pNRQCD for the study of nonequilibrium evolution of quarkonium in a medium

N. Brambilla, M. Escobedo

EQUILIBRIUM QUANTITIES: Potentials, energies, dissociation T, how to use lattice input

in this same framework based on
pNRQCD +open quantum system+ lattice

Results obtained in collaboration with :
M. Escobedo, A. Vairo, J. Soto (van der Griend) N.B. (master equations);
A. Vairo, M. Escobedo, J. Ghiglieri, P. Petreczky, M. Berwein, J. Soto, N. B. (equilibrium properties) ;
J. Weber , P. Petreczky, V. Leino, A. Vairo, N.B. (lattice inside theTUMQCD collaboration created to work at the interface between lattice and EFTs)

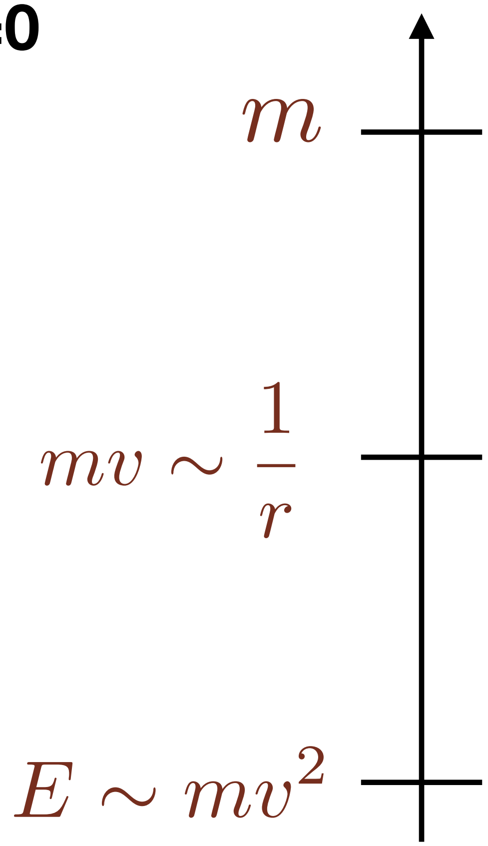
The finite T potential

For long time the in medium static potential was identified with some free energies defined from suitable correlators. This is no more so: the potential describes the real-time evolution of the $Q\bar{Q}$ pair, which is, in general, not the case for the free energies; it also has an imaginary part coming from the quarkonium dissociation through scattering with the partons in the medium [illegible]

The finite T potential: how to obtain it

in pNRQCD the **potential has a clear definition:** it a **matching coefficient** and comes from the integration of all scales from mv up to (and not included) the energy mv^2

T=0



$$m \gg \Lambda_{QCD}$$

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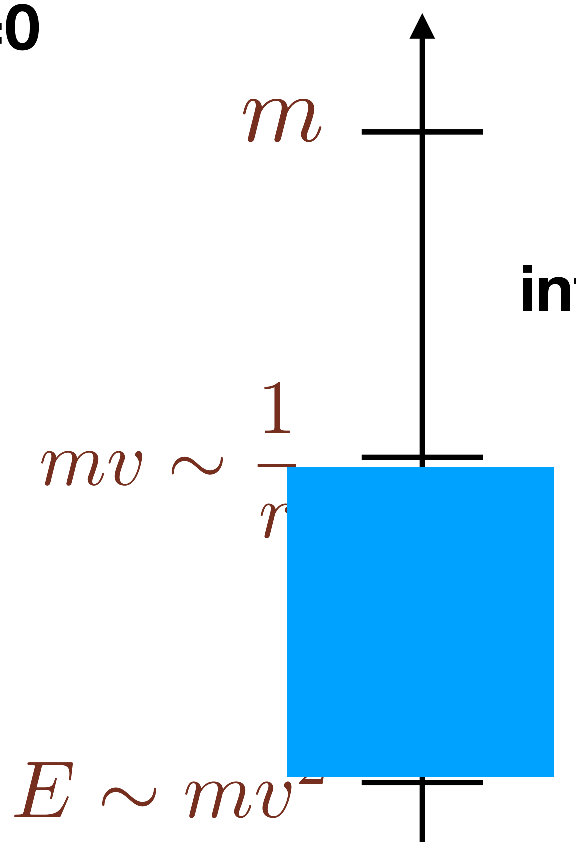

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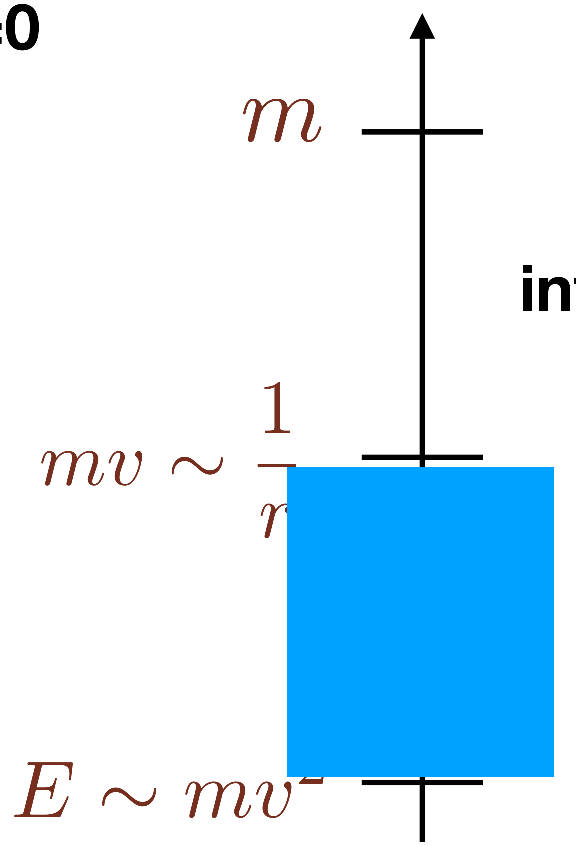

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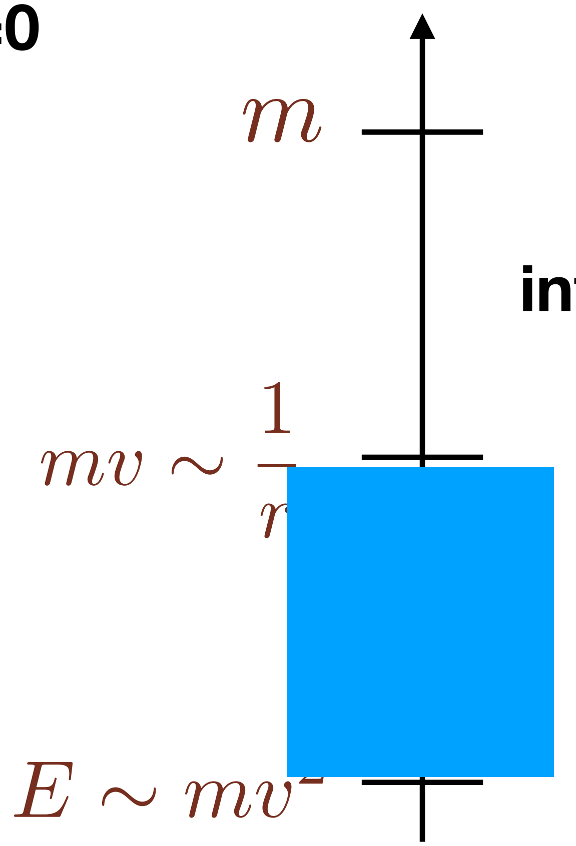

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if $mv \sim \Lambda_{QCD}$ **the potential is nonperturbative given in terms of Wilson loops to be evaluated on the lattice**

