Wrap-up

Friday session of 3rd EMMI Workshop
anti-matter, hyper-matter and exotica production at the LHC

3. Many-particle theory, spectral function

Equation of state

$$n_\tau^{\rm tot}(T,\mu_n,\mu_p) = \frac{1}{\Omega}\sum_{p_1,\sigma_1}\int\frac{d\omega}{2\pi}\frac{1}{e^{(\omega-\mu_\tau)/T}+1}S_\tau(1,\omega)$$

Spectral function

$$S_\tau(1,\omega;T,\mu_n,\mu_p) \qquad E(1)=\hbar^2p_1^2/2m_1$$

Green function G,
Self-energy Σ

$$S(1,\omega)=2{\rm Im}\,G(1,\omega+i0)=2{\rm Im}\frac{1}{\omega-E(1)-\Sigma(1,\omega+i0)}$$

$$S_\tau(1,\omega)=\frac{2{\rm Im}\Sigma(1,\omega-i0)}{(\omega-E(1)-{\rm Re}\Sigma(1,\omega))^2+({\rm Im}\Sigma(1,\omega-i0))^2}$$

Expansion for small damping (Im Σ)

$$S(1,\omega)\approx\frac{2\pi\delta(\omega-E^{\rm quasi}(1))}{1-\frac{d}{dz}{\rm Re}\,\Sigma(1,z)|_{z=E^{\rm quasi}(1)}}-2{\rm Im}\,\Sigma(1,\omega+i0)\frac{d}{d\omega}\frac{\mathcal{P}}{\omega-E^{\rm quasi}(1)}$$

Quasiparticle energy

$$E^{\rm quasi}(1)=E(1)+{\rm Re}\,\Sigma(1,z)|_{z=E^{\rm quasi}(1)}$$

Correlations (bound states) in Im Σ
Cluster decomposition, Bethe-Salpeter equation

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium of quasiparticle clusters:
self-energy and Pauli blocking

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.Eq.

Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

chemical & physical picture

Cluster virial approach:
all bound states (clusters)
scattering phase shifts of all pairs

Correlated medium:
phase space occupation by all bound states
in-medium correlations, quantum condensates

EoS at low densities from HIC

PRL **108,** 172701 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

Laboratory Tests of Low Density Astrophysical Nuclear Equations of State

L. Qin,[1] K. Hagel,[1] R. Wada,[2,1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[6] M. Huang,[6] J. Wang,[6] H. Zheng,[1] S. Kowalski,[7] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] Z. Majka,[11] and Y. G. Ma[12]

Yields of clusters from HIC: p, n, d, t, h, α

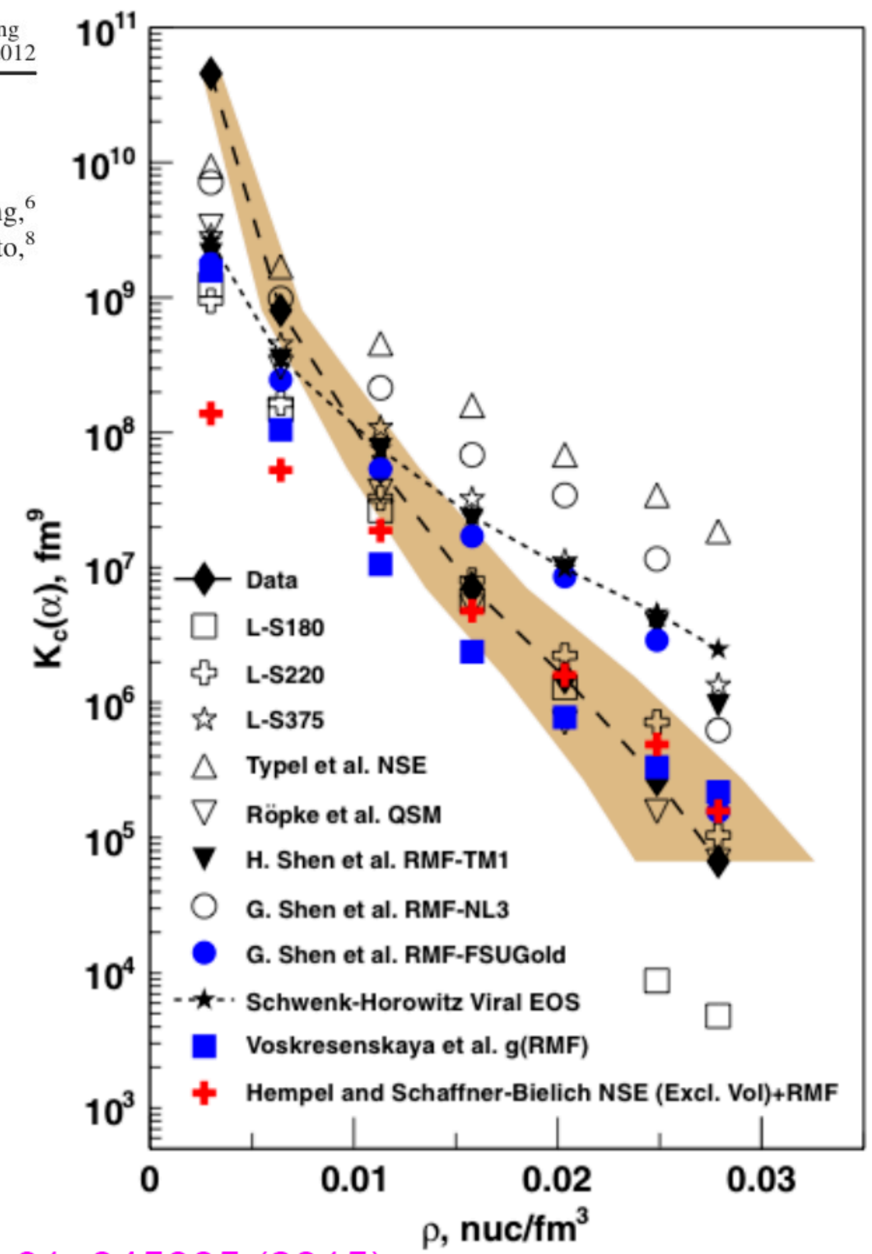
chemical constants

$$K_c(A,Z) = \rho_{(A,Z)}/[(\rho_p)^Z(\rho_n)^N]$$

inhomogeneous,
non-equilibrium

QS, excluded volume

M. Hempel, K. Hagel, J. Natowitz, G. Ropke, S. Typel, Phys. Rec. C 91, 045805 (2015)



Formation of light clusters in heavy ion reactions, transport codes

PHYSICAL REVIEW C, VOLUME 63, 034605

Medium corrections in the formation of light charged particles in heavy ion reactions

C. Kuhrts,[1] M. Beyer,[1,*] P. Danielewicz,[2] and G. Röpke[1]
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[2]*NSCL, Michigan State University, East Lansing, Michigan 48824*
(Received 13 September 2000; published 12 February 2001)

Wigner distribution

cluster mean-field potential

$$\partial_t f_X + \{\mathcal{U}_X, f_X\} = \mathcal{K}_X^{\rm gain}\{f_N, f_d, f_t, \ldots\}(1 \pm f_X) - \mathcal{K}_X^{\rm loss}\{f_N, f_d, f_t, \ldots\} f_X,$$

$$X = N, d, t, \ldots .$$

loss rate

in-medium breakup transition operator

$$\mathcal{K}_d^{\rm loss}(P,t) = \int d^3k \int d^3k_1\, d^3k_2\, d^3k_3 |\langle k_1 k_2 k_3 | U_0 | kP \rangle|^2_{dN \to pnN} \times f_N(k_1,t) f_N(k_2,t) f_N(k_3,t) f_N(k,t) + \cdots . \qquad (3)$$

breakup cross section

$$\sigma_{\rm bu}^0(E) = \frac{1}{|v_d - v_N|} \frac{1}{3!} \int d^3k_1\, d^3k_2\, d^3k_3 |\langle kP | U_0 | k_1 k_2 k_3 \rangle|^2 \times 2\pi \delta(E' - E)(2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3), \qquad (4)$$

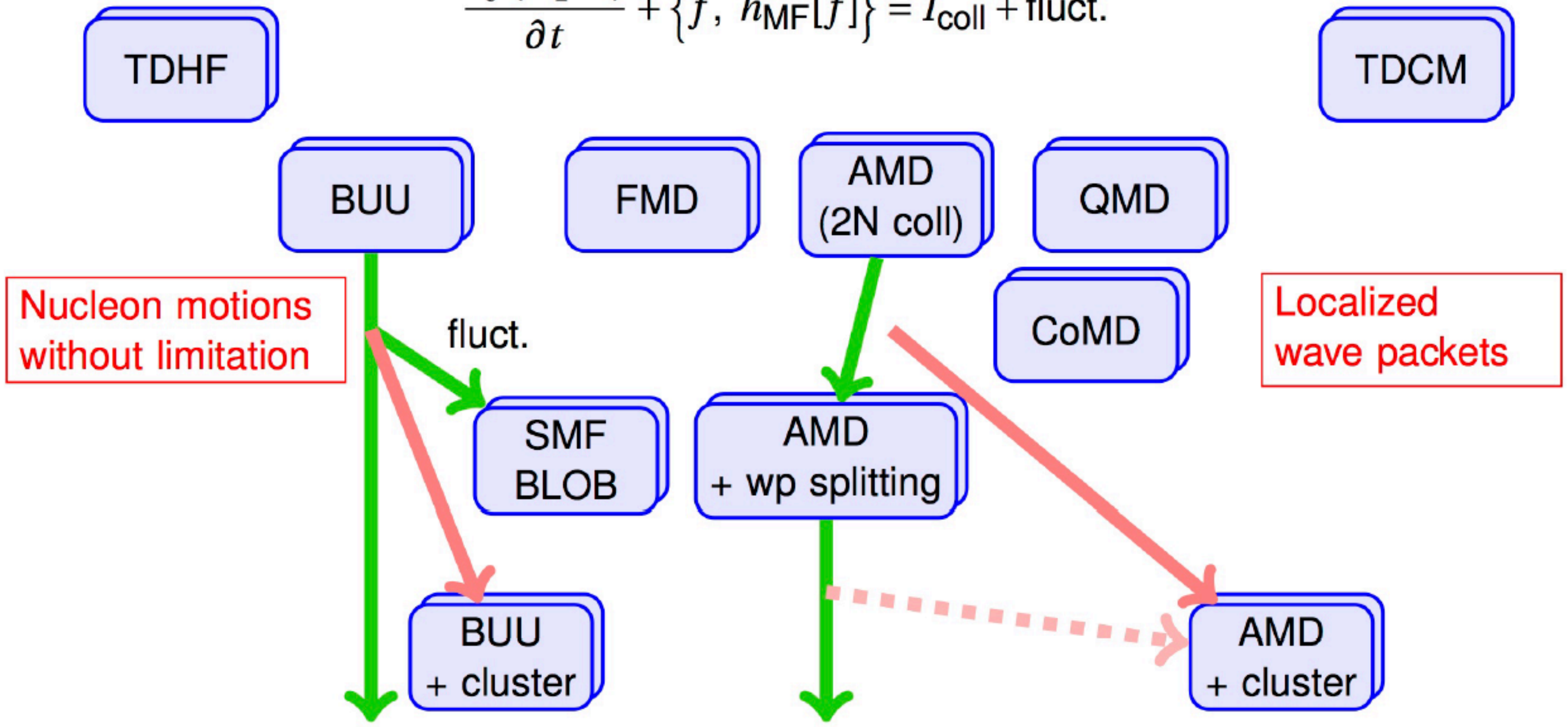
P. Danielewicz and Q. Pan, Phys. Rev. C 46, 2002 (1992)

