Exotica production in Heavy Ion Collision

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1. Few words on multiquark states
2. Are there compact multiquark states? X(3872)
3. Exotica production in heavy ion collision
4. Remark on recent X(3872) measurements by CMS

Anowledgements:

Quark Model : W. Park, A. Park, S. Cho, S. Noh

ExHIC : Sungtae Cho, T. Hyodo, D. Jido. Che Ming Ko, Su Houng Lee, K. Morita, M. Nielsen, A. Ohnishi, T. Sekihara, Taesoo Song. S. Yasui, K. Yazaki

Few words on “Multiquark states”

X(3872) $I^G\left(J^{PC}\right)=0^+\left(1^{++}\right)$

$B^{\pm} \rightarrow K^{\pm}\pi^+\pi^- J/\psi$

$M = 3872.0 \pm 0.6 \pm 0.5\,\text{MeV}$

$\Gamma < 1.2\ \text{MeV}$

\- 2003 -

Zc(3900) $I^G\left(J^{PC}\right)=1^+\left(1^{+-}\right)$

$M = 3887.2 \pm 2.3\ \text{MeV}$

$\Gamma = 28.2 \pm 2.6\ \text{MeV}$

\- 2013 - BESSIII $e^+e^- \rightarrow \pi^+\pi^- J/\psi$

Pentaquark - Pc

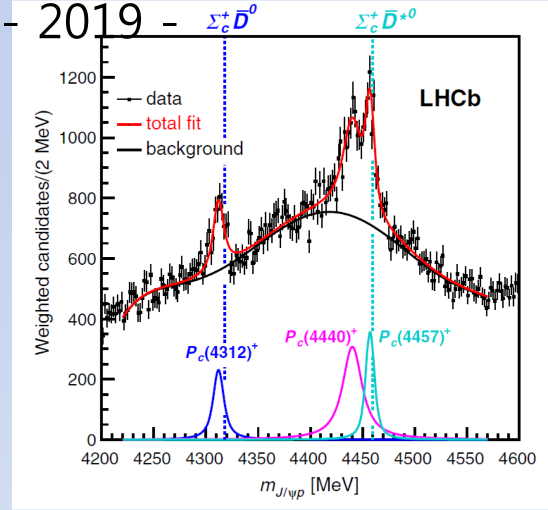
\- 2015 - $\Lambda_b^0 \rightarrow J/\psi\ p\, K^-$

$M_1 = 4380 \pm 8 \pm 29\,\text{MeV}$

$\Gamma_1 = 205 \pm 18 \pm 86\,\text{MeV}$

$M_2 = 4449.8 \pm 1.7 \pm 2.5\,\text{MeV}$

\- 2019 -



d*(2380) $I\left(J^P\right)=0\left(3^+\right)$ $\Gamma = 70\ \text{MeV}$

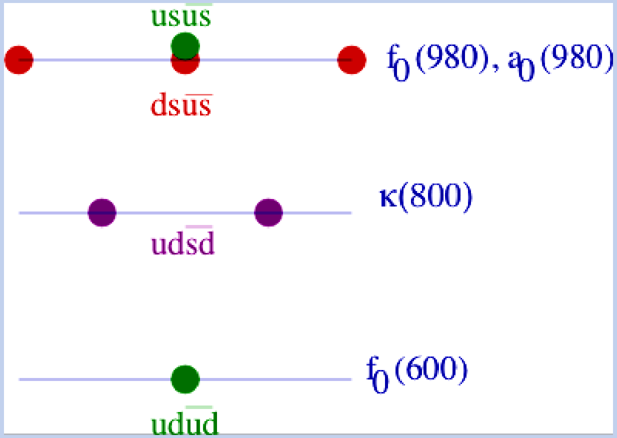
\- WASA-at-COSY [H. Clement]-

Revival of an old topic

☞ Tetraquark:

- scalar tetraquark (Jaffe 76)

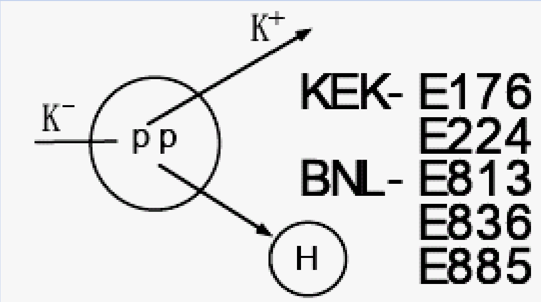
→ no direct proof of structure



☞ Dibaryon

- H (ududss) dibaryon (Jaffe 77):

→ experimentally not found

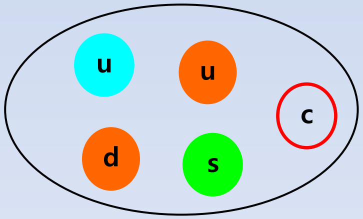


☞ Pentaquark

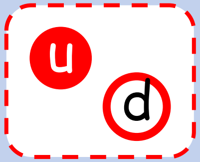
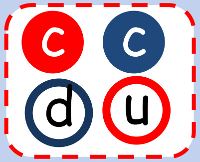
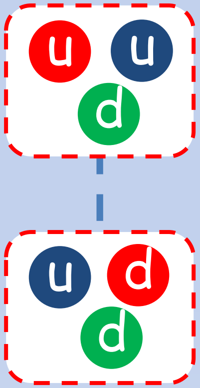
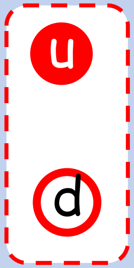
- $P_{\bar{c}s}$ (Gignoux, Silvestre-Brac, Richard 87)
- $P_{\bar{c}s}$ (udus$\bar{c}$) (Lipkin 87)

→ Fermilab E791 : not found

$$P^0_{\bar{c}s} \to K^{*0} K^- p$$



Normal meson, compact multiquark, molecules, resonances

	Normal meson	Compact multiquark	Molecules	Resonance
Geometrical configuration				
Examples	Nucleon, pion, kaon	Flavor exotic?	X(3872) ?	K*, ρ meson

- Are Pc, X, d* compact or loosely bound molecular states ?
- Where are the compact flavor Exotics ?
- How can we distinguish compact vs molecular states ?

II: Are there compact Multiquark states ?

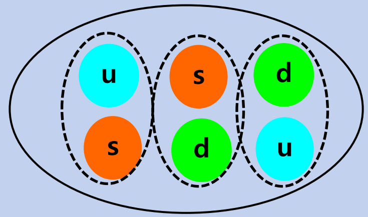
A.Park, W. Park, S. Cho, SHL :

PRD92 (2015) 014037, …….. PRC100 (2019) 055201

Why H –dibaryon ?

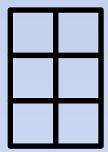
☞ Jaffe (1977) in Bag model: A strong color-spin attraction in Flavor 1 of SU(3)

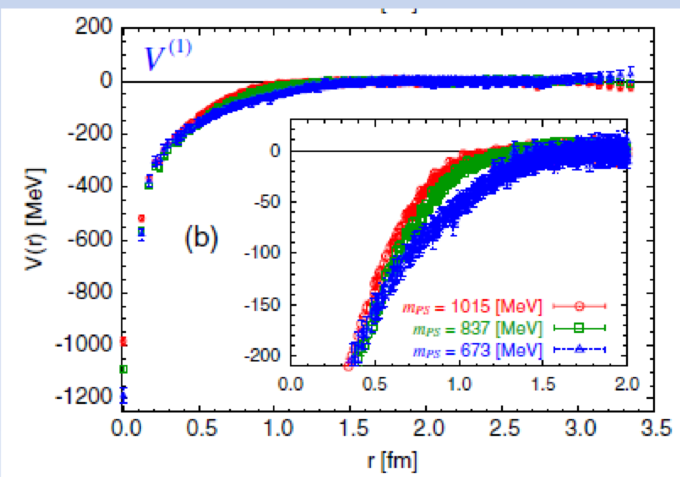
$$\left|\psi_{6-quark}\right\rangle =$$



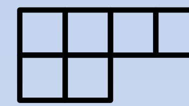
☞ HAL QCD Lattice : Attractive Baryon-Baryon potential in the Flavor=1 channel

