Theory of quarkonia in QCD matter

Suppression and (re)generation
of quarkonium in heavy-ion collisions at the LHC

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Quarkonia as 'hard probes' (1)

Heavy quarks are produced in pairs in the early stages of URHIC. Their number remains constant.

Formation time of a $Q\bar{Q}$ pair is small $\qquad \Delta t \sim \frac{1}{2M_Q}$

J/Ψ	$M_c \simeq 1.5$ Gev	$\Delta t \simeq 0.07$ fm/c
Υ	$M_b \simeq 4.5$ Gev	$\Delta t \simeq 0.02$ fm/c

Mass is large compared to the typical temperature

$$M_Q \gg T$$

Dynamics of heavy quarks is non-relativistic

$$H = \frac{P^2}{M_Q} + V(r)$$

Quarkonia as 'hard probes' (2)

Initial suggestion (MS 86): screening of the potential

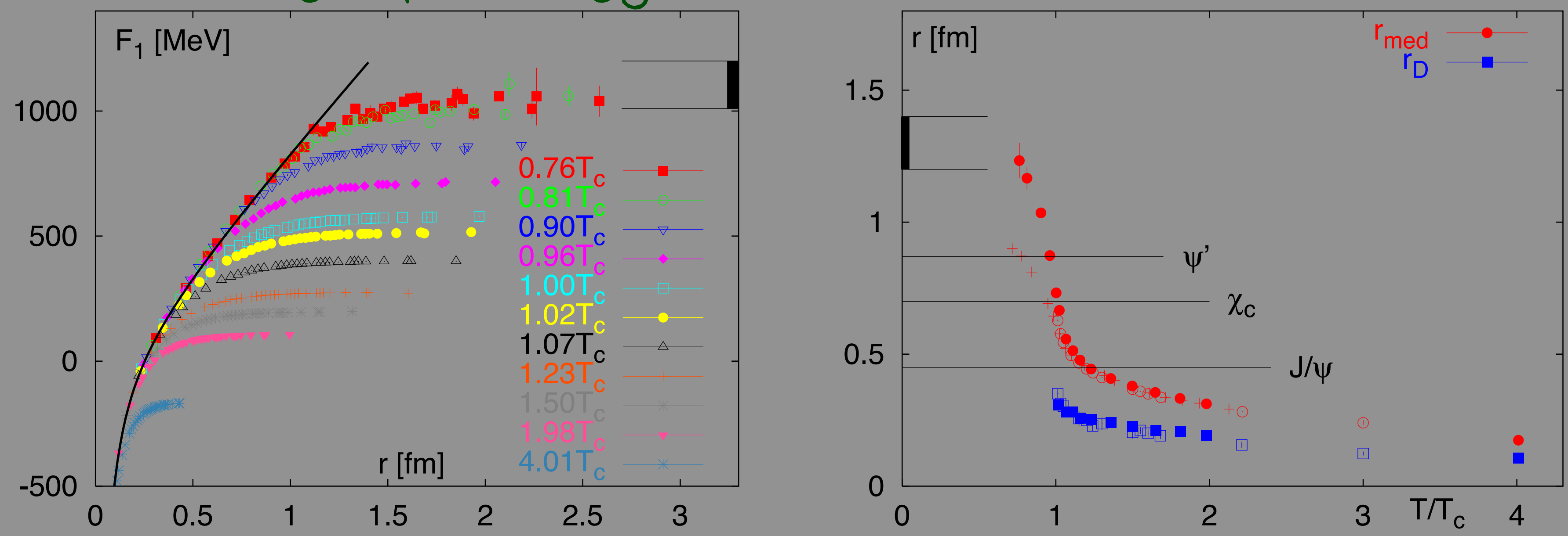
$$H = \frac{P^2}{M_Q} + V(r)$$

$$V(r) = -\frac{\alpha}{r} \exp^{-r m_D(T)} + \sigma(T) r$$

This picture predicts a suppression of bound states at high temperature, the most 'fragile' ones (bigger, less bound) disappearing first as the temperature increases ("sequential suppression").

Picture receives support from lattice calculations

Singlet free energy



from Kaczmarek, Zantow (2005)

However, this is an oversimplification of what turns out to be a rather complicated many-body problem

Why is it a difficult problem ?