AMD (Akira Ono)

Various transport theories

Based on the one-body distribution function $f(\mathbf{r},\mathbf{p},t)$ ⇔ One-body density matrix $\rho(\mathbf{r},\mathbf{r}')$

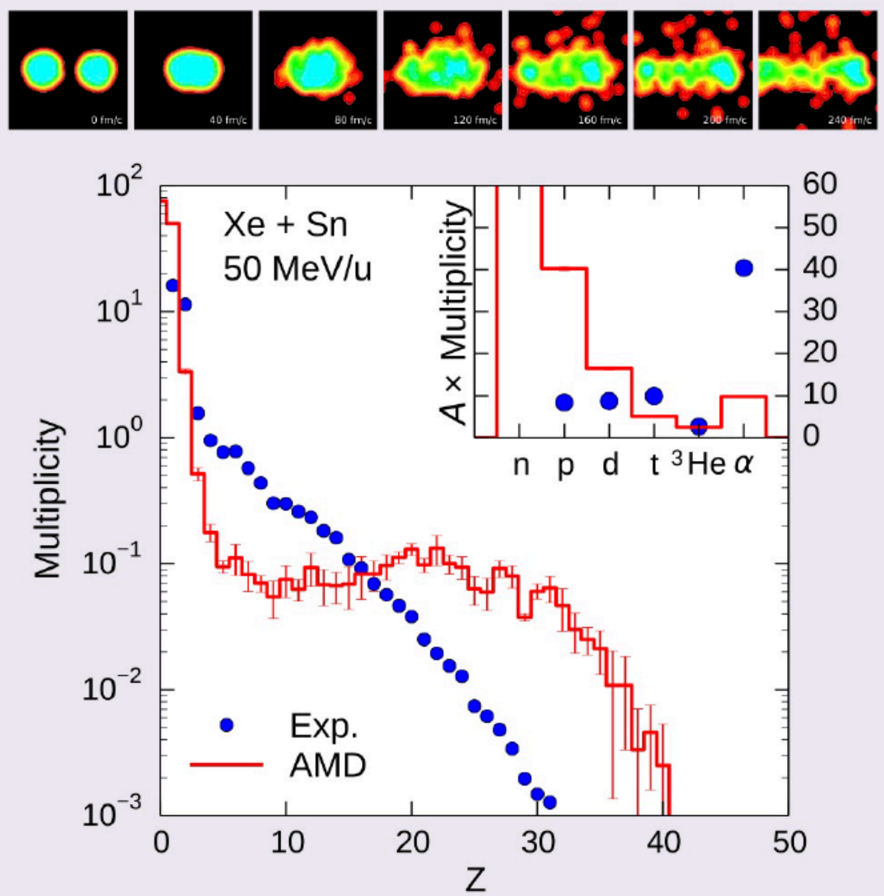
$$\frac{\partial f(\mathbf{r},\mathbf{p},t)}{\partial t} + \left\{f,\ h_{\rm MF}[f]\right\} = I_{\rm coll} + \text{fluct.}$$



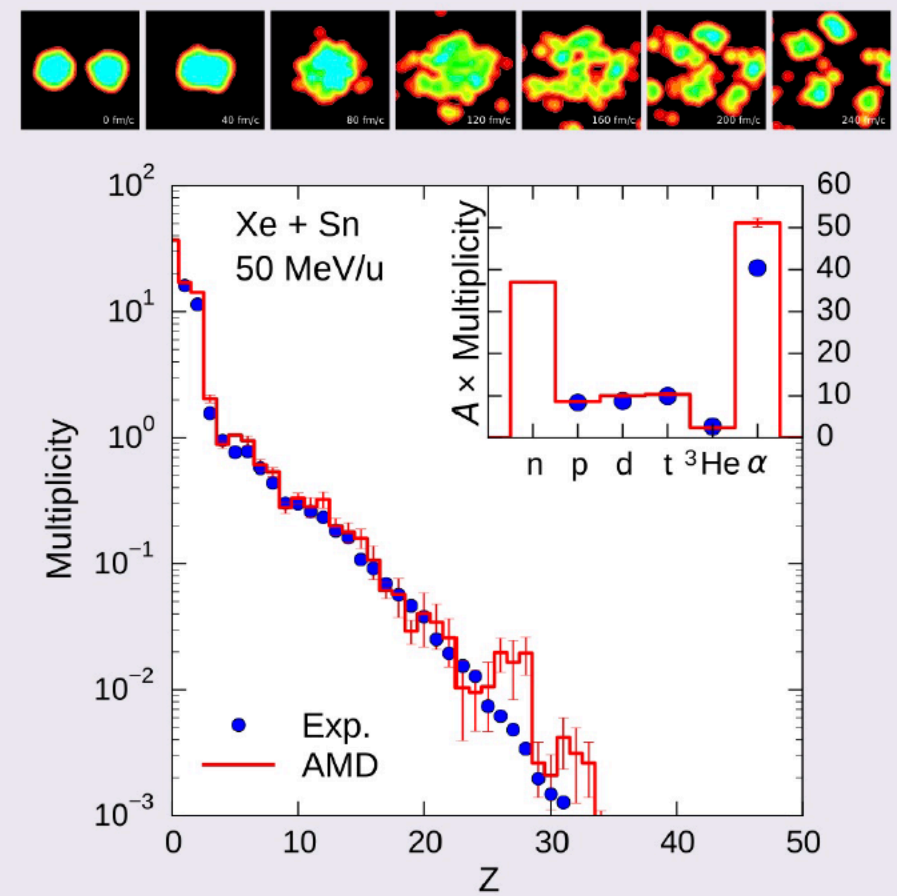
- Fluctuation/branching is a way to handle many-body correlations.
- Not many models treat cluster correlations explicitly.

Effect of cluster correlations: central Xe + Sn at 50 MeV/u

Without clusters



With clusters



QCD phase diagram and critical point

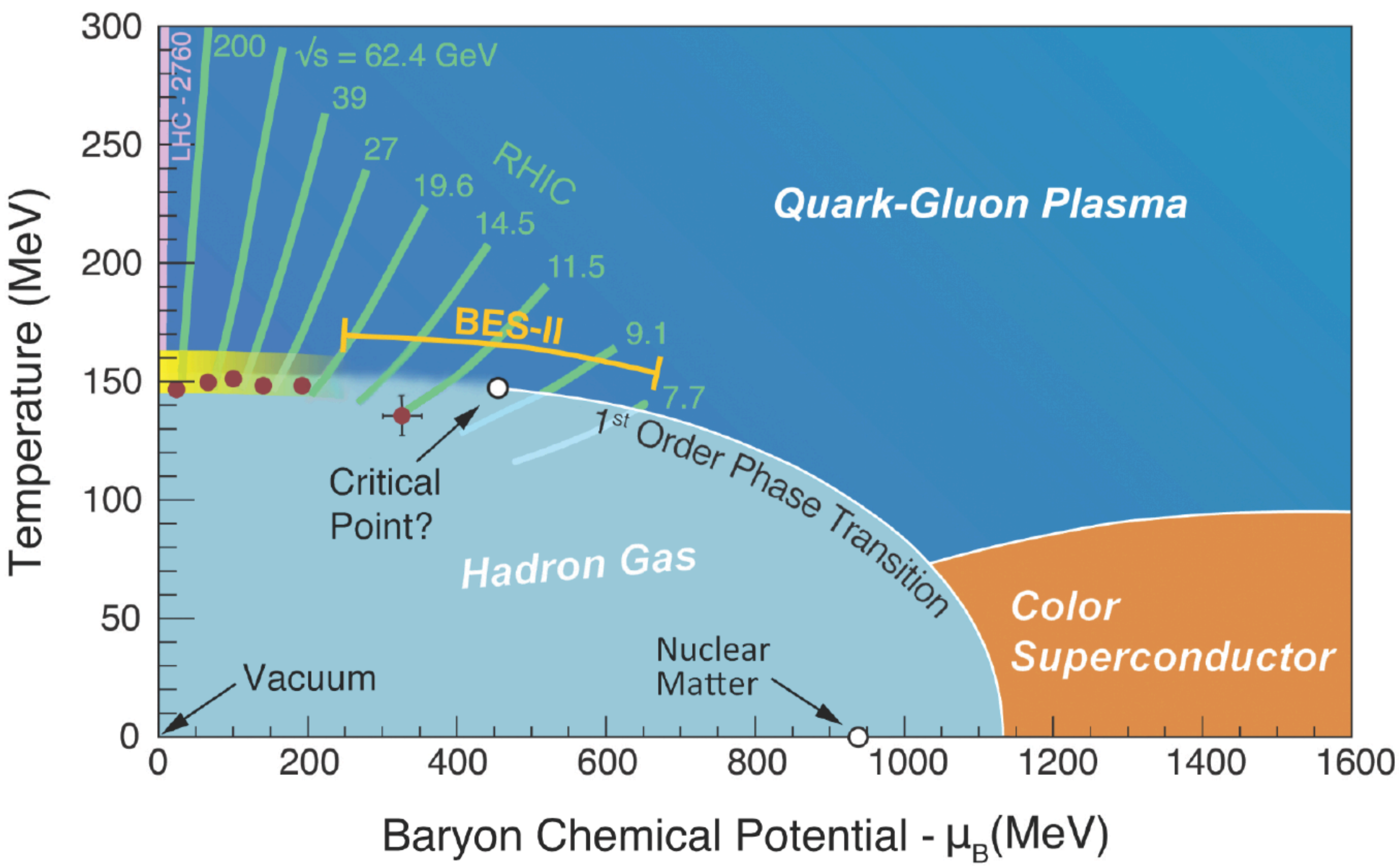
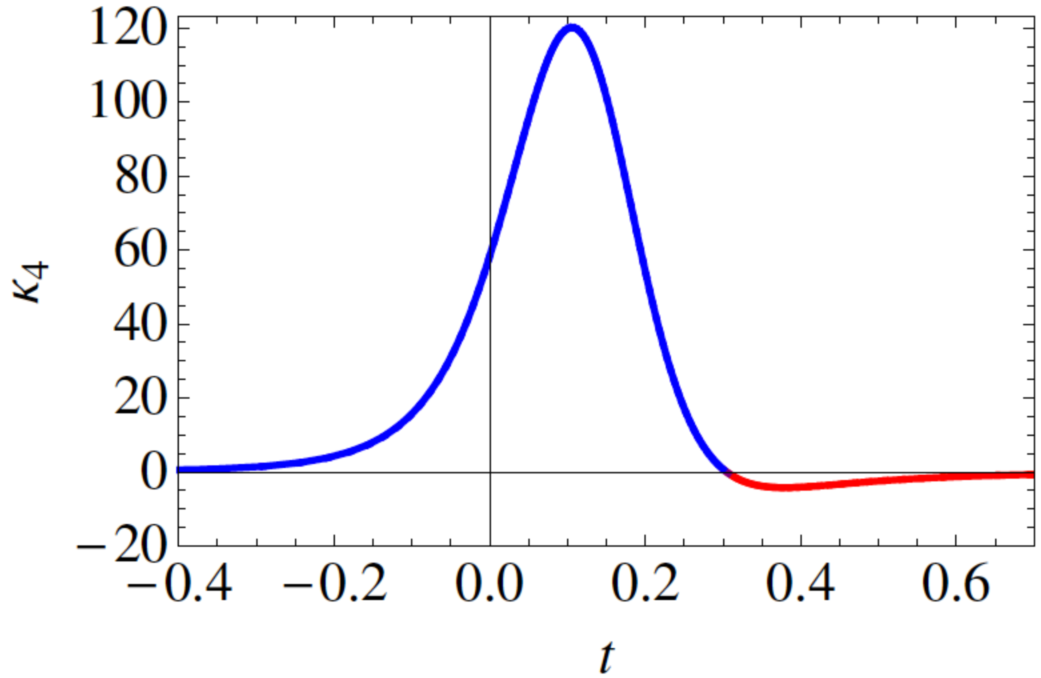


