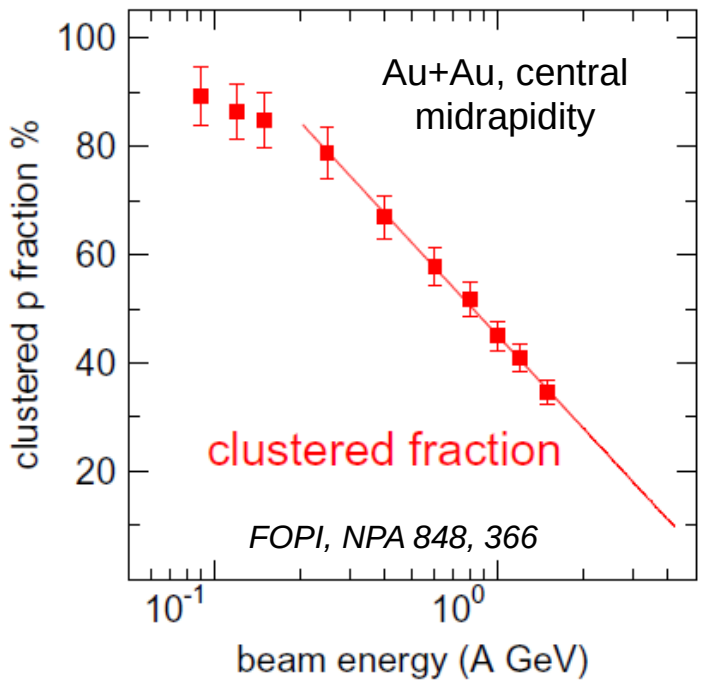
Overview over the different possibilities to determine clusters dynamically in transport models

Joerg Aichelin

Thanks to G.Coci, M. Winn and E. Bratkovskaya

- Why cluster production is an interesting subject?
- Hybrid models and models for dynamical cluster formation
- Why n-body theories are needed and how to construct them
- How clusters can be identified?
- Perspectives

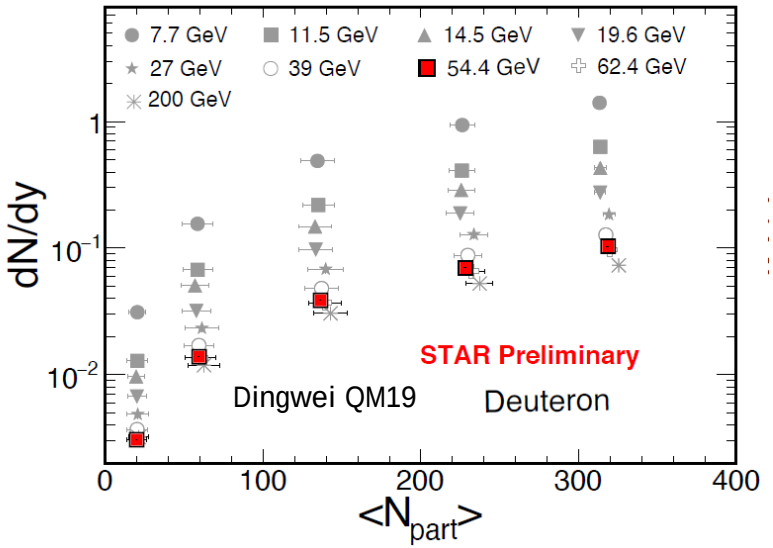
Clusters in HICs

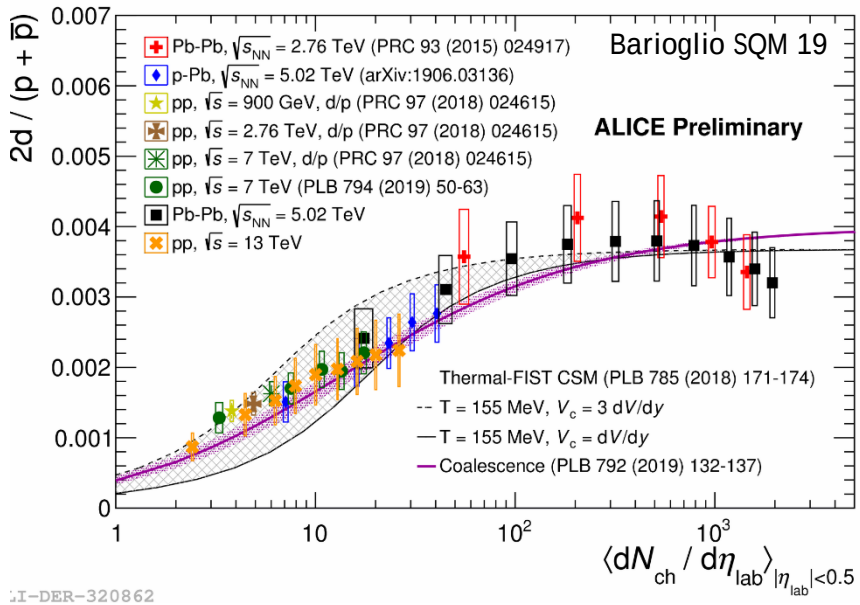


- Clusters are very abundant at low energy;
- at 3 AGeV in central Au+Au collisions ~20% of the baryons are in clusters!
- cluster production continues to STAR energies 1% - 0.3% of the nucleons are bound in d at y_{cm}
- decrease slightly up to LHC energies

→ **midrapdity clusters exist at all beam energies where temperatures T>100 MeV**

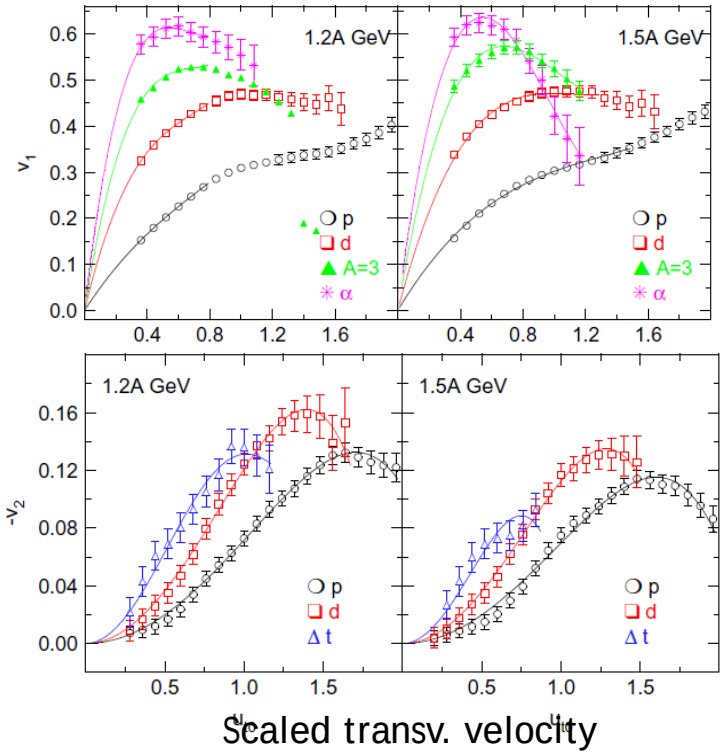
➔ **production mechanism is heavily discussed**





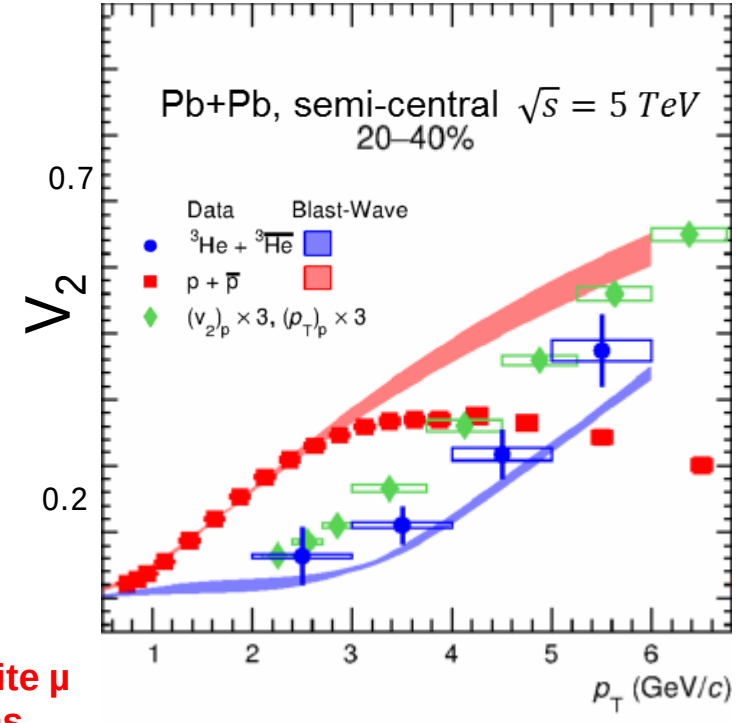
There is more than multiplicity of clusters

Au+Au, semi-central *FOPI, NPA 876,1*



Baryons in clusters have quite different properties (v_1 , v_2, dn/dp_T)

and explore therefore different phase space regions:



In addition, cluster open new physics opportunities

- **possible signals of a 1st order phase transition at finite μ**
- **fluctuations of the phase space densities of nucleons**
- **hyper-nucleus formation at mid as well as target/proj. rapidities**

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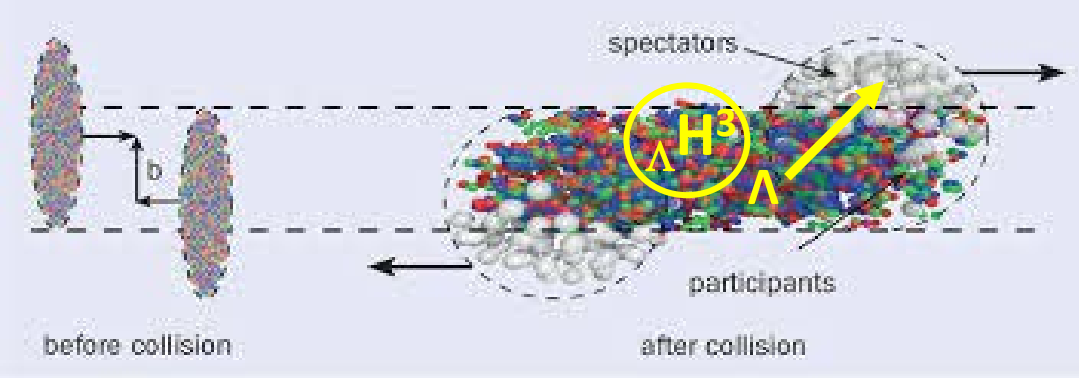
Why should we study hyper-matter production?

