

Inelastic Quarkonium Reaction Rates — Duke Approach

Xiaojun Yao

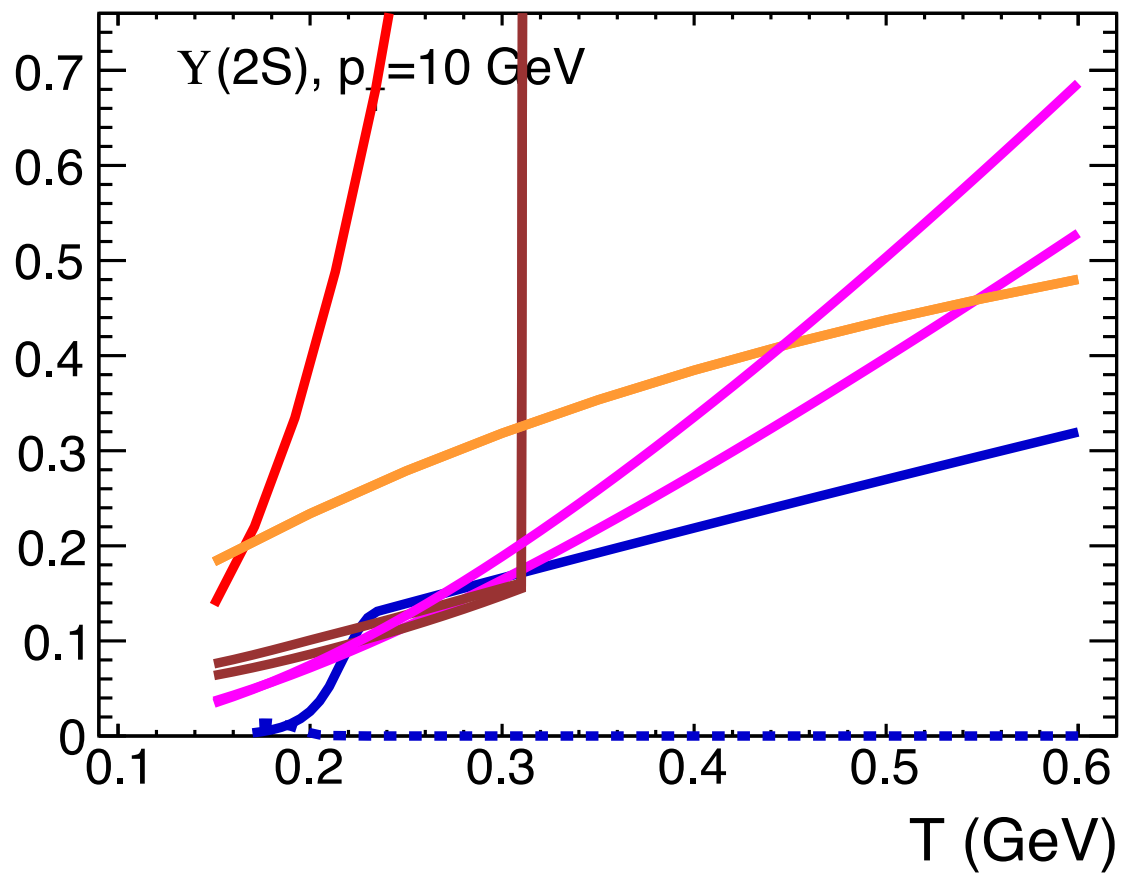
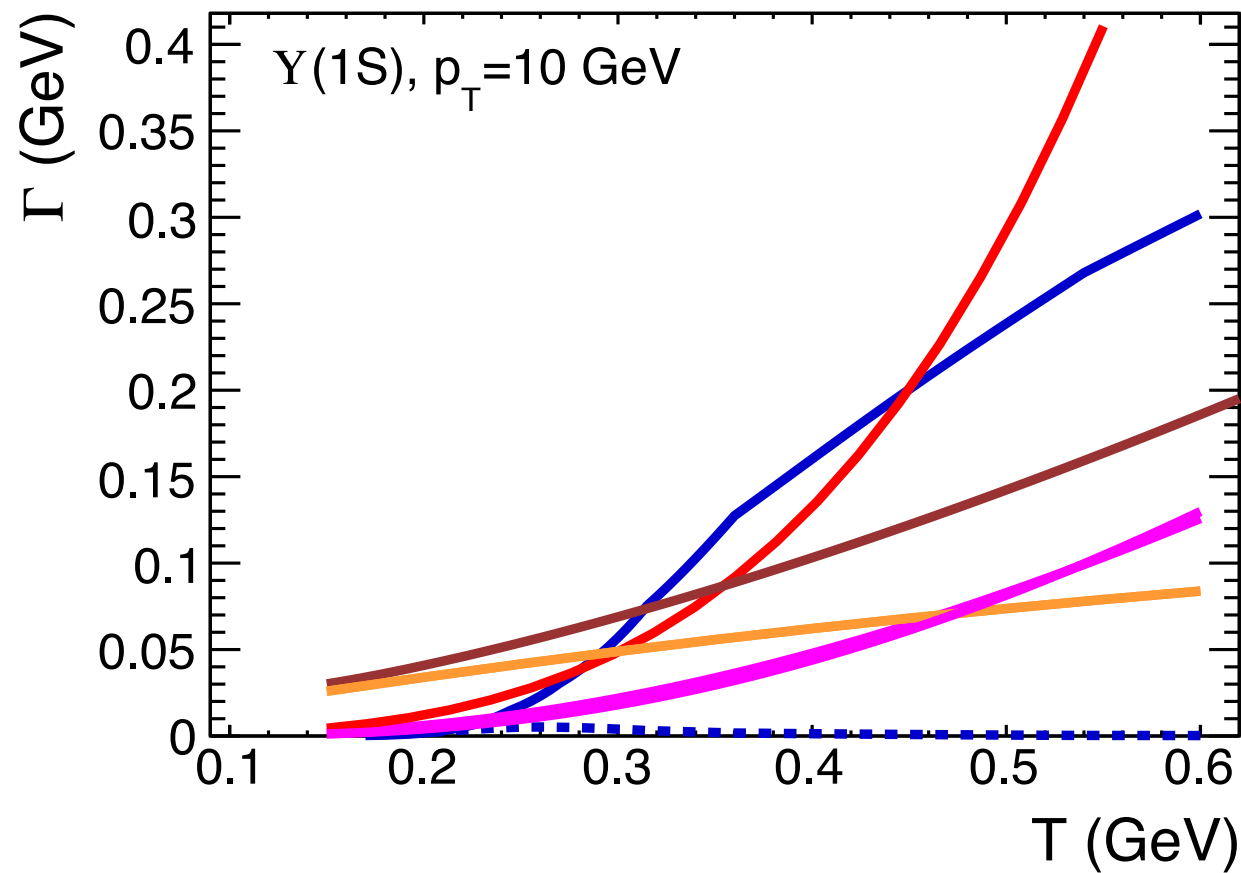