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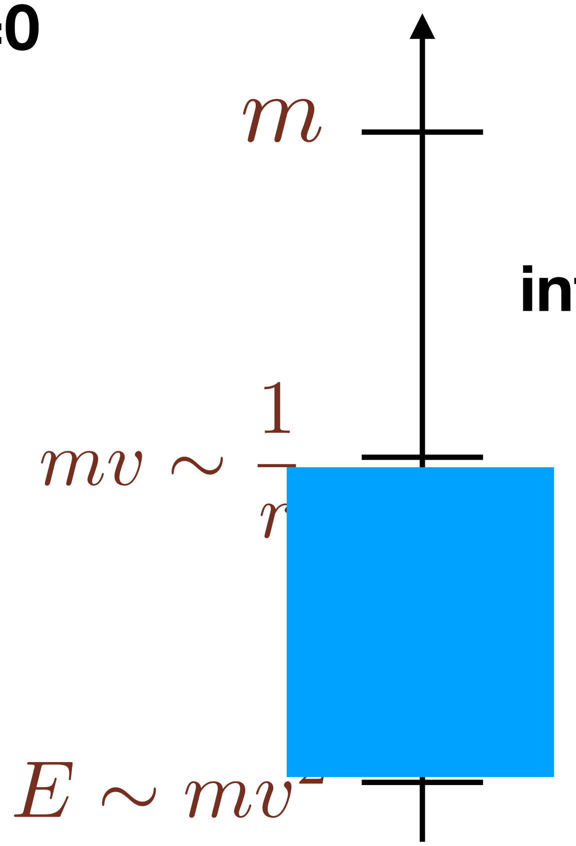

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$m \gg \Lambda_{QCD}$

Notice:

1)one can calculate 1/m and 1/m^2 corrections systematically

2)the potential comes from QCD and depends on QCD parameters

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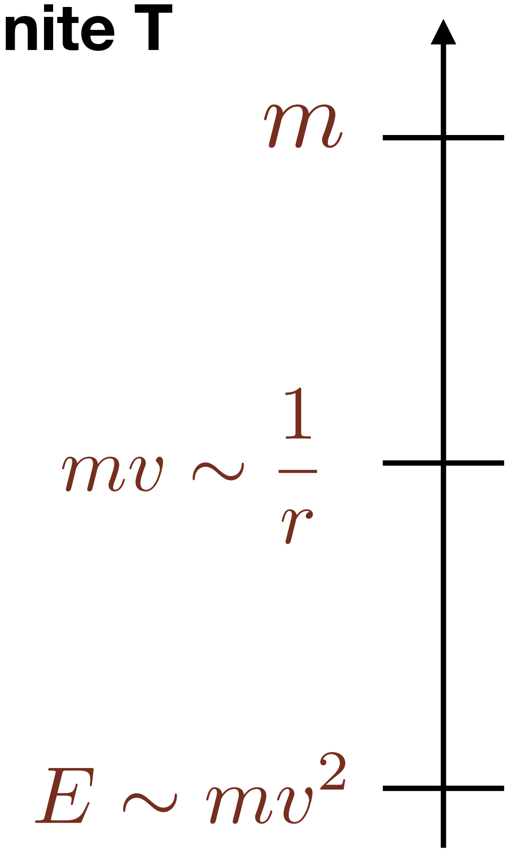
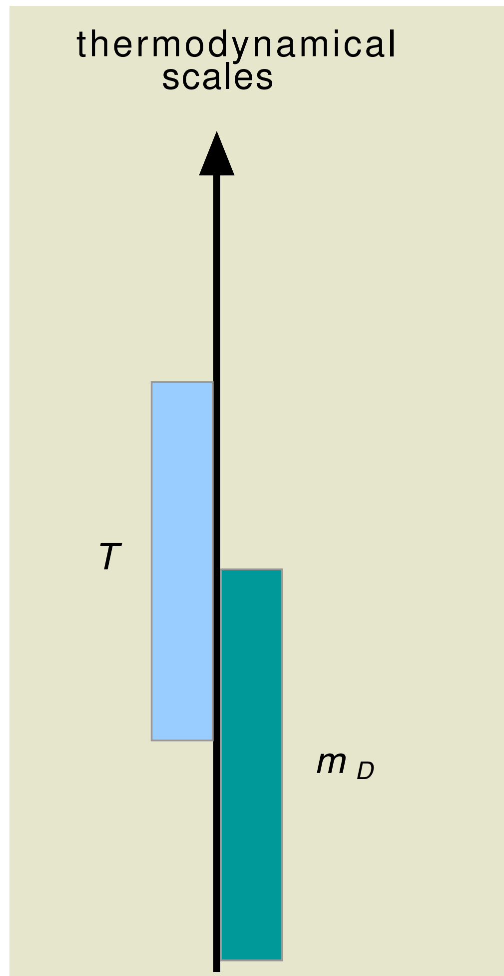
Finite T





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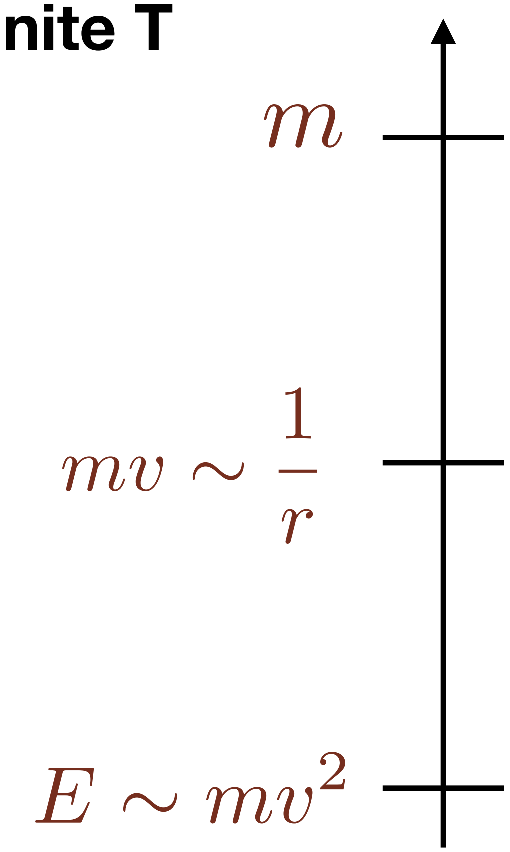

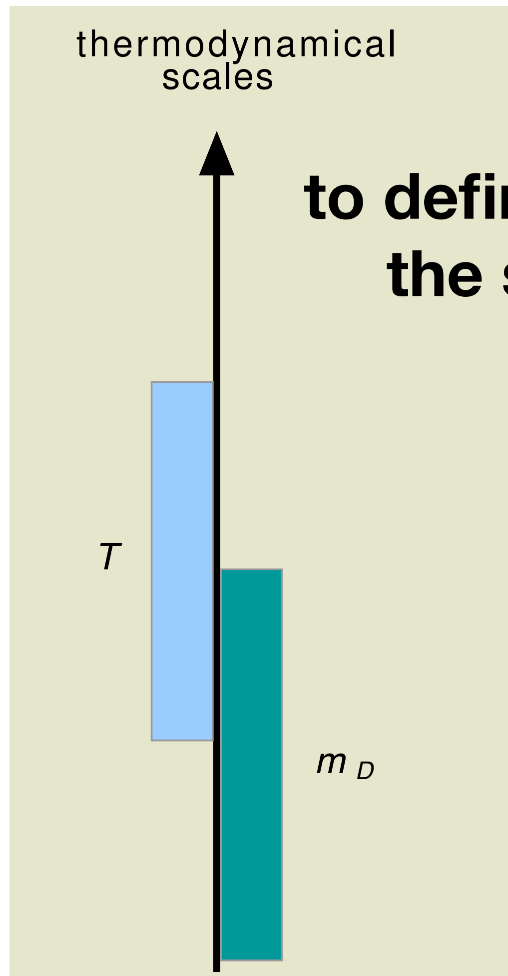

to define the potential we have to integrate out all the scales bigger than E including T and m_d