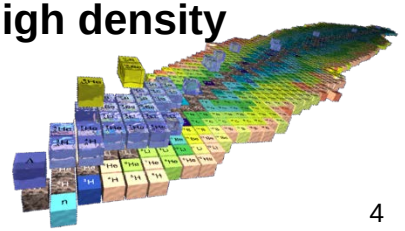


- **Access to the nuclear dynamics:**

different mechanisms for hyper-nucleus production vs. rapidity:
- at mid-rapidity : Λ – hyper-nuclei test the phase-space distribution of baryons in the expanding participant matter
- at target/projectile y: Λ-absorption by spectators - elucidates the physics at the interface between spectator and projectile matter

Hyper-nuclei as bound objects:

- **give access to the third dimension of the nuclear chart (strangeness)**
- **give information on hyperon-nucleon and hyperon-hyperon interactions**
- **important e.g. for neutron stars (production of hypermatter at high density and low temperature)**
- **new field of hyperon spectroscopy**



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Modeling of cluster production in heavy-ion collisions

We need two tools:

- a dynamical simulation of a heavy-ion reactions (including a late stage of baryons and mesons)
- a model which identify clusters

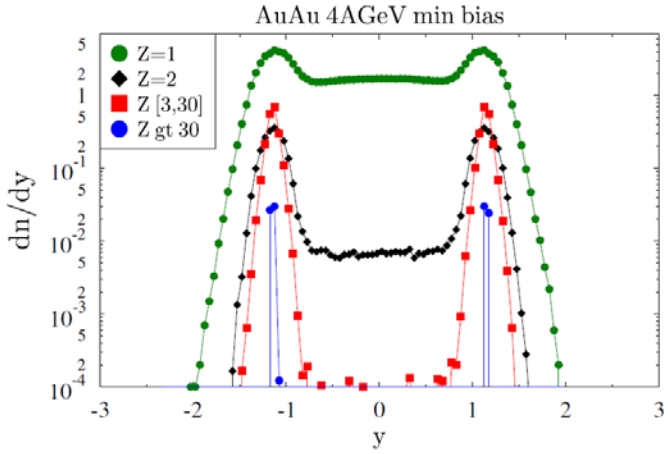
There are two ways:

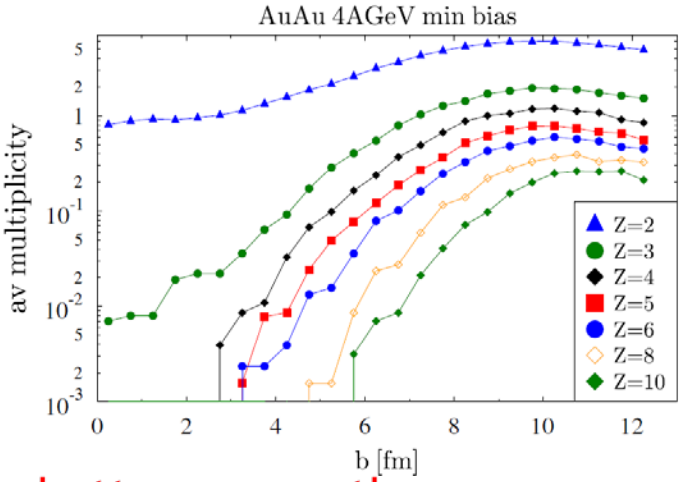
- hybrid model of cluster production - sudden transition from a dynamical model to clusterization via coalescence or statistical model

- dynamical cluster formation
  formation of clusters continuously during the time evolution

There are two types of clusters:

Midrapidity cluster dominating at small b (mostly newly formed)
Proj/target cluster dominating at larger b (initial final state correlations)





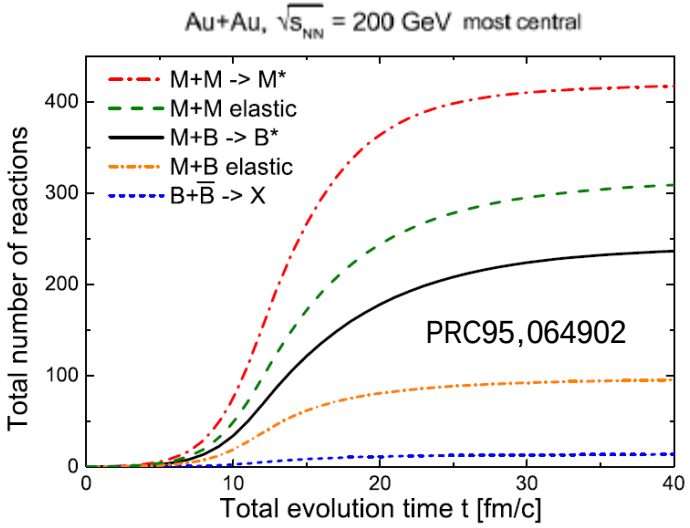
Transition smooth in b and $\sqrt{s}$: better we use the same approach

Hybrid models of cluster production

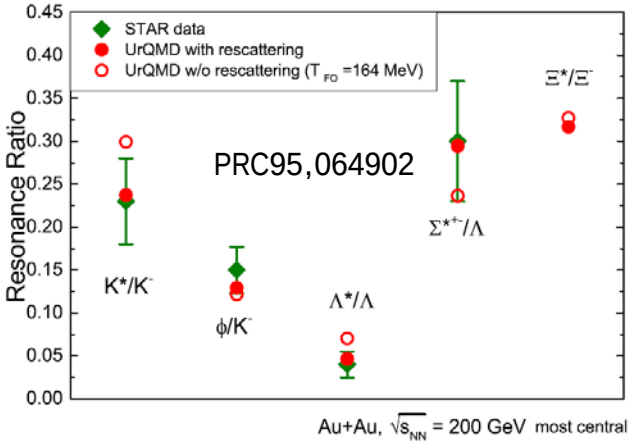
All hybrid models assume that heavy-ion reactions have three phases:

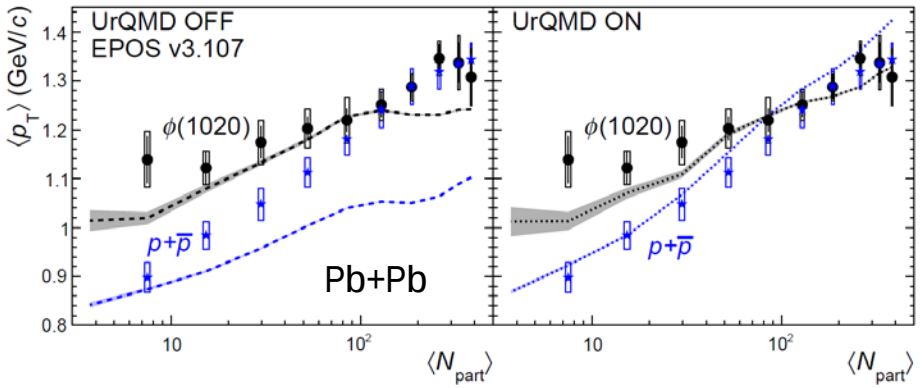
- a phase in which particles collide frequently
  a part of the system comes close to (local or global) equilibrium
- a sudden formation of clusters (given by a local temperature or time)
- a free streaming of clusters to the detector without further interactions

Dynamical models (UrQMD) do not show such a sudden transition but a very smooth fading away of the interactions. Late stage: MB -> B* dominant



PRC93,014911

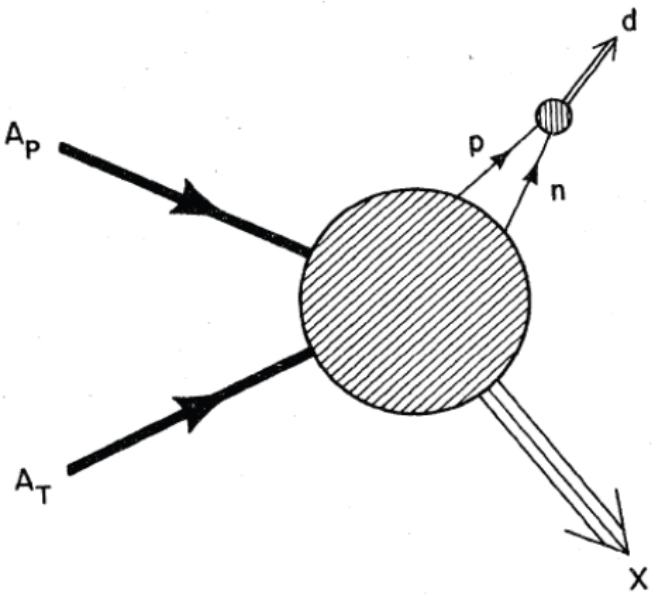




The sudden formation of clusters

Statistical model: describes very well the multiplicities in central collisions
but not the spectra (yield V,T,μ)
difficult to imagine how the cluster production takes place
d: E_b =2.2 MeV , rms radius = 1.7 fm
does not survive in heat bath of T>100 MeV
"ice in fire","snowball in hell"

Coalescence: goes back to Butler and Pearson PR129,836 (p+A)



d-production is a 3-body process
momentum has to be transferred to the third body

QM: in a static potential ~ $1/p^2$

$$n_d(p) \sim \frac{1}{p^2}\, n_n^2\left(\frac{p}{2}\right)$$

The sudden formation of clusters

In later approaches the three body character of the d-production has been neglected

