Breakup-rate calculation in the KSU approach

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Suppression and (re)generation of quarkonium in heavy-ion collisions at the LHC

December 17, 2019

KENT STATE UNIVERSITY

U.S. DEPARTMENT OF ENERGY

Anisotropic QGP

- If the shear viscosity is non-zero, the **longitudinal expansion** of the QGP makes the one-particle distribution function **momentum-space anisotropic** (local rest frame).

- In **second-order viscous hydrodynamics** this momentum anisotropy is encoded in the shear viscous correction, $\pi^{\mu\nu}$. For classical statistics one has, e.g.

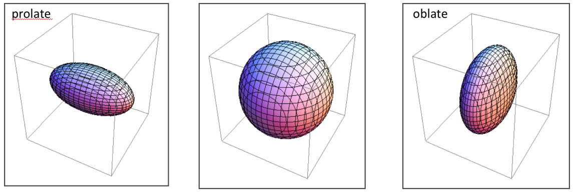
$$f(x,p) = f_{\rm eq}\!\left(\frac{p^\mu u_\mu}{T}\right)\left[1 + \frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2(\mathcal{E}+\mathcal{P})T^2}\right]$$

- Alternatively, in **anisotropic hydrodynamics** one encodes the momentum-space anisotropy in the tensor $\Xi^{\mu\nu}$ [generalized Romatschke-Strickland (RS) form]
  M. Alqahtani, M. Nopoush, and MS, 1712.03282

$$f(x,p) = f_{\rm eq}\!\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)}, \frac{\mu(x)}{\lambda(x)}\right) + \delta\tilde{f}(x,p)$$

- In 0+1d and assuming zero chemical potential, the leading-order term simplifies to the original RS form

$$f(x,p) = f_{\rm eq}\left(\sqrt{\mathbf{p}^2 + \xi(\tau)(\mathbf{p}\cdot\hat{\mathbf{n}})^2}/\lambda(\tau)\right)$$



$-1 < \xi < 0$ $\xi = 0$ $\xi > 0$

The anisotropic heavy quark potential

One can express the heavy-quark potential in terms of the *static* advanced, retarded, and Feynman propagators

real part imaginary part

$$V(\mathbf{r},\xi) = -\frac{g^2 C_F}{2}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\left(e^{i\mathbf{p}\cdot\mathbf{r}}-1\right)\left[\tilde{\mathcal{D}}_R^{00}+\tilde{\mathcal{D}}_A^{00}+\tilde{\mathcal{D}}_F^{00}\right]_{\omega\to 0}$$

The real part can be written as

$$\mathrm{Re}[V(\mathbf{r},\xi)] = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3}\left(e^{i\mathbf{p}\cdot\mathbf{r}}-1\right)\frac{\mathbf{p}^2+m_\alpha^2+m_\gamma^2}{(\mathbf{p}^2+m_\alpha^2+m_\gamma^2)(\mathbf{p}^2+m_\beta^2)-m_\delta^4}$$

with <u>direction-dependent masses</u>, e.g.

$$m_\alpha^2 = -\frac{m_D^2}{2p_\perp^2\sqrt{\xi}}\left(p_z^2\arctan\sqrt{\xi}-\frac{p_z\mathbf{p}^2}{\sqrt{\mathbf{p}^2+\xi p_\perp^2}}\arctan\frac{\sqrt{\xi}p_z}{\sqrt{\mathbf{p}^2+\xi p_\perp^2}}\right)$$

Gluon propagator in an anisotropic plasma: Romatschke and MS, hep-ph/0304092
Real part of the anisotropic potential calculation: Dumitru, Guo, and MS, 0711.4722; M. Nopoush, Y. Guo, and MS, 1706.08091

Complex-valued Potential

- Anisotropic potential can be parameterized as a Debye-screened potential with a direction-dependent Debye mass
- The potential also has an imaginary part coming from the Landau damping of the exchanged gluon
- Generalized imaginary part from isotropic case
  Laine et al hep-ph/0611300
- Used this as a model for the free energy (F) and also obtained internal energy (U) from this.

$$V_{\rm screened}(r,\theta,\xi,\Lambda) = -C_F\alpha_s \frac{e^{-\mu(\theta,\xi,\Lambda)r}}{r}$$

