Quarkonium masses and spectral functions from lattice QCD

FASTSUM Collaboration

- Bayesian Approach
- Maximum Entropy Method
- FASTSUM approach/parameters
- **Lighter quarks** (new-ish)
- **Finer lattices** (new-ish)

The Task

Given data D

Find fit F by maximising $P(F|D)$

Bayesian Methods

Need to maximise $P(F|D)$

Bayes Theorem: $P(F|D)P(D) = P(D|F)P(F) \;\; = \;\; P(D \cap F)$

“priors"

i.e. $P(F|D) = \dfrac{P(D|F)P(F)}{P(D)}$

But $P(D|F) \sim e^{-\chi^2} \;\; \longrightarrow \;\;$ minimising $\chi^2 \neq$ maximising $P(F|D)$

$\longrightarrow$ *Maximum Likelihood Method* wrong??

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Implicit prior: *P(F=elephant) = 0*

So we always have "priors" ….

E.g. for T=0 we have: $F(\omega) = \sum_i \delta(\omega - M_i) e^{-M_i t}$

Maximum Entropy Method

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$P(F) \sim e^{\alpha S}$ (using minimal assumptions)

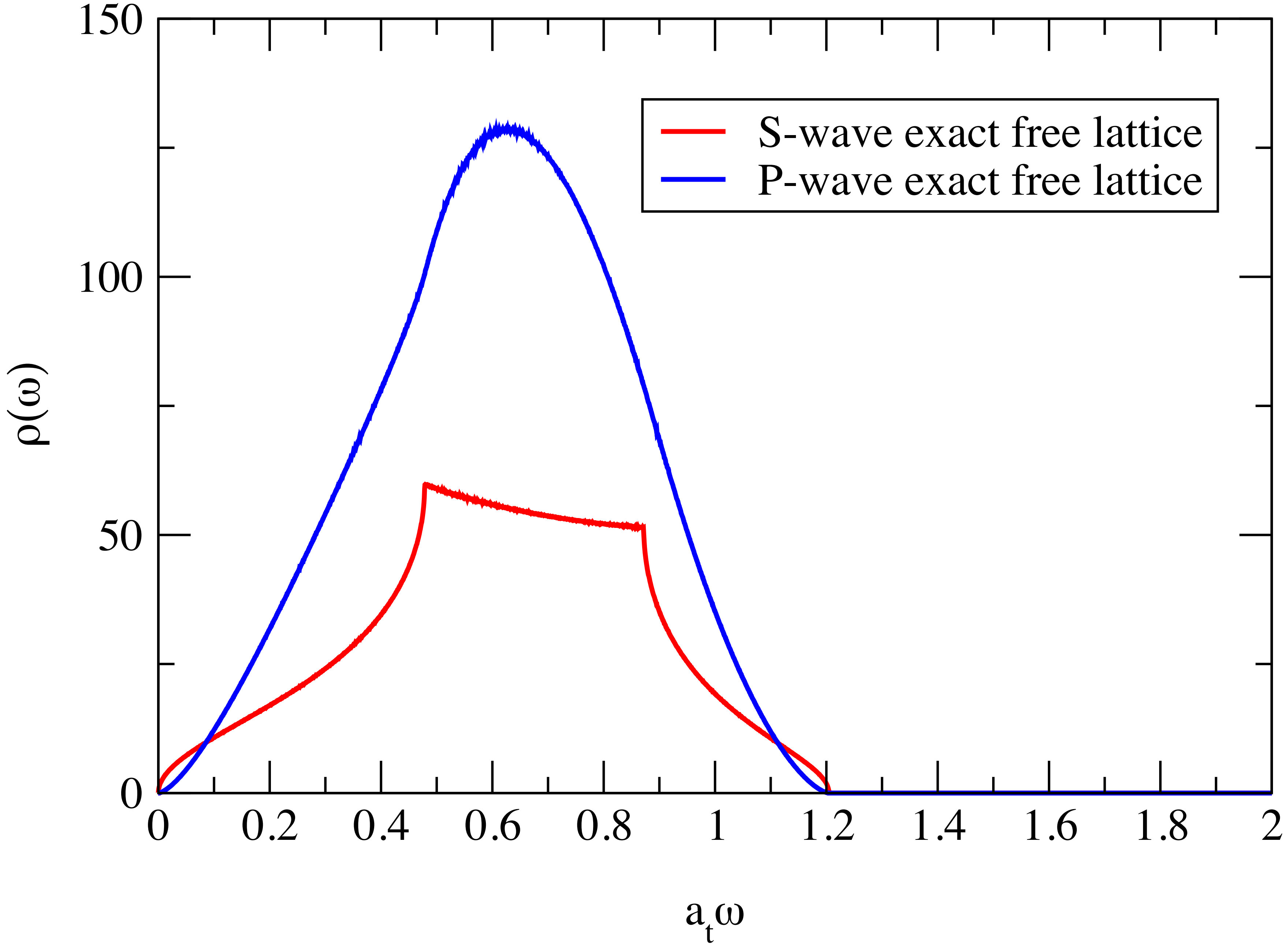
Shannon-Jaynes Entropy: $S = \displaystyle\int_0^\infty \frac{d\omega}{2\pi}\left[\rho(\omega) - m(\omega) - \rho(\omega)\ln\frac{\rho(\omega)}{m(\omega)}\right]$

Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459

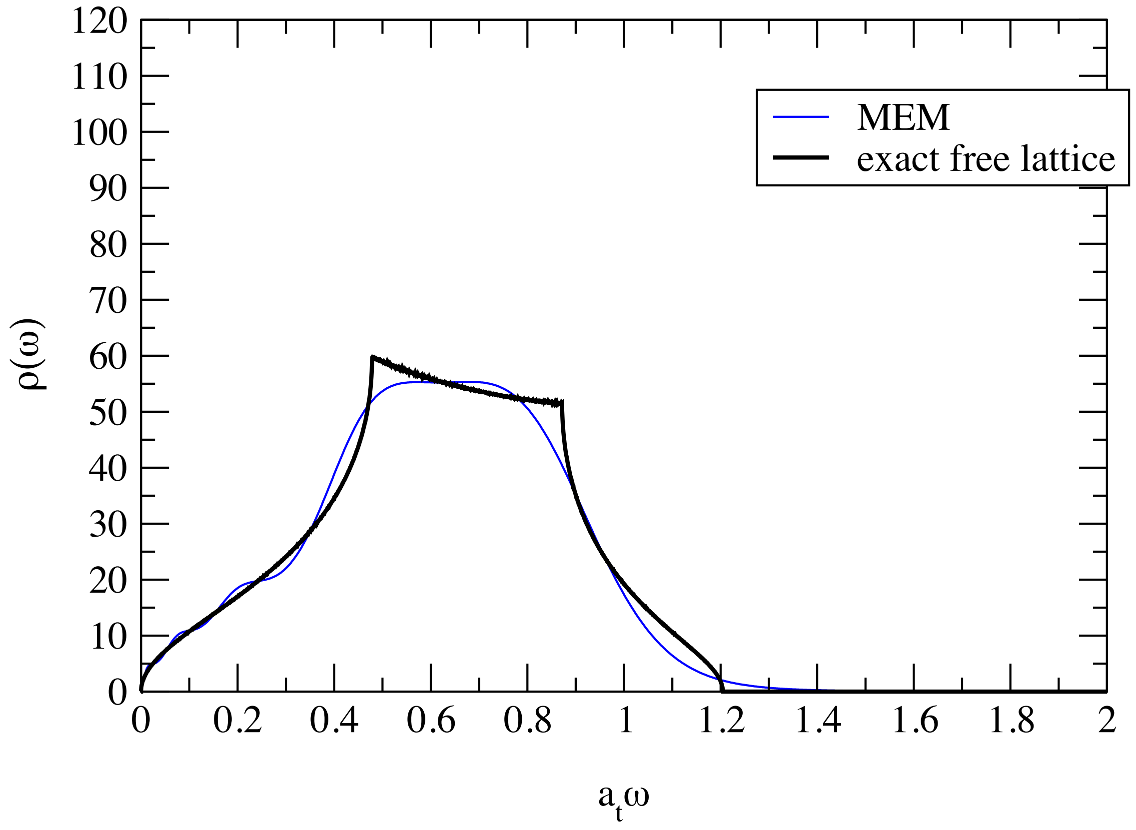
Maximum Entropy Method Tests

Theoretical Free Lattice Spectral Function (NRQCD)