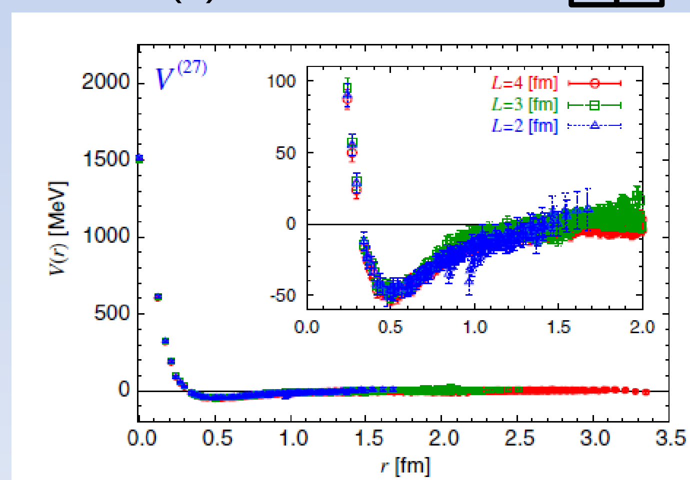
SU(3) flavor 1 state





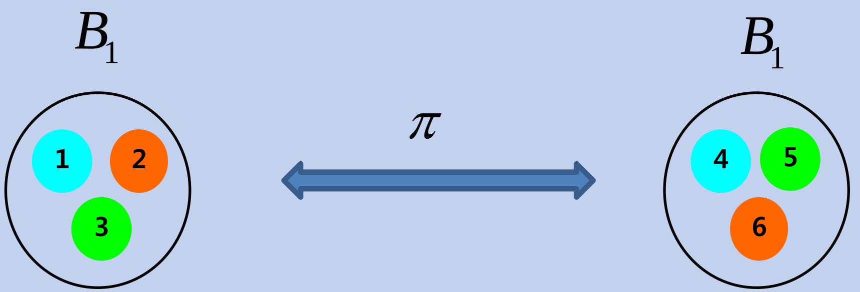
SU(3) flavor 27 state





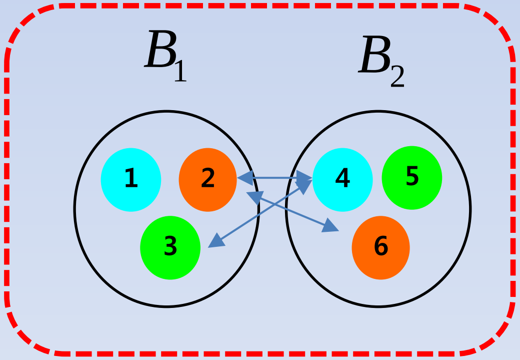
Nuclear force at short distance

☞ Long distance force > 0.5 fm → meson exchange



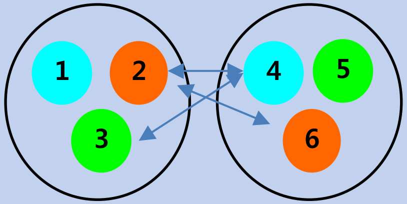
☞ Short distance force < 0.5 fm → quark dynamics (Oka, Yazaki)

$$\left|\psi_{6-quark}\right\rangle =$$



N- quarks in a compact configurations - quark model

$$H = \sum_{i=1}^{N}\left(m_i + \frac{p_i^2}{2m_i}\right) - \sum_{i<j}^{N}\left(\lambda_i^c\lambda_j^c\right)V_{ij}^C\left(r_{ij}\right) - \sum_{i<j}^{N}\frac{\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i\sigma_j\right)}{m_i m_j}V_{ij}^{SS}\left(r_{ij}\right)$$



☞ Color-Color interaction is no important for short range N-N interaction

$$\sum_{i<j}^{N}\left(\lambda_i^c\lambda_j^c\right) = \text{Total color } - N\left(\text{quark charge}\right) = 0 - \frac{2}{3}N \qquad N = N_{B_1} + N_{B_2}$$

☞ Color-spin interaction for 2 body:

$$K = -\sum_{i<j}^{N}\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i^s\sigma_j^s\right) \longrightarrow$$

	Q-Q				*Q-$\bar{Q}$*			
Color	A	S	A	S	1	8	1	8
Flavor	A	A	S	S				
Spin	A(0)	S(1)	S(1)	A(0)	0	0	1	1
K	-8	-4/3	8/3	4	-16	2	16/3	-2/3

☞ $M_\Delta - M_P \approx 290 \text{ MeV} \propto K_\Delta\left(3\times\frac{8}{3}\right) - K_P\left(-8\right) = 16$

- Comparison with lattice (HAL QCD) – NN interaction

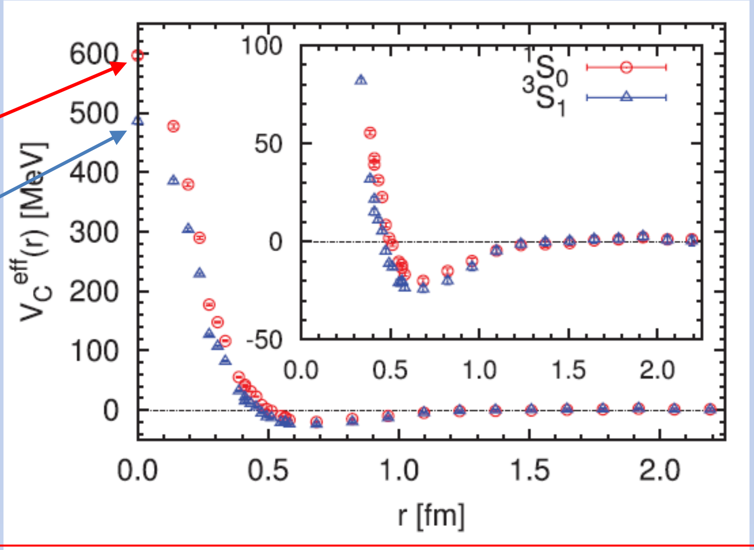
A.Park, SHL, Inoue, Hatsuda, arXiv:1907.06351
A.Park, SHL PRC100(2019)055201

☞ NN force in SU(2) spin 1 vs spin 0 channel

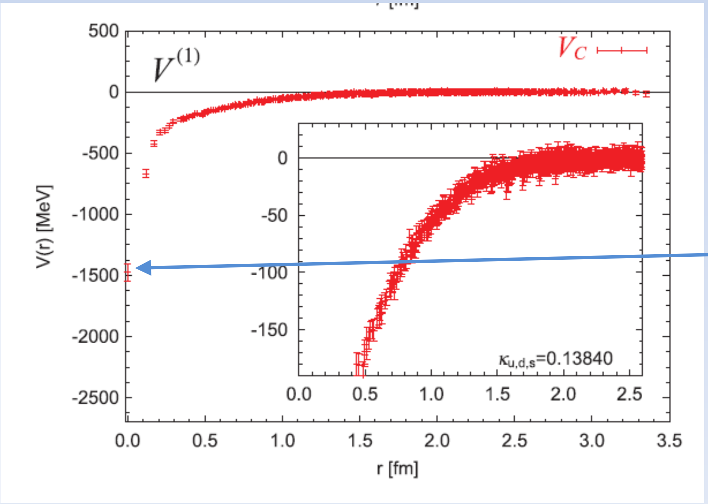
$$K_{2-N} = K_{6-quark} - \left(K_{1N} + K_{1N}\right)$$

$$\frac{K_{2-N}^{S=0}}{K_{2-N}^{S=1}} = 1.29$$

→ comparison

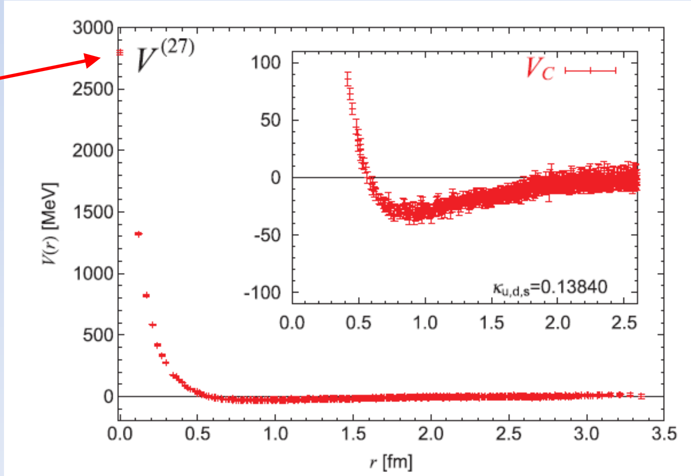


☞ H dibaryon channel: Flavor 1 vs Flavor 27



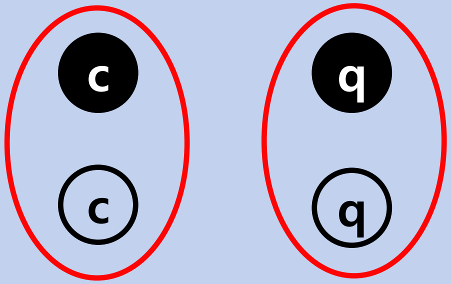
$$\frac{K_{2-N}^{F=27}}{K_{2-N}^{F=1}} = -3$$

