Inelastic quarkonium reaction rates "Saclay" approach

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17th of December, 2019

Motivation

Original motivation:

- To derive from a QFT computation an evolution equation which minimizes the free energy (instead of maximaxim the entropy). Maximum entropy will always tend to have very few singlets at large times.
- In order to do this we found that it is crucial to take into account the binding energy of the singlet and its relation to the temperature.
- We focus on Upsilon. The case of charmonium is different because typically the binding energy is of the order of the decay width.

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Approximations

- One gluon exchange. No chromomagnetic fields (Coulomb gauge).
- Our approach is valid if the binding energy (or the gap) is much bigger than the decay width. Leads to rate equation (instead of quantum master equation that has to be solved numerically).
- Octet potential is zero in the large N_c limit.

Reaction rate

Crucial point, they decay of singlets into octets is suppressed at low temperatures

$$\frac{\Gamma_{s\to o}}{\Gamma_{o\to s}} = e^{\frac{E_s - E_o}{T}},$$

consequence of fluctuation-dissipation theorem.

$$\Gamma_s = 2g^2 C_F \int_{\mathbf{p}} \mathrm{e}^{-\frac{E^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}}{T}} \int_{\mathbf{q}} \Delta^{>}(E^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}, \mathbf{q}) |\langle \mathrm{s}| \sin\left(\frac{\mathbf{q}\hat{\mathbf{r}}}{2}\right) |\mathrm{o}, \mathbf{p}\rangle|^2 ,$$

- $|s\rangle$ and E^s are respectively the wave function and the binding energy of the bound state. They are obtained solving the Schrödinger equation with a screened potential. Therefore, include a non-trivial dependence with the temperature.
- $|o, \mathbf{p}\rangle$ and $E^0_{\mathbf{p}}$ are the wave function and the energy of a quark-antiquark pair in an octet state. In the large N_c limit, $|o, \mathbf{p}\rangle$ is a plain wave and $E^0_{\mathbf{p}} = \frac{p^2}{M}$.

Reaction rate

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- g^2 is $4\pi\alpha_s(2\pi T)$

$$\Delta^{>}(w, \mathbf{q}) = \frac{\mathrm{e}^{w/T}}{\mathrm{e}^{w/T} - 1} \sigma(w, \mathbf{q}) ,$$

where

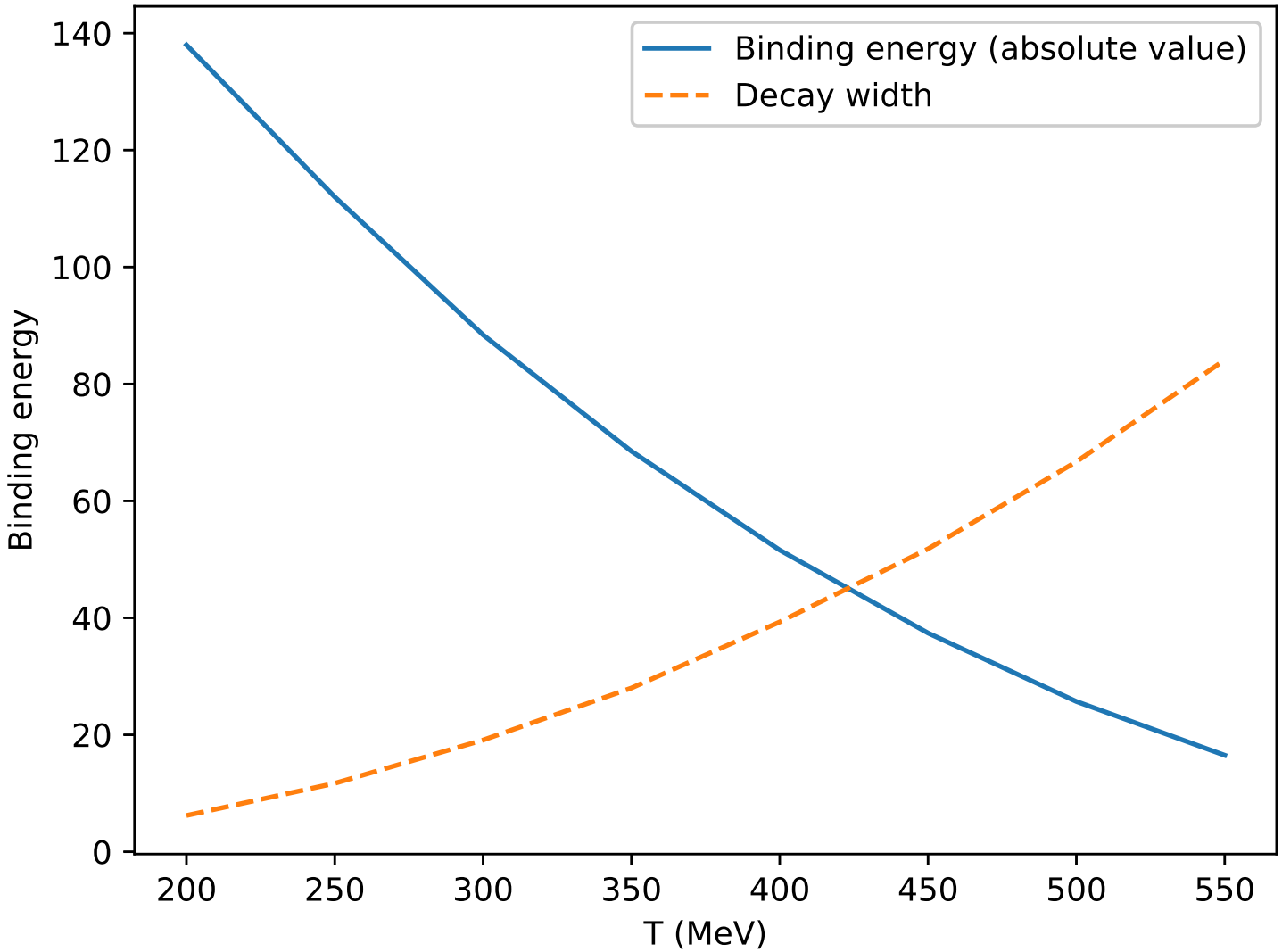
$$\sigma(\omega, \mathbf{q}) = \frac{2\mathrm{Im}\Pi_L(\omega, \mathbf{q})}{(q^2 + \mathrm{Re}\Pi_L(\omega, \mathbf{q}))^2 + (\mathrm{Im}\Pi_L(\omega, \mathbf{q}))^2} ,$$

