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Inelastic reaction rates in the Osaka / Stavanger approach

Alexander Rothkopf
Faculty of Science and Technology
Department of Mathematics and Physics
University of Stavanger

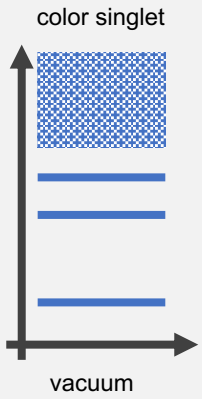
References:
T. Miura, Y. Akamatsu, M. Asakawa, A.R. arXiv:1908.06293
Y. Akamatsu, Phys.Rev. D91 (2015) no.5, 056002
Y. Akamatsu, A.R. Phys.Rev. D85 (2012) 105011

In-medium reactions

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- In-medium effects: mod. Hamiltonian + radiation field (not necessarily in form of a potential)

Quarkonium in-medium physics

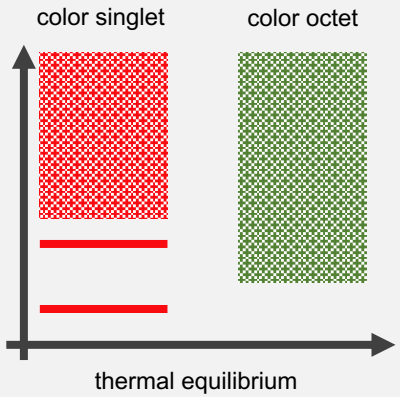


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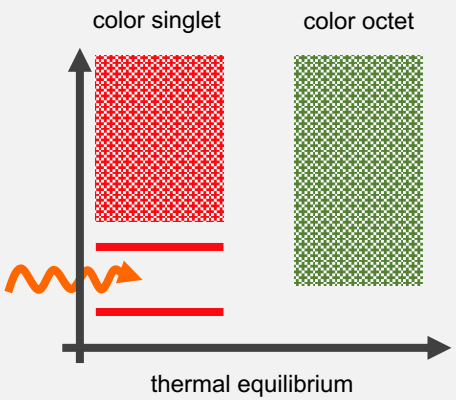


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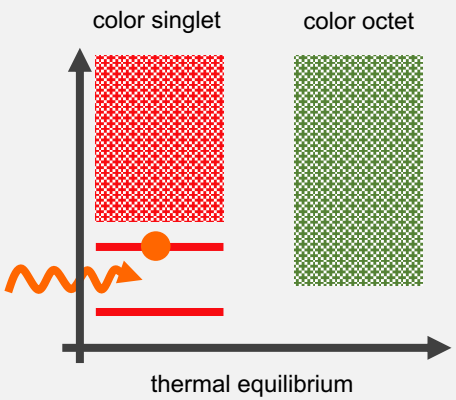


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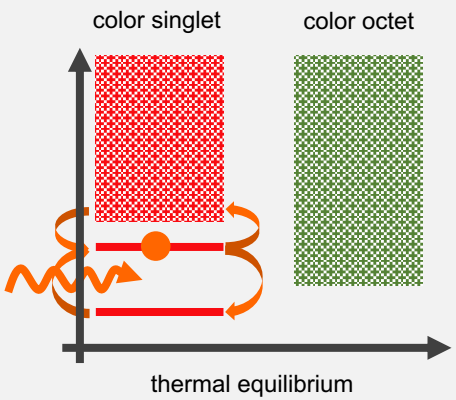


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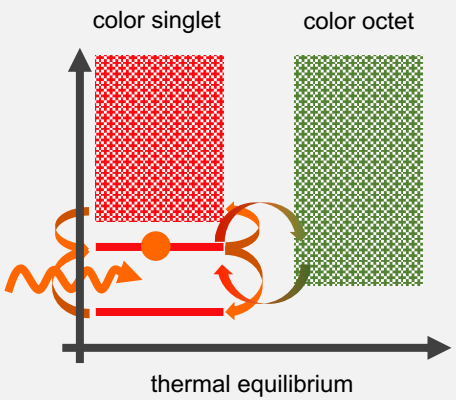


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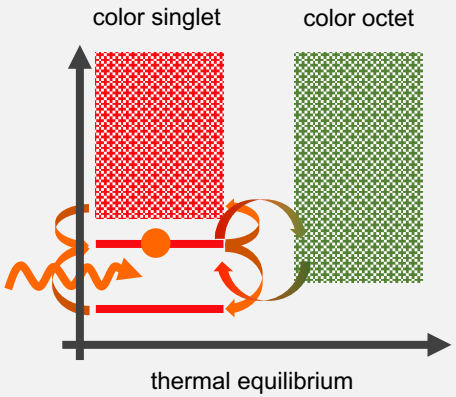


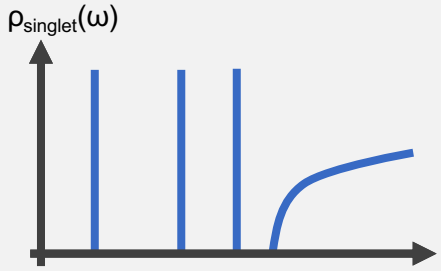
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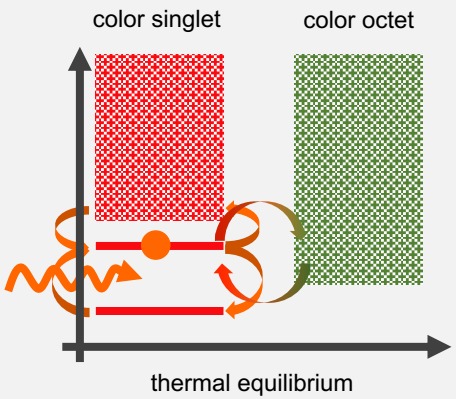
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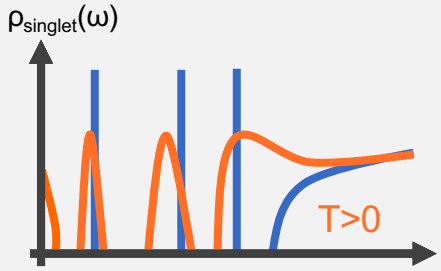
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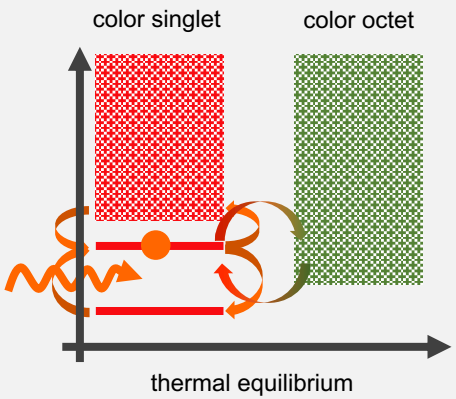
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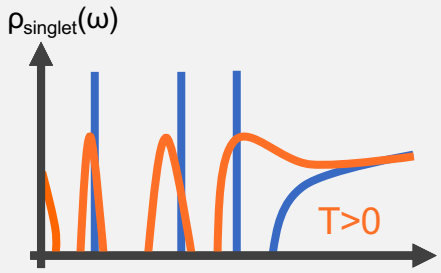
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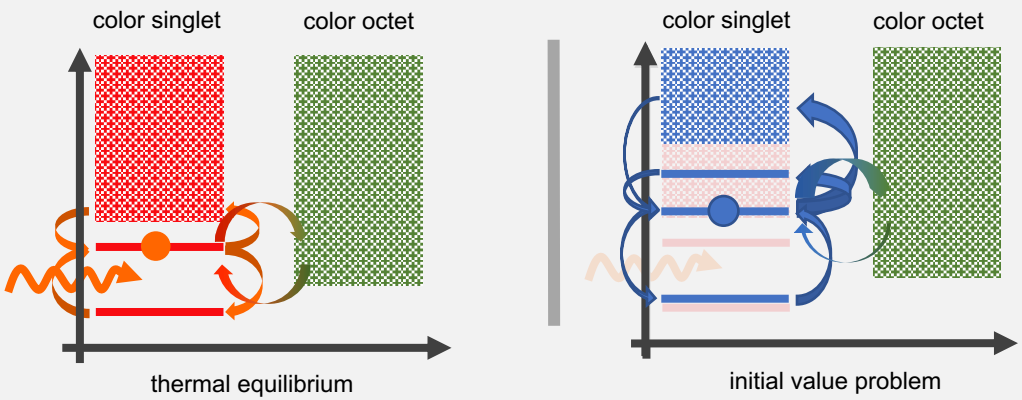
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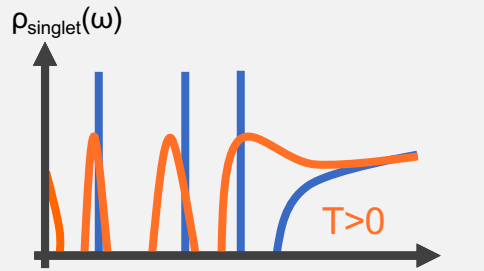
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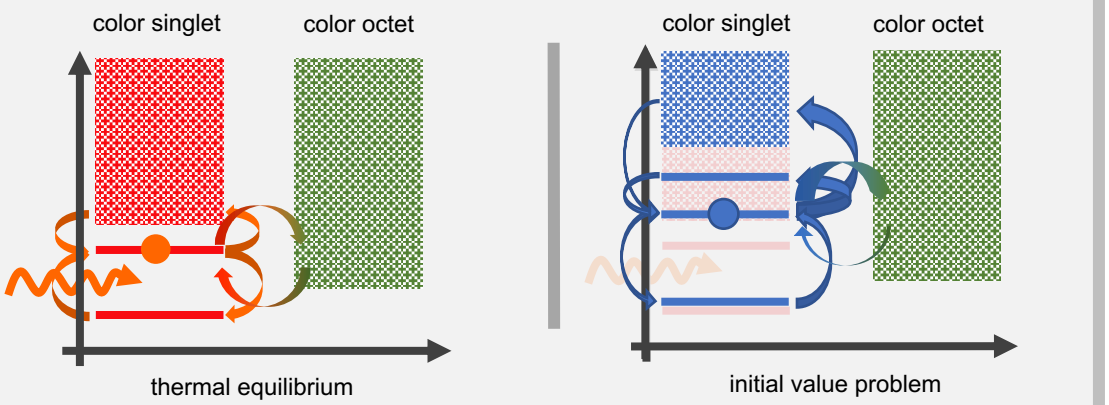
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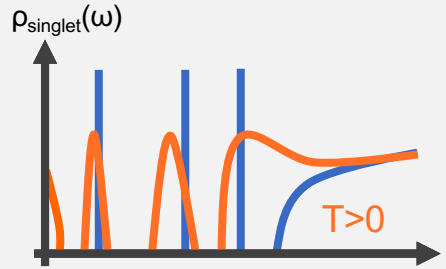
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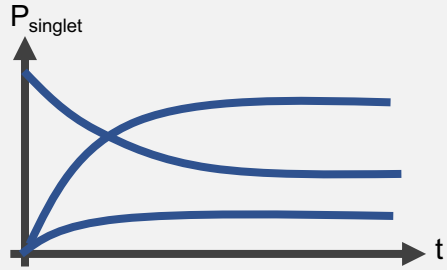
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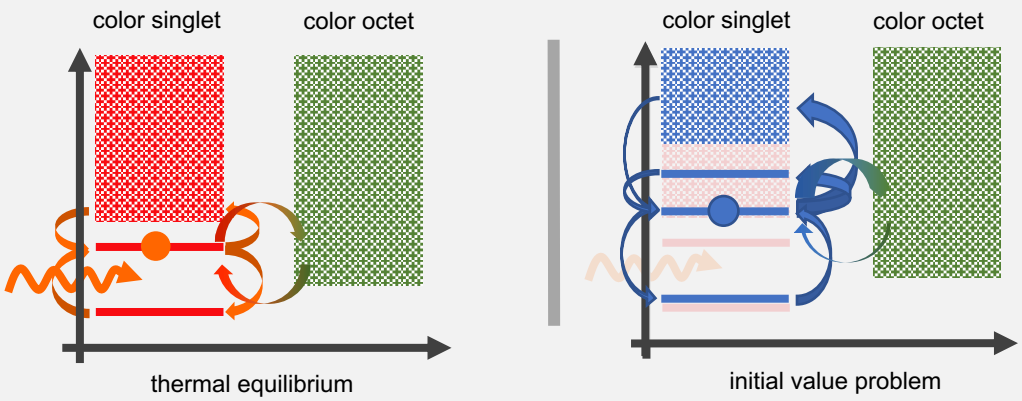
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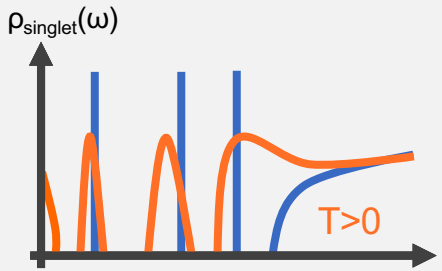
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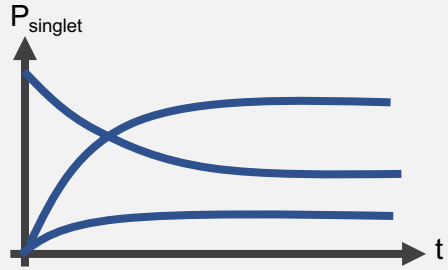
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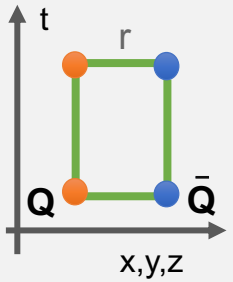


