Color screening, quarkonium properties at T>0 and heavy quark diffusion coefficient

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Color screening at high temperature
TUMQCD, Bazavov, Brambilla, PP, Vairo, Weber, Phys. Rev. D 98 (2018) 054511

Bottomonium correlators of point meson operators in NRQCD
S. Kim, PP, A. Rothkopf, PRD91 (2015) 054511
S. Kim, PP, A. Rothkopf, JHEP11(2018) 088

Bottomonium correlators from extended meson operators in NRQCD
Larsen, Meinel, Mukherjee, PP, PRD100 (2019) 074506; PLB800 (2020) 135119

Heavy quark diffusion coefficient, TUMQCD, work in progress

Color screening of QCD force and deconfinement

$Q\bar{Q}$ interactions in the vacuum

$$V(r) \sim \sigma r, \; r \to \infty$$

$Q\bar{Q}$ interactions in QGP

$$V(r) \sim \frac{\alpha_s}{r} \exp(-m_D r), \; r \to \infty$$

For quantitative description one needs
to introduce the free energy of static $Q\bar{Q}$ pair

$$P = \frac{1}{N} \mathrm{tr} L, \; \langle P \rangle = \exp(-F_Q(T)/T)$$

Order parameter for deconfinement
if no light quarks are present

$$L(x) = \mathcal{P} \exp \left(-ig \int_0^{1/T} d\tau A_0(x, \tau) \right)$$

$$\exp(-F_{Q\bar{Q}}(r,T)/T) = \frac{1}{9} \langle \mathrm{tr} L(r) \mathrm{tr} L^\dagger(0) \rangle$$

free energy of $Q\bar{Q}$ pair

$$F_{Q\bar{Q}} \sim \exp(-Mr)/r^d$$

Singlet free energy:

$$\exp(-F_S(r,T)/T) = \frac{1}{3} \mathrm{tr} \langle L(r) L^\dagger(0) \rangle$$

$$F_S^{LO}(r,T) = -C_F \frac{\alpha_s}{r} \exp(-m_D r) - C_F \alpha_s m_D$$

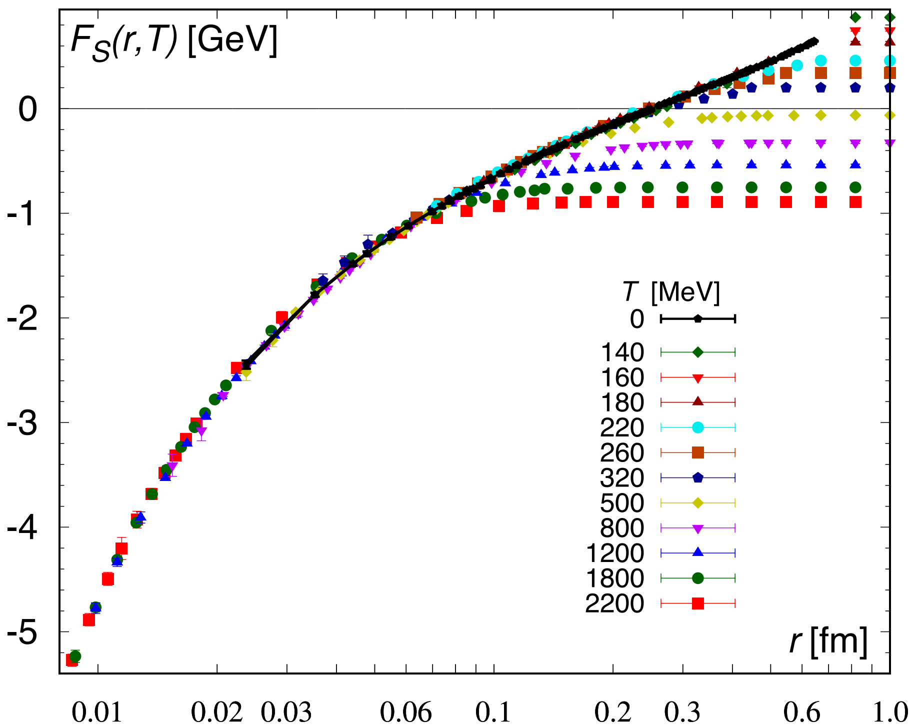
Coulomb gauge

These quantities needs renormalization but it is the same
as for the $T = 0$ potential

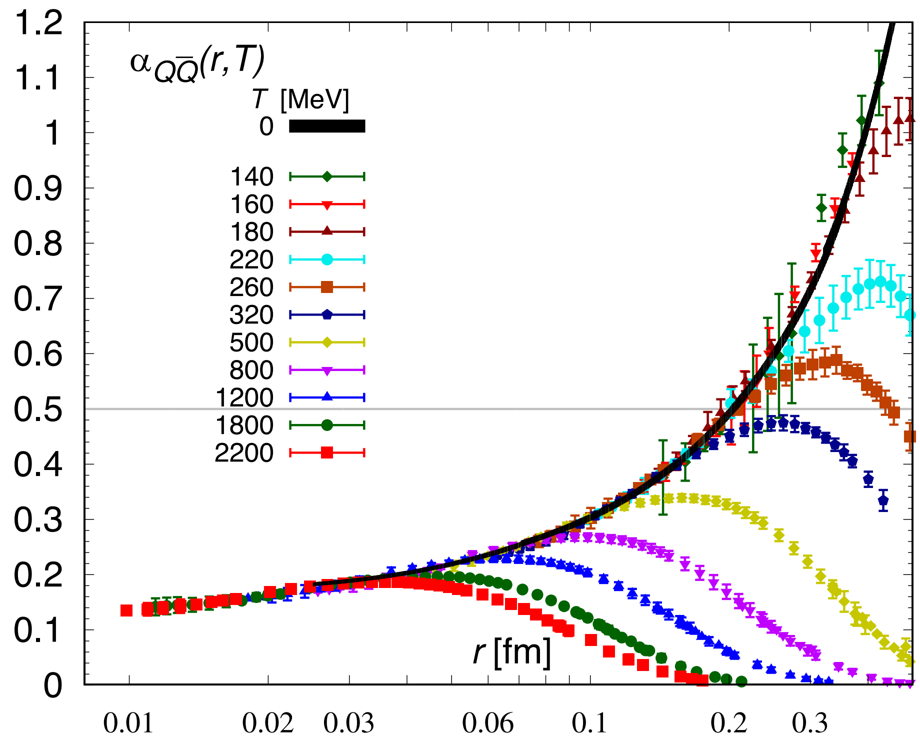
Singlet free energy of static quark anti-quark and screening

Continuum extrapolated lattice results:

singlet $Q\bar{Q}$ free energy, $F_S(r,T)$



effective coupling $\alpha_{Q\bar{Q}} = r^2 \dfrac{\partial F_S(r,T)}{\partial r}$