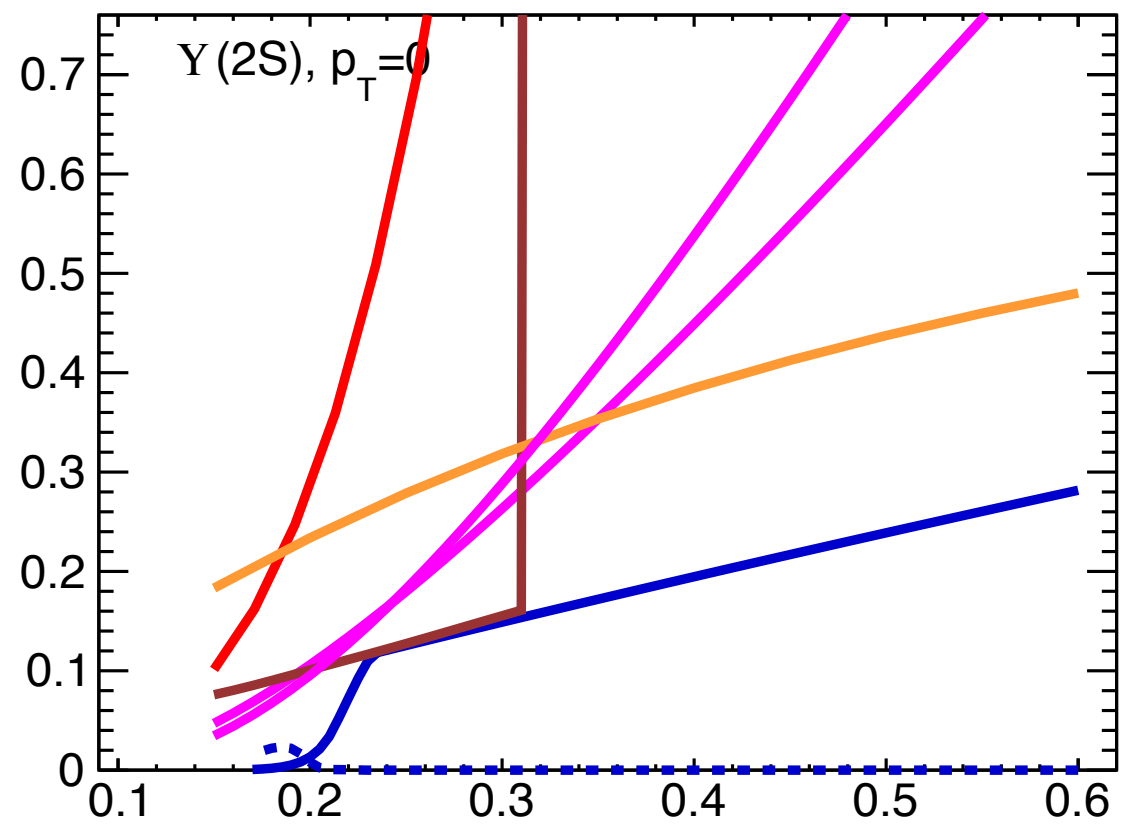
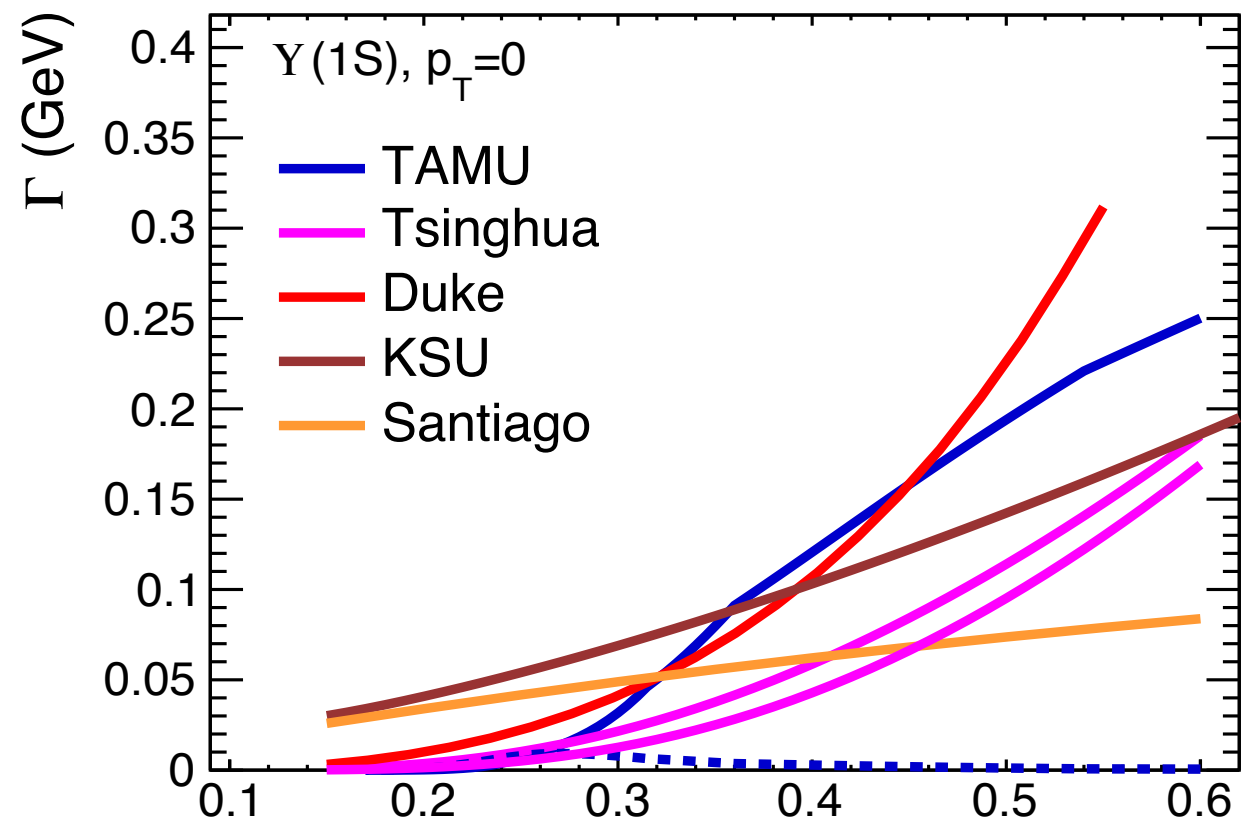
Collaborators: Berndt Müller, Steffen Bass, Thomas Mehen, Weiyao Ke, Yingru Xu