MS, 1106.2571; Bazow and MS, 1112.2761

Internal Energy

$$\begin{aligned} V_{\rm R}(\mathbf{r}) = &-\frac{\alpha}{r}\,(1+\mu\, r)\exp\left(-\mu\, r\right) \\ &+\frac{2\sigma}{\mu}\left[1-\exp\left(-\mu\, r\right)\right] \\ &-\sigma\, r\,\exp(-\mu\, r) - \frac{0.8\,\sigma}{m_Q^2\, r} \end{aligned}$$

Dumitru, Guo, Mocsy, and MS, 0901.1998

$$V_{\rm I}(\mathbf{r}) = -C_F\alpha_s p_{\rm hard}\left[\phi(\hat r) - \xi\left(\psi_1(\hat r,\theta)+\psi_2(\hat r,\theta)\right)\right]$$

Dumitru, Guo, and MS, 0711.4722 and 0903.4703
Burnier, Laine, Vepsalainen, arXiv:0903.3467 (aniso)

$$\alpha_s = 0.29$$

$$\begin{aligned} \phi(\hat r) &= 2\int_0^\infty dz \frac{z}{(z^2+1)^2}\left[1-\frac{\sin(z\,\hat r)}{z\,\hat r}\right], \\ \psi_1(\hat r,\theta) &= \int_0^\infty dz \frac{z}{(z^2+1)^2}\left(1-\frac{3}{2}\left[\sin^2\theta\frac{\sin(z\,\hat r)}{z\,\hat r}+(1-3\cos^2\theta)G(\hat r,z)\right]\right), \\ \psi_2(\hat r,\theta) &= -\int_0^\infty dz \frac{\frac{4}{3}z}{(z^2+1)^3}\left(1-3\left[\left(\frac{2}{3}-\cos^2\theta\right)\frac{\sin(z\,\hat r)}{z\,\hat r}+(1-3\cos^2\theta)G(\hat r,z)\right]\right), \end{aligned}$$

