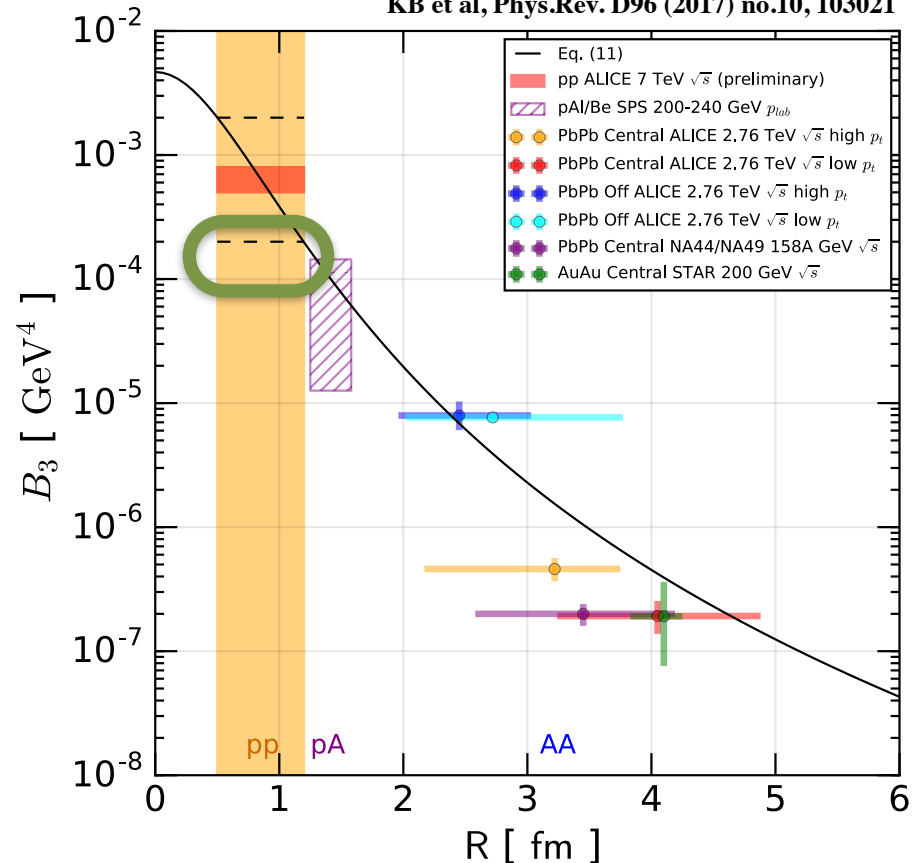


For pp we had no B_3 until Sep 26, 2017

Relevant for cosmic rays: low p_t



KB et al, Phys.Rev. D96 (2017) no.10, 103021



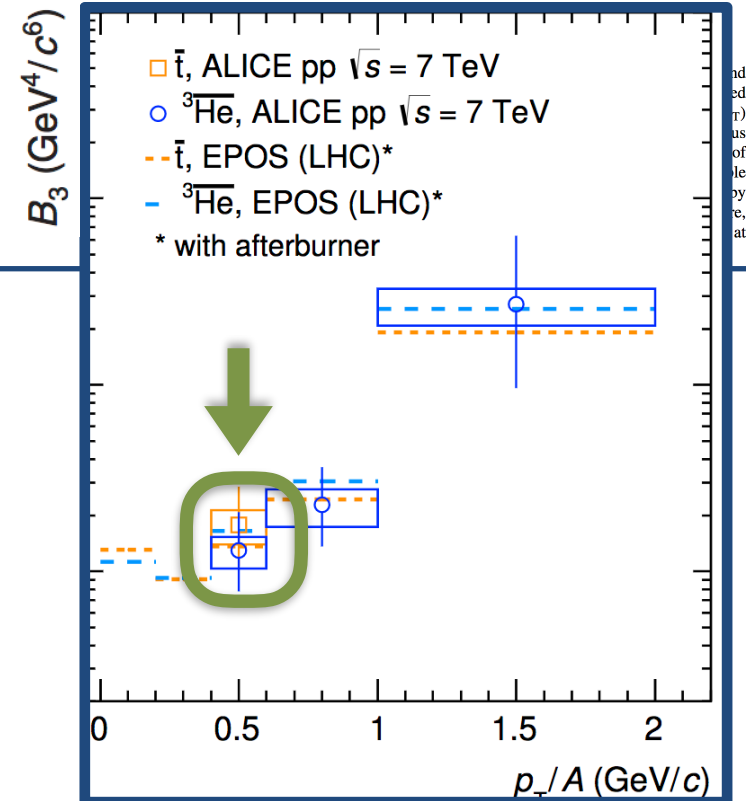
ALICE, PRC97, 024615 (2018)



