pNRQCD for the study of nonequilibrium evolution of quarkonium in a medium

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pNRQCD and open quantum system, singlet and octet density master evolution equations, Lindblad equations

Results obtained in collaboration with :

M. Escobedo, A. Vairo, J. Soto (van der Griend) N.B. (master equations);

based on

(1) N. Brambilla, M.A. Escobedo, A. Vairo and P. Vander Griend
Transport coefficients from in medium quarkonium dynamics
`arXiv:1903.08063`

(2) N. Brambilla, M.A. Escobedo, J. Soto and A. Vairo
Heavy quarkonium suppression in a fireball
Phys. Rev. **D97** (2018) 074009 `arXiv:1711.04515`

(3) N. Brambilla, M.A. Escobedo, J. Soto and A. Vairo
Quarkonium suppression in heavy-ion collisions: an open quantum system approach
Phys. Rev. **D96** (2017) 034021 `arXiv:1612.07248`

Quarkonium in a fireball

- After the heavy-ion collisions, heavy quark-antiquarks propagate freely up to 0.6 fm.
- From 0.6 fm to the freeze-out time t_F they propagate in the medium.
- We assume the medium infinite, homogeneous and isotropic.
- We assume the heavy quarks comoving with the medium.
- We assume the medium to be locally in thermal equilibrium,

  i.e., the temperature T of the medium changes (slowly) with time:

$$T = T_0 \left(\frac{t_0}{t}\right)^{v_s^2}, \quad t_0 = 0.6 \text{ fm}, \quad v_s^2 = \frac{1}{3} \text{ (sound velocity)}$$

Quarkonium as a Coulombic bound state

The lowest quarkonium states (1S bottomonium and charmonium, 2S bottomonium) are the most tightly bound. For these we assume the hierarchy of energy scales

$$M \gg \frac{1}{r} \sim M\alpha_s \gg T \sim gT \gg \text{any other scale}, \qquad v \sim \alpha_s$$

Density matrices

- Subsystem: heavy quarks/quarkonium
- Environment: quark gluon plasma

We may define a density matrix in pNRQCD for the heavy quark-antiquark pair in a singlet and octet configuration:

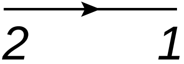
$$\begin{aligned}
\langle \mathbf{r}', \mathbf{R}' | \rho_s(t';t) | \mathbf{r}, \mathbf{R} \rangle &\equiv \mathrm{Tr}\{\rho_{\text{full}}(t_0)\, S^\dagger(t, \mathbf{r}, \mathbf{R}) S(t', \mathbf{r}', \mathbf{R}')\} \\
\langle \mathbf{r}', \mathbf{R}' | \rho_o(t';t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{8} &\equiv \mathrm{Tr}\{\rho_{\text{full}}(t_0)\, O^{a\dagger}(t, \mathbf{r}, \mathbf{R}) O^b(t', \mathbf{r}', \mathbf{R}')\}
\end{aligned}$$

$t_0 \approx 0.6$ fm is the time formation of the plasma.

The system is in non-equilibrium because through interaction with the environment (quark gluon plasma) singlet and octet quark-antiquark states continuously transform in each other although the number of heavy quarks is conserved: $\mathrm{Tr}\{\rho_s\} + \mathrm{Tr}\{\rho_o\} = 1$.

Evolution of the density matrix in pNRQCD. Singlet

$$\partial_t \rho_s = -i[h_s, \rho_s] - i\Sigma\rho_s + i\rho_s\Sigma^\dagger + \mathcal{F}(\rho_o)$$



We use the non-equilibrium quantum field theory formalism to perform this computation. Tree level is equivalent to the $T = 0$ LO evolution

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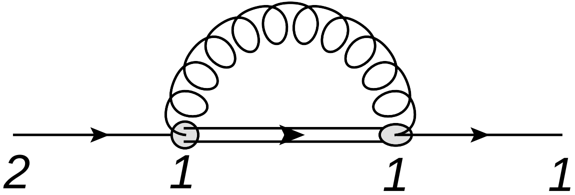

Self-energy diagram contributes to screening and to the decay width.

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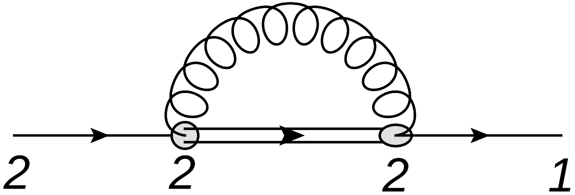

Hermitian conjugate of the previous diagram. We can reorganize this to diagrams in a redefinition of h_s and a term that represents the decay width.

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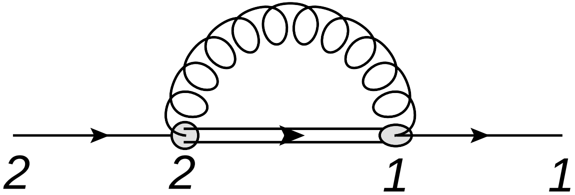

The number of singlets is increased due to the octets in the medium that absorb a gluon.

Evolution of the octet

Very similar reasoning.

$$\partial_t \rho_o = -i[H_o, \rho_o] - \frac{1}{2}\{\Gamma, \rho_o\} + \mathcal{F}_1(\rho_s) + \mathcal{F}_2(\rho_o)$$

- Computationally costly system of coupled equations. The density matrix contains many information, especially in the non-Abelian case (QCD).
- Total number of heavy particles is conserved. $Tr(\rho_s) + Tr(\rho_o)$ is a constant of the evolution.
- Only if $T \gg E$ the interaction with the medium can be considered local in time and the evolution is Markovian.