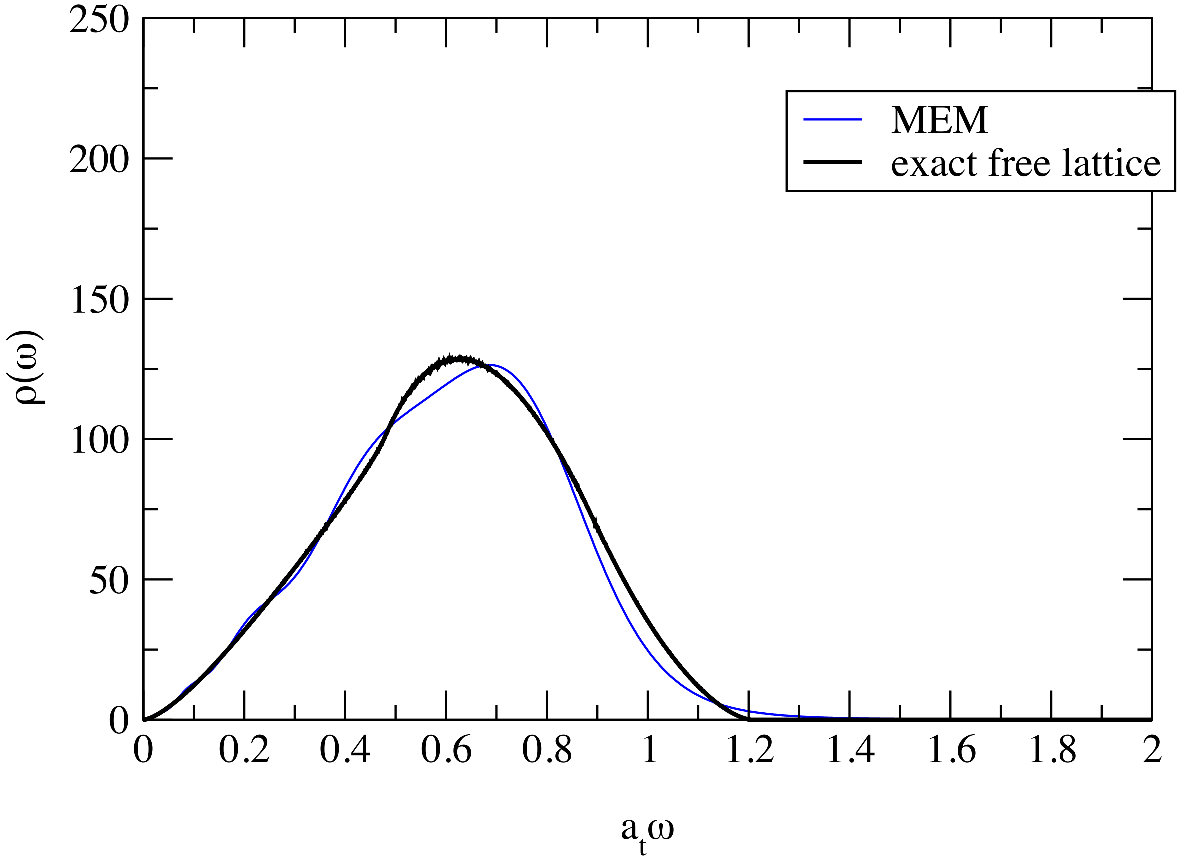
$$\rho(\omega)^{\text{theory}} = \sum_{k_x, k_y, k_z} \delta(\omega - \hat{E}(k_x, k_y, k_z))$$



MEM Result - S-wave

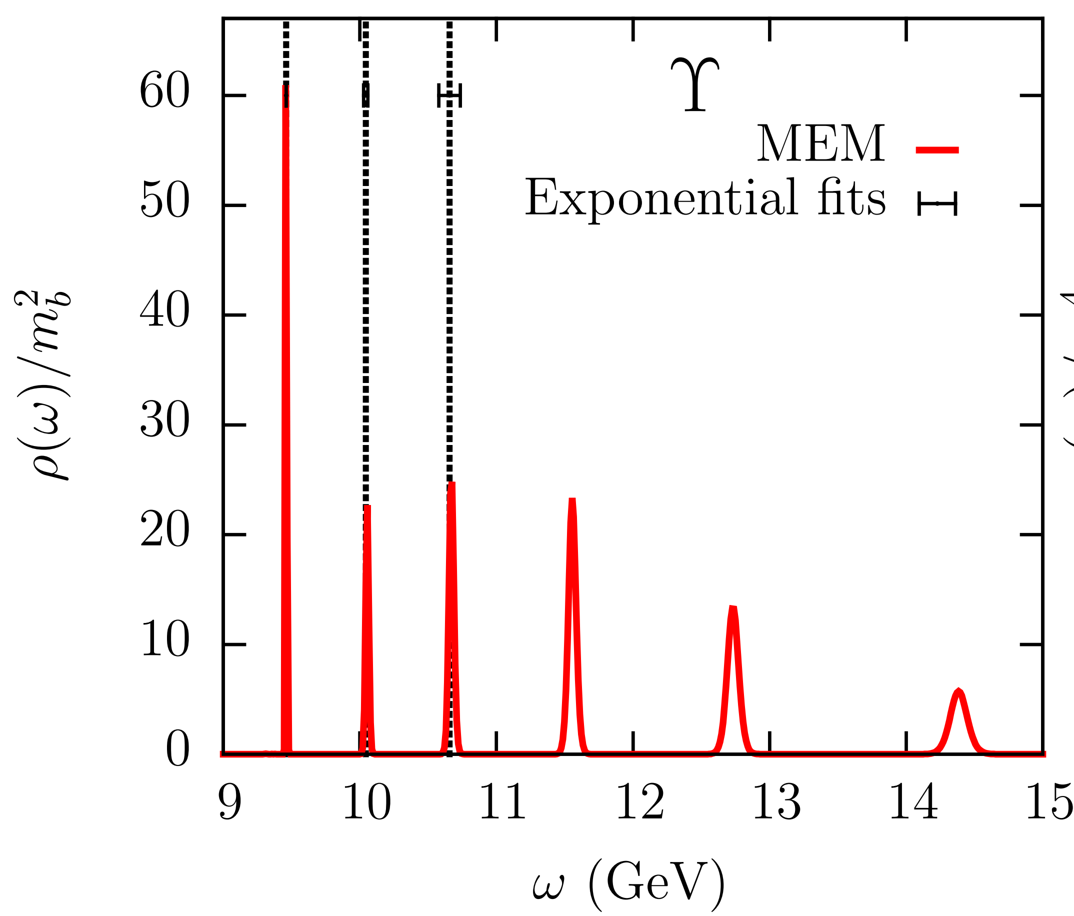


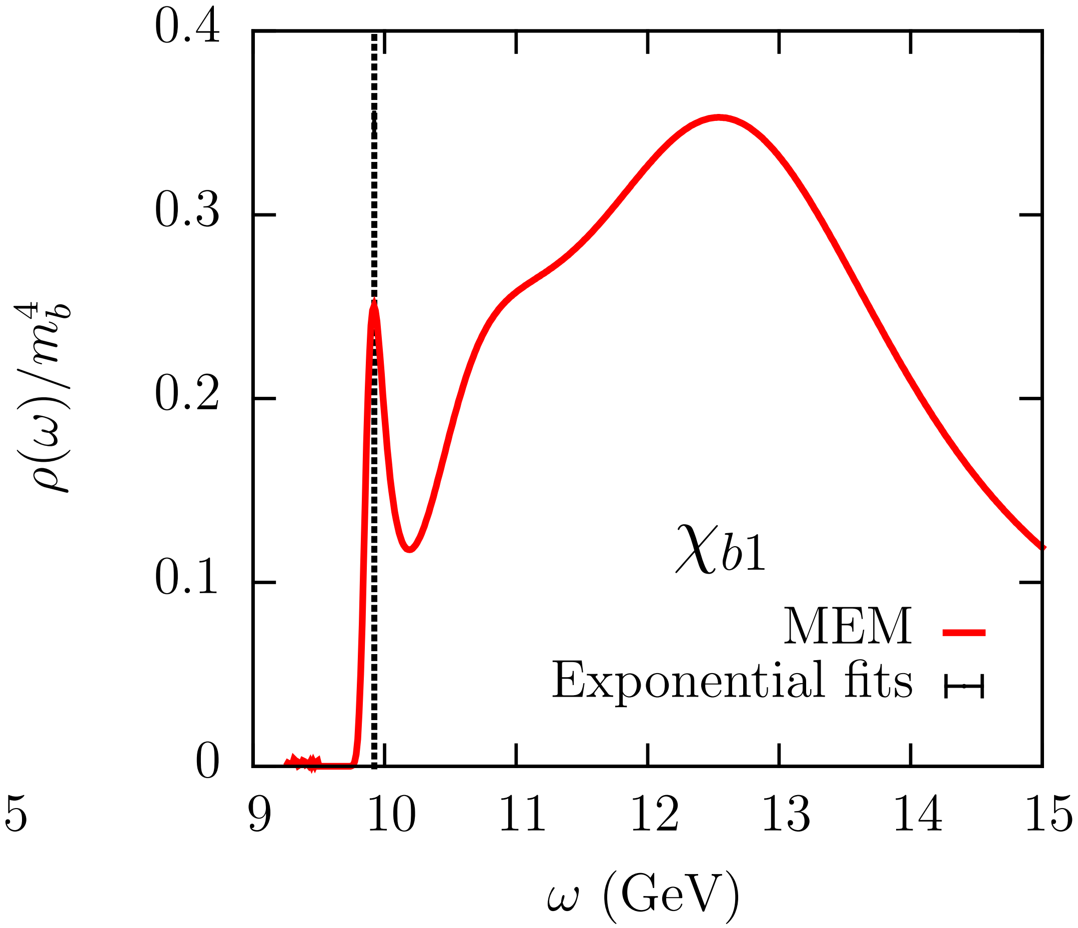
MEM Result - P-wave



T=0 spectral functions

$$G(\tau) = \int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega}{2\pi} K(\tau,\omega)\,\rho(\omega), \qquad K(\tau,\omega) = e^{-\omega\tau}.$$