- Schwarzschild and Zupanic PR129 854:

d is produced if in a sphere of momentum space ~p_0^3 around a nucleon we find another nucleon:

$$n_d(p) \sim p_0^3 \; n_n^2\left(\frac{p}{2}\right)$$

- Kapusta PRC16,1493

d is produced in a fireball of a given volume V

$$n_d(p) \sim V \; n_n^2\left(\frac{p}{2}\right)$$

- Bond et al. , PLB 71, 43

Sudden approximation in QM: sudden transition from a strongly interacting system to a noninteracting system

$$n_d(p) \sim \frac{1}{V} n_n^2\left(\frac{p}{2}\right)$$

- Scheibl et al. PRC59,1585

Overlap of the Wigner density of the d with that of p and n

$$\frac{dN_d}{d^3P_d} = \frac{3}{(2\pi)^3}\int d^3r_d \int \frac{d^3r\, d^3q}{(2\pi)^3}\mathcal{D}(\boldsymbol{r},\boldsymbol{q}) \times f_p(q_+, r_+)\, f_n(q_-^*; r_-)\,.$$

$$r_\pm = r_d \pm \tfrac{1}{2}r$$

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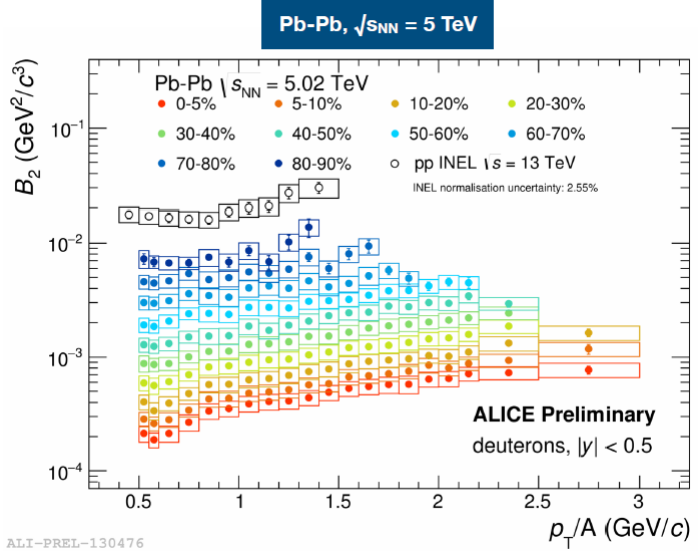
So it is not evident what we can learn from the experimental ratio for A=2

$$B(\mathbf{p}, V, p_0, V_{NN}) = \frac{n_d(\mathbf{p})}{n_n^2(\mathbf{p}/2)} \qquad B_A = \frac{E_A \frac{d^3N_A}{dp_A^3}}{\left(E_p \frac{d^3N_p}{dp_p^3}\right)^A}$$

because it depends on the model assumptions

In addition: for large nuclei the coalescence model does not work

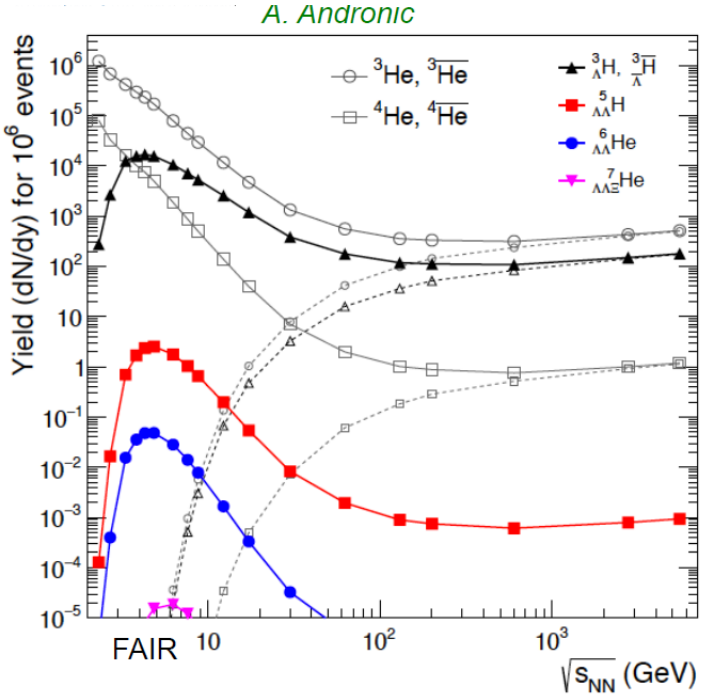
→ no general framework for cluster production



Additional caveats :

- ❑ before the sudden freeze out: d do not exist
- ❑ after sudden freeze out d cannot be produced (3-body process)
- ❑ theoretical results depend strongly on the sudden freeze out time
- ❑ Freeze out time depends on the fragment size if one wants to reproduce the data
  Gossiaux, Keane (EOS) et al. PRC51, 3357

More general approach needed if one wants to exploit the physics potential of cluster production



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Dynamical cluster production

Why does one need a new model ?

Present microscopic approaches:

- **VUU(1985), BUU(1985), HSD(1996), PHSD(2008), SMASH(2016) solve the time evolution of the one-body phase-space density in a mean field**
  → no dynamical fragments

- **UrQMD is a n-body model but makes clusterization via coalescence and a statistical fragmentation model**

- **QMD is a n-body model but is limited to energies < 1.5 AGeV**
  → describes fragments at SIS energies,
  but conceptually not adapted for NICA/FAIR energies and higher

In order to understand the microscopic origin of cluster formation one needs:
- **a realistic model for the dynamical time evolution of HICs**
- **dynamical modelling of cluster formation based on interactions**

Dynamical modelling of cluster formation is a complex task which involves: the fundamental nuclear properties, quantum effects, variable timescales

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Transport eqs. for N-body theories like (PH)QMD,AMD,FMD

Roots in Quantum Mechanics

Remember QM cours when you faced the problem

- we have a Hamiltonian $\hat{H} = -\frac{\hbar^2\nabla^2}{2m} + V$
- the Schrödinger eq.

$$\hat{H}|\psi_j >= E_j|\psi_j >$$

has no analytical solution

- we look for the ground state energy

Walther Ritz

Ritz variational principle:

Assume a trial function $\psi(q,\alpha)$ which contains one adjustable parameter α, which is varied to find the lowest energy expectation value:

$$\frac{d}{d\alpha} < \psi|\hat{H}|\psi >= 0 \rightarrow \alpha_{min}$$

determines α for which $\psi(q,\alpha)$ is closest to the true ground state and $< \psi(\alpha_{min})|\hat{H}|\psi(\alpha_{min}) >= E_0(\alpha_{min})$ closest to true ground state E

Extended Ritz variational principle (Koonin, TDHF)

Take trial wavefct with time dependent parameters and solve

$$\delta \int_{t_1}^{t_2} dt < \psi(t)|i\frac{d}{dt} - H|\psi(t) >= 0. \qquad (1)$$

QMD trial wavefct for particle i with p_{oi} (t) and q_{oi} (t)

$$\psi_i(q_i, q_{0i}, p_{0i}) = Cexp[-(q_i - q_{0i} - \frac{p_{0i}}{m}t)^2/4L] \cdot exp[ip_{0i}(q_i - q_{0i}) - i\frac{p_{oi}^2}{2m}t]$$

For N particles:

$$\psi_N = \prod_{i=1}^{N} \psi_i(q_i, q_{0i}, p_{0i}) \qquad \text{QMD}$$

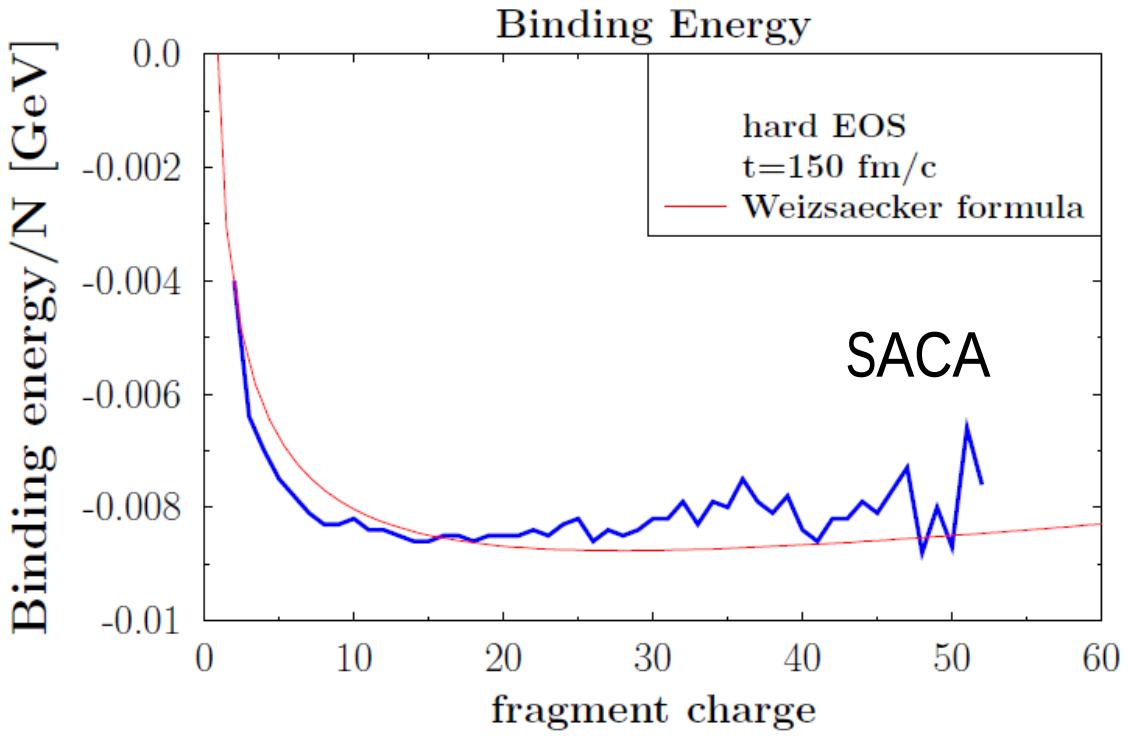
$$\psi_N^F = Slaterdet[\prod_{i=1}^{N} \psi_i(q_i, q_{0i}, p_{0i})] \qquad \text{AMD / FMD}$$

For the QMD trial wavefct eq. (1) yields

$$\frac{dq}{dt} = \frac{\partial < H >}{\partial p} \quad ; \quad \frac{dp}{dt} = -\frac{\partial < H >}{\partial q}$$

For Gaussian wavefct
eq. of motion very similar
to Hamilton's eqs.
(but only for Gaussians !!)