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Heavy-quark potential at T>0

Non-perturbatively:
static singlet to singlet evolution appears to be amenable to potential description



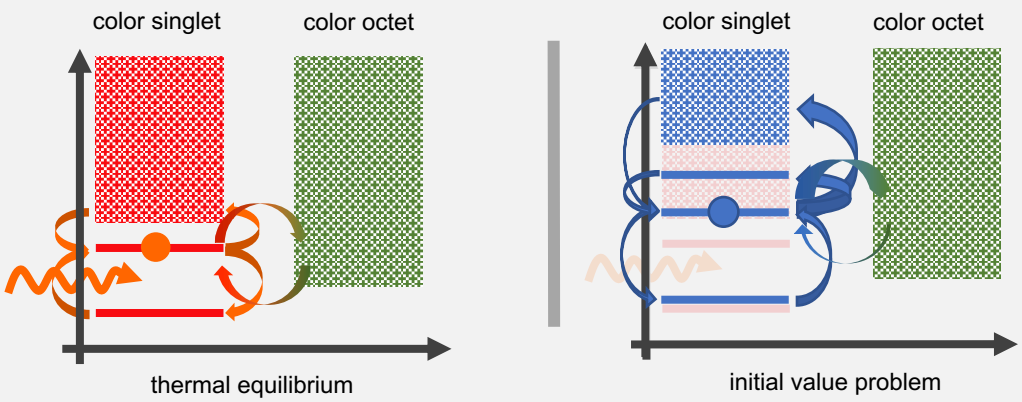
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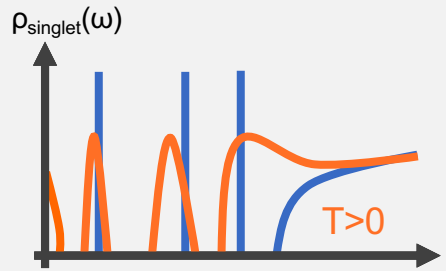
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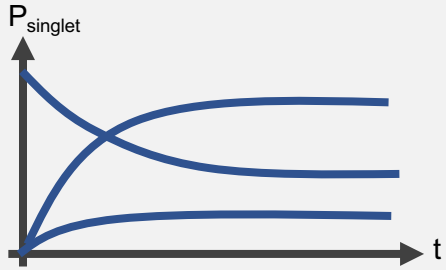
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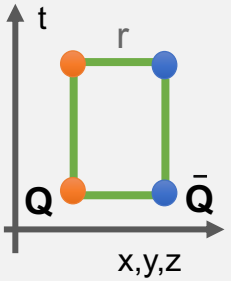
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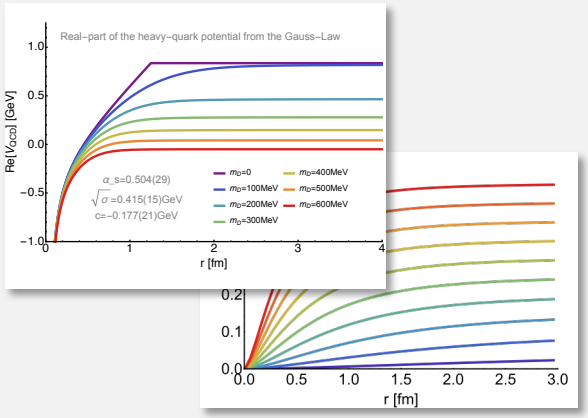
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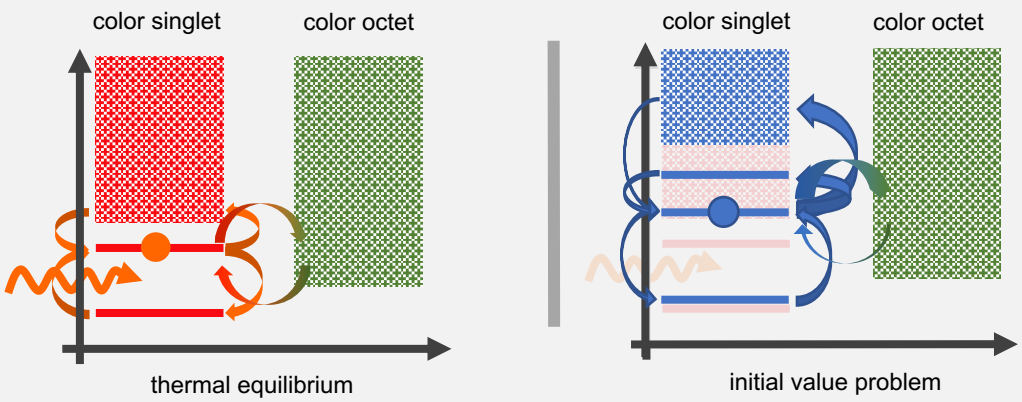
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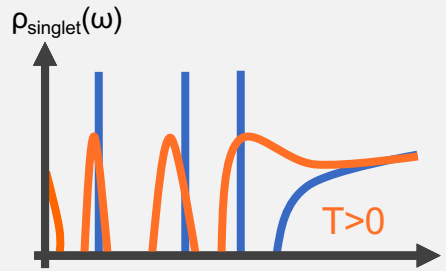
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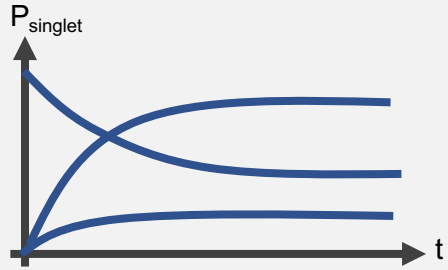
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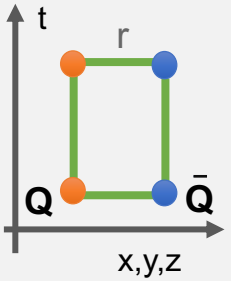


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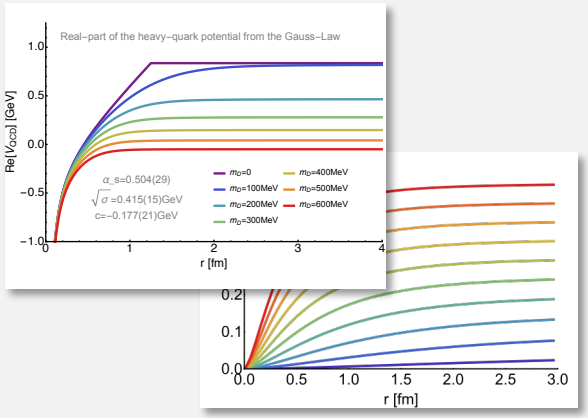
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$$\rho(\omega) = \lim_{r\to 0}\int dt e^{i\omega t}\langle\psi(0,r)\psi(t,r)\rangle$$

Motivation: Osaka/Stavanger approach

University of Stavanger

- Main goal: derive a **potential based** e.o.m. for quarkonium wavefunctions from QCD

$$i\partial_t \psi(t) = F[\psi, Re[V], Im[V], \ldots]$$

  - **conceptual**: connect imaginary part of the EFT potential to microscopic quarkonium dynamics
  - **technical**: range of validity of phenomenological models
  - **practical**: provide path for incremental improvement of current approaches

- Based on **open-quantum-systems** picture (no semiclassical approximation)

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Deterministic Schrödinger

$$i\partial_t\psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\right]\psi$$