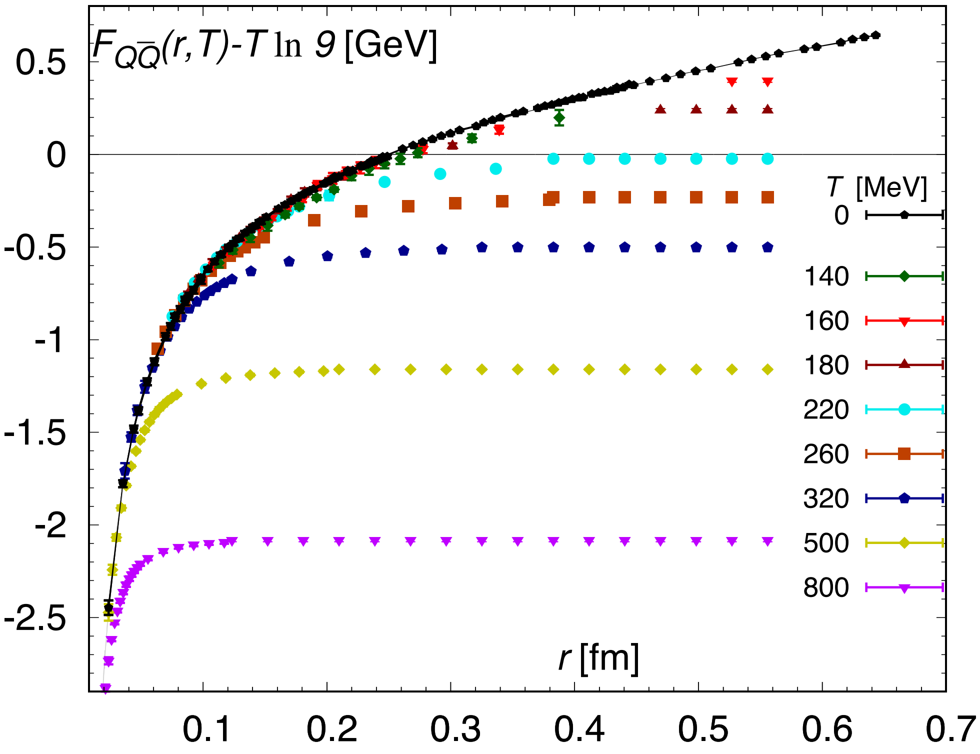


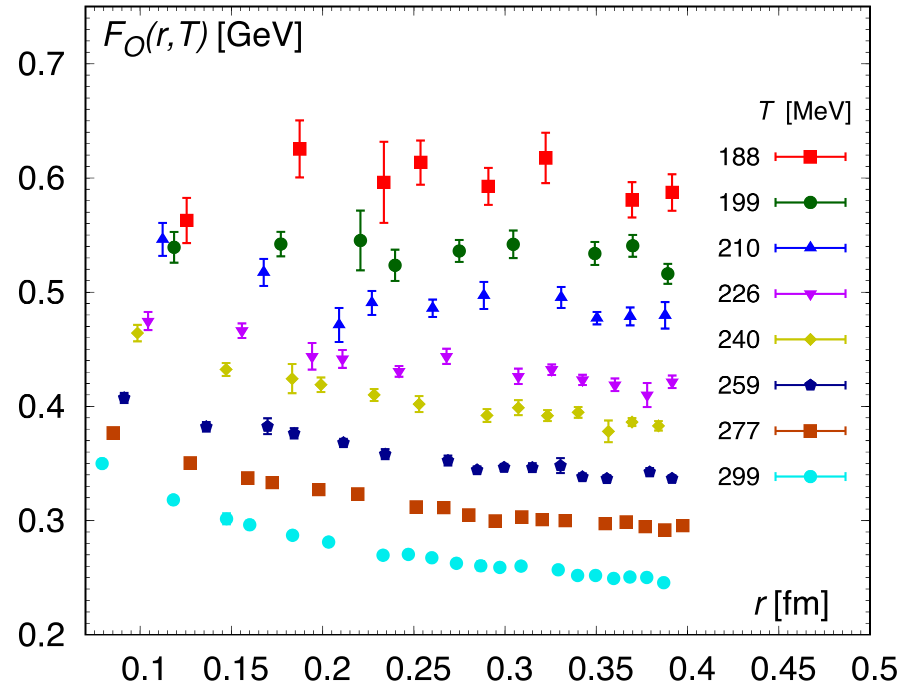
- At short distances, $r < 0.4/T$ the $Q\bar{Q}$ interaction is like at $T = 0$
- $\alpha_{Q\bar{Q}}(r,T)$ has the maximum at $r \simeq 0.4/T$: $\alpha_{Q\bar{Q}}(r \simeq 0.4/T) = \alpha_{Q\bar{Q}}^{max}$
- $\alpha_{Q\bar{Q}}^{max} > 0.5 \Rightarrow$ QGP is strongly coupled for $T < 300$ MeV

Free energy of static quark anti-quark pair

For $rT \ll 1$ potential non-relativistic QCD (pNRQCD) can be used

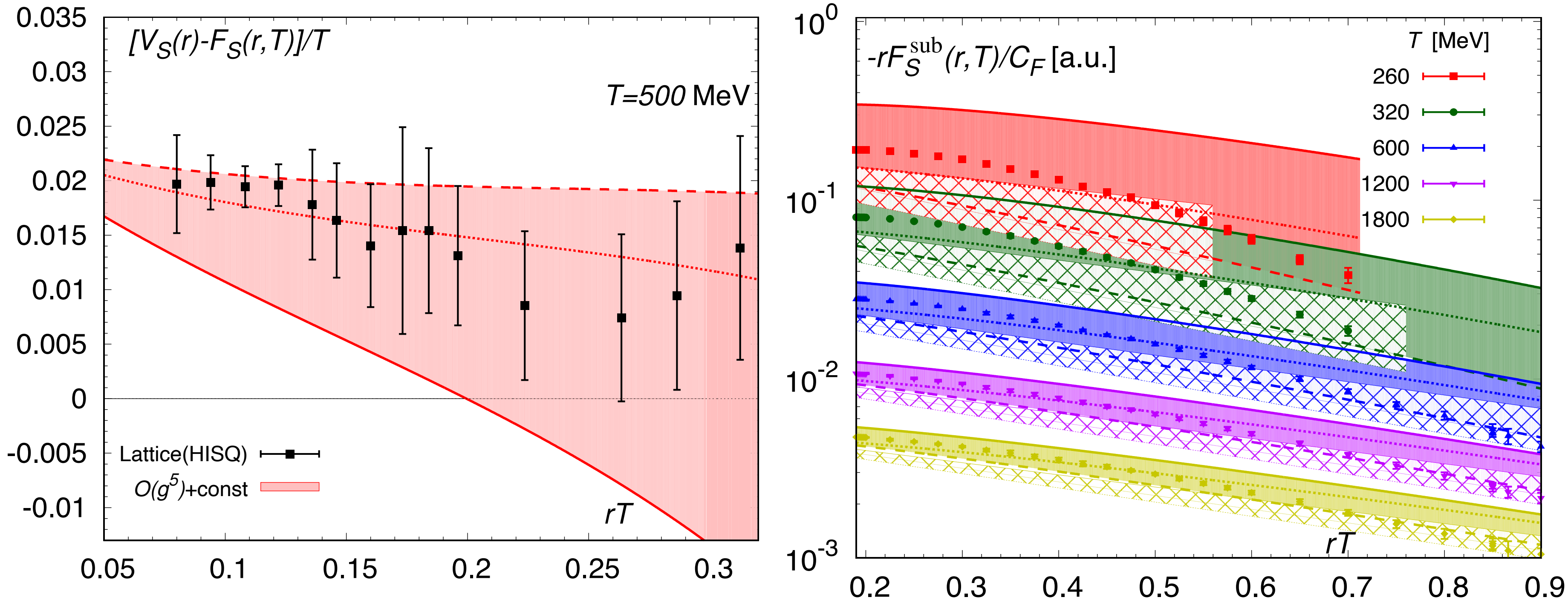
$$e^{-F_{Q\bar{Q}}(r,T)/T} = \frac{1}{9} e^{-V_s(r)/T} + L_8 \frac{8}{9} e^{-V_o(r)/T}$$





Large temperature dependence even at very short distances is due to color octet contribution

Free energy of static quark anti-quark in weak coupling

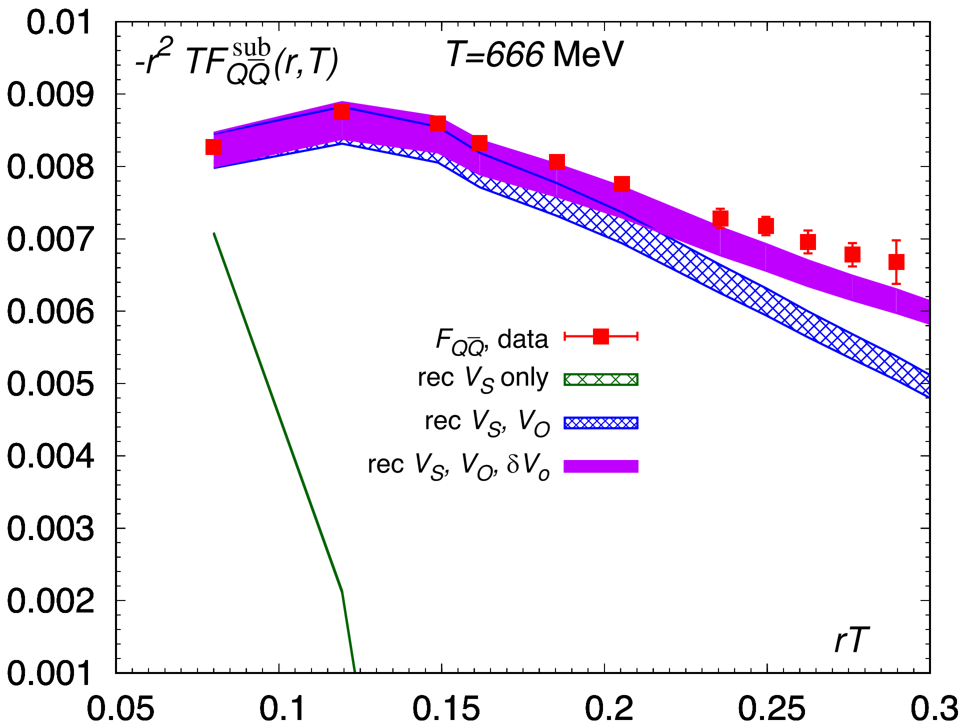


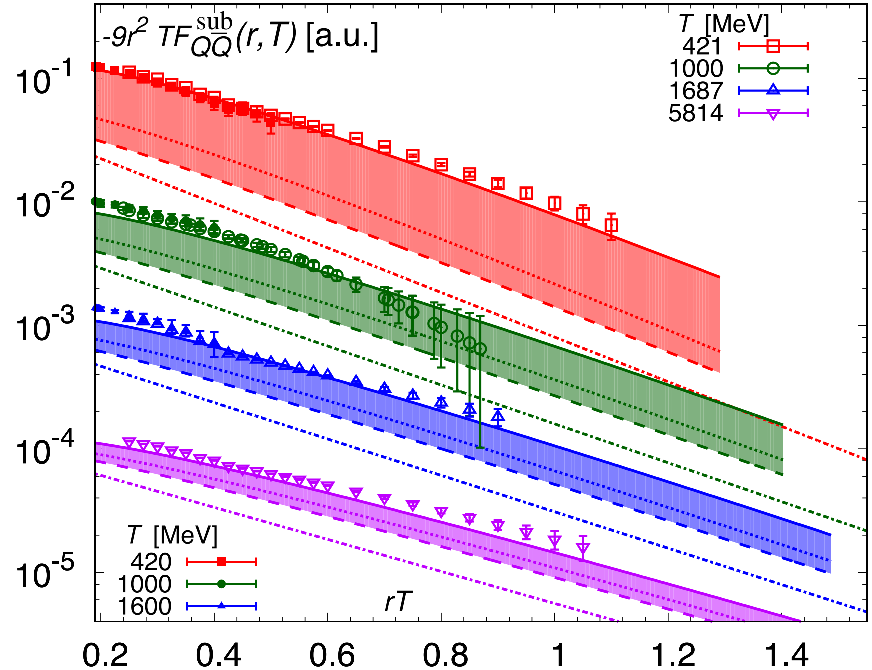
The free energy of quark anti-quark pair at short distances can be descrtibed by the weak coupling results within *T=0* pNRQCD

