Wroclaw, Dec. 6, 2019

Bound states in a hot environment

Gerd Röpke, Rostock

Outline

1. Correlations in nuclear systems
2. Freeze-out approximation
3. Thermodynamic equilibrium: the spectral function
4. Beth-Uhlenbeck equation (S-matrix approach)
5. Application to HIC
6. Weakly bound nuclei and astrophysics

1. Correlations and bound states

- Nuclear systems (nuclei, nuclear matter, neutron stars, HIC,…): dense, strongly interacting many-particle systems (also: dense plasmas, warm dense matter, QGP, etc.)
- Ideal quantum gases? Correlations, formation of bound states. Quantum condensates, correlations in the continuum.
  Quantum statistical approach is needed.
- Single-particle approximation: Quasiparticles, mean-field approximation, effective mass, shell-model, transport codes
- How to include correlations, in particular bound states?

  Talk of Elena Bratkovskaya: PHQMD: ‘price to pay for profit’
- In nuclei: Hoyle state, molecule-like states (^{9}Be,…). Pairing,…
- In nuclear matter (astrophysics): Saha equation, nuclear statistical equilibrium, … pasta phases, Beth-Uhlenbeck equation
- In HIC: cluster formation, freeze-out – coalescence? Transport codes

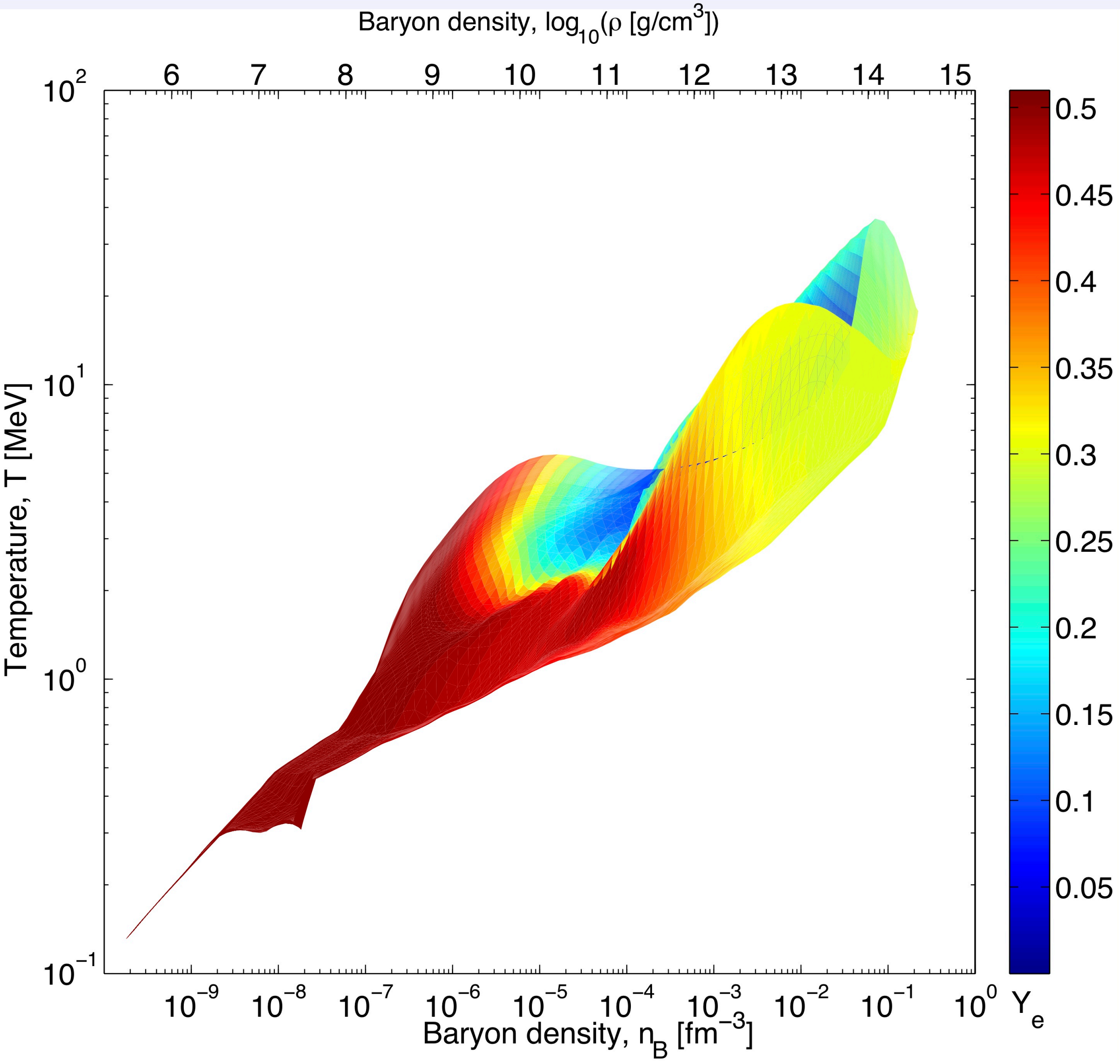
  N-body correlations are important for cluster formation!

Nuclear matter phase diagram

Core collapse supernovae

Relevant Parameters:

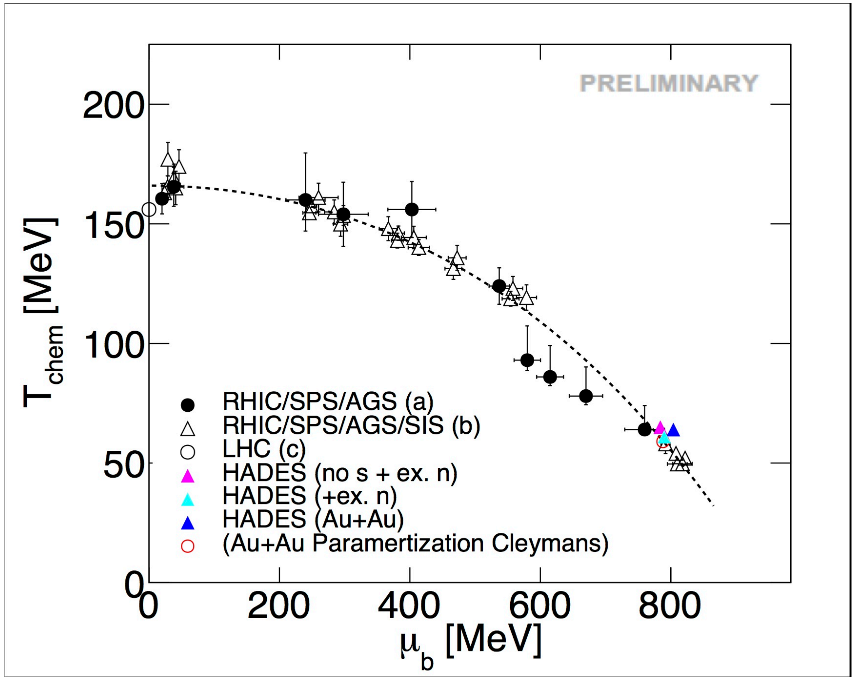
- **density**:
  $10^{-9} \lesssim \varrho/\varrho_{\rm sat} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\rm sat} \approx 2.5 \cdot 10^{14}$ g/cm^3
  ($n_{\rm sat} = \varrho_{\rm sat}/m_n \approx 0.15$ fm^{-3})
- **temperature**:
  0 MeV $\leq k_B T \lesssim 50$ MeV
  ($\hat{=}\ 5.8 \cdot 10^{11}$ K)
- **electron fraction**:
  $0 \leq Y_e \lesssim 0.6$



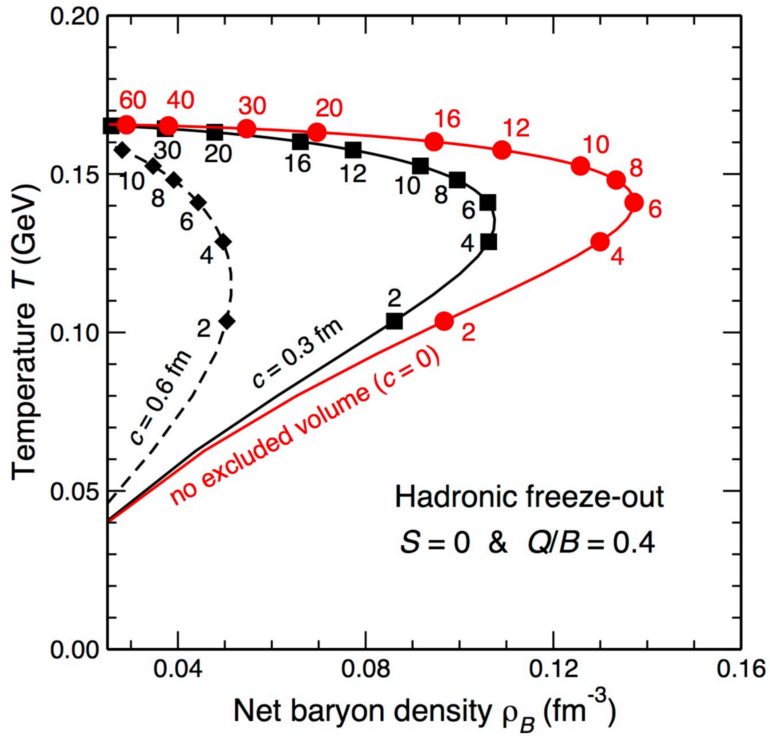
T. Fischer, GSI Darmstadt
U Wroclaw

Freeze-out in the phase diagram

Talk given by Manuel Lorenz



Jørgen Randrup, Jean Cleymans:
Freezeout density (2016)



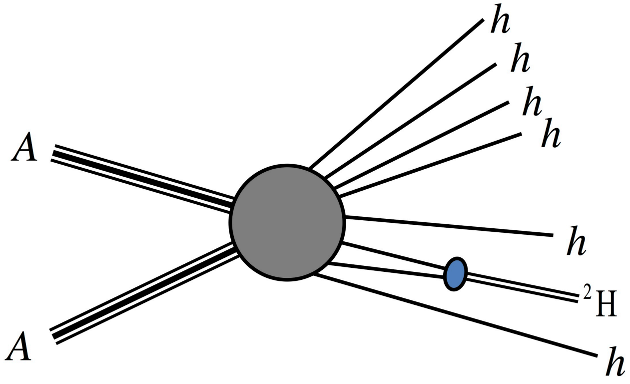
Nonequilibrium evolution of the fireball.
Where the clusters are formed? Very early? Late?

2. How to form clusters?

Nuclear reactions, nonequilibrium process

Talk given by Stanisław Mrówczyński

Final state interaction – conventional approach to production of light nuclei



Binding energy of a deuteron is ε_B = 2.2 MeV.

A characteristic time of deuteron formation is $1/\varepsilon_B$ = 100 fm/c.

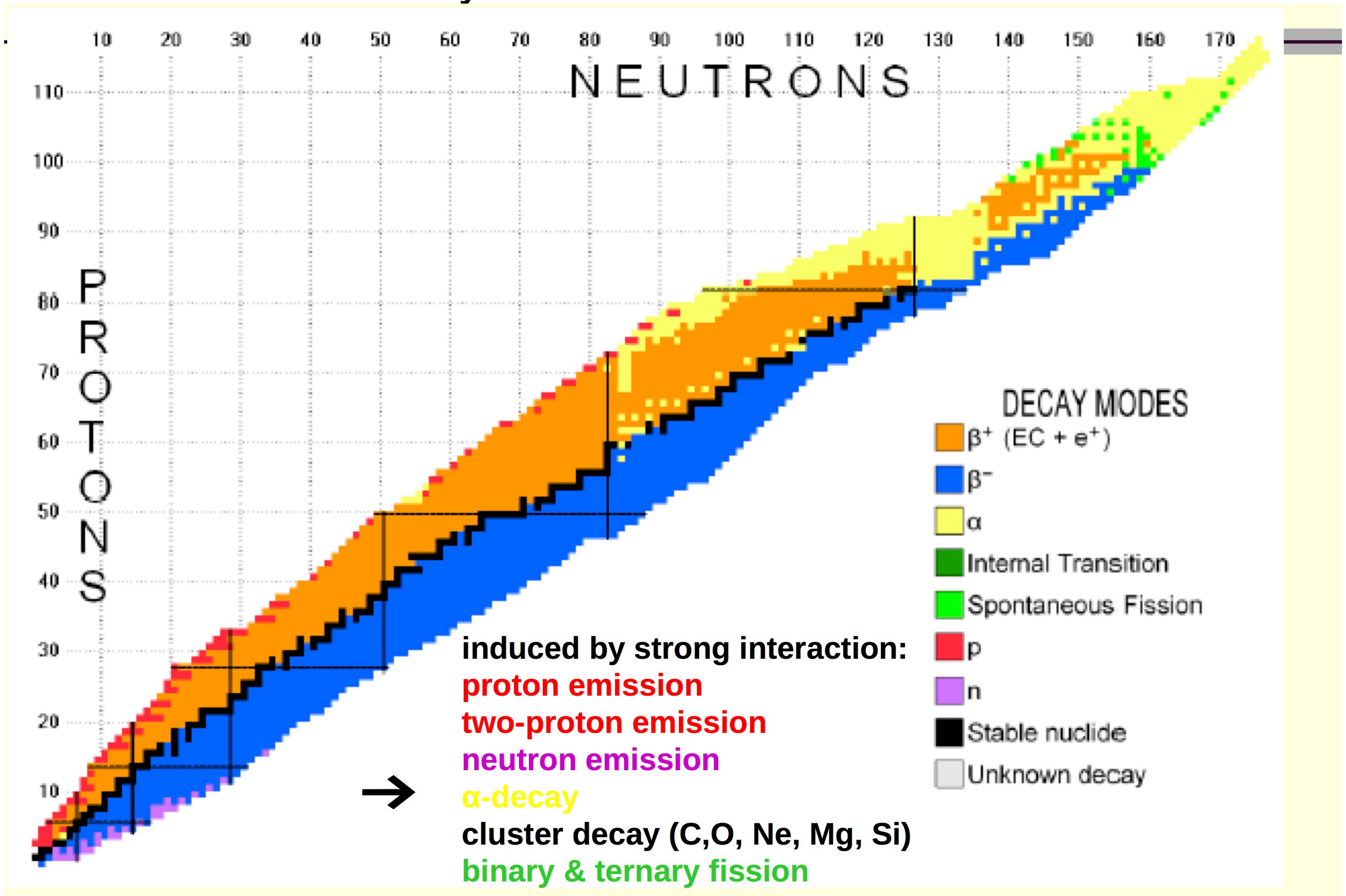
resonances, ^{4}Li ?