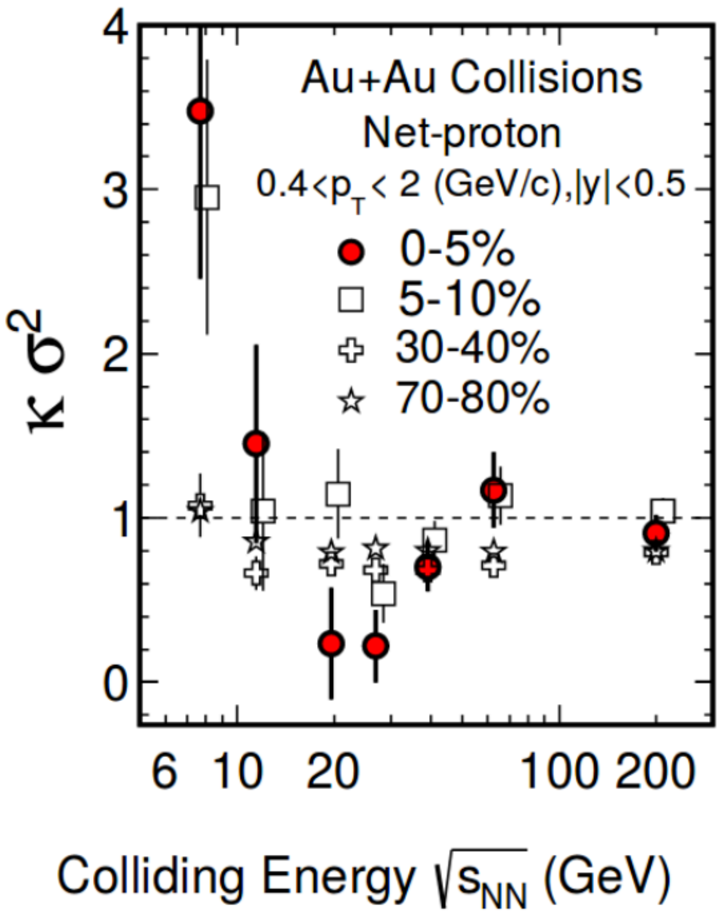
Image: S. Mukherjee (Brookhaven National Lab.)

Critical mode couples to baryons

M. Stephanov, 2011

$$\mathcal{L}_{eff} = g\sigma p\bar{p}$$





$$C_2 = \langle \delta N^2_{p-\bar{p}} \rangle$$

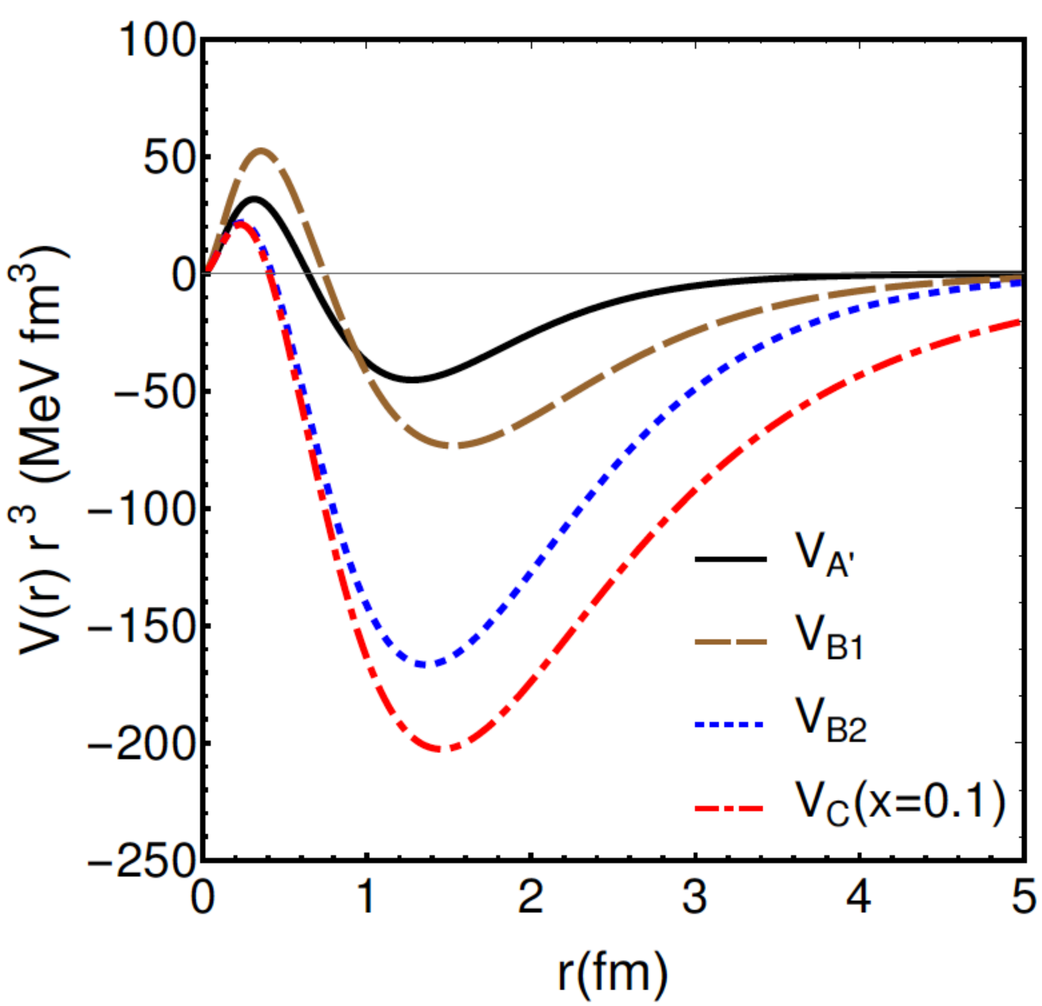
$$C_4 = \langle \delta N^4_{p-\bar{p}} \rangle - 3\langle \delta N^2_{p-\bar{p}} \rangle^2$$

$$\kappa\sigma^2 = C_4 / C_2$$

STAR Coll., 2015

NN potential modifications

- Close to T_c a very light σ enhances the attraction
- ***NN* potential should be affected by the presence of the QCD critical point!**
- We consider more and more attractive potentials:

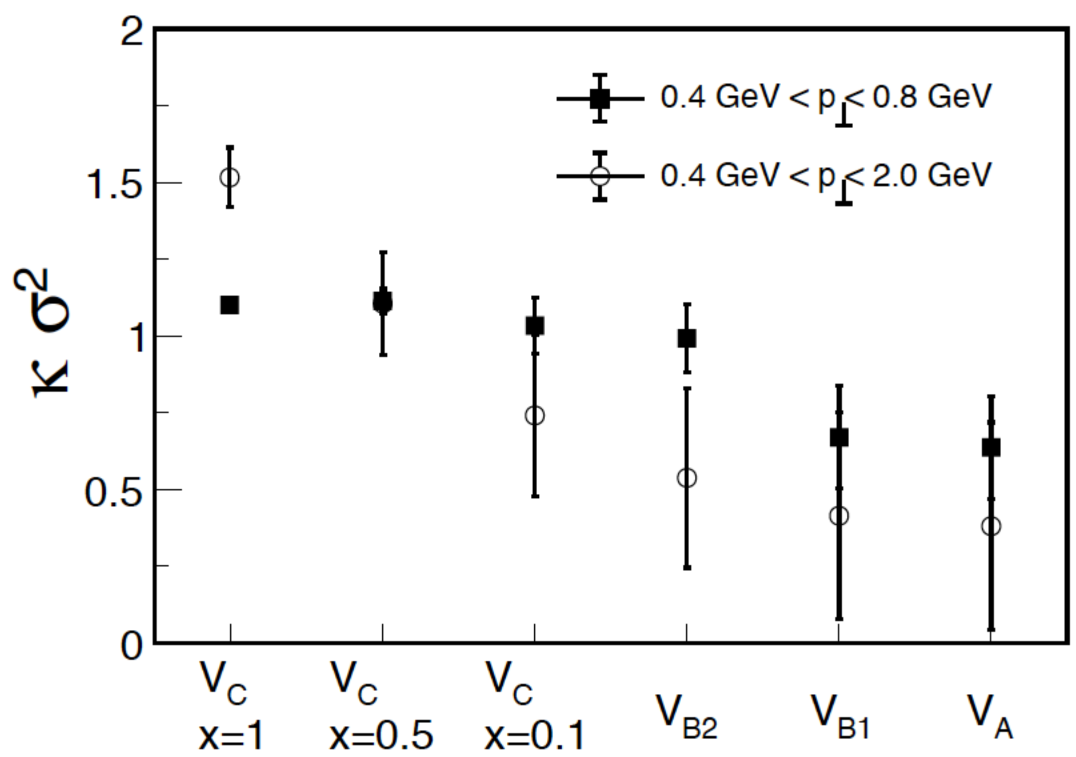


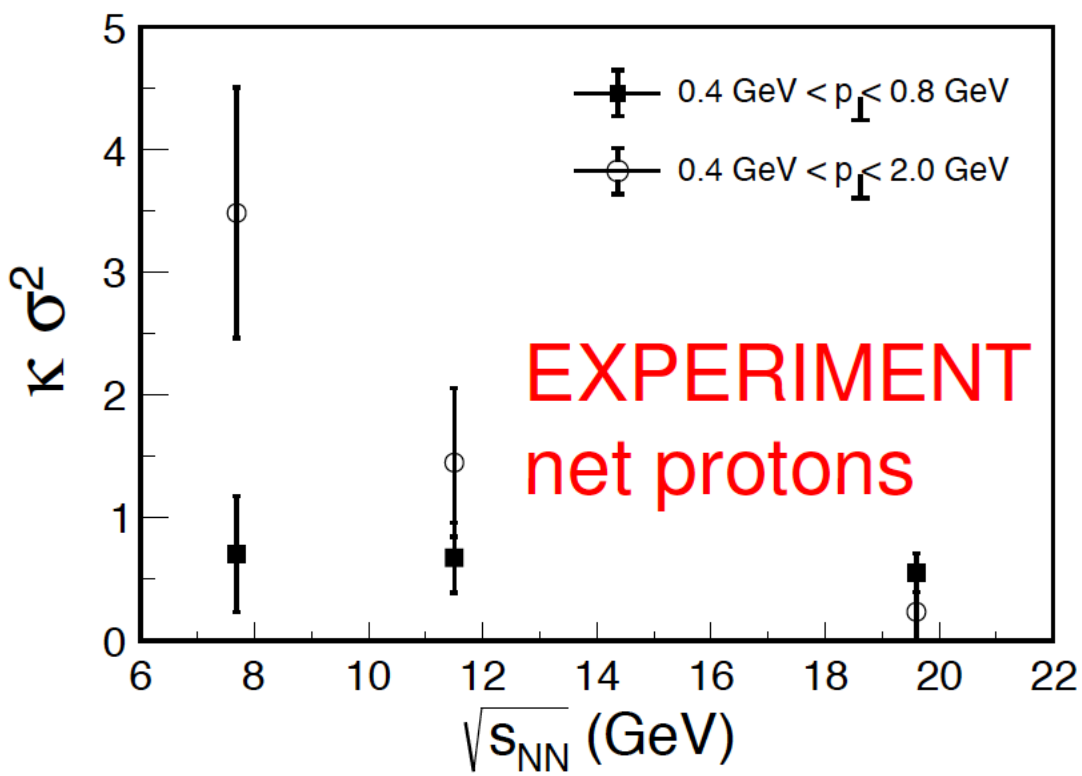
- V_A: Serot-Walecka with MF parameters
- $V_{A'}$: extra repulsion $\alpha_\omega \to 1.4\alpha_\omega$
- V_{B1}: $V_{A'}$ with $m_\sigma^2 \to m_\sigma^2/2$, $\alpha_\sigma \to \alpha_\sigma/2$
- V_{B2}: $V_{A'}$ with $m_\sigma^2 \to m_\sigma^2/2$
- V_C: very light critical mode $V_C(x) = (1-x)V_{B2} + xV_{A'}(m_\sigma^2 \to m_\sigma^2/6)$

Higher-order moments

Few-body correlations should contribute to proton moments

Scaled kurtosis: $\kappa\sigma^2 = C_4/C_2$

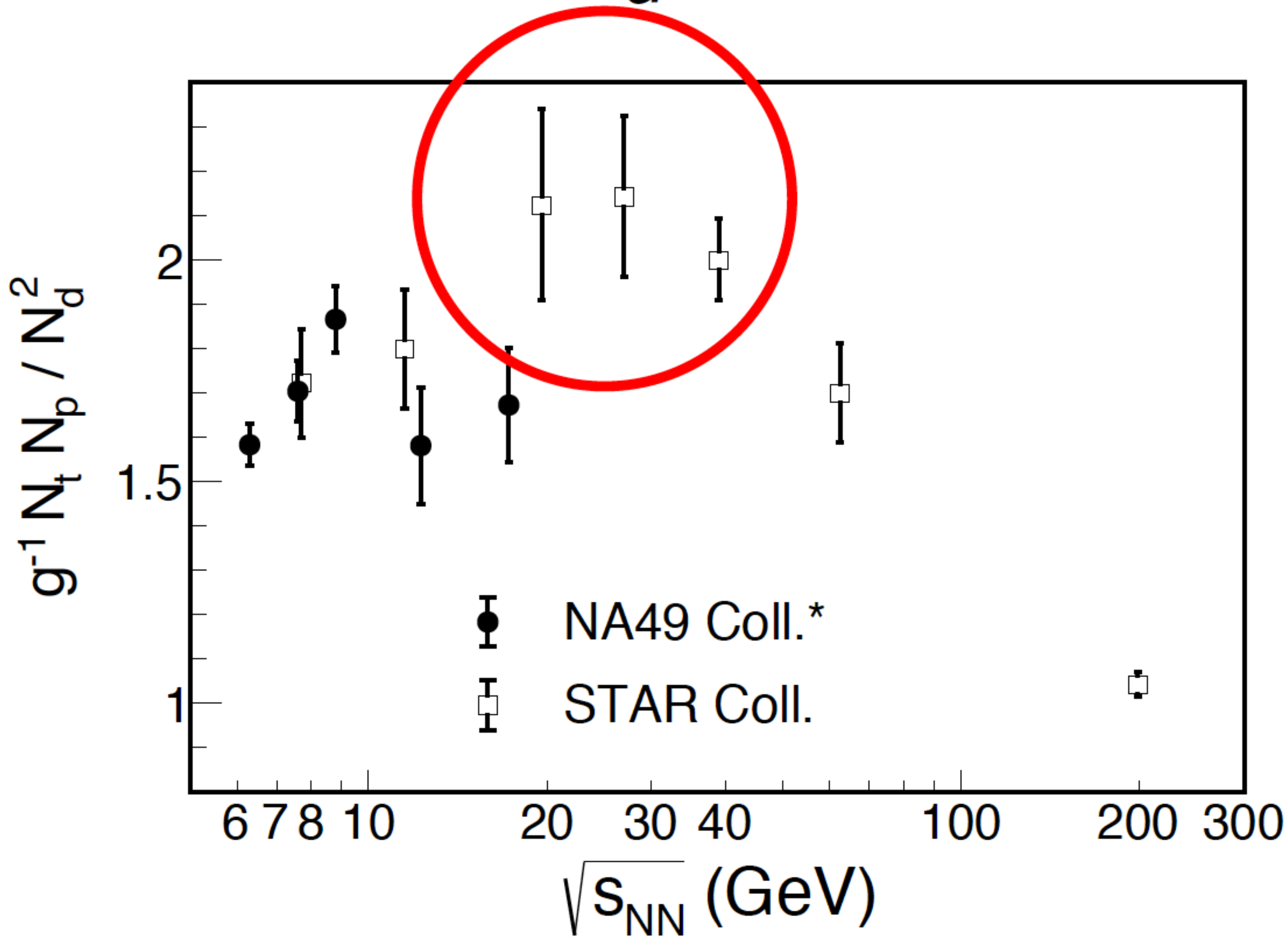




Expected increase with enhanced attraction, esp. in the wider $p_\perp$ window.

Triton-proton/deuteron ratio

$$g^{-1}\frac{N_t N_p}{N_d^2} \sim \left\langle e^{-\frac{V(x)}{T}} \right\rangle \qquad (g = 0.29)$$