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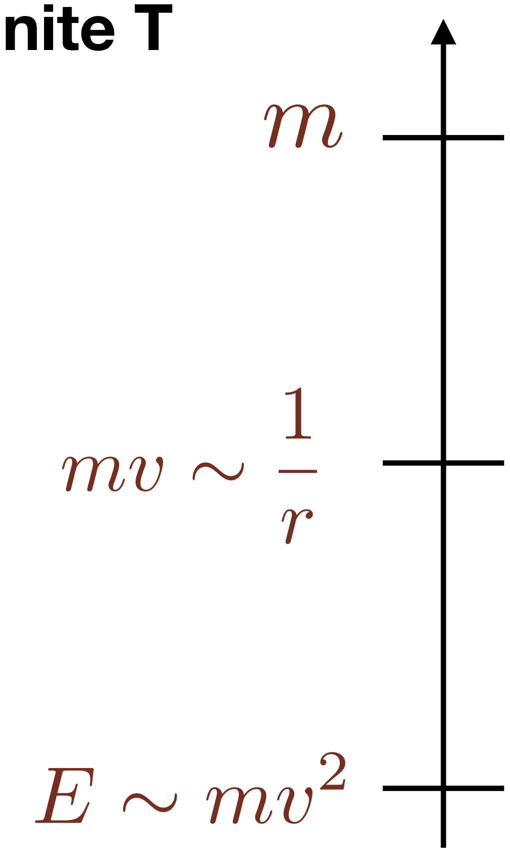

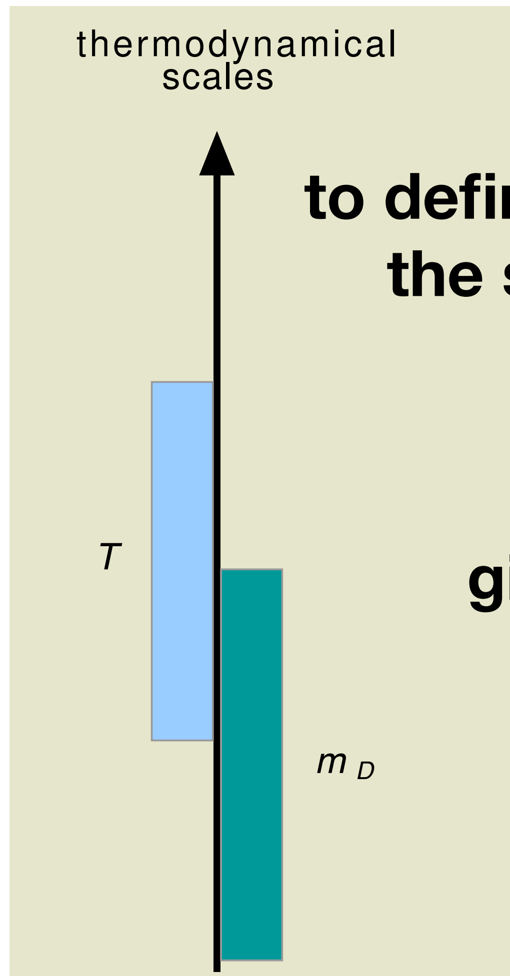

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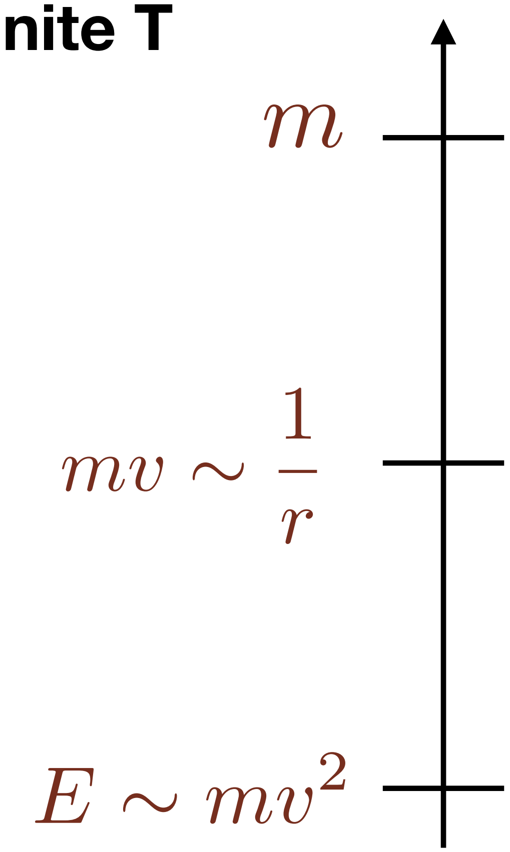

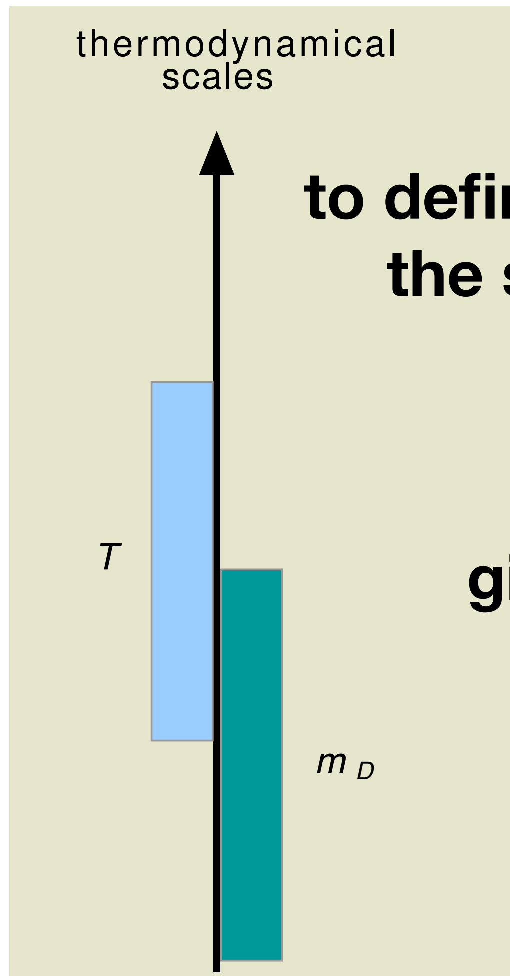

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$$m \gg \Lambda_{QCD}$$

Notice:

The potential V(r,T) dictates thought the Schroedinger equation the real time evolution of the QQbar in the medium

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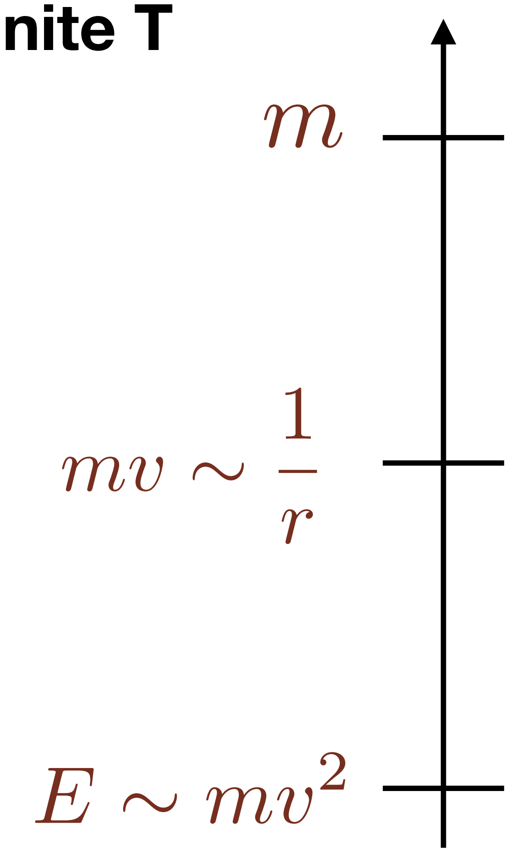