energy scale of particle production - particle's mass >> energy scale of formation of a nucleus - binding energy

fast process | slow process

α decay of heavy nuclei

Decay modes of nuclei



induced by strong interaction:

proton emission

two-proton emission

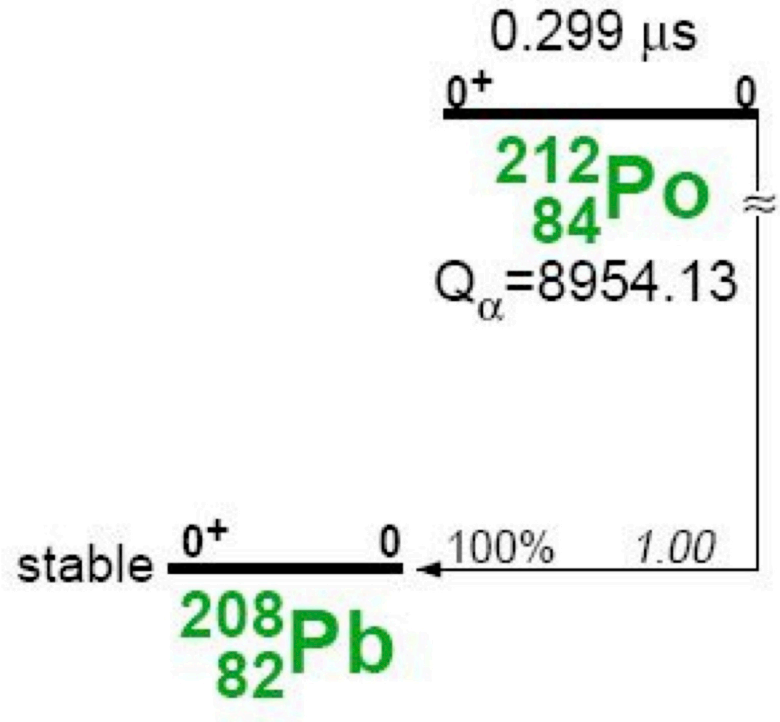
neutron emission

→ **α-decay**

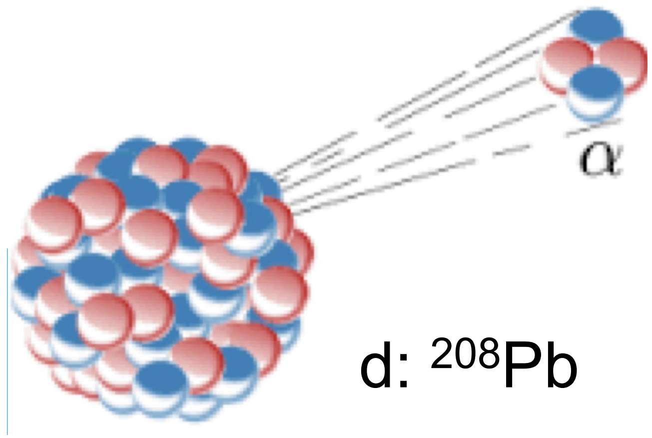
cluster decay (C,O, Ne, Mg, Si)

binary & ternary fission

Preformation: α decay of ^{212}Po



m: ^{212}Po



d: ^{208}Pb

^{212}Po: α on top of ^{208}Pb

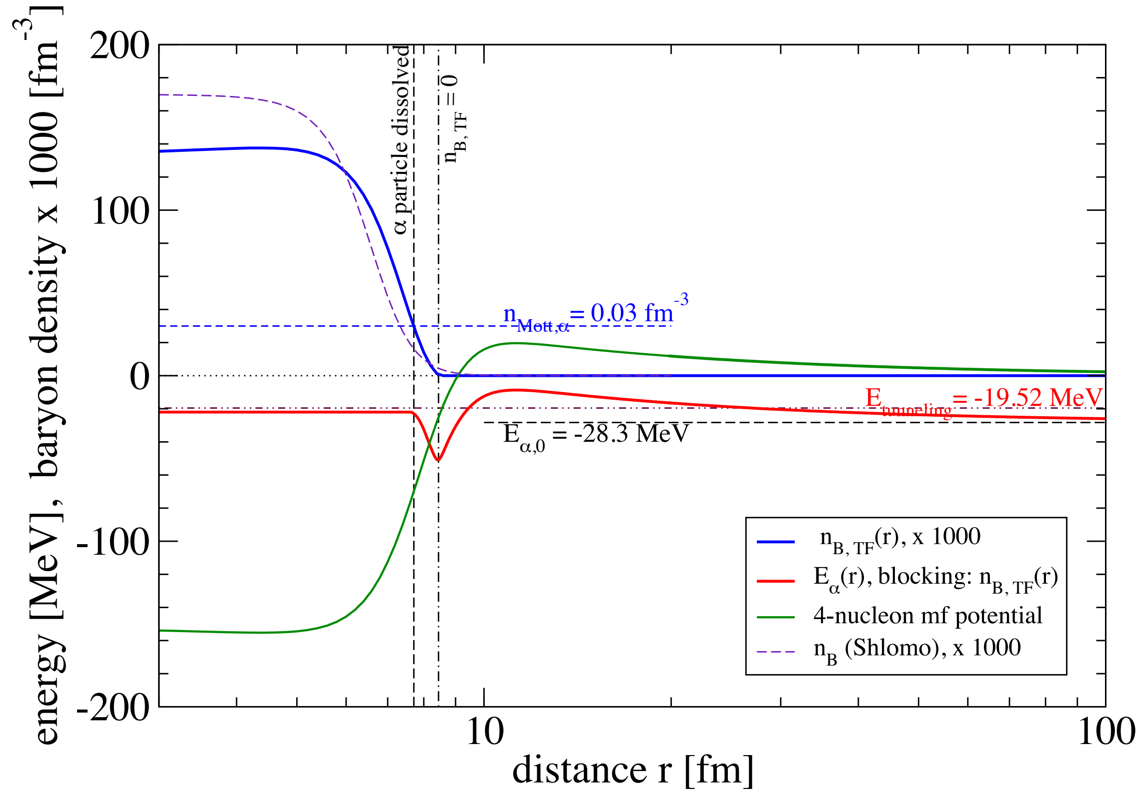
Where the α is formed? Preformation factor?

Local effective potential W(**R**)
with respect to the ^{208}Pb core.

Woods-Saxon potential
of 2 neutrons and 2 protons
including Coulomb repulsion.

Density in Thomas-Fermi
approximation
with chemical potential fixed
by the total nucleon number

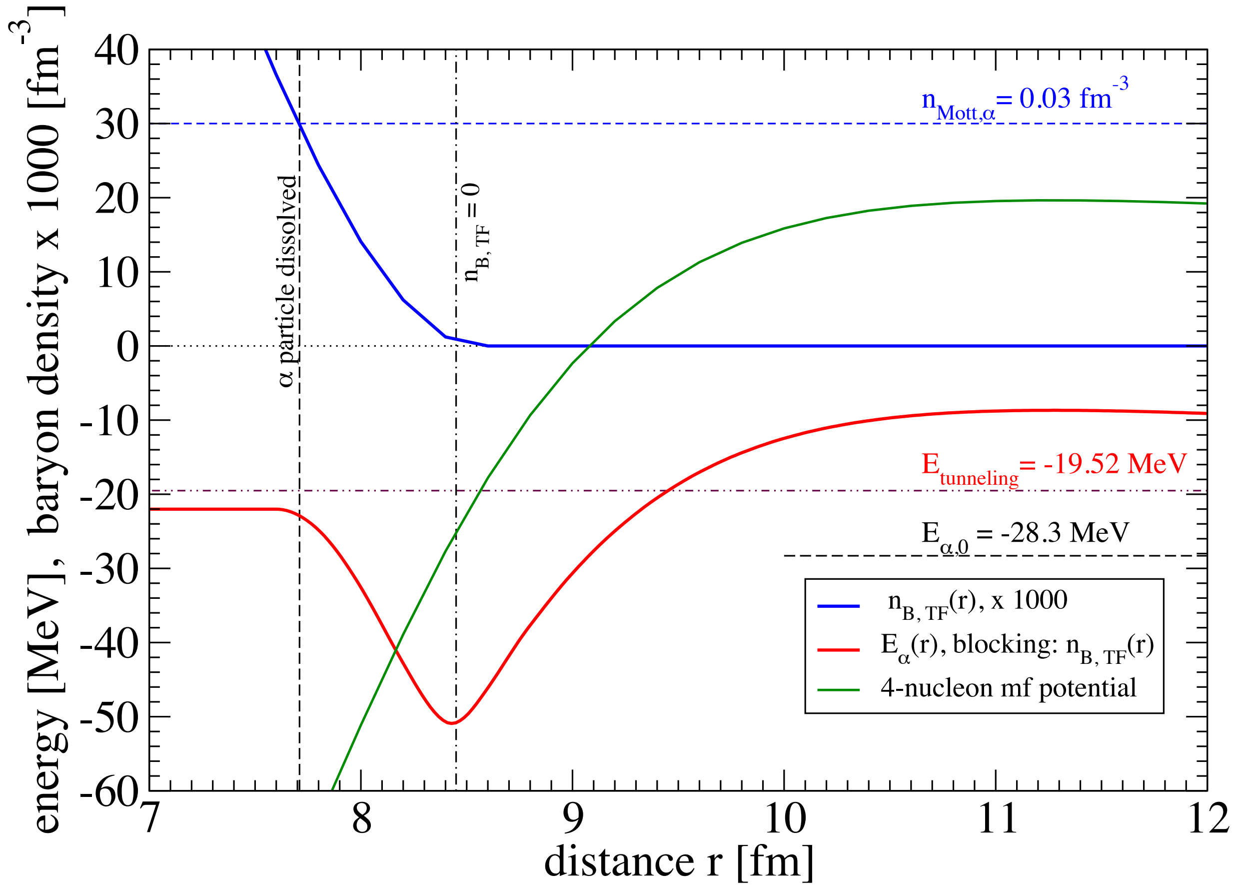
Pauli-blocking of the α particle



G. R. et al., PRC **90**, 034304 (2014),
C. Xu et al., PRC **93**, 011306(R) (2016)

^{212}Po: α on top of ^{208}Pb

Bound state (quartet) in a dense environment



Alpha Decay to Doubly Magic Core in
Quartetting Wave Function Approach
arXiv1912.01151: ^{104}Te

Nonequilibrium statistical operator (NSO)

Nonequilibrium – (local) thermodynamic equilibrium: freeze-out concept

principle of weakening of initial correlations (Bogoliubov, Zubarev)

$$\rho_\epsilon(t) = \epsilon \int_{-\infty}^{t} e^{\epsilon(t_1 - t)} U(t, t_1) \rho_{\mathrm{rel}}(t_1) U^\dagger(t, t_1)\, dt_1$$

time evolution operator $U(t, t_0)$

relevant statistical operator $\rho_{\mathrm{rel}}(t)$ maximum of information entropy

selection of the set of relevant observables $\{B_n\}$

self-consistency relations $\mathrm{Tr}\{\rho_{\mathrm{rel}}(t) B_n\} \equiv \langle B_n \rangle^t_{\mathrm{rel}} = \langle B_n \rangle^t$

extended von Neumann equation

$$\frac{\partial}{\partial t} \varrho_\varepsilon(t) + \frac{i}{\hbar} [H, \varrho_\varepsilon(t)] = -\varepsilon \left(\varrho_\varepsilon(t) - \varrho_{\mathrm{rel}}(t) \right)$$

$\varrho(t) = \lim_{\varepsilon \to 0} \varrho_\varepsilon(t)$ after thermodynamic limit

Relevant statistical operator

State of the system in the past $\mathrm{Tr}\{\rho(t)B_n\} = \langle B_n \rangle^t$

Construction of the relevant statistical operator at time t

$$S_{\mathrm{rel}}(t) = -k_{\mathrm{B}}\,\mathrm{Tr}\{\rho_{\mathrm{rel}}(t)\log\rho_{\mathrm{rel}}(t)\}$$

-> maximum

$$\delta[\mathrm{Tr}\{\rho_{\mathrm{rel}}(t)\log\rho_{\mathrm{rel}}(t)\}] = 0$$

$$\mathrm{Tr}\{\rho_{\mathrm{rel}}(t)B_n\} \equiv \langle B_n \rangle^t_{\mathrm{rel}} = \langle B_n \rangle^t$$

Generalized Gibbs distribution

$$\rho_{\mathrm{rel}}(t) = \exp\left\{-\Phi(t) - \sum_n \lambda_n(t)B_n\right\}$$

$$\Phi(t) = \log\mathrm{Tr}\exp\left\{-\sum_n \lambda_n(t)B_n\right\}$$

$$\frac{\partial S_{\mathrm{rel}}(t)}{\partial t} = \sum_n \lambda_n(t)\langle \dot{B}_n \rangle^t$$

But: von Neumann equation?
Entropy?

3. Many-particle theory, spectral function

Equation of state

$$n_\tau^{\rm tot}(T,\mu_n,\mu_p) = \frac{1}{\Omega}\sum_{p_1,\sigma_1}\int\frac{d\omega}{2\pi}\frac{1}{e^{(\omega-\mu_\tau)/T}+1}S_\tau(1,\omega)$$

Spectral function

$$S_\tau(1,\omega;T,\mu_n,\mu_p) \qquad E(1)=\hbar^2 p_1^2/2m_1$$

Green function G,
Self-energy Σ

$$S(1,\omega) = 2{\rm Im}\,G(1,\omega+i0) = 2{\rm Im}\frac{1}{\omega-E(1)-\Sigma(1,\omega+i0)}$$

$$S_\tau(1,\omega) = \frac{2{\rm Im}\Sigma(1,\omega-i0)}{(\omega-E(1)-{\rm Re}\Sigma(1,\omega))^2+({\rm Im}\Sigma(1,\omega-i0))^2}$$

Expansion for small damping (Im Σ)

$$S(1,\omega)\approx\frac{2\pi\delta(\omega-E^{\rm quasi}(1))}{1-\frac{d}{dz}{\rm Re}\,\Sigma(1,z)|_{z=E^{\rm quasi}(1)}} - 2{\rm Im}\,\Sigma(1,\omega+i0)\frac{d}{d\omega}\frac{\mathcal{P}}{\omega-E^{\rm quasi}(1)}$$

Quasiparticle energy

$$E^{\rm quasi}(1) = E(1) + {\rm Re}\,\Sigma(1,z)|_{z=E^{\rm quasi}(1)}$$

Correlations (bound states) in Im Σ
Cluster decomposition, Bethe-Salpeter equation

Different approximations

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Different approximations

medium effects

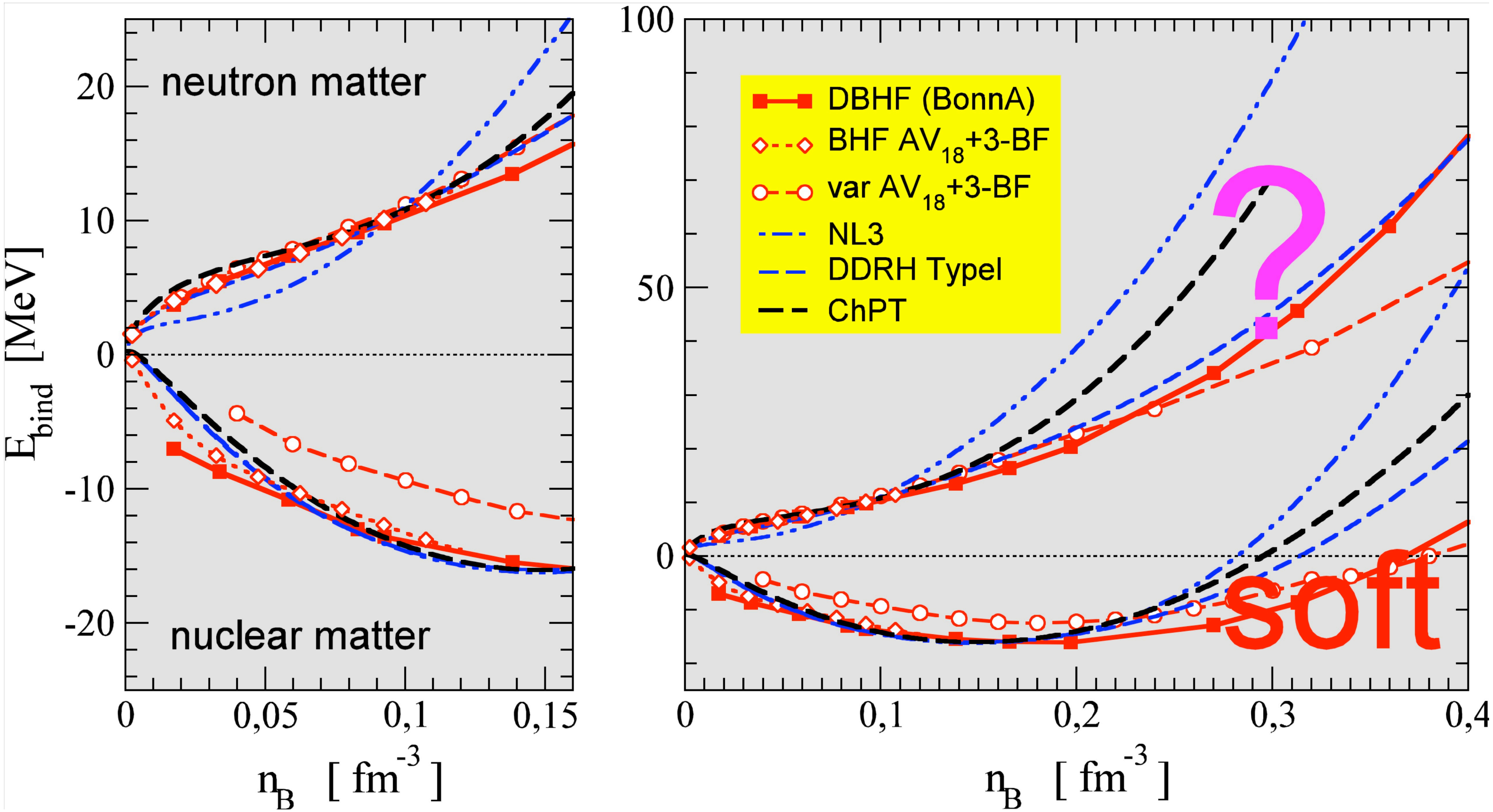
Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

Quasiparticle picture: RMF and DBHF



But: cluster formation

Incorrect low-density limit

C. Fuchs et al.;
J.Margueron et al., Phys.Rev.C **76**,034309 (2007)

Different approximations

medium effects

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

bound state formation

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

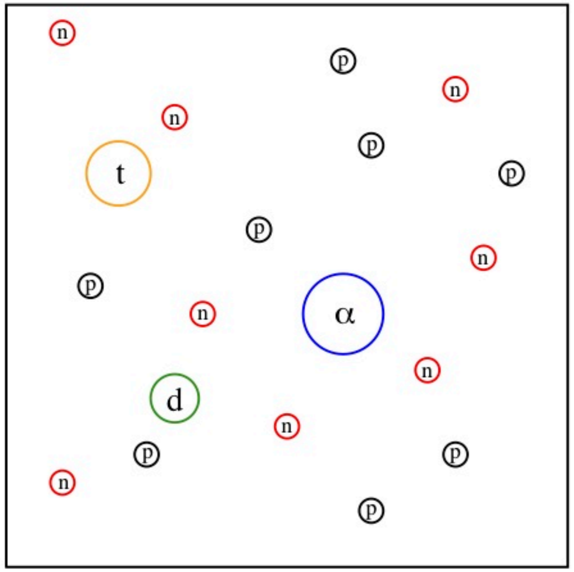
Inclusion of the light clusters (d,t,^{3}He,^{4}He)