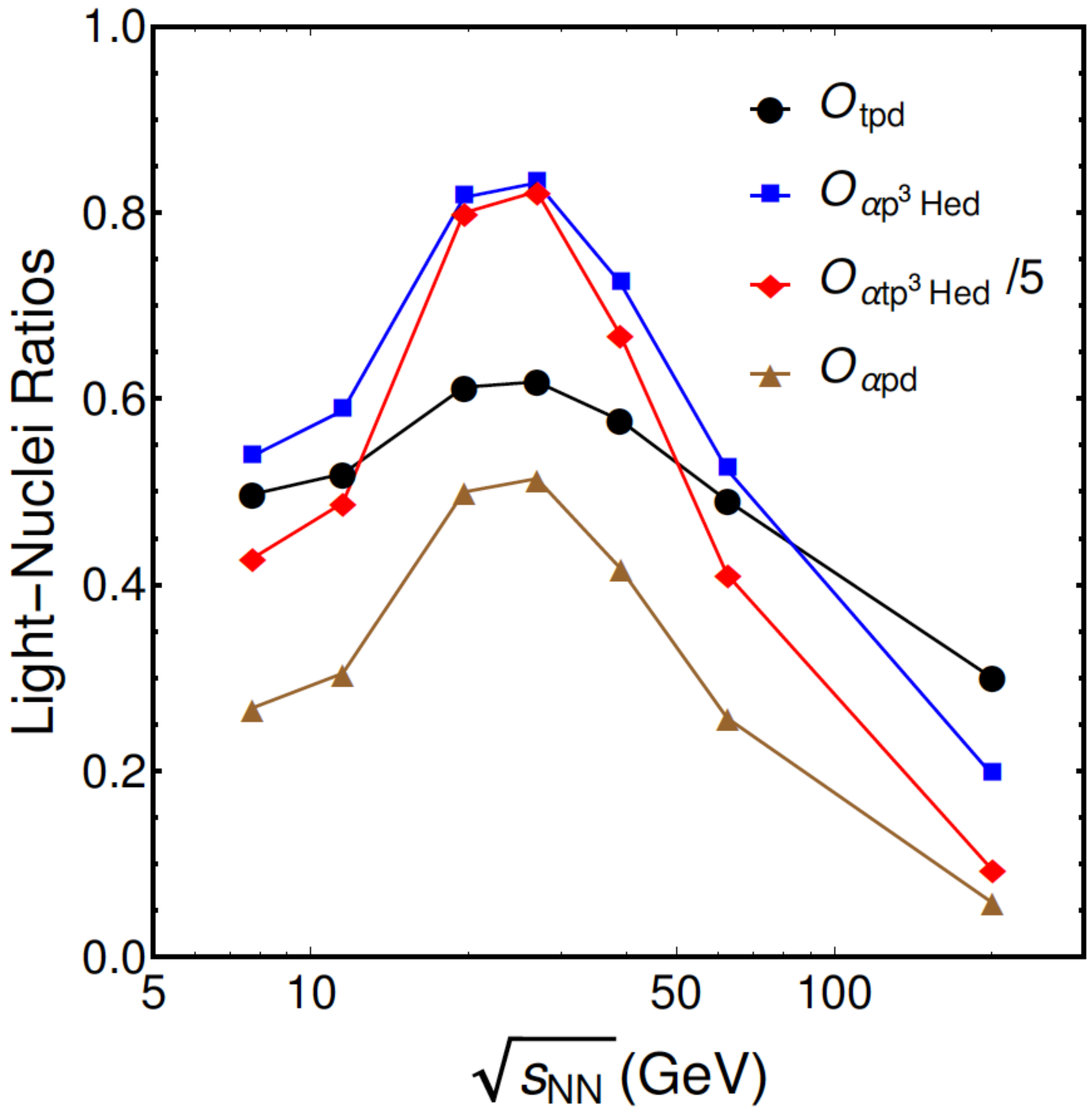


*Sun, Chen, Ko, Xu 2017, based on NA49 Coll. data

STAR Collaboration, preliminary 0%-10% (QM2018, arXiv:1909.07028)

Nuclei ratios

If clustering effects around T_c are the main source of the peak...
explore ^{4}He at the same energies!

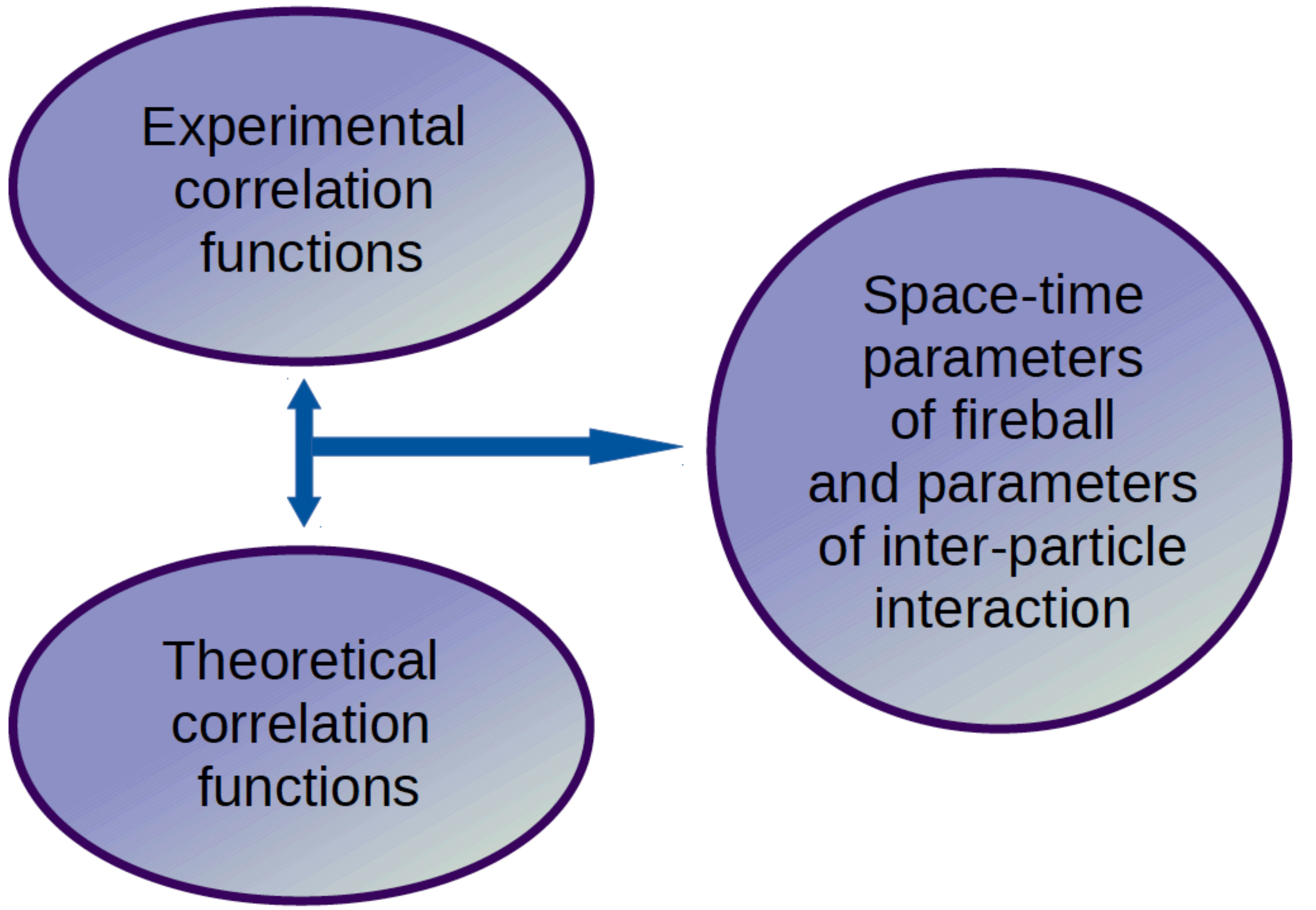


$$\mathcal{O}_{\alpha p^3\mathrm{He}d} = \frac{N_\alpha N_p}{N_{^3\mathrm{He}} N_d} \sim \langle e^{-\mathbf{2}V(x)/T} \rangle$$

$$\mathcal{O}_{\alpha t p^3\mathrm{He}d} = \frac{N_\alpha N_t N_p^2}{N_{^3\mathrm{He}} N_d^3} \sim \langle e^{-\mathbf{3}V(x)/T} \rangle$$

$$\mathcal{O}_{\alpha p d} = \frac{N_\alpha N_p^2}{N_d^3} \sim \langle e^{-\mathbf{3}V(x)/T} \rangle$$

Motivation


Experimental correlation functions
Theoretical correlation functions
Space-time parameters of fireball and parameters of inter-particle interaction

Test of improved sum rule

Pions $\pi^+ \pi^-$

Point-like source $\boxed{D_r(\mathbf{r}) = \delta^{(3)}(\mathbf{r})}$

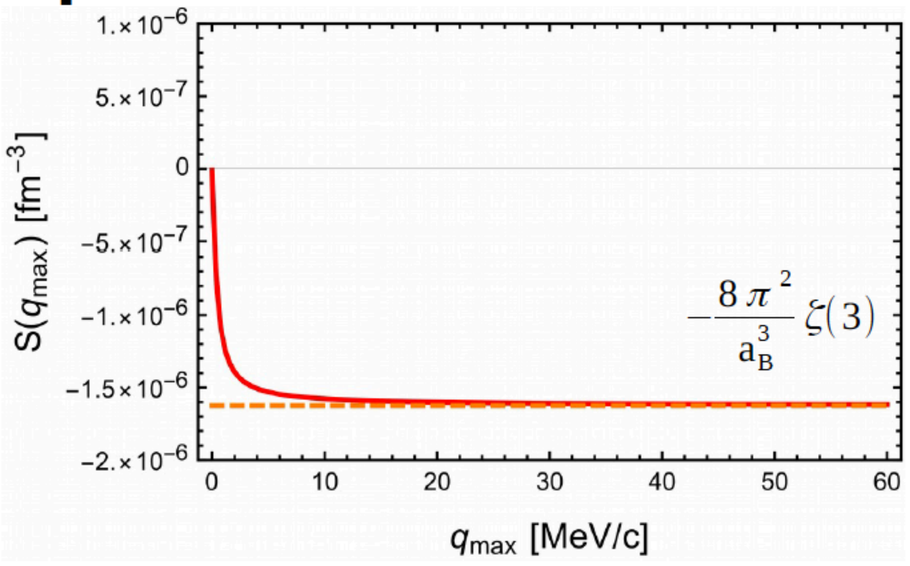
Expected result:

$$\int d^3q \left(G^{+-}(\mathbf{q}) + G^{++}_{--}(\mathbf{q}) - 2\right) = -\frac{8\pi^2}{a_B^3}\zeta(3)$$

$\widetilde{R}(\mathbf{q}) = G^{++}_{--}(\mathbf{q})$ - the correlation function for point-like source and pions treated as nonidentical particles

Problem: integrand expanded in powers of $\left[\frac{2\pi}{(a_B q)}\right]$ reveals divergent term.
It has to be removed from integrand.