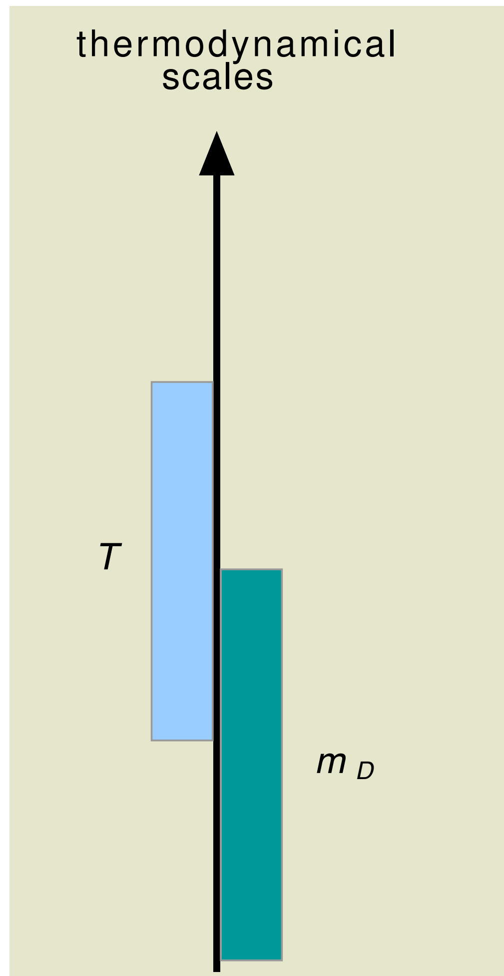

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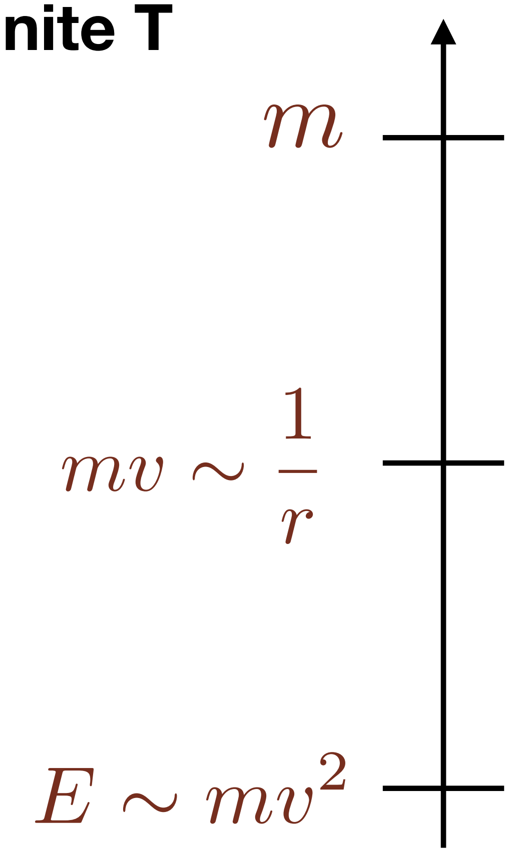

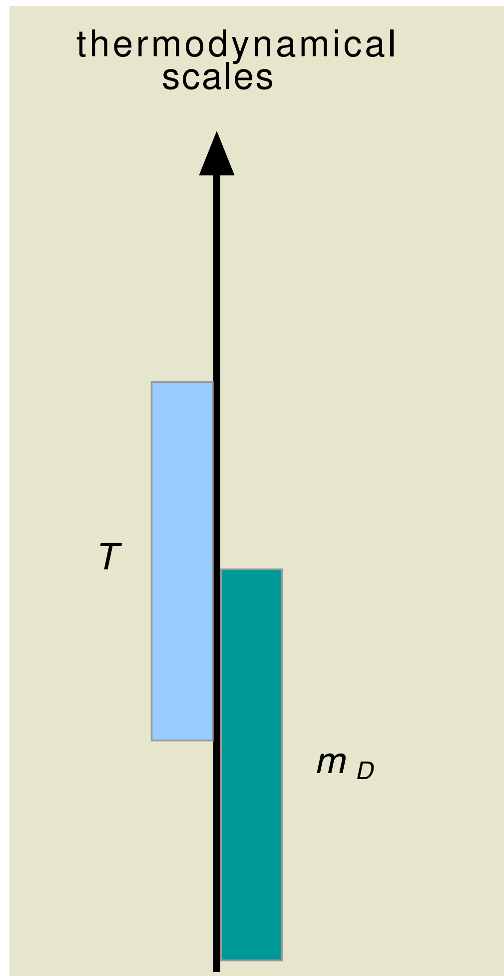

We assume that bound states exist for

- $T \ll m$
- $1/r \sim mv \gtrsim m_D$

We neglect smaller thermodynamical scales.

Inside these constraints we consider all the possible scales hierarchies

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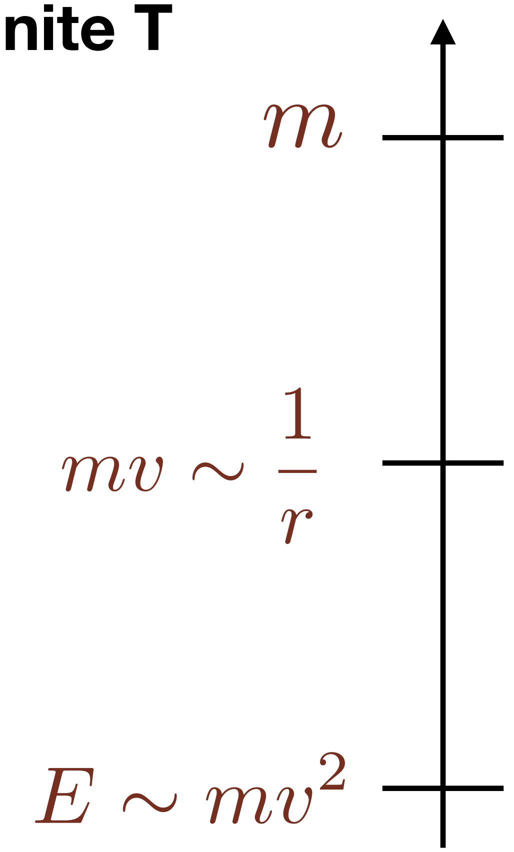