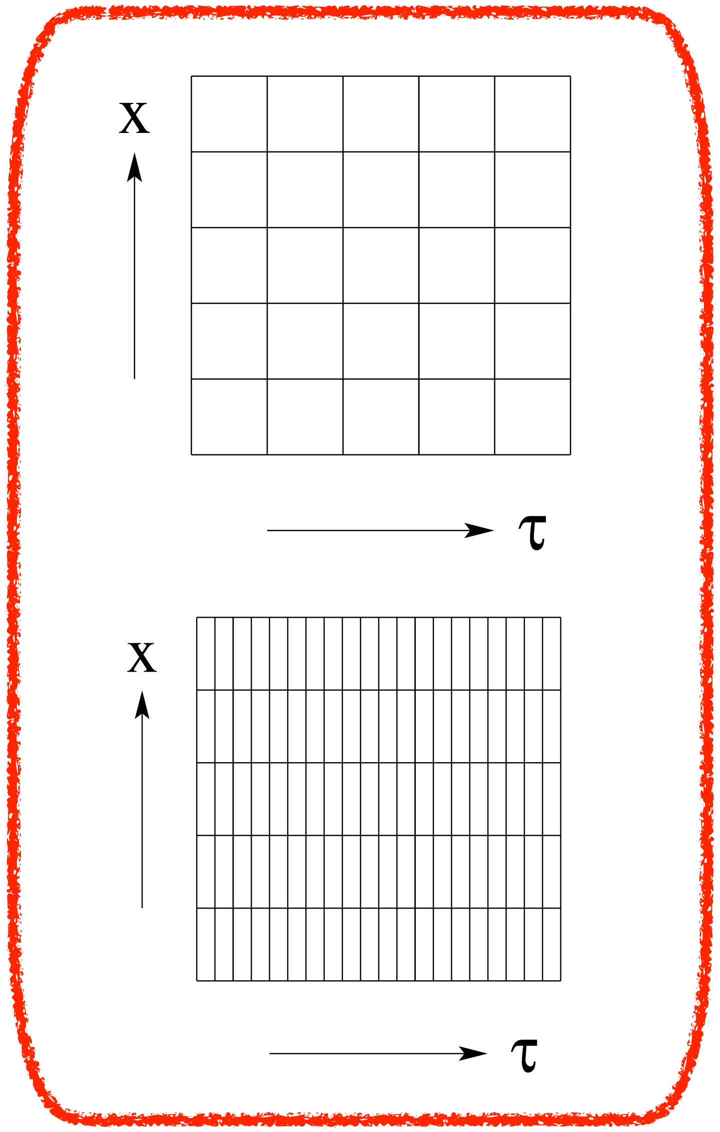




Generation 2

Results

Lattice Parameters - Generation 2



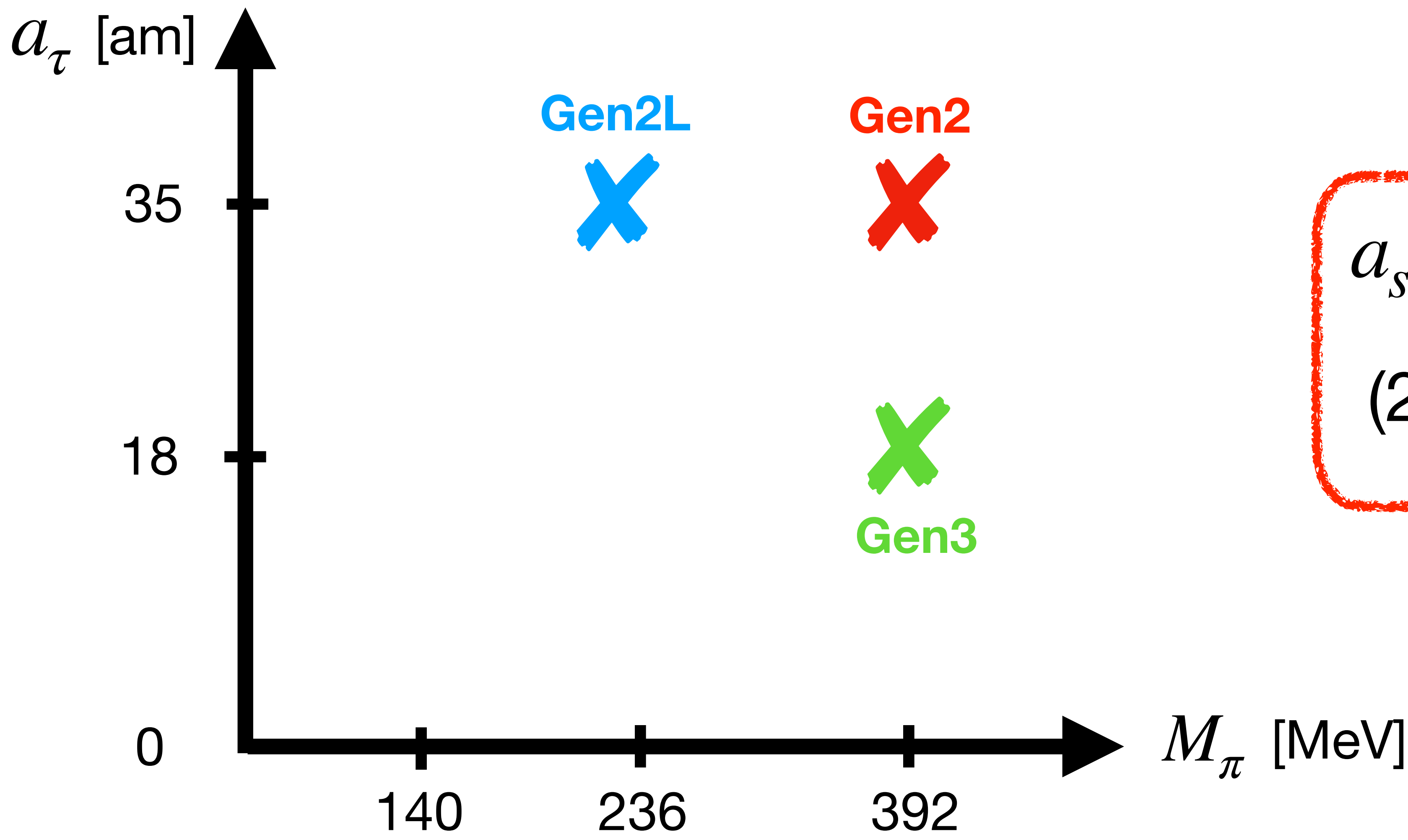
N_s	N_τ	T [MeV]	T/T_c	N_{src}	N_{cfg}
24	128	44	0.24	16	139
24	40	141	0.76	4	501
24	36	156	0.84	4	501
24	32	176	0.95	2	1000
24	28	201	1.09	2	1001
24	24	235	1.27	2	1001
24	20	281	1.52	2	1000
24	16	352	1.90	2	1001