CERN-EP-2017-255
September 26, 2017

Production of deuterons, tritons, ^3He nuclei and their anti-nuclei in pp collisions at $\sqrt{s} = 0.9, 2.76$ and 7 TeV

ALICE Collaboration

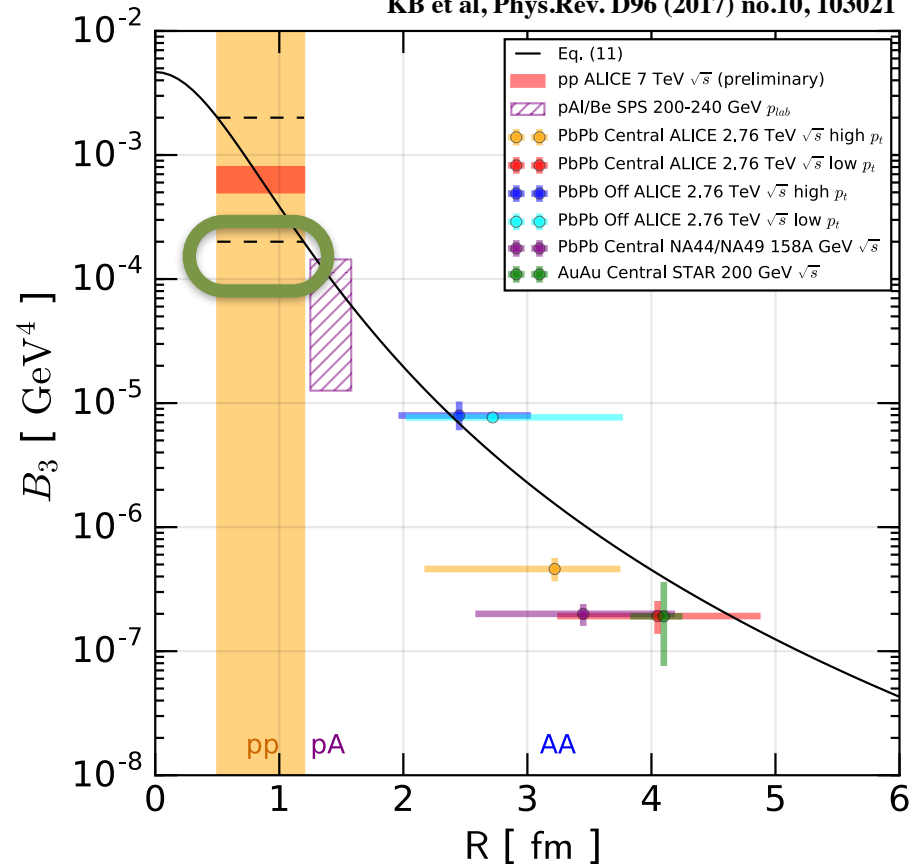


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KB et al, Phys.Rev. D96 (2017) no.10, 103021