- The formation of bound state is not instantaneous. Nor is the establishment of a screening cloud.

  typical formation time:

$$\tau_0 \sim \frac{1}{2M}\frac{4}{\alpha^2}$$

$$\tau_{\Upsilon} \sim 1 \text{ fm/c}$$

$$\tau_{J/\Psi} \sim 2.5 \text{ fm/c}$$

- As the bound state "forms" interactions with the medium take place.

- The formation of bound state is affected both by screening and collisions with the plasma constituents

- The effect of the medium does not reduce to an instantaneous modification of the potential: a complete dynamical treatment is essential.

Why is it interesting ?

- New data of high quality and precision, for two distinct systems, charmonium and bottomonium
- New phenomena observed ("regeneration")
- New theoretical developments: lattice, spectral functions, imaginary part of potential, effective field theories NRQCD, pNRQCD, etc... Mainly, possibility has emerged to treat in a coherent fashion both screening and collisional effects
- More broadly, connections with other interesting issues: modifications of bound state properties in medium, production and survival of fragile states in a hot environment, etc

Several conceptual issues remain to be clarified

In the rest of this talk, focus on a simple problem

Put a number of $Q\bar{Q}$ pairs in color singlet states into a quark-gluon plasma in equilibrium at temperature T, and study their evolution.

Typical questions:

How does the system evolves towards equilibrium? How does the equilibrium state looks like? Can we find robust features of the dynamics? For instance, can we identify various regimes as a functions of external parameters (T, M, ...)

Towards an effective theory for heavy quarks in QGP

$$H = H_Q + H_{\rm pl} + H_{\rm int} \qquad\qquad H_{\rm int} \sim J_Q \cdot A_{\rm pl}$$

Protoptype of an "open quantum system"

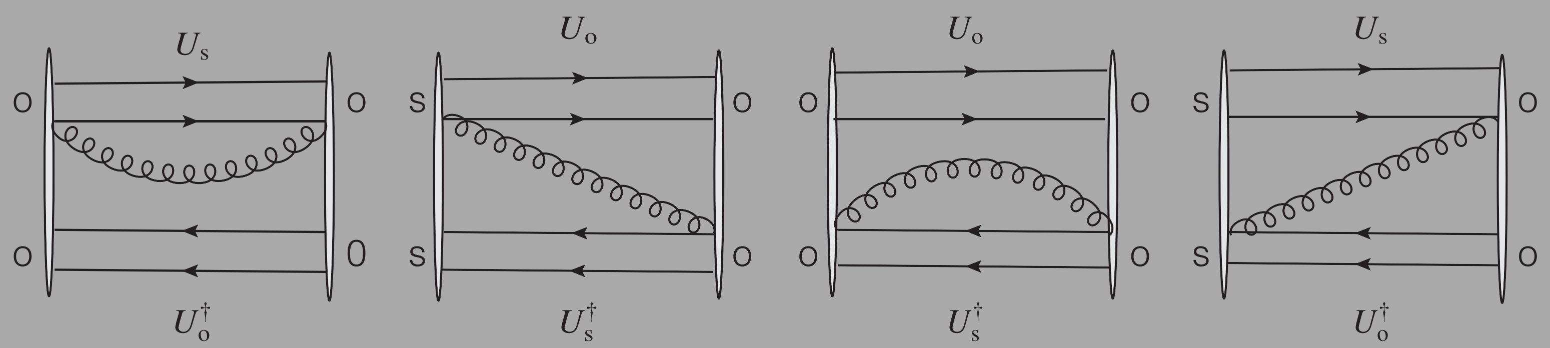
- Our goal: construct an effective theory for the HQ, by eliminating the plasma dof's.