$$T = 1/L_\tau$$
$$= 1/(a_\tau N_\tau)$$

M_π = 392(4) MeV

(2+1) flavour

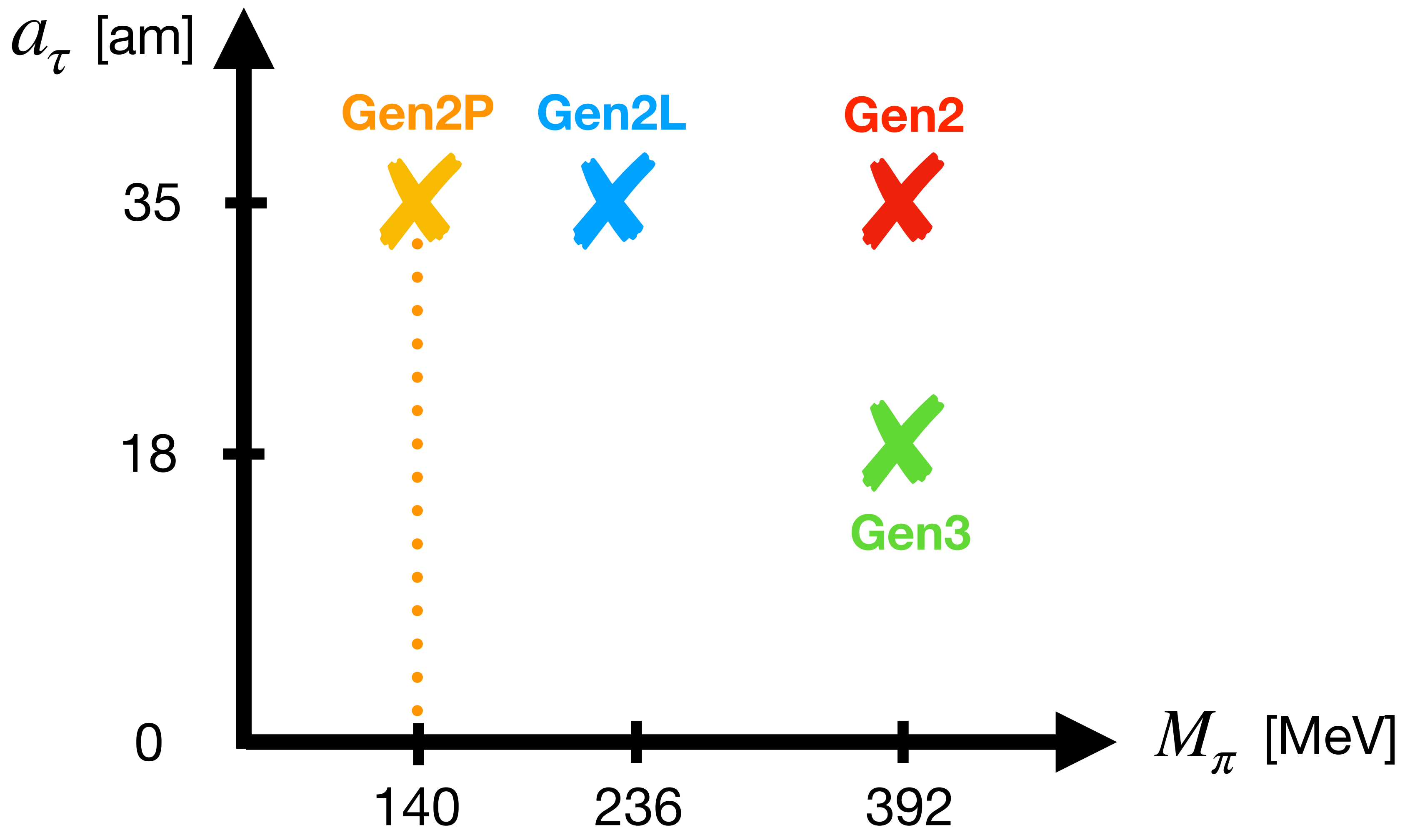
Lattice Parameters



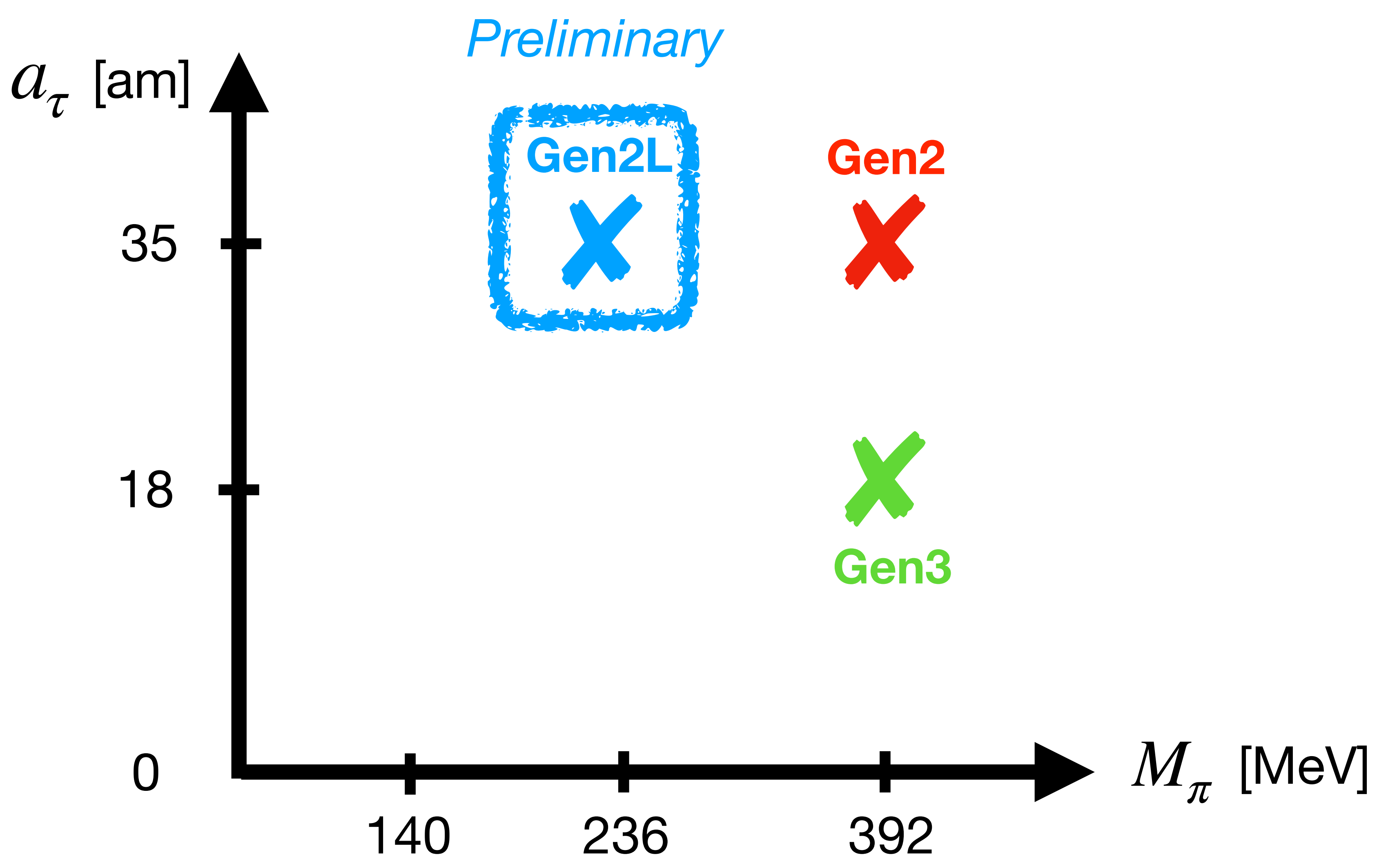
$a_s \sim 0.12$ fm

(2+1) flavour