where Π_L is the usual HTL self-energy of the temporal gluon.
Temperature dependence encoded in the value of the Debye mass.

Summary

Non trivial temperature dependence encoded in various terms. It is not possible to give a simple analytical formula.

Temperature dependence



Temperature dependence. Parametrization.

We are able to find a formula that reproduces the temperature dependence of the decay width. Obtained by fitting our numerically obtained results. Useful for phenomenological analyses.

$$\Gamma(1S) = aT + bT^2\,,$$

where $a = -0.04257301$ and $b = 0.00035313\,MeV^{-1}$.

Remarks

Charmonium vs Bottomonium

We did not study charmonium. This is because the condition that the decay width is much smaller than the binding energy is not fulfilled for LHC temperatures.

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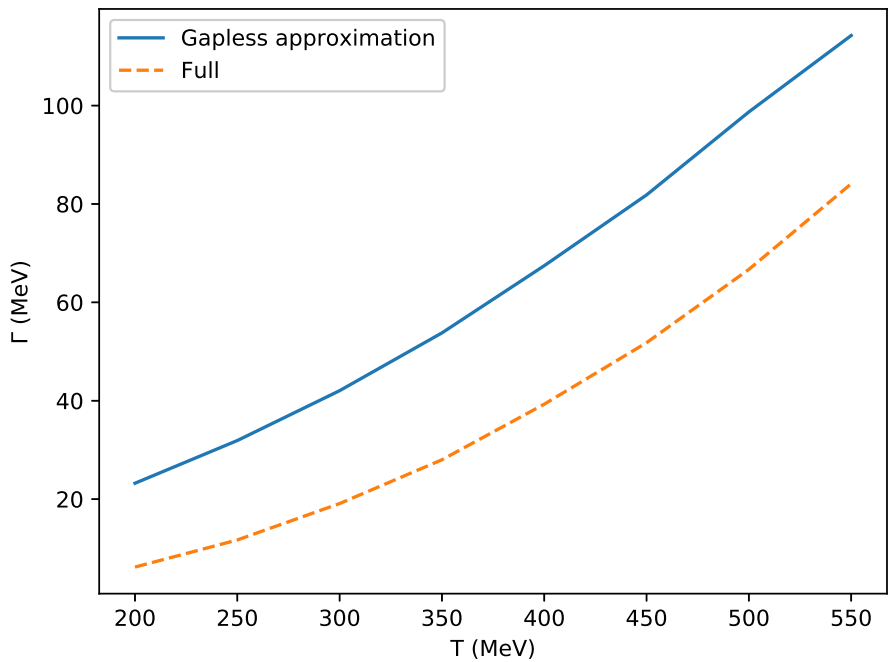
Effect of the gap in the decay width



This is important as it illustrates the energy dependence of the decay width.