- Tool of choice: reduced density matrix for heavy quarks

$$\mathcal{D}_Q(t) = \mathrm{Tr}_{\mathrm{pl}}\mathcal{D}(t)$$

- D(t) Obeys equation of motion of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D}_Q(t) = -i[H_Q, \mathcal{D}_Q(t)] + \int_0^{t-t_0} \mathrm{d}\tau\, \mathcal{L}(\tau)\mathcal{D}(t-\tau)$$



- This does not reduce to changing the HQ hamiltonian: $H_Q \to H_Q^{\mathrm{eff}}$
- Non hamiltonian dynamics involved (dissipation, transport, etc)

Typical approximations

(i) Weak coupling between HQ and the plasma $H_{\rm int} \sim J_Q \cdot A_{\rm pl}$

simple response functions (correlators) are
sufficient to describe the effective HQ dynamics

$$\Delta(1,2) \equiv \langle A_{\rm pl}(1) A_{\rm pl}(2) \rangle_T = {\rm Tr}\left[A_{\rm pl}(1) A_{\rm pl}(2) \mathcal{D}_{\rm pl} \right]$$

NB. Most (all?) approaches rely on this approximation

(ii) The response of the plasma is mostly 'static' $(\omega = 0)$

plasma response characterized by a single energy scale, the Debye mass

$$m_D = CT \qquad (C \simeq 2)$$

In strict weak coupling $C = g$

soft gluon exchanges

$$q \lesssim m_D \ll M$$

small energy transfer

$$\frac{q^2}{M} \sim \frac{m_D^2}{M} \ll m_D$$

(iii) semi-classical approximation

$$M \gg T \qquad \lambda_Q \sim \frac{1}{\sqrt{MT}} \ll \frac{1}{T}$$

density matrix
nearly diagonal
in coordinate space

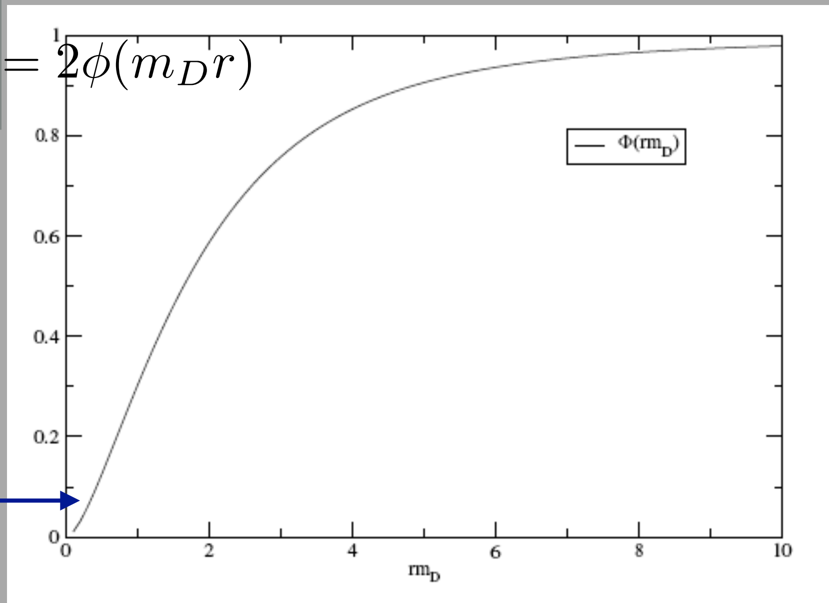
Static response and "Optical potential"

(*first obtained by M. Laine et al hep-ph/ 0611300)

$$\mathcal{V}(r) = V(r) + iW(r)$$

$$\Delta^R(\omega = 0, r) = -V(r) \qquad \Delta^<(\omega = 0, r) = -W(r)$$

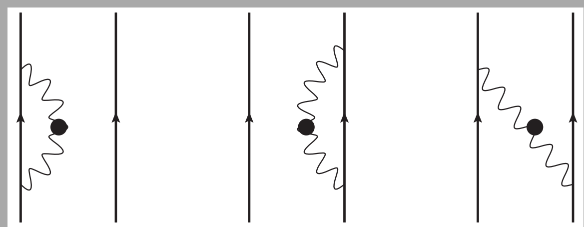
$$\Gamma(\boldsymbol{r}) = W(\boldsymbol{r}) - W(0) = 2\phi(m_D r)$$



At large distance the imaginary part is twice the damping rate of the heavy quark

$$\Phi(x) = \frac{x^2}{3}\left(-\ln x + \frac{4}{3} - \gamma_E\right)$$

At short distance, interference produces cancellation: a small dipole does not "see" the electric field fluctuations.