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law



Ideal mixture of reacting nuclides

$$n_p(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A \mu_p - (A - Z_A)\mu_n\}$$

$$n_n(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A,\nu,K} (A - Z_A) f_A\{E_{A,\nu K} - Z_A \mu_p - (A - Z_A)\mu_n\}$$

mass number *A*,
charge Z_A,
energy $E_{A,\nu,K}$,
ν internal quantum number,
~*K* center of mass momentum

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

Chemical equilibrium, mass action law,
Nuclear Statistical Equilibrium (NSE)

Different approximations

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

medium effects

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

low density limit

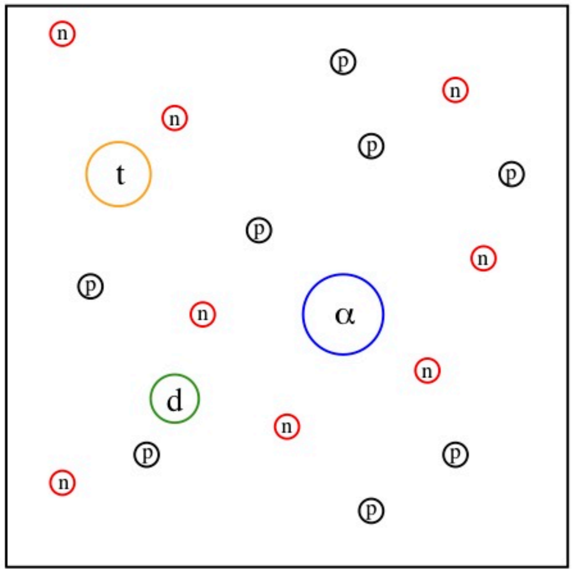
saturation density

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

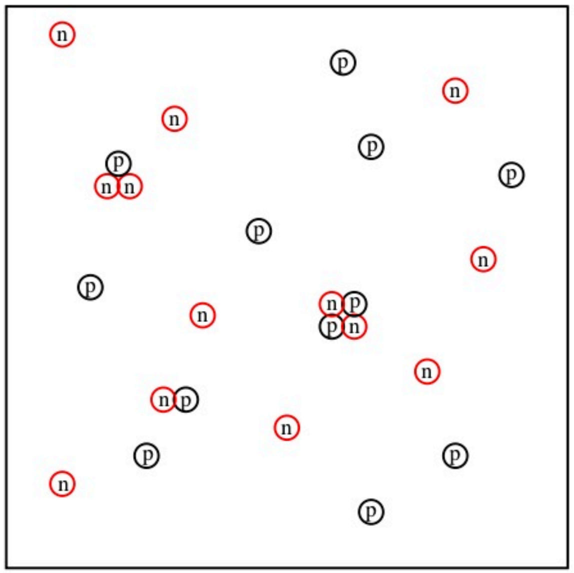
Mass action law



Interaction between the components
internal structure: Pauli principle
"excluded volume"

Physical picture:

"elementary" constituents
and their interaction



Quantum statistical (QS) approach,
quasiparticle concept, virial expansion

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium with quasiparticle clusters:
self-energy and Pauli blocking

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

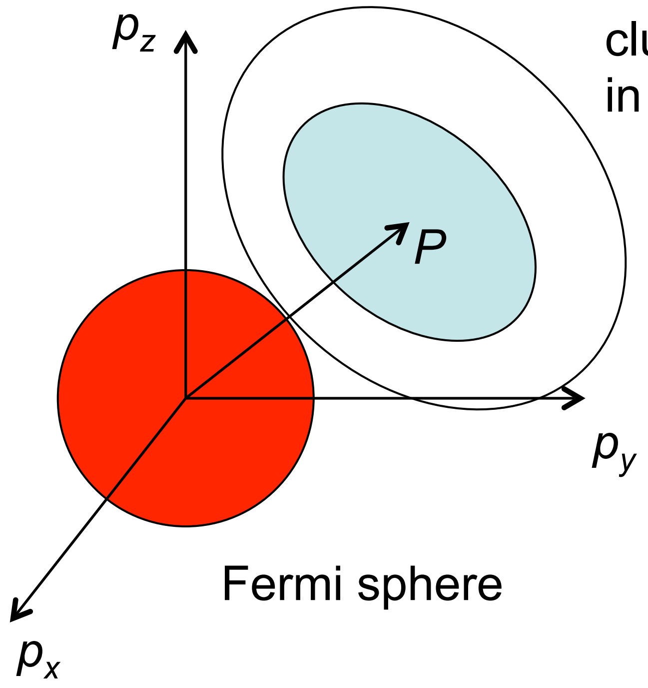
$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

Pauli blocking – phase space occupation



cluster wave function (deuteron, alpha,…)
in momentum space

P - center of mass momentum

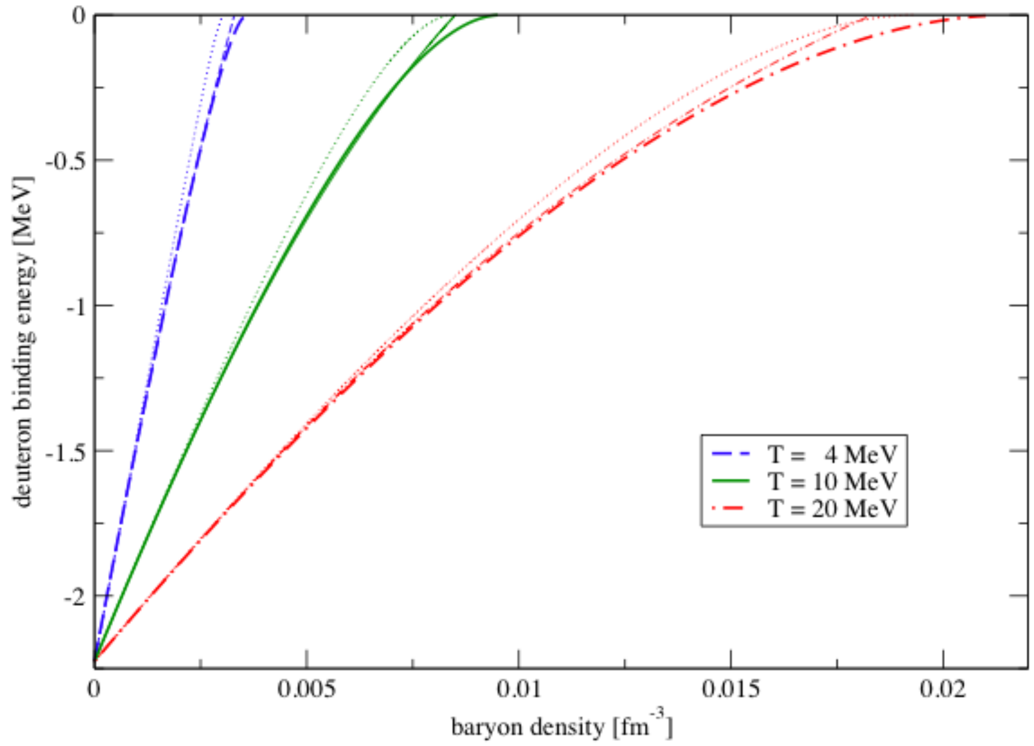
The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum *P*

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Shift of the deuteron bound state energy

Dependence on nucleon density, various temperatures,
zero center of mass momentum

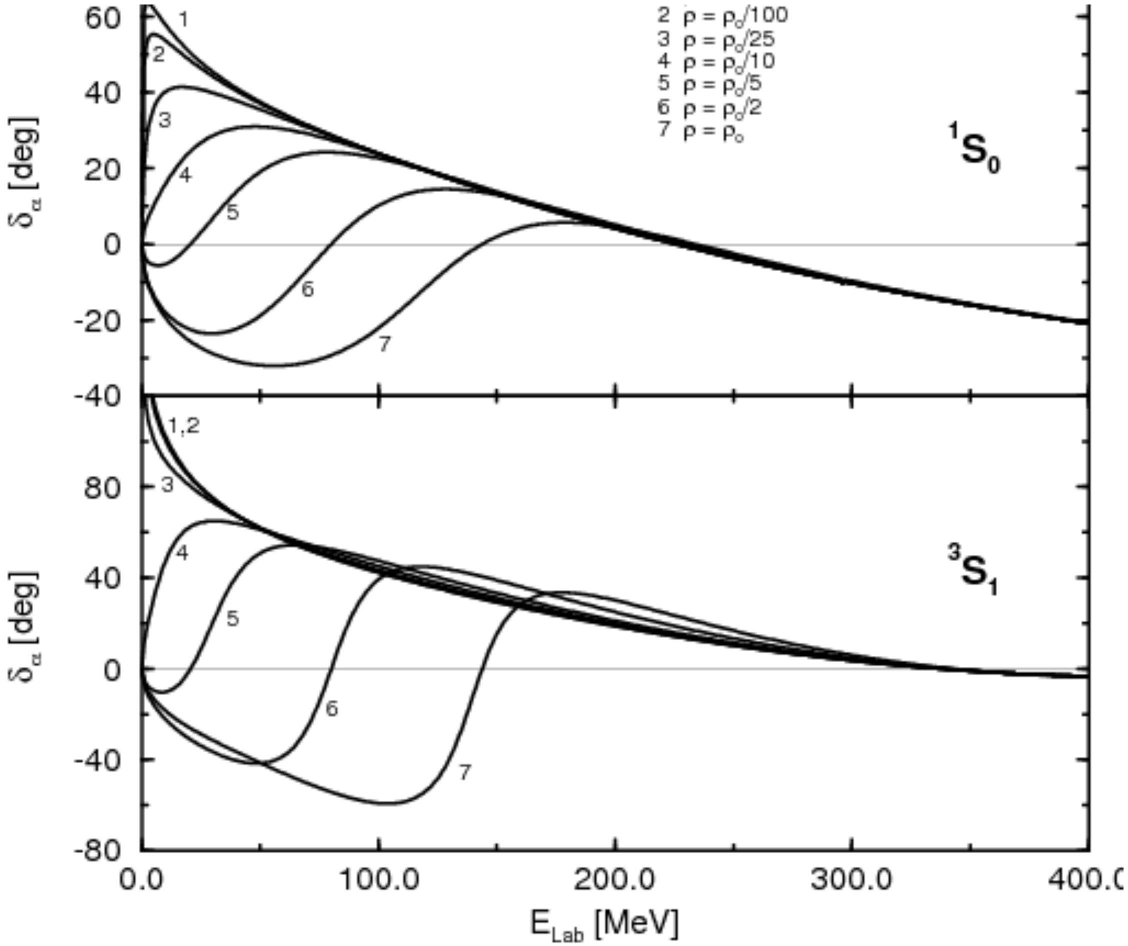


thin lines:

fit formula

G.R., Nucl. Phys. A **867**, 66 (2011)

Scattering phase shifts in matter



Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

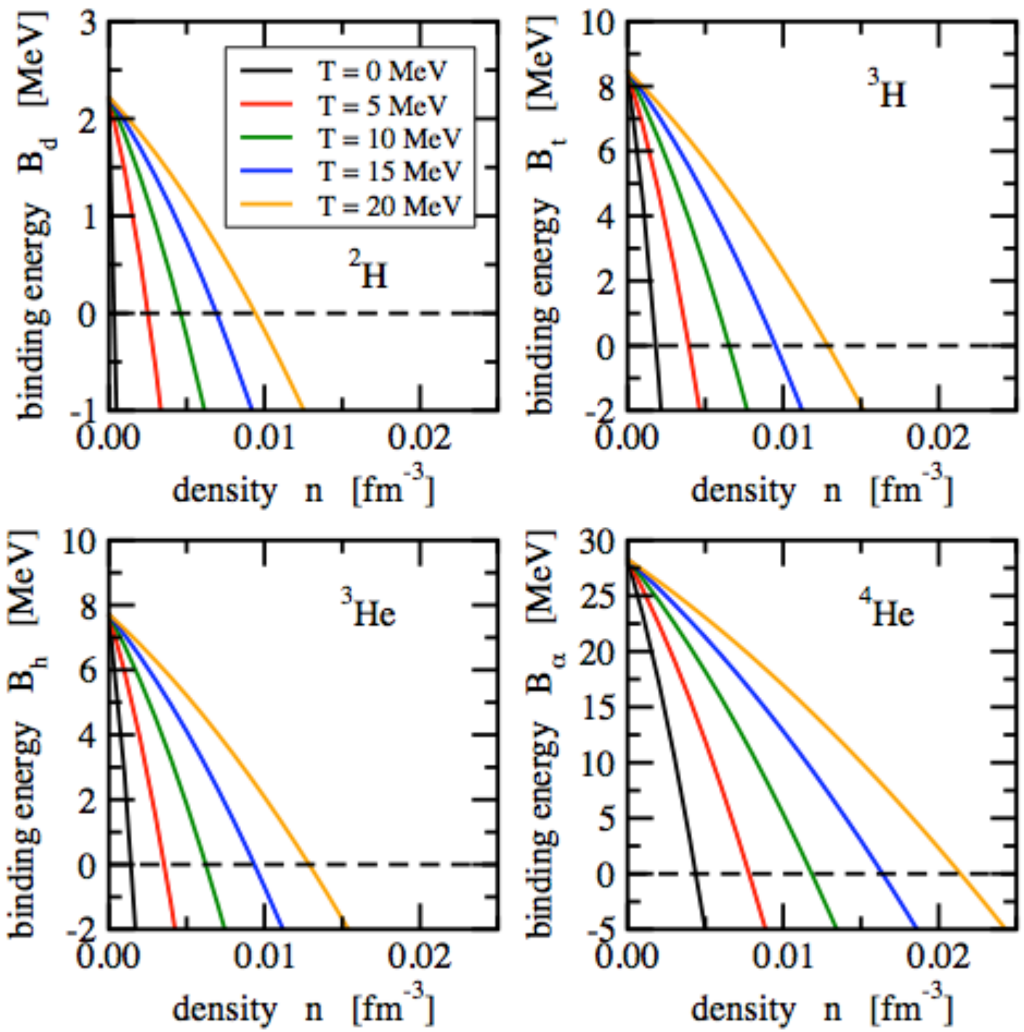
mass number A
charge Z_A
energy $E_{A,\nu,K}$

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

ν: internal quantum number
excited states, continuum correlations

- Medium effects: correct behavior near saturation
self-energy and Pauli blocking shifts of binding energies,
Coulomb corrections due to screening (Wigner-Seitz, Debye)

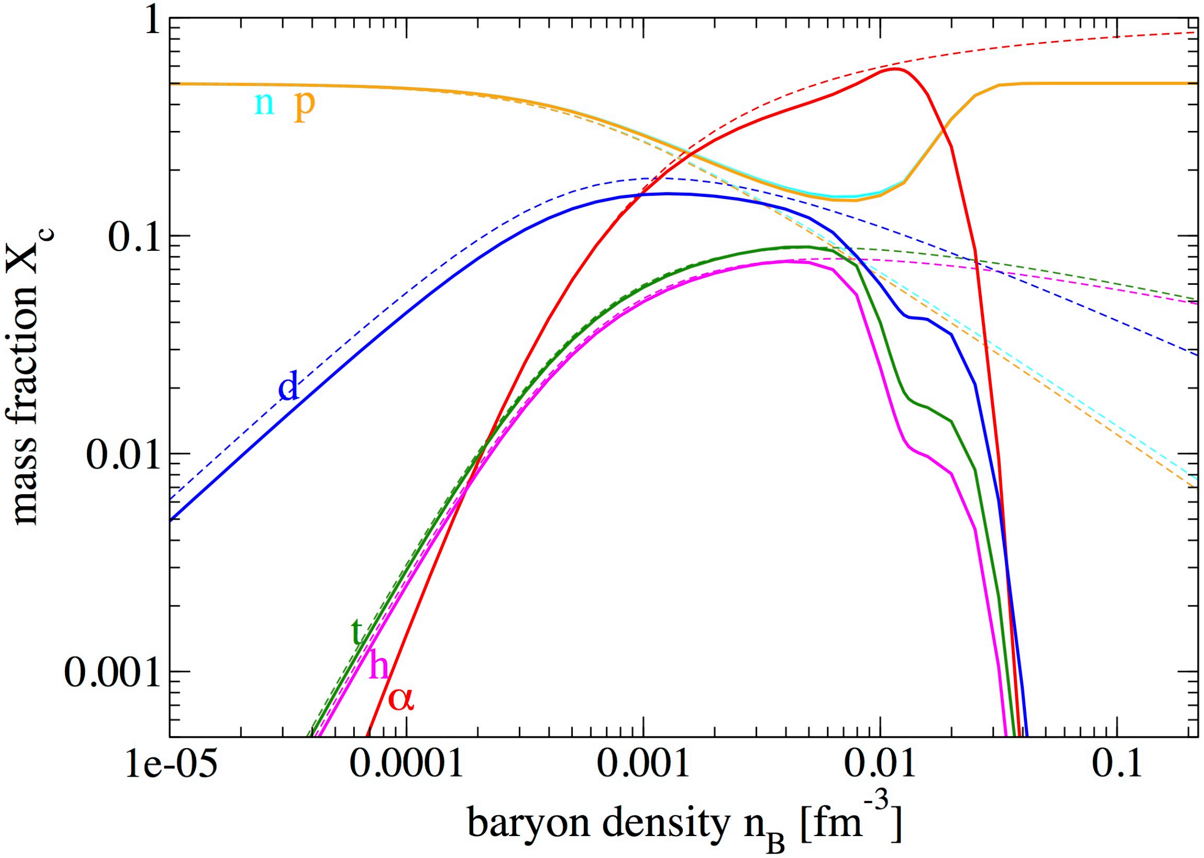
Shift of Binding Energies of Light Clusters



Symmetric matter

G.R., PRC **79**, 014002 (2009)
S. Typel et al.,
PRC **81**, 015803 (2010)

Light Cluster Abundances



Composition of symmetric matter in dependence on the baryon density n_B, T = 5 MeV.
Quantum statistical calculation (full) compared with NSE (dotted).