The singlet free energy at large distances can be understood in terms of weak coupling calculations in EQCD

Burnier, Laine,Vepsalainen, JHEP 1001 (2010) 054

Free energy of static quark anti-quark in weak coupling





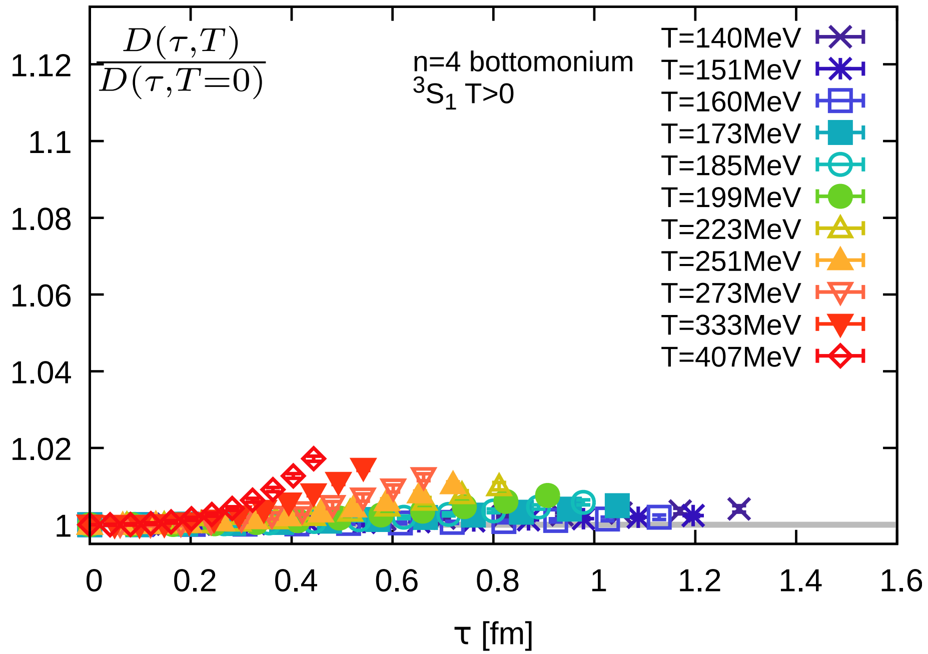
The free energy of quark anti-quark pair at short distances can be descrtibed by the weak coupling results within T=0 pNRQCD

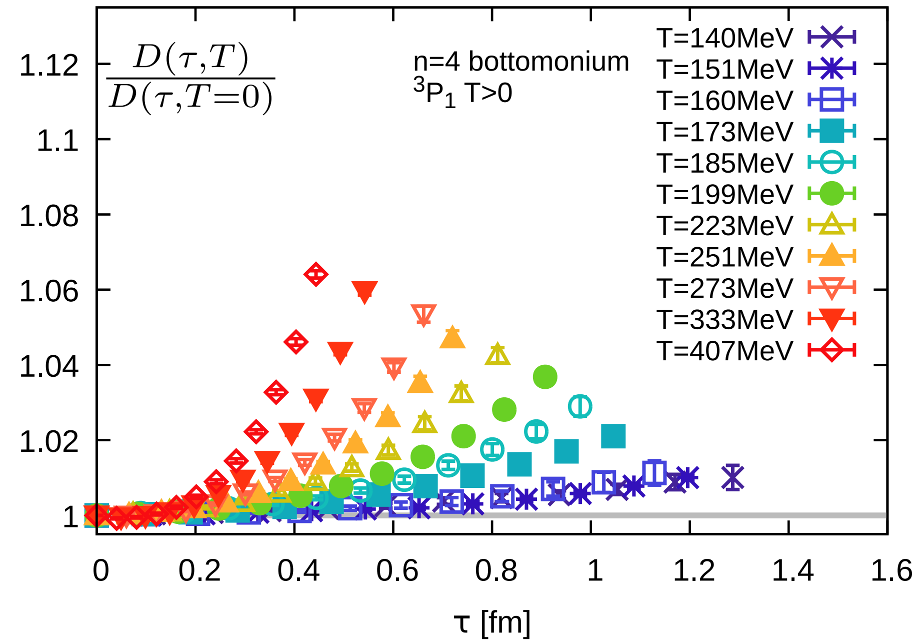
$$e^{-F_{Q\bar{Q}}(r,T)/T} = \frac{1}{9} e^{-V_s(r)/T} + L_8 \frac{8}{9} e^{-V_o(r)/T}$$

Brambilla, Ghiglieri, PP, Vairo, PRD82 (2010) 074019

The free energy at large distances can be understood in terms of weak coupling calculations in EQCD

Temperature Dependence of the Bottomonium Correlators





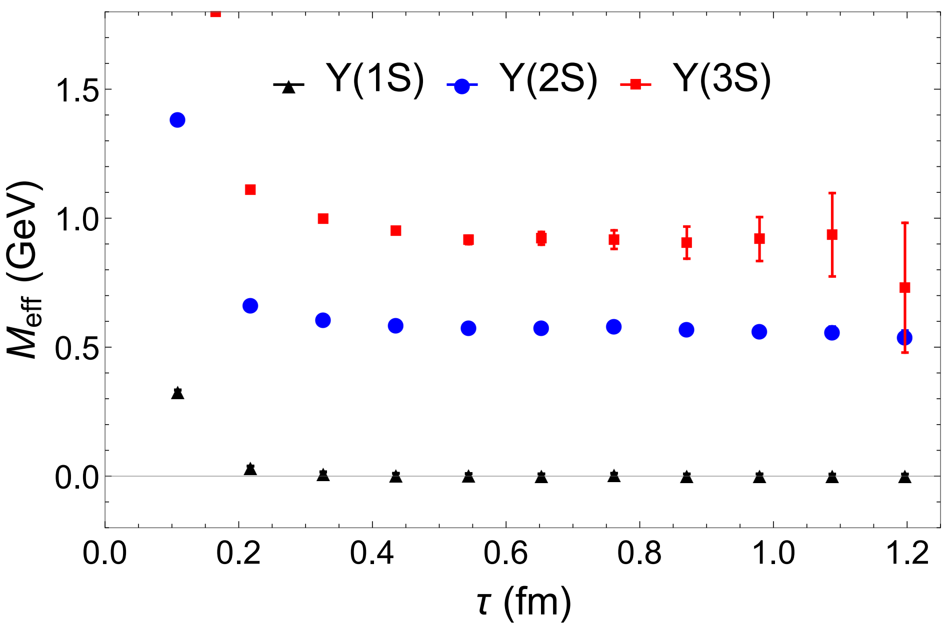
change in Υ correlator $< 2\%$

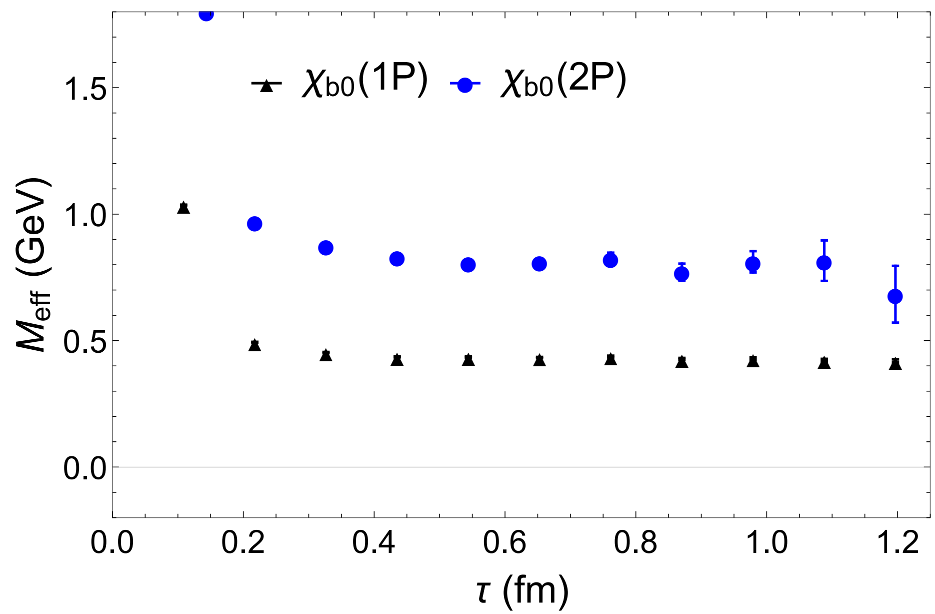
change in χ_{b1} correlator $< 7\%$

$\Rightarrow$ hints for sequential melting pattern: stronger medium modification of χ_{b1} spectral function than for Υ spectral function