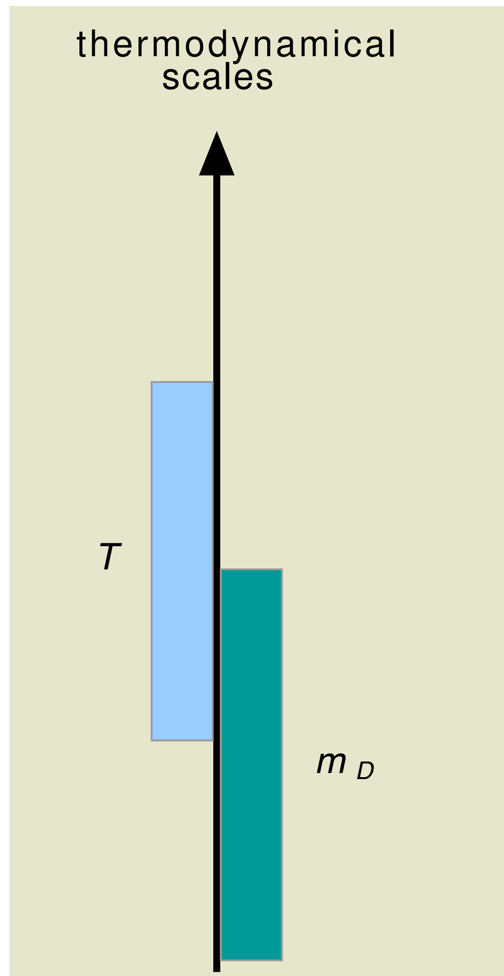

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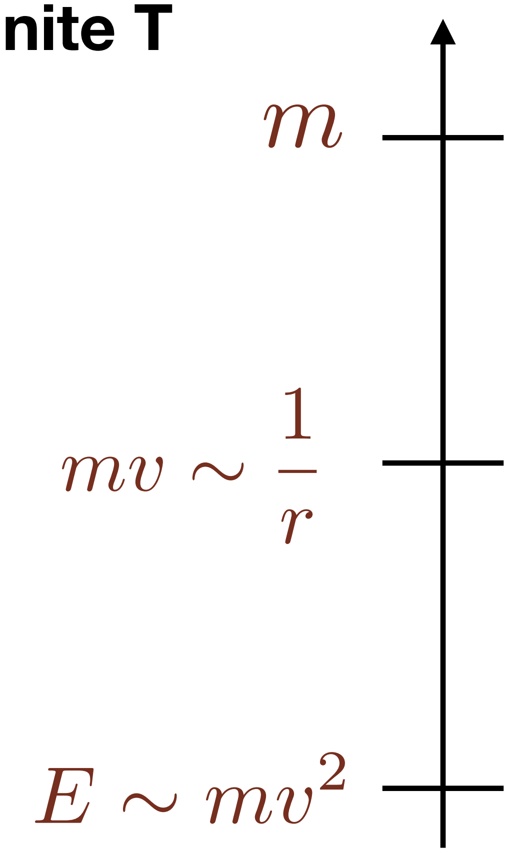

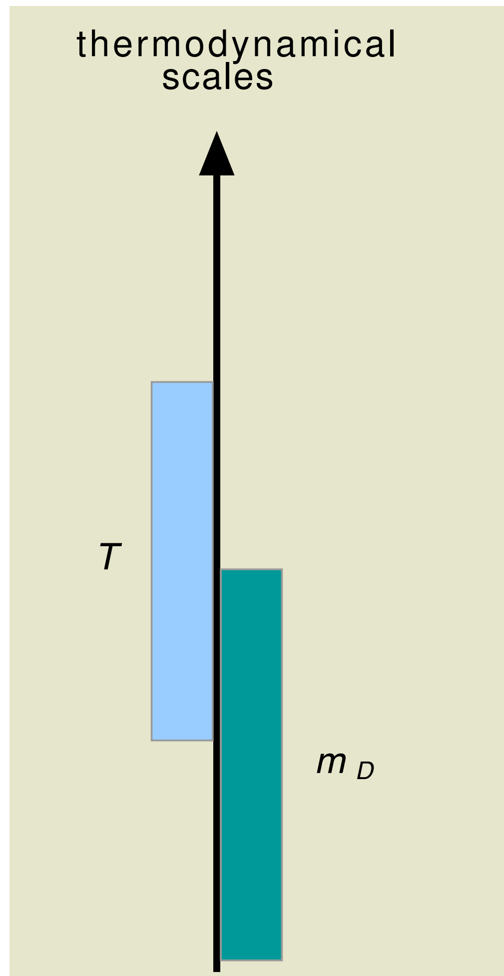

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for the nonperturbative regime -> lattice calculation of the Wilson loop

The thermal part of the potential has a real and an imaginary part

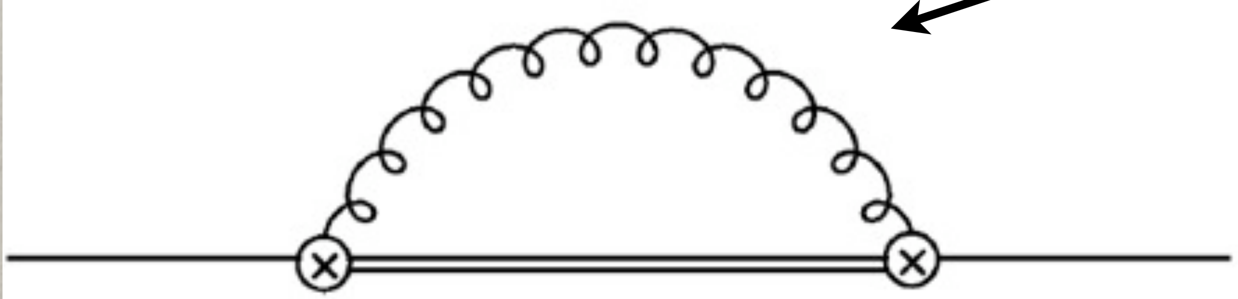
$\mathrm{Re}V_S\,(r,T)$

$\mathrm{Im}V_S\,(r,T)$

thermal width of $Q\bar{Q}$

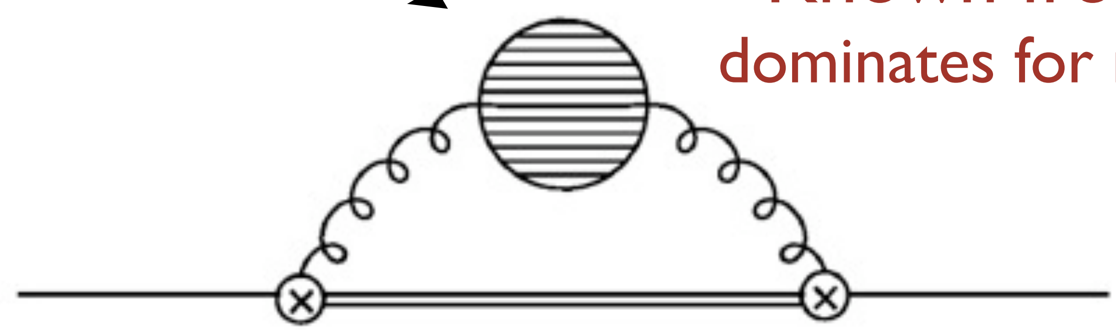
New effect, specific of QCD
dominates for E/m_D>>1

Known from QED
dominates for m_D/E>>1



Singlet-to-octet

N.B Ghiglieri, Petreczky, Vairo 2008



Landau damping

Laine et al 07, Escobedo Soto 07

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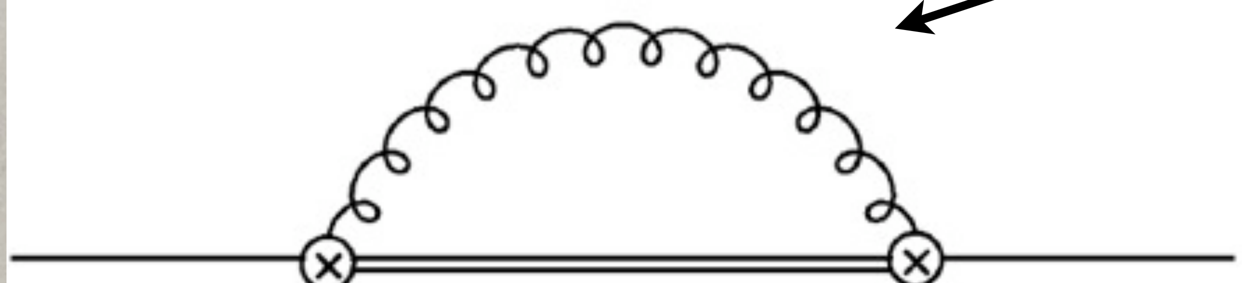