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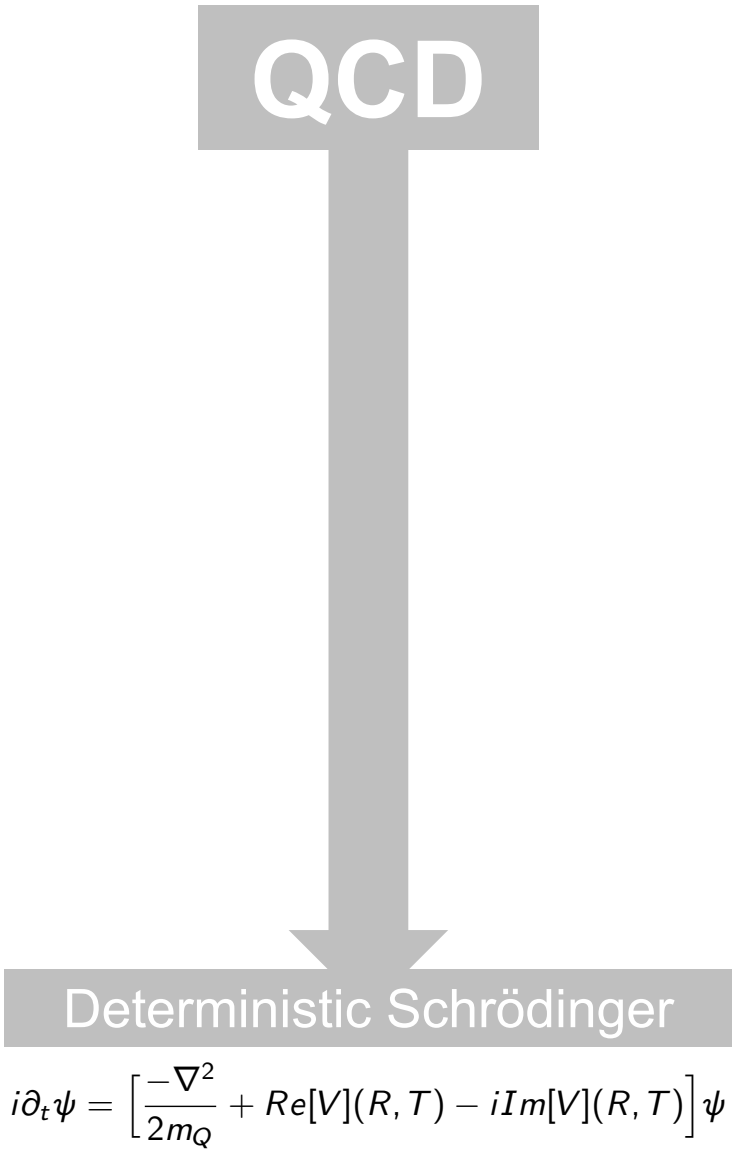
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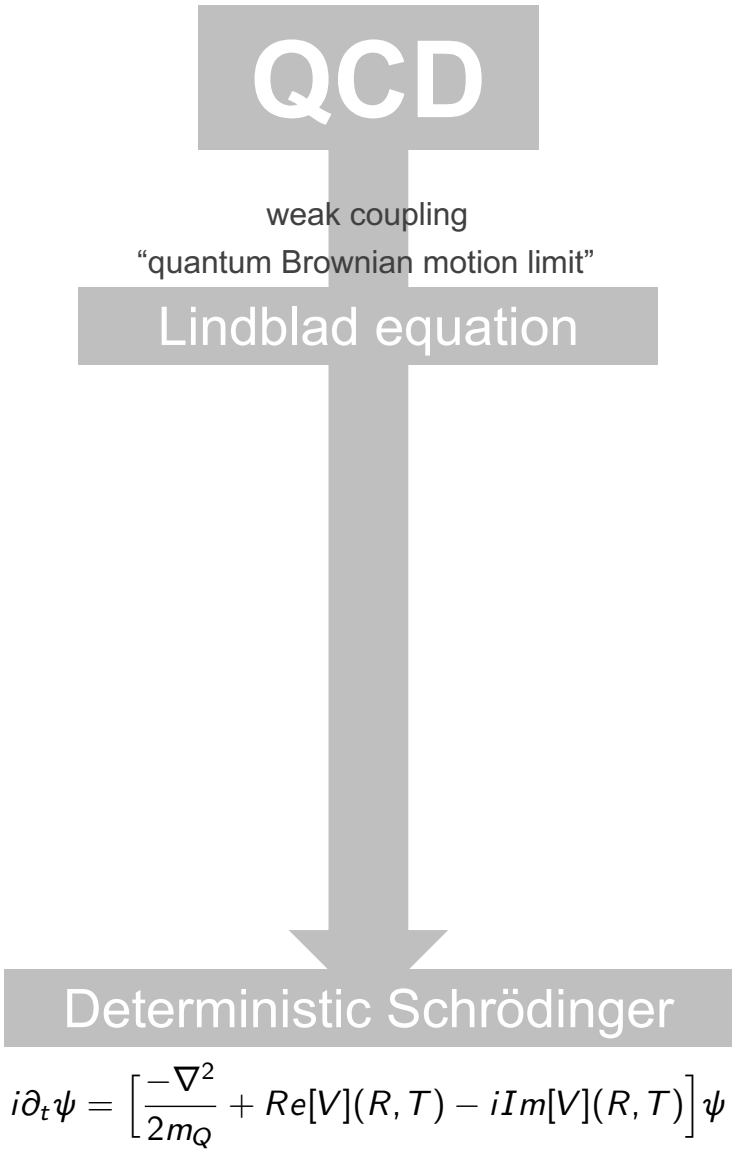
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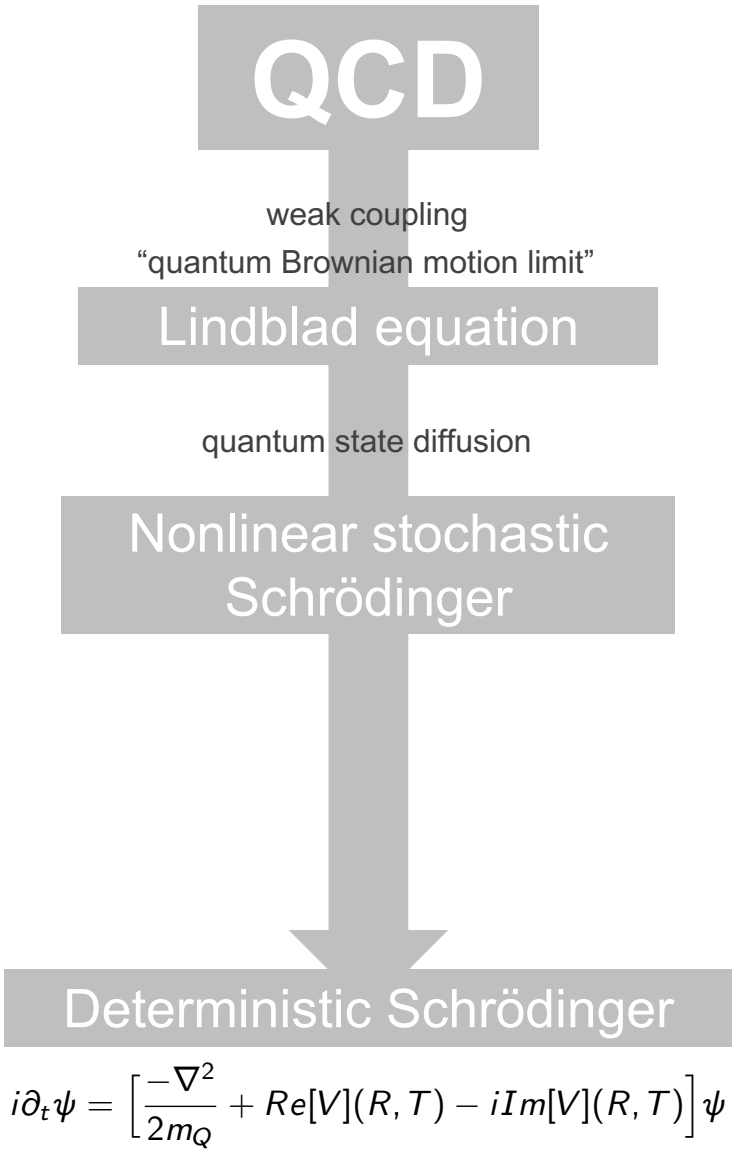
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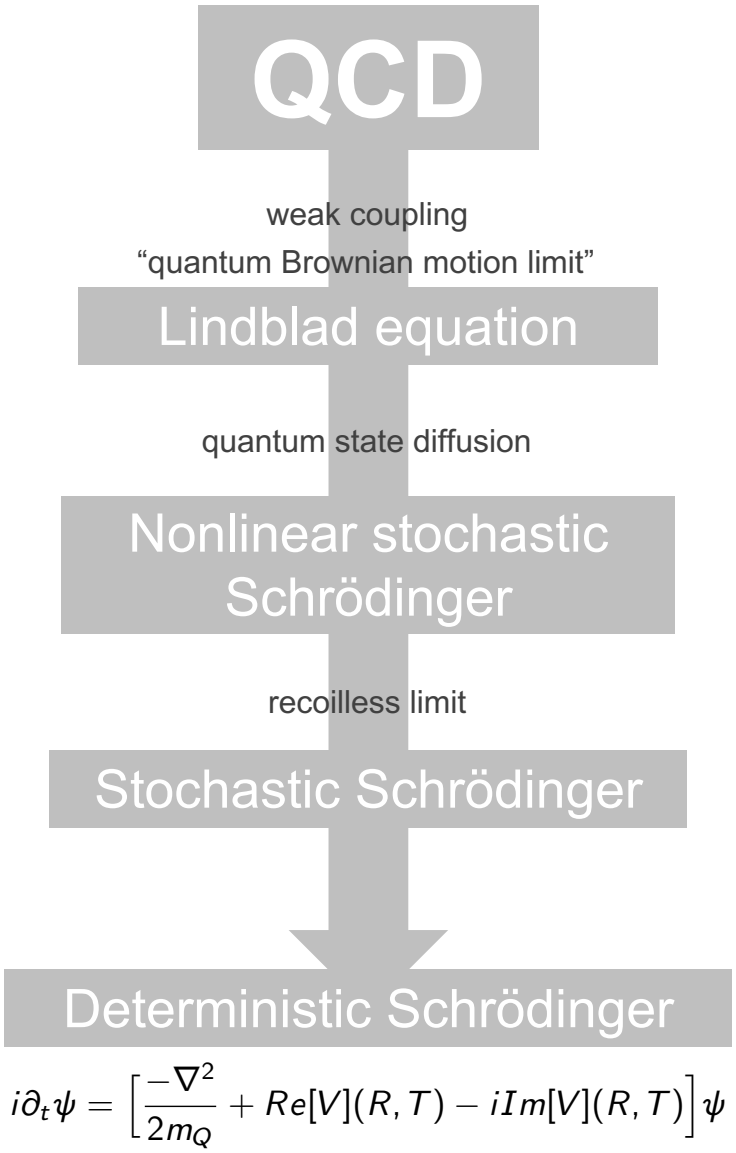
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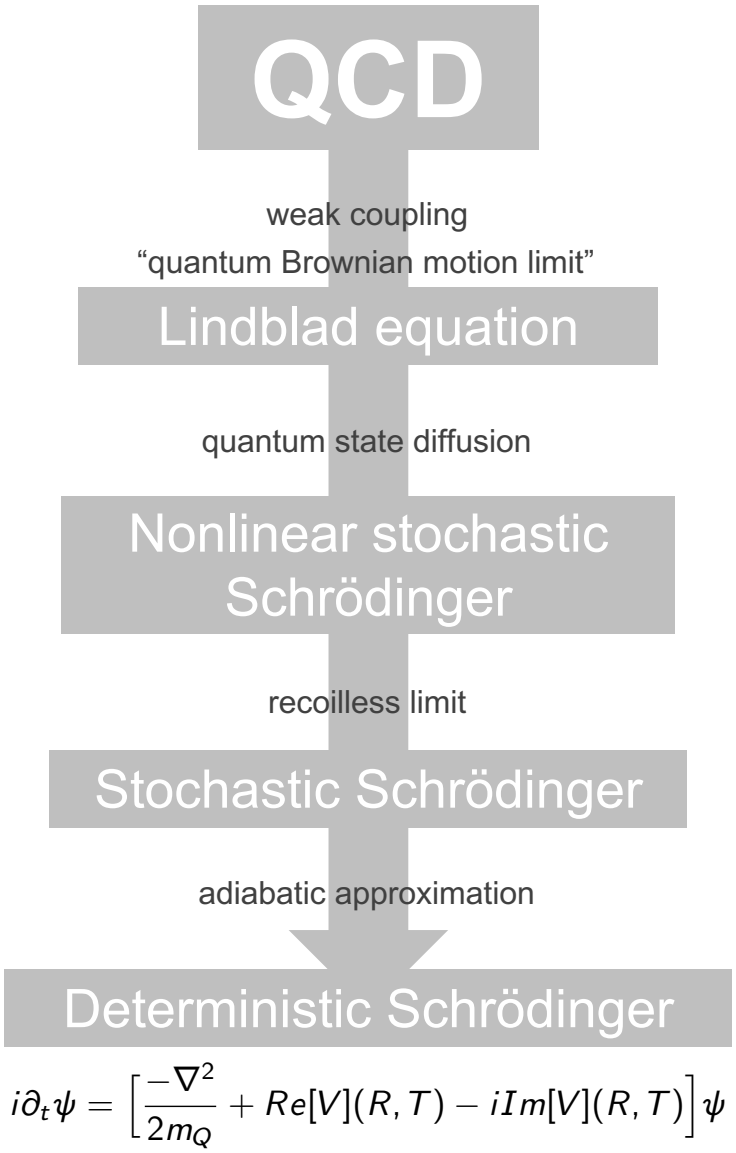
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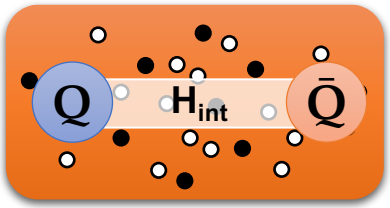