Time scales

- Environment correlation time $\tau_E \sim \frac{1}{T}$.
- System intrinsic time scale $\tau_S \sim \frac{1}{E}$.
- System relaxation time $\tau_R \sim \frac{1}{\Sigma} \sim \frac{1}{\alpha_s a_0^2 \Lambda^3}$ where Λ is either T or E depending on the temperature regime.

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- Because $\frac{1}{r} \gg T, E$, it follows that τ_R is the longest time scale. Markovian approximation.
- If $T \gg E$ then $\tau_S \gg \tau_E$, the interaction with the medium is instantaneous from the point of view of the system. Lindblad equation limit.

The $\frac{1}{r} \gg T, m_D \gg E$ regime

Brambilla, M.A.E., Soto and Vairo (2017-2018)

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k \left(C_k(\kappa)\rho C_k^\dagger(\kappa) - \frac{1}{2}\{C_k^\dagger(\kappa) C_k(\kappa), \rho\}\right)$$

$$\kappa = \frac{g^2}{6\,N_c} \mathrm{Re} \int_{-\infty}^{+\infty} ds \, \langle \, \mathrm{T} \, E^{a,i}(s, \mathbf{0}) E^{a,i}(0, \mathbf{0}) \rangle$$

The heavy quark diffusion coefficient

$$\gamma = \frac{g^2}{6\,N_c} \mathrm{Im} \int_{-\infty}^{+\infty} ds \, \langle \, \mathrm{T} \, E^{a,i}(s, \mathbf{0}) E^{a,i}(0, \mathbf{0}) \rangle$$

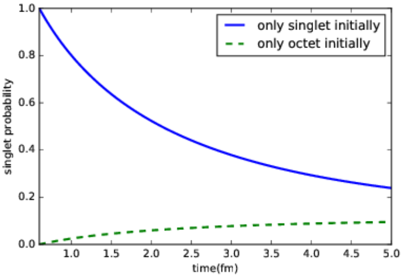
No lattice QCD information on γ but we observe that we reproduce data better if it is small. For comparison, in pQCD

$$\gamma = -2\zeta(3)\, C_F \left(\frac{4}{3} N_c + n_f\right) \alpha_s^2(\mu_T)\, T^3 \approx -6.3\, T^3$$

Approach to equilibrium in the static limit

We assume that $\frac{\kappa}{T^3}$ is a constant. Then, $\Gamma(T) = \Gamma(T_0)\left(\frac{T}{T_0}\right)^3$.
In the static limit we can solve the equation analytically using Bjorken evolution, the approach to equilibrium is power like.

$$\rho_s(t) = \frac{1}{9} - \frac{8}{9}\left(\frac{\rho_o(t_0)}{8} - \rho_s(t_0)\right)\left(\frac{t_0}{t}\right)^{\frac{9}{8}\Gamma(T_0)t_0}$$



Initial conditions

At any point in time, our density matrix is diagonal by boxes in colour space

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Physical insight

- The production of a heavy quark pair is a high energy process.
- The production of a pair in a singlet state is suppressed by a factor $\alpha_s(m)$ with respect to that of the octet.

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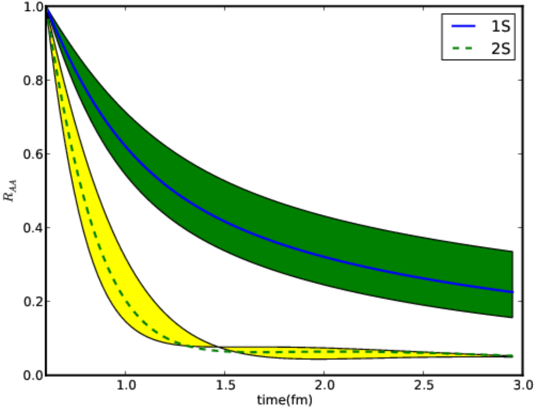
Initial conditions

- We take as initial condition $\rho_s(0) = A|\mathbf{r} = \mathbf{0}\rangle\langle\mathbf{r} = \mathbf{0}|$ and $\rho_o(0) = \frac{\delta}{\alpha_s(m)}\rho_s(0)$. Then, we evolve the density matrix in the vacuum from $t = 0$ to $t = t_0 = 0.6\, fm$.
- δ is a free parameter. However, we observe that the results depend very little on its value.

R_{AA}

We compute R_{AA} (before feed-down) using the formula

$$R_{AA}(n) = \frac{\langle n|\rho(t_f)|n\rangle}{\langle n|\rho(t_0)|n\rangle}$$



Quantum coherence

- The evolution equations that we are able to deduce by systematically exploiting the separation between energy scales are quantum.
- Approximations that can lead to simplifications (like semi-classical equations), as for example the rotating wave approximation or multiple-scale analysis, are not fully compatible with a systematic expansion of ratios of energy scales.
- At the moment, it is difficult to determine quantitatively the importance of quantum coherence.
- Quantum coherence might force us to change how we think about initial conditions. Not in terms of probabilities, but in terms of density matrix.

Future improvements

Beyond Bjorken evolution

- Due to the high numerical cost of our computations, we have only implemented Bjorken evolution. This overestimates suppression.
- We are studying how to reduce this cost. When this is done we will implement more realistic hydro.
- We have calculated the binding energy and decay width in an anisotropic plasma. (Biondini, Brambilla, Escobedo and Vairo (2017)).

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Quarkonium in a hot wind

We have also studied how the potential, the binding energy and the decay width are modified when quarkonium is moving with a finite velocity with respect to the medium. This might be important for the R_{AA} dependence with transverse momentum. (Escobedo, Giannuzzi, Mannarelli and Soto (2013)).

Future improvements II

- Use more realistic initial conditions.
- Use more recent determinations of κ and γ and include a more complex temperature dependence.
- Relax the assumption $T \gg E$.