For one heavy quark

$$\partial_t \langle \boldsymbol{r}|\mathcal{D}_Q|\boldsymbol{r}'\rangle = \cdots - \Gamma(\boldsymbol{r}-\boldsymbol{r}')\langle \boldsymbol{r}|\mathcal{D}_Q|\boldsymbol{r}'\rangle$$

Semi-classical expansion for heavy quark motion

- Equation for the density matrix ⟶ Langevin equation

- Langevin equation for the relative motion

$$\frac{M}{2}\ddot{\boldsymbol{r}}^i = -\gamma_{ij}\boldsymbol{v}^j - \boldsymbol{\nabla}^i V(\boldsymbol{r}) + \xi^i(\boldsymbol{r},t)$$

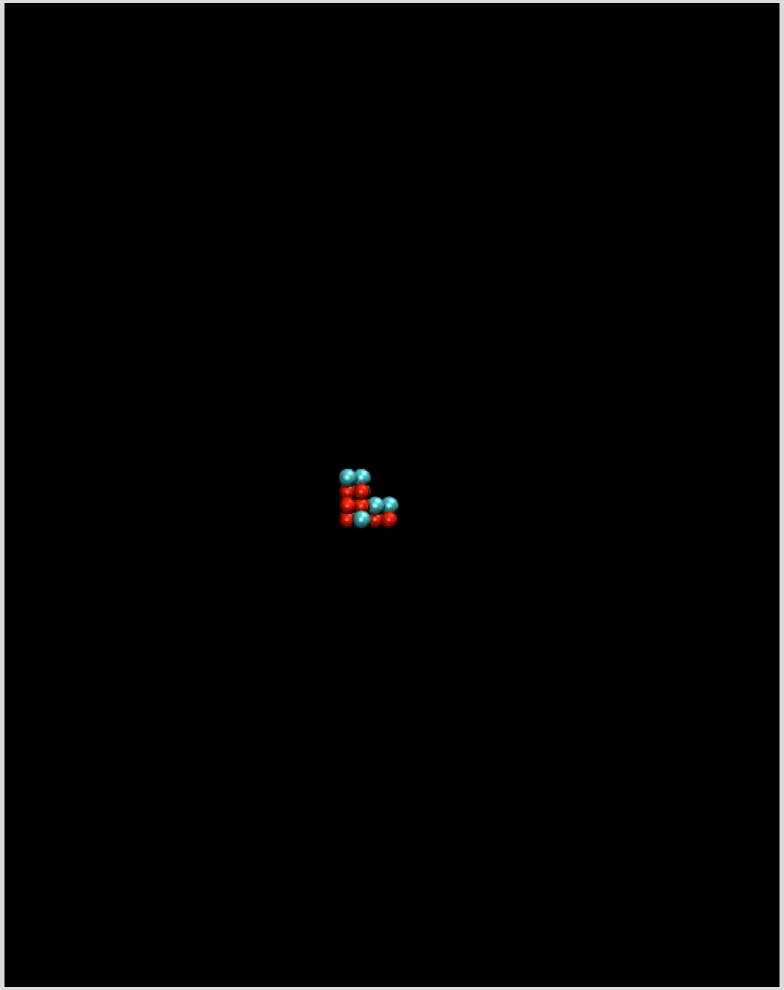
$$\gamma_{ij}(\boldsymbol{r}) = \frac{1}{2T}\eta_{ij}(\boldsymbol{r}) \qquad \langle \xi^i(\boldsymbol{r},t)\xi^i(\boldsymbol{r},t')\rangle = \eta_{ij}(\boldsymbol{r})\delta(t-t')$$