Correlators of Extended Meson Operators at T=0

$$aM_{\text{eff}}(t) = \ln[C_\alpha(t)/C_\alpha(t+1)]$$





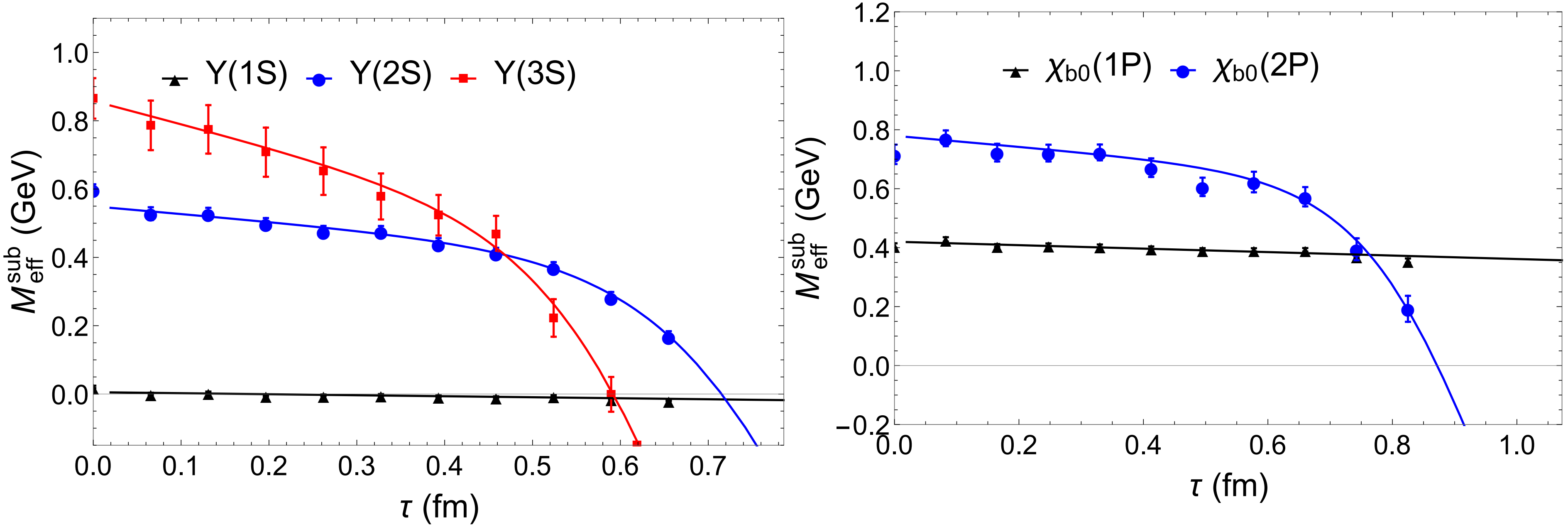
$$C_\alpha(\tau, T) = \int_{-\infty}^{\infty} d\omega \rho_\alpha(\omega, T) e^{-\omega\tau} \qquad \rho_\alpha(\omega, T) = \rho_\alpha^{\text{med}}(\omega, T) + \rho_\alpha^{\text{high}}(\omega)$$

$$\rho_\alpha^{\text{med}}(\omega, T=0) = A_\alpha \delta(\omega - M_\alpha) \Rightarrow C_\alpha(\tau, T=0) = A_\alpha e^{-M_\alpha \tau} + C_\alpha^{\text{high}}(\tau)$$

Determine A_α,M_α from single exponential fit for $\tau > 0.6\text{fm}$ and then $C_\alpha^{\text{high}}(\tau)$

Correlators of Extended Meson Operators at T>0

$$C_\alpha^{\rm sub}(\tau,T) = C_\alpha(\tau,T) - C_\alpha^{\rm high}(\tau) \Rightarrow aM_{\rm eff}^{\rm sub}(\tau,T) = \ln\left(C_\alpha^{\rm sub}(\tau,T)/C_\alpha^{\rm sub}(\tau+a,T)\right)$$



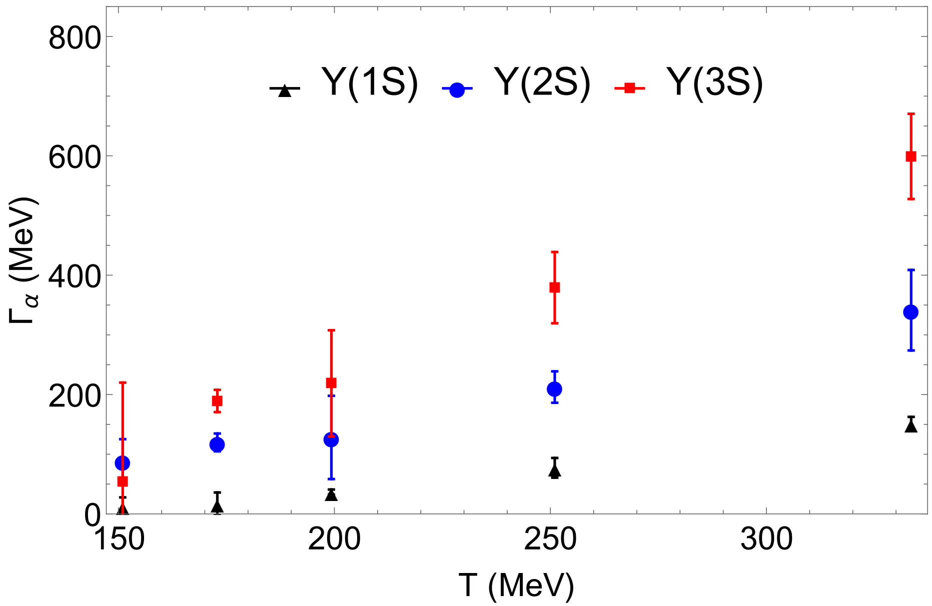
Fit $M_{\rm eff}^{\rm sub}(\tau,T)$ using a simple Ansatz:

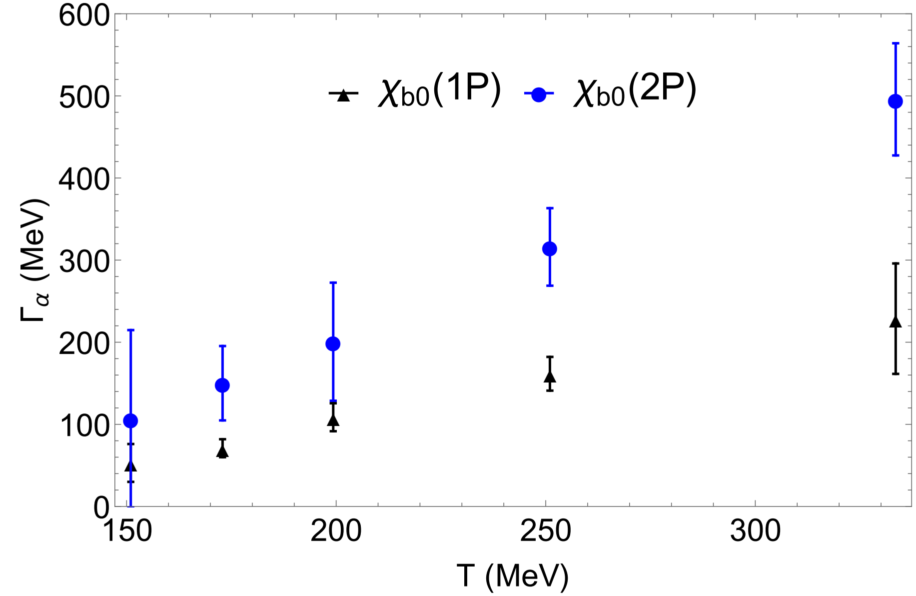
$$\rho_\alpha^{\rm med}(\omega,T) = A_\alpha^{\rm cut}(T)\,\delta\left(\omega-\omega_\alpha^{\rm cut}(T)\right) + A_\alpha(T)\exp\left(-\frac{[\omega-M_\alpha(T)]^2}{2\Gamma_\alpha^2(T)}\right)$$

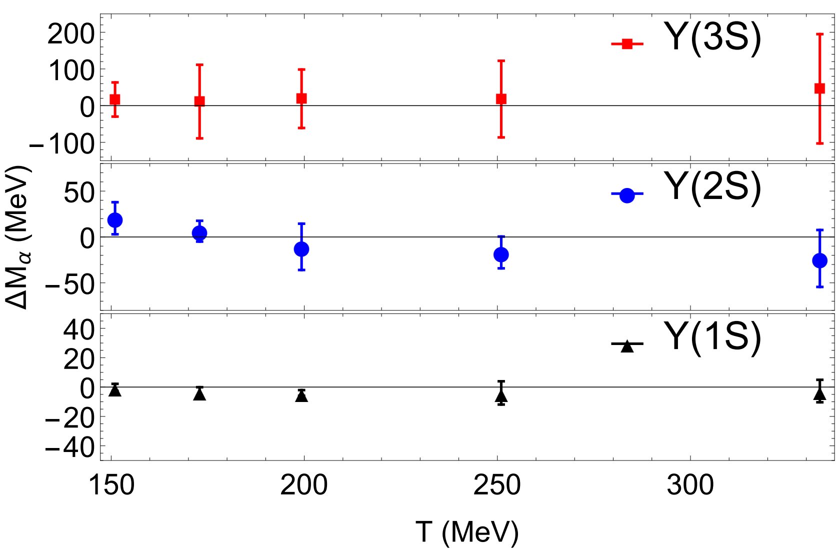
Low energy tail

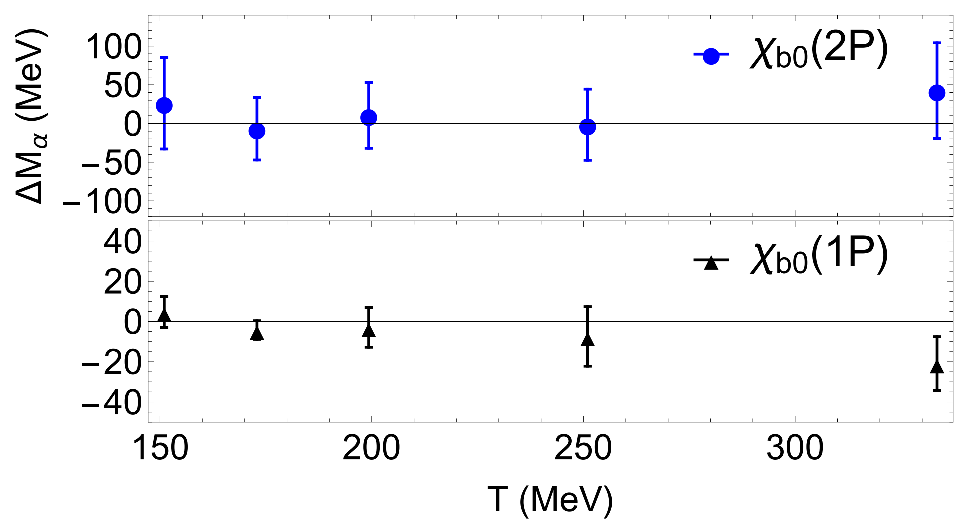
$$\Rightarrow M_\alpha(T), \Gamma_\alpha(T)$$

