

Coupled Channel Description of Charmed Exotica

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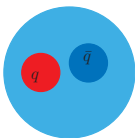
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3rd EMMI Workshop: Anti-matter, hyper-matter and exotica
production at the LHC 2019
4 December, 2019

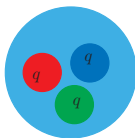
- Introduction
- $X(3872)$, $X(3940)$ and $X(4160)$
- $Y(4260)$
- $Z_c(3900)$

Hadron Spectroscopy

- Ordinary hadrons

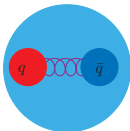


Meson ($q\bar{q}$)

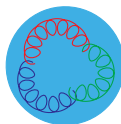


Baryon (qqq)

- Gluonic excitations

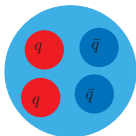


Hybrid meson:
 $q\bar{q}$ with gluonic excitations,



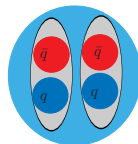
Glueball:
only gluons, no valence quarks

- Multiquark states



Tetraquark:

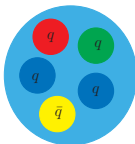
Two quarks and two antiquarks,



Hadronic molecule:

composed of two or more color-neutral hadrons

- Pentaquark

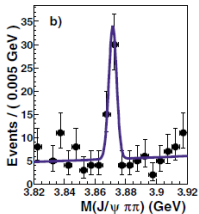


four quarks and one antiquark bound together

State	$J^{PC}(J^{PC})$	M (MeV)	Γ (MeV)	S-wave threshold(s) (MeV)	Observed mode(s) (branching ratios)
X(3872)	$0^+(1^{++})$	3871.69 ± 0.17	< 1.2	$D^{*+}D^+ + c.c.(-8.15 \pm 0.20)$ $D^{*0}\bar{D}^0 + c.c.(0.00 \pm 0.18)$	$B \rightarrow K[D^{*0}\bar{D}^0](> 24\%)$ $B \rightarrow K[D^0\bar{D}^0\pi^0](> 32\%)$ $B \rightarrow K[J/\psi\pi^+\pi^-](> 2.6\%)$ $B \rightarrow K[J/\psi\pi^+\pi^0]$ $p\bar{p} \rightarrow [J/\psi\pi^+\pi^-] \dots$ $p p \rightarrow [J/\psi\pi^+\pi^-] \dots$ $B \rightarrow K[J/\psi\omega](> 1.9\%)$ $B \rightarrow [J/\psi\eta](> 6 \times 10^{-3})$ $B \rightarrow [\psi(2S)\gamma](> 3.0\%)$
X(3940)	$?^?(?^?)$	3942.0 ± 9	37^{+27}_{-17}	$D^*\bar{D}^*(-75.1 \pm 9)$	$e^+e^- \rightarrow J/\psi[D\bar{D}^*]$
X(4160)	$?^?(?^?)$	4156^{+28}_{-25}	139^{+100}_{-60}	$D^*\bar{D}^*(139^{+28}_{-25})$	$e^+e^- \rightarrow J/\psi[D^*\bar{D}^*]$
Z _c (3900)	$1^+(1^{++})$	3886.6 ± 2.4	28.1 ± 2.6	$D^*\bar{D}^*(10.8 \pm 2.4)$	$e^+e^- \rightarrow \pi[D\bar{D}^* + c.c.]$ $e^+e^- \rightarrow \pi[J/\psi\pi]$
Z _c (4020)	$1(?^?)$	4024.1 ± 1.9	13 ± 5	$D^*\bar{D}^*(7.0 \pm 2.4)$	$e^+e^- \rightarrow \pi[D^*\bar{D}^*]$ $e^+e^- \rightarrow \pi[h_c\pi]$ $e^+e^- \rightarrow \pi[\psi'\pi]$
Y(4260)	$?^?(1^{--})$	4251 ± 9	120 ± 12	$D_1\bar{D}^* + c.c.(-38.2 \pm 9.1)$ $Z_{c,00}(53.6 \pm 9.0)$	$e^+e^- \rightarrow J/\psi\pi\pi$ $e^+e^- \rightarrow \pi D\bar{D}^* + c.c.$ $e^+e^- \rightarrow X_{c,00}$ $e^+e^- \rightarrow X(3872)\gamma$
Y(4360)	$?^?(1^{--})$	4346 ± 6	102 ± 10	$D_1\bar{D}^* + c.c.(-85 \pm 6)$	$e^+e^- \rightarrow \psi(2S)\pi^+\pi^-$
Y(4660)	$?^?(1^{--})$	4643 ± 9	72 ± 11	$\psi(2S)f_0(980)(-33 \pm 21)$ $\Lambda_c^+\Lambda_c^-(70 \pm 6)$	$e^+e^- \rightarrow \psi(2S)\pi^+\pi^-$
Z _c (4430) ⁺	$?^?(1^+)$	4478^{+15}_{-18}	181 ± 31	$\psi(2S)[\rho(17^{+15}_{-18})]$	$B \rightarrow K[\psi(2S)\pi^+]$ $B \rightarrow K[J/\psi\pi^+]$
Z _c (4200) ⁺	$?^?(1^+)$	4196^{+15}_{-22}	370^{+100}_{-32}		$\bar{B}^0 \rightarrow K^-[J/\psi\pi^+]$
Z _c (4050) ⁺	$?^?(?^?)$	4051^{+24}_{-40}	82^{+30}_{-28}	$D^*\bar{D}^*(34^{+24}_{-20})$	$\bar{B}^0 \rightarrow K^-[\chi_{c1}\pi^+]$
Z _c (4250) ⁺	$?^?(?^?)$	4248^{+100}_{-50}	177^{+100}_{-70}	$\chi_{c1}\rho(-37^{+100}_{-30})$	$\bar{B}^0 \rightarrow K^-[\chi_{c1}\pi^+]$
X(4140) (Aaij et al., 2017a, 2017b)	$0^+(1^{++})$	$4146.5 \pm 4.5^{+4.6}_{-2.8}$	$83 \pm 21^{+21}_{-14}$	$D_s D_s^*(-66.1^{+5.2}_{-5.2})$	$B^+ \rightarrow K^+[J/\psi\phi]$
X(4274) (Aaij et al., 2017a, 2017b)	$0^+(1^{++})$	$4273.3 \pm 8.3^{+17.2}_{-1.6}$	$56 \pm 11^{+8}_{-11}$	$D_s^* \bar{D}_s^*(-49.1^{+19.1}_{-0.1})$	$B^+ \rightarrow K^+[J/\psi\phi]$
X(4500) (Aaij et al., 2017a, 2017b)	$0^+(0^{++})$	$4506 \pm 11^{+12}_{-15}$	$92 \pm 21^{+21}_{-20}$	$D_{s0}^*(2317)\bar{D}_{s0}^*(2317)(-129^{+16}_{-16})$	$B^+ \rightarrow K^+[J/\psi\phi]$
X(4700) (Aaij et al., 2017a, 2017b)	$0^+(0^{++})$	$4704 \pm 10^{+14}_{-24}$	$120 \pm 31^{+42}_{-33}$	$D_{s0}^*(2317)\bar{D}_{s0}^*(2317)(69^{+12}_{-20})$	$B^+ \rightarrow K^+[J/\psi\phi]$