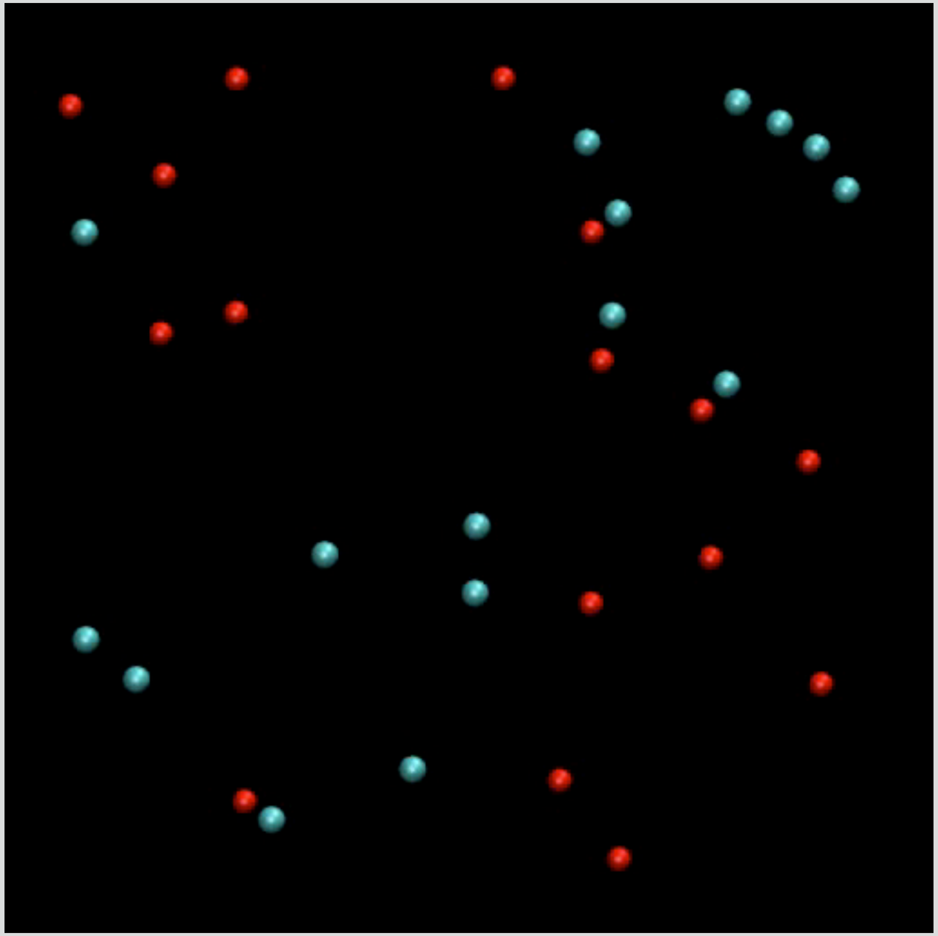
Non trivial noise

- For an isotropic plasma

$$\eta_{ij}(\boldsymbol{r}) = \delta_{ij}\eta(\boldsymbol{r}) \qquad \eta(\boldsymbol{r}) = \frac{1}{6}\left(\nabla^2 W(0) + \nabla^2 W(\boldsymbol{r})\right)$$

- All ingredients of the dynamics are calculated from the "potential" and its imaginary part





J-P.B, D. de Boni, P. Faccioli and G. Garberoglio, NPA (2016)

Typical scales in Coulomb bound states

Bohr radius $$a_0 \sim \frac{2}{M\alpha}$$

Momentum $$p_0 \sim \frac{1}{a_0} \sim \frac{M}{2}\mathrm{v}_0 \qquad \mathrm{v}_0 \sim \alpha$$

$\Upsilon \quad \mathrm{v}_0^2 \sim 0.1$

$J/\Psi \quad \mathrm{v}_0^2 \sim 0.3$

Energy $$M\mathrm{v}_0^2 \sim M\alpha^2$$

(p) NRQCD hierarchy of scales

$$\Lambda_{QCD} \lesssim M\alpha^2 \ll \alpha M \ll M \qquad\qquad M \gg T$$

Typical time scale for HQ motion in a bd state

$$\tau_0 \sim \frac{1}{2M}\frac{4}{\alpha^2}$$

When does melting occur ?