G. R., PRC **92**, 054001 (2015)

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium
of quasiparticle clusters:
self-energy and Pauli blocking

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.Eq.

Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

chemical & physical picture

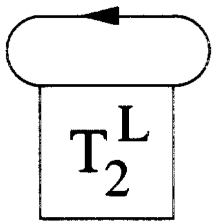
Cluster virial approach:
all bound states (clusters)
scattering phase shifts of all pairs

Correlated medium:
phase space occupation by all bound states
in-medium correlations, quantum condensates

4. Beth-Uhlenbeck formula

low density limit:

$$G_2^L(12,1'2',i\lambda) = \sum_{n\mathbf{P}} \Psi_{n\mathbf{P}}(12) \frac{1}{i\omega_\lambda - E_{n\mathbf{P}}} \Psi^*_{n\mathbf{P}}(12)$$

$\Sigma =$ 

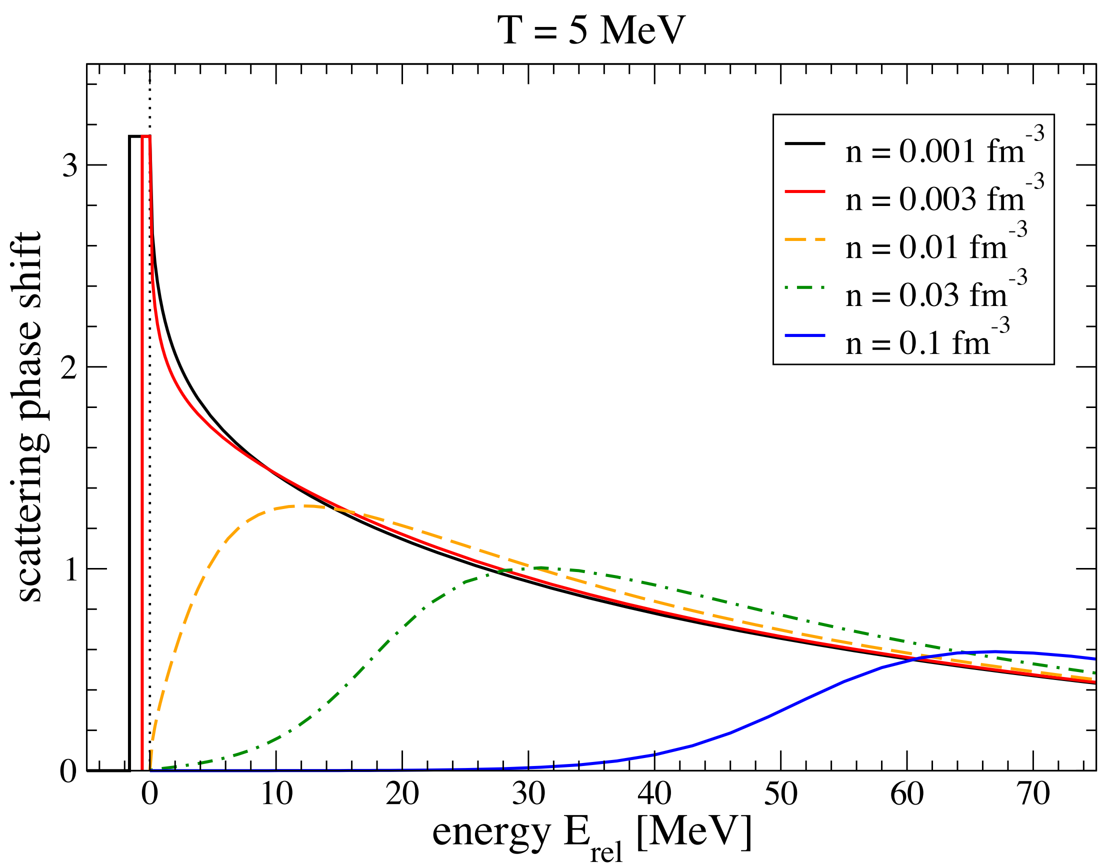
$$n(\beta,\mu) = \sum_1 f_1(E^{\text{quasi}}(1)) + \sum_{2,n\mathbf{P}}^{\text{bound}} g_{12}(E_{n\mathbf{P}})$$

$$+ \sum_{2,n\mathbf{P}} \int_0^\infty \mathrm{d}k\; \delta_{\mathbf{k},\mathbf{p}_1-\mathbf{p}_2} g_{12}(E^{\text{quasi}}(1)+E^{\text{quasi}}(2)) 2\sin^2\delta_n(k) \frac{1}{\pi} \frac{\mathrm{d}}{\mathrm{d}k}\delta_n(k)$$

- generalized Beth-Uhlenbeck formula
  correct low density/low temperature limit:
  mixture of free particles and bound clusters

Deuteron-like scattering phase shifts

Virial coeff. $\propto e^{-E_d^0/T} - 1 + \frac{1}{\pi T}\int_0^\infty dE\, e^{-E/T}\left\{\delta_c(E) - \frac{1}{2}\sin[2\delta_c(E)]\right\}$

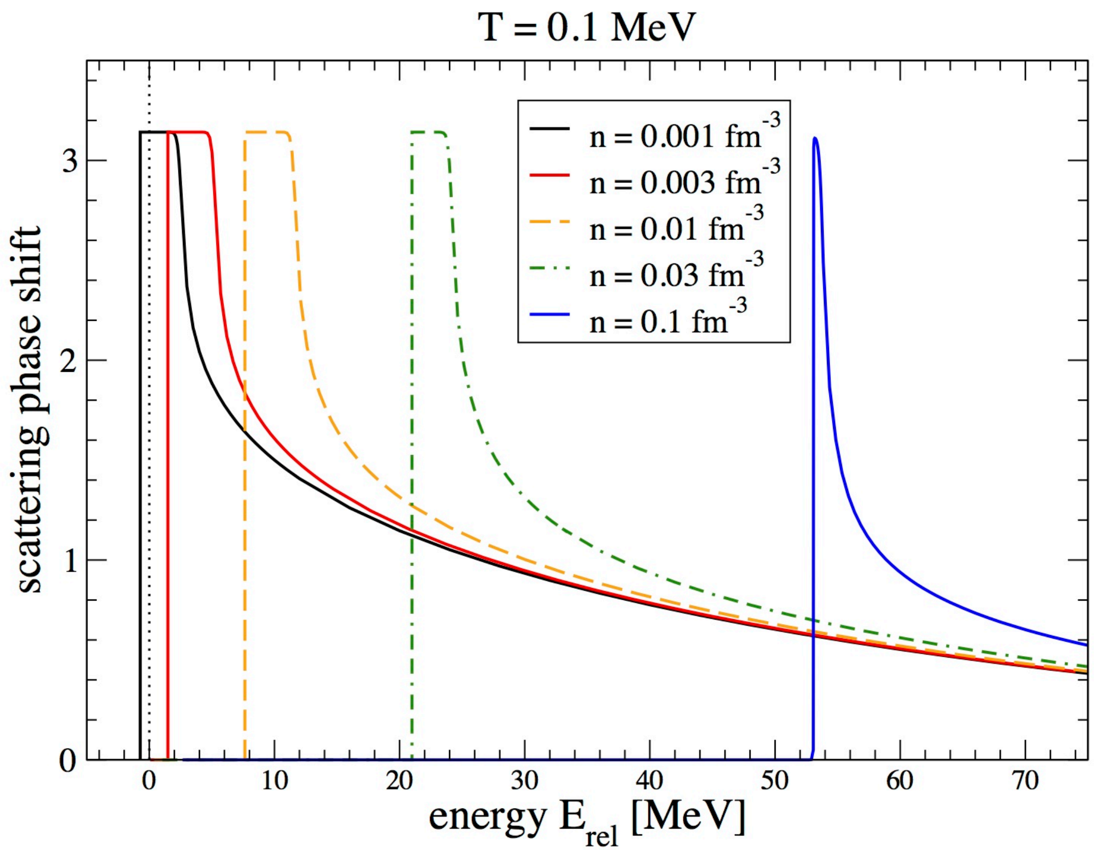


deuteron bound state -2.2 MeV

G. Roepke, J. Phys.: Conf. Series 569, 012031 (2014).

Deuteron-like scattering phase shifts

Virial coeff. $\propto e^{-E_d^0/T} - 1 + \frac{1}{\pi T}\int_0^\infty dE\, e^{-E/T}\left\{\delta_c(E) - \frac{1}{2}\sin[2\delta_c(E)]\right\}$



Tamm-Dancoff

deuteron bound state -2.2 MeV

G. Roepke, J. Phys.: Conf. Series 569, 012031 (2014)
Phys. Part. Nucl. 46, 772 (2015) [arXiv:1408.2654]

EOS: continuum contributions

Partial density of channel A,c at P (for instance, $^3S_1$= d):

$$z_{A,c}^{\rm part}(\mathbf{P};T,\mu_n,\mu_p) = e^{(N\mu_n+Z\mu_p)/T}\left\{\sum_{\nu_c}^{\rm bound} g_{A,\nu_c}\, e^{-E_{A,\nu_c}(\mathbf{P})/T}\,\Theta\left[-E_{A,\nu_c}(\mathbf{P})+E_{A,c}^{\rm cont}(\mathbf{P})\right]+z_{A,c}^{\rm cont}(\mathbf{P})\right\}$$

separation: bound state part – continuum part ?

$$z_c^{\rm part}(\mathbf{P};T,n_B,Y_p) = e^{[N\mu_n+Z\mu_p-NE_n(\mathbf{P}/A;T,n_B,Y_p)-ZE_p(\mathbf{P}/A;T,n_B,Y_p)]/T}$$
$$\times g_c\left\{\left[e^{-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)/T}-1\right]\Theta\left[-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)\right]+v_c(\mathbf{P};T,n_B,Y_p)\right\}$$

parametrization (d – like):

$$v_c(\mathbf{P}=0;T,n_B,Y_p) \approx \left[1.24+\left(\frac{1}{v_{T_I=0}(T)}-1.24\right)e^{\gamma_c n_B/T}\right]^{-1}.$$

$$v_d^0(T) = v_{T_I=0}^0(T) \approx 0.30857+0.65327\; e^{-0.102424\; T/\mathrm{MeV}}$$

G. Roepke, PRC 92,054001 (2015)

Cluster virial expansion

G.R., N. Bastian, D. Blaschke, T. Klaehn, S. Typel, H. Wolter, NPA **897**, 70 (2013)

Density effects?

The Beth-Uhlenbeck equation is identical

with the Dashen, Ma, Bernstein approach.

Talk given by Peter Braun-Munzinger:

the proton anomaly and the Dashen, Ma, Bernstein S-matrix approach

thermal yield of an (interacting) resonance with mass M, spin J, and isospin I

$$\langle R_{I,J} \rangle = d_J \int_{m_{th}}^{\infty} dM \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\pi} B_{I,J}(M) \times \frac{1}{e^{(\sqrt{p^2+M^2}-\mu)/T} + 1},$$

need to know derivatives of phase shifts with respect to invariant mass

$$B_{I,J}(M) = 2 \frac{d\delta_J^I}{dM}.$$

A. Andronic, pbm, B. Friman, P.M. Lo, K. Redlich, J. Stachel, arXiv:1808.03102, Phys.Lett.B792 (2019)304

5. Heavy ion collisions

EoS at low densities from HIC

PRL **108**, 172701 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

Laboratory Tests of Low Density Astrophysical Nuclear Equations of State

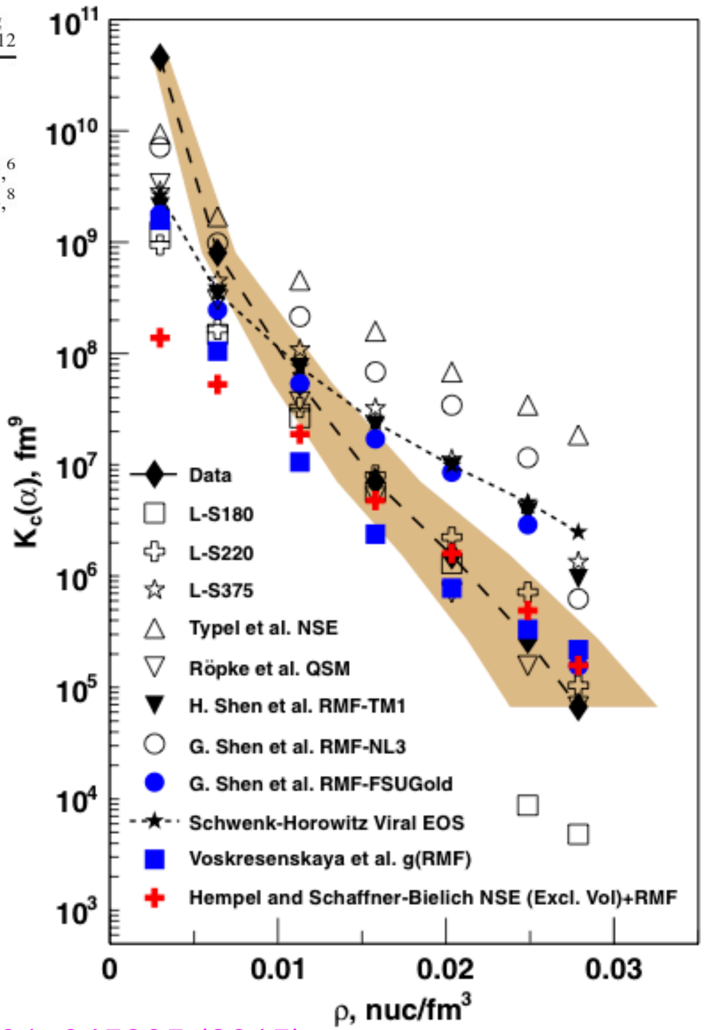
L. Qin,[1] K. Hagel,[1] R. Wada,[2,1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[6] M. Huang,[6] J. Wang,[6] H. Zheng,[1] S. Kowalski,[7] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] Z. Majka,[11] and Y. G. Ma[12]