N-body models can produce cluster with the right E_{bind}



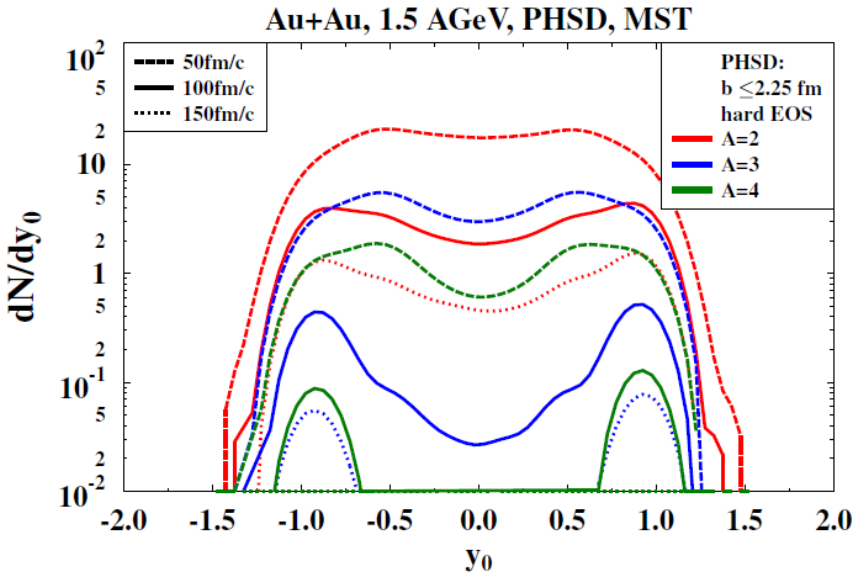
There are two kinds of fragments

- formed from spectator matter
  close to beam and target rapidity
  initial-final state correlations
  HI reaction makes spectator matter unstable

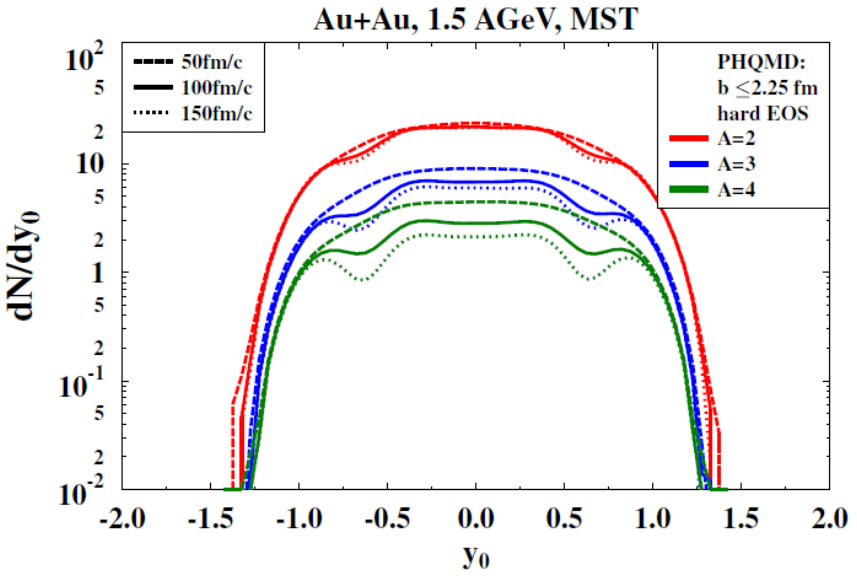
- formed from participant matter
  created during the expansion of the fireball
  "ice" (E_{bind} ≈8 MeV/N) in "fire"(T≥ 100 MeV)
  origin not known yet
  seen from SIS to RHIC
  (quantum effects may be important)

QMD vs. MF

mean field propagation
all two or more body correlation suppressed



QMD propagation
correlations present



QMD propagation: number of clusters are stable vs. time
(MST finds at 50 fm/c almost the same clusters as at 150fm/c)

MF propagation (per construction not suited for cluster studies):

- **number of fragments is strongly time dependent**
- **fragments disappear with time**
- **midrapidity fragments disappear early, projectile/target fragments later**
  (as expected from the underlying theory)

→ no common time for coalescence

PHQMD

The goal: to develop a unified n-body microscopic transport approach for the description of heavy-ion dynamics and dynamical cluster formation from low to ultra-relativistic energies

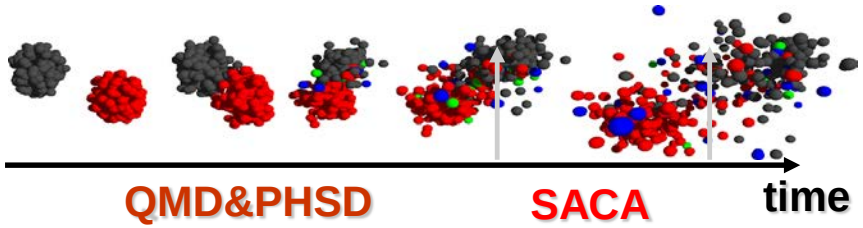
Realization: combined model PHQMD = (PHSD & QMD) & SACA

Parton-Hadron-Quantum-Molecular Dynamics

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

Clusters recognition:
SACA (Simulated Annealing Clusterization Algorithm)
vs. MST (Minimum Spanning Tree)



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Methods to identify clusters in models which propagate nucleons:

Static approaches:

means cluster multiplicity determined at a fixed time or temp

-- coalescence (early, assumption: no coll. later)

-- statistical model (V,T,N) very late $\rho << \rho_0$

Dynamical approaches:

means cluster multiplicity is function of time

-- minimum spanning tree (correlation in coord. space)

-- simulated annealing (correlation in mom and coord. space)

-- time dep. perturbation theory using Wigner densities

I. Minimum Spanning Tree

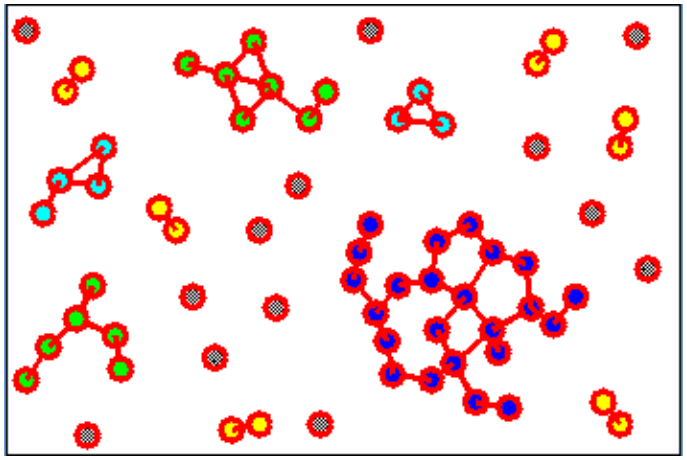
I. Minimum Spanning Tree (MST) is a cluster recognition method applicable for the (asymptotic) final state where coordinate space correlations may only survive for bound states.

The MST algorithm searches for accumulations of particles in coordinate space:

1. Two particles are bound if their distance in coordinate space fulfills

$$\left|\vec{r}_i - \vec{r}_j\right| \leq 2.5\, fm$$

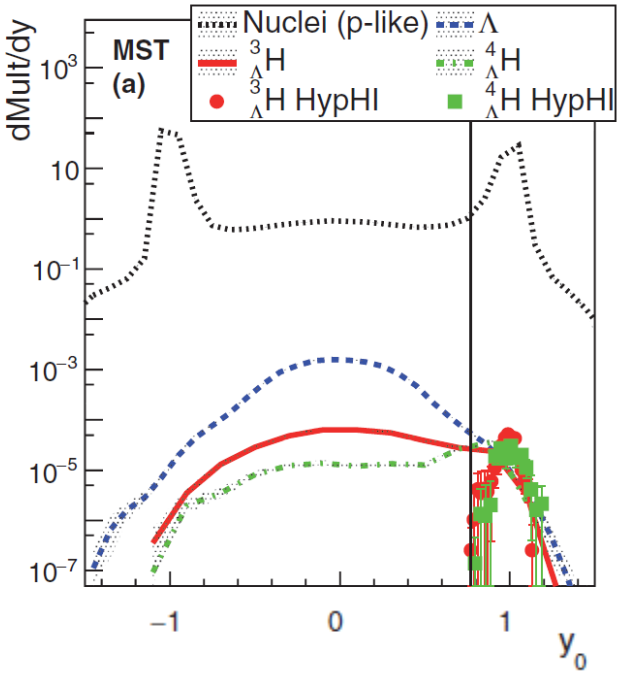
2. A particle is bound to a cluster if it is bound with at least one particle of the cluster.