EMMI Rapid Reaction Task Force (RRTF)

Suppression and (re)generation of quarkonium in heavy-ion collisions at the LHC

GSI, Dec. 17, 2019





Overview

- EFT potential NRQCD: nonrelativistic expansion
- Contributing Feynman diagrams
 - Real gluon induced process (LO in g for QGP)
 - Virtual gluon induced process (NLO in g for QGP)
- Dependence on momentum, temperature, binding energy

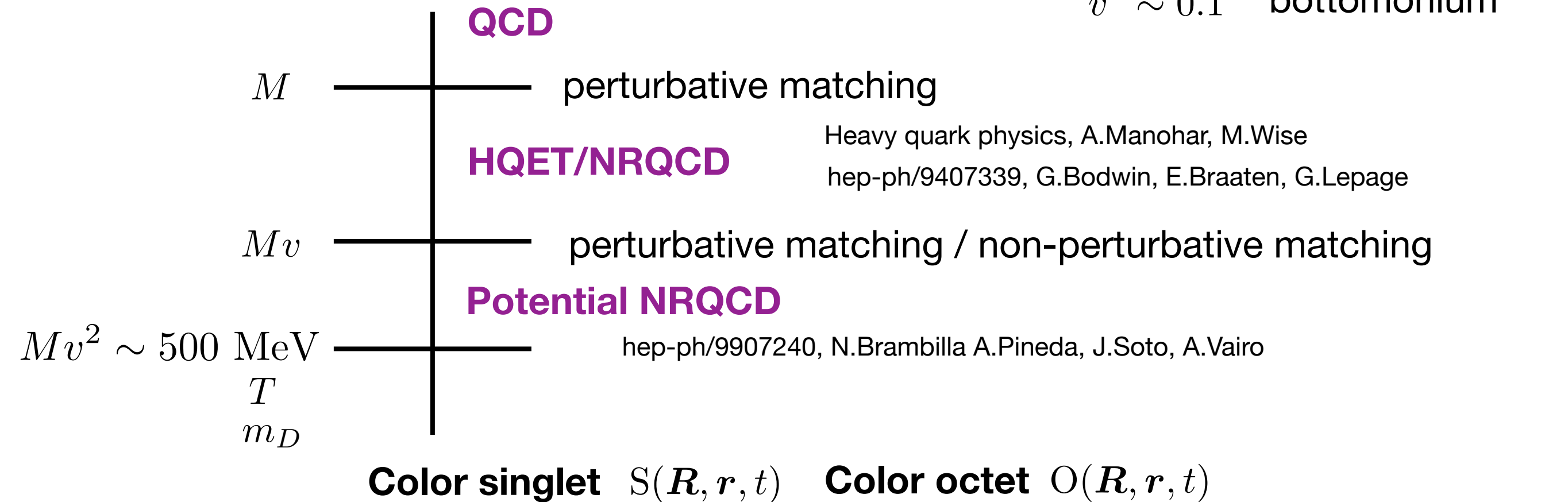
The potential NRQCD

Separation of scales

$$M \gg Mv \gg Mv^2 \gtrsim T \gtrsim m_D$$

$$v^2 \sim 0.3 \quad \text{charmonium}$$

$$v^2 \sim 0.1 \quad \text{bottomonium}$$



$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r \text{Tr} \left(S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O + V_A (O^\dagger \mathbf{r} \cdot g\mathbf{E} S + \text{h.c.}) + \frac{V_B}{2} O^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, O \} + \dots \right)$$

NR and multipole expansions

$$H_s = \frac{(i\nabla_{\text{cm}})^2}{4M} + \frac{(i\nabla_{\text{rel}})^2}{M} + V_s^{(0)} + \frac{V_s^{(1)}}{M} + \frac{V_s^{(2)}}{M^2} + \dots \longrightarrow H_{s,o} = \frac{(i\nabla_{\text{rel}})^2}{M} + V_{s,o}^{(0)}$$

$$H_o = \frac{(i\mathbf{D}_{\text{cm}})^2}{4M} + \frac{(i\nabla_{\text{rel}})^2}{M} + V_o^{(0)} + \frac{V_o^{(1)}}{M} + \frac{V_o^{(2)}}{M^2} + \dots$$

- **Virial theorem**
- **No hyperfine splitting**

The potential NRQCD: Dipole Interaction

Separation of scales $M \gg Mv \gg Mv^2 \gtrsim T \gtrsim m_D$

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r \text{Tr} \left(S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O + \boxed{V_A (O^\dagger \mathbf{r} \cdot g\mathbf{E} S + \text{h.c.})} + \frac{V_B}{2} O^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, O \} + \dots \right)$$

Dipole interaction $r \sim \frac{1}{Mv}$

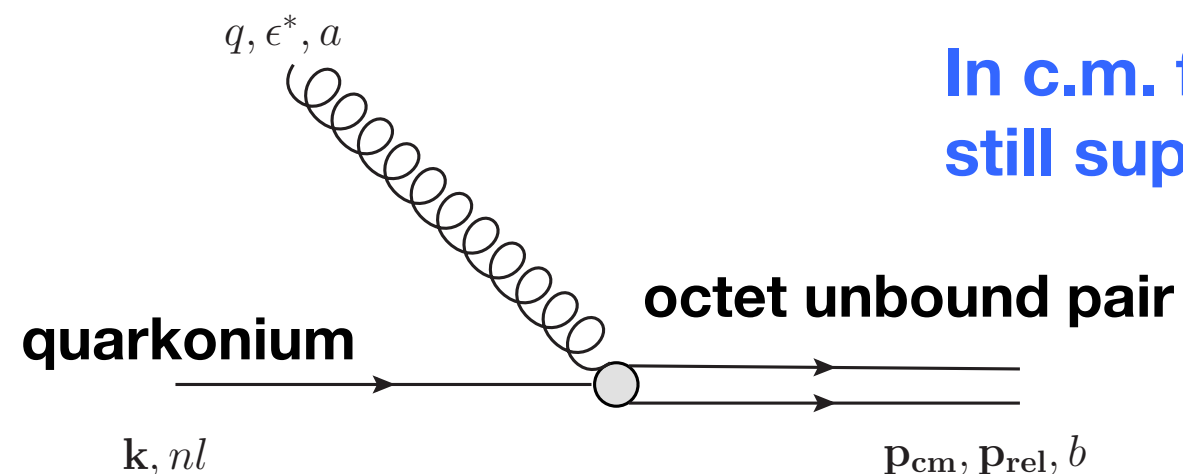
Weak coupling between quarkonium and QGP: quarkonium small in size

When at rest in medium $rT \sim v$ suppressed

Moving quarkonium? in c.m. frame, gluon more energetic

Require $rT \frac{1}{\sqrt{1 - v_{\text{med}}^2}} \sim \sqrt{v}$

In c.m. frame, dipole vertex and c.m. energy still suppressed if $v_{\text{med}} \lesssim \sqrt{1 - v}$



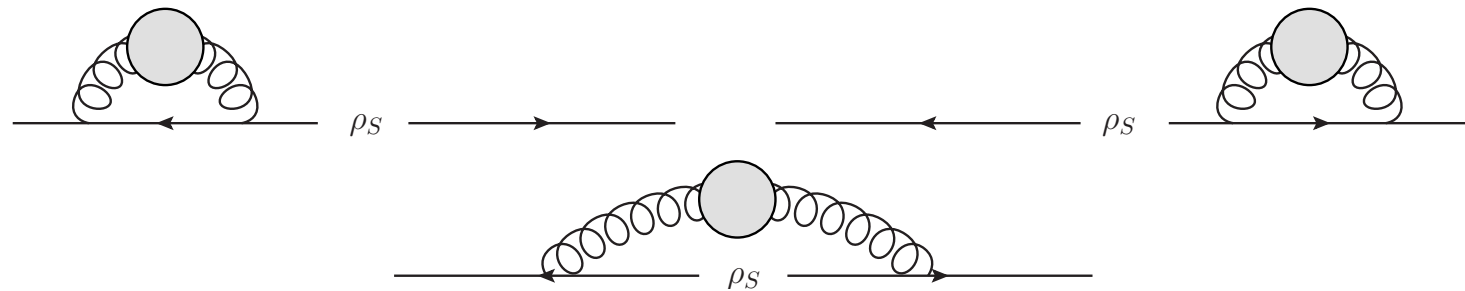
$$g \sqrt{\frac{T_F}{N_c}} \delta^{ab} (q^0 g^{\mu i} - q^i g^{\mu 0}) \langle \Psi_{\mathbf{p}_{\text{rel}}} | r^i | \psi_{nl} \rangle$$