Summary of the phenomenological method

Solve the 3d Schrödinger EQ with complex-valued potential

Publicly available code since 2009

https://sourceforge.net/projects/quantumfdtd/files/quantumfdtd/

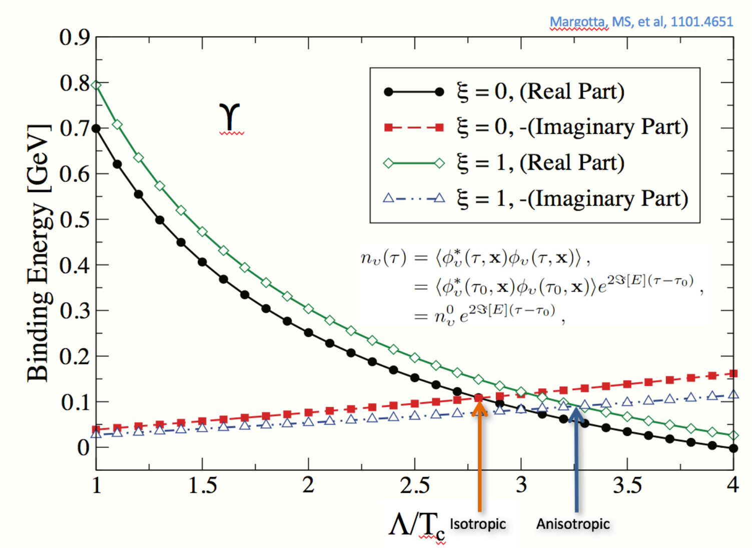
↓

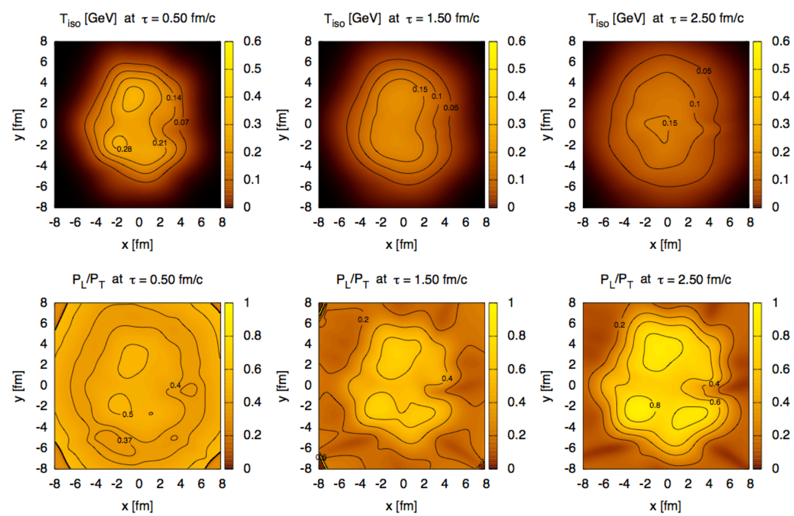
Obtain real and imaginary parts of the binding energies for the $\Upsilon(1s)$, $\Upsilon(2s)$, $\Upsilon(3s)$, χ_{b1}, and χ_{b2} as function of energy density and anisotropy.

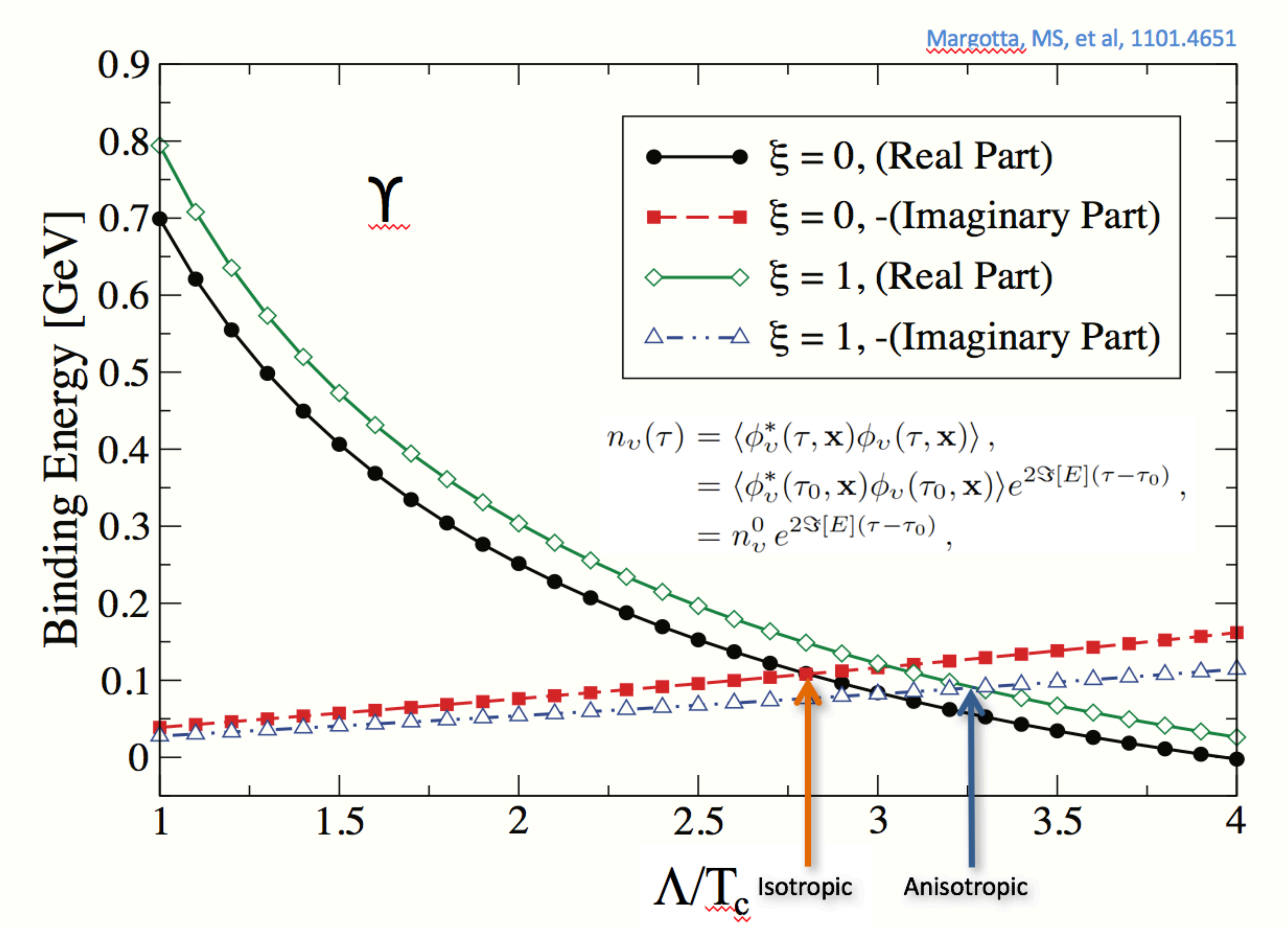
Yager-Elorriaga and MS, 0901.1998;
Margotta, MS, et al, 1101.4651