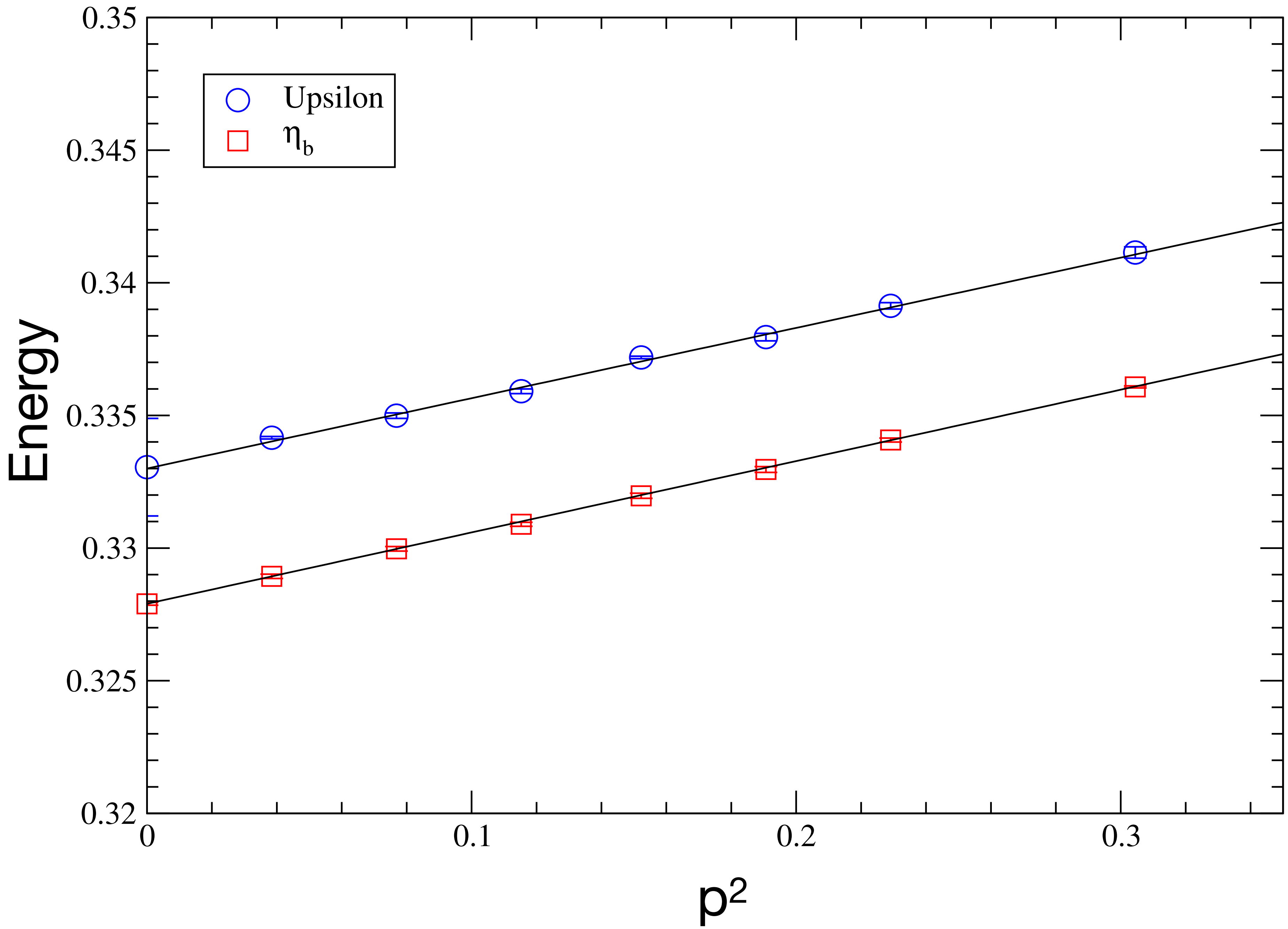
Lattice Parameters



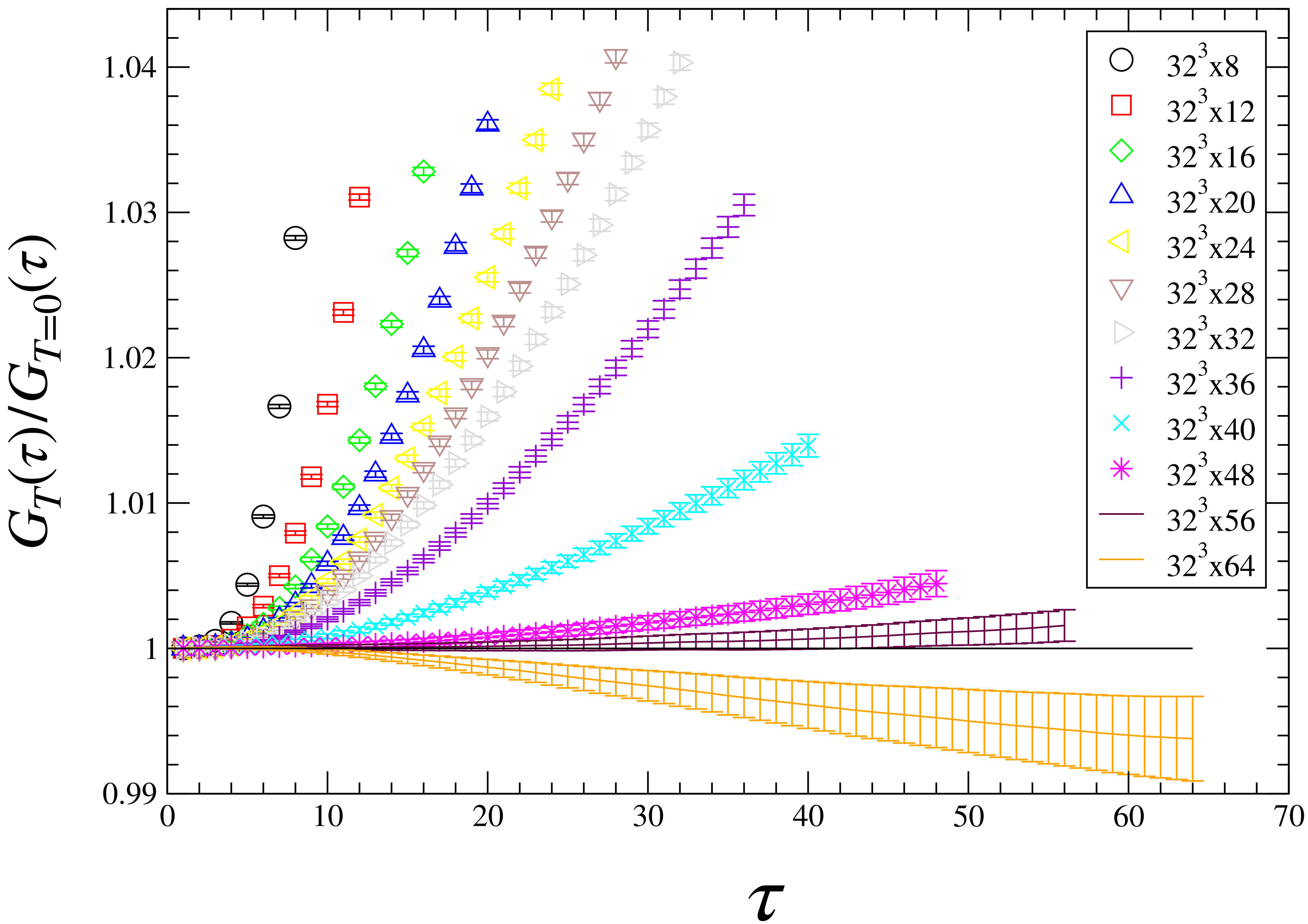
Gen2L Results



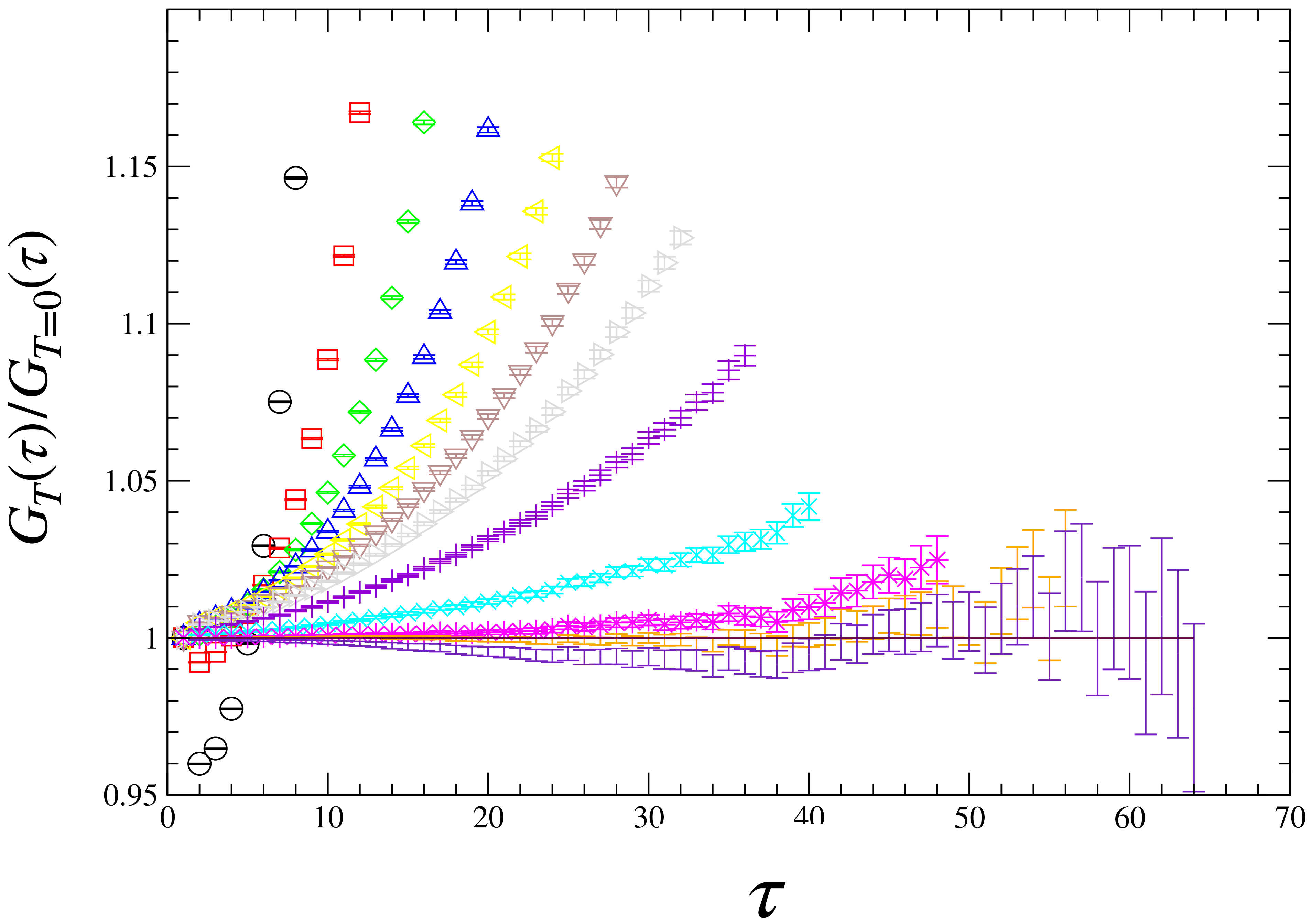
Dispersion Relations



Correlation Ratios: Upsilon

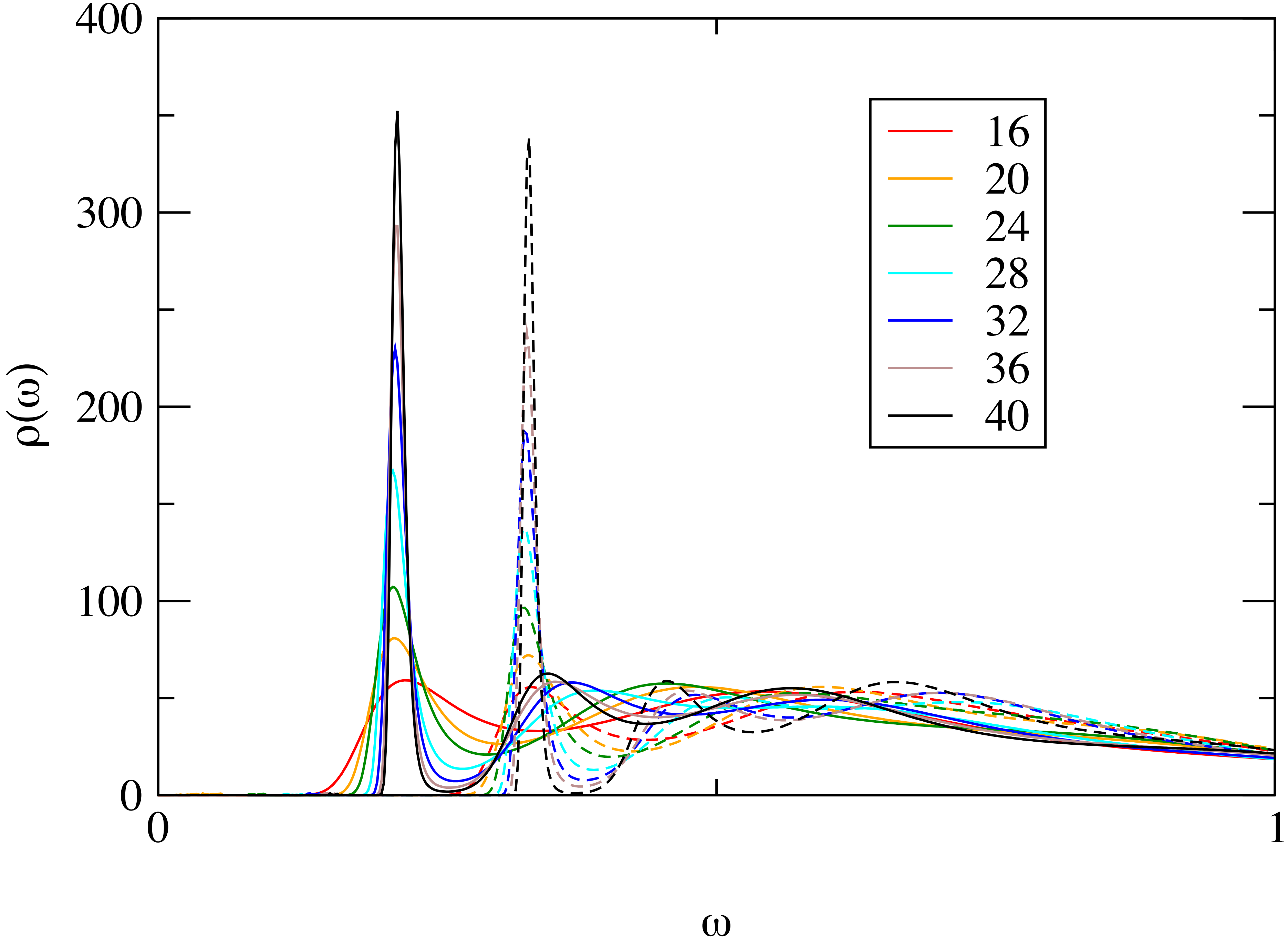