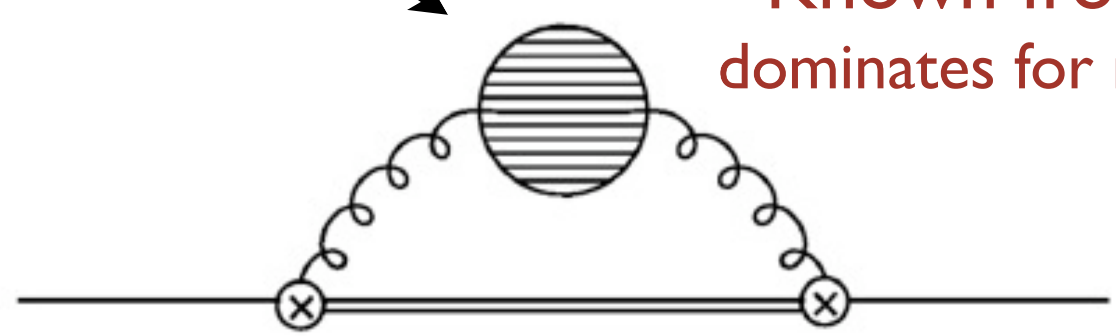

Singlet-to-octet

N.B Ghiglieri, Petreczky, Vairo 2008

(gluo dissociation)

N. B. Escobedo, Ghiglieri , Vairo 2011

Landau damping

Laine et al 07, Escobedo Soto 07

(inelastic parton scattering)

N. B. Escobedo, Ghiglieri , Vairo 2013

The singlet static potential and the static energy

you always have a real and an imaginary part

- Temperature effects can be other than screening

$T > 1/r$ and $1/r \sim m_D \sim gT$

exponential screening but $\mathrm{Im}V \gg \mathrm{Re}V$

$T > 1/r$ and $1/r > m_D \sim gT$ or $\frac{1}{r} \gg T \gg E$

no exponential screening but
power-like T corrections

$T < E_{bin}$

no corrections to the potential,
corrections to the energy

for the detailed form
of the potentials in each regime see:

N.B Ghiglieri, Petreczky,
Vairo Phys.Rev. D78 (2008) 014017

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for the detailed form of the potentials in each regime see: **N.B Ghiglieri, Petreczky, Vairo Phys.Rev. D78 (2008) 014017**

imaginary parts in the potential have subsequently been found also for a strongly coupled plasma on the lattice (A. Rothkopf et al, Petreczky, Weber..) and in strings calculations

Lessons that can be useful to interpret lattice data

- We can not expect a simple function to parametrize thermal modifications at all temperature regimes. Good theoretical reasons to expect the case $\frac{1}{r} \gg T$ to be very different from $T \gg \frac{1}{r}$.
- At very small temperatures, non-local terms will be needed to describe the evolution of quarkonium, which by construction are not present in the static limit.

Change in the paradigm of dissociation

Change in the paradigm of dissociation

- The imaginary part is bigger than the real part before the screening exp{-m_D r} sets in

->the imaginary part is responsible for QQbar dissociation

$$T \gg 1/r \gg m_D \gg V$$

- Quarkonium dissociates at a temperature such that $\mathrm{Im}\, V_s(r) \sim \mathrm{Re}\, V_s(r) \sim \alpha_s/r$:

$$E_{\mathrm{binding}} \sim \Gamma$$

$$\pi T_{\mathrm{dissociation}} \sim m g^{4/3}$$

- Escobedo Soto arXiv:0804.0691
  Laine arXiv:0810.1112

- The interaction is screened when $\langle 1/r \rangle \sim m_D$, hence

$$\pi T_{\mathrm{screening}} \sim m g \gg \pi T_{\mathrm{dissociation}}$$

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The bottomonium ground state , which is a weakly coupled non-relativistic bound state: $mv \sim m\alpha_s$, $mv^2 \sim m\alpha_s^2 \gtrsim \Lambda_{QCD}$, produced in the QCD medium of heavy-ion collisions at the LHC may possibly realize the hierarchy

$$m \approx 5 \text{ GeV} > m\alpha_s \approx 1.5 \text{ GeV} > \pi T \approx 1 \text{ GeV} > m\alpha_s^2 \approx 0.5 \text{ GeV} \gtrsim m_D, \Lambda_{QCD}$$

T_dissociation in the $\Upsilon(1S)$ case is about 450 MeV.

bottomonium 1S below the melting temperature T_d

The complete mass and width up to $\mathcal{O}(m\alpha_s^5)$

$$\delta E_{1S}^{(\mathrm{thermal})} = \frac{34\pi}{27}\alpha_s^2\, T^2 a_0 + \frac{7225}{324}\frac{E_1\alpha_s^3}{\pi}\left[\ln\left(\frac{2\pi T}{E_1}\right)^2 - 2\gamma_E\right] + \frac{128 E_1\alpha_s^3}{81\pi} L_{1,0} - 3a_0^2\left\{\left[\frac{6}{\pi}\zeta(3) + \frac{4\pi}{3}\right]\alpha_s\, T\, m_D^2 - \frac{8}{3}\zeta(3)\,\alpha_s^2\, T^3\right\}$$

$$\Gamma_{1S}^{(\mathrm{thermal})} = \frac{1156}{81}\alpha_s^3 T + \frac{7225}{162} E_1\alpha_s^3 + \frac{32}{9}\alpha_s\, T m_D^2\, a_0^2\, I_{1,0} - \left[\frac{4}{3}\alpha_s T m_D^2\left(\ln\frac{E_1^2}{T^2} + 2\gamma_E - 3 - \ln 4 - 2\frac{\zeta'(2)}{\zeta(2)}\right) + \frac{32\pi}{3}\ln 2\,\alpha_s^2\, T^3\right] a_0^2$$

where $E_1 = -\frac{4m\alpha_s^2}{9}$, $a_0 = \frac{3}{2m\alpha_s}$ and $L_{1,0}$ (similar $I_{1,0}$) is the Bethe logarithm.

- Brambilla Escobedo Ghiglieri Soto Vairo JHEP 1009 (2010) 038

Consistent with lattice calculations of spectral functions

- Aarts Allton Kim Lombardo Oktay Ryan Sinclair Skullerud JHEP 1111 (2011) 103

electromagnetic decays

figure out what our results imply for the electromagnetic decays to lepton pairs or to two photons. First of all, the masses of the heavy quarkonium states increase quadratically with the temperature at leading order (first line of (7.1)), which would translate into the same functional increase in the energy of the outgoing leptons and photons if produced by the quarkonium in the plasma. Second, since electromagnetic decays occur at short distances ($\sim 1/m \ll 1/T$), the standard NRQCD factorization formulas hold, and, at leading order, all the temperature dependence is encoded in the wave function at the origin. The leading temperature correction to it comes from first-order quantum-mechanical perturbation theory of the first term of (4.25). The size of this correction is $\sim n^4T^2/(m^2\alpha_{\rm s})$. Hence, a quadratic dependence on the temperature should also be observed in the frequency in which leptons or photons are produced by the quarkonium in the plasma. Finally, at leading order, a decay width linear with temperature is developed (first line of (7.3)), which implies a tendency to decay to the continuum of colour-octet states. Hence, a smaller number of vector and pseudoscalar ground states is expected to be in the sample with respect to the zero temperature case.

for the J psi is 93 Kev that corresponds to 10^3 fm , QGP gone!