- Thermal velocity equals velocity in bound state

$$v_Q \sim \sqrt{\frac{T}{M}} \sim v_0 \sim \alpha \qquad T \sim M\alpha^2$$

- Or momentum of thermal particles matches HQ bd state momentum

$$T \sim M\alpha$$

- The bd state typical time scales matches that of plasma response

$$\frac{\tau_{\rm pl}}{\tau_0} \sim \frac{M}{m_D}\frac{\alpha^2}{2} \sim 1$$

Binding energy is of the order of the Debye mass

Emergence of two regimes

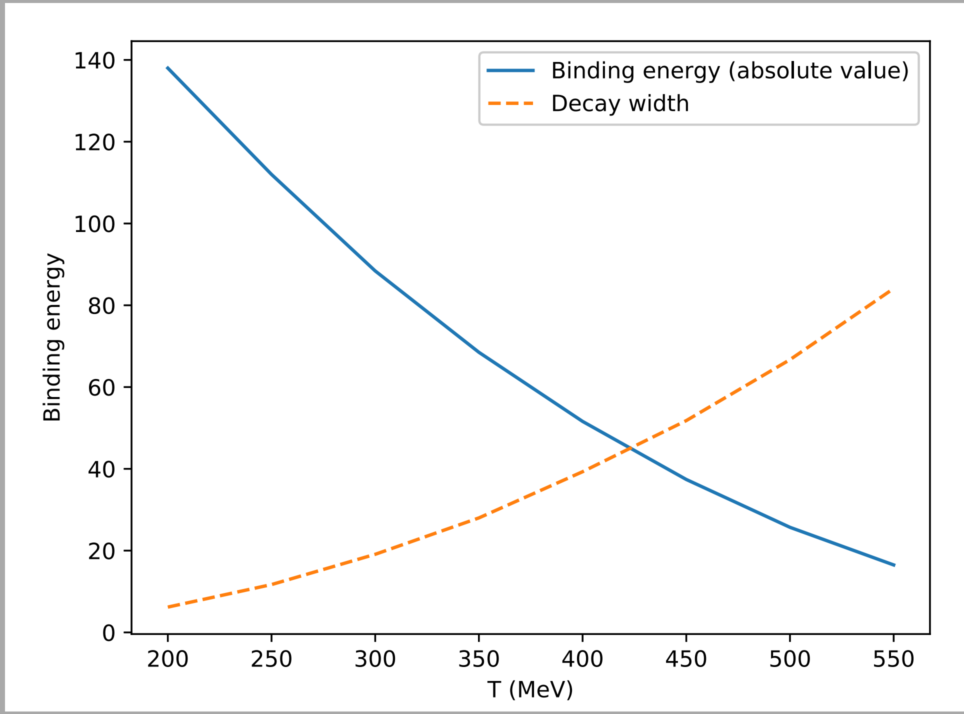
Competition between binding (and screening) and collisional effects (decay width)

$$V \sim \frac{\alpha}{r} \sim \alpha^2 M \qquad \Gamma \sim \alpha T \Phi(m_D r) = \alpha T \Phi(m_D/\alpha M)$$

Low T

Bound states

Rate equations



High T

Collision dominated

Langevin dynamics

$$\Phi \sim 1 \qquad T \sim \alpha M$$

$$\Phi \sim m_D^2/\alpha^2 M^2 \qquad m_D \sim CT \qquad T \sim \alpha \frac{M}{C^{2/3}}$$

"Optical potential" is energy dependent

Simple two-level model

(two massive quarks separated by r)

$N_c^2 - 1$

V_o ——— octet

V_s ——— singlet

$$Z = \mathrm{e}^{-\frac{V_\text{s}}{T}} + (N_c^2 - 1)\mathrm{e}^{-\frac{V_\text{o}}{T}}$$

$$\Delta V(r) = V_\text{o} - V_\text{s}$$

Low T
$$F \approx V_\text{s} - T(N_c^2 - 1)\mathrm{e}^{-\frac{\Delta V}{T}}$$
"Binding" dominates

High T
$$F = -T \ln N_c^2 + \frac{V_\text{s} + (N_c^2 - 1)V_\text{o}}{N_c^2}$$
Entropy dominates