The open quantum systems picture

University of Stavanger

- Need a general approach to describe quarkonium coupled to a thermal medium
  - Overall system is closed, hermitean Hamiltonian: von Neumann equation

$$H = H_{Q\bar{Q}} \otimes I_{med} + I_{Q\bar{Q}} \otimes H_{med} + H_{int} \qquad \frac{d\rho}{dt} = -i[H, \rho]$$



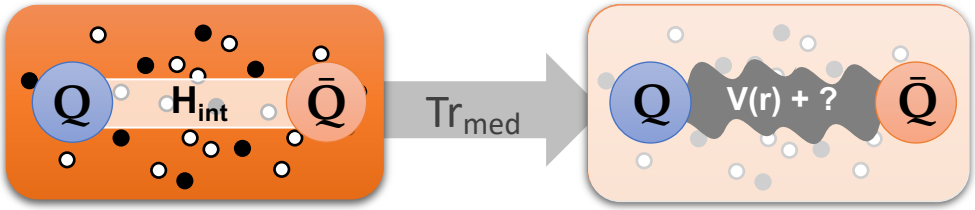
$$H_{\text{int}} = \sum_m \Sigma_m \otimes \Xi_m$$

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$$H_{\text{int}} = \sum_m \Sigma_m \otimes \Xi_m$$

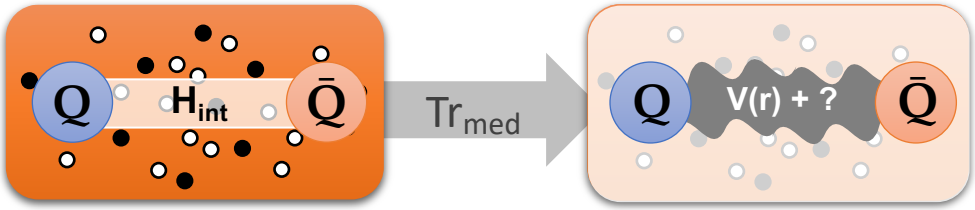
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The open quantum systems picture

University of Stavanger

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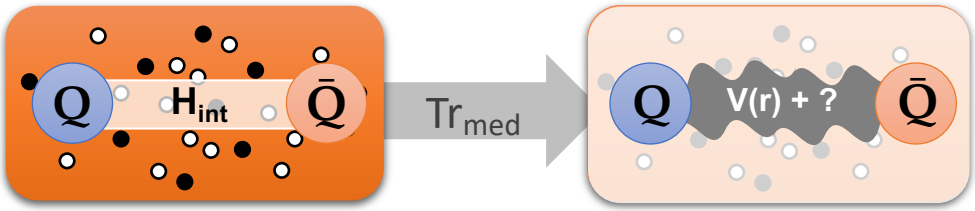
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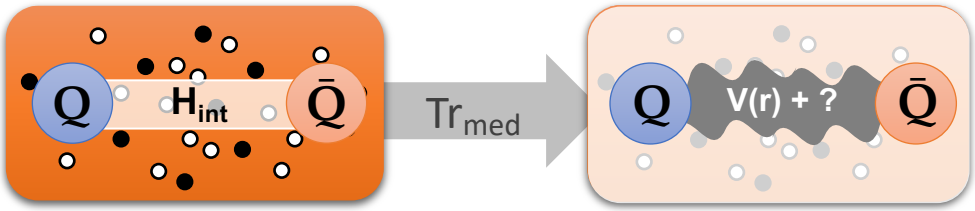
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$\langle n|\rho_{Q\bar{Q}}|n\rangle > 0, \forall n$

$\rho_{Q\bar{Q}}^\dagger = \rho_{Q\bar{Q}}, \quad \text{Tr}[\rho_{Q\bar{Q}}] = 1$

Quarkonium Lindblad eq. at high T

University of Stavanger

T. Miura, Y.Akamatsu, M. Asakawa, A.R., arXiv:1908.06293

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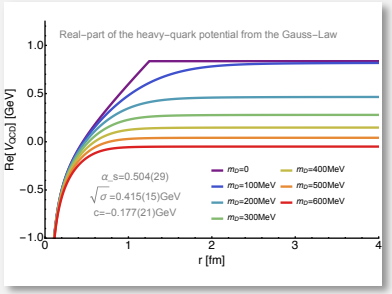
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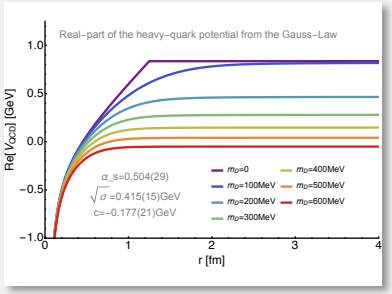
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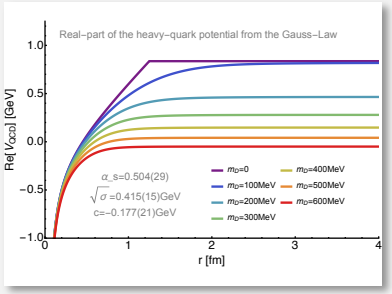
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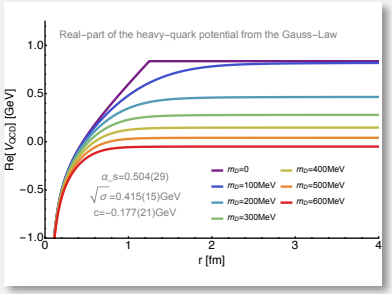
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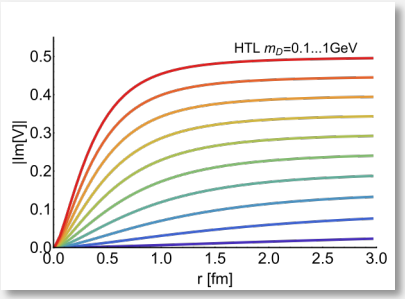
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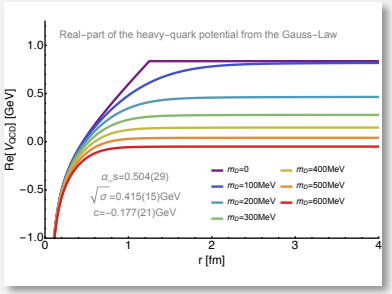
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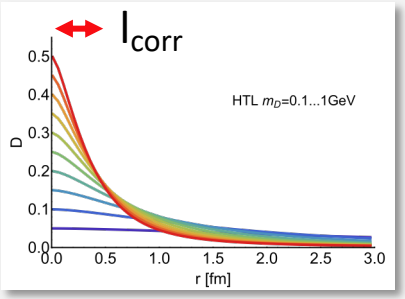
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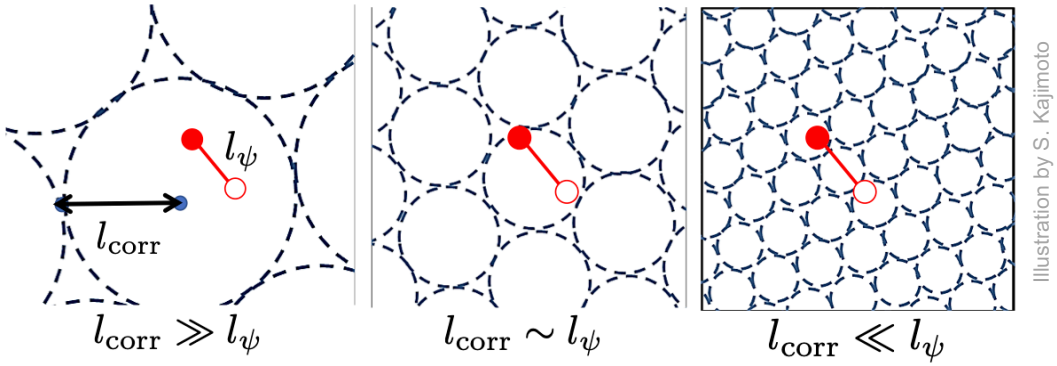
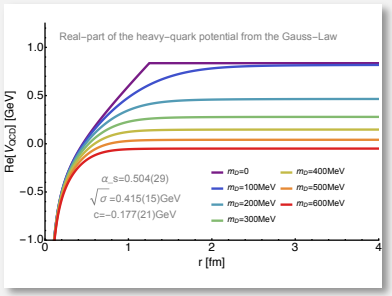