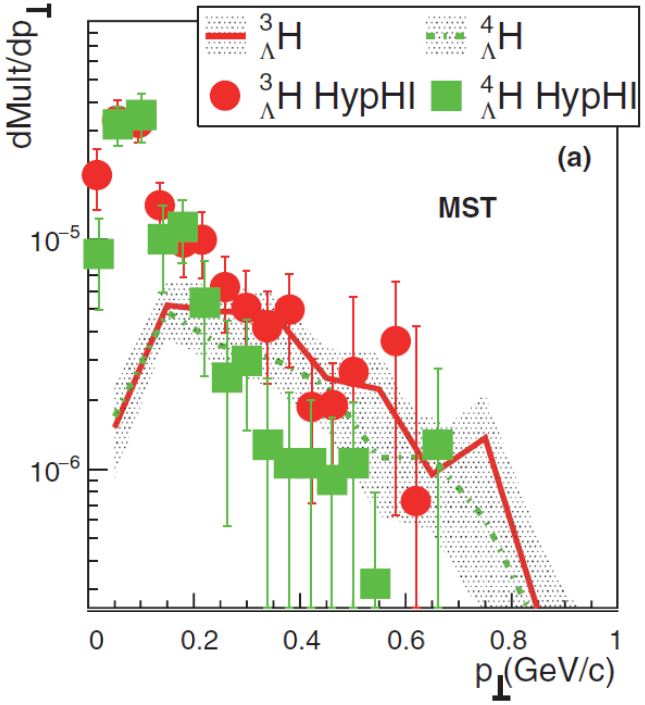
Additional momentum cuts (coalescence) change little:
large relative momentum -> finally not at the same position

Example: hyper-nuclei of HypHi (PLB747,129)

$^{6}Li + ^{12}C$ at 2A GeV



Le Fevre et al. PRC100,034904



Rapdity and p_T spectra of hyper-clusters are reproduced despite of the complicated physics:

- Modeling of Λ production
- Interface between participants and spectators
- Absorption of Λ by spectators

II.SACA or ECRA now FRIGA

If we want to identify fragments earlier one has to use momentum space info as well as coordinate space info

Idea by Dorso et al. (Phys.Lett.B301:328,1993) :

a) Take the positions and momenta of all nucleons at time t.
b) Combine them in all possible ways into all kinds of fragments or leave them as single nucleons
c) Neglect the interaction among clusters
d) Choose that configuration which has the highest binding Energy

Simulations have shown that the most bound configuration is the precursor of the final fragment distribution

(large persistent coefficient)

How does this work?

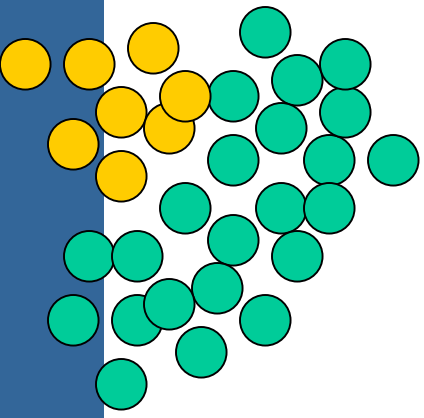
Simulated Annealing Procedure:

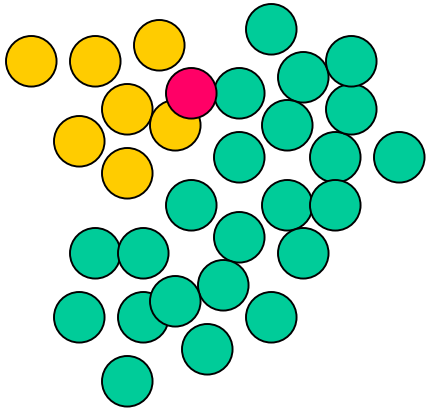
SACA: PLB301,328; J.Comp.Phys.162,245, NPA619,375

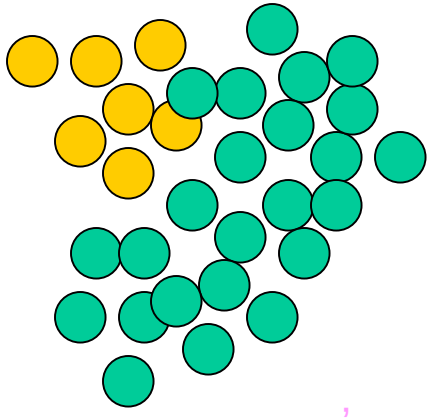
now FRIGA :**Nuovo Cim. C39,399 (including symmetry and pairing energy)**

Take randomly 1 nucleon out of a fragment

Add it randomly to another fragment







$E=E^{1}_{kin}+E^{2}_{kin}+V^{1}+V^{2}$

$E'=E^{1'}_{kin}+E^{2'}_{kin}+V^{1'}+V^{2'}$

There is no interaction between clusters-> no energy conserv.

V is the nucleon-nucleon interaction

(mom. dep Skyrme, Coulomb, (Pairing, Symmetry) energy)

Simulated annealing procedure

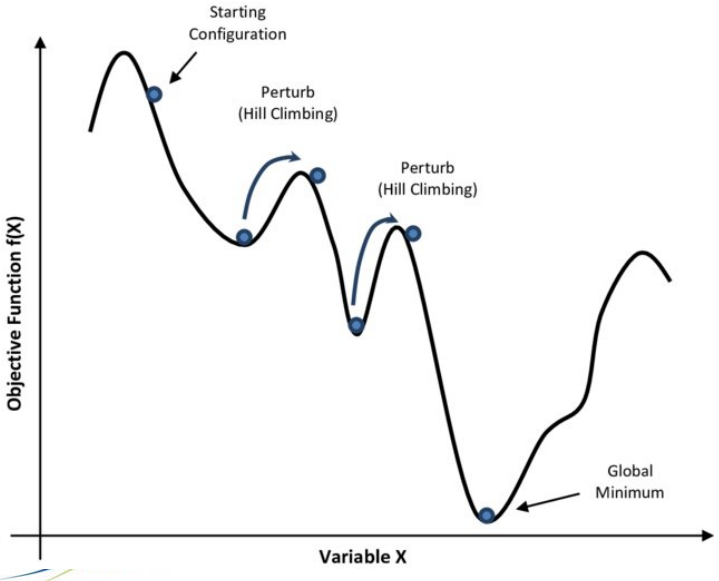
Originally from solid state physics, today applied for all kind of optimization problems

SA = Optimization procedure based on Metropolis'algorithm with Boltzmann probability

Objective function = energy

Goal: find the most bound configuration of nucleons and fragments

Energy between fragments is not taken into account (-> no energy conservation)



P(E'-E) = exp(-(E'-E)/T)

- If E' < E → accept new configuration
- If E' > E → MC accept/reject with P(E'-E)

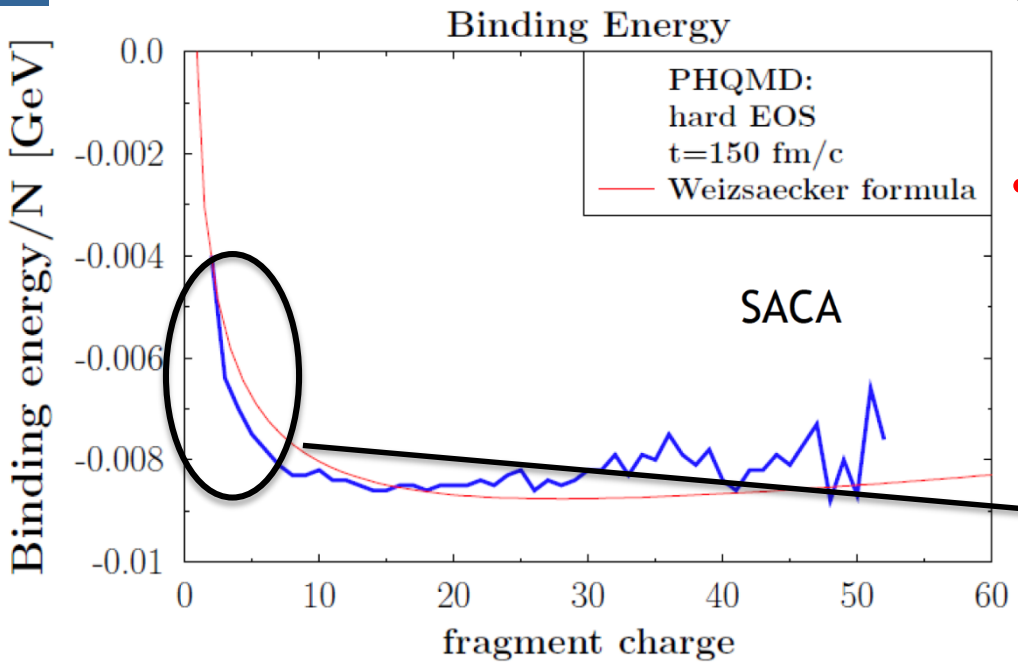
- Control Temperature: T → R T (R < 1)
- The system evolves through quasi-equilibirum states reducing T until it is trapped into the optimal solution (lowest E state). Mathematics assures that this minimum is found.