Correlation Ratios: χ_{b1}



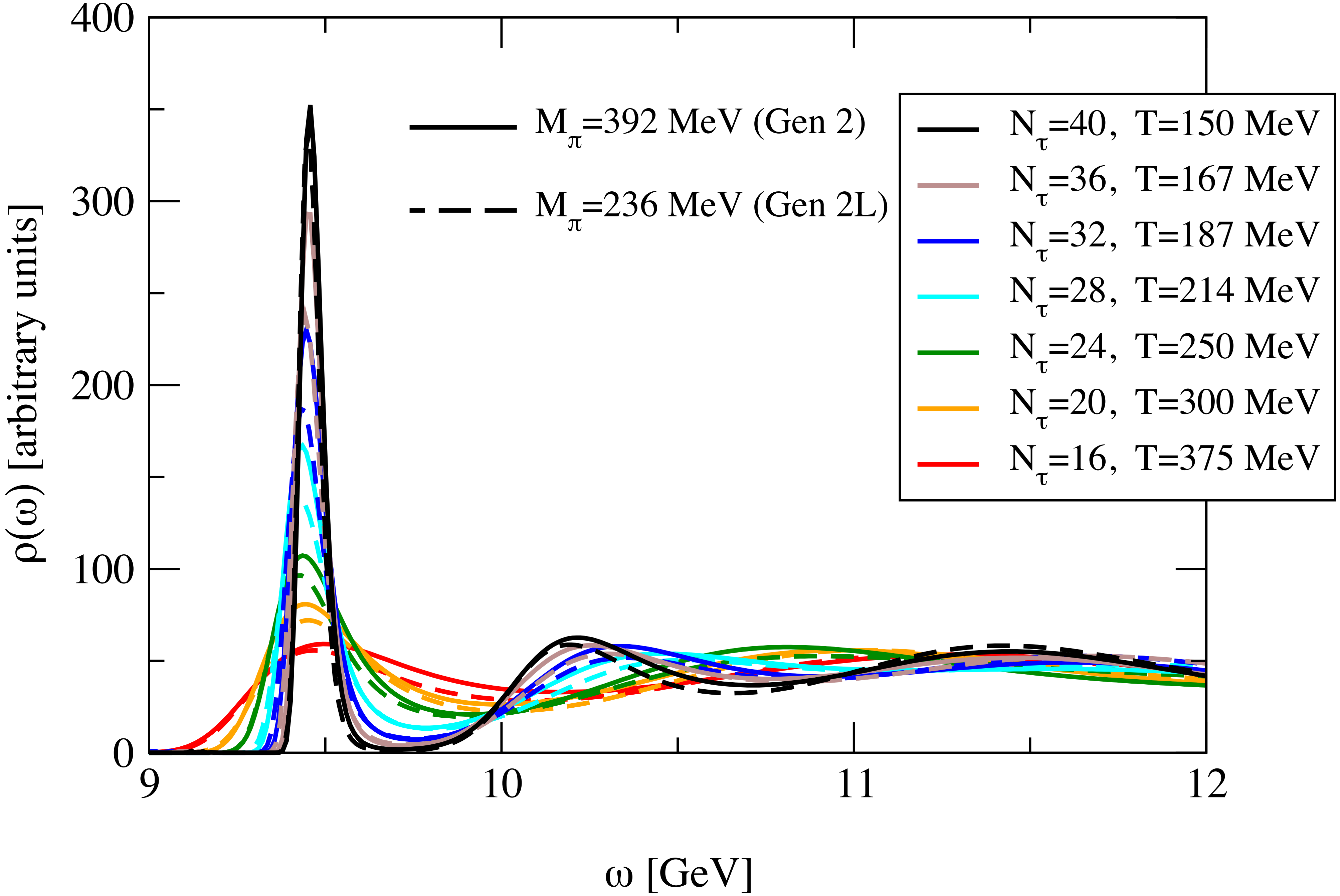
Spectral Functions: Gen2 vs Gen2L [Upsilon]

Raw: un-renormalised



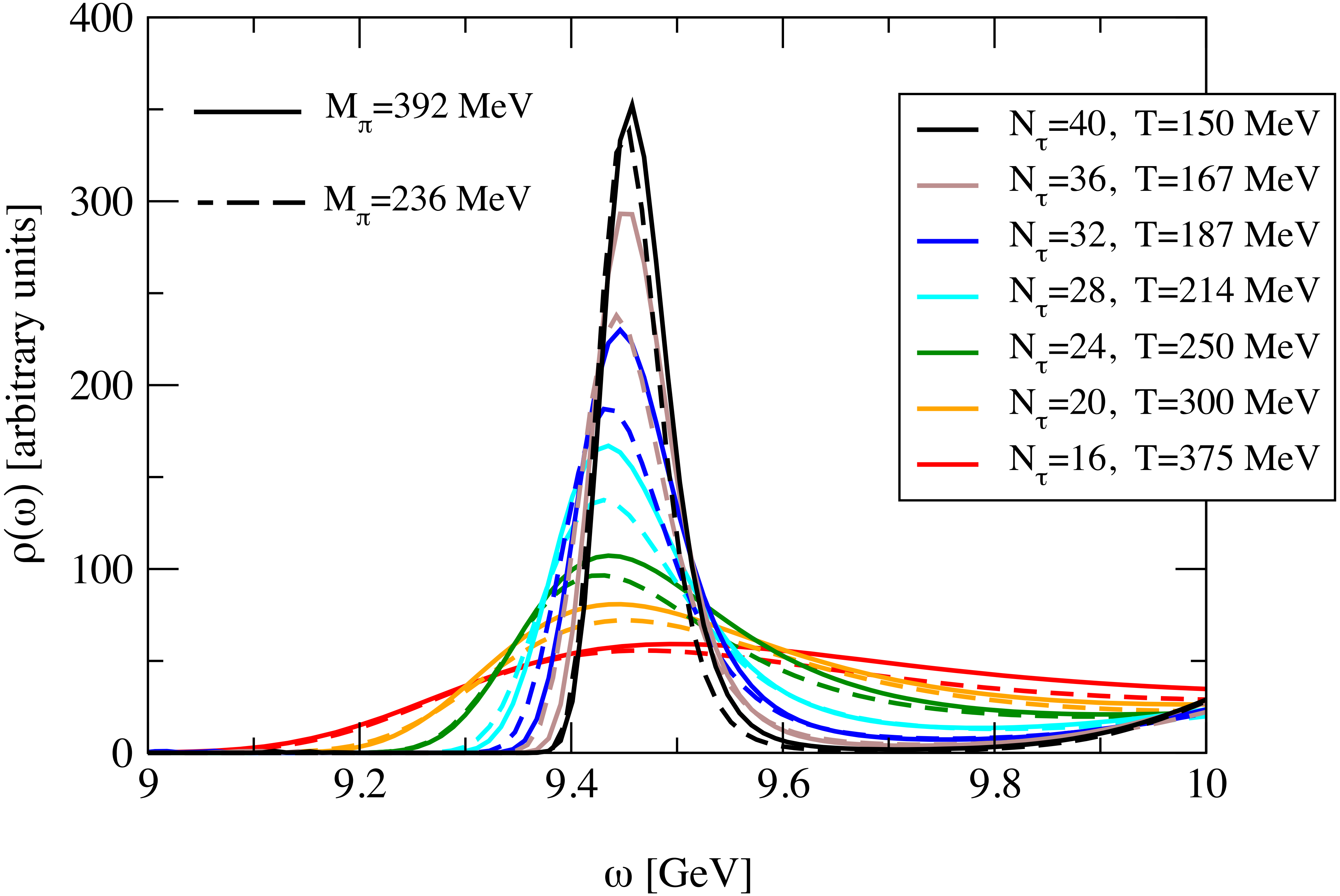
Spectral Functions: Gen2 vs Gen2L [Upsilon]