Thermal width and mass shift of bottomonium









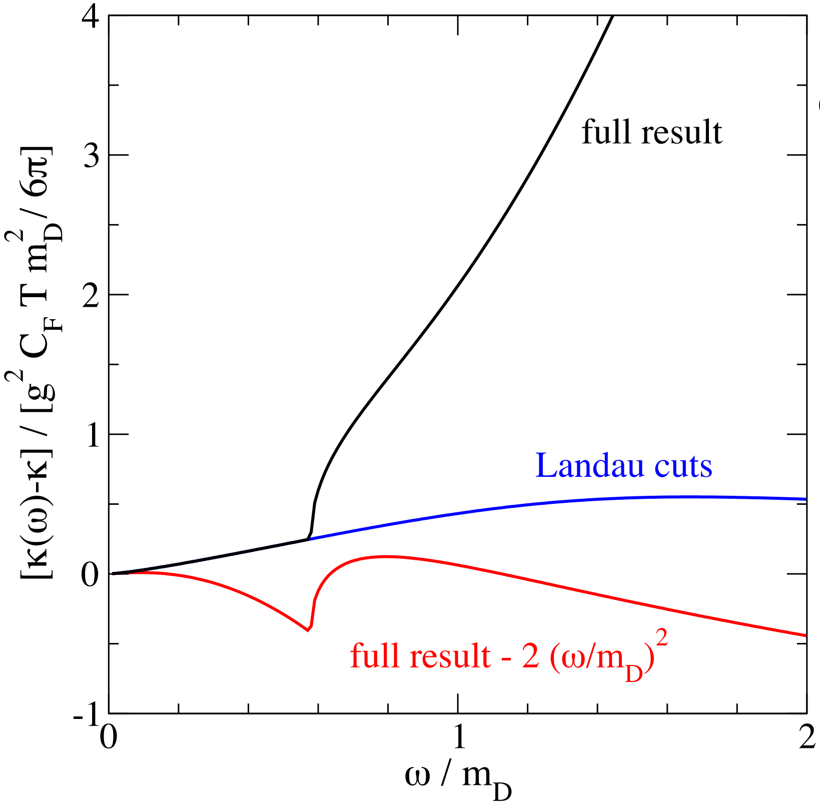
Current-current correlators in the heavy quark limit

$$\kappa = \frac{1}{3T}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty}\frac{M^2}{\chi_2^q}\int_{-\infty}^{\infty}\mathrm{d}t\, e^{i\omega(t-t')}\int \mathrm{d}^3\vec{x}\left\langle \frac{1}{2}\left\{\frac{\mathrm{d}\hat{J}^i(t,\vec{x})}{\mathrm{d}t},\frac{\mathrm{d}\hat{J}^i(t',\vec{0})}{\mathrm{d}t'}\right\}\right\rangle\right]$$

$$\frac{\mathrm{d}\hat{J}^i}{\mathrm{d}t} = \frac{1}{M}\left\{\hat{\phi}^\dagger gE^i\hat{\phi} - \hat{\theta}^\dagger gE^i\hat{\theta}\right\} + \mathcal{O}\Big(\frac{1}{M^2}\Big)$$

$t \to i\tau$

$$G_E(\tau) = \frac{1}{3\chi_2^q T}\sum_i\int \mathrm{d}^3x\Big\langle\Big[\phi^\dagger gE_i\phi - \theta^\dagger gE_i\theta\Big](\tau,\vec{x})\,\Big[\phi^\dagger gE_i\phi - \theta^\dagger gE_i\theta\Big](0,\vec{0})\Big\rangle$$



Integrate out ϕ, θ

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\Big\langle \mathrm{ReTr}\Big[U(\beta,\tau)\,gE_i(\tau,\vec{0})\,U(\tau,0)\,gE_i(0,\vec{0})\Big]\Big\rangle}{\Big\langle \mathrm{ReTr}[U(\beta,0)]\Big\rangle}$$

$$G_E(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega)\frac{\cosh\left(\tau-\frac{1}{2T}\right)\omega}{\sinh\frac{\omega}{2T}}$$

Transport coefficient ~ intercept of the spectral function not its width

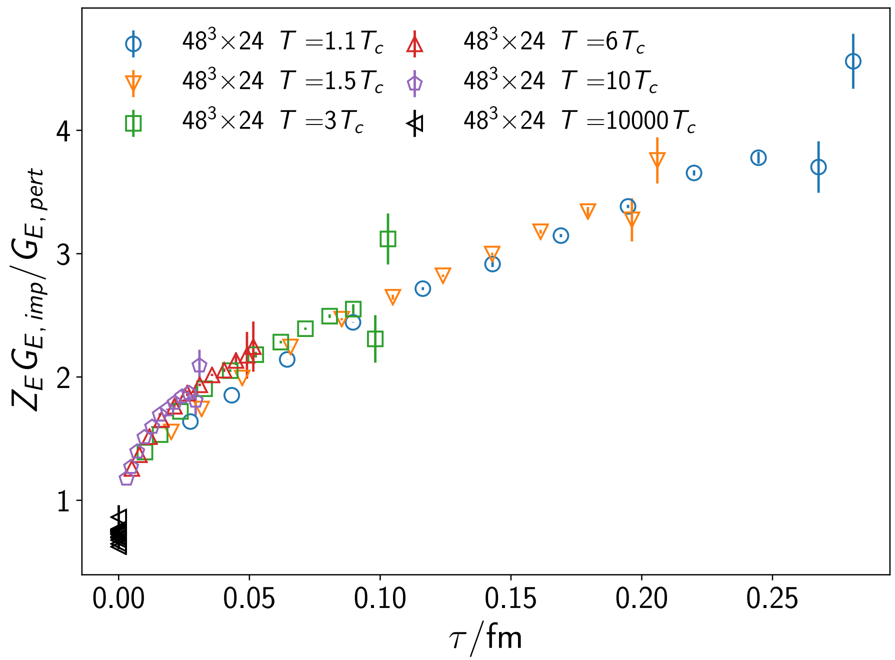
$$\kappa = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega)$$

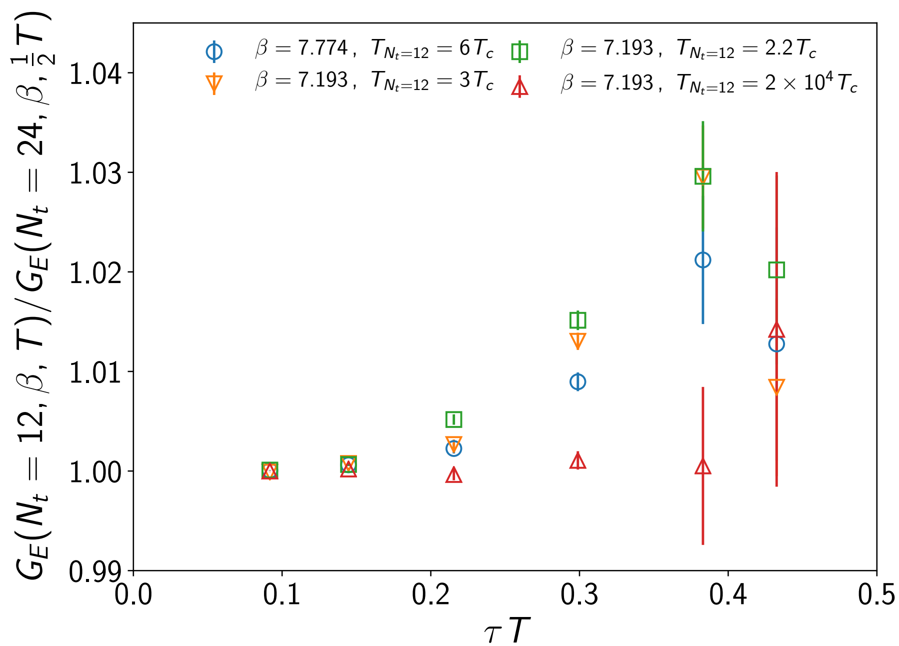
Caron-Huot, Laine, Moore, JHEP 0904 (2009) 053