From Open Quantum System to Transport Equation

Lindblad equation:

Correction to Hamiltonian

$$\rho_S(t) = \rho_S(0) - i \left[t H_S + \sum_{a,b} \sigma_{ab}(t) L_{ab}, \rho_S(0) \right] + \sum_{a,b,c,d} \gamma_{ab,cd} \left(L_{ab} \rho_S(0) L_{cd}^\dagger - \frac{1}{2} \{ L_{cd}^\dagger L_{ab}, \rho_S \} \right)$$



$$\frac{T_F}{N_C} g^2 \langle E_{i_1}^{b_1}(\mathbf{R}_1, t_1) E_{i_2}^{b_2}(\mathbf{R}_2, t_2) \rangle_T$$

Recombination

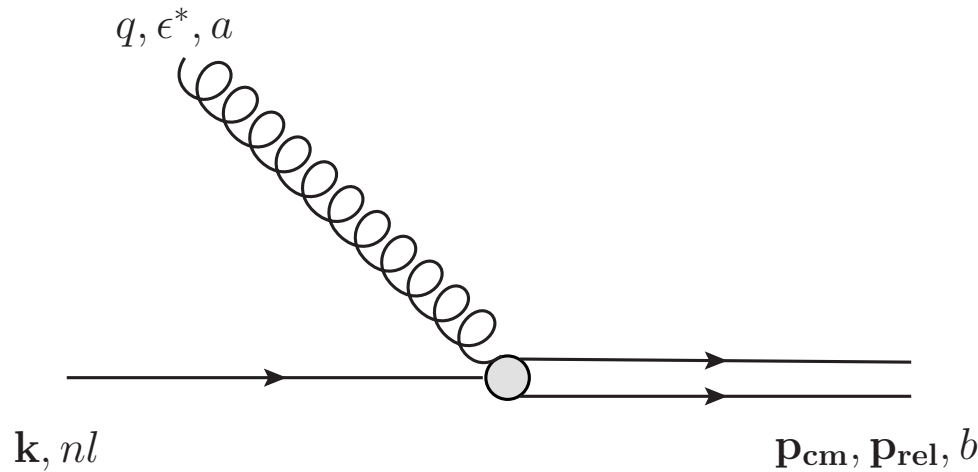
Dissociation

Boltzmann transport equation

$$\frac{\partial}{\partial t} f_{nls}(\mathbf{x}, \mathbf{k}, t) + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_{nls}(\mathbf{x}, \mathbf{k}, t) = C_{nls}^{(+)}(\mathbf{x}, \mathbf{k}, t) - C_{nls}^{(-)}(\mathbf{x}, \mathbf{k}, t)$$

LO Medium Description

$$\frac{T_F}{N_C} g^2 \langle E_{i_1}^{b_1}(\mathbf{R}_1, t_1) E_{i_2}^{b_2}(\mathbf{R}_2, t_2) \rangle_T$$



$$E_k = -|E_{nl}|$$

$$E_p = \frac{p_{\text{rel}}^2}{M}$$

binding energy

E&p conservation

$$C_{nl}^{(-)} \equiv \int \frac{d^3 p_{\text{cm}}}{(2\pi)^3} \frac{d^3 p_{\text{rel}}}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3 2q} n_B(q) (2\pi)^4 \delta^3(\mathbf{k} - \mathbf{p}_{\text{cm}} + \mathbf{q}) \delta(E_k - E_p + q)$$

Phase space measure

$$\frac{2}{3} C_F q^2 g^2 |\langle \psi_{nl} | \mathbf{r} | \Psi_{\mathbf{p}_{\text{rel}}} \rangle|^2 f_{nl}(\mathbf{x}, \mathbf{k}, t)$$

Amplitude squared

For Coulomb, neglect octet potential → Peskin-Bhanot

$$C_{nl}^{(+)} \equiv \frac{1}{9} \int \frac{d^3 p_{\text{cm}}}{(2\pi)^3} \frac{d^3 p_{\text{rel}}}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3 2q} (1 + n_B(q)) f_Q(\mathbf{x}_1, \mathbf{p}_1, t) f_{\bar{Q}}(\mathbf{x}_2, \mathbf{p}_2, t)$$

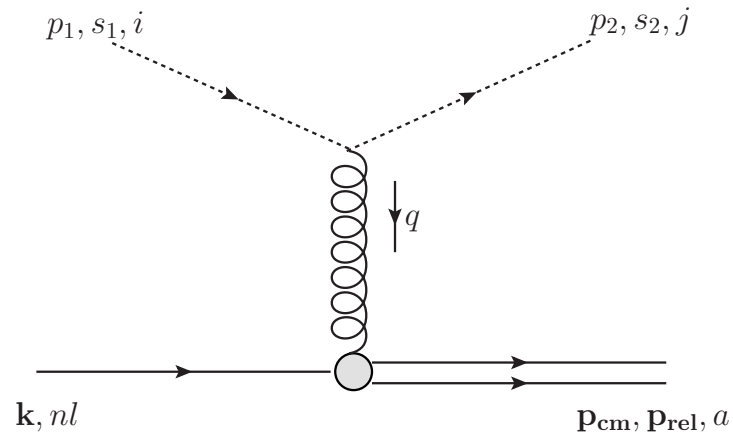
Real-time heavy quark distributions

$$(2\pi)^4 \delta^3(\mathbf{k} - \mathbf{p}_{\text{cm}} + \mathbf{q}) \delta(-|E_{nl}| + q - \frac{p_{\text{rel}}^2}{M}) \frac{2}{3} C_F q^2 g^2 |\langle \psi_{nl} | \mathbf{r} | \Psi_{\mathbf{p}_{\text{rel}}} \rangle|^2$$