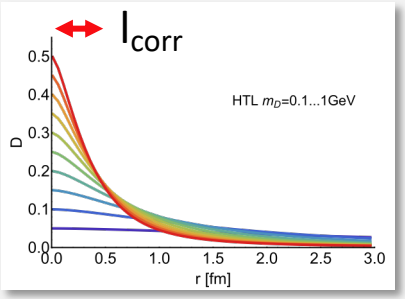
Illustration by S. Kajimoto

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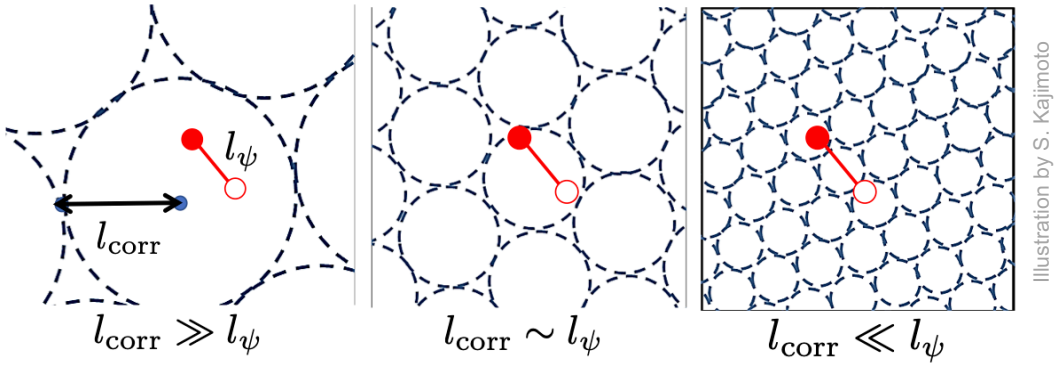
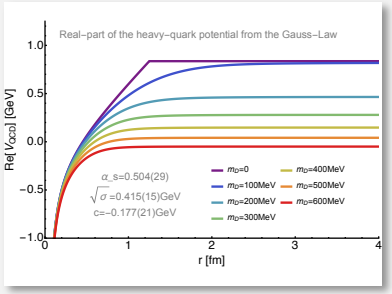


Illustration by S. Kajimoto

$$\rho(\mathbf{x}_1, \mathbf{x}_2, t) \sim \rho(\mathbf{x}_1, \mathbf{x}_2, 0)\exp\Big[-\big(D(0) - D(\mathbf{x}_1 - \mathbf{x}_2)\big)t\Big]$$

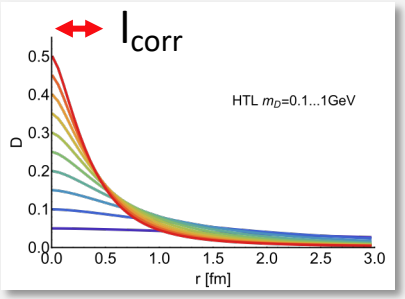
decay of off-diagonal terms = decoherence

$$Re[V] = -\alpha_S \frac{e^{-m_D r}}{r}$$



$$\tilde{D}(k) = g^2 T \frac{\pi m_D^2}{k(k^2 + m_D^2)^2}$$

Im[V]



$$D(r) = Im[V](r) - Im[V](\infty)$$

Solving in the coordinate basis

University of Stavanger

Full dissipative dynamics

Quantum state diffusion: T. Miura, Y.Akamatsu, M. Asakawa, A.R., arXiv:1908.06293
unravel any Lindblad into a nonlinear stochastic Schrödinger equation

$$
\begin{aligned}
|d\psi\rangle &= |\psi(t+dt)\rangle - |\psi(t)\rangle \\
&= -iH|\psi(t)\rangle dt + \sum_n \begin{pmatrix} 2\langle L_n^\dagger\rangle_\psi L_n - L_n^\dagger L_n \\ -\langle L_n^\dagger\rangle_\psi \langle L_n\rangle_\psi \end{pmatrix} |\psi(t)\rangle dt \\
&\quad + \sum_n \left(L_n - \langle L_n\rangle_\psi\right) |\psi(t)\rangle d\xi_n,
\end{aligned}
$$

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Recoilless limit

Colored Stochastic Potential: Y. Akamatsu, Phys.Rev. D91 (2015) no.5, 056002
see also: R.Sharma, A. Tiwari arXiv:1912.07036
without dissipation dynamics unravel into linear stochastic Schrödinger

$$H(\mathbf{r},t) = -\frac{1}{2\mu}\nabla_{\mathbf{r}}^2 + V(\mathbf{r})(t^a \otimes t^{a*}) + \Theta(\mathbf{r},t)$$

$$\Theta(\mathbf{r},t) = \theta^a(\mathbf{R}+\mathbf{r}/2,t)(t^a\otimes 1) - \theta^a(\mathbf{R}-\mathbf{r}/2,t)(1\otimes t^{a*})$$

$$\langle\theta^a(\mathbf{x},t)\theta^b(\mathbf{x}',t')\rangle = D(\mathbf{x}-\mathbf{x}')\delta(t-t')\delta^{ab}$$

Solving in the coordinate basis

University of Stavanger

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Colored Stochastic Potential: Y. Akamatsu, Phys.Rev. D9
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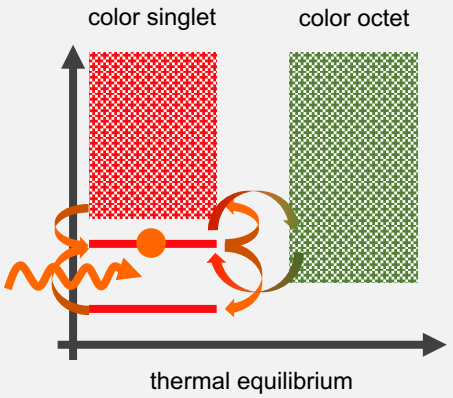
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Density matrix from WF ensembles

$$\rho_{Q\bar{Q}}(\mathbf{r},\mathbf{s}) = \langle\psi(\mathbf{r},t)\psi^*(\mathbf{s},t)\rangle$$

$$\rho_{Q\bar{Q}}(n,m) = \int d^3r d^3s\, \phi_n^*(\mathbf{r})\langle\psi(\mathbf{r},t)\psi^*(\mathbf{s},t)\rangle\phi_m(\mathbf{s})$$



Transitions among basis states well defined from matrix elements of Lindblad operators L

Conclusion

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- At high T - derivation of quarkonium **Lindblad equation** from QCD
- **Fully quantum** evolution based on Re[V] and Im[V]
- Reveals interplay of **screening** (m_D) and **decoherence** (l_{corr})
- Straight forward simulation prescriptions in **coordinate space** available
- Transition rates well defined with respect to a certain basis from L_{ka}

Feynman Vernon influence functional

University of Stavanger

- Derivation via path integral formalism: **Feynman-Vernon influence functional**

for details see Y. Akamatsu, Phys.Rev. D87 (2013) 4, 045016 and PRD91 (2015) 5, 056002

$$Z = \int \mathcal{D}[Q, A, q]_{1,2} \langle Q_1, A_1, q_1 | \rho_{\text{init}}^{Q\bar{Q}} \otimes \rho_{\text{therm}}^{\text{med}} | Q_2, A_2, q_2 \rangle_{t=0} \exp\left[iS[Q_1, A_1, q_1] - iS[Q_2, A_2, q_2] \right]$$

Feynman Vernon influence functional

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medium - QQ interaction

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medium - QQ interaction

$$e^{iS_{IF}} = \int \mathcal{D}[A, q]_{1,2} \langle A_1, q_1 | \rho_{\text{therm}}^{\text{med}} | A_2, q_2 \rangle_{t=0} \exp\left[iS[A_1, q_1] - iS[A_2, q_2] - i \int \left(g\rho_1^{a0} A_1^{a0} + g\rho_2^{a0} A_2^{a0} \right) \right]$$

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University of Stavanger

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$$iS_{IF} = -\frac{g^2}{2} \int_{t_0} d^4x d^4y (\rho_1^a, \rho_2^a)_{(x)} \begin{bmatrix} G^F_{ab,00} & -G^<_{ab,00} \\ -G^>_{ab,00} & G^{\tilde{F}}_{ab,00} \end{bmatrix}_{(x-y)} \begin{pmatrix} \rho_1^b \\ \rho_2^b \end{pmatrix}_{(y)}$$

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- After Markov approx. & time coarse graining & Ladder approx. two relevant quantities emerge:

$$S_{\text{IF}} = S_{\text{pot}} + S_{\text{fluct}} + S_{\text{diss}}$$

$$iS_{\text{pot}} = -\frac{i}{2} \int_{t_0} dt \int d^3x d^3y V(\vec{x} - \vec{y}) (\rho_1^a, \rho_2^a)_{(t,\vec{x})} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} \rho_1^a \\ \rho_2^a \end{pmatrix}_{(t,\vec{y})},$$

$$iS_{\text{fluct}} = -\frac{1}{2} \int_{t_0} dt \int d^3x d^3y D(\vec{x} - \vec{y}) (\rho_1^a, \rho_2^a)_{(t,\vec{x})} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{pmatrix} \rho_1^a \\ \rho_2^a \end{pmatrix}_{(t,\vec{y})},$$

$$iS_{\text{diss}} = -\frac{i}{4T} \int_{t_0} dt \int d^3x d^3y D(\vec{x} - \vec{y}) (\rho_1^a, \rho_2^a)_{(t,\vec{x})} \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} \dot{\rho}_1^a \\ \dot{\rho}_2^a \end{pmatrix}_{(t,\vec{y})},$$