Yields of clusters from HIC: p, n, d, t, h, α

chemical constants

$$K_c(A,Z) = \rho_{(A,Z)}/[(\rho_p)^Z(\rho_n)^N]$$

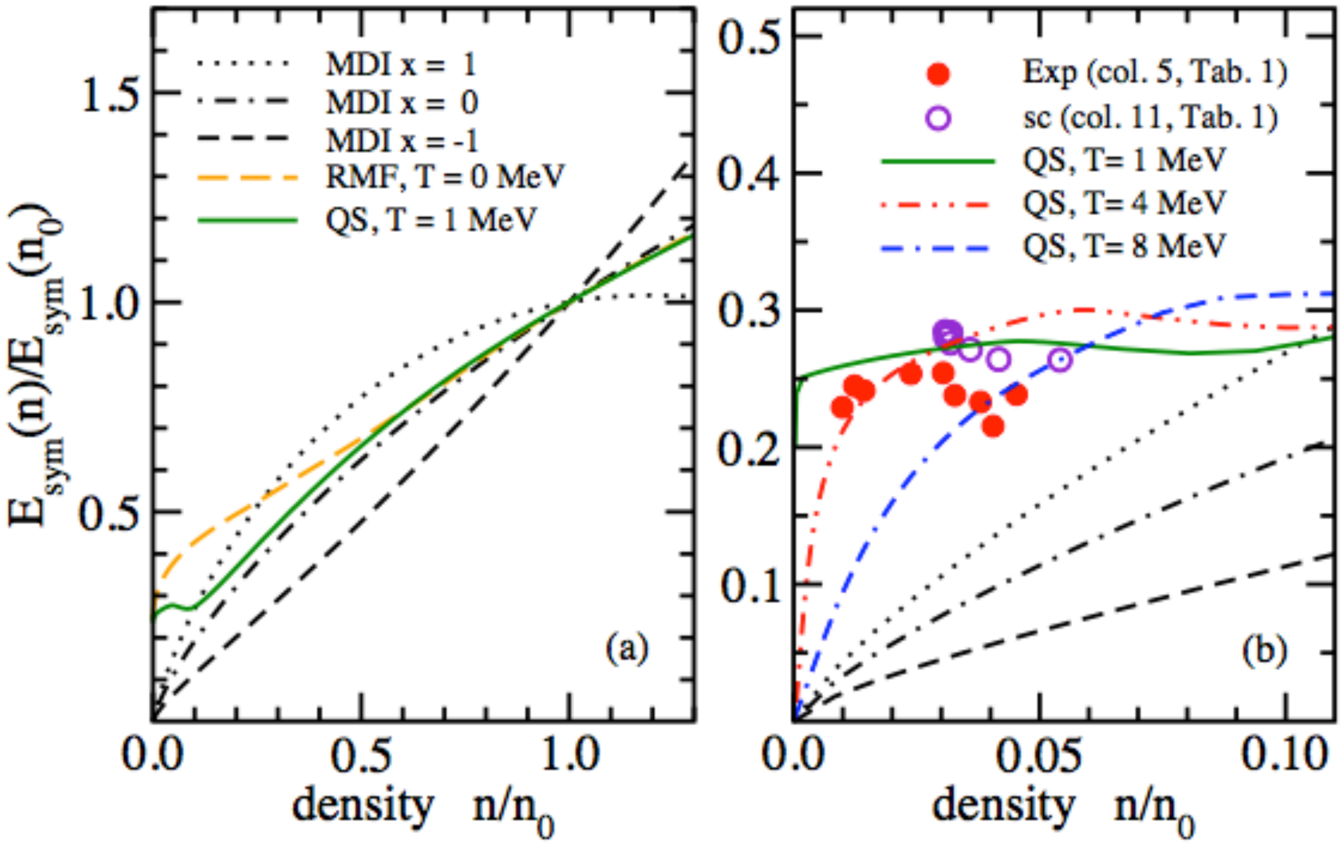
inhomogeneous,
non-equilibrium



QS, excluded volume

M. Hempel, K. Hagel, J. Natowitz, G. Ropke, S. Typel, Phys. Rec. C 91, 045805 (2015)

Symmetry Energy

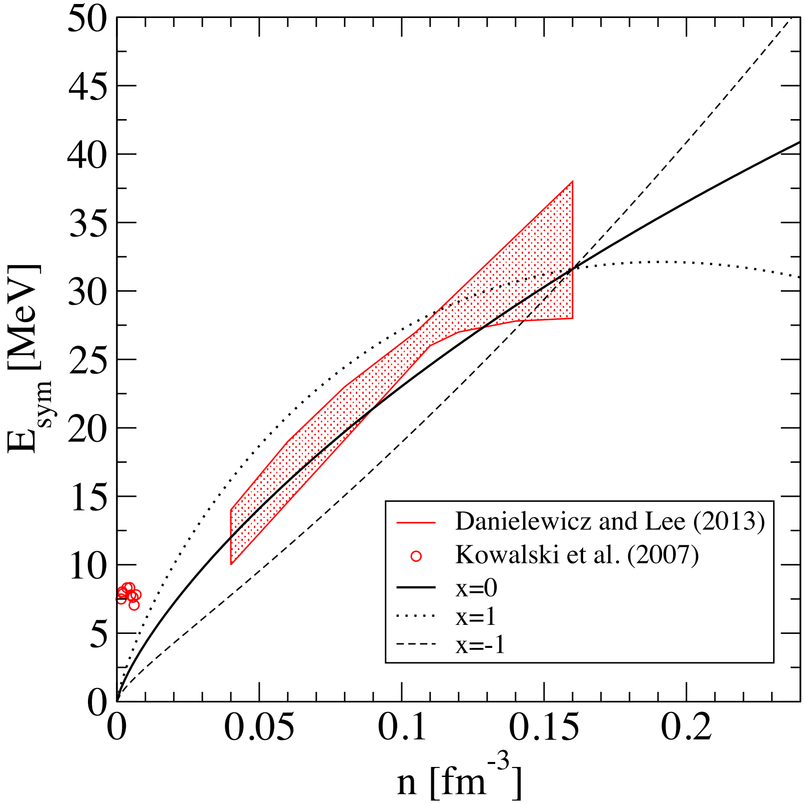


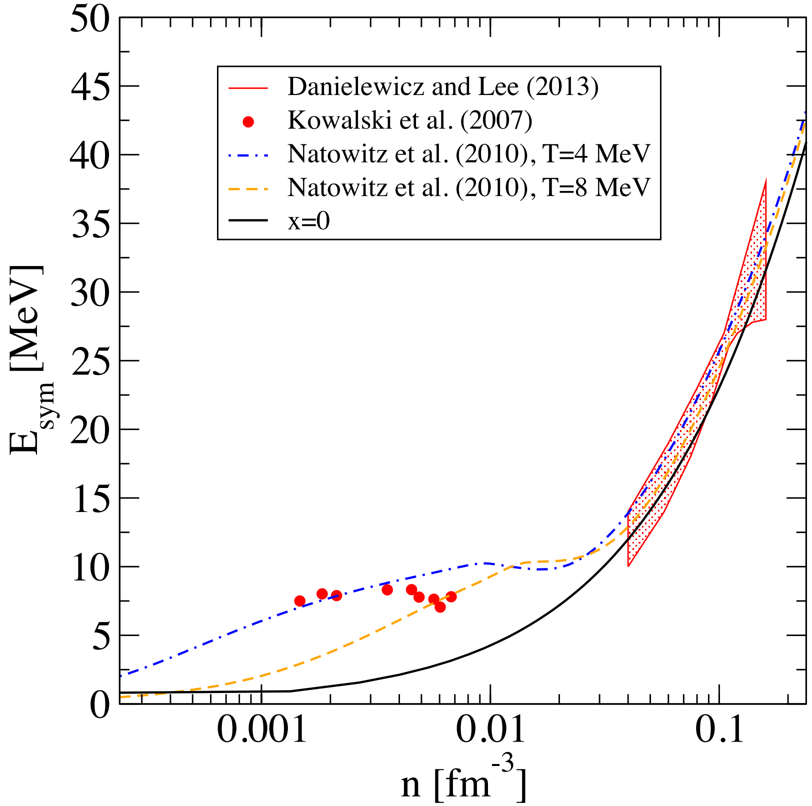
Scaled internal symmetry energy as a function of the scaled total density.
MDI: Chen et al., QS: quantum statistical, Exp: experiment at TAMU

J.Natowitz et al. PRL, May 2010

Symmetry energy: low density limit

correlations (bound states) → larger values for the symmetry energy





K. Hagel et al., Eur. Phys. J. A (2014) 50: 39

Formation of light clusters in heavy ion reactions, transport codes

PHYSICAL REVIEW C, VOLUME 63, 034605

Medium corrections in the formation of light charged particles in heavy ion reactions

C. Kuhrts,[1] M. Beyer,[1,*] P. Danielewicz,[2] and G. Röpke[1]
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[2]NSCL, Michigan State University, East Lansing, Michigan 48824
(Received 13 September 2000; published 12 February 2001)

Wigner distribution

cluster mean-field potential

$$\partial_t f_X + \{\mathcal{U}_X, f_X\} = \mathcal{K}_X^{\text{gain}}\{f_N, f_d, f_t, \ldots\}(1 \pm f_X) - \mathcal{K}_X^{\text{loss}}\{f_N, f_d, f_t, \ldots\} f_X,$$

$$X = N, d, t, \ldots .$$

loss rate

in-medium
breakup transition operator

$$\mathcal{K}_d^{\text{loss}}(P,t) = \int d^3k \int d^3k_1\, d^3k_2\, d^3k_3 |\langle k_1 k_2 k_3 | U_0 | kP\rangle|^2_{dN\to pnN} \times f_N(k_1,t) f_N(k_2,t) f_N(k_3,t) f_N(k,t) + \cdots . \quad (3)$$

breakup cross section

$$\sigma_{\text{bu}}^0(E) = \frac{1}{|v_d - v_N|} \frac{1}{3!} \int d^3k_1\, d^3k_2\, d^3k_3 |\langle kP|U_0|k_1 k_2 k_3\rangle|^2 \times 2\pi\delta(E' - E)(2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3), \quad (4)$$

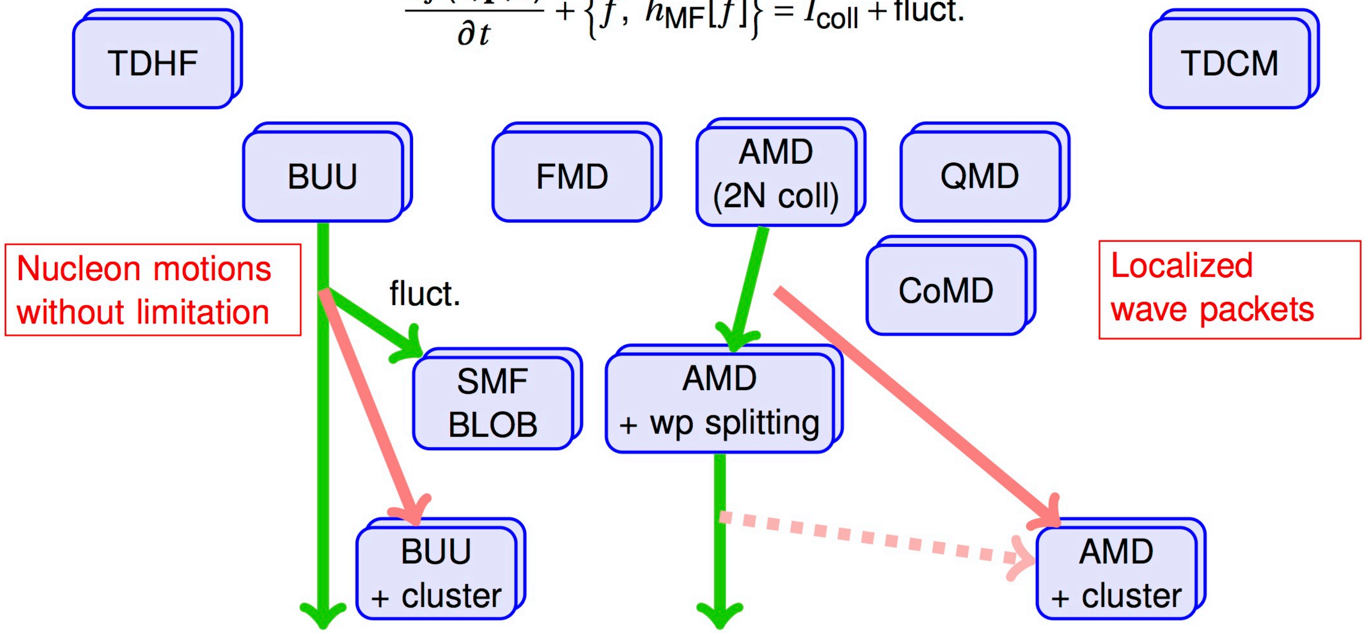
P. Danielewicz and Q. Pan, Phys. Rev. C 46, 2002 (1992)

AMD (Akira Ono)

Various transport theories

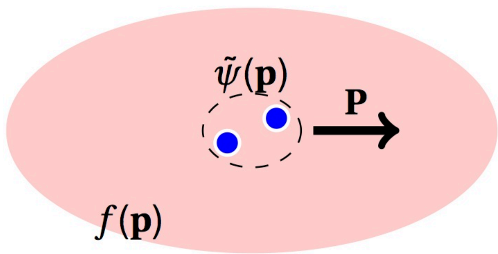
Based on the one-body distribution function $f(\mathbf{r},\mathbf{p},t)$ ⇔ One-body density matrix $\rho(\mathbf{r},\mathbf{r}')$

$$\frac{\partial f(\mathbf{r},\mathbf{p},t)}{\partial t} + \left\{f,\ h_{\mathrm{MF}}[f]\right\} = I_{\mathrm{coll}} + \text{fluct.}$$



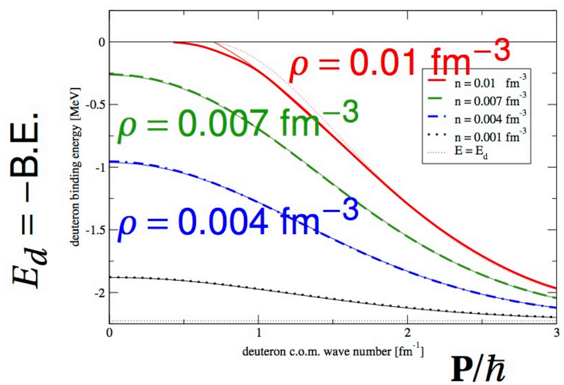
- Fluctuation/branching is a way to handle many-body correlations.
- Not many models treat cluster correlations explicitly.

A cluster in medium & Clusterized nuclear matter



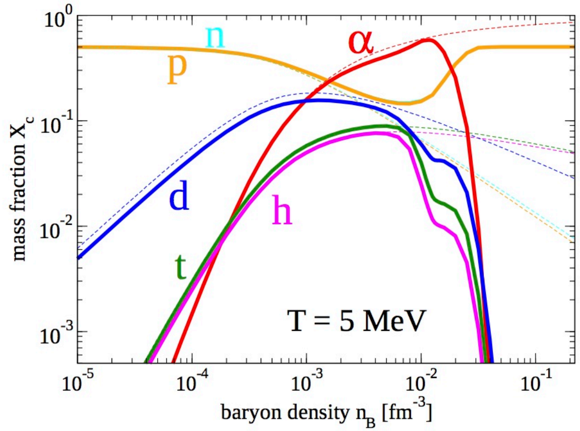
Equation for a deuteron in uncorrelated medium

$$\left[e(\tfrac{1}{2}\mathbf{P}+\mathbf{p})+e(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\right]\tilde{\psi}(\mathbf{p})$$
$$+\left[1-f(\tfrac{1}{2}\mathbf{P}+\mathbf{p})-f(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\right]\int\frac{d\mathbf{p}'}{(2\pi)^3}\langle\mathbf{p}|v|\mathbf{p}'\rangle\tilde{\psi}(\mathbf{p}')$$
$$=E\tilde{\psi}(\mathbf{p})$$



Momentum ($\mathbf{P}$) dependence of B.E.

Röpke, NPA867 (2011) 66.



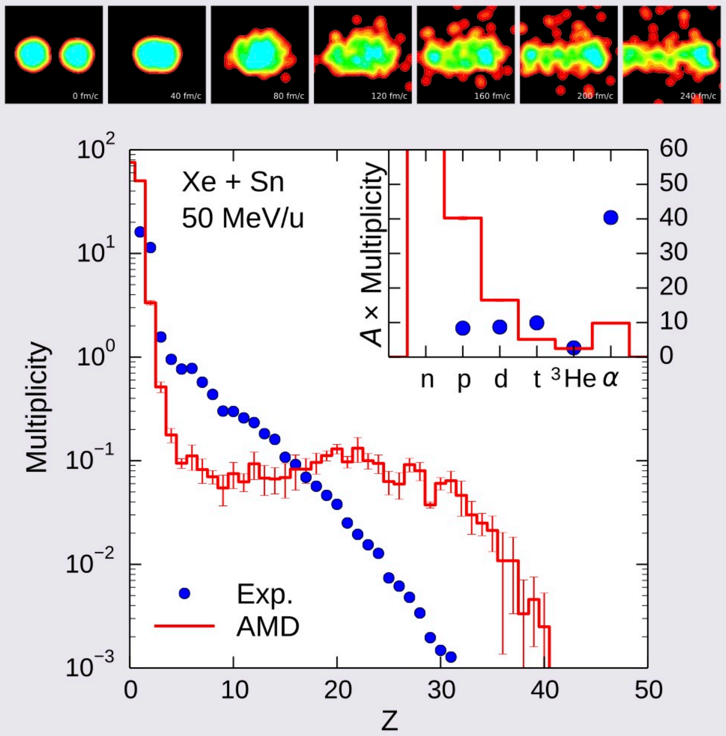
QS for symmetric nuclear matter

Röpke, PRC 92 (2015) 054001.