(HAL QCD Collaboration)



Quark wave function for X(3872) and Zc(3900)

- A tetra-quark state $\left(c\bar{c}\right)\otimes\left(q\bar{q}\right)$ with quarks in s-wave



X(3872) $I^G\left(J^{PC}\right)=0^+\left(1^{++}\right)$

$$\psi_1=\left(c\bar{c}\right)^{S=1}_{C=1}\otimes\left(q\bar{q}\right)^{S=1}_{C=1}$$

$$\psi_2=\left(c\bar{c}\right)^{S=1}_{C=8}\otimes\left(q\bar{q}\right)^{S=1}_{C=8}$$

Z_c(3900) $I^G\left(J^{PC}\right)=1^+\left(1^{+-}\right)$

$$\phi_1=\left(c\bar{c}\right)^{S=0}_{C=1}\otimes\left(q\bar{q}\right)^{S=1}_{C=1}$$

$$\phi_2=\left(c\bar{c}\right)^{S=0}_{C=8}\otimes\left(q\bar{q}\right)^{S=1}_{C=8}$$

$$+\left(S=1\leftrightarrow S=0\right)$$

Color-spin interaction of X(3872) and $Z_c(3900)$

X(3872) $I^G\left(J^{PC}\right)=0^+\left(1^{++}\right)$

$$(c\bar{c})\otimes(q\bar{q})$$

$$\sum_{i,j}\frac{K_{ij}}{m_i m_j}=\begin{pmatrix}\frac{16}{3}\frac{1}{m_c^2}+\frac{16}{3}\frac{1}{m_c^2} & 0 \\ 0 & -\frac{2}{3}\frac{1}{m_c^2}-\frac{2}{3}\frac{1}{m_q^2}-\frac{36}{3}\frac{1}{m_c m_q}\end{pmatrix}$$

$(1\otimes 1)_c(V\otimes V)_{S=1}$

$(8\otimes 8)_c(V\otimes V)_{S=1}$

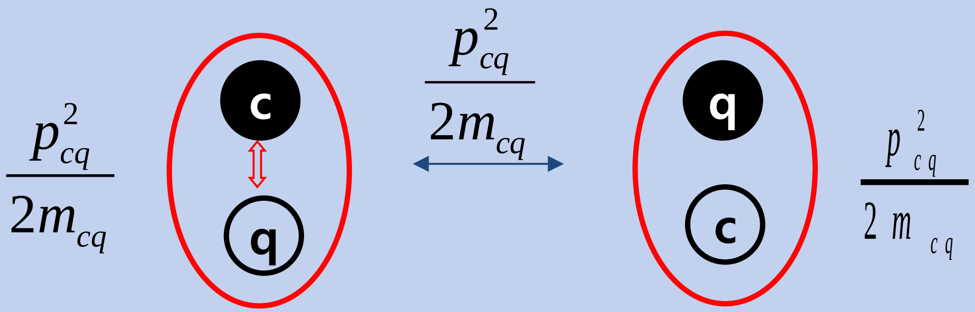
~ -100 MeV

- K factor for lowest 2-body threshold $\bar{D}D^* = -\frac{32}{3}\frac{1}{m_c m_q}$ ~ -60 MeV
- K factor in diquark antidiquark picture $(cq)^{S=0}_{C=\bar{3}}\otimes(\overline{cq})^{S=0}_{C=3}=-\frac{16}{3}\frac{1}{m_c m_q}$ ~ -36 MeV
- *Corresponding* K factor in $Z_c(3900)$ is small $(c\bar{c})^{S=1}_{C=8}\otimes(q\bar{q})^{S=0}_{C=8}=-\frac{2}{3}\frac{1}{m_q^2}+2\frac{1}{m_c^2}$

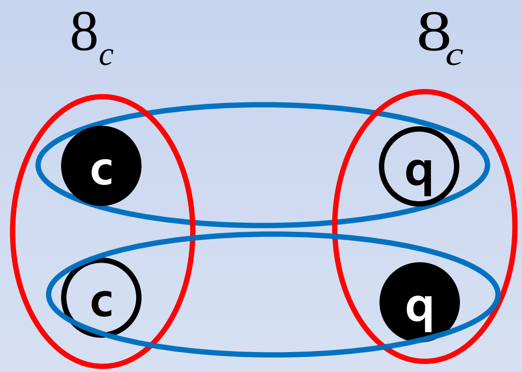
~ -9 MeV

II Kinetic term vs additional attraction

- Additional Kinetic term in compact configuration > 100 MeV
  → favors more than one heavy quark



- Additional attraction should be strong enough to overcome additional kinetic energy → favors more flavor



☞ *Attraction seems not strong enough but only experiment will tell*

→ *Heavy Ion collision will have many heavy quarks*

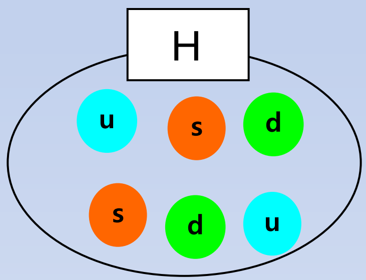
$$E_{CS} = -\sum_{i<j}^{n} \frac{\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i \sigma_j\right)}{m_i m_j} V_{ij}^{SS}\left(r_{ij}\right) = -\sum_{i<j}^{n} \left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i \sigma_j\right) C_{ij}$$

K

$$\begin{cases} \Delta - N = 16C_{uu},\ \Sigma^* - \Sigma + \Xi^* - \Xi = 32C_{us}, \\ \Omega_c^* - \Omega_c = 16C_{sc}\ ,\ \ \Sigma_c^* - \Sigma_c = 16C_{uc}\ , \\ 2\Omega + \Delta - \left(2\Xi^* + \Xi\right) = 8C_{ss} + 8C_{uu} \end{cases}$$

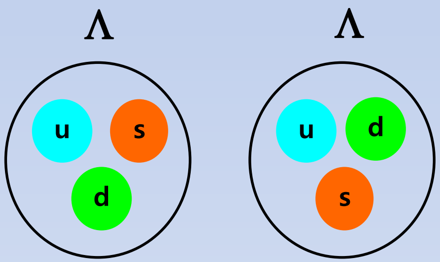
Need to Work out (color) x (spin) x (flavor) wave function (W. Park, A. Park, Cho, Lee)

☞ H dibaryon channel: Flavor Breaking effect weakens attraction in Flavor 1



$$\leftarrow E_{Kin} = \frac{p_{BB}^2}{2\mu_{BB}} \approx 100 \text{ MeV}$$

VS



E_{CS}	**H dibaryon**	$\Lambda\ +\ \Lambda$	**Δ E_{CS}**	**Δ E_{kin}**
$m_{u,d} = m_s$	$-24C_{uu}$	$-8C_{uu} - 8C_{uu}$	-145 MeV	+100 MeV
$m_{u,d} \neq m_s$	$\left(-5C_{uu} - 22C_{us} + 3C_{ss}\right)$	$-8C_{uu} - 8C_{uu}$	-61 MeV	+100 MeV

Probably molecular configurations

Less attractive →

☞ Tetraquark X($q\bar{q}c\bar{c}$)

$\Delta E_{CS}\left(m_{ud}, m_s, m_c\right) = -40 \text{ MeV}$ → probably molecular type

☞ Pentaquark Pc (uudc$\bar{c}$) (LHCb)

$\Delta E_{CS}\left(m_{ud}, m_s, m_c\right) = -34 \text{ MeV}$ → Can not be a compact state (Park,Park,Cho, Lee PRD95(17))

→ probably molecular type

☞ Dibaryon d*(2380) (uuuuuu) (WASA at COSY)