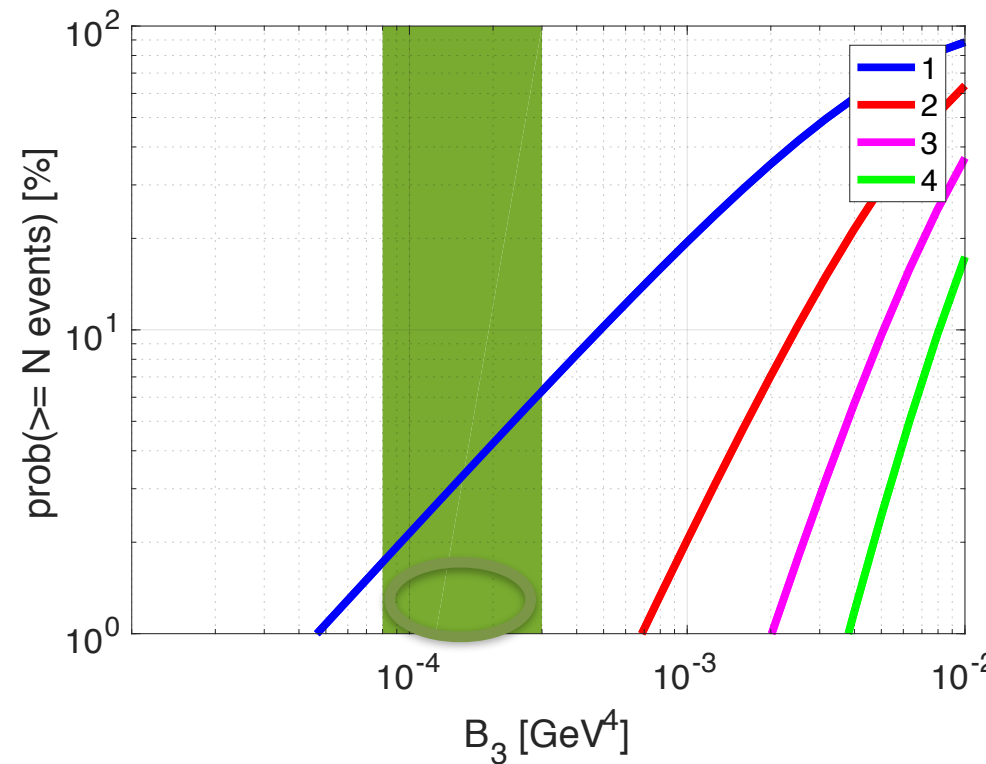
ALICE, PRC97, 024615 (2018)



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Implications of ALICE results
for astrophysics:

1 anti-He3 at AMS02,
in 5-year exposure: plausible.

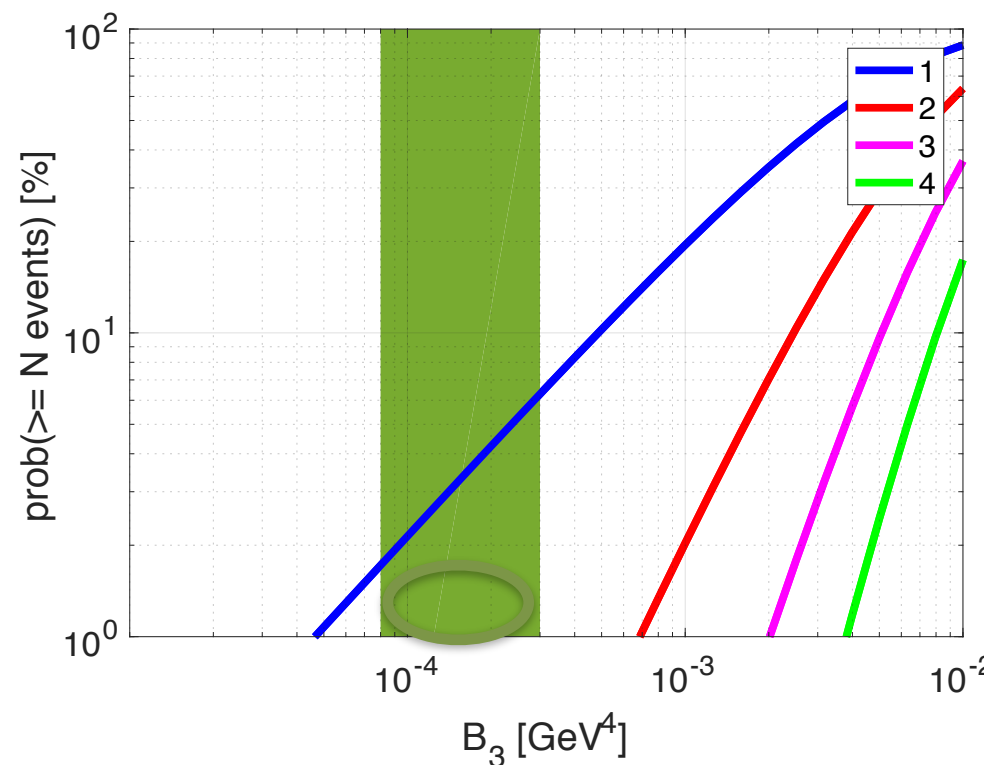
6 anti-He3 events: *not plausible*.