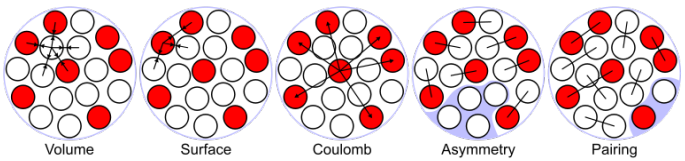
- The choice of initial T is system-dependent:
- Large initial T → many configurations spanned.

nucleons in a fragment, identified after passing time, are almost the same as in the final fragment

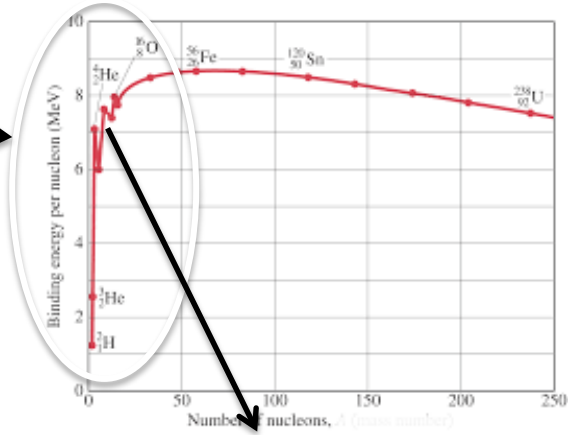
Potential interaction in SACA

V_{Skyrme} used for IQMD and PHQMD propagation as well as for the binding energy
→ Weizsäcker formula





- Correct description of heavy clusters
- Binding energy is not precise enough for small clusters (A < 10)



Without quantum corrections
SACA is not yet suited for small clusters

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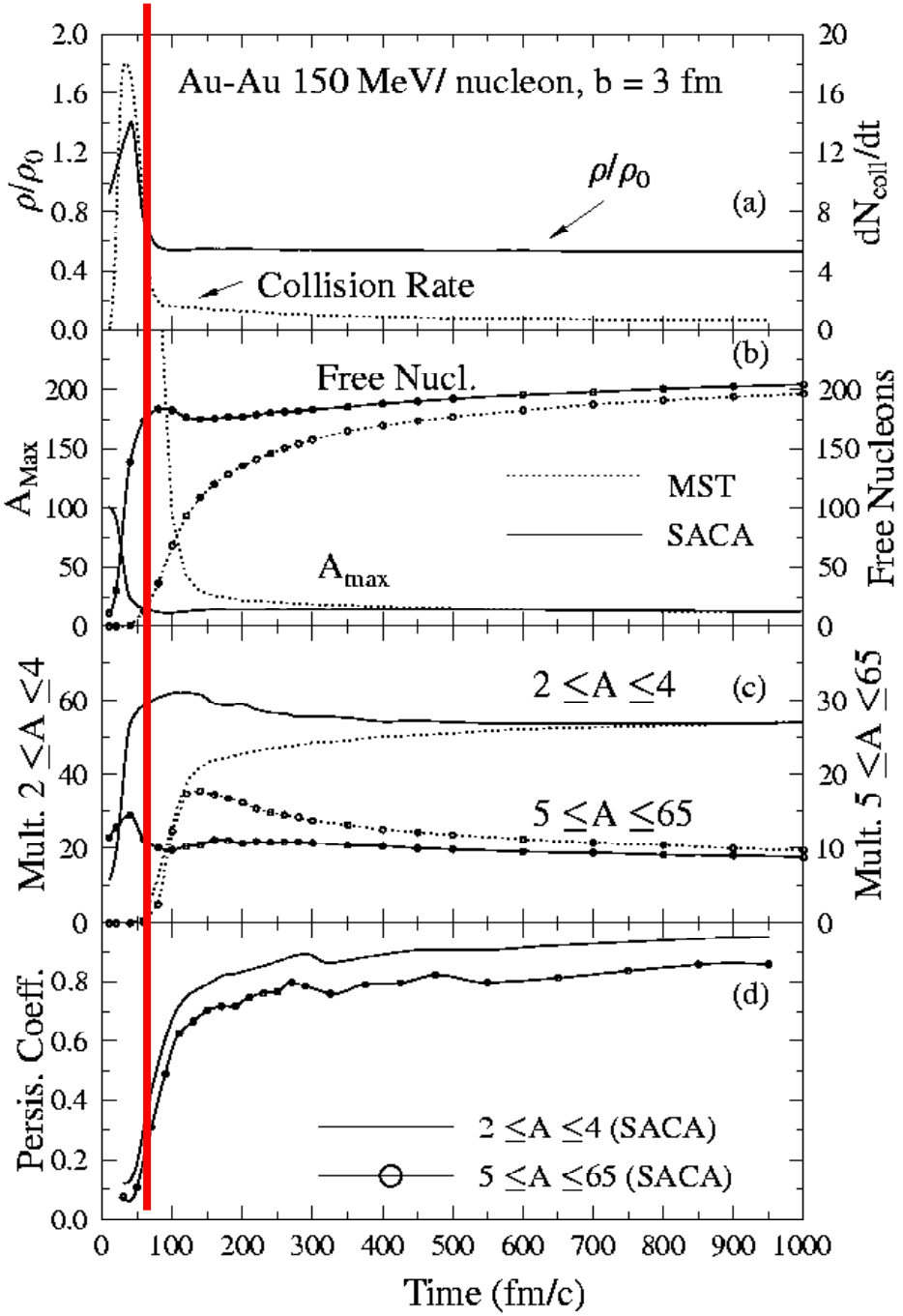
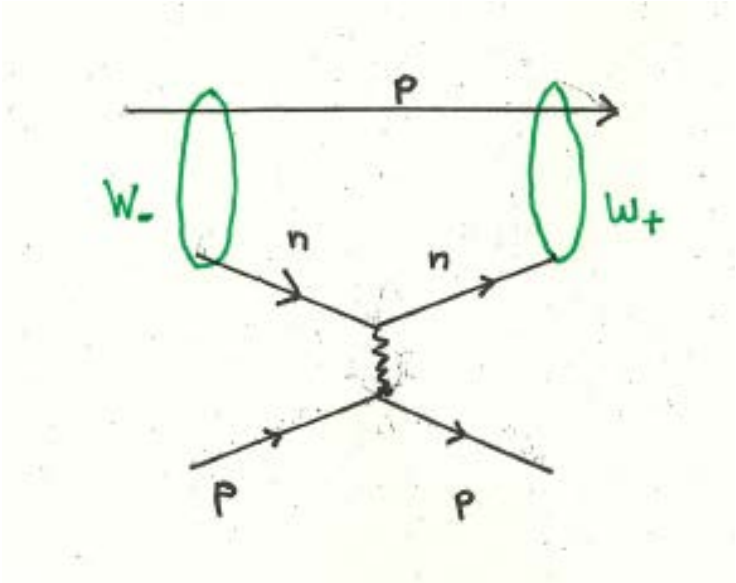


SACA can really identify the fragment pattern very early as compared to the Minimum Spanning Tree (MST) which assumes that two nucleons form a fragment if they are closed than r_{max} .

At $1.5t_{pass}$ Amax and multiplicities of intermediate mass fragments are determined

III. Wigner density formalism (Remler (NPA 402, 596))



Deuteron wave function

$$\Psi_d(\mathbf{r},\mathbf{R}) \propto exp^{-(\mathbf{r}-\mathbf{r_0})^2 L} exp^{-(\mathbf{R}-\mathbf{R_0})^2 L/4}$$

Deuteron Wigner density

$$\rho_d^W(\mathbf{r},\mathbf{p}) \propto exp^{-(\mathbf{r}-\mathbf{r_0})^2 L} exp^{-(\mathbf{p}-\mathbf{p_0})^2/L\hbar}$$

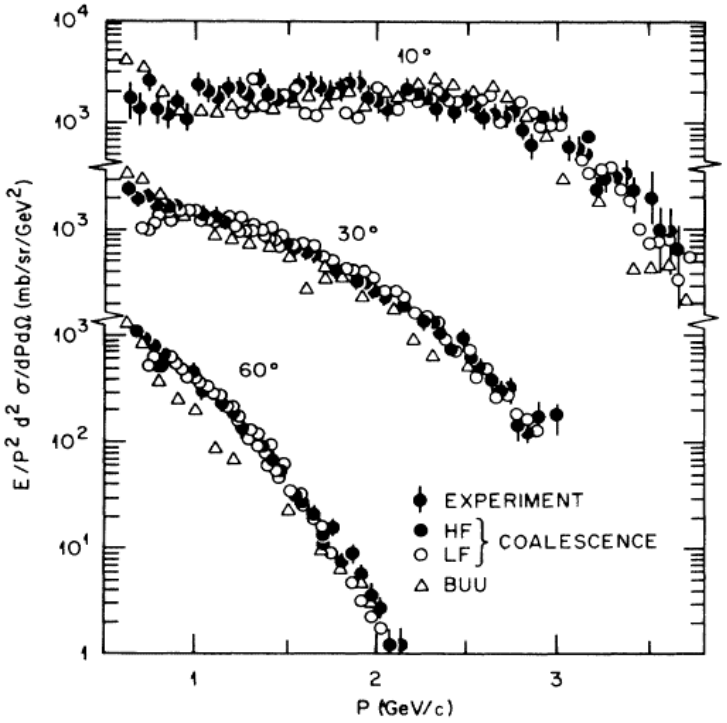
Yields for the rate

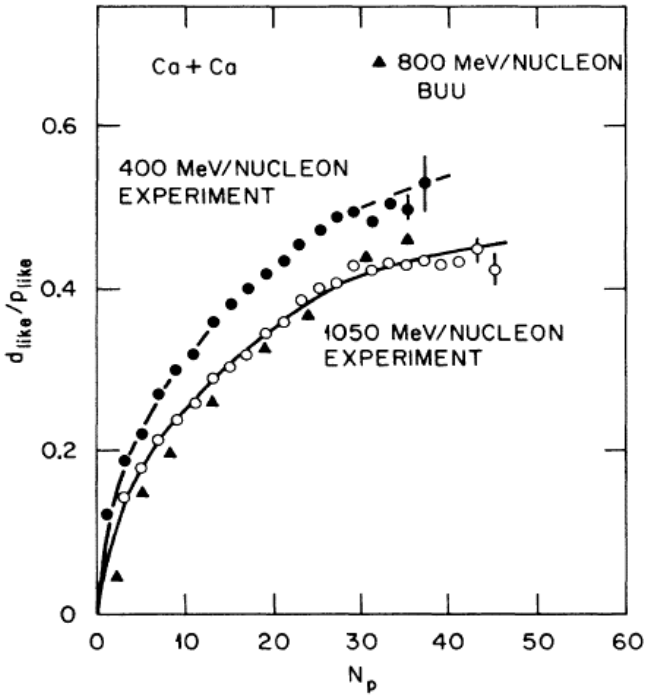
$$\Gamma(t) = \underbrace{\sum_{i=1,2}\sum_{j\geq 3}}_{\text{coll between n or p and rest}} \delta(t-t_{ij}(\nu)) \overbrace{\int \prod_i \frac{d^3p_i d^3x_i}{h^3} \underbrace{\rho_d^W(\mathbf{p_1},\mathbf{x_1},\mathbf{p_2},\mathbf{x_2})}_{\text{Wigner density of d}} [\rho_N^W(t+\epsilon) - \rho_N^W(t-\epsilon)]}^{W^+}$$

$$= \sum_{i=1,2}\sum_{j>3} \delta(t-t_{ij}(\nu))[\rho_d^W(t+\epsilon) - \rho_d^W(t-\epsilon)]$$