$\Delta E_{CS}\left(m_{ud}, m_s, m_c\right) = 0 \text{ MeV}$ → Can not be a compact state (Park,Park,Lee PRD92(15))

→ probably molecular type

Possible compact Flavor exotics? W.Park, S. Cho, Lee PRD99(2019)094023

Most attractive → Same origin as $P_{\bar{c}s}(udus\bar{c})$ color spin of q^4 : [31]

☞ Pentaquark Psc$\bar{c}$ (udsc$\bar{c}$) $I = 0,\ S = 1/2$ $\Lambda + J/\psi$

$$\Delta E_{CS}(m_{ud}, m_s, m_c) = -124 \text{ MeV}$$

☞ Pentaquark Pcc$\bar{s}$ (udcc$\bar{s}$) $I = 0,\ S = 1/2$ $\Xi_{cc}^{+} + K^{+}$

$$\Delta E_{CS}(m_{ud}, m_s, m_c) = -135 \text{ MeV}$$

☞ Tetraquark T(ud$\bar{c}\bar{c}$) $I = 0,\ S = 1$ $D^0 + D^{*+}$ or $\bar{D}^0 + \bar{D}^{*-}$

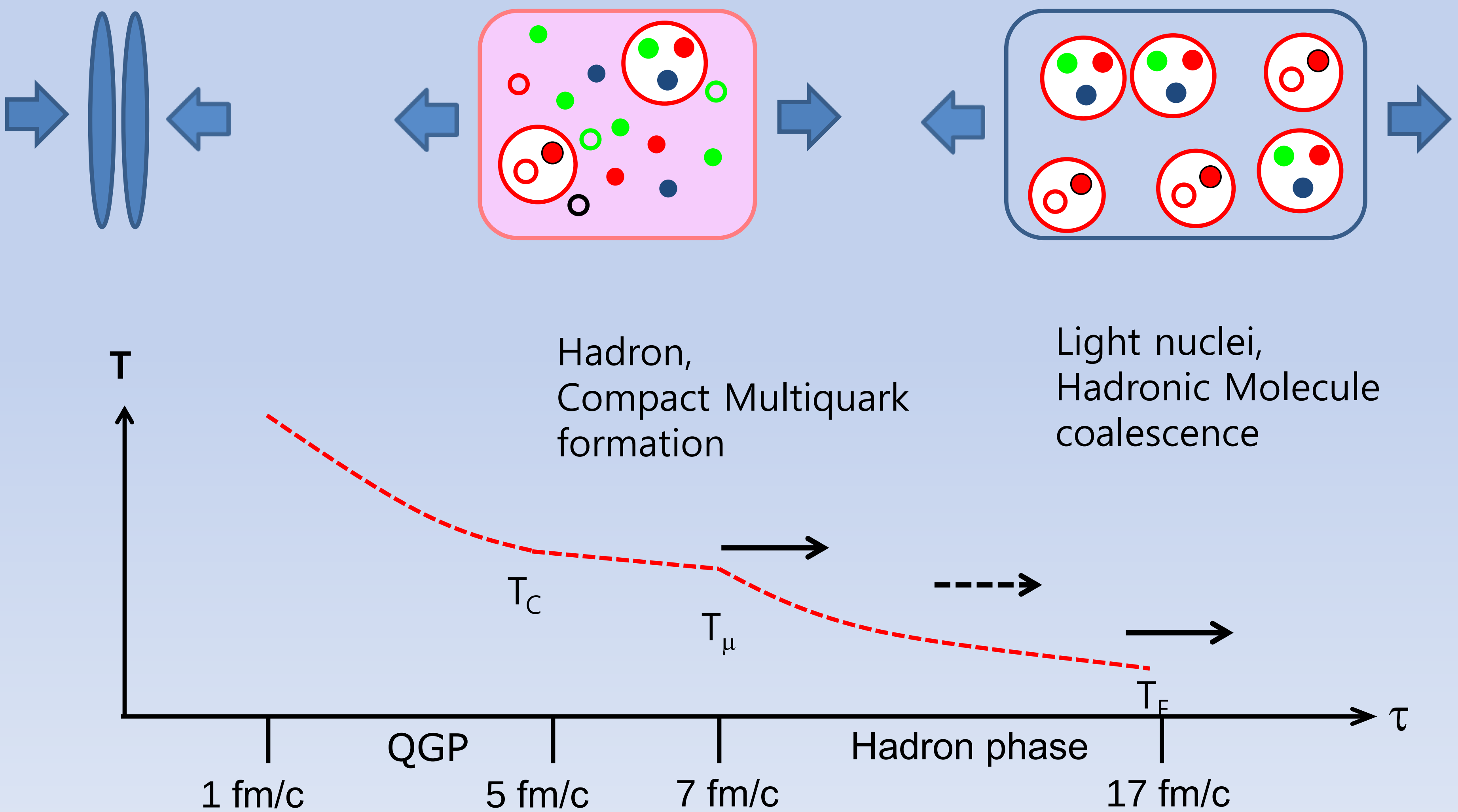
$$\Delta E_{CS}(m_{ud}, m_s, m_c) = -150 \text{ MeV}$$

III: Exotica production from Heavy Ion Collision

- ExHIC collaboration (Theory):
PRL 106(2011)212001, PRC38 (2011) 064910, PPNP95 (2017) 279

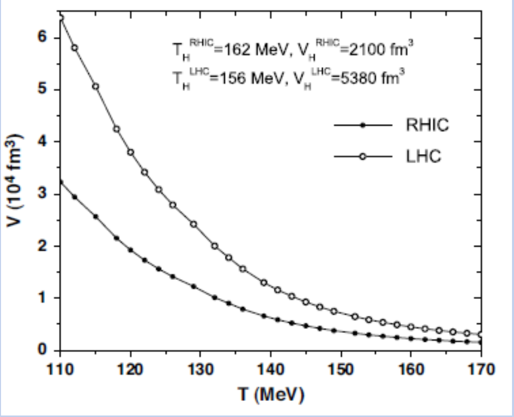
- X(3872) and Tcc related with S. Cho, J. Hong,
PRC88 (2013) 054901, PRC98(2018)104913

Particle production in heavy ion collision



Production of Exotica (ExHIC PRL2011, PPNP 2017)

- Model central rapidity, central collision using Lattice EOS



- Normal and compact multi-quark formation at T_C

  Coalescence Parameters:

  fit production of normal hadrons

  from statistical model

$$N_h^{\rm coal} = g_h \prod_{j=1}^{n} \frac{N_j}{g_j} \prod_{i=1}^{n-1} \frac{\int d^3y_i d^3k_i f_i(k_i) f^W(y_i, k_i)}{\int d^3y_i d^3k_i f_i(k_i)}$$

$$f_s^W(y_i, k_i) = 8\exp\left(-\frac{y_i^2}{\sigma_i^2} - k_i^2\sigma_i^2\right) \qquad \sigma_i = 1/\sqrt{\mu_i \omega}$$

$$m_{u,d} = 300 \text{ MeV}$$
$$m_s = 500 \text{ MeV}$$
$$m_c = 1500 \text{ MeV}$$
$$m_b = 4700 \text{ MeV}$$