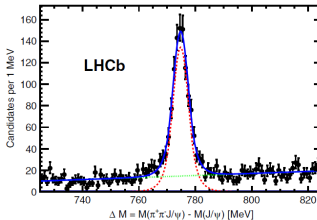
F.-K. Guo, C. Hanhart, Ulf-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, *Rev.Mod.Phys.* 90 (2018) no.1, 015004

$B \rightarrow KX(3872); X(3872) \rightarrow \pi^+\pi^- J/\psi$



Belle, PRL91,262001(2003)

$B \rightarrow KX(3872); X(3872) \rightarrow \pi^+\pi^- J/\psi$



LHCb, PRD92,011102 (2015)

In Coupled Channels:

- D. Gamermann, E. Oset, Eur. Phys. J A33(2007),
- D. Gamermann, E. Oset, Eur. Phys. J A36(2008),
- D. Gamermann, E. Oset, PRD80 (2009),
- D. Gamermann, J. Nieves, E. Oset, E. Ruiz Arriola, PRD81 (2010),
- F. Aceti, R. Molina, E. Oset, PRD86 (2012)

The Lagrangian:

$$\mathcal{L} = \frac{-1}{4f^2} \text{Tr} (J_\mu \mathcal{J}^\mu), \quad J_\mu = (\Phi)\Phi - \Phi\Phi, \quad \mathcal{J}_\mu = (\mathcal{V}_\nu)\mathcal{V}^\nu - \mathcal{V}_\nu\mathcal{V}^\nu \quad (1)$$