Temperature dependence of the electric correlator

Brambilla, Leino, PP, Vairo, arXiv:1912.00689

The temperature dependence is very small

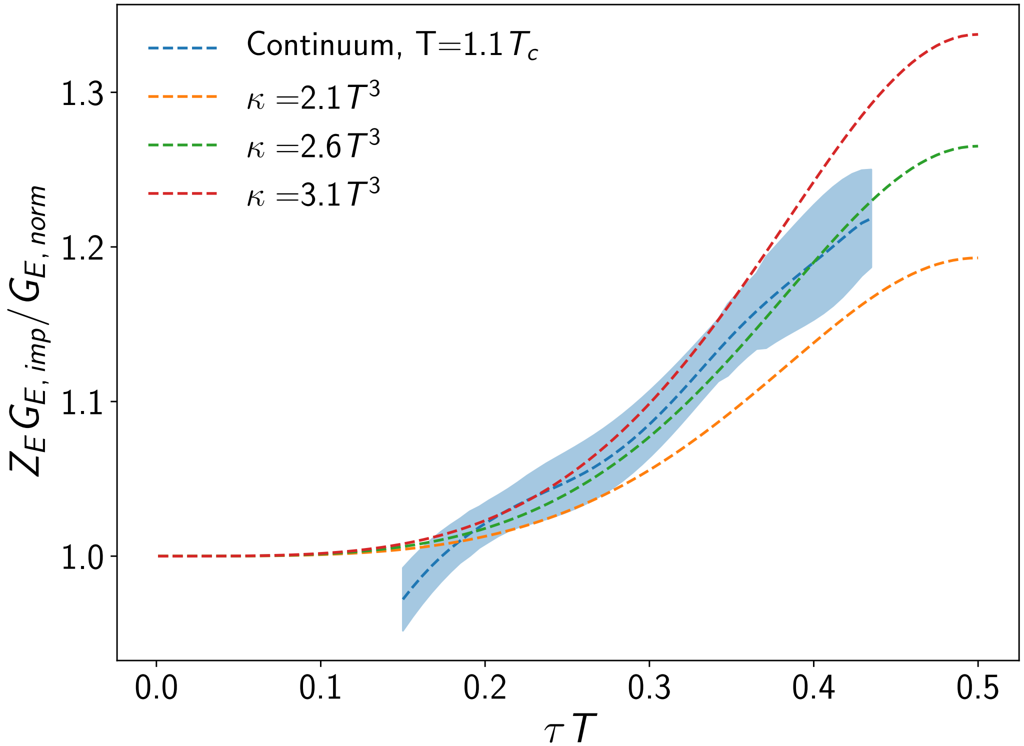




Fitting the electric correlator

Brambilla, Leino, PP, Vairo (TUMQCD) , arXiv:1912.00689

Take into account the running of the strong coupling constant and include the linear part at small energy



Preliminary estimates:

$$2.28 < \frac{\kappa}{T^3} < 3.57 \text{ for } \quad T = 1.1T_c\,,$$
$$1.99 < \frac{\kappa}{T^3} < 2.69 \text{ for } \quad T = 1.5T_c\,,$$
$$1.05 < \frac{\kappa}{T^3} < 2.26 \text{ for } \quad T = 3T_c\,,$$
$$0 < \frac{\kappa}{T^3} < 1.5 \text{ for } \quad T = 6T_c\,,$$
$$0 < \frac{\kappa}{T^3} < 0.91 \text{ for } \quad T = 10T_c\,,$$
$$0 < \frac{\kappa}{T^3} < 0.39 \text{ for } \quad T = 10^4T_c\,.$$

Summary

- Bottomonium spectral functions of point meson operators are dominated by the continuum and therefore the corresponding Euclidean time correlation functions show small *T*-dependence:

$\Rightarrow$ Difficult to reconstruct in-medium bottomonium properties especially for P-states
$\Rightarrow$ No sensitivity to *2S* and *3S* bottomonium states

- Using appropriately chosen extended meson operators it is possible to reduce the relative contribution of continuum part of the spectral function and reconstruct the spectral functions for *1S*, *2S*, *3S*, *1P* and *2P* bottomonium states

The correlators of extended meson operators show significant *T*-dependence consistent with thermal broadening

Using simple form for the bottomonium spectral function we can extract the thermal widths of different bottomonium states and we see that

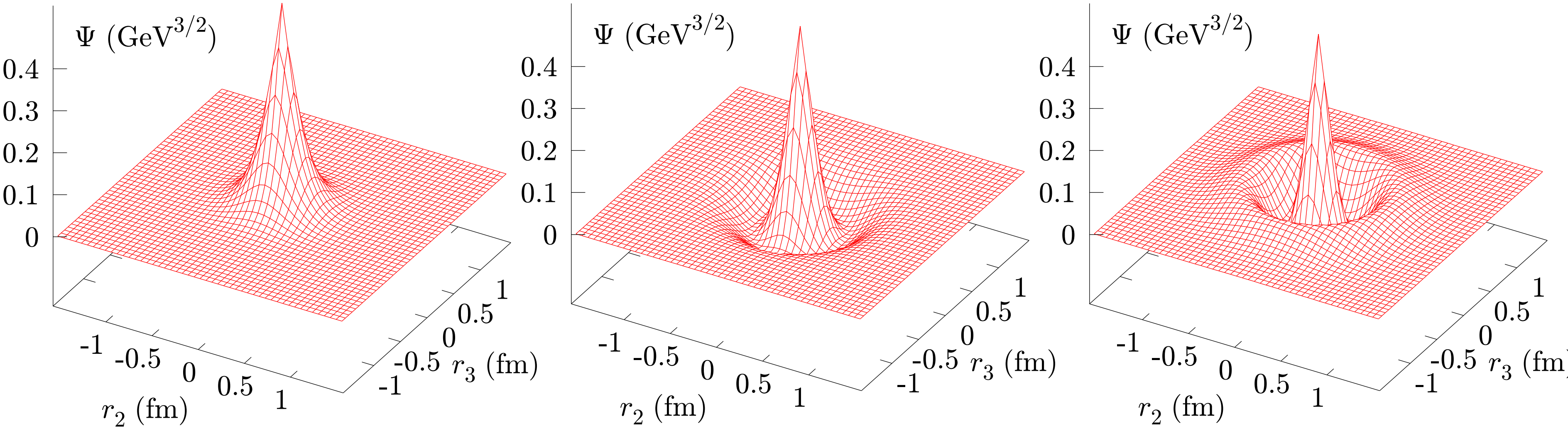
$$\Gamma_{3S}(T) > \Gamma_{2P}(T) > \Gamma_{2S}(T) > \Gamma_{1P}(T) > \Gamma_{1S}(T)$$

The momentum heavy quark diffusion constant has significant T-dependence and at high temperature is consistent with the weak coupling results

Backup slides

Extended Meson Operators

$O_i(\mathbf{x},t) = \sum_{\mathbf{r}} \Psi_i(\mathbf{r})\bar{q}(\mathbf{x}+\mathbf{r},t)\Gamma q(\mathbf{x},t)\,.$ $\Psi_i(\mathbf{r})$ from potential model with Cornell potential



Good overlap with bottomonium states but

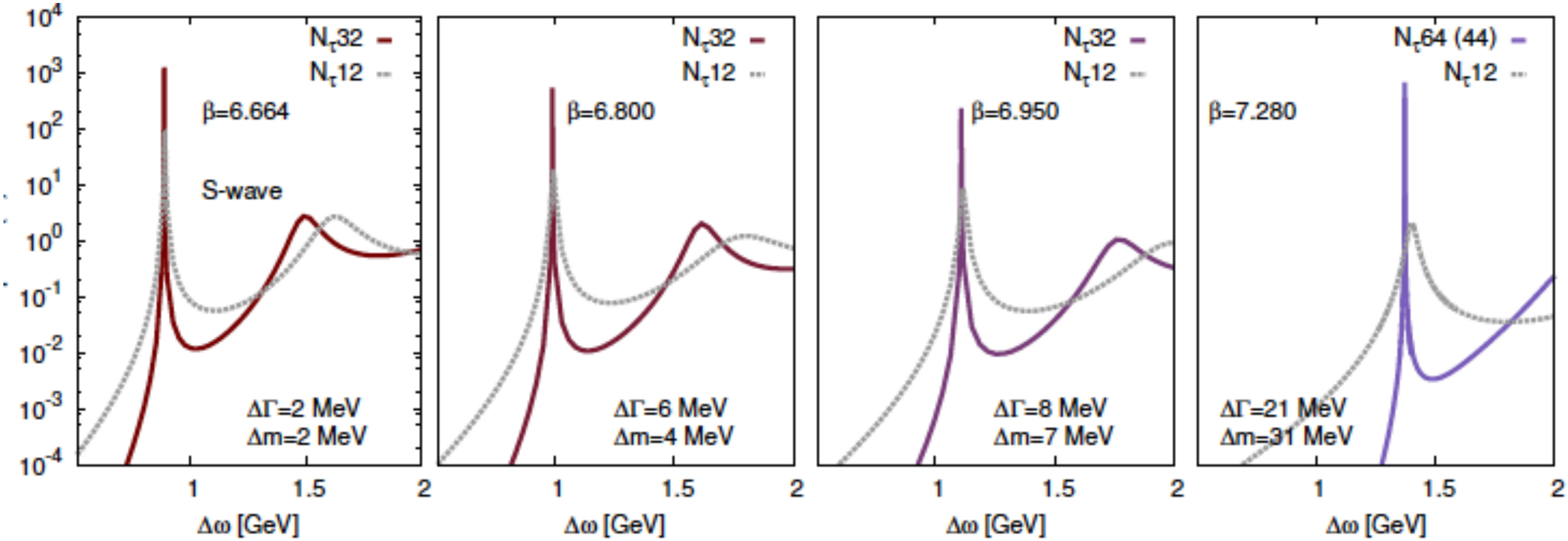
$G_{ij}(t) = \langle O_i(t) O_j^\dagger(0)\rangle \neq 0$ for $i \neq j$

$O_i \to \tilde{O}_\alpha = \Omega_{\alpha j} O_j$ such that
$O_\alpha(t)\tilde{O}_\beta^\dagger(0)\rangle \propto \delta_{\alpha,\beta}$
$\Omega_{\alpha j}$ can be obtained as
$G_{ij}(t)\Omega_{\alpha j} = \lambda_\alpha(t,t_0) G_{ij}(t_0)\Omega_{\alpha j}.$