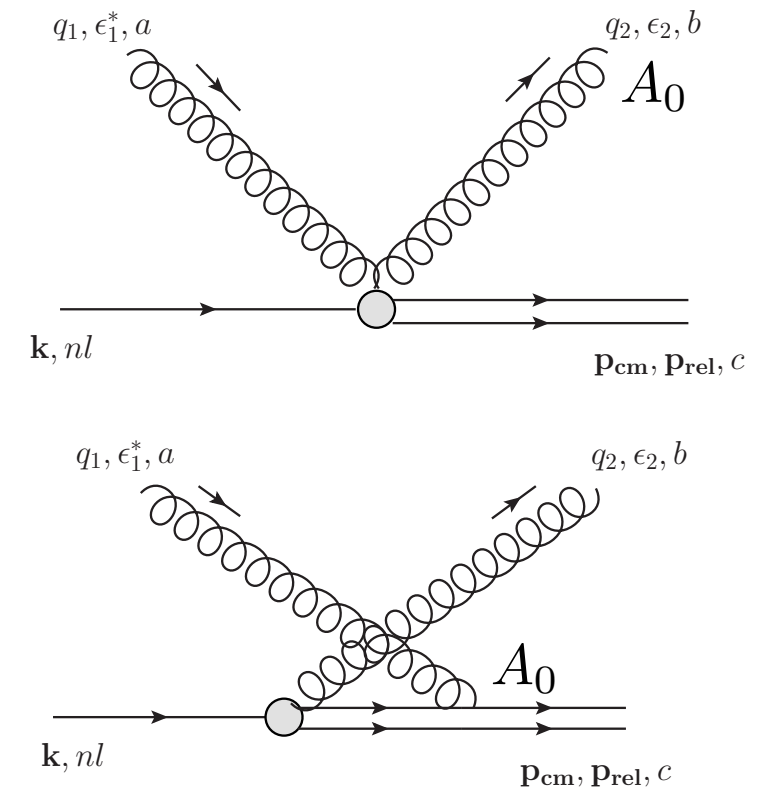
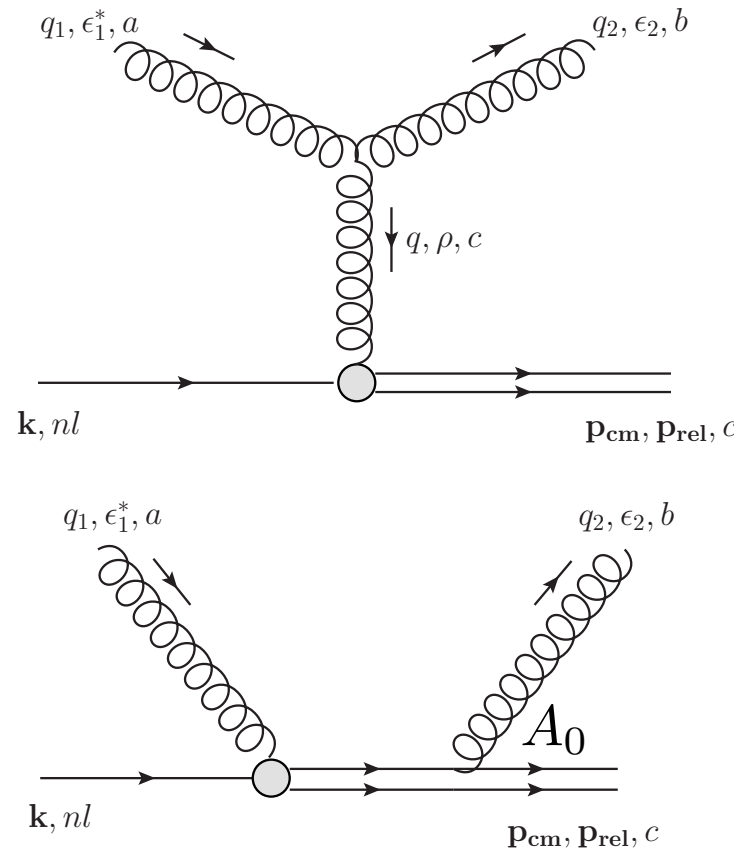
NLO Medium Description

$$\frac{T_F}{N_C} g^2 \langle E_{i_1}^{b_1}(\mathbf{R}_1, t_1) E_{i_2}^{b_2}(\mathbf{R}_2, t_2) \rangle_T$$

Light quark



Gluon



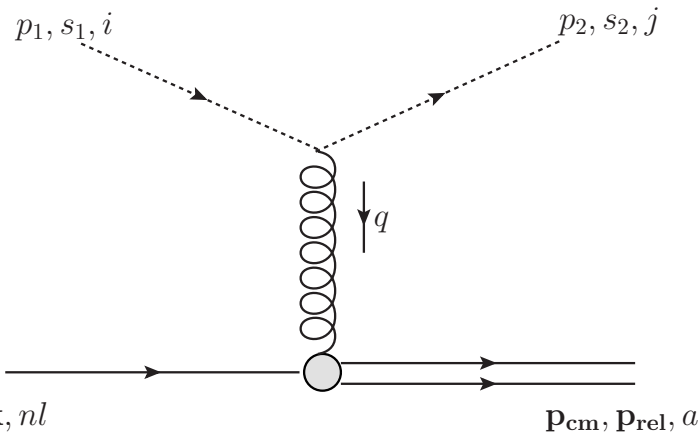
Ward identity

$$i(q_1)_\mu \sum \mathcal{M}^{\mu\nu}(\epsilon_2)_\nu = 0 + \mathcal{O}(v^2)$$

Reaction rates written as phase integral + E&p conservation + amplitude squared
Cross sections are not needed

Infrared Safety

$$\sum |\mathcal{M}_{(b)}|^2 \equiv \sum_{a,i,j} \sum_{s_1,s_2} \sum_{u,\bar{u},d,\bar{d}} |\mathcal{M}_{(b)}|^2 = \frac{16}{3} g^4 V_A^2 T_F C_F |\langle \Psi_{\mathbf{p}_{\text{rel}}} | \mathbf{r} | \psi_{nl} \rangle|^2$$

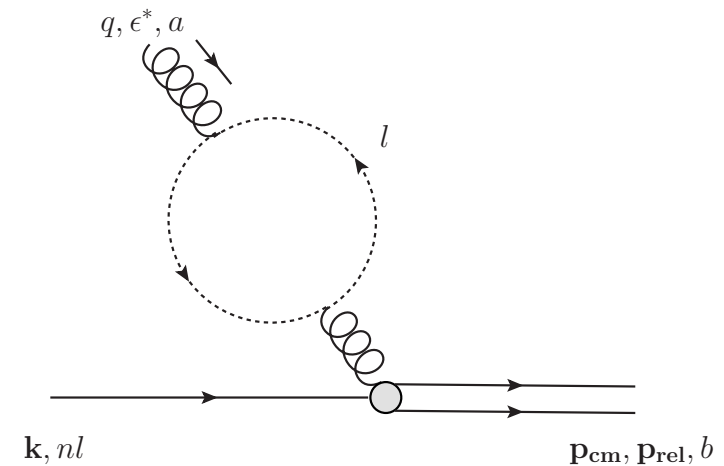
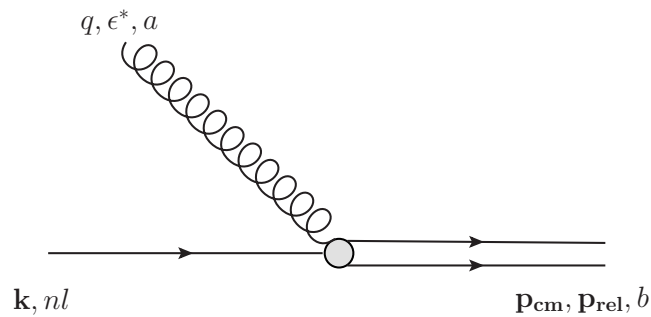


$$\left[\frac{p_1 p_2 + \mathbf{p}_1 \cdot \mathbf{p}_2}{q^2} + \frac{2(q^0)^2 (p_1 p_2 - \mathbf{p}_1 \cdot \hat{\mathbf{q}} \cdot \mathbf{p}_2 \cdot \hat{\mathbf{q}})}{((q^0)^2 - \mathbf{q}^2 + i\epsilon)^2} \right]$$