The pseudoscalar and vector mesons fields:

$$\Phi = \begin{pmatrix} \pi^0 + \eta + \frac{\eta_C}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\pi^0 + \eta + \frac{\eta_C}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{12}} + \frac{\eta_C}{\sqrt{12}} & D_s^- \\ D^0 & D^+ & D_s^+ & \frac{-3\eta_C}{\sqrt{12}} \end{pmatrix} \quad (2)$$

$$\mathcal{V}_\mu = \begin{pmatrix} \frac{\rho_\mu^0}{\sqrt{12}} + \frac{\omega_\mu}{\sqrt{12}} + \frac{J/\psi_\mu}{\sqrt{12}} & \rho_\mu^+ & K_\mu^{*+} & \bar{D}_\mu^{*0} \\ \rho_\mu^{*-} & \frac{-\rho_\mu^0}{\sqrt{12}} + \frac{\omega_\mu}{\sqrt{12}} + \frac{J/\psi_\mu}{\sqrt{12}} & K_\mu^{*0} & D_\mu^{*-} \\ K_\mu^{*-} & \bar{K}_\mu^{*0} & \frac{-2\omega_\mu}{\sqrt{12}} + \frac{J/\psi_\mu}{\sqrt{12}} & D_{S\mu}^{*-} \\ D_\mu^{*0} & D_\mu^{*+} & D_{S\mu}^{*+} & \frac{-3J/\psi_\mu}{\sqrt{12}} \end{pmatrix}. \quad (3)$$

$I^G(J^{PC})=0^+(1^{++})$, $X(3872) \Rightarrow (\bar{K}K^* + c.c.), (\bar{D}D^* + c.c.), (\bar{D}_sD_s^* - c.c.)$

- The Bethe-Salpeter equation in coupled channels: $t = [1 + V\hat{G}]^{-1}(-V)\vec{\epsilon} \cdot \vec{\epsilon}'$



- The two meson loop function:

$$G_I(\sqrt{s}) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(P-q)^2 - M_I^2 + i\epsilon} \frac{1}{q^2 - m_I^2 + i\epsilon}$$

- In the dimensional regularization scheme the loop function:

$$\begin{aligned} G_I(\sqrt{s}) = & \frac{1}{16\pi^2} \left\{ a(\mu) + \ln \frac{M_I^2}{\mu^2} + \frac{m_I^2 - M_I^2 + s}{2s} \ln \frac{m_I^2}{M_I^2} \right. \\ & + \frac{q_I}{\sqrt{s}} \left[\ln(s - (M_I^2 - m_I^2) + 2q_I\sqrt{s}) \right. \\ & + \ln(s + (M_I^2 - m_I^2) + 2q_I\sqrt{s}) \\ & - \ln(-s + (M_I^2 - m_I^2) + 2q_I\sqrt{s}) \\ & \left. \left. - \ln(-s - (M_I^2 - m_I^2) + 2q_I\sqrt{s}) \right] \right\} \end{aligned}$$

- q_I determined at the center of mass frame, $q_I = \frac{\sqrt{[s - (M_I - m_I)^2][s - (M_I + m_I)^2]}}{2\sqrt{s}}$
- $\mu \Rightarrow$ a scale parameter in this scheme, $a(\mu) \Rightarrow$ the subtraction constant

X(3872)

Pole position:

C	Mass (MeV)	S	$I^G(J^{PC})$	RE(\sqrt{s}) (MeV)	IM(\sqrt{s}) (MeV)	Resonance ID
	3867.5	0	$0^+(1^{++})$	3837.57	-0.00	X(3872)

Residues for the C=0,S=0, $I^P=0^+$ sector:

Channel	(3837.57-i0.00) MeV $ g_i $ (GeV)
$(D\bar{D}^* + c.c.)$	13.61
$(D_s\bar{D}_s^* - c.c.)$	10.58
$(KK^* + c.c.)\downarrow$	0.03

PDG:

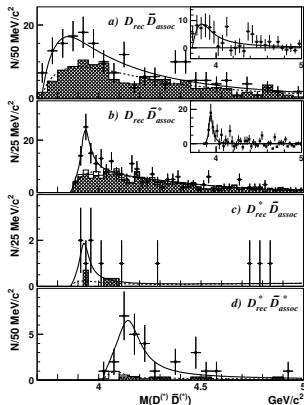
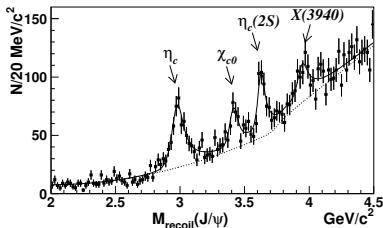
$X(3872) \Rightarrow \chi_{c1}(3872), m = 3871.69 \pm 0.17 \text{ MeV}$

M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) and 2019 update.