Reconstructing Spectral Functions at T>0

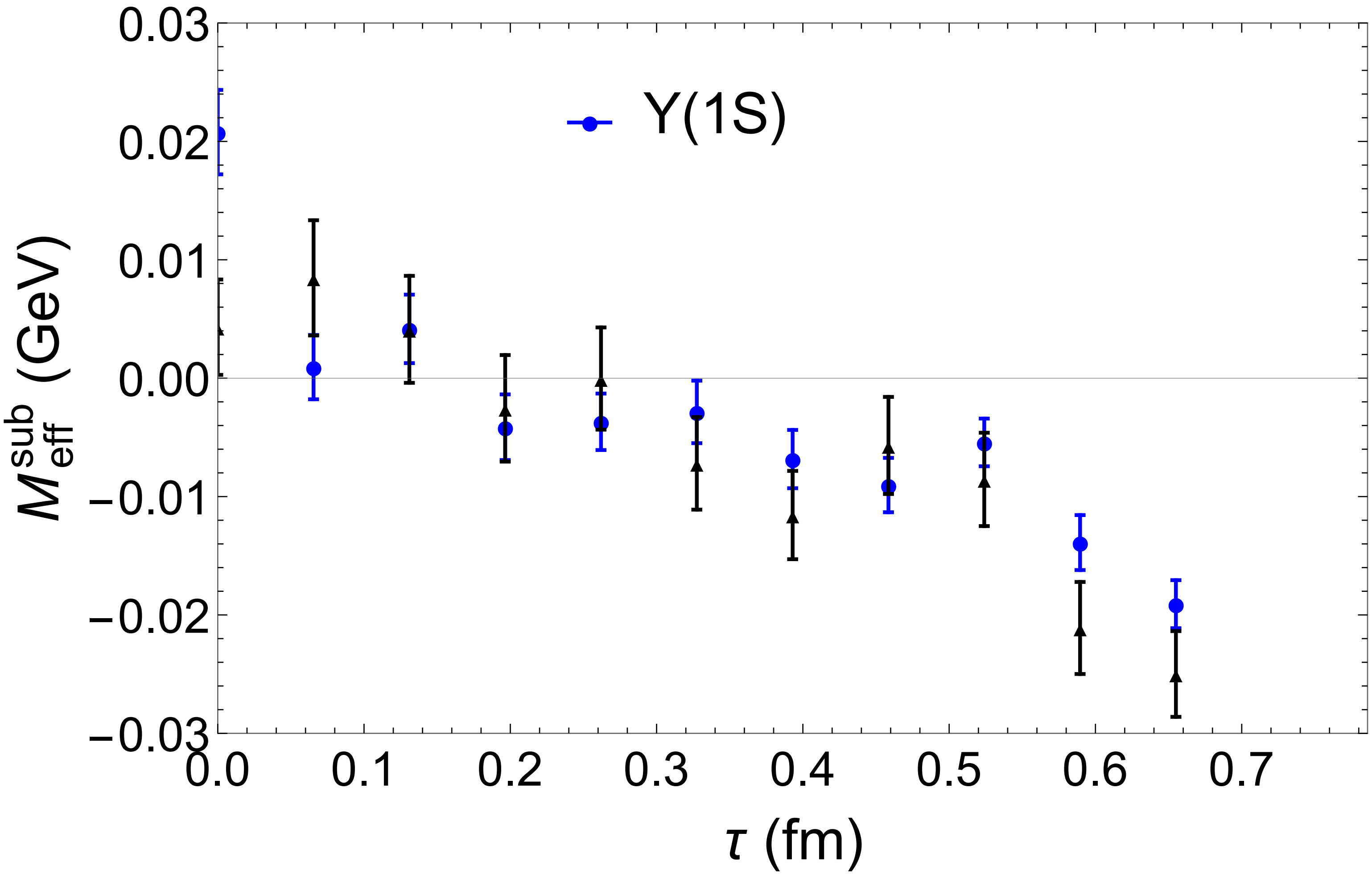
Two main problems:
1) $\tau < 1/T \Rightarrow$ limited temporal extent at high T
2) relatively small number of time slices ($N_\tau = 12$ in our study)

Study these effects at $T = 0$ by using only the first 12 data points:



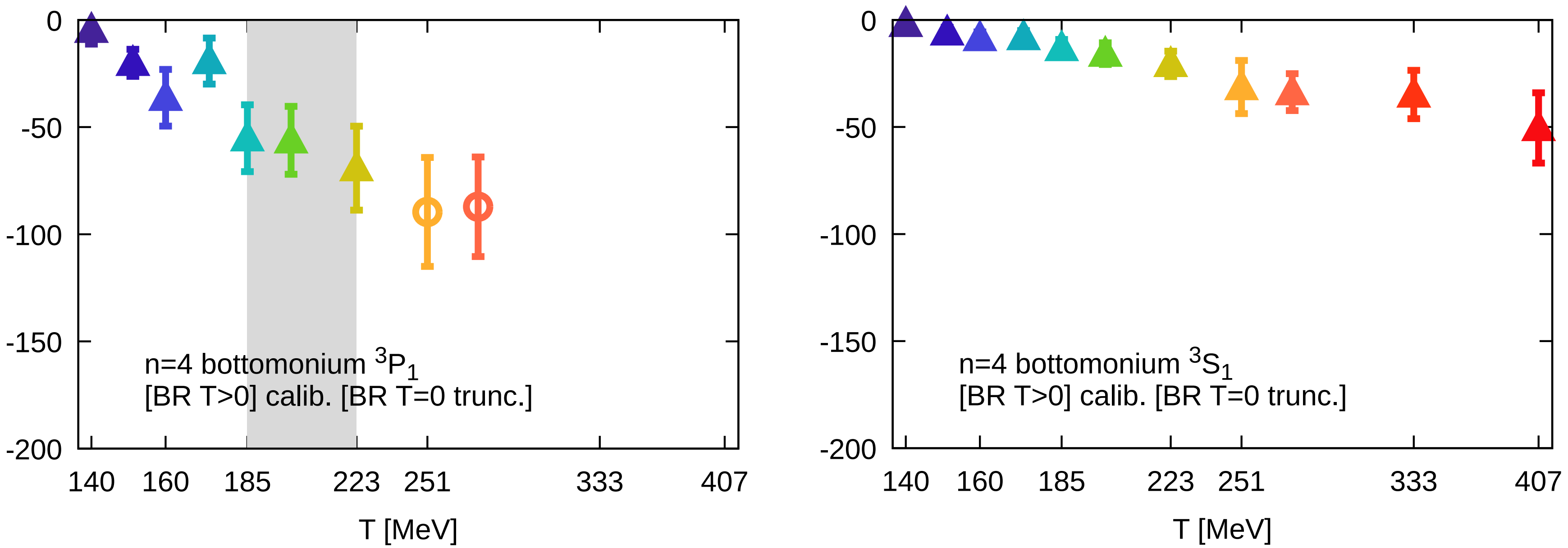
Decreasing $\tau_{max} = 1/T$ leads to broadening of the bound state peak

(to be taken into account in comparison $T = 0$ and $T > 0$ spectral functions)