- Molecule formation at

  kinetic Freeze out T_F

  Coalescence size: Molecule radius

	RHIC		LHC (2.76 TeV)		LHC (5.02 TeV)		RHIC	LHC (5 TeV)
	Sc. 1	Sc. 2	Sc. 1	Sc. 2	Sc. 1	Sc. 2	Refs [14,15]	
T_H (MeV)	162		156				175	
V_H (fm^3)	2100		5380				1908	5152
μ_B (MeV)	24		0				20	0
μ_s (MeV)	10		0				10	0
γ_c	22		39		50		6.40	15.8
γ_b	4.0×10^7		8.6×10^8		1.4×10^9		2.2×10^6	3.3×10^7
T_C (MeV)	162	166	156	166	156	166	175	
V_C (fm^3)	2100	1791	5380	3533	5380	3533	1000	2700
ω (MeV)	590	608	564	609	564	609	550	
ω_s (MeV)	431	462	426	502	426	502	519	
ω_c (MeV)	222	244	219	278	220	279	385	
ω_b (MeV)	183	202	181	232	182	234	338	
$N_u = N_d$	320	302	700	593	700	593	245	662
$N_s = N_{\bar{s}}$	183	176	386	347	386	347	150	405
$N_c = N_{\bar{c}}$	4.1		11		14		3	20
$N_b = N_{\bar{b}}$	0.03		0.44		0.71		0.02	0.8
T_F (MeV)	119		115				125	
V_F (fm^3)	20355		50646				11322	30569
N_K	67.5		134				142[a]	363[a]
$N_{\bar{K}}$	59.6		134				127[a]	363[a]
N_N	20		32				62[a]	150[a]
N_Δ	18		28				–	–
N_Λ	3.8		6.5				–	–
N_Ξ	2.6		4.4				4.7	13
N_Ω	0.37		0.62				0.81	2.3
$N_D = N_{\bar{D}}$	1.5		4.0		5.2		1.0	6.9
$N_{D^*} = N_{\bar{D}^*}$	2.0		5.4		6.9		1.5	10
$N_{D_1} = N_{\bar{D}_1}$	0.20		0.49		0.63		0.19	1.3
$N_B = N_{\bar{B}}$	8.1×10^{-3}		0.12		0.20		5.3×10^{-3}	0.21
$N_{B^*} = N_{\bar{B}^*}$	1.9×10^{-2}		0.27		0.45		1.2×10^{-2}	0.49
N_{Λ_c}	0.17		0.36		0.46		–	–
N_{Σ_c}	0.2		0.41		0.52		–	–
$N_{\Sigma_c^*}$	0.28		0.56		0.71		–	–
N_{Ξ_c}	0.11		0.25		0.32		0.10	0.65

[a] Values contain feed down contributions.

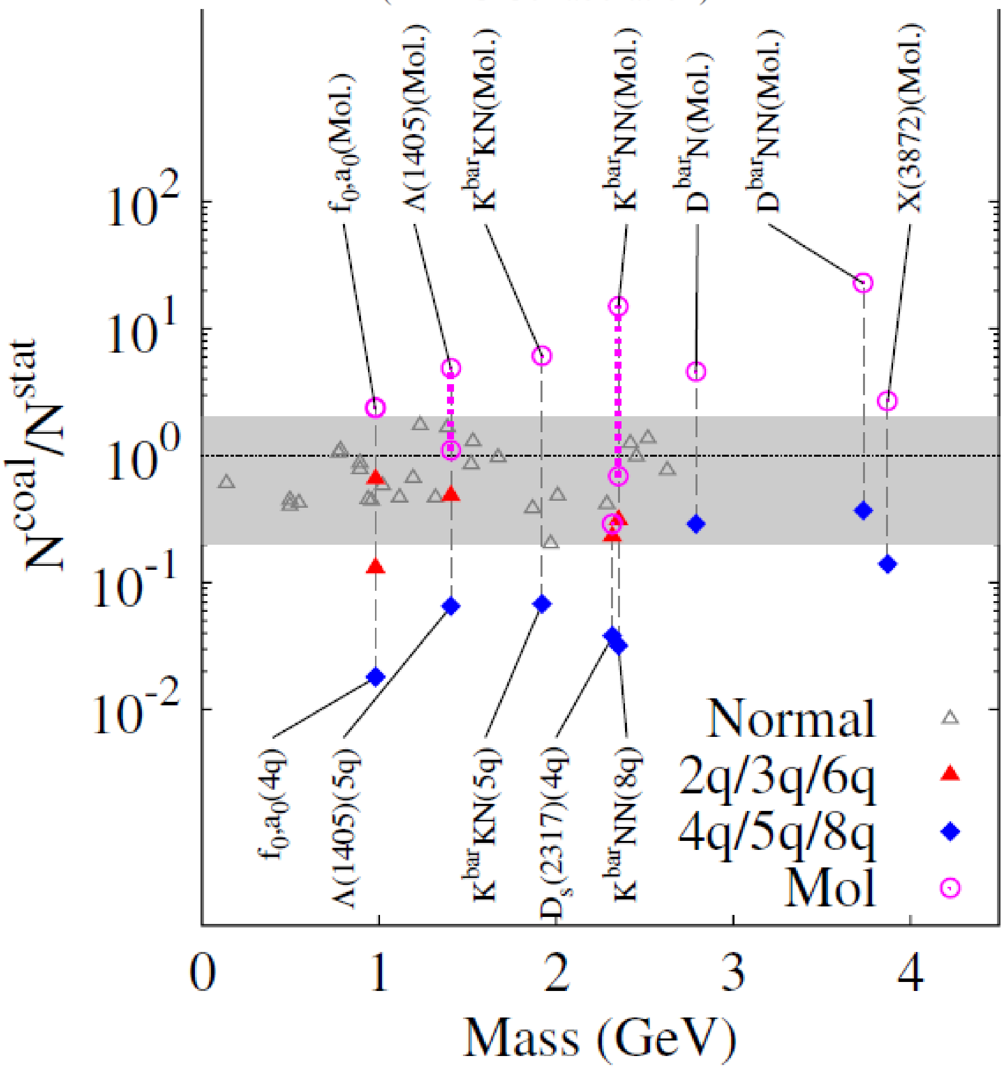
PRL 106, 212001 (2011) PHYSICAL REVIEW LETTERS week ending 27 MAY 2011

Identifying Multiquark Hadrons from Heavy Ion Collisions

Sungtae Cho,[1] Takenori Furumoto,[2,3] Tetsuo Hyodo,[4] Daisuke Jido,[2] Che Ming Ko,[5] Su Houng Lee,[1,2] Marina Nielsen,[6] Akira Ohnishi,[2] Takayasu Sekihara,[2,7] Shigehiro Yasui,[8] and Koichi Yazaki[2,3]

(ExHIC Collaboration)