X(3940) and X(4160)

Belle, Phys.Rev.Lett. 98 (2007) 082001. X(3940); $e^+e^- \rightarrow J/\psi X(3940)$: $X(3940) \rightarrow D^* \bar{D}$.

Belle, Phys.Rev.Lett. 100 (2008) 202001. X(4160); $e^+e^- \rightarrow J/\psi D^* \bar{D}$.



PDG: X(3940), $I^G(J^{PC}) = ?^? (?^{??})$, $m \simeq 3942 \text{ MeV}$, $\Gamma \sim 37 \pm 8 \text{ MeV}$ the
 X(4160), $I^G(J^{PC}) = ?^? (?^{??})$, $m \simeq 4156 \text{ MeV}$, $\Gamma \sim 139 \pm 21 \text{ MeV}$

The Bethe-Salpeter equation:

$$T = (\hat{1} - VG)^{-1} V. \quad (4)$$

The amplitude close to a pole:

$$T_{ij} \approx \frac{g_i g_j}{s - s_p} \quad (5)$$

Couplings g_i in units of MeV for $I = 0, J = 0$.

$$\sqrt{s}_{pole} = 3943 + i7.4, I^G[J^{PC}] = 0^+[0^{++}]$$

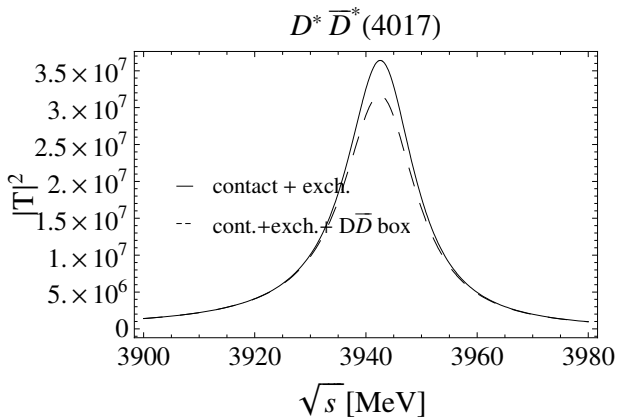
$D^* \bar{D}^*$	$D_s^* \bar{D}_s^*$	$K^* \bar{K}^*$	$\rho\rho$	$\omega\omega$
18810 - i682	8426 + i1933	10 - i11	-22 + i47	1348 + i234
$\phi\phi$	$J/\psi J/\psi$	$\omega J/\psi$	$\phi J/\psi$	$\omega\phi$
-1000 - i150	417 + i64	-1429 - i216	889 + i196	-215 - i107

Couplings g_i in units of MeV for $I = 0, J = 2$ (second pole).

$$\sqrt{s}_{pole} = 4169 + i66, I^G[J^{PC}] = 0^+[2^{++}]$$

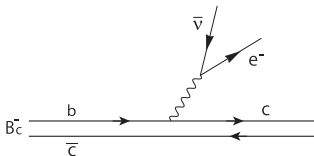
$D^* \bar{D}^*$	$D_s^* \bar{D}_s^*$	$K^* \bar{K}^*$	$\rho\rho$	$\omega\omega$
1225 - i490	18927 - i5524	-82 + i30	70 + i20	3 - i2441
$\phi\phi$	$J/\psi J/\psi$	$\omega J/\psi$	$\phi J/\psi$	$\omega\phi$
1257 + i2866	2681 + i940	-866 + i2752	-2617 - i5151	1012 + i1522

$|T|^2$ for $I = 0$ and $J = 0$:



- $B_c^- \rightarrow \bar{\nu} e^- X(3940) (X(4160))$
- $X(3940) (0^{++}), X(4160) (2^{++}) \Rightarrow$ **dynamically generated** from vector–vector interaction ($D^* \bar{D}^*, D_s^* \bar{D}_s^*$) in the charm sector.
- To produce X states, we consider three steps:

First step : $B_c^- \rightarrow \bar{\nu} e^- (\bar{c}c)$ at quark level

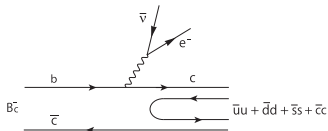


bc weak transition

- If we want to see **two mesons**, the $\bar{c}c$ quarks must **hadronize** into two mesons components

Hadronization: second step

- Introduce an extra $q\bar{q}$ pair with vacuum quantum numbers; $\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c$