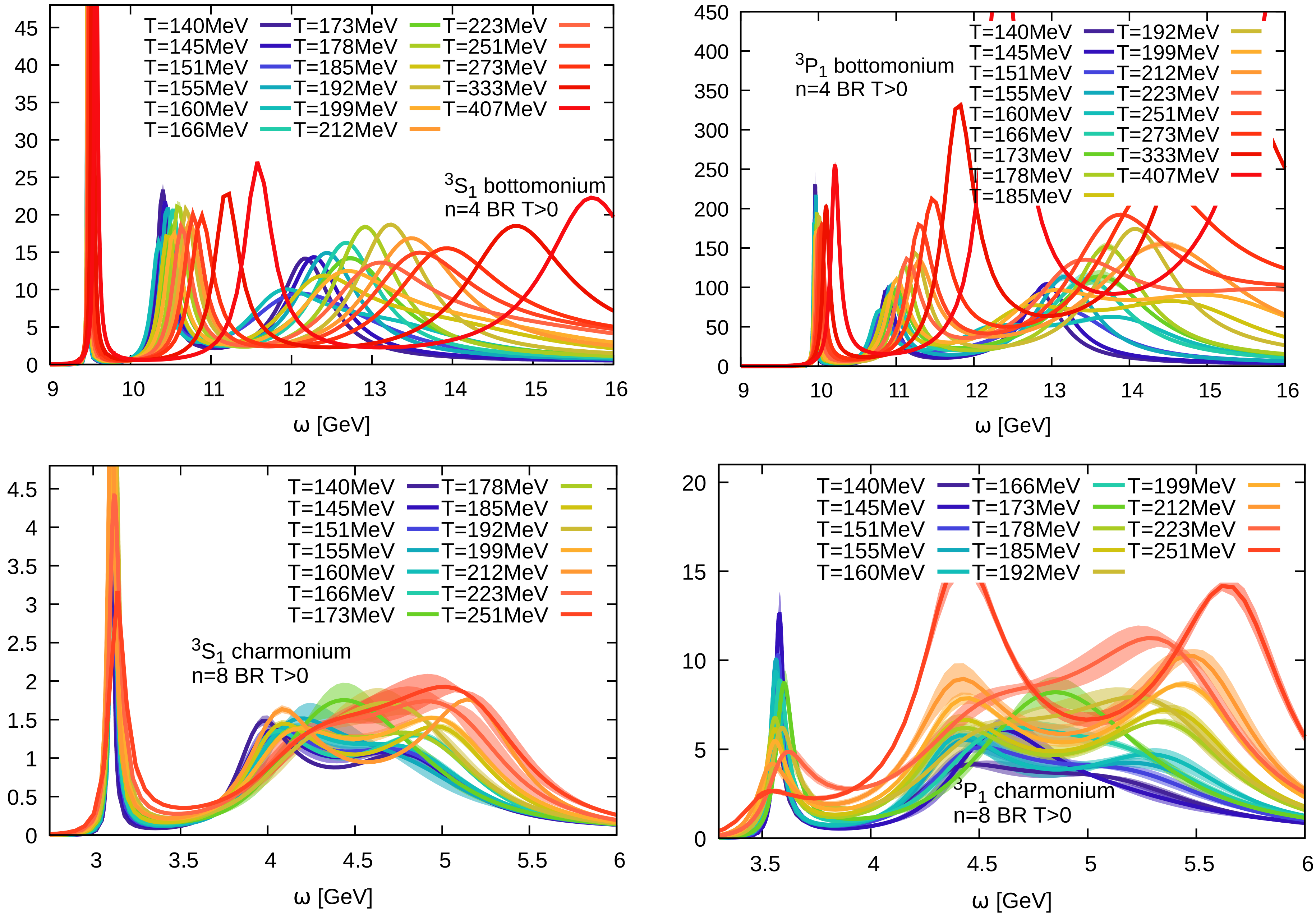


Υ(1S)
$M_{\rm eff}^{\rm sub}$ (GeV)
τ (fm)
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

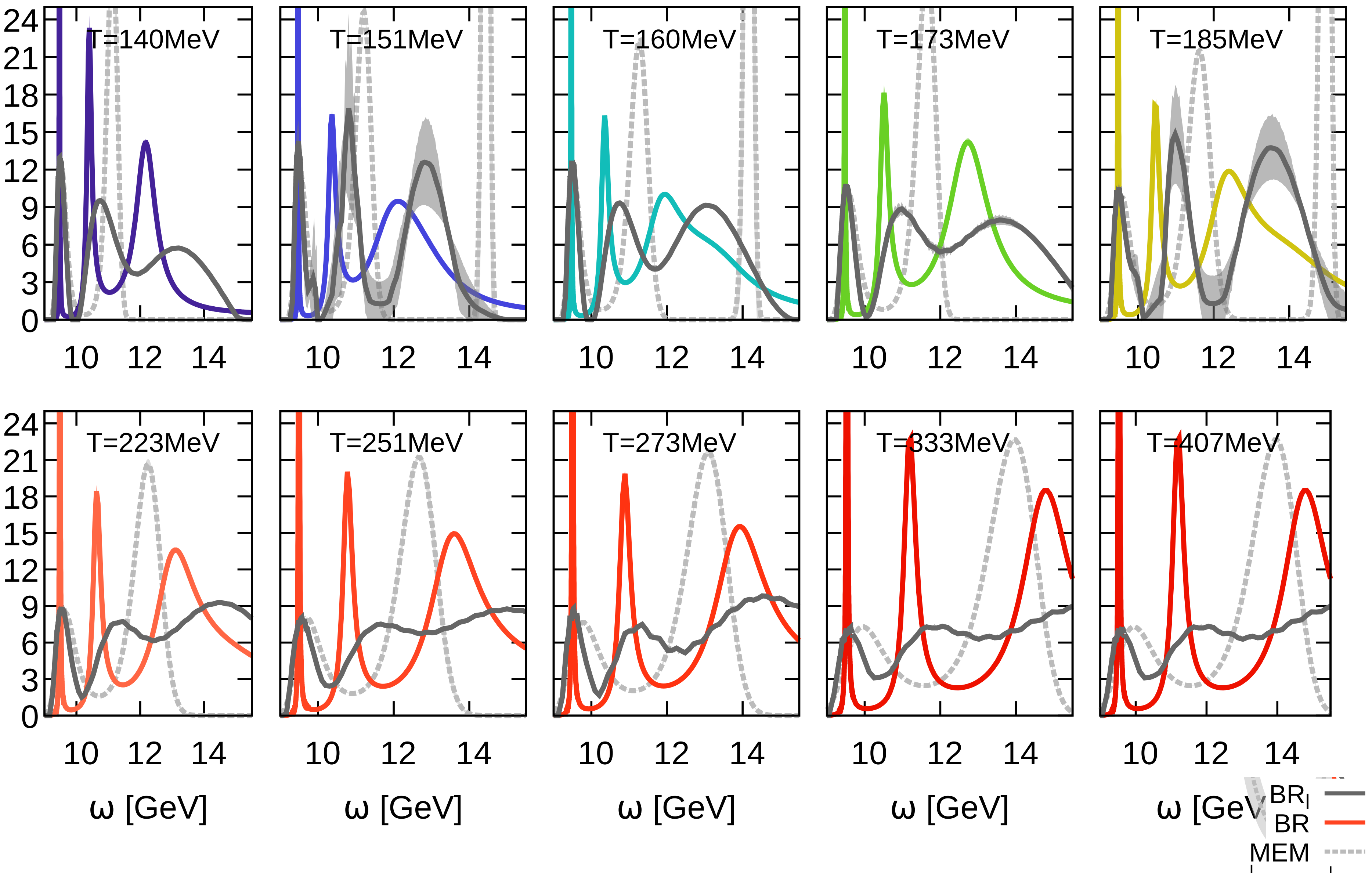
In Medium Bottomonium Mass Shifts
From Bayesian analysis



Quarkonium Spectral Functions at T>0

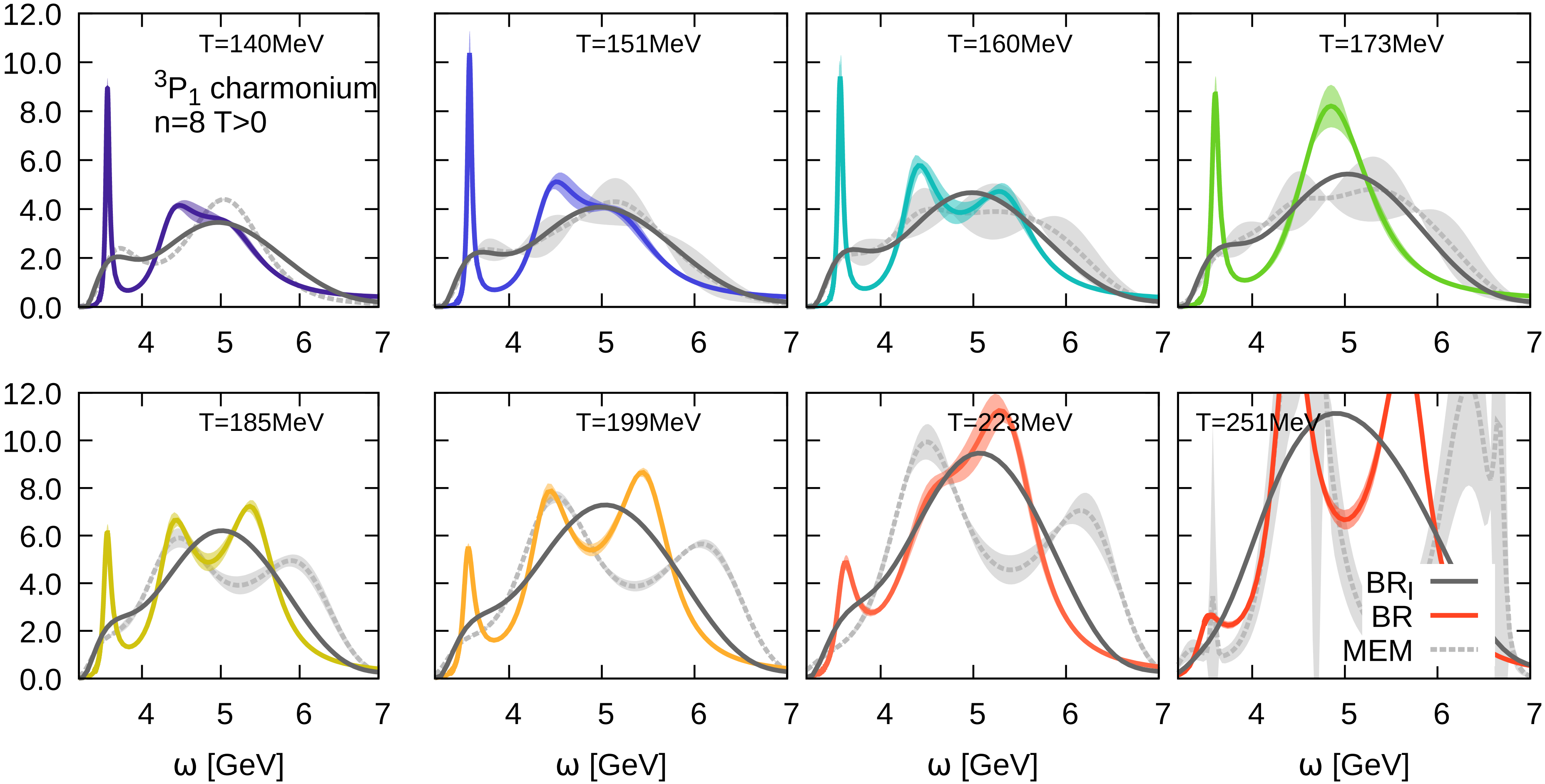


Bottomonium Spectral Functions with Different Methods



Υ spectral functions

Charmonium Spectral Functions with Different Methods



χ_c spectral functions