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University of Stavanger

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$$iS_{\text{diss}} = -\frac{i}{4T} \int_{t_0} dt \int d^3x d^3y D(\vec{x} - \vec{y}) (\rho_1^a, \rho_2^a)_{(t,\vec{x})} \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} \dot{\rho}_1^a \\ \dot{\rho}_2^a \end{pmatrix}_{(t,\vec{y})},$$

$$V(\vec{r})\delta_{ab} \equiv -g^2 \text{Re}\tilde{G}^{\text{R}}_{ab,00}(0, \vec{r}),$$

$$\tilde{G}^{\text{R}}_{ab,00}(0, \vec{r}) = -\frac{e^{-\omega_{\text{D}} r}}{4\pi r}$$

$$D(\vec{r})\delta_{ab} \equiv -g^2 T \frac{\partial}{\partial \omega} \sigma_{ab,00}(0, \vec{r}),$$

$$\sigma_{ab,00}(\omega, \vec{x} - \vec{y}) \equiv \int dt e^{-i\omega(x^0 - y^0)} \langle [A_0^a(x), A_0^b(y)] \rangle$$

$$\frac{\partial}{\partial \omega} \sigma_{ab,00}(0, \vec{r}) = \int \frac{d^3k}{(2\pi)^3} \frac{\pi \omega_{\text{D}}^2 e^{i\vec{k}\cdot\vec{r}}}{k(k^2 + \omega_{\text{D}}^2)^2}$$

Color stochastic potential

University of Stavanger

Stochastic SE in the **recoilless limit** including SU(3) color d.o.f.:

$$H(\mathbf{r},t) = -\frac{1}{2\mu}\nabla_{\mathbf{r}}^2 + V(\mathbf{r})(t^a \otimes t^{a*}) + \Theta(\mathbf{r},t)$$

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Based on single gluon exchange: noise governed by D function always corresponds to a singlet <-> octet transition.

Color stochastic potential

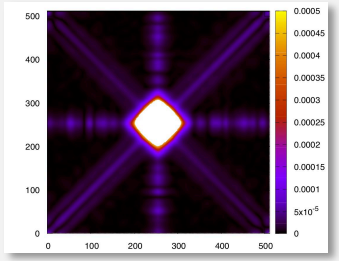
University of Stavanger

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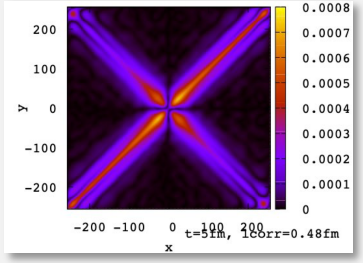
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color singlet density matrix



color octet density matrix



Color stochastic potential

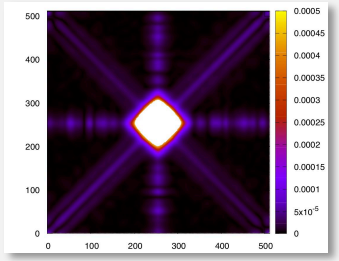
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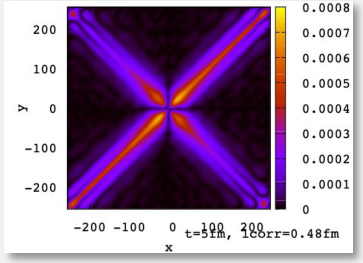
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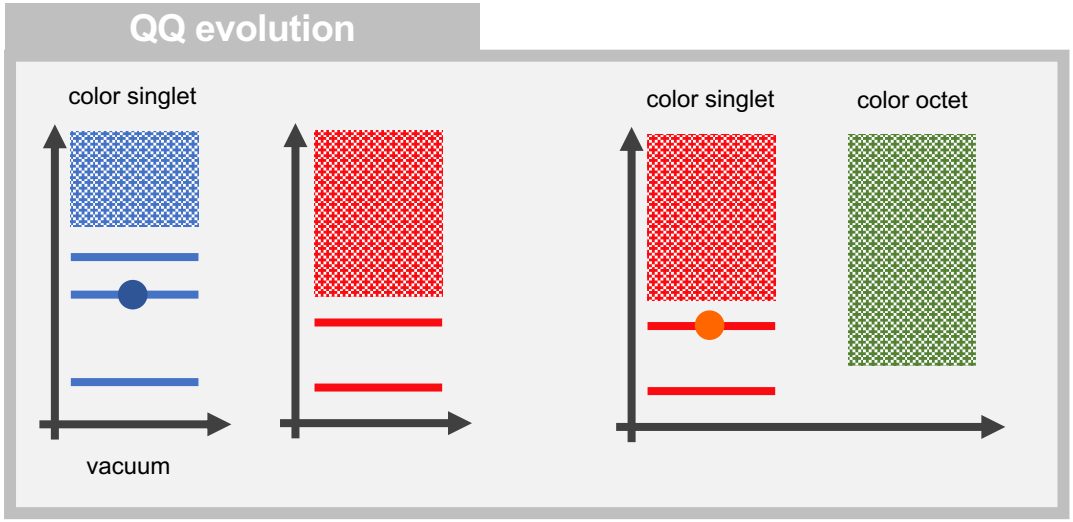
$$W(R,p) = \int dr \rho(x,y) e^{-ipr}$$

Wigner distribution around origin shows **non-positive contributions**

Genuine **quantum nature** of quarkonium **remains relevant**

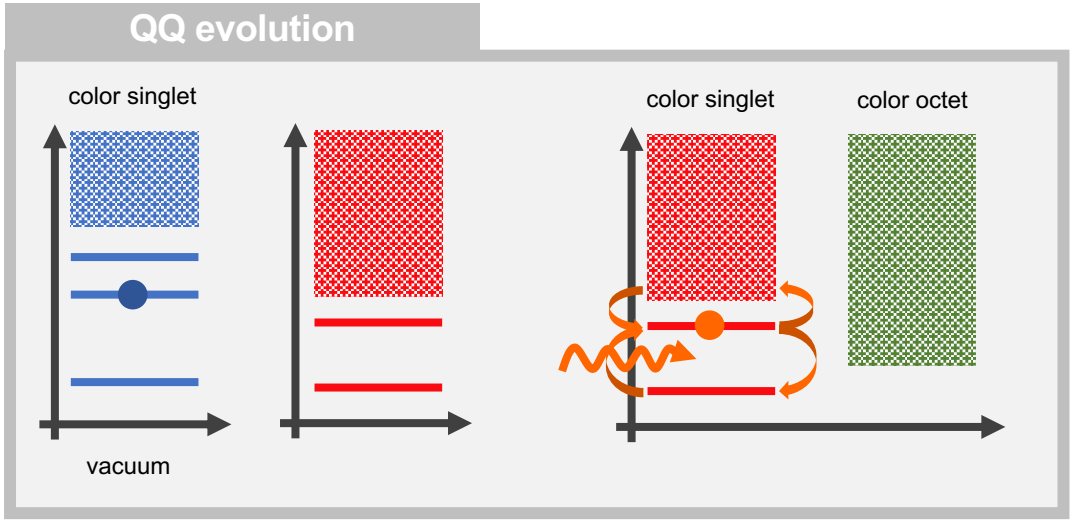
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- In-medium effects: mod. Hamiltonian + radiation field (not necessarily in form of a potential)



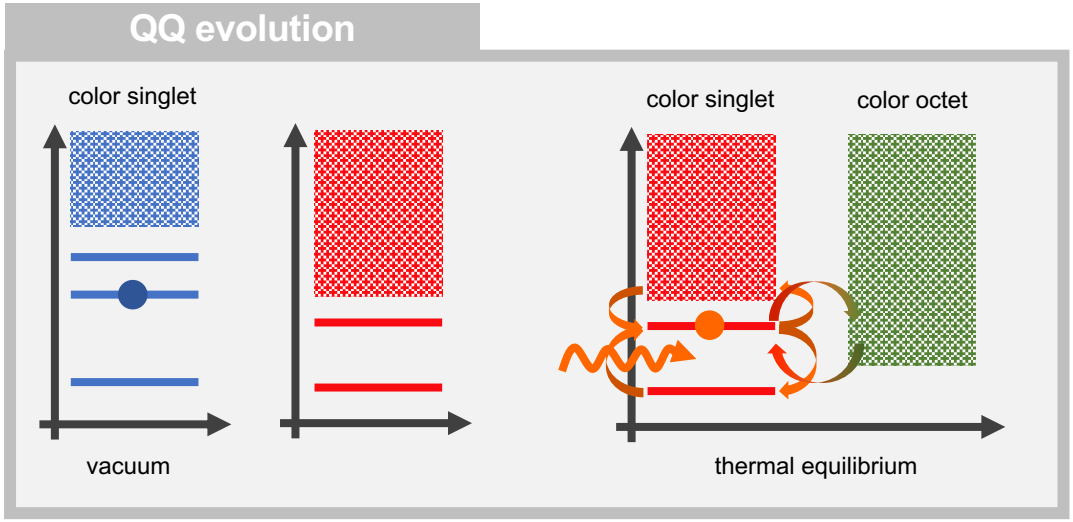
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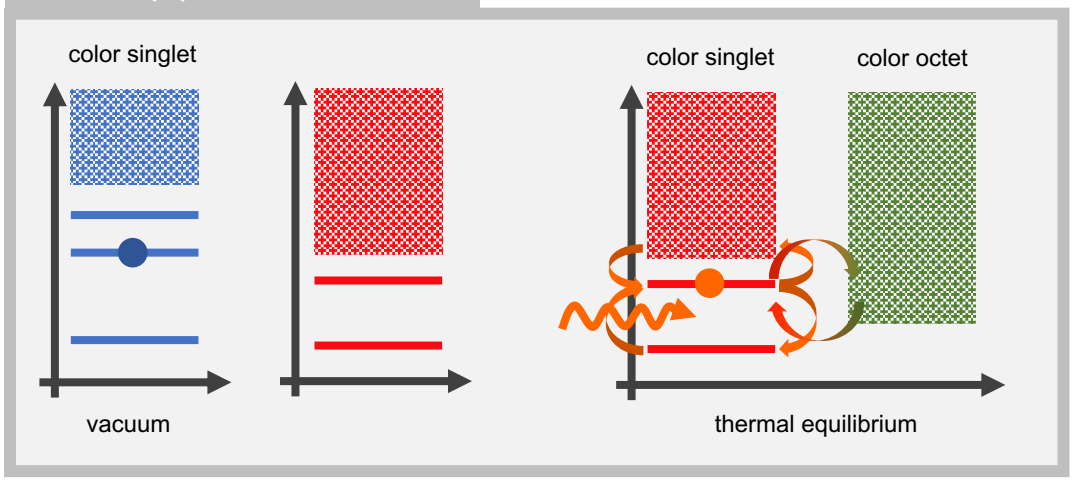
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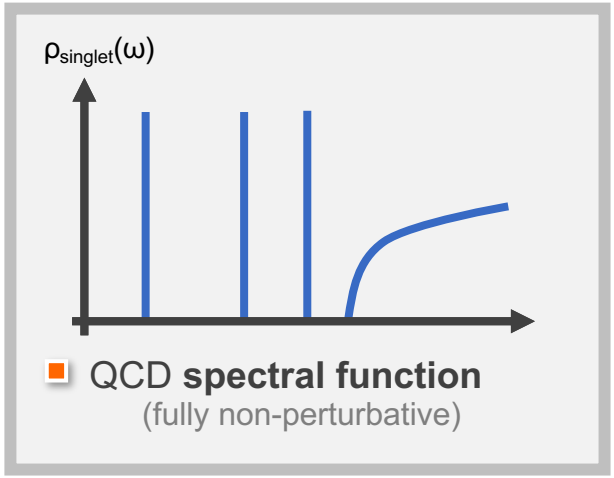