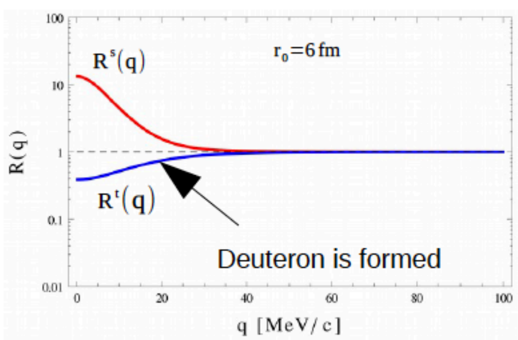
$$\int d^3q \left(G^{+-}(\mathbf{q}) + G^{++}_{--}(\mathbf{q}) - 2 - \frac{2\pi^2}{3 a_B^2 q^2}\right) = -\frac{8\pi^2}{a_B^3}\zeta(3)$$



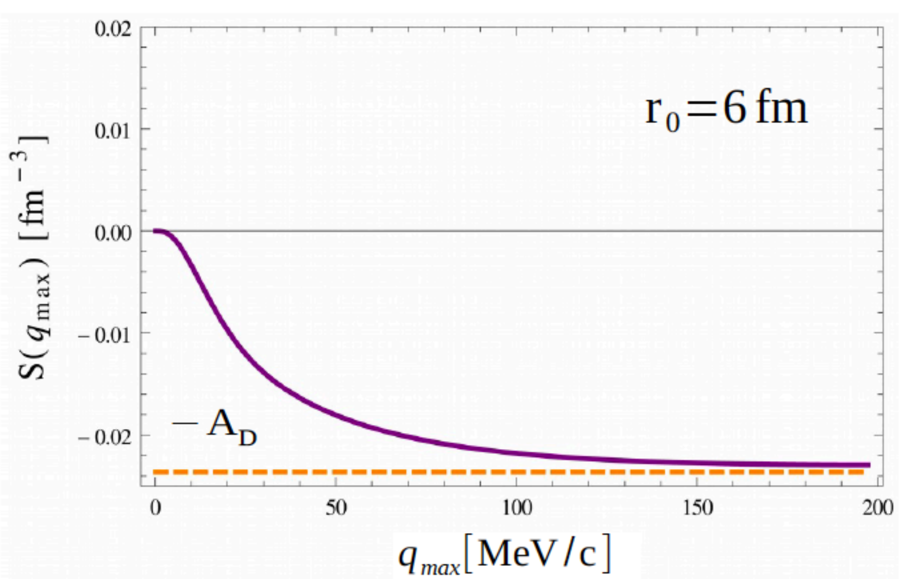
New sum rule works perfectly well !

Improved sum rule as a tool to test models

Neutron – proton

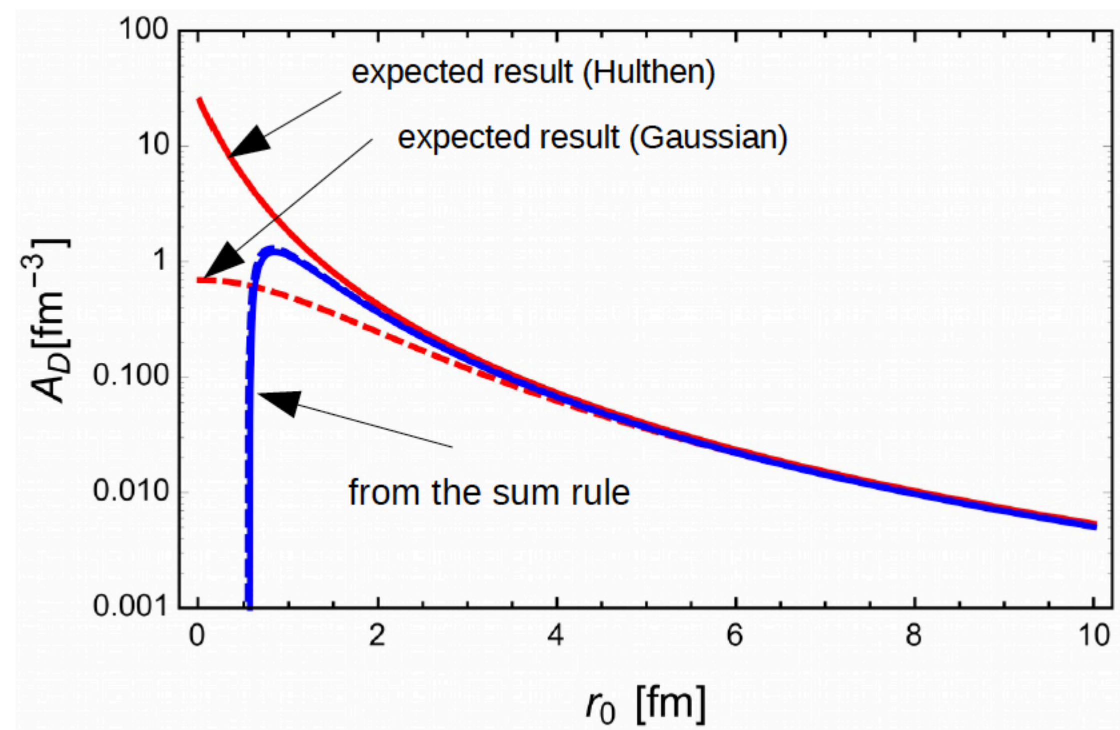


$$\widetilde{R}(\mathbf{q})=R^s(\mathbf{q})$$



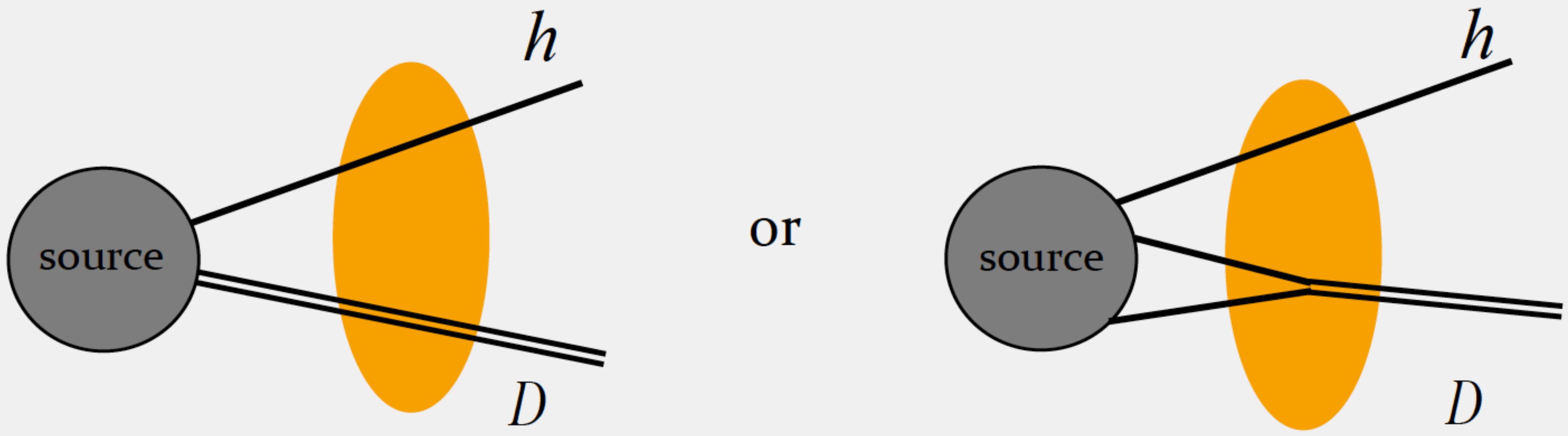
Expected result: $\int d^3q(R^t(q)-R^s(q))=-A_D$

The improved sum rule allows us to check the various models

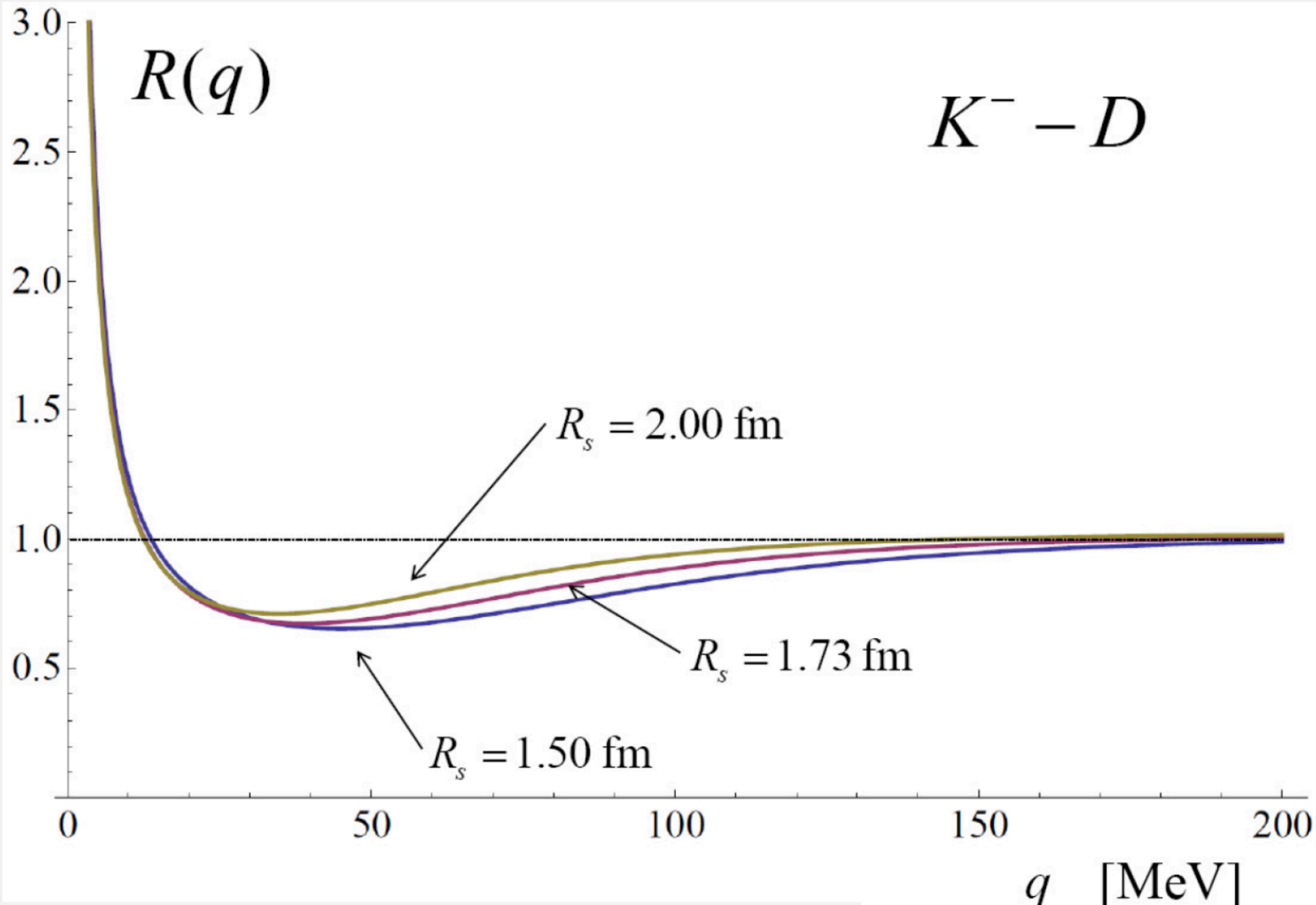


How to falsify one of the models?

- Hadron-deuteron correlations carry information about a source of deuterons.
- A measurement of K^--D or p-D correlation functions is suggested to falsify the thermal or coalescence model.