Easy to apply at SIS energies

Ca+Ca 800 AMeV (PRC35,1291)





At higher energies: role of baryonic resonances ?
role of mesons
role of d* ->Dima's talk

Conclusions

Hybrid models (where one changes the modelling of the system)
very useful to parametrize the data
results are difficult to interpret

Dynamical models
need a n-body approach for the dynamics of the nucleons

Minimum spanning tree (only applicable for t → ∞)

Simulated annealing (SACA,FRIGA)
can identify fragments during the HI reaction
→ allow for identifying when and how fragments are formed
not easy to be applied for small clusters

Wigner density: tool based on quantum mechanics
only for small clusters

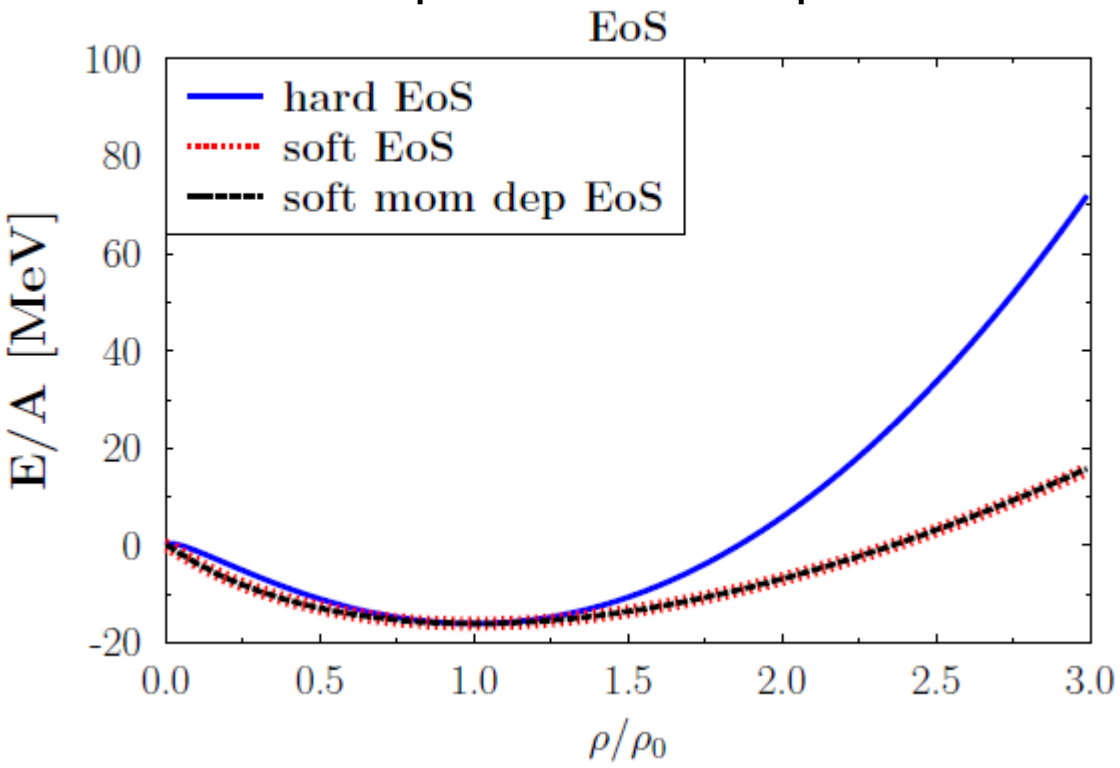
Models are quite successful to interpret cluster data
But still a lot of work to be done to apply them from SIS18 to LHC

How they can be applied in the context of a modern transport approach
➔ wait for the next talks

Backup slides

How to fix the strength of the potential?

In infinite matter a potential corresponds to an equation of state (EoS)

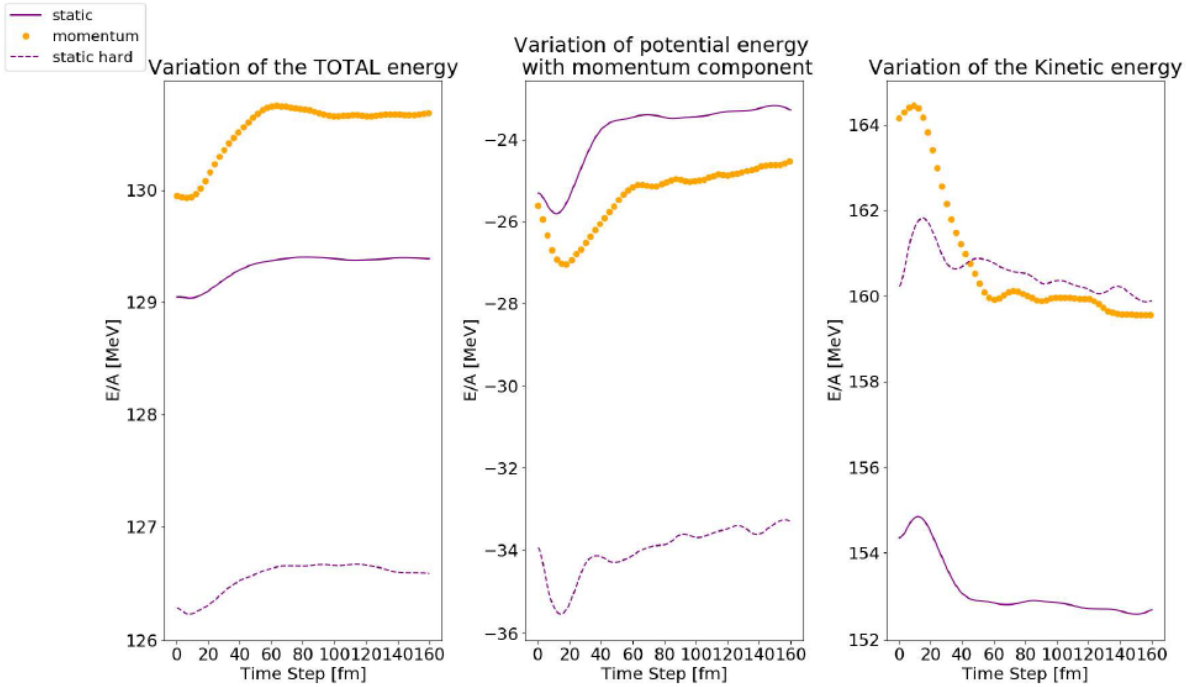


Equation of state cannot be calculated:
Brückner G-matrix is a low density expansion:
Expansion parameter : $a \cdot k_F$ a=hard core range (.6 *fm*)

$k_F = p_F / \hbar = 1.28\ (\rho/\rho_0)^{1/3}\ \frac{1}{fm}$

fixing by experiments (K^+ ,v_1 , v_2 , cluster……)

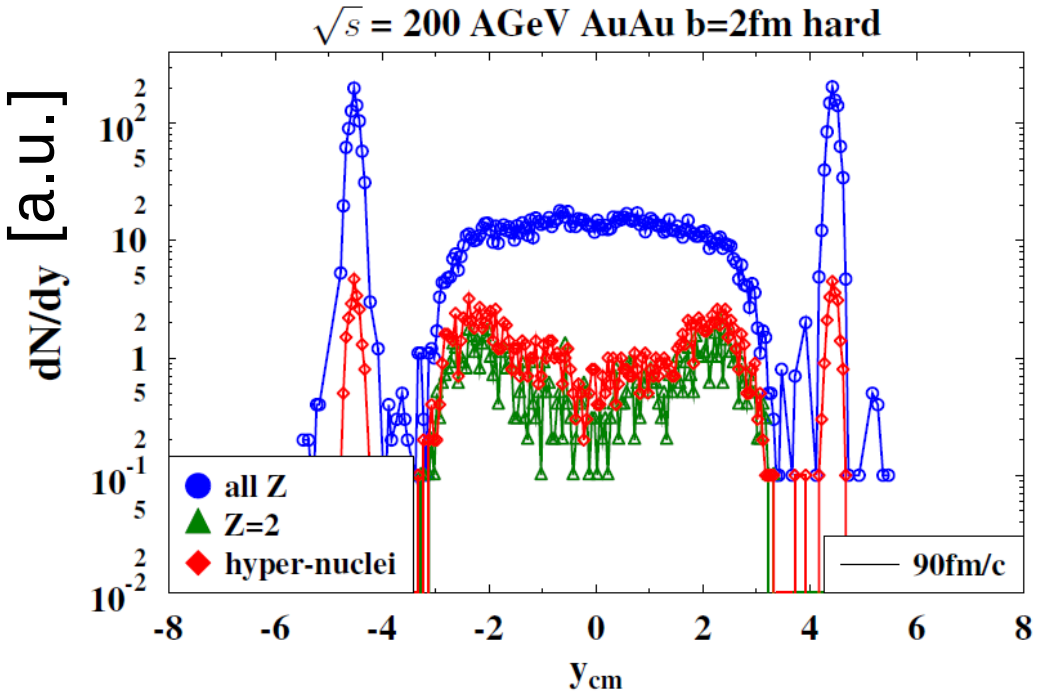
Energy conservation of the numcerical realization
Au+Au 600 AMeV



First Results of PHQMD

At RHIC

hyper-nuclei also from spectator matter
Z=2 fragments at midrapidity
very preliminary