- The $q\bar{q}$ matrix M :

$$M \equiv q\bar{q}^T = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} & u\bar{c} \\ d\bar{u} & d\bar{d} & d\bar{s} & d\bar{c} \\ s\bar{u} & s\bar{d} & s\bar{s} & s\bar{c} \\ c\bar{u} & c\bar{d} & c\bar{s} & c\bar{c} \end{pmatrix}$$

- In terms of vector mesons:

$$M \rightarrow V \equiv \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & \bar{D}^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & \bar{D}_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}.$$

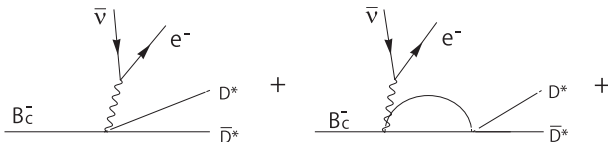
$$c(\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c)\bar{c} = \sum_{i=1}^4 M_{4i}M_{i4} = (M^2)_{44}$$

$$\Rightarrow (V \cdot V)_{44} = D^{*0}\bar{D}^{*0} + D^{*+}\bar{D}^{*-} + D_s^{*+}\bar{D}_s^{*-} + J/\psi J/\psi.$$

- The production vertex is written as

$$(V \cdot V)_{44} \rightarrow \sqrt{2}|D^*\bar{D}^*; l=0\rangle + |D_s^*\bar{D}_s^*; l=0\rangle. \quad (6)$$

- Next step:** These mesons are allowed to undergo **final state interaction** \Rightarrow X resonances appear.

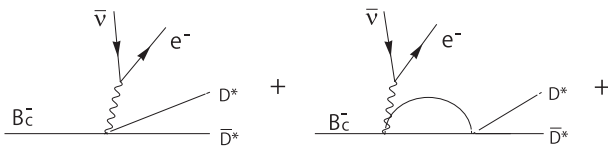


a) tree level

b) rescattering



Rescattering process



a) tree level

b) rescattering

- Hadronization amplitude V'_{had}
 $D^* \bar{D}^*$ states for $l = 0, J = 0$

$$V'_{\text{had}} = C(\sqrt{2} + \sqrt{2} G_{D^* \bar{D}^*} t_{D^* \bar{D}^*, D^* \bar{D}^*} + G_{D_s^* \bar{D}_s^*} t_{D_s^* \bar{D}_s^*, D^* \bar{D}^*}). \quad (7)$$

$D_s^* \bar{D}_s^*$ state for $l = 0, J = 2$

$$V'_{\text{had}} = C'(1 + \sqrt{2} G_{D^* \bar{D}^*} t_{D^* \bar{D}^*, D_s^* \bar{D}_s^*} + G_{D_s^* \bar{D}_s^*} t_{D_s^* \bar{D}_s^*, D^* \bar{D}^*}). \quad (8)$$

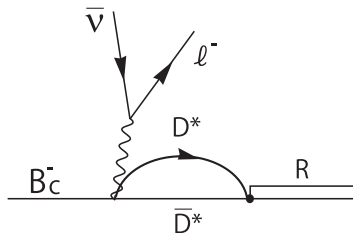
- Scattering amplitudes t :

$$t_{D_s^* \bar{D}_s^*, D^* \bar{D}^*} = \frac{g_{R, D_s^* \bar{D}_s^*} g_{R, D^* \bar{D}^*}}{M_{\text{inv}}^2 - M_R^2 + iM_R \Gamma_R}, \quad (9)$$

$\Rightarrow g_{R, D^* \bar{D}^*}, g_{R, D_s^* \bar{D}_s^*} \Rightarrow$ the couplings of the resonance to these channels

Coalescence process

Production of resonances R after rescattering process:



- The hadronization factor V_{had} :

$$V_{\text{had}} = C(\sqrt{2} G_{D^* \bar{D}^*} g_{R, D^* \bar{D}^*} + G_{D_s^* \bar{D}_s^*} g_{R, D_s^* \bar{D}_s^*}), \quad (10)$$

$\Rightarrow g_{R, D^* \bar{D}^*}, g_{R, D_s^* \bar{D}_s^*} \Rightarrow$ the couplings of the resonance to these channels