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- In-medium effects: mod. Hamiltonian + radiation field (not necessarily in form of a potential)

QQ evolution

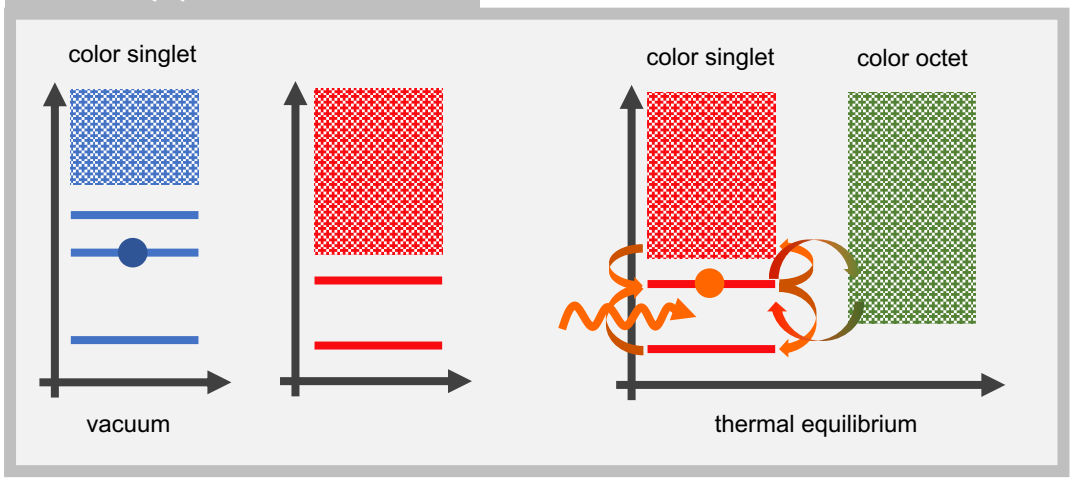


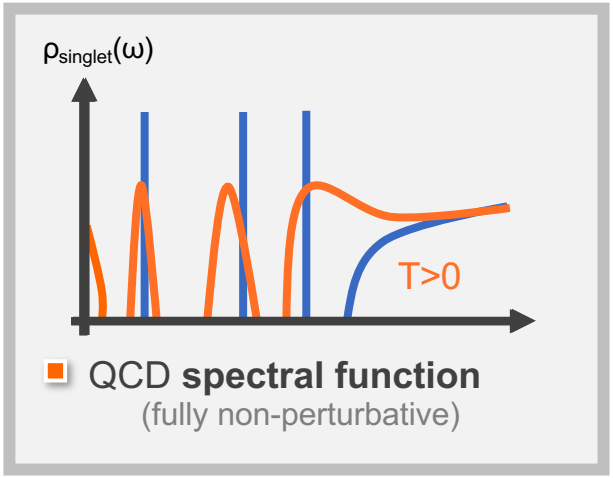


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- In-medium effects: mod. Hamiltonian + radiation field (not necessarily in form of a potential)

QQ evolution





- QCD **spectral function**
  (fully non-perturbative)

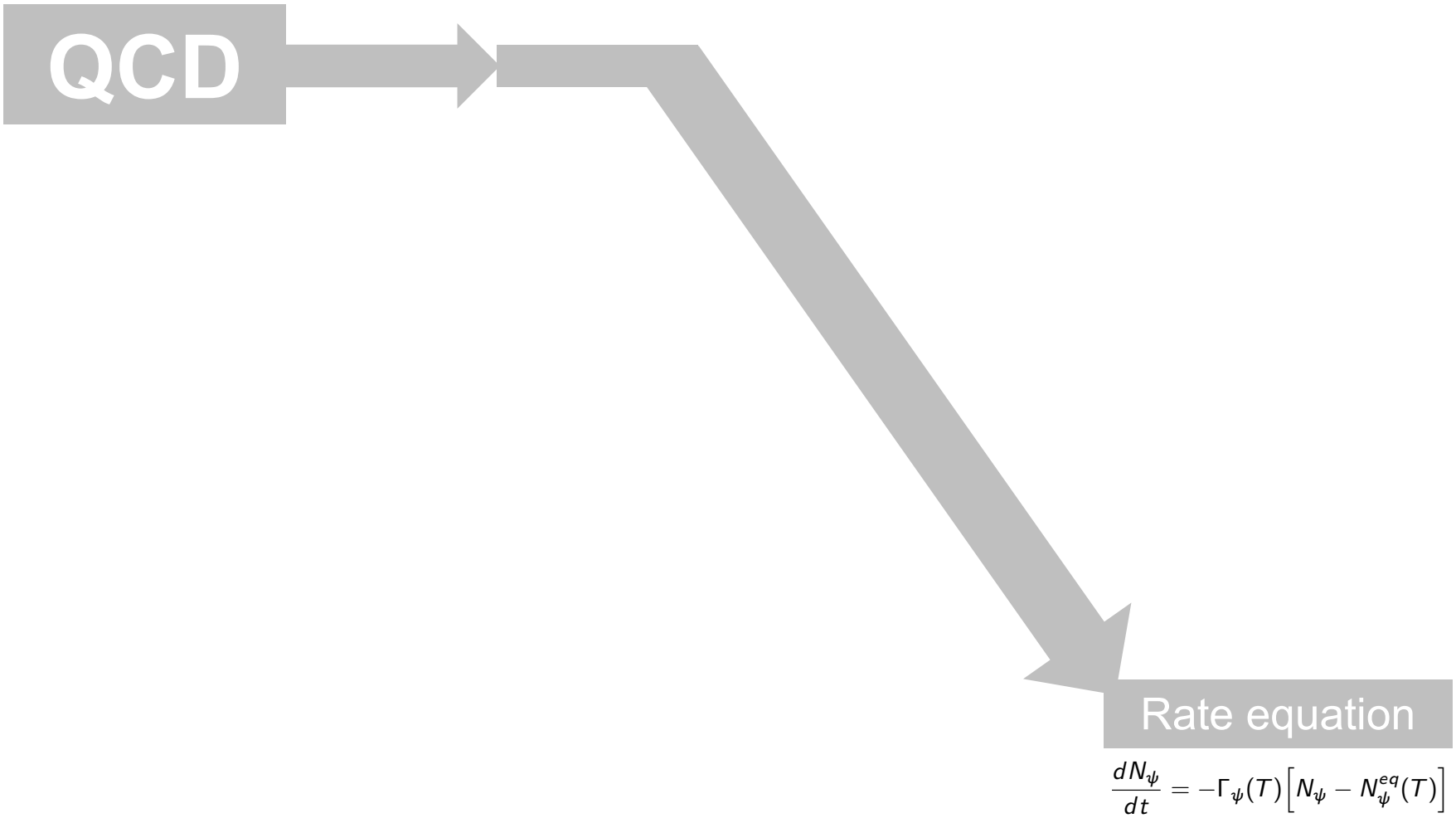
From QCD towards phenomenology

University of Stavanger

QCD

From QCD towards phenomenology

University of Stavanger



From QCD towards phenomenology

University of Stavanger

QCD → pNRQCD

weak coupling

$$\mathrm{Tr}\left[\psi_O^\dagger \mathbf{r} g\mathbf{E} \psi_S + \psi_S^\dagger \mathbf{r} g\mathbf{E} \psi_O\right]$$

$$\Sigma_m \equiv Re\left[\langle \psi_S(\mathbf{R},t)|r_i|\psi_O^a(\mathbf{R},t)\rangle \quad \Xi_m \equiv \sqrt{\frac{T_F}{N_C}} g E_i^a(\mathbf{R},t)\right.$$

$$f_{nl}(\mathbf{X},\mathbf{K},t) \equiv \int \frac{d^3K'}{(2\pi)^3} e^{i\mathbf{K}'\cdot\mathbf{x}} \langle \mathbf{K}+\frac{\mathbf{K}'}{2}, nl, 1|\hat{\rho}_{Q\bar{Q}}(t)|\mathbf{K}-\frac{\mathbf{K}'}{2}, nl, 1\rangle$$

X. Yao, T. Mehen PRD99 (2019) 096028

→ Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\left[N_\psi - N_\psi^{eq}(T)\right]$$

From QCD towards phenomenology

University of Stavanger

QCD

→ **pNRQCD**

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X. Yao, T. Mehen PRD99 (2019) 096028

→ **Boltzmann equation**

gradient expansion & "**quantum optical limit**"

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{X},\mathbf{K},t) + \mathbf{v}\cdot\nabla_{\mathbf{X}} f_{nl}(\mathbf{X},\mathbf{K},t) = \mathcal{C}_{nl}^{(rec)}(\mathbf{X},\mathbf{K},t) - \mathcal{C}_{nl}^{(diss)}(\mathbf{X},\mathbf{K},t)$$

→ **Rate equation**

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\left[N_\psi - N_\psi^{eq}(T)\right]$$

From QCD towards phenomenology

University of Stavanger

QCD

pNRQCD

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X. Yao, T. Mehen PRD99 (2019) 096028

Boltzmann equation

gradient expansion & "**quantum optical limit**"