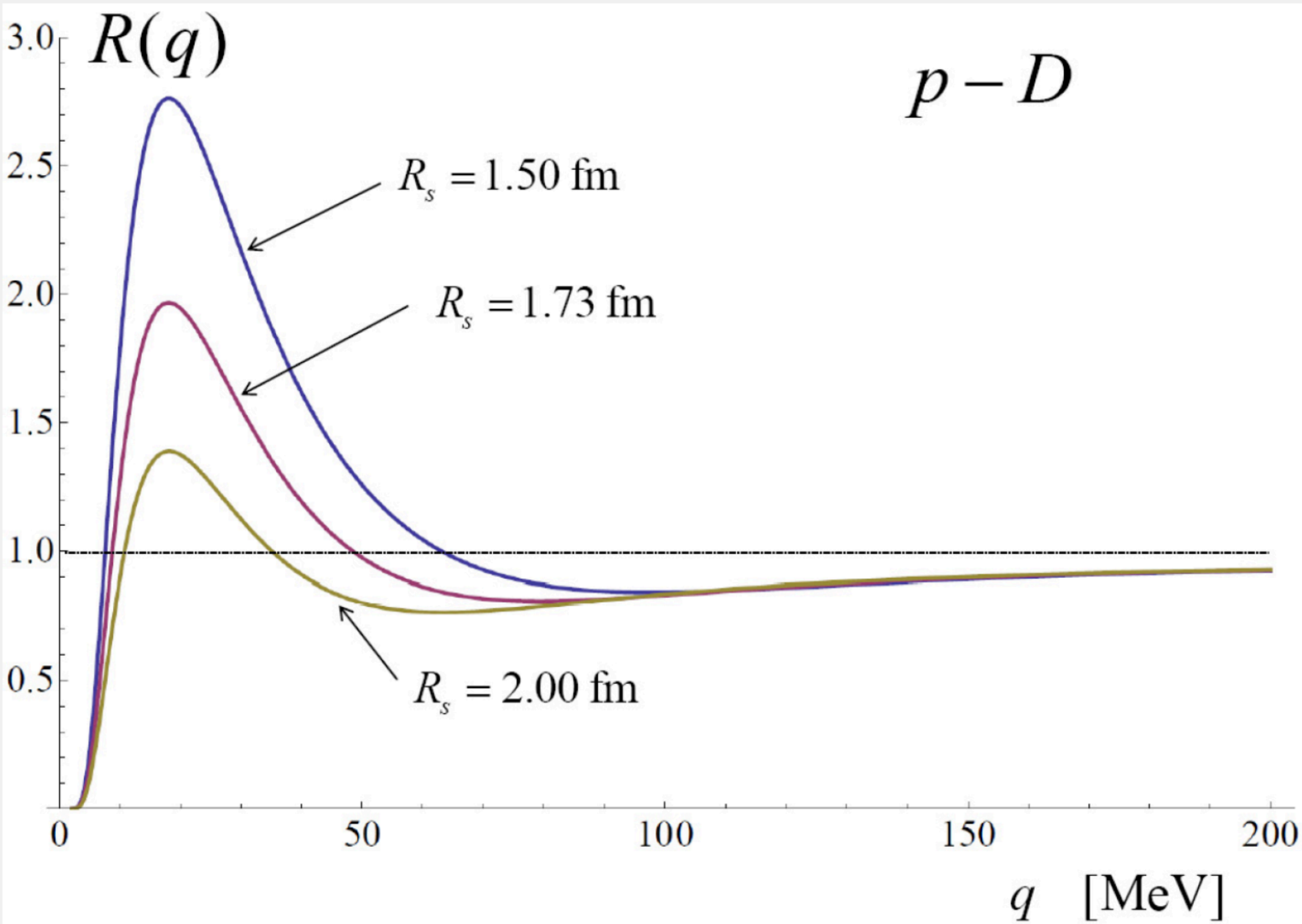


K^--D correlation functions



Measurement of *h*-*D* correlation function can tell us whether deuterons are directly emitted from a fireball or deuterons are formed due to final state interactions.

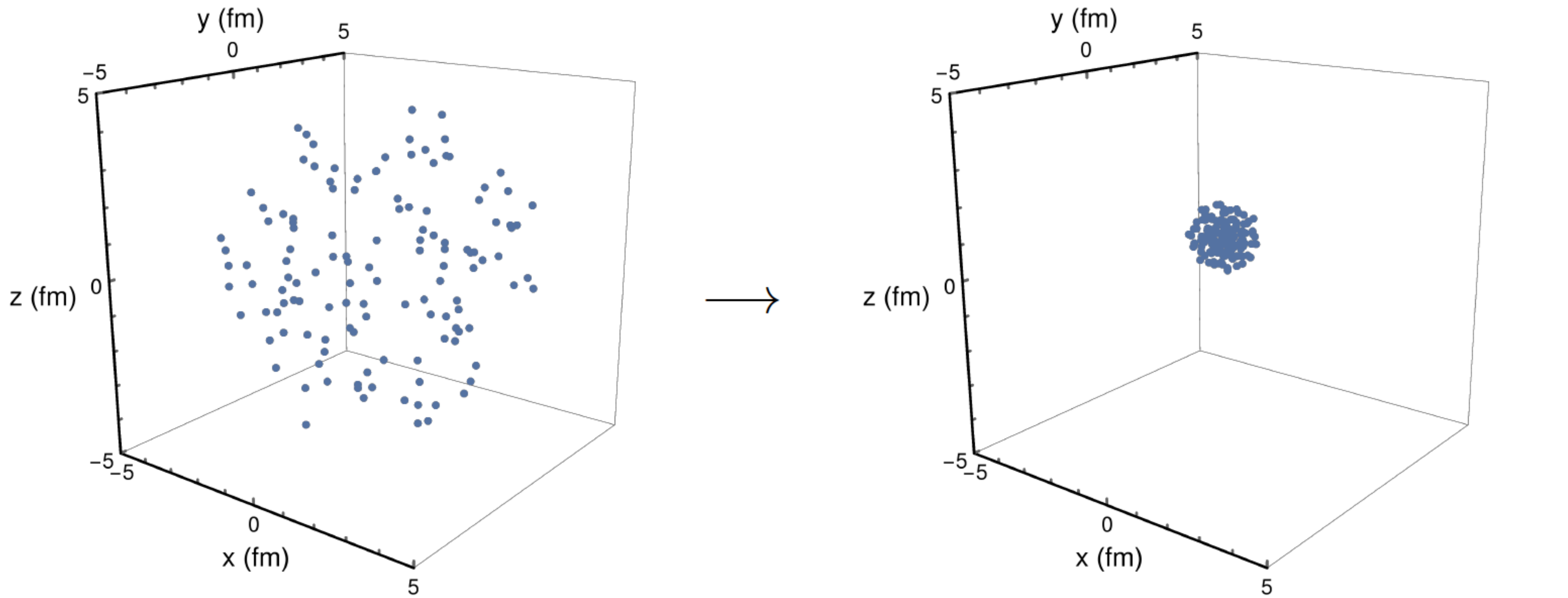
p-D correlation functions

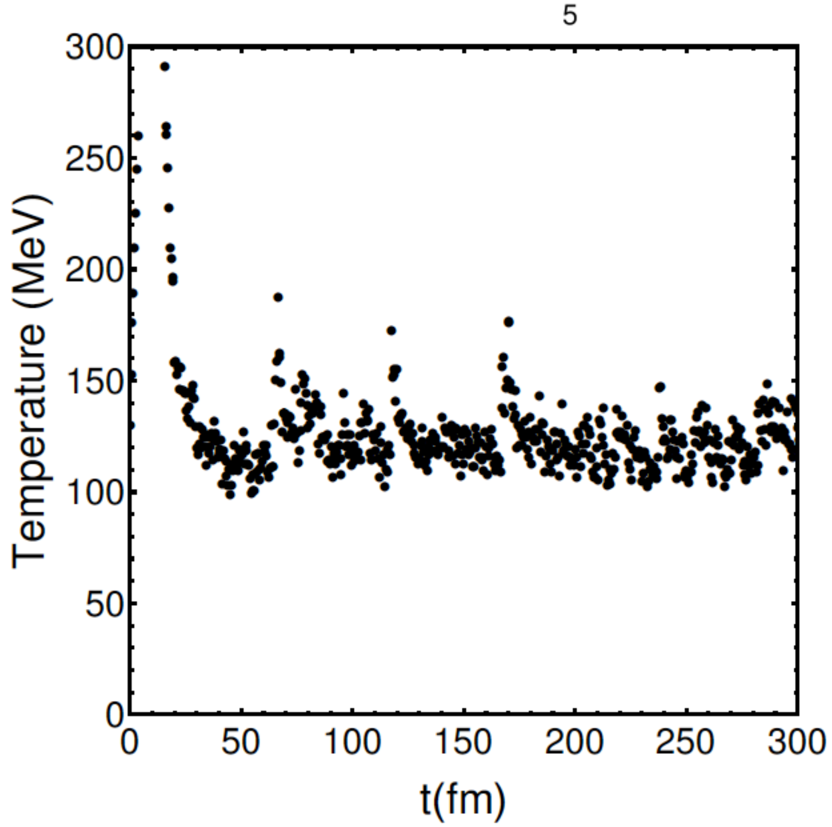


p-D correlation function shows a sufficient sensitivity to a size of particle source to falsify the thermal or coalescence model.