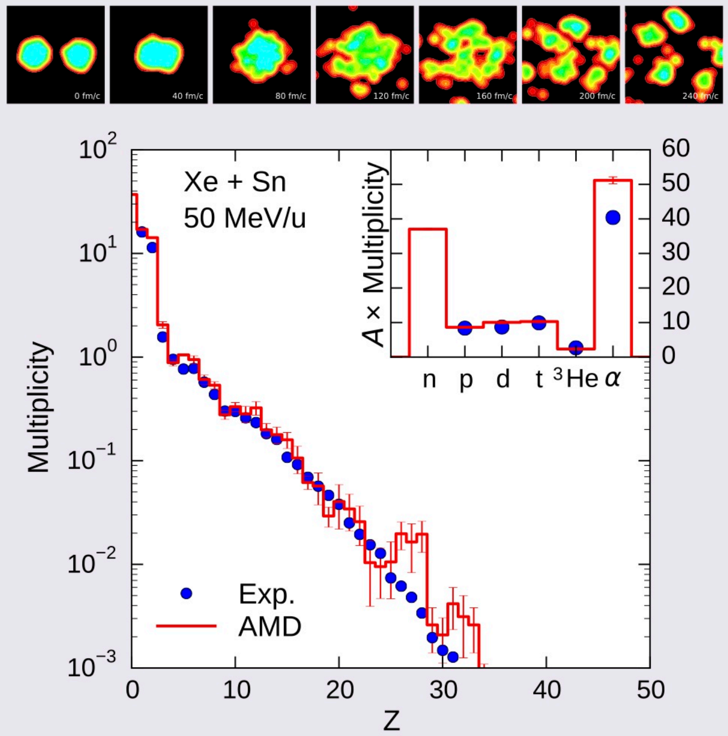
from A. Ono

Effect of cluster correlations: central Xe + Sn at 50 MeV/u

Without clusters

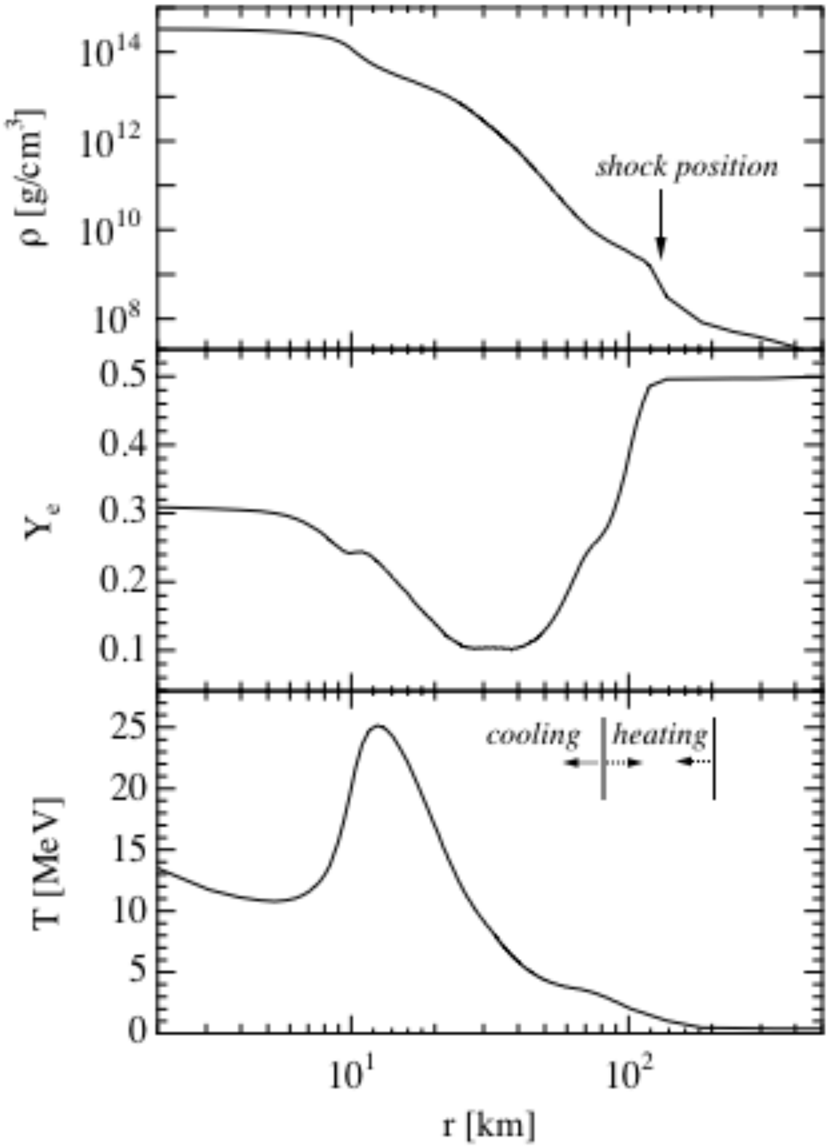


With clusters



6. Weakly bound nuclei, astrophysics

Core-collapse supernovae



Density.

electron fraction, and

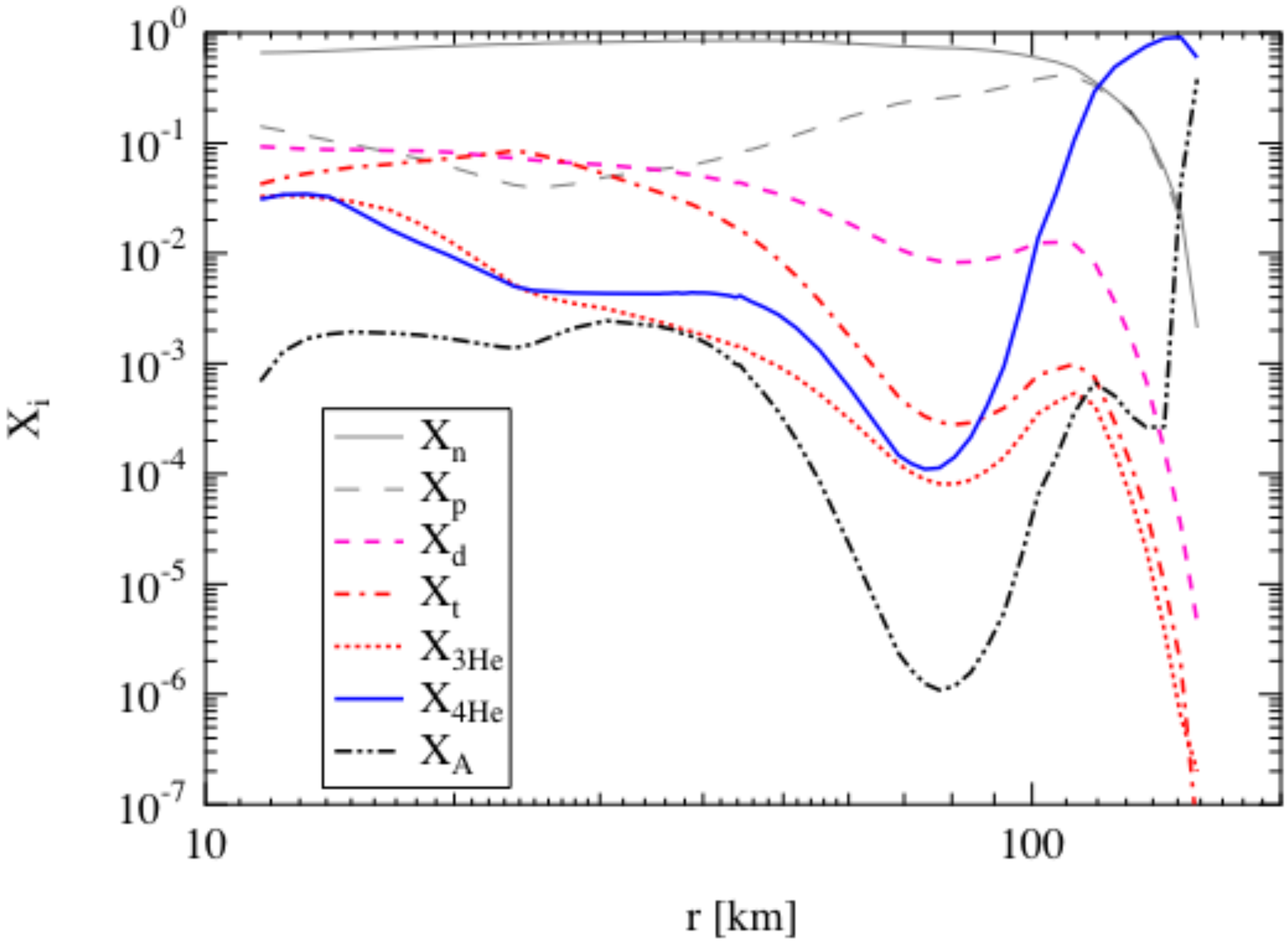
temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

K.Sumiyoshi et al.,
Astrophys.J. **629**, 922 (2005)

Composition of supernova core



Mass fraction X
of light clusters
for a post-bounce
supernova core

K.Sumiyoshi,
G. R.,
PRC **77**,
055804 (2008)

Asymmetric nuclear light clusters in supernova matter

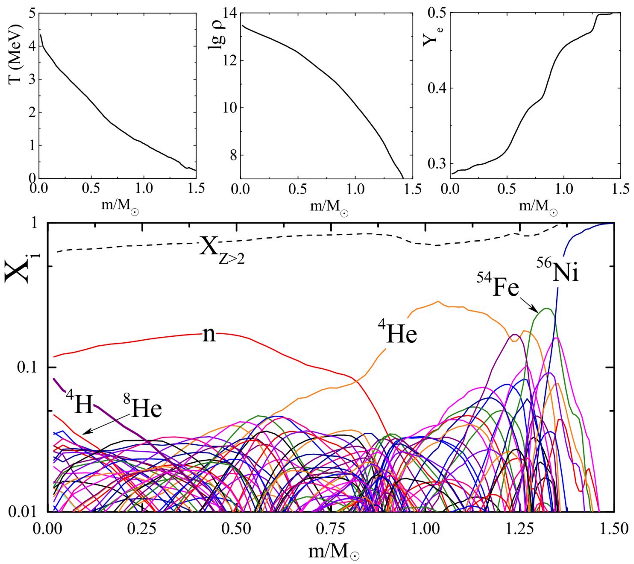


Figure 1. Upper three panels, from left ro right: temperature T (in MeV), log of density ρ (in $\mathrm{g \cdot cm^{-3}}$) and electron fraction Y_e as a functions of mass coordinate m. Lower panel: mass fractions of of nuclei X_i as a function of m. The black dashed line marked $X_{Z>2}$ shows the total mass fraction of elements with $Z > 2$. EoS is pure NSE.

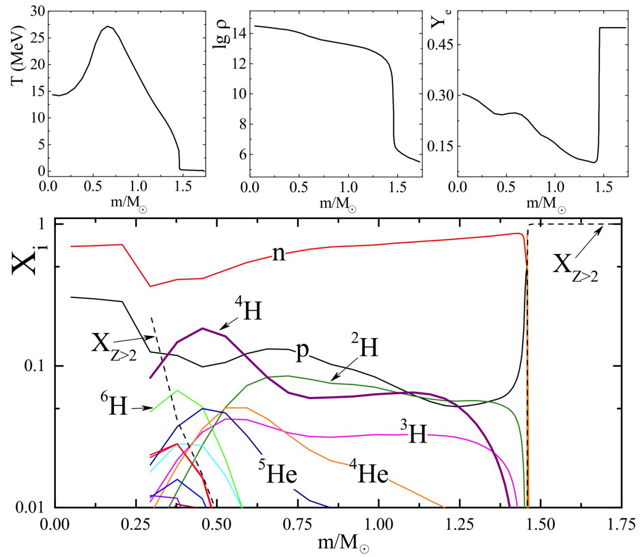


Figure 7. Upper three panels, from left ro right: temperature T (in MeV), log of density ρ (in $\mathrm{g \cdot cm^{-3}}$) and electron fraction Y_e as a functions of mass coordinate m. Lower panel: mass fractions X_i of of hydrogen and helium isotopes as a function of m. The black dashed line marked $X_{Z>2}$ shows the total mass fraction of all rest nuclei. Stellar profile corresponds to 200 ms after bounce approximately, calculations according to modified HS EoS.

A. V. Yudin, M. Hempel, S. I. Blinnikov, D. K. Nadyozhin, I. V. Panov,
Monthly Notices of the Royal Astronomical Society **483**, 5426 (2019)

Example: ^{5}He

Partial density
$$n_{^5\mathrm{He}} = 8\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2} b_{\alpha n}(T) e^{(-E_\alpha+3\mu_n+2\mu_p)/T}$$

virial coefficient

nuclear stat. equ.
$$b_{\alpha n}^{\mathrm{NSE}}(T) = \frac{5^{3/2}}{2} e^{(-E_{^5\mathrm{He}}+E_{^4\mathrm{He}})/T}$$

generalized Beth-Uhlenbeck
$$b_{\alpha n}^{\mathrm{gBU}}(T) = \left(\frac{5}{4}\right)^{1/2} \frac{1}{\pi T}\int_0^\infty dE_{\mathrm{lab}}\, e^{-4E_{\mathrm{lab}}/5T}\left\{\delta_{\alpha n}^{\mathrm{tot}}(E_{\mathrm{lab}}) - \frac{1}{2}\sin[2\delta_{\alpha n}^{\mathrm{tot}}(E_{\mathrm{lab}})]\right\}$$

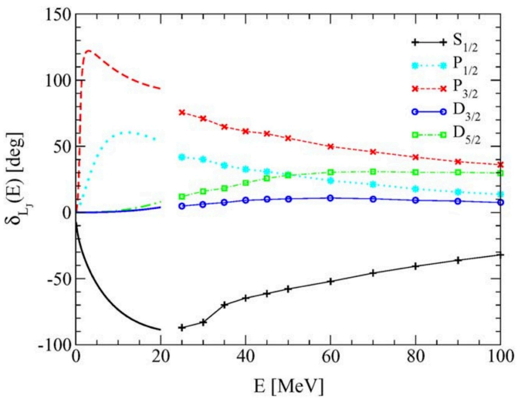
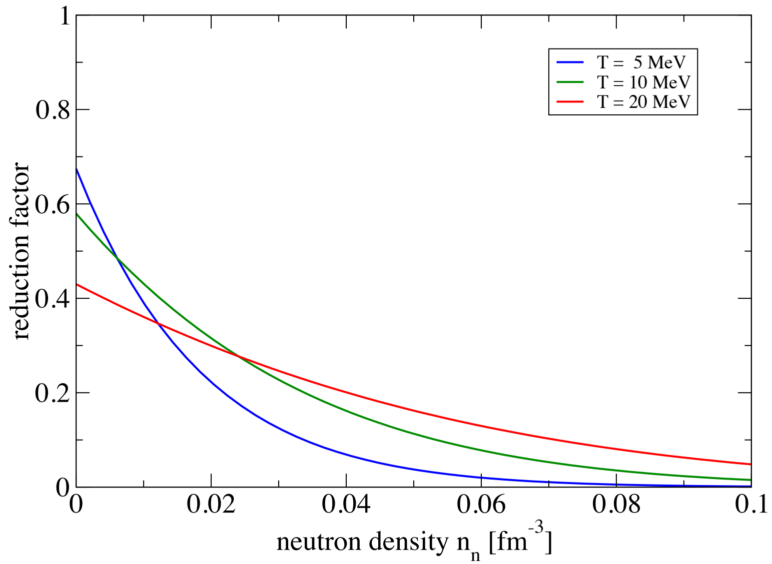


Fig. 2. (Color online.) The phase shifts for elastic neutron-alpha scattering $\delta_{L_J}(E)$ versus laboratory energy E. As discussed in the text, the solid lines are from Arndt and Roper [37] and the symbols are from Amos and Karataglidis [38]. For clarity, we do not show the F-waves included in our results for $b_{\alpha n}$.

C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)



ratio generalized Beth-Uhlenbeck/NSE

Conclusions

- Nuclear systems: strongly interacting, quantum
- Correlations (bound states) are of relevance
- Continuum correlations, resonances
- In-medium corrections, Beth-Uhlenbeck equation
- Inhomogeneous, nonequilibrium systems
- Cluster (pre)formation, early “initial” correlations
- Of interest also for the quark substructure

Thanks

to D. Blaschke, Y. Funaki, M. Hempel, H. Horiuchi, J. Natowitz,
Z. Ren, A. Sedrakian, P. Schuck, S. Shlomo,
A. Tohsaki, S. Typel, H. Wolter, C. Xu, T. Yamada, B. Zhou
for collaboration