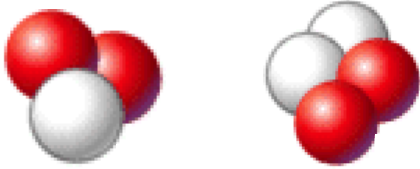


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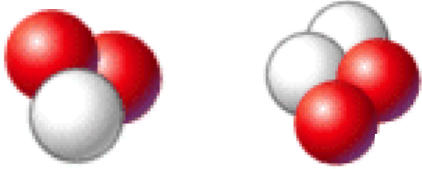
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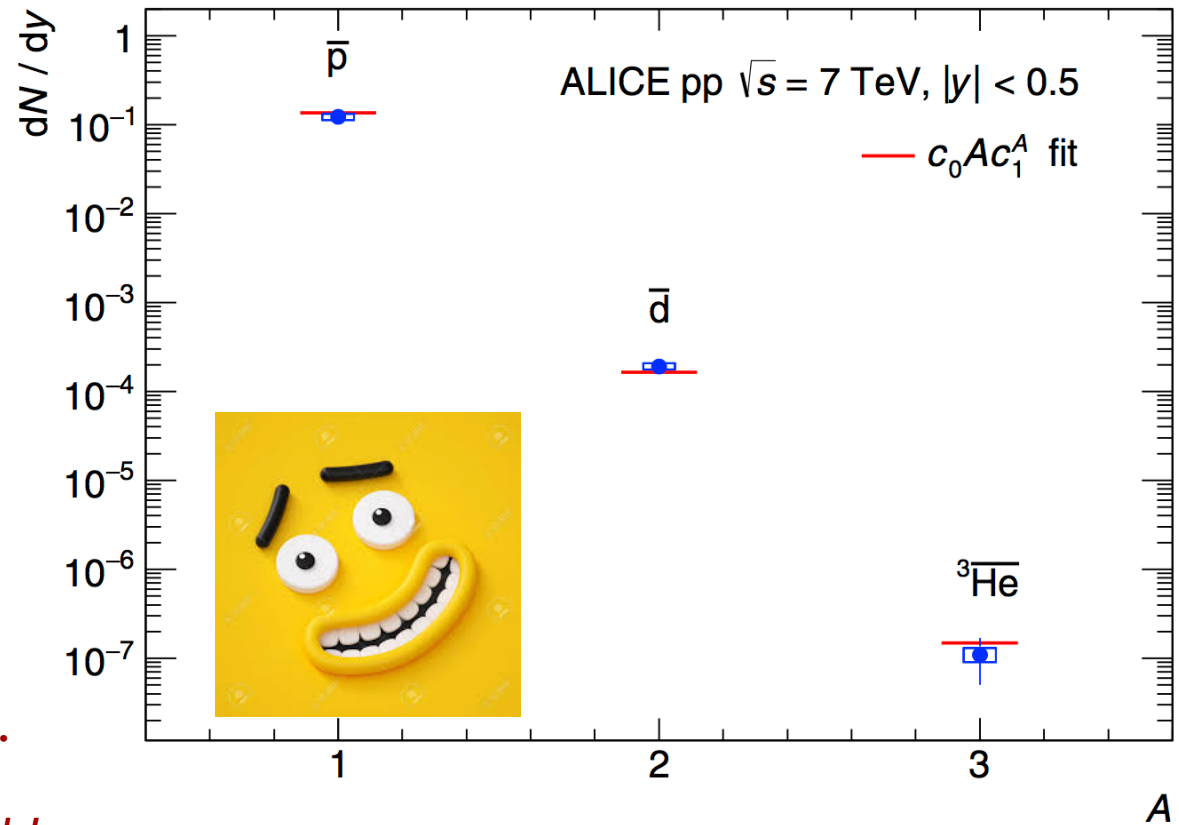