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C^{rec} connection to open HF: keep track of unbound QQ

Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\left[N_\psi - N_\psi^{eq}(T)\right]$$

From QCD towards phenomenology

University of Stavanger

QCD

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J.P. Blaizot, M. Escobedo JHEP 1806 (2018) 034

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Im[V]

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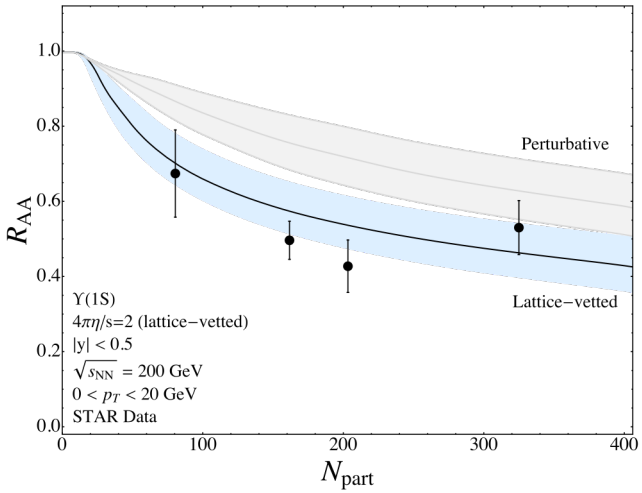
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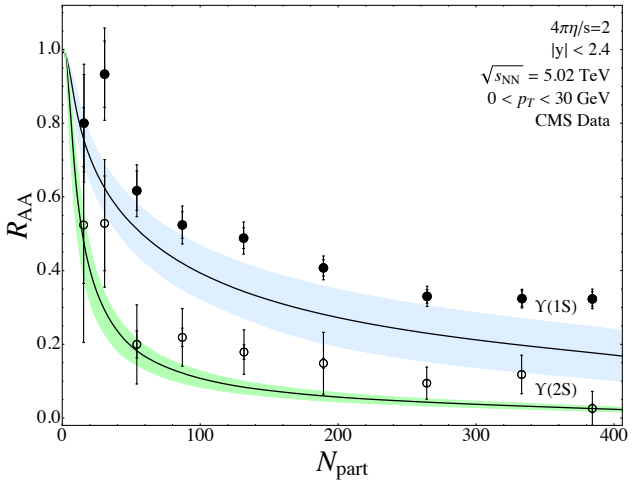
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Krouppa et. al., PRDD97 (2018) 016017
[STAR Collaboration] Z. Ye
Nucl.Part.Phys.Proc. 289-290 (2017) 401





- At RHIC: bottom not equilibrated & good agreement. At LHC: underestimates yields

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- Include the effects of fluctuations by taking the **first order gradient expansion**

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$$U = \exp\Big[-i \int dt \Big(\underbrace{-\frac{\nabla^2}{m_Q} + V(r)}_{S_{\text{pot}}} + \underbrace{\eta(t, \frac{\mathbf{r}}{2}) - \eta(t, -\frac{\mathbf{r}}{2})}_{\Theta(r,t)} \Big) \Big]$$

$$\langle \eta(t, \mathbf{x}) \eta(t', \mathbf{y}) \rangle = \delta(t - t') D(\mathbf{x} - \mathbf{y})$$

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$$U = \exp\Big[-i \int dt \Big(\underbrace{-\frac{\nabla^2}{m_Q} + V(r)}_{S_{\text{pot}}} + \underbrace{\eta(t, \frac{\mathbf{r}}{2}) - \eta(t, -\frac{\mathbf{r}}{2})}_{\Theta(r,t)} \Big) \Big]$$

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The stochastic potential

University of Stavanger

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- Fully unitary microscopic dynamics, no thermalization

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University of Stavanger

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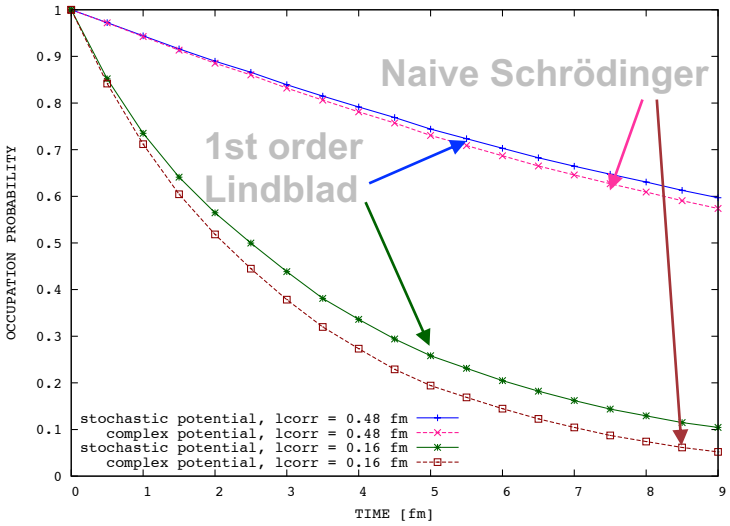
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S.Kajimoto, Y.Akamatsu, M. Asakawa, A.R., PRD97 (2018) 014003

The stochastic potential

University of Stavanger

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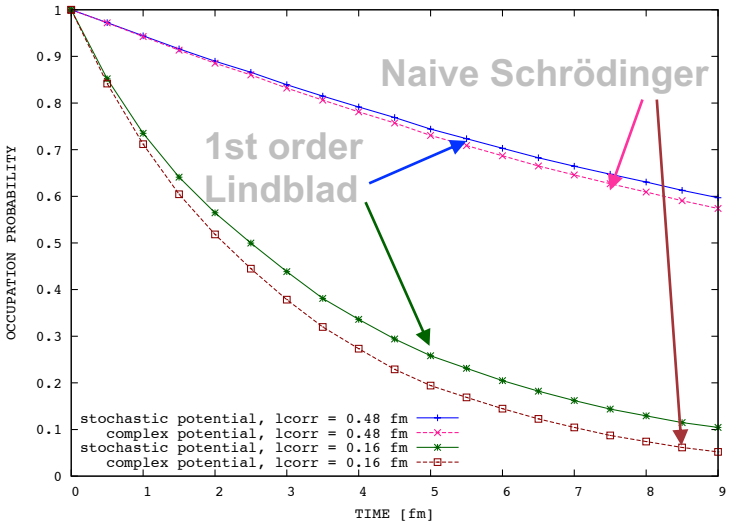
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S.Kajimoto, Y.Akamatsu, M. Asakawa, A.R., PRD97 (2018) 014003

- Fully unitary microscopic dynamics, no thermalization

$$i\partial_t \langle \psi_{Q\bar{Q}}(t)\rangle = \Big(-\frac{\nabla^2}{M} + Re[V] - i|Im[V]| \Big)\langle \psi_{Q\bar{Q}}(t)\rangle$$

Towards full Lindblad dynamics (I)

University of Stavanger

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- Unravel dynamics in wavefunction stochastic dynamics: **Quantum State Diffusion**

T. Miura, Y.Akamatsu, M. Asakawa, A.R., arXiv:1908.06293

$$|d\psi\rangle = |\psi(t+dt)\rangle - |\psi(t)\rangle$$
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$$-\sqrt{\frac{D(\mathbf{k})}{2}}\left[1 - \frac{\mathbf{k}}{4m_Q T}\cdot\left(\frac{1}{2}\mathbf{P}_{CM} - \mathbf{p}\right)\right] e^{-i\mathbf{k}\cdot\mathbf{r}/2}\left(1 \otimes T^a\right)$$

$$\tilde{D}(k) = g^2 T \frac{\pi m_D^2}{k(k^2+m_D^2)^2}, \quad m_D = gT\sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$$

weak coupling parameters, V(r) Debye form

Towards full Lindblad dynamics (I)

University of Stavanger

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- First “derivation” of phenomenological models based on nonlinear Schrödinger equation

c.f. e.g. R. Katz, P. Gossiaux Annals Phys. 368 (2016) 267

Towards full Lindblad dynamics (I)

University of Stavanger

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- First genuine Lindblad implementation: previous works could not maintain positivity of ρ

D. De Boni, JHEP 1708 (2017) 064

Towards full Lindblad dynamics (II)

University of Stavanger

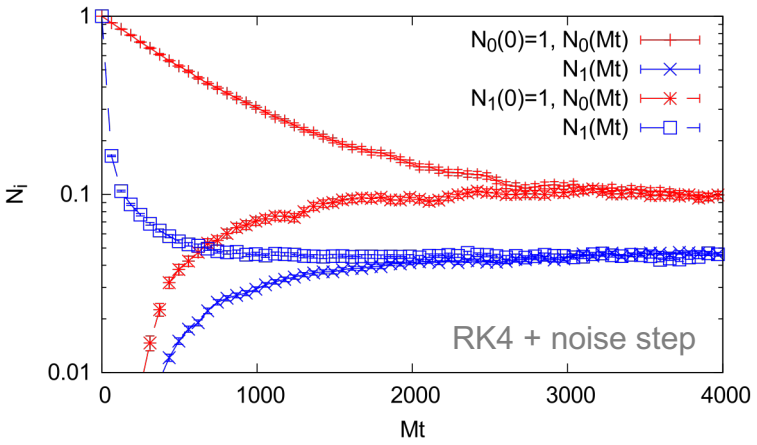
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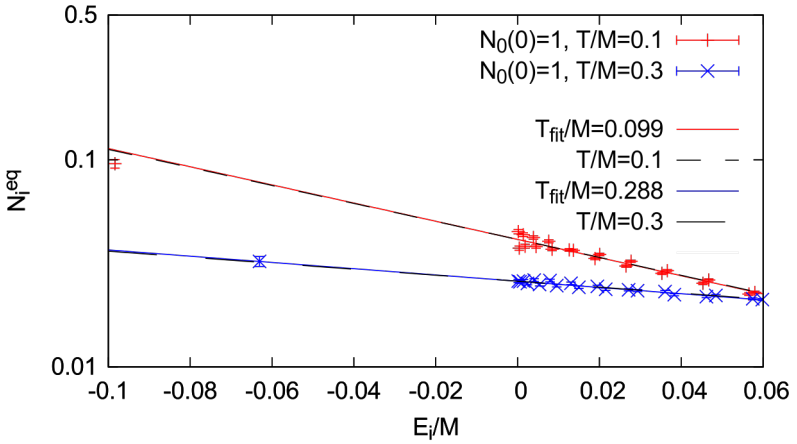
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Proof of principle in 1d





- Encouraging: admixtures become independent of initial conditions at late times
- Distribution of states at late times agrees well with Boltzmann and yields consistent T