N. B, M. Escobedo, J. Soto, A. Vairo JHEP 1009 (2010) 038

Free energies from Polyakov loop and Polyakov loop correlator

Bibliography: lattice

(1) A. Bazavov, N. Brambilla, P. Petreczky, A. Vairo and J. Weber [TUMQCD coll.]
Color screening in 2+1 flavor QCD
arXiv:1804.10600

(2) A. Bazavov, N. Brambilla, P. Petreczky, A. Vairo and J. Weber [TUMQCD coll.]
Polyakov loop in 2+1 flavor QCD from low to high temperatures
Phys. Rev. D93 (2016) 114502 arXiv:1603.06637

Bibliography: perturbation theory

(1) M. Berwein, N. Brambilla, P. Petreczky and A. Vairo
Polyakov loop correlator in perturbation theory
Phys. Rev. D96 (2017) 014025 arXiv:1704.07266

(2) M. Berwein, N. Brambilla, P. Petreczky and A. Vairo
Polyakov loop at next-to-next-to-leading order
Phys. Rev. D93 (2016) 034010 arXiv:1512.08443

(3) M. Berwein, N. Brambilla and A. Vairo
Renormalization of Loop Functions in QCD
Phys. Part. Nucl. 45 (2014) 656 arXiv:1312.6651

(4) M. Berwein, N. Brambilla, J. Ghiglieri and A. Vairo
Renormalization of the cyclic Wilson loop
JHEP 1303 (2013) 069 arXiv:1212.4413

(5) N. Brambilla, J. Ghiglieri, P. Petreczky and A. Vairo
The Polyakov loop and correlator of Polyakov loops at next-to-next-to-leading order
Phys. Rev. D82 (2010) 074019 arXiv:1007.5172

many interesting results from the Polyakov loop

- The Polyakov loop has been computed up to order g^5.
- The (subtracted) $Q\bar{Q}$ free energy has been computed at short distances up to corrections of order $g^7(rT)^4$, g^8.
- The (subtracted) $Q\bar{Q}$ free energy has been computed at screening distances up to corrections of order g^8.
- The singlet free energy has been computed at short distances up to corrections of order $g^4(rT)^5$, g^6.
- The singlet free energy has been computed at screening distances up to corrections of order g^5.

- Lattice calculations are consistent with weak-coupling expectations.
- Crossover temperature to the quark-gluon plasma is $153^{+6.5}_{-5}$ MeV from the entropy of the Polyakov loop.
- Screening sets in at $rT \approx 0.3$-0.4 (observable dependent), consistent with a screening length of about $1/m_D$.
- Asymptotic screening masses are about $2m_D$ (observable dependent).
- First determination of the color octet $Q\bar{Q}$ free energy.

Still the free energy is not the object to be taken as the potential in the Schroedinger equation

(the singlet free energy may provide a good approximation of the real part of the potential)

In the regime $\frac{1}{r} \gg T \gg E$ **we obtained evolution equation for the singlet and octet density matrix**

- In this temperature regime, the influence of the medium in the evolution of quarkonium can be encoded in two non-perturbative parameters.
  - The heavy quark diffusion parameter κ. It can be related with the decay width.
  - A parameter γ that encodes the modification of the real part of the potential due to the medium. It can be related with the thermal energy shift.

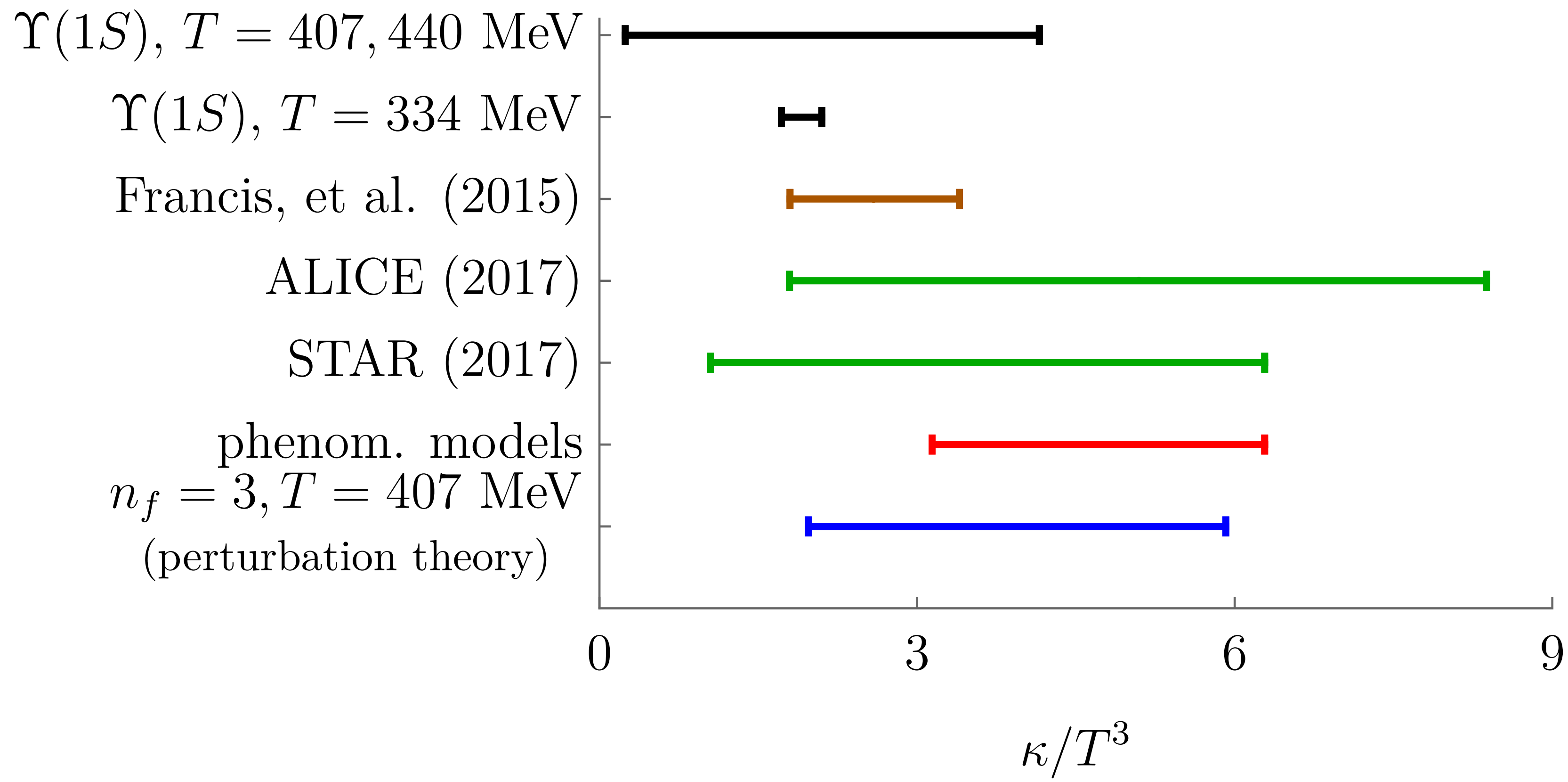
- Therefore, we can use lattice data on thermal modifications of quarkonium to get information about the evolution equation.

N.B., M. Escobedo, A. Vairo, P. vander Griend Phys.Rev. D100 (2019) no.5, 054025

Use lattice data from Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair and Skullerud (2011) and Kim, Petreczky and Rothkopf (2018). Unquenched.