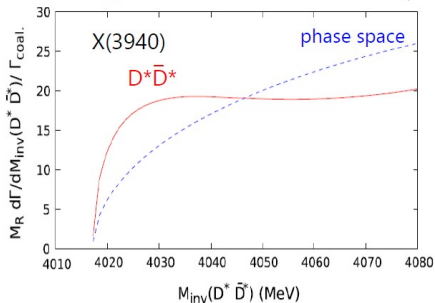
Differential decay width:

$$\frac{d\Gamma_i}{dM_{\text{inv}}} = \frac{|G_F V_{bc} V'_{\text{had},i}|^2}{32\pi^5 m_{B_c}^3 M_{\text{inv}}^{(i)}} \int dM_{\text{inv}}^{(\nu e)} P^{\text{cm}} \tilde{p}_\nu \tilde{p}_i M_{\text{inv}}^{(\nu e)2} \left(\tilde{E}_{B_c} \tilde{E}_i - \frac{\tilde{p}_{B_c}^2}{3} \right)$$

$$\frac{M_R}{\Gamma_{\text{coal}}} \frac{d\Gamma_i}{dM_{\text{inv}}}$$

Decay widths Γ_{coal} for the resonance $L = 0$ and $J = 0$ state

$$\Gamma_{\text{coal}} = \frac{|G_F V_{bc} V_{\text{had}}|^2}{8\pi^3 m_{B_c}^3 m_R} \int dM_{\text{inv}}^{(\nu e)} P_R^{\text{cm}} \tilde{p}_\nu |M_{\text{inv}}^{(\nu e)}|^2 \left(\tilde{E}_{B_c} \tilde{E}_R - \frac{\tilde{p}_{B_c}^2}{3} \right)$$



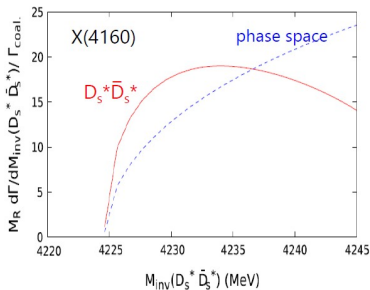
Shape of $D^* \bar{D}^*$ invariant mass distribution is different from the phase space.

\Leftarrow due to the presence of a resonance below threshold that couples strongly to the observed channel

Differential decay width for the $L = 2$ state:

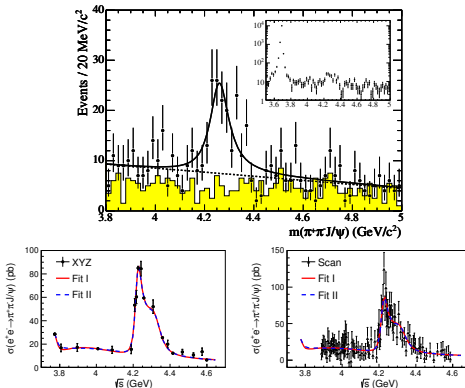
$$\frac{d\Gamma_i}{dM_{\text{inv}}} = \frac{|G_F V_{bc} V'_{\text{had},i}|^2}{32\pi^5 m_{B_c}^3 M_{\text{inv}}^{(i)}} \int dM_{\text{inv}}^{(\nu e)} \underline{(P_R^{\text{cm}})^5} \tilde{p}_\nu \tilde{p}_i M_{\text{inv}}^{(\nu e)2} \left(\tilde{E}_{B_c} \tilde{E}_i - \frac{\tilde{p}_{B_c}^2}{3} \right)$$

$$\Gamma_{\text{coal}} = \frac{|G_F V_{bc} V_{\text{had}}|^2}{8\pi^3 m_{B_c}^3 m_R} \int dM_{\text{inv}}^{(\nu e)} \underline{(P_R^{\text{cm}})^5} \tilde{p}_\nu |M_{\text{inv}}^{(\nu e)}|^2 \left(\tilde{E}_{B_c} \tilde{E}_R - \frac{\tilde{p}_{B_c}^2}{3} \right)$$



Y(4260)

$e^+e^- \rightarrow Y(4260) \rightarrow \pi^+\pi^- J/\psi$, BaBar, Phys.Rev.Lett. 95 (2005) 142001

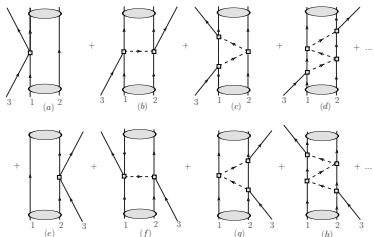


$e^+e^- \rightarrow Y(4260) \rightarrow \pi^+\pi^- J/\psi$, BESIII, Phys.Rev.Lett. 118 (2017) no.9, 092001

$D_1 \bar{D} + c.c.$ hadronic molecule:

G-J Ding, Phys.Rev. D79 (2009) 014001, M. Cleven et al., Phys.Rev. D90 (2014) no.7, 074039, X-G Wu et al., Phys.Rev. D89 (2014) no.5, 054038,