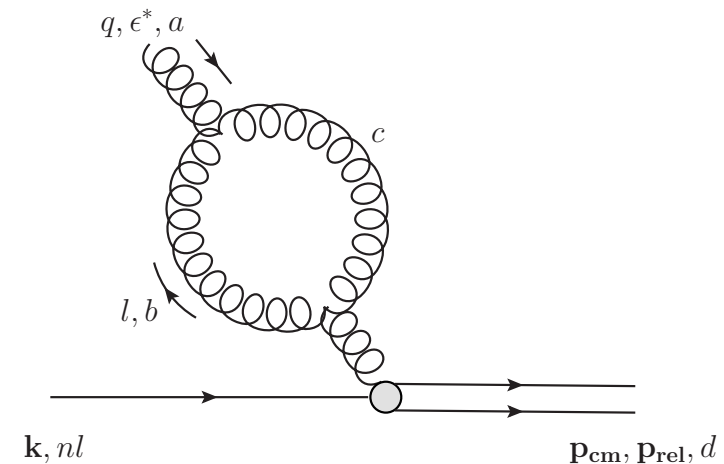
**Infrared safe:
finite binding energy**

**Soft safe: finite binding energy
Collinear divergent**

Collinear divergence cancelled by interference



+ more



Dependence on Parameters

- Temperature: only in distributions n_B , $1+n_B$, $n_B(1+n_B)$, $n_F(1-n_F)$
- Binding energy: part of energy conservation, give minimum energy required for reactions to happen, serve as IR regulator in t-channel processes
- Quarkonium momentum: calculate rates in rest frame of quarkonium or heavy quark pair, then boost to lab frame; medium is boosted, gluon more energetic (make sure NR expansion valid, charmonium pT range)
- Dissociation rate of $Y(2S)$ blows up $\rightarrow$ breakdown of multipole expansion; interpretation of $Y(2S)$ changes

Conclusions

- The potential NRQCD provides controlled way to calculate reaction rates when there is a separation of scales; NR is crucial for Schrodinger equation
- NLO corrections in multipole expansion? r^2 terms of pNRQCD are known, need to calculate rates; other ways to go beyond dipole
- New process in NLO in multipole expansion: diffusion



$$3\kappa \equiv \int \frac{d^3 k_2}{(2\pi)^3} \int \frac{d^3 q_1}{2q_1(2\pi)^3} \int \frac{d^3 q_2}{2q_2(2\pi)^3} n_B(q_1)(1 + n_B(q_2)) \\ (q_1 - q_2)^2 (2\pi)^4 \delta^3(\mathbf{k}_1 + \mathbf{q}_1 - \mathbf{k}_2 - \mathbf{q}_2) \delta(q_1 - q_2) \sum |\mathcal{M}|^2$$

Diffusion coefficient of $Y(1S) \sim 0.05$, but could be important for J/ψ and $Y(2S)$ because of the breakdown of multipole expansion, include it when extracting dissociation rate from lattice?

Coupled with Transport of Open Heavy Flavor

heavy quark

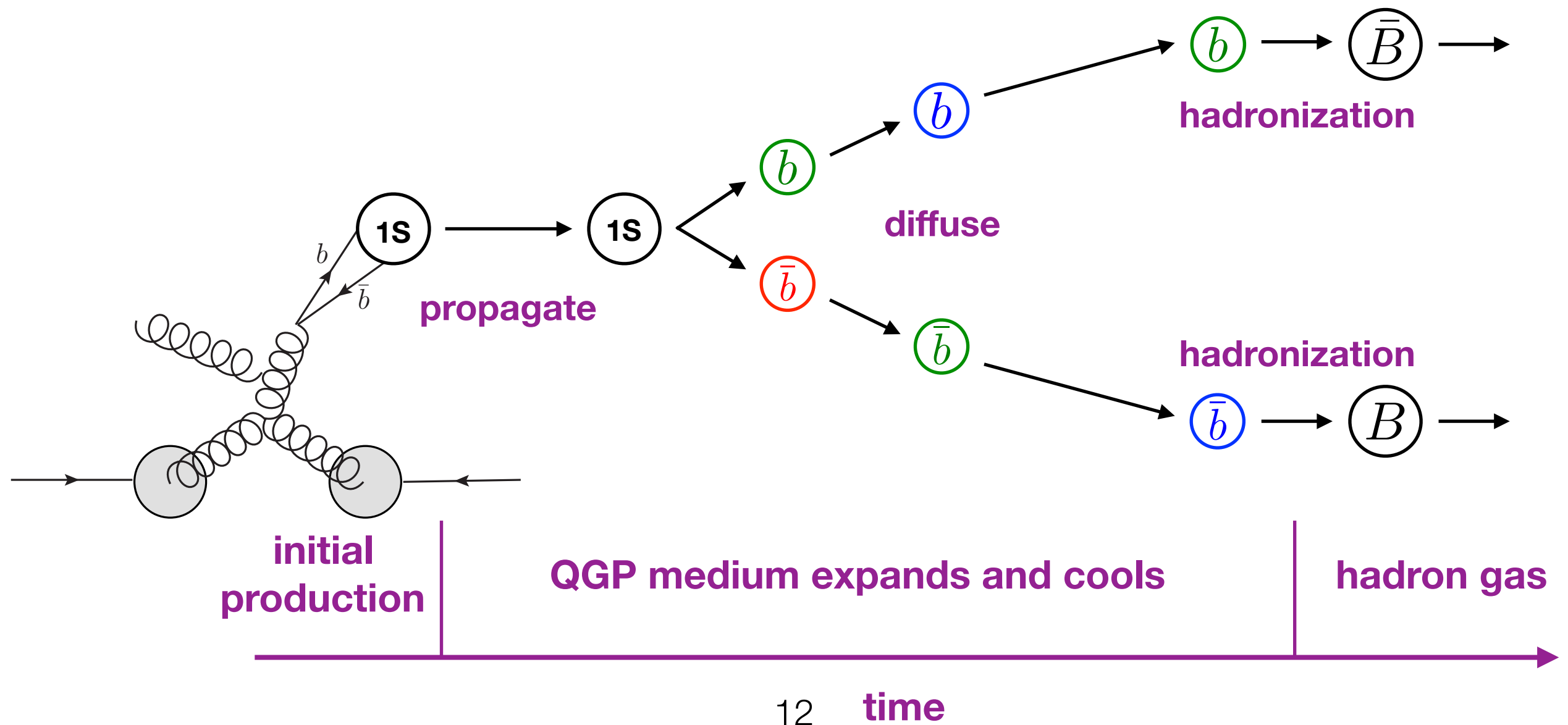
$$\left(\frac{\partial}{\partial t} + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}}\right) f_Q(\mathbf{x}, \mathbf{p}, t) = \mathcal{C}_Q - \mathcal{C}_Q^+ + \mathcal{C}_Q^-$$

anti-heavy quark

$$\left(\frac{\partial}{\partial t} + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}}\right) f_{\bar{Q}}(\mathbf{x}, \mathbf{p}, t) = \mathcal{C}_{\bar{Q}} - \mathcal{C}_{\bar{Q}}^+ + \mathcal{C}_{\bar{Q}}^-$$

each quarkonium state
nl = 1S, 2S, 1P etc.