and the new data from R. Larsen, S. Meinel, S. Mukherjee, P. Petreczy 2019, 2020

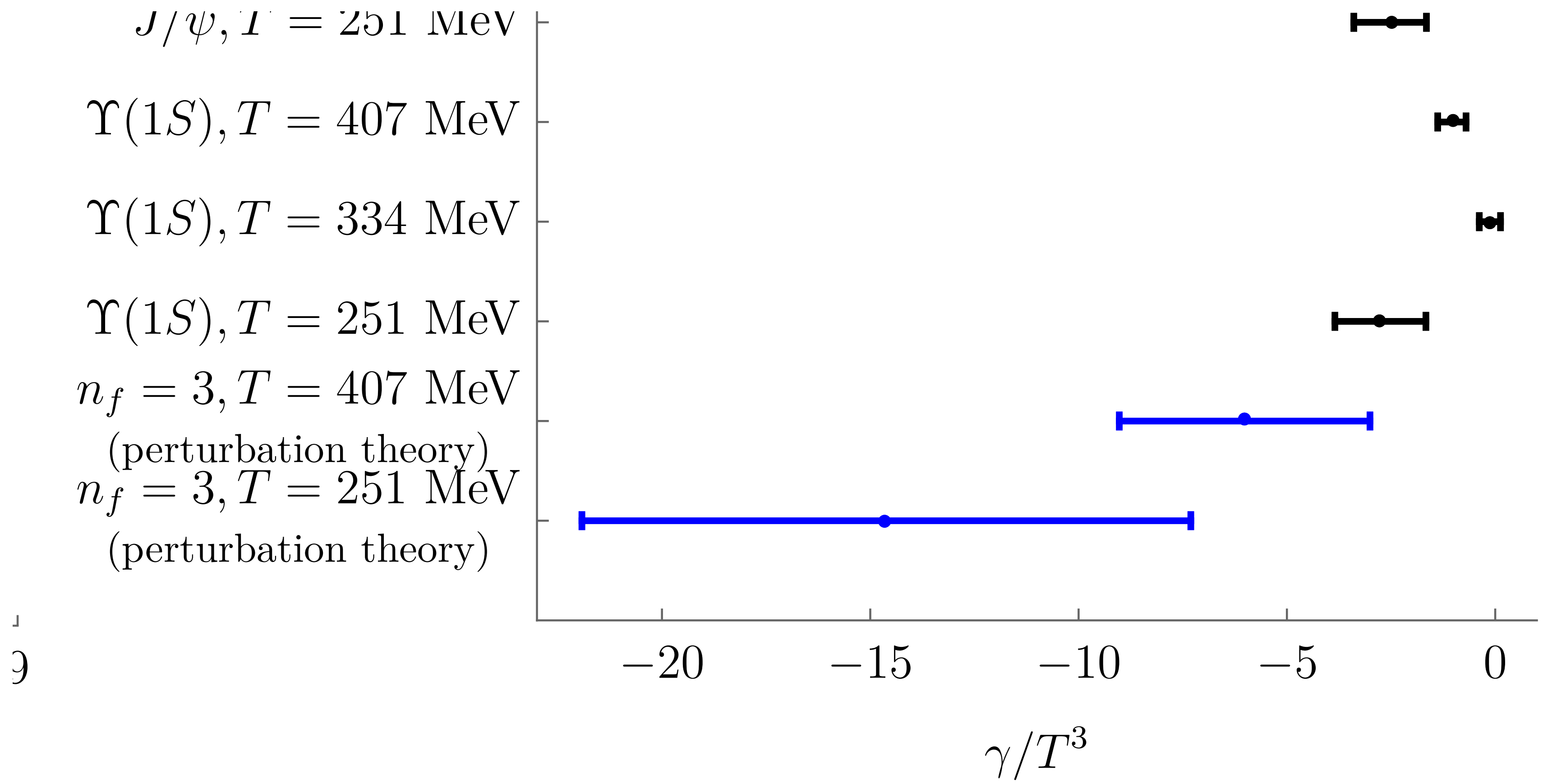
The decay width is $\Gamma = \kappa\langle r^2\rangle$



Extractions of κ: from unquenched lattice measurements of the thermal width of the $\Upsilon(1S)$ (black) with newest lattice data ($T = 334$ MeV) with comparison to other measurements.

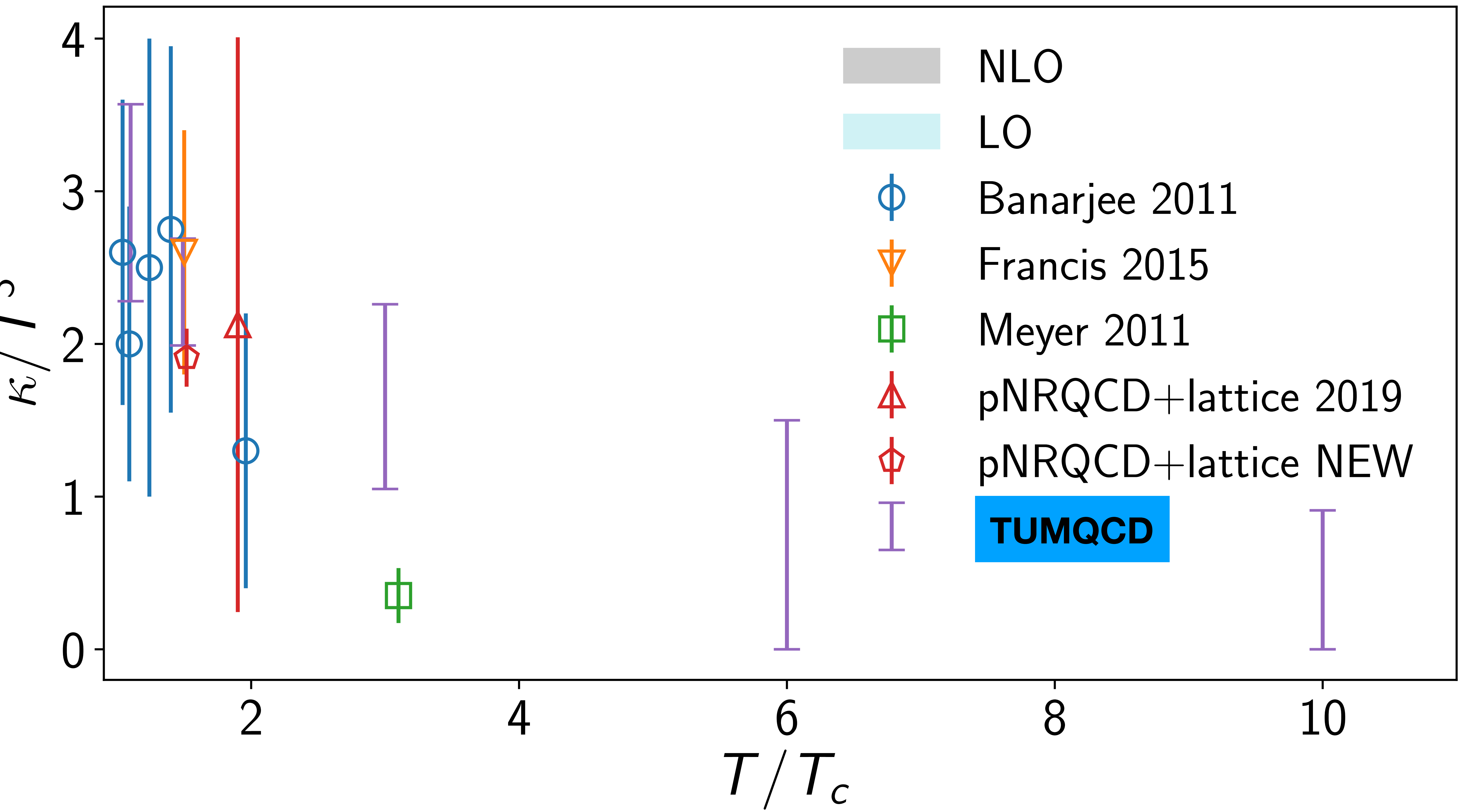
plots from P. vander Griend 2019

- The thermal mass shift is $\delta M = \frac{1}{2}\gamma\langle r^2\rangle$.



Extractions of γ: from unquenched lattice measurements of the thermal mass shift of the J/ψ and $\Upsilon(1S)$ at temperatures from 251 to 407 MeV (black) with newest data ($T = 334$ MeV) and from perturbation theory (blue).

plots from P. vander Griend 2019



4
3
2
1
0
κ / T^3
2
4
6
8
10
T / T_c
NLO
LO
Banarjee 2011
Francis 2015
Meyer 2011
pNRQCD+lattice 2019
pNRQCD+lattice NEW
TUMQCD