- Clusters : $\rho D \Rightarrow D_1(2420) (I = 1/2)$ $\bar{D}(\rho D) D_1(2420)$
- $\rho D (D_1(2420)) \Rightarrow \pi D^*, D\rho, KD_S^*, D_S K^*, \eta D^*, D\omega, \eta_c D^*, DJ/\psi, (I = 1/2)$ and $\pi D^*, D\rho, (I = 3/2)$ (D. Gamermann, E. Oset, D. Strottman, and M. J. Vicente Vacas, PRD76(2007), D. Gamermann and E. Oset, EPJA33(2007))
- The Faddeev equations under the Fixed Center Approximation (FCA):



- T_1 : all diagrams beginning with interaction in particle 1.
- T_2 : all diagrams beginning with interaction in particle 2.

$$T_1 = t_1 + t_1 G_0 T_2, \quad T_2 = t_2 + t_2 G_0 T_1 \quad (11)$$

$$T = T_1 + T_2 = \frac{\tilde{t}_1 + \tilde{t}_2 + 2 \tilde{t}_1 \tilde{t}_2 G_0}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2}.$$

The function G_0 :

$$G_0(s) = \int \frac{d^3\vec{q}}{(2\pi)^3} F_R(q) \frac{1}{q^{02} - \vec{q}^2 - m_3^2 + i\epsilon}, \quad q^0(s) = \frac{s + m_3^2 - M_R^2}{2\sqrt{s}}.$$

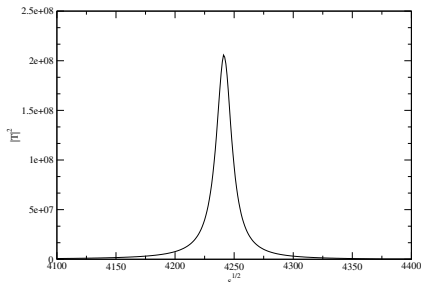
$F_R(q)$ is the cluster form factor

$$F_R(q) = \frac{1}{\mathcal{N}} \int_{|\vec{p}| < \Lambda', |\vec{p} - \vec{q}| < \Lambda'} d^3\vec{p} \frac{1}{2E_1(\vec{p})} \frac{1}{2E_2(\vec{p})} \frac{1}{M_R - E_1(\vec{p}) - E_2(\vec{p})} \quad (12)$$
$$\frac{1}{2E_1(\vec{p} - \vec{q})} \frac{1}{2E_2(\vec{p} - \vec{q})} \frac{1}{M_R - E_1(\vec{p} - \vec{q}) - E_2(\vec{p} - \vec{q})},$$
$$\mathcal{N} = \int_{|\vec{p}| < \Lambda'} d^3\vec{p} \left(\frac{1}{2E_1(\vec{p})} \frac{1}{2E_2(\vec{p})} \frac{1}{M_R - E_1(\vec{p}) - E_2(\vec{p})} \right)^2,$$

(J.Yamagata-Sekihara, J. Nieves, E. Oset Phys. Rev. D 83,014003 (2011))

Y(4260)

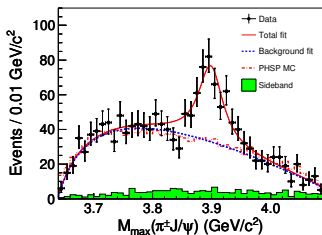
- Clusters : $\rho D \Rightarrow D_1(2420) (I = 1/2)$ $\bar{D}(\rho D)_{D_1(2420)}$
- $\rho D(D_1(2420)) \Rightarrow \pi D^*, D\rho, KD_S^*, D_S K^*, \eta D^*, D\omega, \eta_c D^*, DJ/\psi, (I = 1/2)$ and $\pi D^*, D\rho, (I = 3/2)$ (D. Gamermann, E. Oset, D. Strottman, and M. J. Vicente Vacas, PRD76(2007), D. Gamermann and E. Oset, EPJA33(2007))



$\bar{D}(\rho D)_{D_1(2420)}, I = 0, m \sim 4241 \text{ MeV}, \Gamma \sim 25 - 30 \text{ MeV}$
PDG: $Y(4260) \Rightarrow \psi(4260), I^G(J^{PC}) = 0^-(1^{--}),$
 $m \simeq 4230 \text{ MeV}, \Gamma \sim 55 \pm 19 \text{ MeV}$