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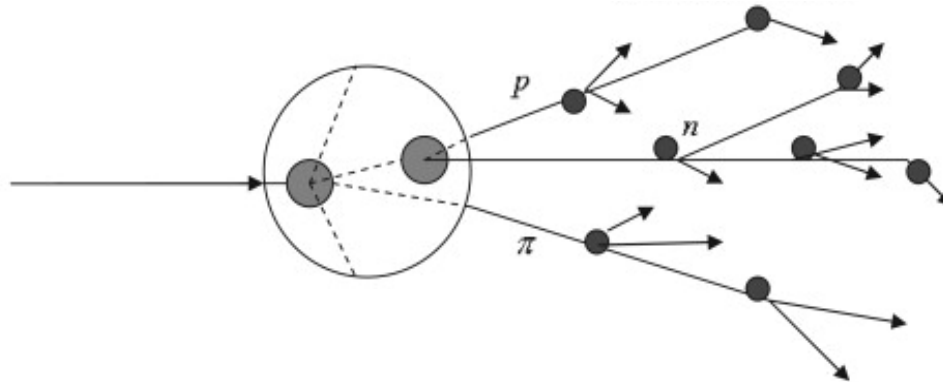


antimatter is produced in collisions of the bulk of the CRs
— protons and He – with interstellar gas.

Need to calculate this background to learn about possible exotic sources.

Problem: we don't know where CRs come from, nor how long they are trapped in the Galaxy, nor how they eventually escape.