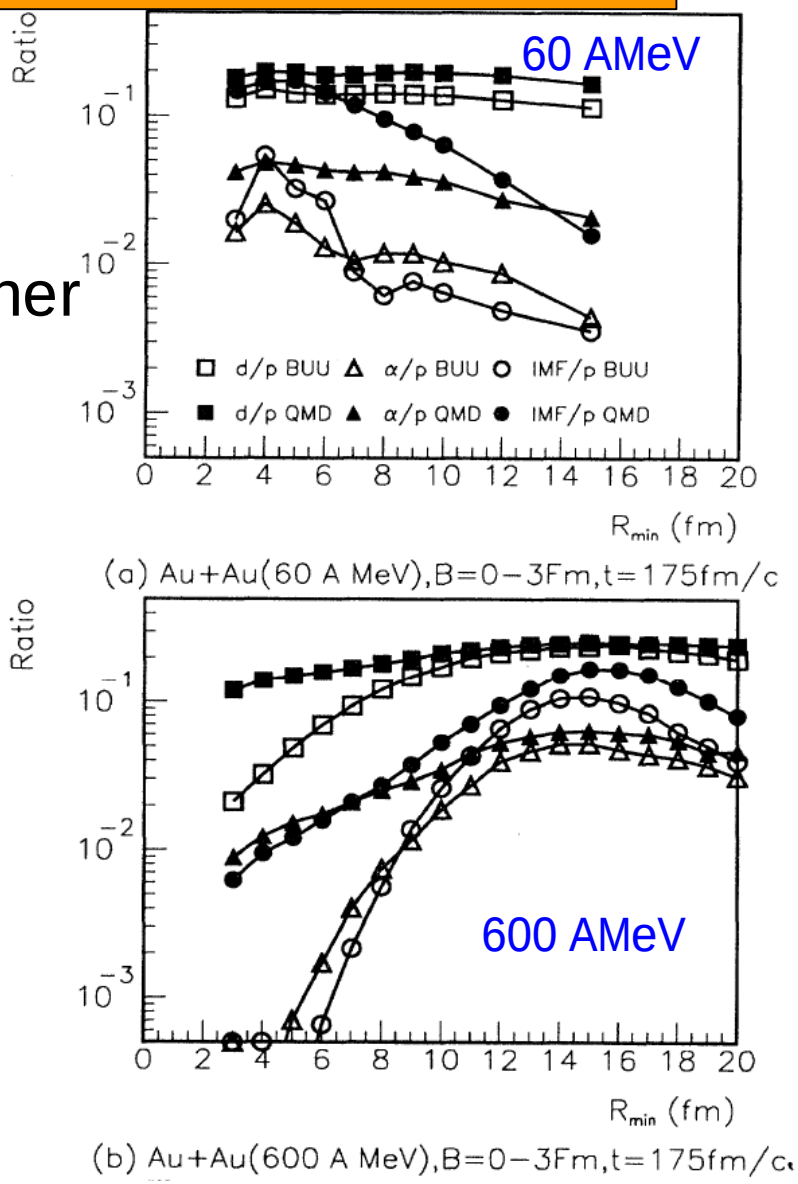
Importance of correlations and fluctuations

MST analysis with variable R_{min}

Two particles i,j are bound together

if $|\vec{r}_i - \vec{r}_j| \leq R_{min}$

BUU obtained by event mixing of QMD events



Gossiaux, Keane (EOS coll) et al
PRC51 (1995) 3357

Bi+Xe, 28 AMeV b=5fm

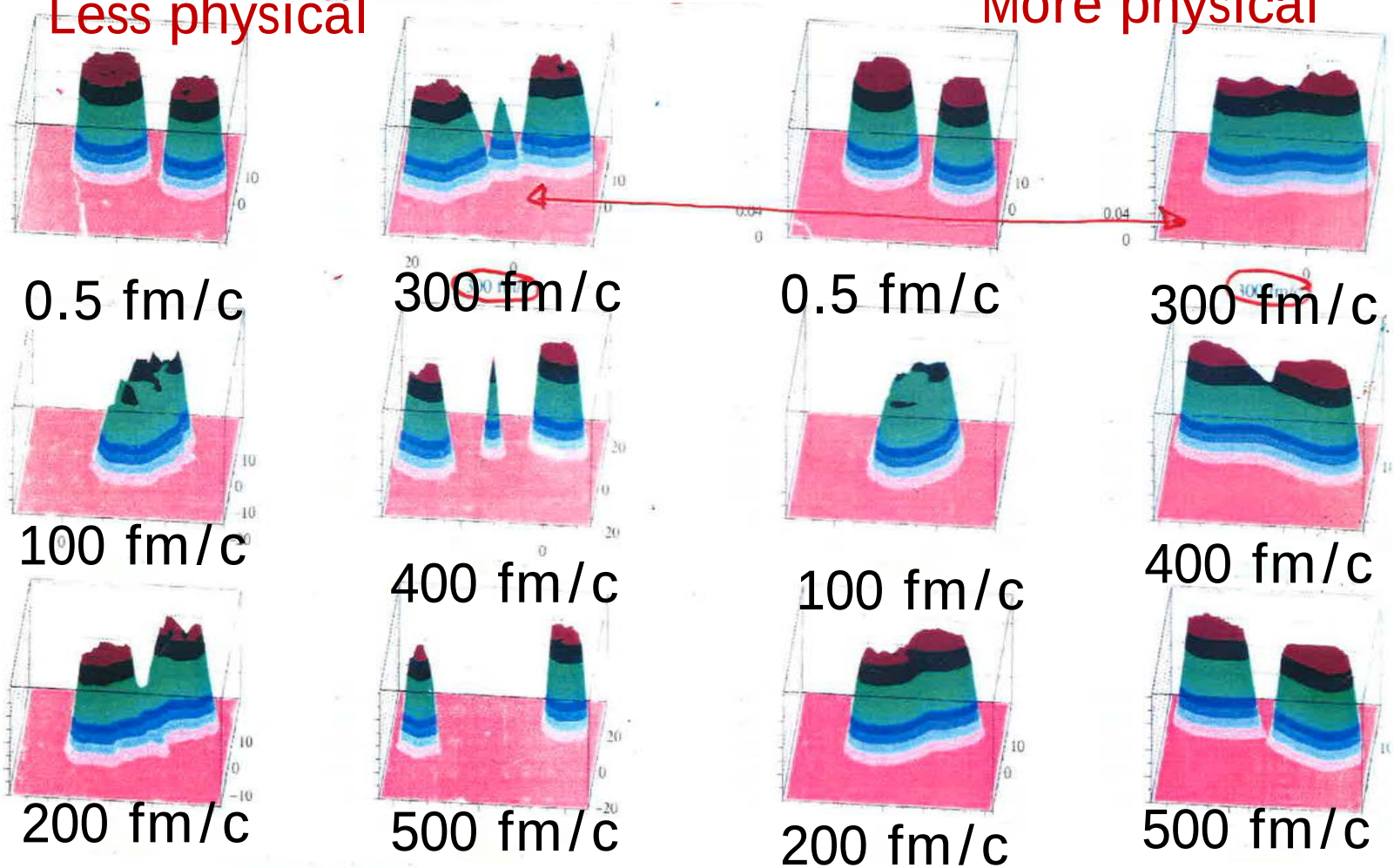
25 test particles/N

W. Bauer
U.Schröder

275 test paricles/N

Less physical

More physical

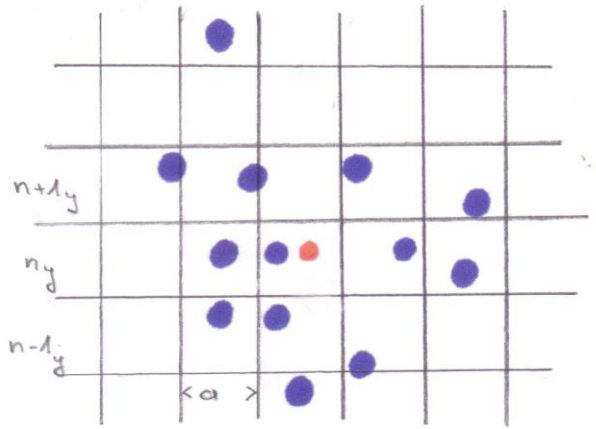


Numbers of test particles must be large enough

When is N sufficiently large?

One uses delta like forces: F(r) = δ(r) (Skyrme) but then point-like test particles f= Σ δ(r-r_i(t)) do almost never interact. Solution: one uses grids (and introduces the grid size **a** which plays a similar role as the width in QMD).

Euler

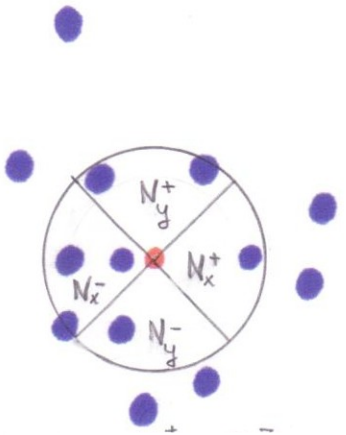


n_x-1 n_x n_x+1

$$F_x = \frac{U_{n_x+1}(\rho) - U_{n_x+1}(\rho)}{2a}$$

Result different if number of test particles is finite (usually N=100)

Lagrange



$$F_x = \frac{U_{x\ right}(\rho) - U_{x\ left}(\rho)}{2a}$$

Average distance between nucleons 2fm. Grid size ≈ 1fm (surface). Therefore very many test particles necessary to avoid numerical fluctuations: 100tp->12 in a cell->30% fluctuation

VUU, BUU, HSD, SMASH solve a Boltzmann type eq.

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla f + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}$$

Same interaction, not possible classically

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = \int\int g I(g,\Omega)[f(\mathbf{p'}_A,t) f(\mathbf{p'}_B,t) - f(\mathbf{p}_A,t) f(\mathbf{p}_B,t)]\, d\Omega\, d^3\mathbf{p}_A\, d^3\mathbf{p}_B.$$

v · differential cross section

Only the test particle method made it possible to solve the BUU equations in complex situations

Test particle method -> replace integrals by sums (MC) integration

$$f(\mathbf{r},\mathbf{p},t) = \sum_{i=1}^{N\to\infty} \delta(\mathbf{r} - \mathbf{r}_i(t))\ \delta(\mathbf{p} - \mathbf{p}_i(t))$$

test particle ≠ nucleon

If N small unphysical fluctuations

What means N ->∞ in reality ?