Production rate normalized to statistical model



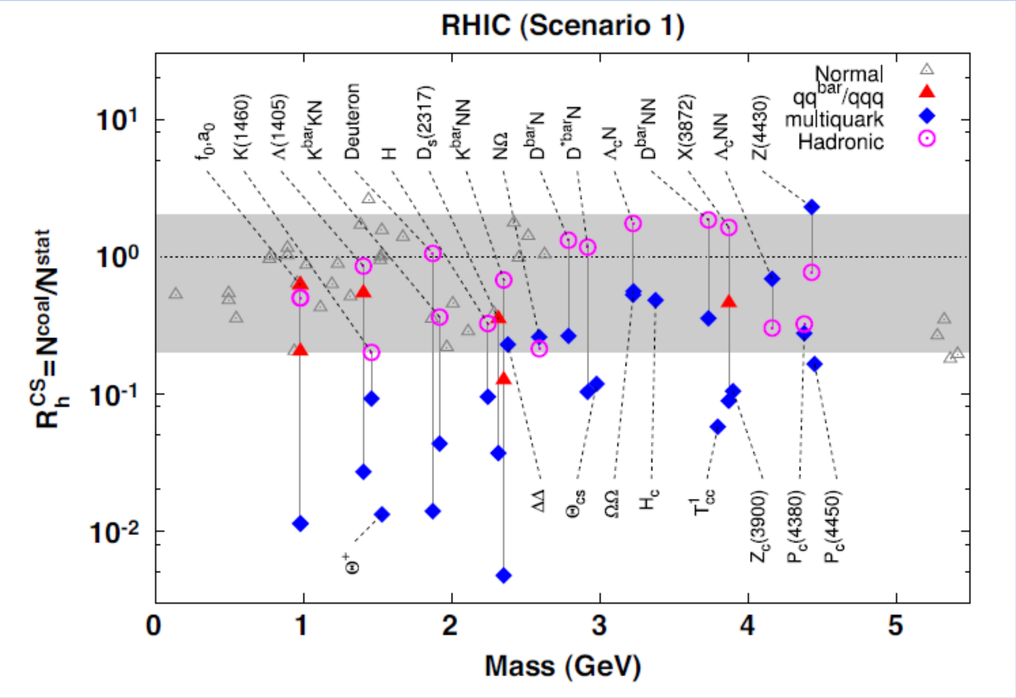
Normal Hadron

Molecule

Compact multiquark

Production of compact multiquark state

Production rate normalized to statistical model



Normal Hadron

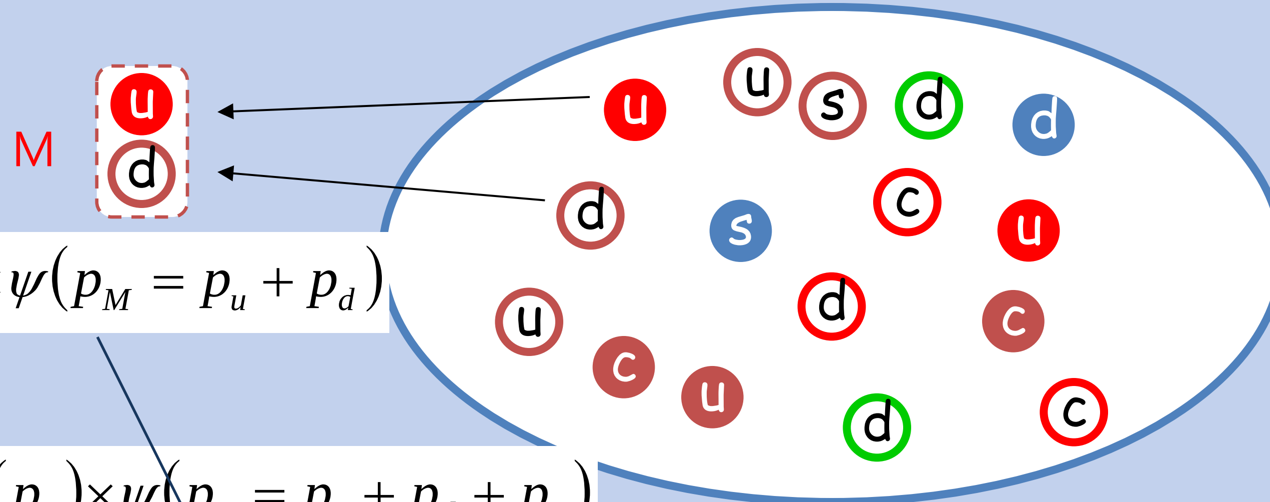
Molecule

Compact multiquark

Progress in Particle and Nuclear Physics 95 (2017) 279–322

Exotic hadrons from heavy ion collisions☆

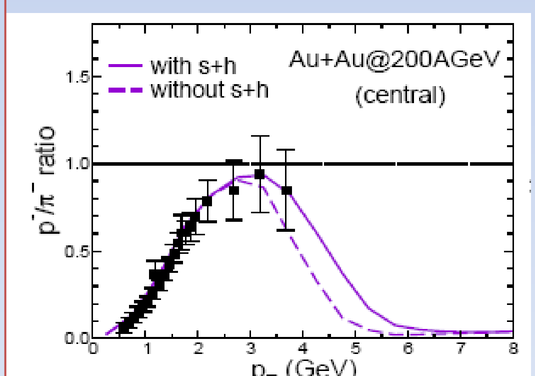
Sungtae Cho[a], Tetsuo Hyodo[b], Daisuke Jido[c], Che Ming Ko[d], Su Houng Lee[e,*], Saori Maeda[f], Kenta Miyahara[g], Kenji Morita[b], Marina Nielsen[h], Akira Ohnishi[b], Takayasu Sekihara[i], Taesoo Song[j], Shigehiro Yasui[f], Koichi Yazaki[k] (ExHIC Collaboration)



$$N_M = C\int f_u(p_u) f_d(p_d) \times \psi(p_M = p_u + p_d)$$

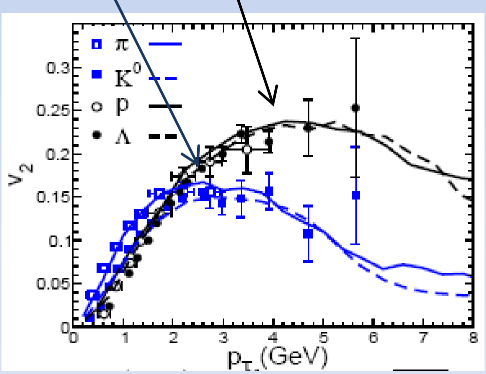
$$N_B = C\int f_u(p_u) f_d(p_d) f_u(p_u) \times \psi(p_B = p_u + p_d + p_u)$$

P_T dependence of ratio



Greco, Ko et al

Quark number scaling of v2



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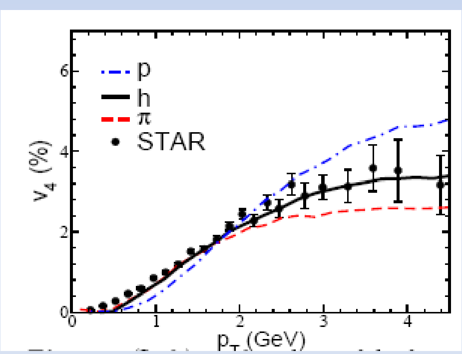
v4



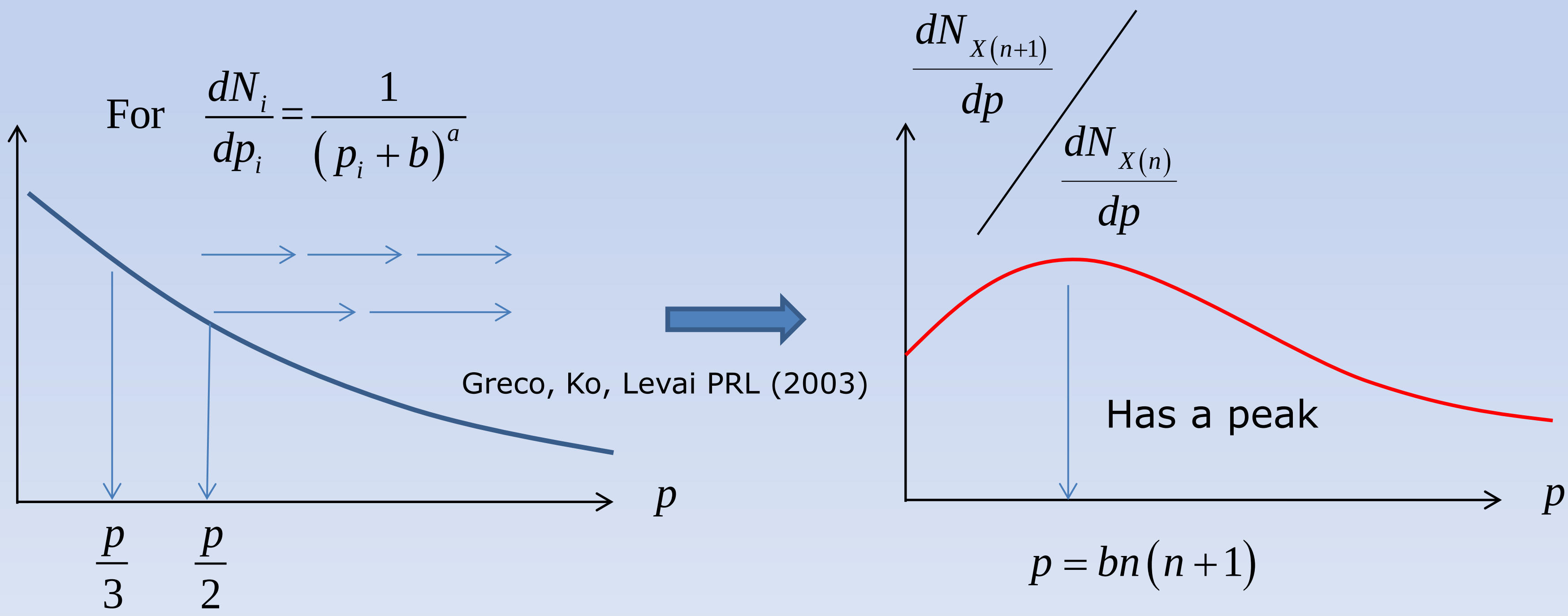
P_T dependence of Ratio : Coalescence model

$$\frac{dN_X}{dp_X} = C\int dx_1 dx_2 dp_1 dp_2 \frac{dN_1}{dp_1}\frac{dN_2}{dp_2} W\left(x_1, x_2, p_1, p_2\right)\delta\left(p_X - p_1 - p_2\right)$$

C M Ko

- Simple approximation $W\left(p_1, p_2\right) \propto \delta\left(p_1 - p_2\right)$,

$$\frac{dN_{X(n)}}{dp_X} = g_X O \frac{dN_1}{dp_1}\left(\frac{dN_2}{Vdp_2}\right), \qquad \text{where } p_1 = p_2 = \frac{p_X}{2}$$