↓

Fold together with the non-EQ spatiotemporal evolution to obtain the **survival probability**.






Margotta, MS, et al, 1101.4651
ϒ
ξ = 0, (Real Part)
ξ = 0, -(Imaginary Part)
ξ = 1, (Real Part)
ξ = 1, -(Imaginary Part)
$n_\upsilon(\tau) = \langle \phi^*_\upsilon(\tau, \mathbf{x}) \phi_\upsilon(\tau, \mathbf{x}) \rangle ,$
$= \langle \phi^*_\upsilon(\tau_0, \mathbf{x}) \phi_\upsilon(\tau_0, \mathbf{x}) \rangle e^{2\Im[E](\tau-\tau_0)} ,$
$= n^0_\upsilon \, e^{2\Im[E](\tau-\tau_0)} ,$
Binding Energy [GeV]
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
1.5
2
2.5
3
3.5
4
Λ/Tc
Isotropic
Anisotropic

Backup

Imaginary part of the potential…

- The potential also has an imaginary part associated with Landau damping of the exchanged gluon. Laine et al hep-ph/0611300 (iso); Burnier, Laine, and Vepsalainen, 0903.3467 (aniso); Dumitru, Guo, and MS, 0903.4703

$$V_{\rm I}(\mathbf{r}) = -C_F \alpha_s p_{\rm hard} \left[\phi(\hat{r}) - \xi \left(\psi_1(\hat{r},\theta) + \psi_2(\hat{r},\theta) \right) \right]$$

- In 0903.3467, Burnier, Laine, and Vepsalainen used a Taylor expansion around $\xi = 0$ and found that, **beyond linear order in ξ**, one encounters **higher order poles which are not integrable → lots of infinities**.

- At the time this result appeared, it seemed to me that this must be related to the existence of plasma instabilities in an anisotropic QGP. This has recently been demonstrated explicitly. M. Nopoush, Y. Guo, and MS, 1706.08091 (JHEP).

- On the bad news front, the end result is similarly depressing; namely there is **un-integrable pinch singularity** which results in the imaginary part of the potential being ill-defined in an anisotropic QGP.

The suppression factor

- Resulting decay rate Γ_T = -2 Im[E_{bind}] is a function of τ, $x_\perp$, and ς (spatial rapidity). First we need to integrate over proper time

$$\bar{\gamma}(\mathbf{x}_\perp, p_T, \varsigma, b) \equiv \int_{\max(\tau_{\rm form}(p_T),\tau_0)}^{\tau_f} d\tau\, \Gamma_T(\tau, \mathbf{x}_\perp, \varsigma, b)$$

- From this we can extract R_{AA}

$$R_{AA}(\mathbf{x}_\perp, p_T, \varsigma, b) = \exp(-\bar{\gamma}(\mathbf{x}_\perp, p_T, \varsigma, b))$$

- Use the overlap density as the probability distribution function for quarkonium production vertices and geometrically average

$$\langle R_{AA}(p_T, \varsigma, b)\rangle \equiv \frac{\int_{\mathbf{x}_\perp} d\mathbf{x}_\perp\, T_{AA}(\mathbf{x}_\perp)\, R_{AA}(\mathbf{x}_\perp, p_T, \varsigma, b)}{\int_{\mathbf{x}_\perp} d\mathbf{x}_\perp\, T_{AA}(\mathbf{x}_\perp)}$$