Thank you

Big clusters, $N = 128$





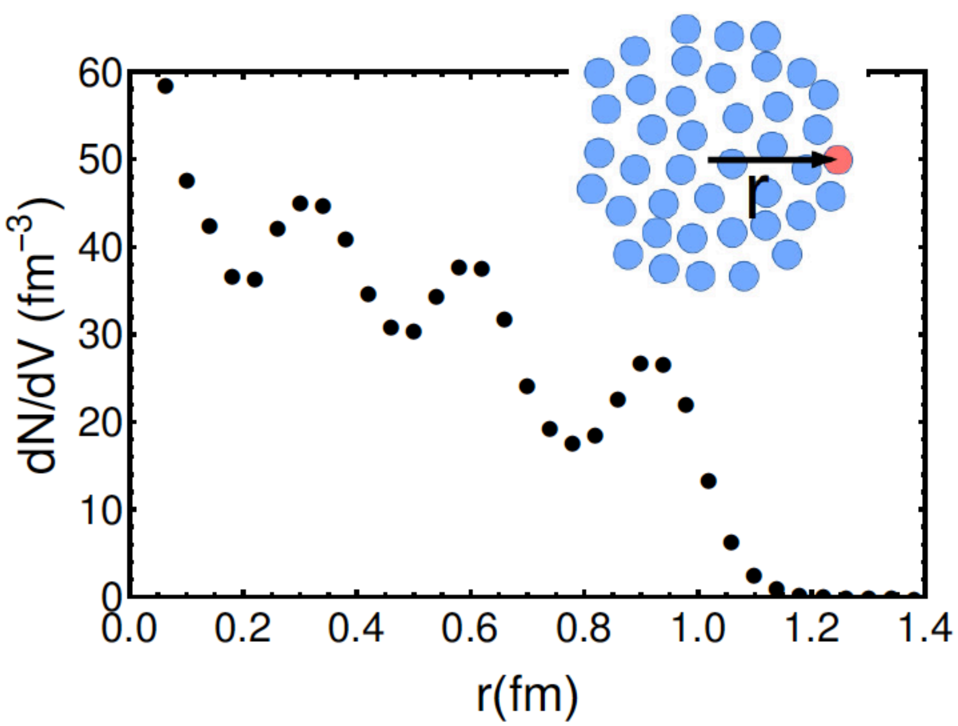
$T = 120$ MeV

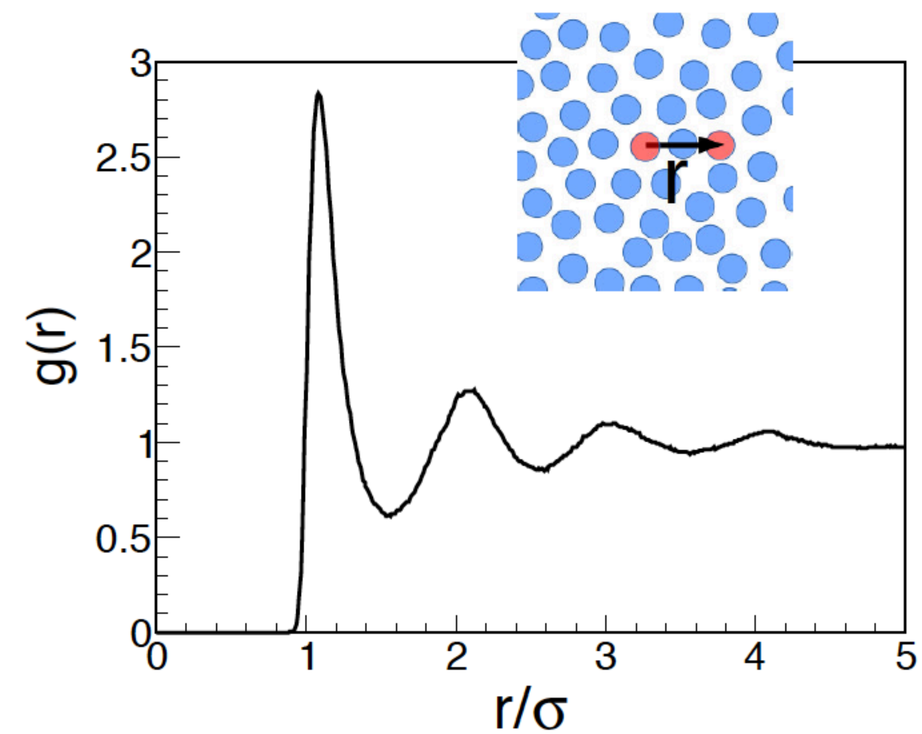
At large N the potential energy always wins over entropy: **clustering effect**.

This is just an illustrative example: unreachable time scales for HICs!

Message: Strongly-correlated systems

- Strongly correlated system ($P/K \simeq \mathcal{O}(N) > 1$): beyond mean field



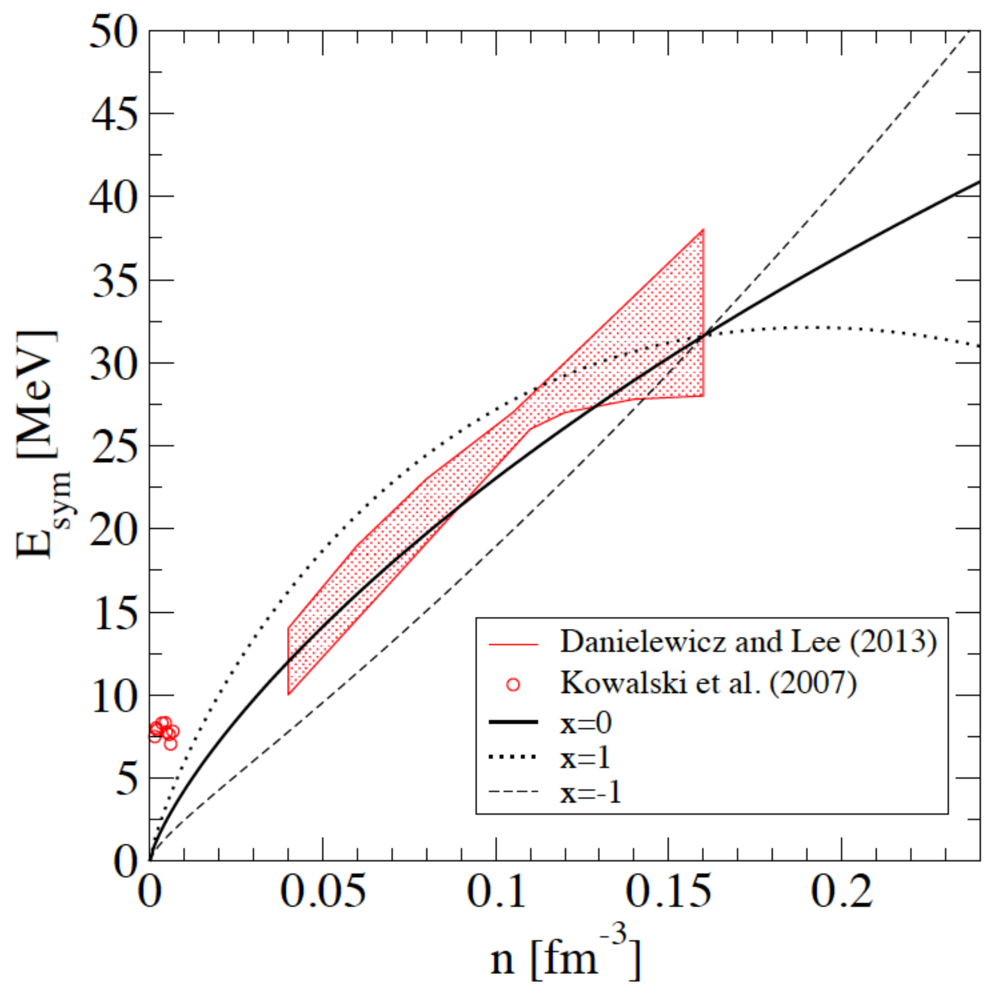


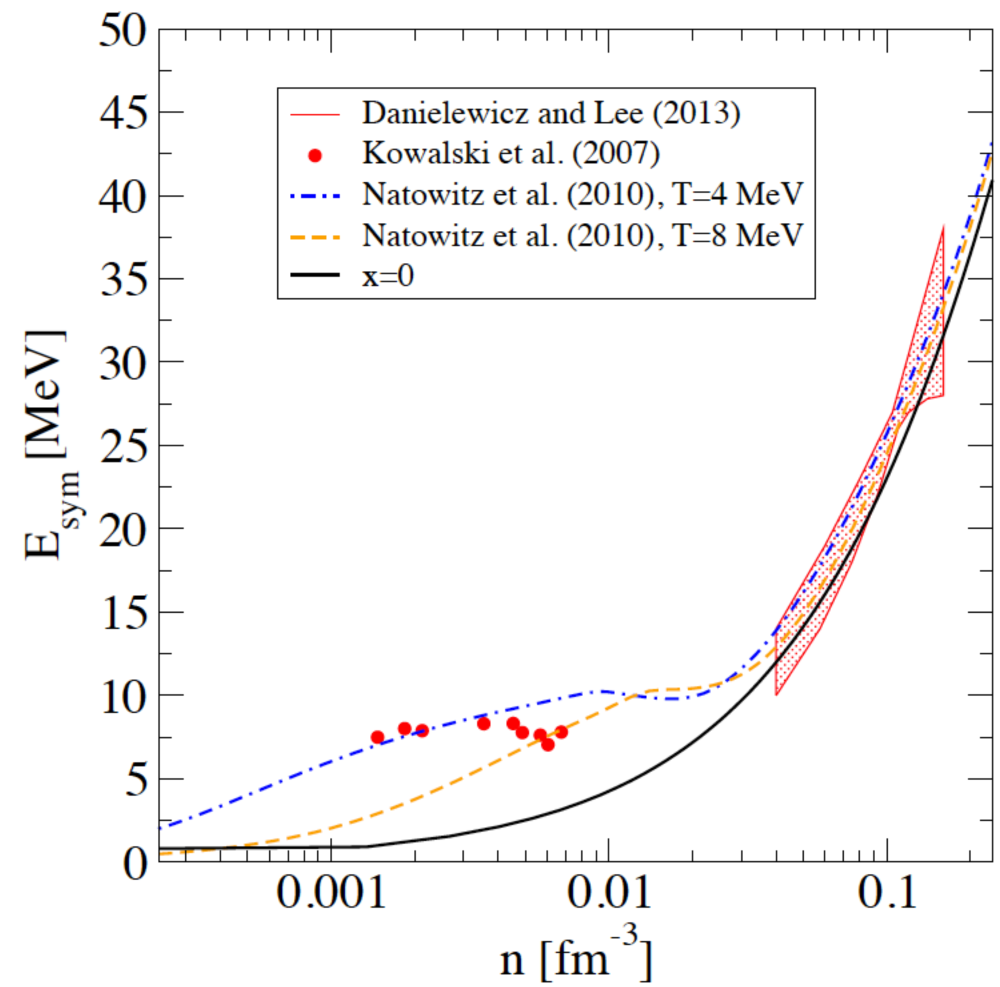
- Infinite systems: internal structure described by **pair correlation function** $g(r)$ e.g. liquid Argon ($N = 108$) via Lennard-Jones potential

- Approaches based on Boltzmann assumptions would NOT capture the whole effect *(similar idea in E. Bratkovskaya's talk)*

Symmetry energy: low density limit

correlations (bound states) → larger values for the symmetry energy





K. Hagel et al., Eur. Phys. J. A (2014) 50: 39