$$\left(\frac{\partial}{\partial t} + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}}\right) f_{nls}(\mathbf{x}, \mathbf{p}, t) = \mathcal{C}_{nls}^+ - \mathcal{C}_{nls}^-$$



Coupled with Transport of Open Heavy Flavor

heavy quark

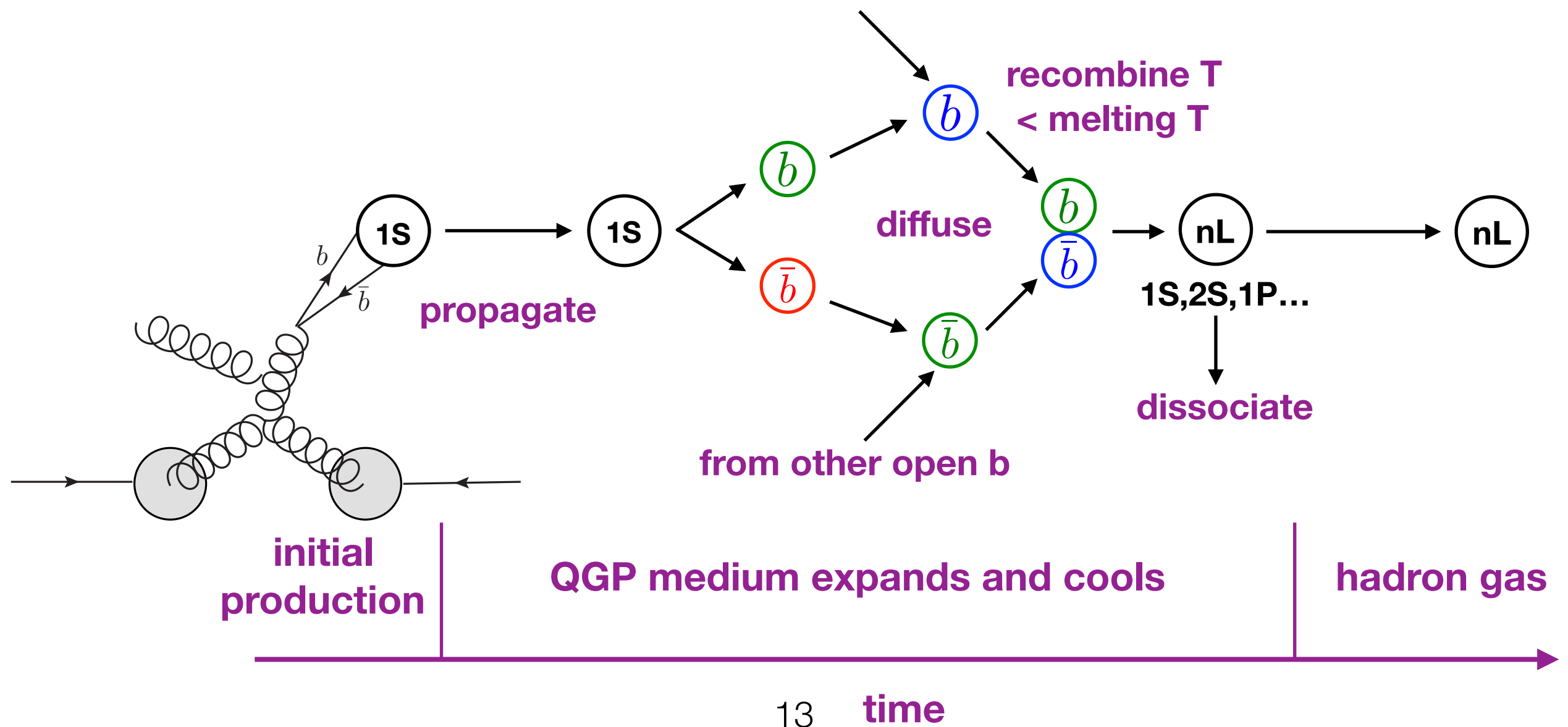
$$\left(\frac{\partial}{\partial t} + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}}\right) f_Q(\mathbf{x}, \mathbf{p}, t) = \mathcal{C}_Q - \mathcal{C}_Q^+ + \mathcal{C}_Q^-$$

anti-heavy quark

$$\left(\frac{\partial}{\partial t} + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}}\right) f_{\bar{Q}}(\mathbf{x}, \mathbf{p}, t) = \mathcal{C}_{\bar{Q}} - \mathcal{C}_{\bar{Q}}^+ + \mathcal{C}_{\bar{Q}}^-$$

each quarkonium state
nl = 1S, 2S, 1P etc.

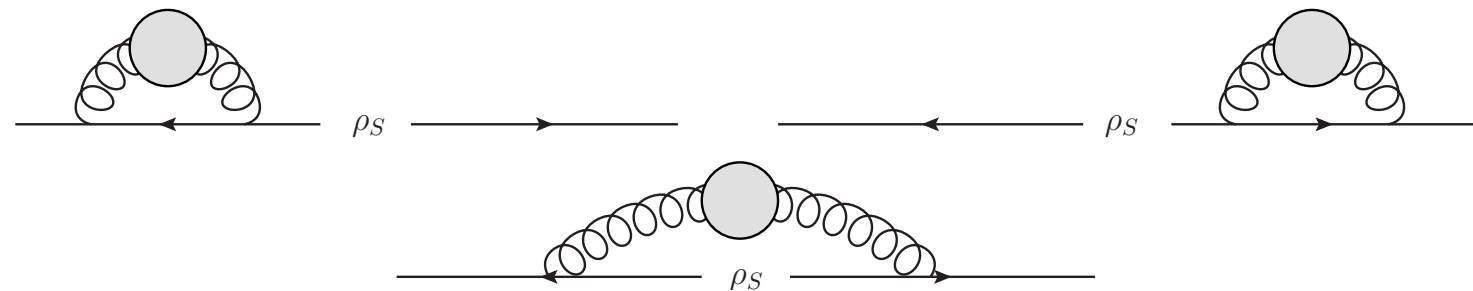
$$\left(\frac{\partial}{\partial t} + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}}\right) f_{nls}(\mathbf{x}, \mathbf{p}, t) = \mathcal{C}_{nls}^+ - \mathcal{C}_{nls}^-$$