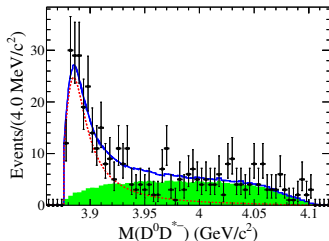
BESIII, Phys. Rev. Lett. 110, 252001 (2013)

$$e^+e^- \rightarrow \pi^+\pi^- J/\psi$$



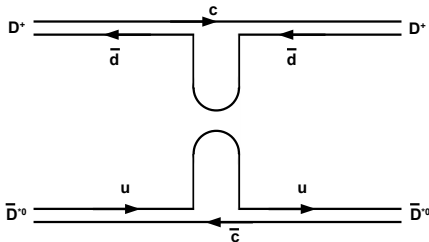
BESIII, Phys.Rev. D92 (2015) no.9, 092006

$$e^+e^- \rightarrow \pi^-(D\bar{D}^*)^+$$



PDG: $Z_c(3900)$, ; $I^G(J^{PC}) = 1^+(1^{+-})$, $m \simeq 3887.2 \pm 2.3 \text{ MeV}$, $\Gamma \sim 28.2 \pm 2.6 \text{ MeV}$

- Possible states \Rightarrow the interaction of $D\bar{D}^* + cc$ in the $I = 1$ channel
- One light meson exchange is OZI forbidden!! This means ρ, ω cancel and π, η, η' cancel if equal masses (or for large q)



- Exchange of a heavy vector $c\bar{c}$ or two pions
- The exchange of two pions \Rightarrow small !!

The heavy vector exchange:

- $I^G(J^{PC}) = 1^+(1^{+-})$; six possible channels $\Rightarrow \pi\omega, \eta\rho, (\bar{K}K^* + c.c.)/\sqrt{2}, (\bar{D}D^* + c.c.)/\sqrt{2}, \eta_c\rho, \pi J/\psi$
- The $PV \rightarrow PV$ interaction \Rightarrow local hidden gauge approach \Rightarrow generalizes the chiral Lagrangians to include vector mesons
- The Lagrangian describing the VPP and VVV vertex:

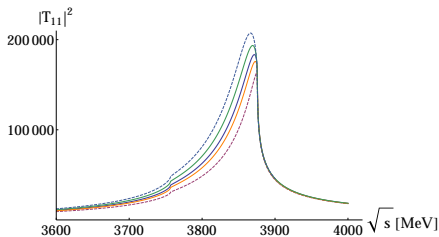
$$\mathcal{L}_{VPP} = -ig\langle V^\mu [P, \partial_\mu P] \rangle, \quad \mathcal{L}_{VVV} = ig\langle (V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu) V^\nu \rangle, \quad (13)$$

$$P = \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}, \quad (14)$$

$$V_\mu = \begin{pmatrix} \frac{\omega}{\sqrt{2}} + \frac{\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & \frac{\omega}{\sqrt{2}} - \frac{\rho^0}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu. \quad (15)$$

$$T = (1 - VG)^{-1} V,$$

$$V_{ij}(s) = -\frac{\vec{\epsilon} \cdot \vec{\epsilon}'}{8f^2} C_{ij} \left[3s - (M^2 + m^2 + M'^2 + m'^2) - \frac{1}{s}(M^2 - m^2)(M'^2 - m'^2) \right] \quad (16)$$



$$q_{max} = 850, 800, 770, 750 \text{ and } 700 \text{ MeV}$$

The peak moves to the left as the cutoff increases,

$\Gamma \sim 50 \text{ MeV} !!! \Rightarrow$ in coupled channels \Rightarrow open channels; $\eta_c \rho, \pi J/\psi$

- Position of the peak of $|T|^2$ corresponding to different values of q_{max} :

q_{max} [MeV]	\sqrt{s} [MeV]
700	3875
750	3873
770	3872
800	3869
850	3867

- The chiral unitary approach with coupled channels provides a simple and natural explanation of charmed exotica
- $X(3872) \Rightarrow D\bar{D}^*$; PDG: $X(3872) \Rightarrow \chi_{c1}(3872)!!!$
- $X(3940) \Rightarrow D^*\bar{D}^*$
- $X(4160) \Rightarrow D_s^*\bar{D}_s^*$
- $Y(4260) \Rightarrow \bar{D}(\rho D)_{D_1(2420)}$ PDG: $Y(4260) \Rightarrow \psi(4260)!!!$
The FCA to the Faddeev equations is an effective tool to deal with multi-hadron interaction
- $Z_c(3900) \Rightarrow \bar{D}D^* (I = 1)$

Thank you for your attention!