Transition occurs when
$$T \sim \Delta V$$

Approach to equilibrium

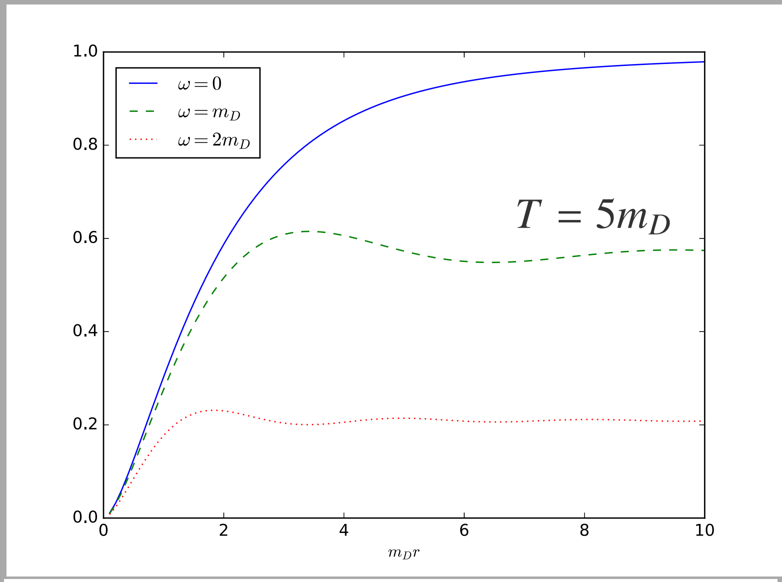
$$\frac{\mathrm{d}p_\text{s}}{\mathrm{d}t} = (N_c^2 - 1)p_\text{o}\Gamma_{\text{o}\to\text{s}} - p_\text{s}\Gamma_{\text{s}\to\text{o}}$$

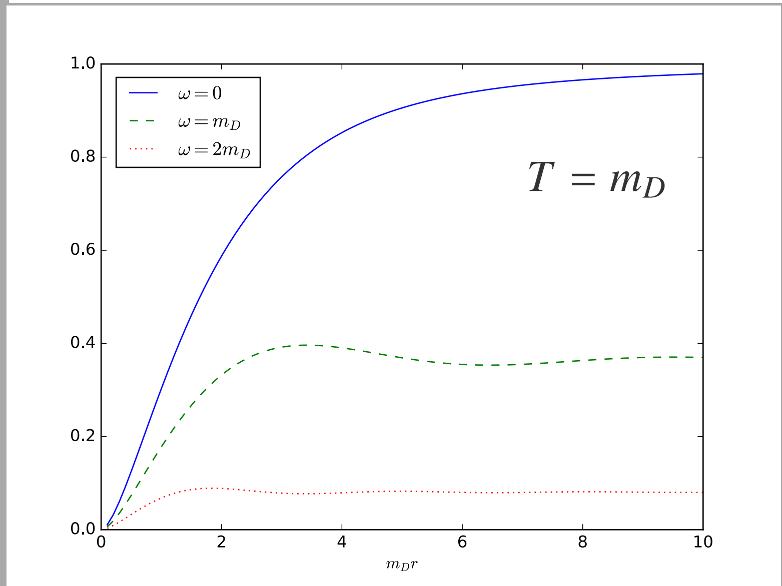
$$\Gamma_{\text{s}\to\text{o}} = g^2 C_F \mathrm{e}^{-\frac{\Delta V}{T}} \int_q \Delta^{>}(\Delta V, q) |\mathcal{S}_{q\cdot\hat{r}}|^2$$

static approximation breaks down

The imaginary part of the potential is energy dependent

$$(4\pi/Tg^2)(W(\omega,r) - W(\omega,0))$$





J-P.B, M. Escobedo (2018)

Final remarks (1)

- A consistent framework is emerging, allowing to treat on par both screening and collisional effects (imaginary potential, energy dependent)
- Two regimes: low and high temperature (fuzzy transition)

$$T \sim M\alpha \qquad M\alpha^2 \sim m_D \qquad T \sim \alpha \frac{M}{C^{2/3}}$$

- Low T: bound states are weakly affected by the plasma, appropriate rate equations should be fine
- High T: binding effects not essential, Langevin dynamics, equilibration, etc
- Intermediate region difficult: bound states melt, combination of Langevin and rate equations seems needed

Final remarks (2)

How do various approaches fit into this general picture?

- Many ways to treat collisional effects (rate equations, Boltzmann, stochastic potentials, Lindblad equation, etc)

- Lattice calculations: do not address dynamics directly, (except, perhaps, for spectral functions), but it can provide input for the dynamics (correlators, potential, transport coefficients)

- Effective field theory (NRQCD, pNRQCD). Restricts the dynamics to pair degrees of freedom (singlet and octet). Provides systematics improvements of potential. Limited by multipole expansion (dipolar interactions)

- In-medium T-matrix. Solidly rooted in well established many-body physics. Efficient bridge to phenomenology. Connection with other approaches would be worth strengthening