Renormalised

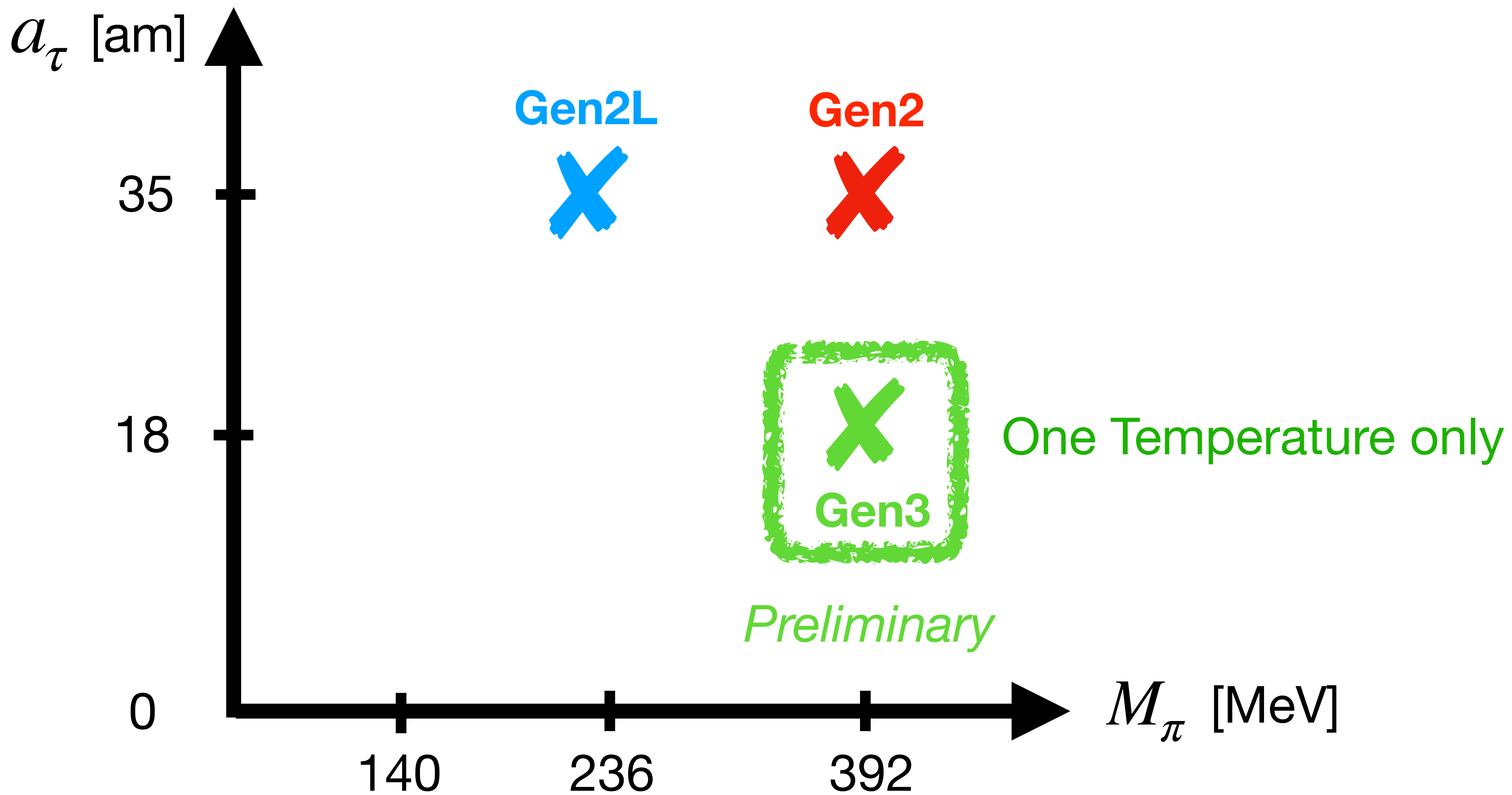


Spectral Functions: Gen2 vs Gen2L [Upsilon]