From Open Quantum System to Transport Equation

Weak coupling to 2nd order: Lindblad equation

$$\rho_S(t) = \rho_S(0) - i \left[t H_S + \sum_{a,b} \sigma_{ab}(t) L_{ab}, \rho_S(0) \right] + \sum_{a,b,c,d} \gamma_{ab,cd} \left(L_{ab} \rho_S(0) L_{cd}^\dagger - \frac{1}{2} \{ L_{cd}^\dagger L_{ab}, \rho_S \} \right)$$



$$H_I = \sum_{\alpha} O_{\alpha}^{(S)} \otimes O_{\alpha}^{(E)}$$

$$\gamma_{ab,cd}(t) \equiv \sum_{\alpha,\beta} \int_0^t dt_1 \int_0^t dt_2 C_{\alpha\beta}(t_1, t_2) \langle a | O_{\beta}^{(S)}(t_2) | b \rangle \langle c | O_{\alpha}^{(S)}(t_1) | d \rangle^*$$

$$\sigma_{ab}(t) \equiv \frac{-i}{2} \sum_{\alpha,\beta} \int_0^t dt_1 \int_0^t dt_2 C_{\alpha\beta}(t_1, t_2) \text{sign}(t_1 - t_2) \langle a | O_{\alpha}^{(S)}(t_1) O_{\beta}^{(S)}(t_2) | b \rangle$$

Boltzmann transport equation

$$C_{\alpha\beta}(t_1, t_2) \equiv \text{Tr}_E(O_{\alpha}^{(E)}(t_1) O_{\beta}^{(E)}(t_2) \rho_E)$$

$|a\rangle$ Eigenstates of H_S

$$L_{ab} = |a\rangle \langle b|$$

Dissociation $-\gamma_{ab,cd} \frac{1}{2} \{L_{cd}^\dagger L_{ab}, \rho_S(0)\}$

For $L_{cd}^\dagger L_{ab} \rho_S$: $|d\rangle = |\mathbf{k}_1, n_1 l_1, 1\rangle$ $|a\rangle = |c\rangle = |\mathbf{p}_{\text{cm}}, \mathbf{p}_{\text{rel}}, a_1\rangle$ $|b\rangle = |\mathbf{k}_3, n_3 l_3, 1\rangle$

Linear order in r : transition between bound singlet & unbound octet

$$\gamma_{ab,cd} = \int d^3 R_1 \int d^3 R_2 \sum_{i_1, i_2, b_1, b_2} \int_0^t dt_1 \int_0^t dt_2 C_{\mathbf{R}_1 i_1 b_1, \mathbf{R}_2 i_2 b_2}(t_1, t_2)$$

$$\langle \mathbf{k}_1, n_1 l_1, 1 | \langle S(\mathbf{R}_1, t_1) | r_{i_1} | O^{b_1}(\mathbf{R}_1, t_1) \rangle | \mathbf{p}_{\text{cm}}, \mathbf{p}_{\text{rel}}, a_1 \rangle$$

$$\langle \mathbf{p}_{\text{cm}}, \mathbf{p}_{\text{rel}}, a_1 | \langle O^{b_2}(\mathbf{R}_2, t_2) | r_{i_2} | S(\mathbf{R}_2, t_2) \rangle | \mathbf{k}_3, n_3 l_3, 1 \rangle$$

$$\langle \Psi_{\mathbf{p}_{\text{rel}}} | r_{i_2} | \psi_{n_3 l_3} \rangle \delta^{a_1 b_2} e^{-i(E_{\mathbf{k}_3} t_2 - \mathbf{k}_3 \cdot \mathbf{R}_2)} e^{i(E_{\mathbf{p}} t_2 - \mathbf{p}_{\text{cm}} \cdot \mathbf{R}_2)}$$

**Weakly-coupled
plasma**

$$\frac{T_F}{N_C} g^2 \langle E_{i_1}^{b_1}(\mathbf{R}_1, t_1) E_{i_2}^{b_2}(\mathbf{R}_2, t_2) \rangle_T$$

$$= \frac{T_F}{N_C} g^2 \delta^{b_1 b_2} \int \frac{d^4 q}{(2\pi)^4} e^{i q_0 (t_1 - t_2) - i \mathbf{q} \cdot (\mathbf{R}_1 - \mathbf{R}_2)} (q_0^2 \delta_{i_1 i_2} - q_{i_1} q_{i_2}) n_B(q_0) (2\pi) \text{sign}(q_0) \delta(q_0^2 - \mathbf{q}^2)$$

Markovian approximation:

$t \longrightarrow \infty$ when doing time integral $\rightarrow$ time length + energy conservation

Recombination $\gamma_{ab,cd} L_{ab} \rho_S(0) L_{cd}^\dagger$

Phase space measure

E&p conservation

$$t \int \frac{d^3 p_{\text{cm}}}{(2\pi)^3} \frac{d^3 p_{\text{rel}}}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3 2q} (1 + n_B(q)) \sum_{a,i} (2\pi)^4 \delta^3(\mathbf{k} - \mathbf{p}_{\text{cm}} + \mathbf{q}) \delta(-|E_{nl}| + q - \frac{\mathbf{p}_{\text{rel}}^2}{M})$$

$$\frac{2T_F}{3N_C} q^2 g^2 \langle \psi_{nl} | r_i | \Psi_{\mathbf{p}_{\text{rel}}} \rangle \int d^3 r \psi_{nl}(\mathbf{r}) r_i \Psi_{\mathbf{p}_{\text{rel}}}^*(\mathbf{r}) f_{Q\bar{Q}}(\mathbf{x}, \mathbf{p}_{\text{cm}}, \mathbf{r}, \mathbf{p}_{\text{rel}}, a, t = 0)$$

$$p_{\text{rel}} \sim a_B^{-1} \sim Mv$$

Amplitude squared, mixed with distribution function
hard to implement numerically

When can we take distribution function out?

Uniformly distributed when $r < \text{Bohr radius } a_B$ Diffusion length $\gg$ Bohr radius

$$p_{\text{rel}} \ll \frac{Mv}{\alpha_s^2 v^2}$$

BBGKY hierarchy $f_{Q\bar{Q}}(\mathbf{x}, \mathbf{p}_{\text{cm}}, \mathbf{r}, \mathbf{p}_{\text{rel}}, a, t) = \frac{1}{9} f_Q(\mathbf{x}_1, \mathbf{p}_1, t) f_{\bar{Q}}(\mathbf{x}_2, \mathbf{p}_2, t)$

$$tC_{nl}^{(+)} \equiv t \frac{1}{9} \int \frac{d^3 p_{\text{cm}}}{(2\pi)^3} \frac{d^3 p_{\text{rel}}}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3 2q} (1 + n_B(q)) f_Q(\mathbf{x}_1, \mathbf{p}_1, t) f_{\bar{Q}}(\mathbf{x}_2, \mathbf{p}_2, t)$$

$$(2\pi)^4 \delta^3(\mathbf{k} - \mathbf{p}_{\text{cm}} + \mathbf{q}) \delta(-|E_{nl}| + q - \frac{\mathbf{p}_{\text{rel}}^2}{M}) \frac{2}{3} C_F q^2 g^2 |\langle \psi_{nl} | \mathbf{r} | \Psi_{\mathbf{p}_{\text{rel}}} \rangle|^2$$

Spin

$$g_s = \frac{3}{4}, \frac{1}{4}$$