# Off-equilibrium effects in transport evolution "Saclay" approach

Miguel A. Escobedo

Instituto Galego de Física de Altas Enerxías

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# Open quantum evolution

General structure

$$\frac{\mathrm{d}D_{\mathrm{s}}}{\mathrm{d}t} = -i[H_{\mathrm{s}}, D_{\mathrm{s}}] + \int_0^{t-t_0} \mathrm{d}\tau \left\{\mathcal{L}^{\mathrm{ss}}(\tau)D_{\mathrm{s}}(t-\tau) + \mathcal{L}^{\mathrm{so}}(\tau)D_{\mathrm{o}}(t-\tau)\right\},$$

$$\frac{\mathrm{d}D_{\mathrm{o}}}{\mathrm{d}t} = -i[H_{\mathrm{o}}, D_{\mathrm{o}}] + \int_{t_0}^{t} dt' \left\{\mathcal{L}^{\mathrm{os}}(\tau)D_{\mathrm{s}}(t-\tau) + \mathcal{L}^{\mathrm{oo}}(\tau)D_{\mathrm{o}}(t-\tau)\right\}.$$

Where, for example

$$\mathcal{L}^{\mathrm{ss}}(\tau)D_{\mathrm{s}}(t') = -g^2 C_F \int_{\mathbf{X},\mathbf{X}'} \Big\{\Delta^{\geq}_{-}(X,X')\,\mathcal{P}_{\boldsymbol{X}}\,U_{\mathrm{o}}(\tau)\,\mathcal{P}_{\boldsymbol{X}'}\,D_{\mathrm{s}}(t')\,U_{\mathrm{s}}^{\dagger}(\tau)$$
$$+\Delta^{\leq}_{-}(X,X')\;U_{\mathrm{s}}(\tau)D_{\mathrm{s}}(t')\mathcal{P}_{\boldsymbol{X}'}U_{\mathrm{o}}^{\dagger}(\tau)\mathcal{P}_{\boldsymbol{X}}\Big\},$$

with $t' = t - \tau$.

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$$\mathcal{L}^{\mathrm{ss}}(\tau) D_\mathrm{s}(t') = -g^2 C_F \int_{\mathbf{X},\mathbf{X}'} \Big\{ \Delta_-^{\geq}(X, X')\, \mathcal{P}_{\boldsymbol{X}}\, U_\mathrm{o}(\tau)\, \mathcal{P}_{\boldsymbol{X}'}\, D_\mathrm{s}(t')\, U_\mathrm{s}^\dagger(\tau)$$

$$+ \Delta_-^{\leq}(X, X')\, U_\mathrm{s}(\tau) D_\mathrm{s}(t') \mathcal{P}_{\boldsymbol{X}'} U_\mathrm{o}^\dagger(\tau) \mathcal{P}_{\boldsymbol{X}} \Big\},$$

with $t' = t - \tau$.

- Markovian evolution can be obtained under the approximation that the non-unitary evolution is a perturbation.
- Expression formed by operators written in two different basis (coordinate and Hamiltonian eigenvalues).

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## Classical limit

- Using a technique called multiple-scale analysis we can derive a rate equation whenever the energy gap is much bigger than the decay width.
- Octet to octet transition is mathematically similar to the QED case, which is well described by a Langevin equation. Moreover, in the large $N_c$ limit the potential is zero.

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In practice we use a semi-classical equation, but we have a very clear criterion to determine the regime of validity, the relation between the energy gap and the decay width. (Intuitively, it must take many wave function oscillations to decay).

# Langevin equation for the octet

- We need as an input the momentum diffusion coefficient $\kappa = \frac{T\gamma M}{2}$, can not be computed in our framework and the total volume of the medium $\Omega$.

$$\frac{\partial p^{\rm o}_{\boldsymbol{p}}}{\partial t} - \gamma\nabla(\mathbf{p}p^{\rm o}_{\boldsymbol{p}}) - \frac{T\gamma M}{2}\Delta^2 p^{\rm o}_{\boldsymbol{p}} =$$
$$-\frac{g^2}{2N_c}\frac{1}{\Omega}\left(p^{\rm o}_{\boldsymbol{p}} - p^{\rm s}{\rm e}^{-\frac{E^{\rm o}_{\boldsymbol{p}}-E^{\rm s}}{T}}\right)\int_{\boldsymbol{q}}\Delta^{>}(\omega^{\rm o}_{\boldsymbol{p}} - E^{\rm s}, \boldsymbol{q})|\langle{\rm s}|\mathcal{S}_{\boldsymbol{q}\cdot\hat{\boldsymbol{r}}}|{\rm o}, \boldsymbol{p}\rangle|^2\,,$$

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- Decay of octet into singlet is suppressed by the inverse of the volume. Very slow equilibration.
- We made the derivation in the dilute limit (two heavy quarks in a thermal environment).

# $\Upsilon(1S)$ survival probability in static QGP brick

| | $\Omega = 1\,\text{fm}^3$ | | | $\Omega = 100\,\text{fm}^3$ | | |
|---|---|---|---|---|---|---|
| | $5\,\text{fm/c}$ | $100\,\text{fm/c}$ | eq. | $5\,\text{fm/c}$ | $100\,\text{fm/c}$ | eq. |
| $T = 200\,\text{MeV}$ | 0.86 | 0.136 | 0.0814 | 0.85 | 0.0438 | 0.00089 |
| $T = 400\,\text{MeV}$ | 0.39 | 0.0515 | 0.0175 | 0.36 | 0.0002 | 0.00018 |

- We used an initial condition with one singlet and no octets.
- Volume dependence indicates that the octet contribution (in the dilute limit) is important at very large times, but not at LHC.
- Therefore, small sensitivity to the parameters that enter the Langevin equation of the octet.

# Homework

Since in the paper we only considered a QGP brick, we needed further assumptions to complete the homework

- Bjorken evolution for the medium.
- We neglected the contribution of the octet.
  - We have indications that it is a small effect.
  - In this way we did not have to know the dependency of the wave function with the temperature (numerically costly).

Under these approximations we get

$$R_{AA}(1S) = \mathrm{e}^{-1.5 a T_0 t_0 \left( \left( \frac{T_0}{T_f} \right)^2 - 1 \right) - 3 b T_0^2 t_0 \left( \frac{T_0}{T_f} - 1 \right)}$$

where $a$ and $b$ are obtained from fitting the decay width with the formula $\Gamma(1S) = aT + bT^2$.