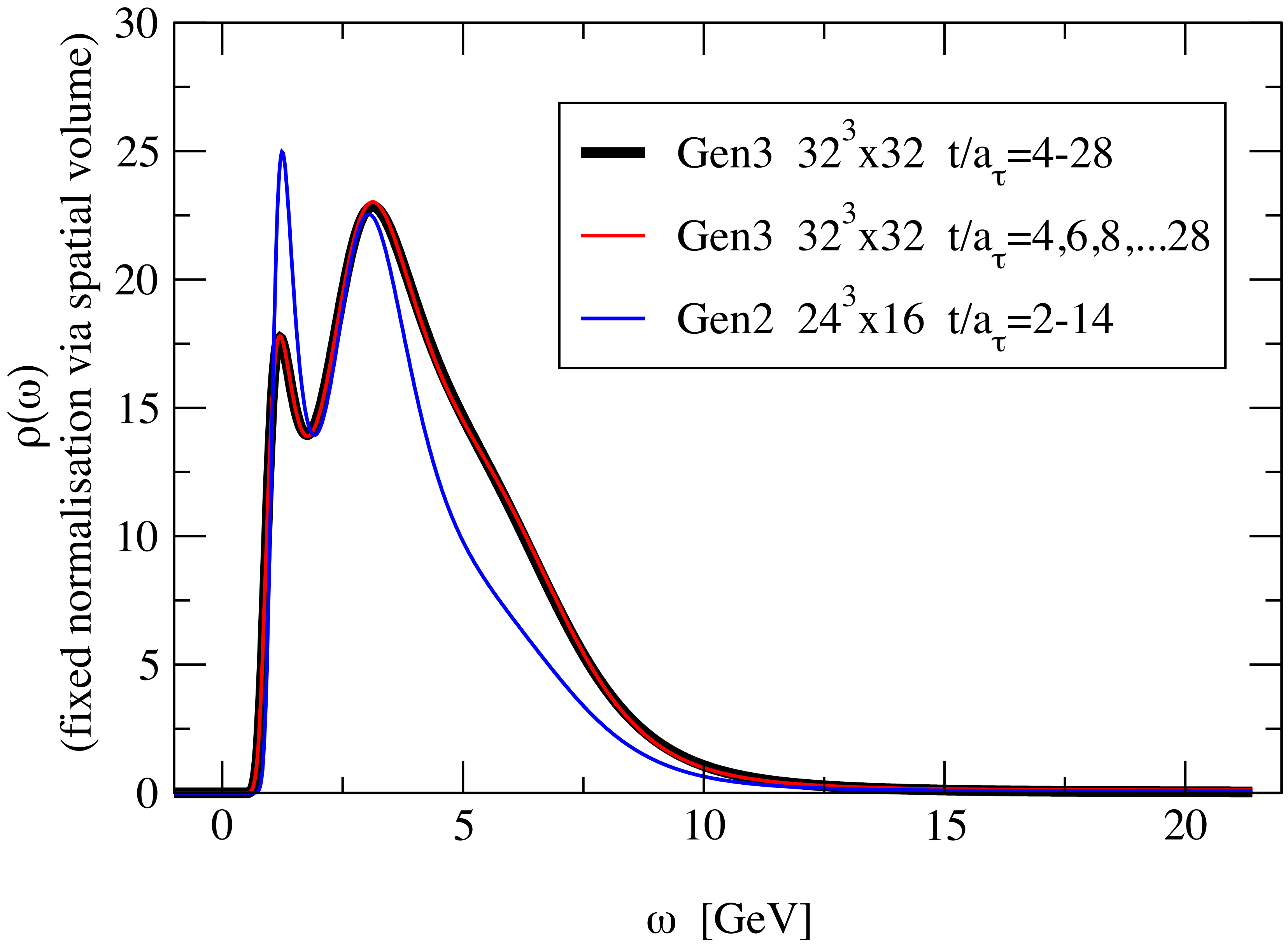
Renormalised: Close-up



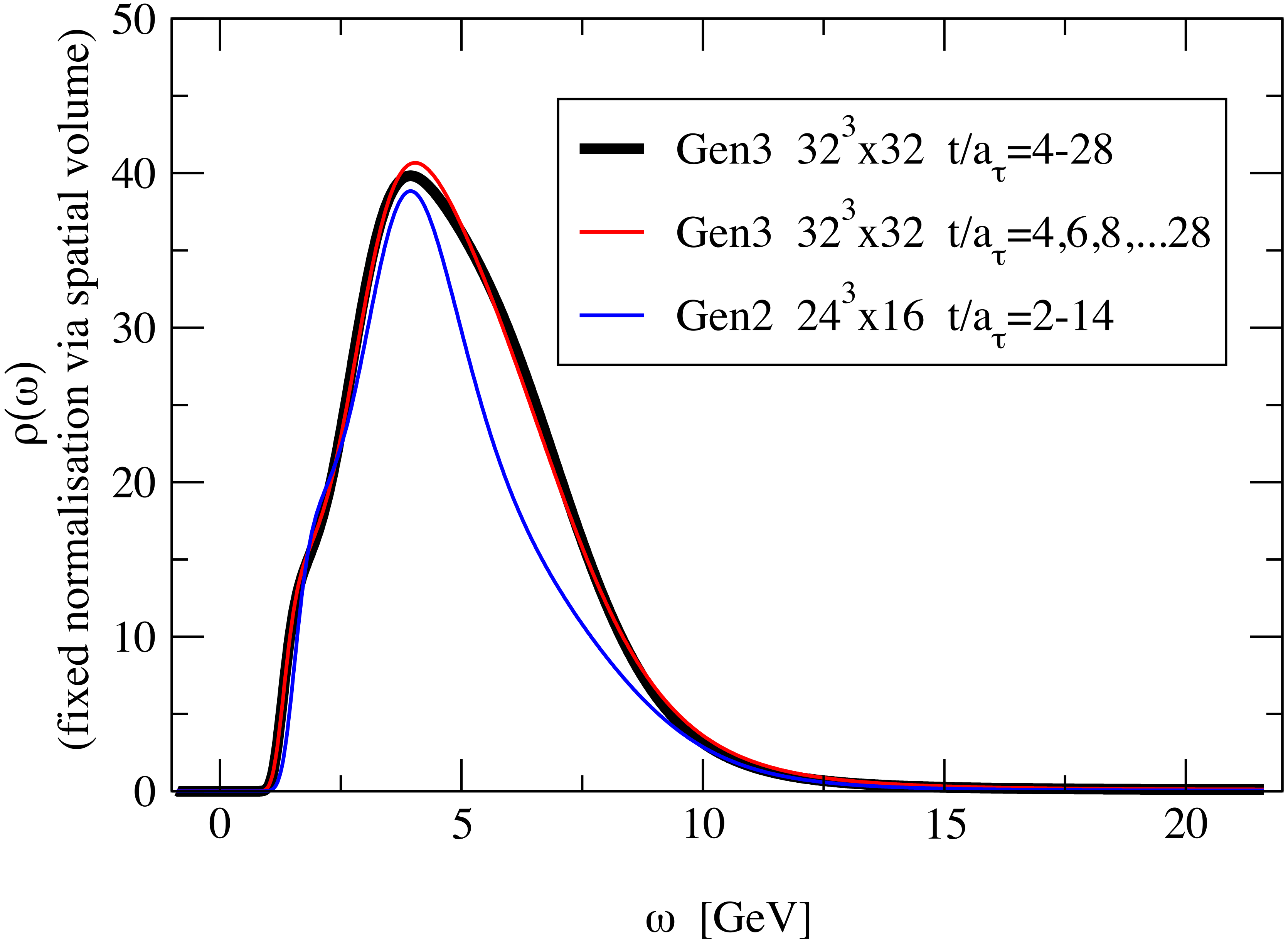
Gen3 Results



Gen2 vs Gen3 [Upsilon]
Towards (temporal) continuum limit



Gen2 vs Gen3 $[\chi_{b1}]$
Towards (temporal) continuum limit



the end…

the end…

Slides to help me answer dumb questions

Slides to help me answer tricky questions

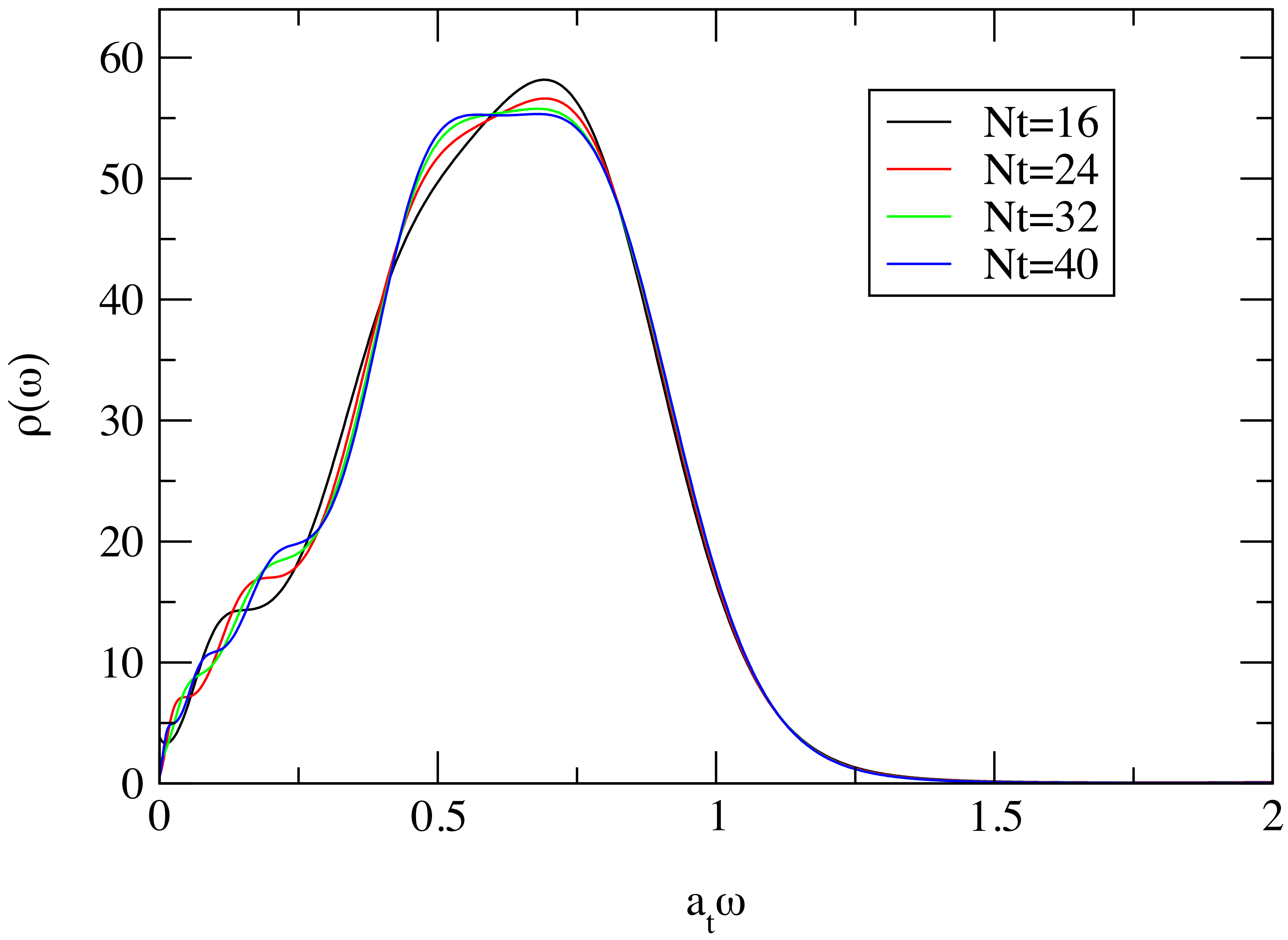
Free Lattice Correlation Functions (NRQCD)

- Use FASTSUM Collaboration's "2nd generation" parameters
- Set links $U_\mu(x) = e^{igaA_\mu(x)} \equiv 1$ i.e. *free*
- Use computer code to generate $C(t)$
- Use same stat errors and t-correlations as interacting case

Normalisation (Sum Rule) for NRQCD:

$$C(t) = \int \rho(\omega) e^{-\omega t} d\omega \quad \longrightarrow \quad C(t=0) = \int \rho(\omega) d\omega$$

N_t systematics - S-wave



MEM: more than you ever wanted to know

gen2_NRQCD_40 sonia_40_ spp_i_000 K=.00000,.00000 # 2

t = 2-38 Err=J Sym=N #cfgs= 502 #cfg/clus= 1