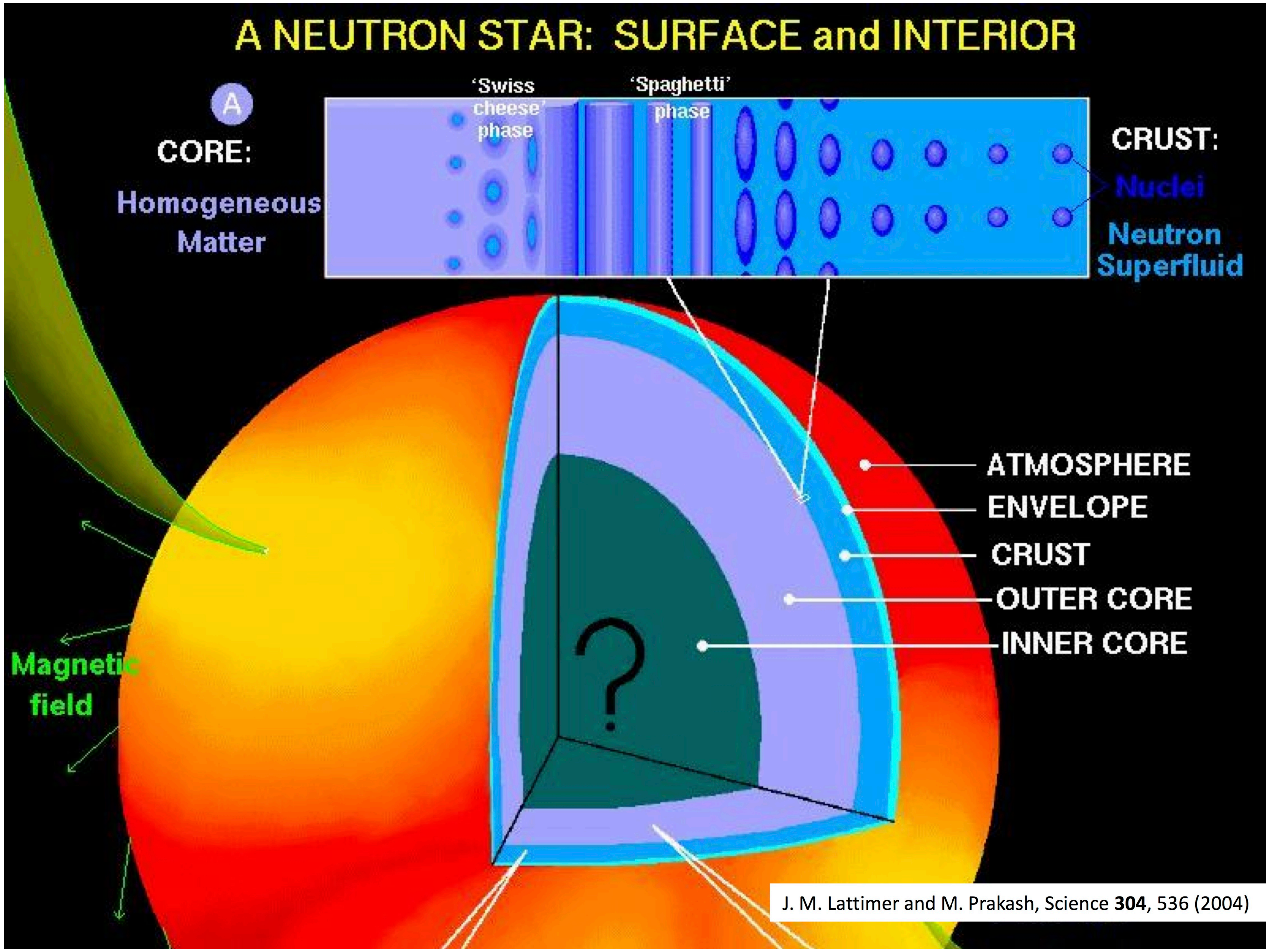
to you

for attention

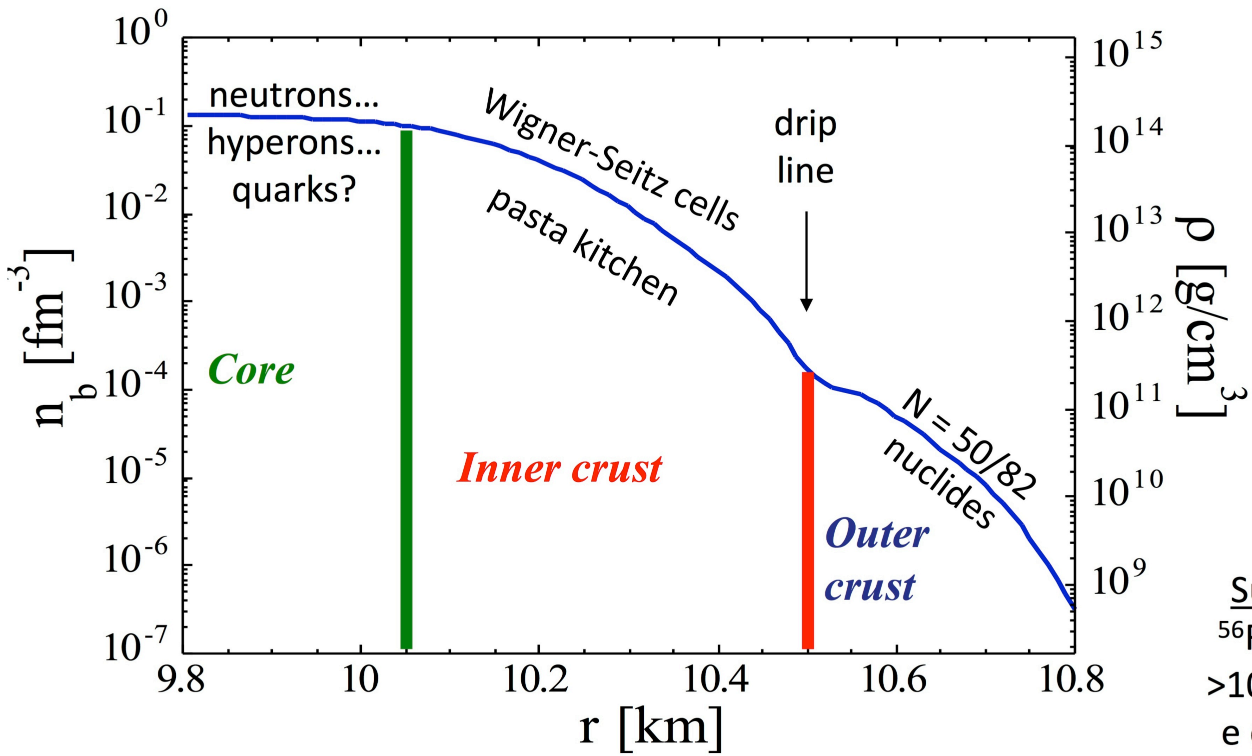
D.G.

7. Neutron stars

Inner crust: pasta structures



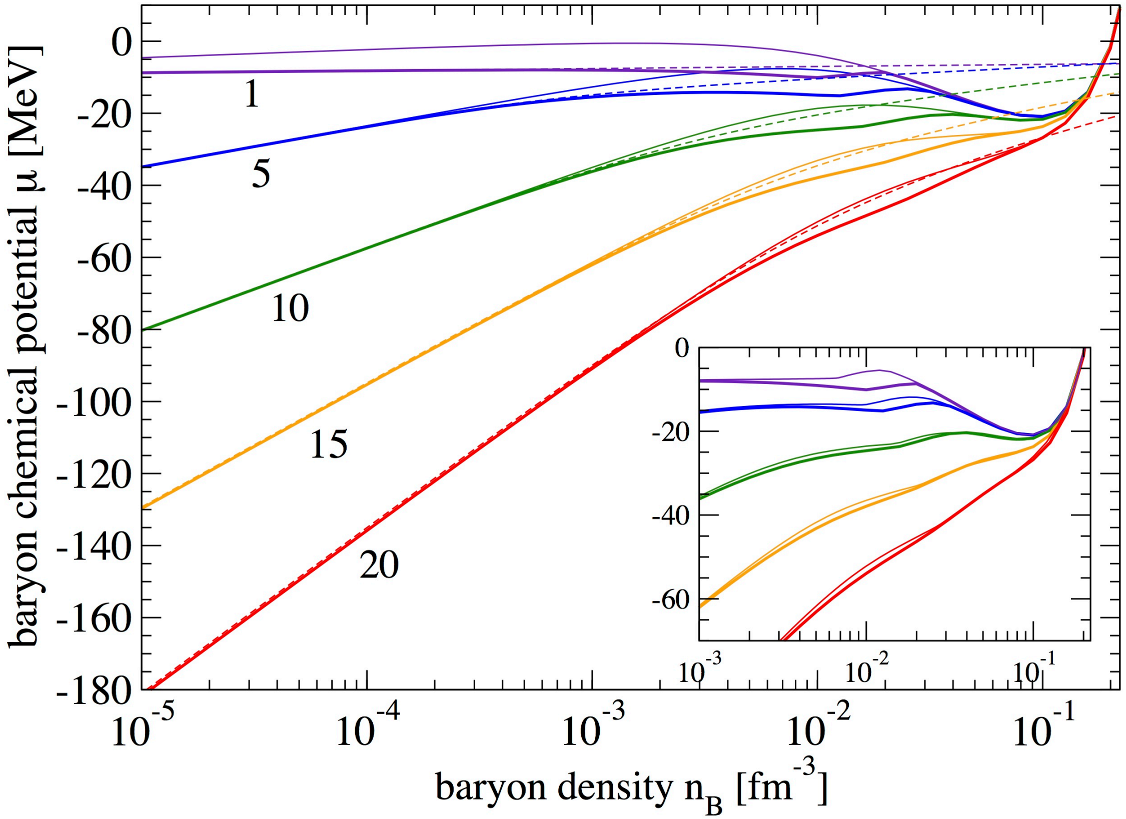
Density of neutron star crust



J. M. Pearson *et al.*, PRC **83**, 065810 (2011)

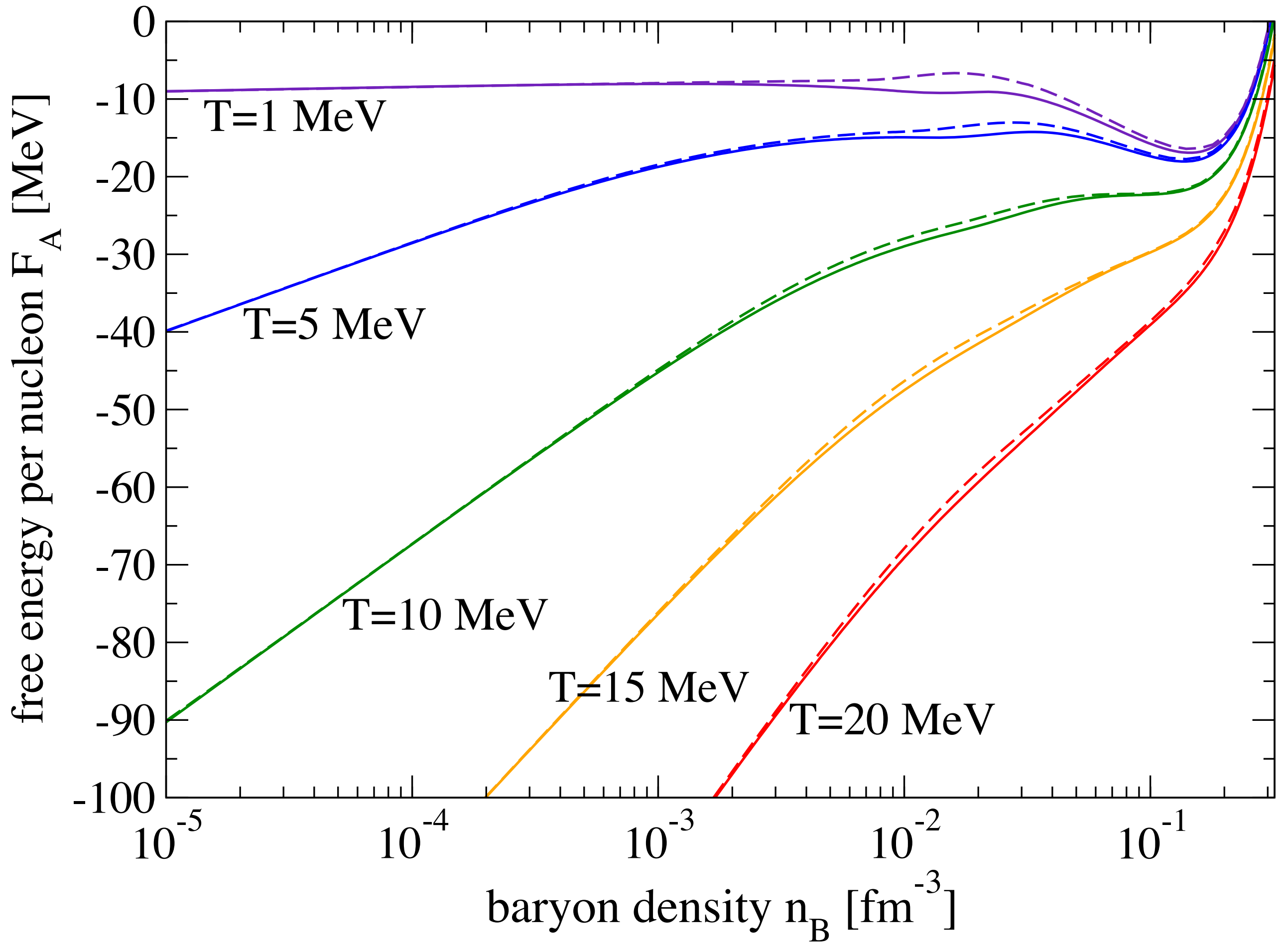
Surface:
^{56}Fe; ^{62}Ni;
$>10^7$ g/cm^3
e degen.:
p + e → n

Equation of state: chemical potential



Chemical potential for symmetric matter. T=1, 5, 10, 15, 20 MeV.
QS calculation compared with RMF (thin) and NSE (dashed).
Insert: QS calculation without continuum correlations (thin lines).

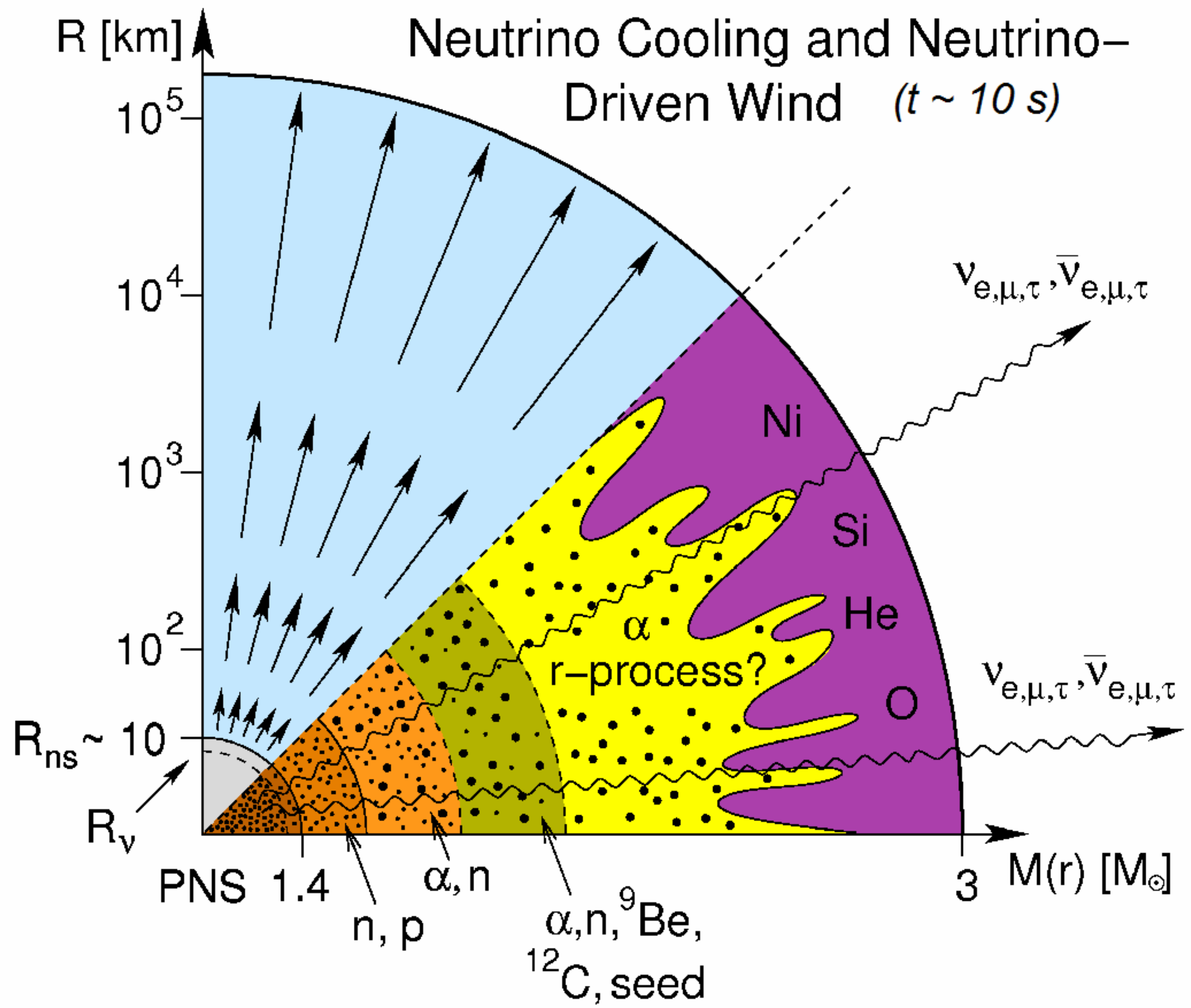
Symmetric matter: free energy per nucleon



Dashed lines: no continuum correlations

G. R., PRC **92**, 054001 (2015)

Supernova explosion



T.Janka

Formation of light clusters in heavy ion reactions, transport codes

PHYSICAL REVIEW C, VOLUME 63, 034605

Medium corrections in the formation of light charged particles in heavy ion reactions

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[2]NSCL, Michigan State University, East Lansing, Michigan 48824
(Received 13 September 2000; published 12 February 2001)

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$$X = N, d, t, \ldots .$$

loss rate

in-medium
breakup transition operator

$$\mathcal{K}_d^{\text{loss}}(P,t) = \int d^3k \int d^3k_1\, d^3k_2\, d^3k_3 |\langle k_1 k_2 k_3 | U_0 | kP\rangle|^2_{dN\to pnN} \times f_N(k_1,t) f_N(k_2,t) f_N(k_3,t) f_N(k,t) + \cdots . \quad (3)$$

breakup cross section

$$\sigma_{\text{bu}}^0(E) = \frac{1}{|v_d - v_N|} \frac{1}{3!} \int d^3k_1\, d^3k_2\, d^3k_3 |\langle kP | U_0 | k_1 k_2 k_3\rangle|^2 \times 2\pi\delta(E' - E)(2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3), \quad (4)$$