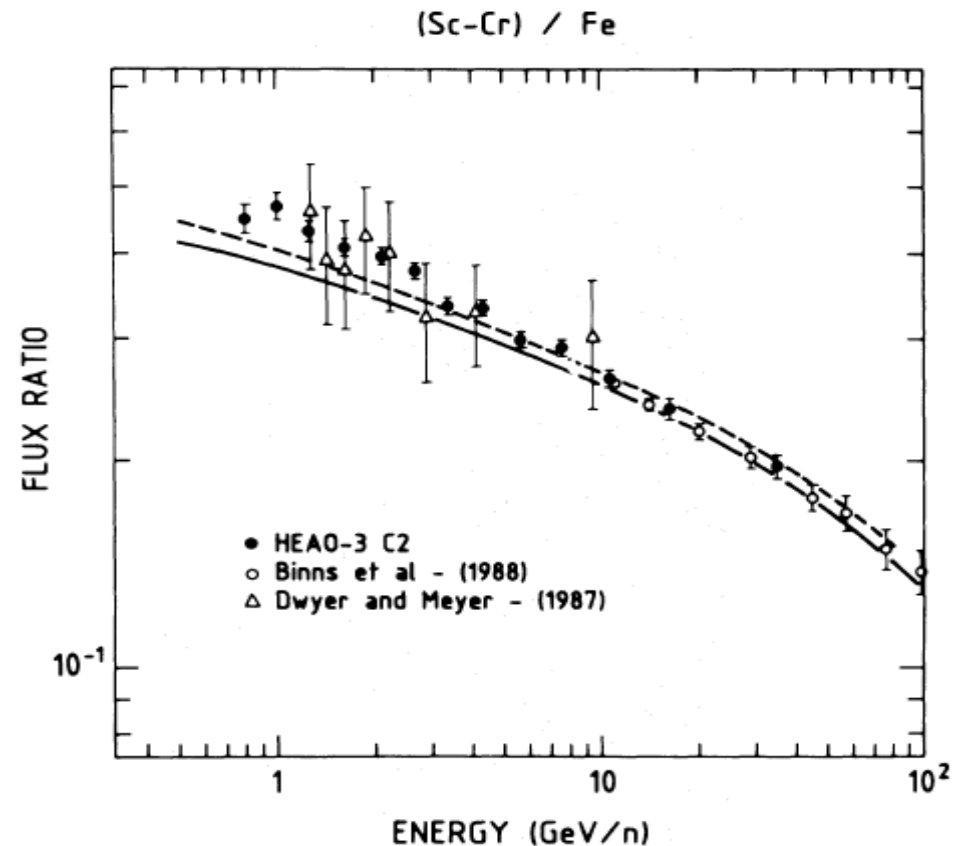
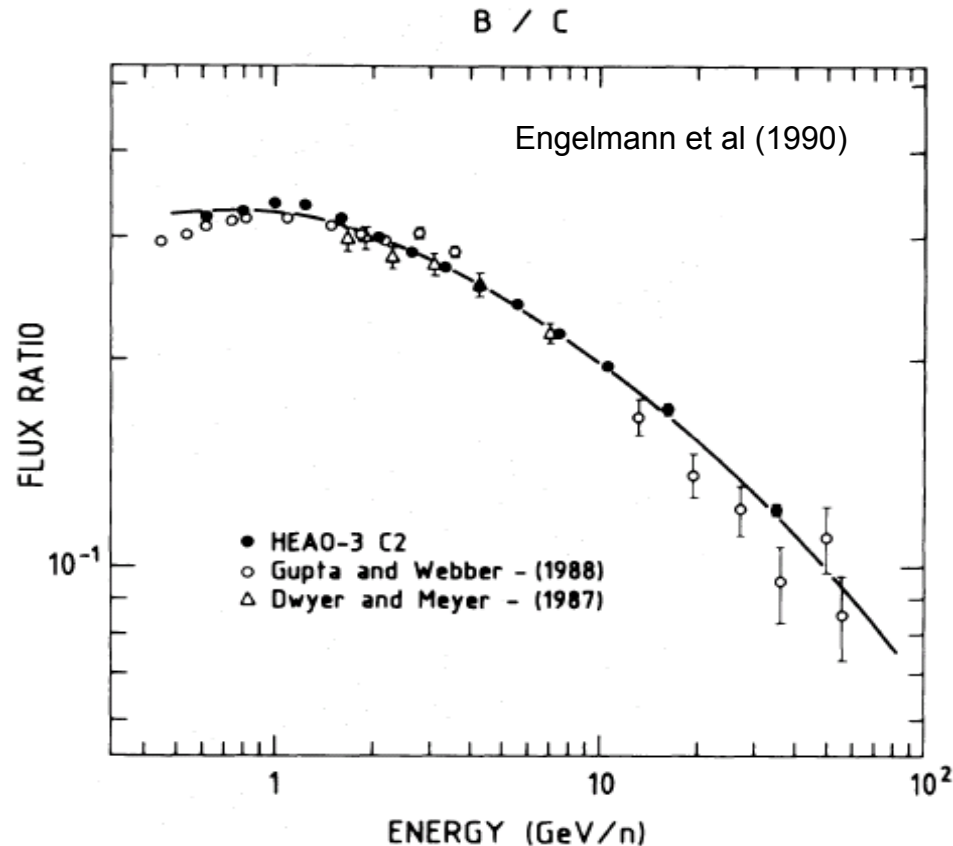
This problem is often under-stated...



antimatter is produced in collisions of the bulk of the CRs
 — protons and He — with interstellar gas.

**For stable, relativistic secondary CR nuclei,
 we have a handle: branching fractions**

$$\frac{n_a(\mathcal{R})}{n_b(\mathcal{R})} \approx \frac{Q_a(\mathcal{R})}{Q_b(\mathcal{R})}$$



Apply this to antiprotons

$$n_{\bar{p}}(\mathcal{R}) \approx \frac{n_B(\mathcal{R})}{Q_B(\mathcal{R})} Q_{\bar{p}}(\mathcal{R})$$