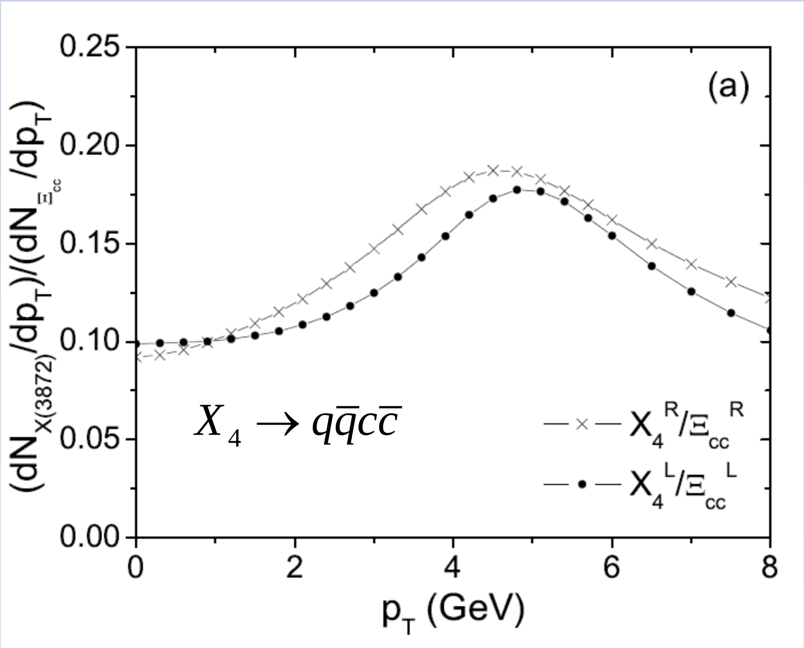


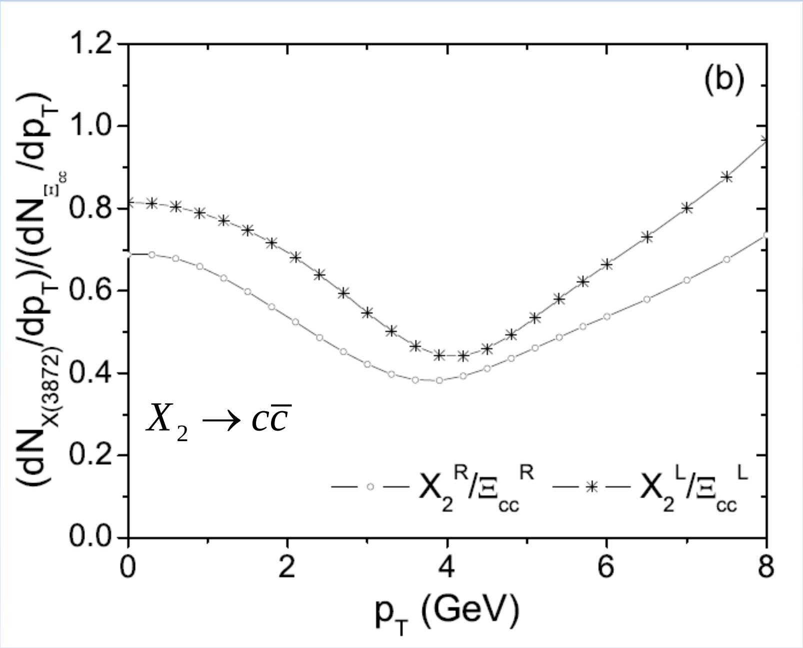
Example: X(3872) 2 quark vs compact 4 quark

- Full calculation: $W(x_1, x_2, p_1, p_2) \propto \exp\left(-\frac{(x_1 - x_2)^2}{\sigma^2} - \sigma^2 (p_1 - p_2)^2\right)$

1. Coalescence probability is smaller for small σ as $O = \frac{\sigma^3}{(\sigma^2 + 1/mT)^{3/2}}$ can provide suppression factor of 2 for each quark when size decreases by a factor 2
   → compact multiquark states are suppressed.

2. Peak structure still correct : Sungtae Cho, SHL arXiv:1908.08333





VI: Remark on X(3872) measurement by CMS

Comments on Recent CMS measurement of X(3872) → details will come, but consider it a DD* molecule

- Molecules will form at kinetic freeze-out: can use final D D* p_T distribution
- Simple approximation $W\left(p_1, p_2\right) \propto \delta\left(p_1 - p_2\right)$

$$\frac{dN_{X\left(D\bar{D}^*\right)}}{dp_X} = g_X O \frac{dN_D}{dp_D}\left(\frac{dN_{D^*}}{Vdp_{D^*}}\right)$$

$$m_{D^0}\left(1864.83\right) + m_{D^{*0}}\left(2006.85\right) = 3871.68 \text{ MeV}$$

$$p_D \approx p_{D^*} \approx \frac{p_X}{2}$$

- One step further, note $m_{D^0}\left(1864.83\right) + m_{D^0}\left(1864.83\right) = 3729.66 \text{ MeV} \approx m_{\psi'}\left(3686\right)\right)$

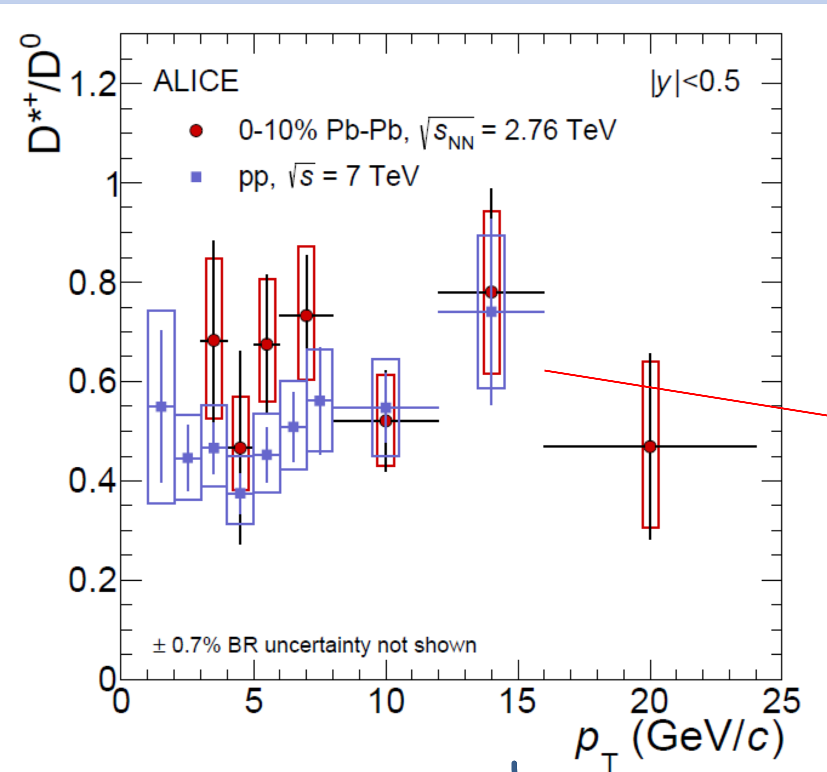
$$\frac{dN_{\psi'\left(D\bar{D}\right)}}{dp_{\psi'}} = g_{\psi'} O' \frac{dN_D}{dp_D}\left(\frac{dN_D}{Vdp_D}\right) \qquad p_D \approx p_{D^*} \approx \frac{p_{\psi'}}{2}$$

- Ratio $R = N_{X\left(D\bar{D}^*\right)} / N_{\psi'\left(D\bar{D}\right)} = \frac{1}{3}\left(O \times N_{\bar{D}^*}\right) / \left(O' \times N_{\bar{D}}\right)$

- Alice data at 2.76 TeV

$$R = N_{X(D\bar{D}^*)} / N_{\psi'(D\bar{D})} = \frac{1}{3} N_{\bar{D}^*} / N_{\bar{D}} \sim 0.2$$