P. Danielewicz and Q. Pan, Phys. Rev. C 46, 2002 (1992)

Mott effect, in-medium cross section

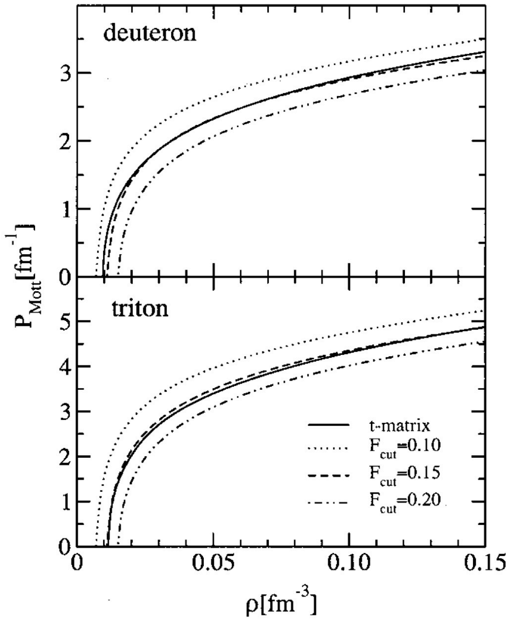


FIG. 1. Deuteron and triton Mott momenta P_{Mott} shown as a function of density ρ at fixed temperature of $T = 10$ MeV. The solid line represents results of the t matrix approach. The dashed, dotted, and dashed-dotted lines represent the deuteron Mott momenta from the parametrization given in Eq. (24) for three different cutoff values F_{cut}.

$$\int d^3 q \; f\left(\mathbf{q}+\frac{\mathbf{P}_{\mathrm{c.m.}}}{2}\right) \; |\phi(\mathbf{q})|^2 \leqslant F_{\mathrm{cut}}$$

C. Kuhrts, PRC **63**,034605 (2001)

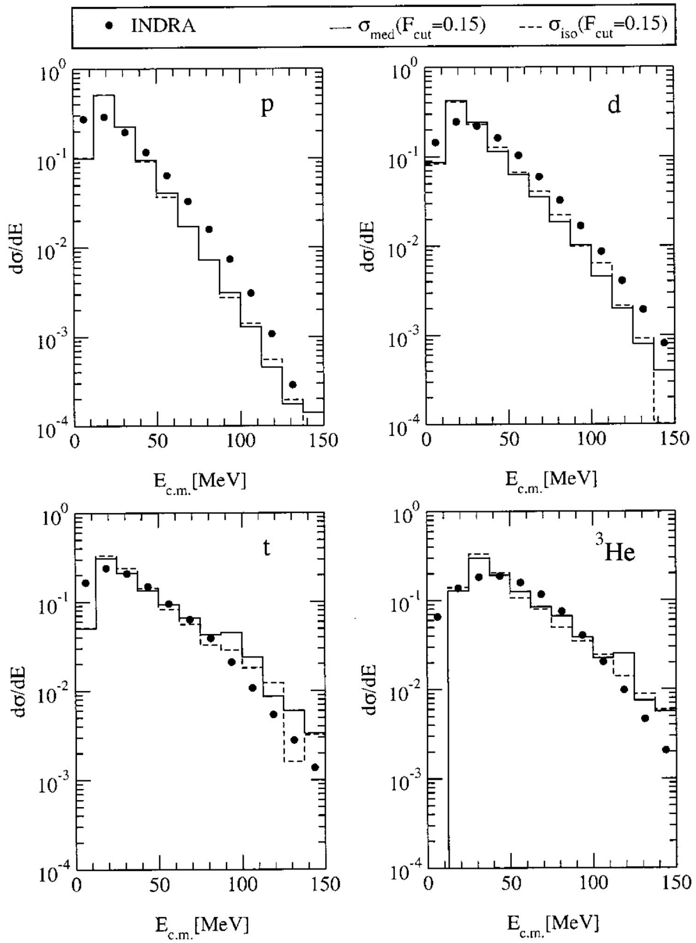


FIG. 5. Renormalized light charged light particle spectra in the center of mass system for the reaction ^{129}Xe+^{119}Sn at 50 MeV/nucleon. The filled circles represent the data of the INDRA Collaboration [21]. The solid line shows the calculations with the in-medium Nd reaction rates, while the dashed line shows a calculation using the isolated Nd breakup cross section; both with $F_{\mathrm{cut}} = 0.15$.

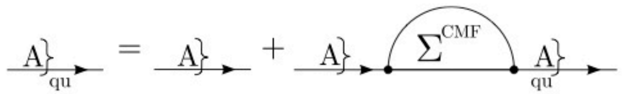
Cluster virial expansion for nuclear matter within a quasiparticle statistical approach

Generalized Beth-Uhlenbeck approach

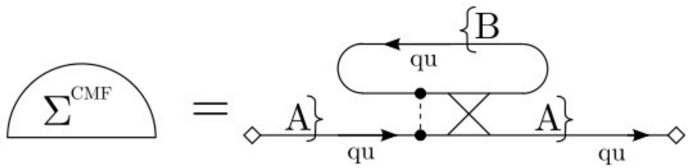
$$n_1^{\mathrm{qu}}(T,\mu_p,\mu_n) = \sum_{A,Z,\nu} \frac{A}{\Omega} \sum_{\substack{\vec{P} \\ P>P_{\mathrm{Mott}}}} f_A\big(E_{A,Z,\nu}(\vec{P};T,\mu_p,\mu_n),\mu_{A,Z,\nu}\big)$$

$$\begin{aligned} n_2^{\mathrm{qu}}(T,\mu_p,\mu_n) \;=\; & \sum_{A,Z,\nu} \sum_{A',Z',\nu'} \frac{A+A'}{\Omega} \sum_{\vec{P}} \sum_{c} g_c \frac{1+\delta_{A,Z,\nu;A',Z',\nu'}}{2\pi} \\ & \times \int_0^\infty dE\, f_{A+A'}\left(E_c(\vec{P};T,\mu_p,\mu_n)+E,\mu_{A,Z}+\mu_{A',Z'}\right) 2\sin^2(\delta_c)\,\frac{d\delta_c}{dE} \end{aligned}$$

Avoid double counting



Generating functional



G.R., N. Bastian, D. Blaschke, T. Klaehn, S. Typel, H. Wolter, NPA **897**, 70 (2013)

Excited light nuclei

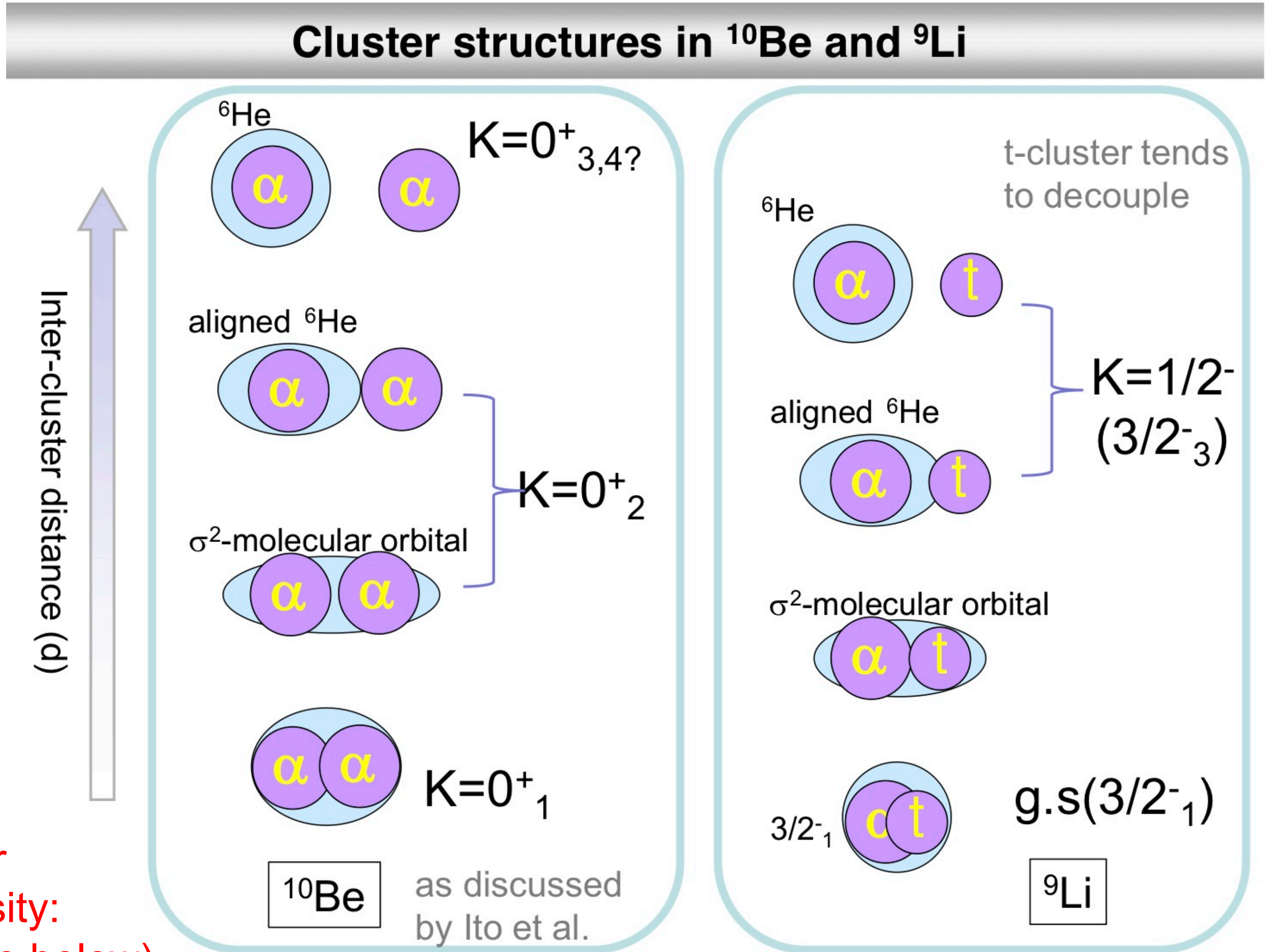
Yoshiko Kanada-En'yo
Cluster2012,Debrecen

decreasing density

systematics in weakly bound light elements

clustering at low densities

clusters disappear at increasing density: Pauli blocking (see below)



Few-particle Schrödinger equation in a dense medium

4-particle Schrödinger equation with medium effects

(self-energy shifts and Pauli blocking)

$$\left(\left[E^{HF}(p_1)+E^{HF}(p_2)+E^{HF}(p_3)+E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

$$+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4)$$

$$+\left\{permutations\right\}$$

$$=E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

Intermediate-mass fragment production

density value of $\rho/\rho_0 = 0.56$ from a previous analysis [26], the temperature and symmetry energy values of $T = 4.6 \pm 0.4$ MeV and $a_{sym} = 23.6 \pm 2.1$ MeV are extracted. These

X. Liu et al., Phys. Rev. C 95, 044601 (2017)

Zhao-Wen Zhang, Lie-Wen Chen,
Phys. Rev. C 95, 064330 (2017);

J. A. Lopez, S. Terrazas Porras,
Nucl. Phys. A 957, 312 (2017)

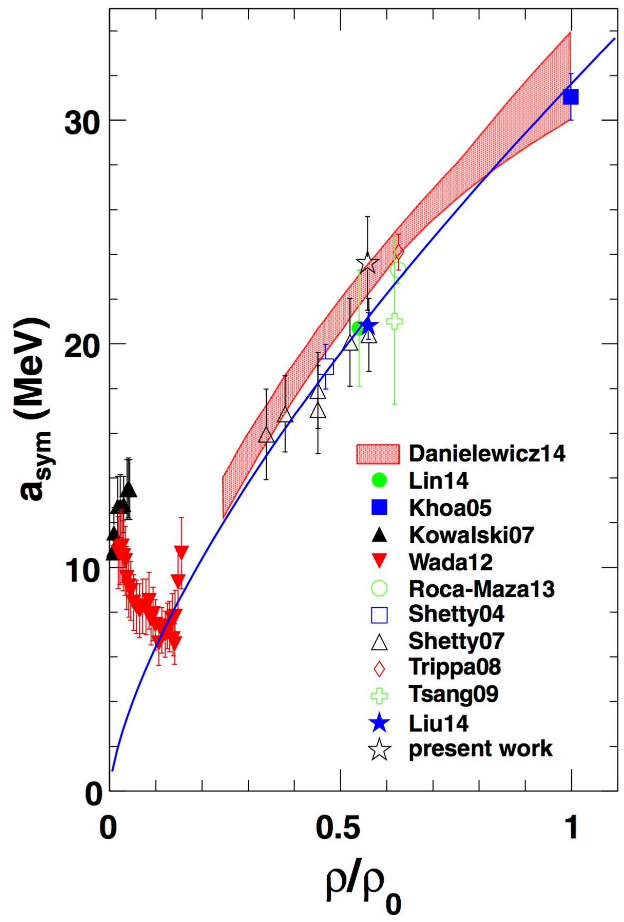
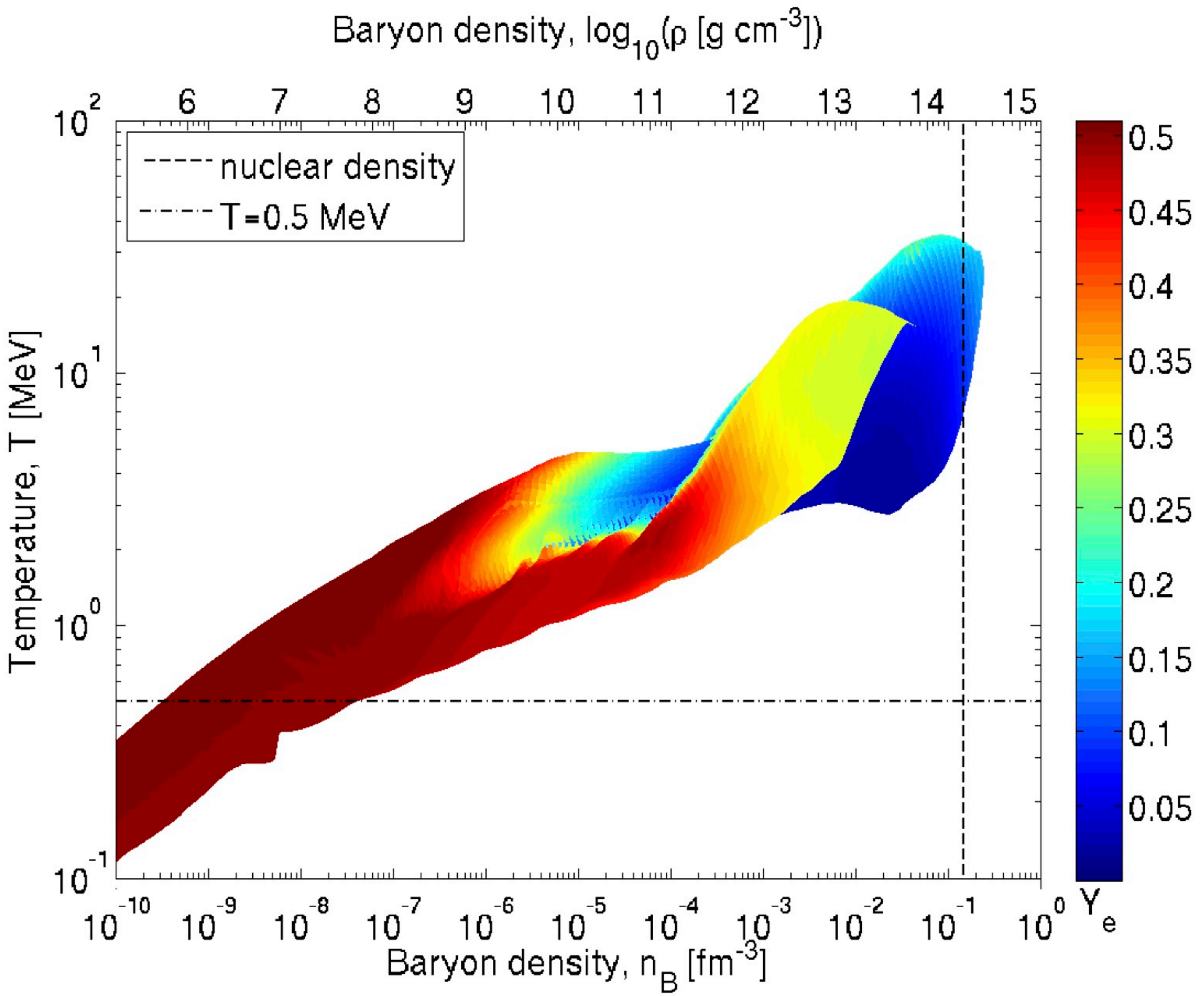


FIG. 10. Summary of the density dependent symmetry energy obtained in the present and previous studies. The line is the fit of the existing data points at $0.1 \leqslant \rho/\rho_0 \leqslant 1.0$ using Eq. (14).

Nuclear matter phase diagram

Exploding supernova



T. Fischer et al., arXiv 1307.6190