No change in R=0.2 from pp to Pb-Pb

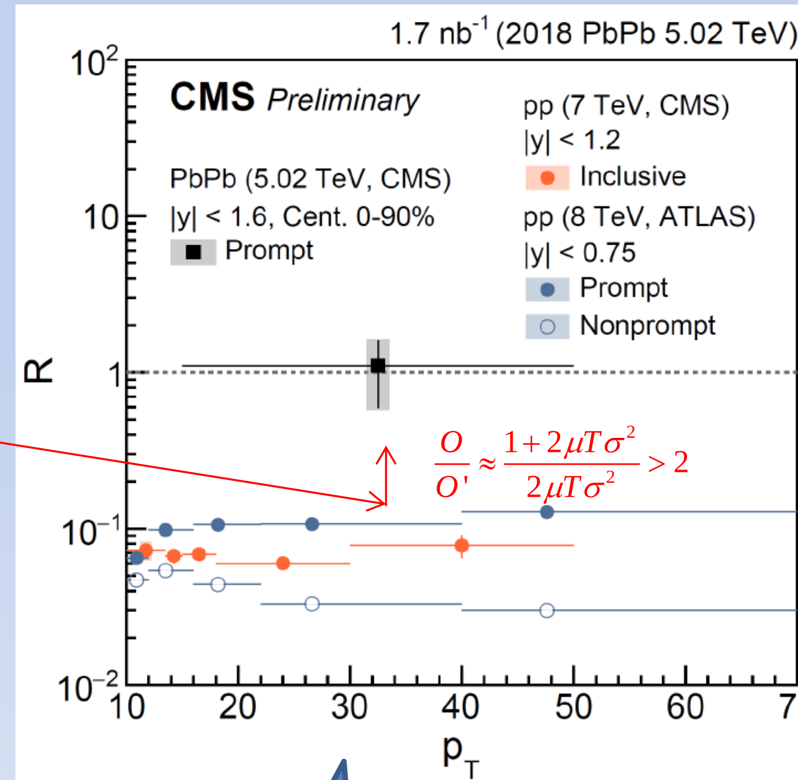


- QM2019 Yen-Jie Lee (MIT)

$$R = N^{(\mathrm{Corr})}_{X(3872)} / N^{(\mathrm{Corr})}_{\psi(2S)} \sim 0.1 \quad \text{pp}$$

to

$$R = N^{(\mathrm{Corr})}_{X(3872)} / N^{(\mathrm{Corr})}_{\psi(2S)} \sim 1 \quad \text{Pb-Pb}$$



$$p_D \approx p_{D^*} \approx \frac{p_X}{2} \approx \frac{p_{\psi'}}{2}$$

Need to go down in p_T

Summary

☞ Quark model candidates on compact multi-quark states: But not conclusive

$$\mathrm{P}_{cc\bar{s}}^{++}\left(udcc\bar{s}\right) \rightarrow \Lambda_c K^- K^+ \pi^+ \quad \text{Note} \quad \Xi_{cc}^{++} \rightarrow \Lambda_c K^- \pi^+ \pi^+ \quad \text{or note} \quad P_{cc\bar{s}}^{++}\left(\Xi_{cc}^{+} K^+ \ \text{ or } \ \Lambda_c D_s^+\right)$$

$$\mathrm{T}_{\mathrm{cc}}^{1}(\mathrm{ud\bar{c}\bar{c}}) \rightarrow (\overline{D}^0 + D^{*-}) \rightarrow K^+\pi^- + K^+\pi^-\pi^-$$

$$\mathrm{P}_{sc\bar{c}}\left(udsc\bar{c}\right) \rightarrow \Lambda + J/\psi$$

☞ Production yield and p_T dependence can discriminate structures

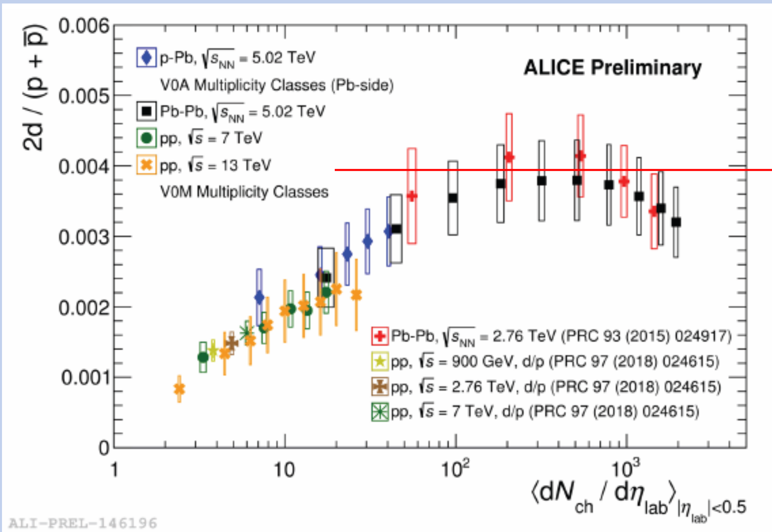
☞ May be we are already seeing proof that X(3871) is a molecule but need more data at lower P_T

☞ Most compact multi-quark candidates have more than one heavy quarks (hidden or open)

But now experiments can observe them

→ Heavy Ion collision could be an EXOTICA FACTROY !!

Production of light nuclei



Should be constant in Statistical model

Coalescence model $$\frac{2d}{p+\bar{p}} = \frac{N}{V_F}\frac{\sigma^3}{\left(1+2m_p T_F \sigma^2\right)}$$

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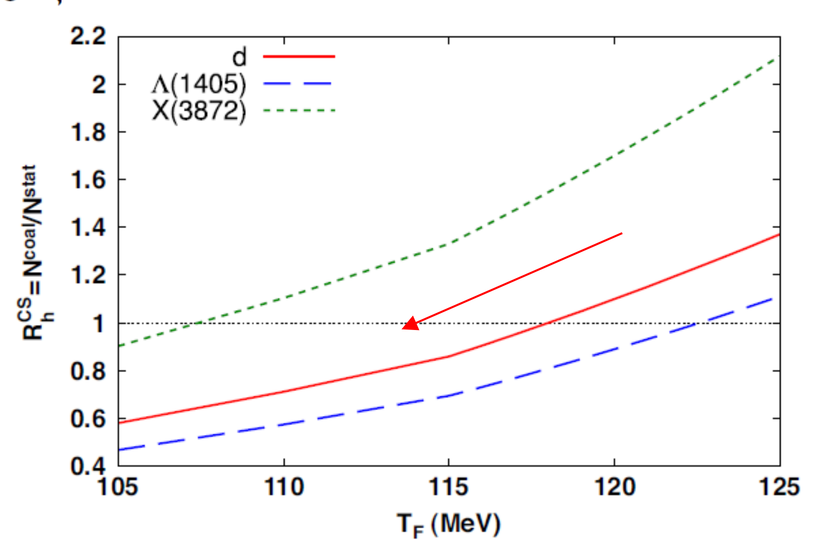
Freezeout condition from hadronic gas

$$\frac{N}{V_F} \propto \frac{1}{\sigma_{fo}^{3/2}} \cdot \frac{1}{N^{1/2}}$$



P_T dependence of Ratio : Coalescence model

$$\frac{dN_X}{dp_X} = C\int dx_1 dx_2 dp_1 dp_2 \frac{dN_1}{Vdp_1}\frac{dN_2}{Vdp_2} W(p_1, p_2)\delta(p_X - p_1 - p_2)$$

- Next approximation $W(p_1, p_2) \propto \exp\left(-\frac{x^2}{\sigma^2} - \sigma^2 k^2\right)$

$$\frac{dN_X}{dp_X} = C\frac{\left(2\sqrt{\pi}\sigma\right)^3}{V}\int dp_1 dp_2 \frac{dN_1}{dp_1}\frac{dN_2}{dp_2}\exp\left(-k^2\sigma^2\right)\delta(p_X - p_1 - p_2)$$

$$(\sigma \to \infty) = \frac{(2\pi)^3}{V}\int dp_1 dp_2 \frac{dN_1}{dp_1}\frac{dN_2}{dp_2}\delta(p_1 - p_2)\delta(p_X - p_1 - p_2)$$

$$\frac{dN_1}{dp_1} = \exp\left(-\frac{p_1^2}{mT}\right) \quad \to \frac{(\sigma)^3}{\left(1 + mT\sigma^2\right)^{3/2}} \text{ type corrections}$$

- Formula by C. M. Ko $\frac{N_d}{N_p} \approx \frac{3N_n}{4\left(mTR^2\right)^{3/2}}\frac{1}{\left(1 + \frac{1}{mT\sigma^2}\right)^{3/2}}$