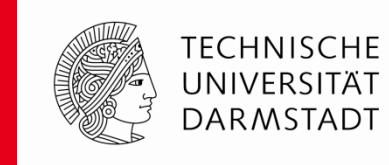


# Space-charge limitation for fast bunch compression in SIS-18

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# Outline



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- Background & motivation
- Coupled envelope equations
- Bunch compression in SIS-18
- Summary



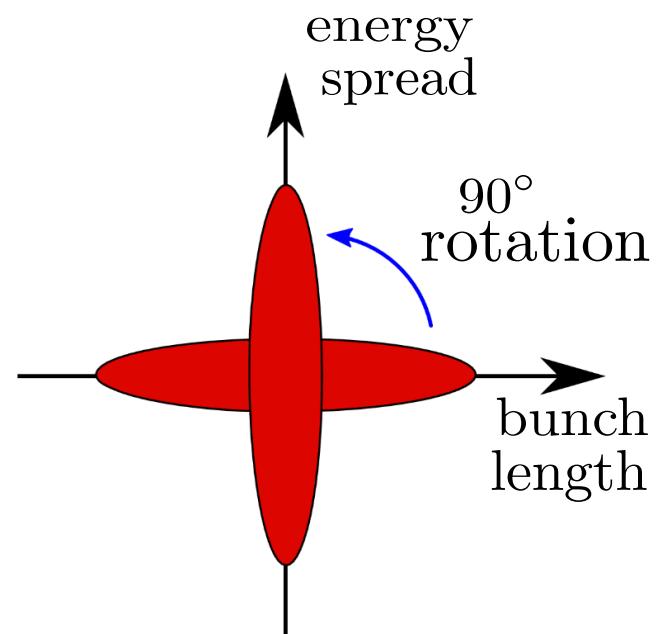
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# Background & Motivation

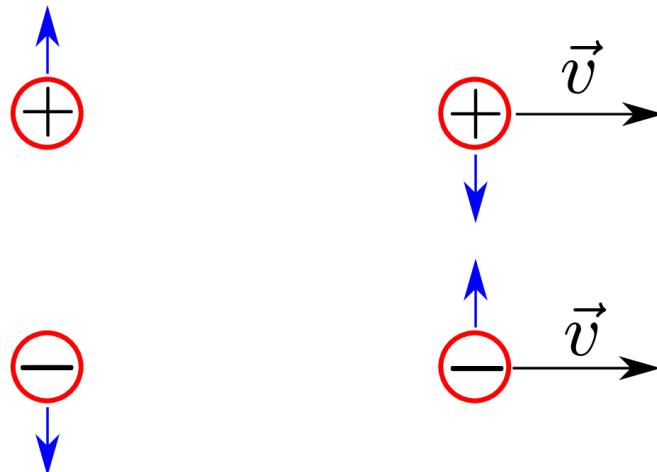


- Bunch compression generates short and intense ion bunches.
  - Generation of dense plasmas
  - Spallation neutron sources
- Scheme: A fast bunch rotation in longitudinal phase space
- During bunch compression, both of
  - { Space charge }
  - { Dispersion }



will be enhanced, and lead to the beam intensity limitation

# Space charge



Electric  
repulsion

Magnetic  
attraction

- Electromagnetic interaction among charged particles  
→ space charge
- Defocusing effect on the beam
- Higher beam current, higher space charge
- Higher beam velocity, lower space charge

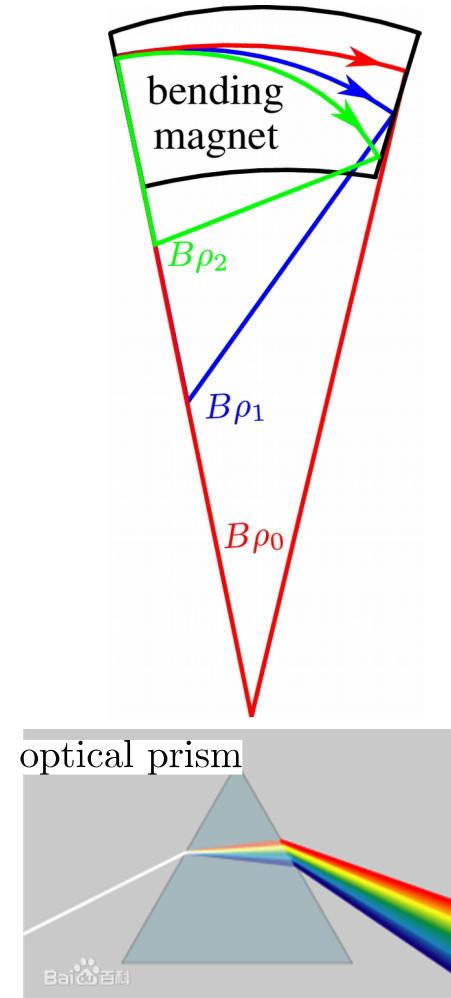
# Dispersion



- Dispersion effects exist in synchrotrons.
- Particles going through arc section (dipoles) travel on different orbits.
- Dispersion function

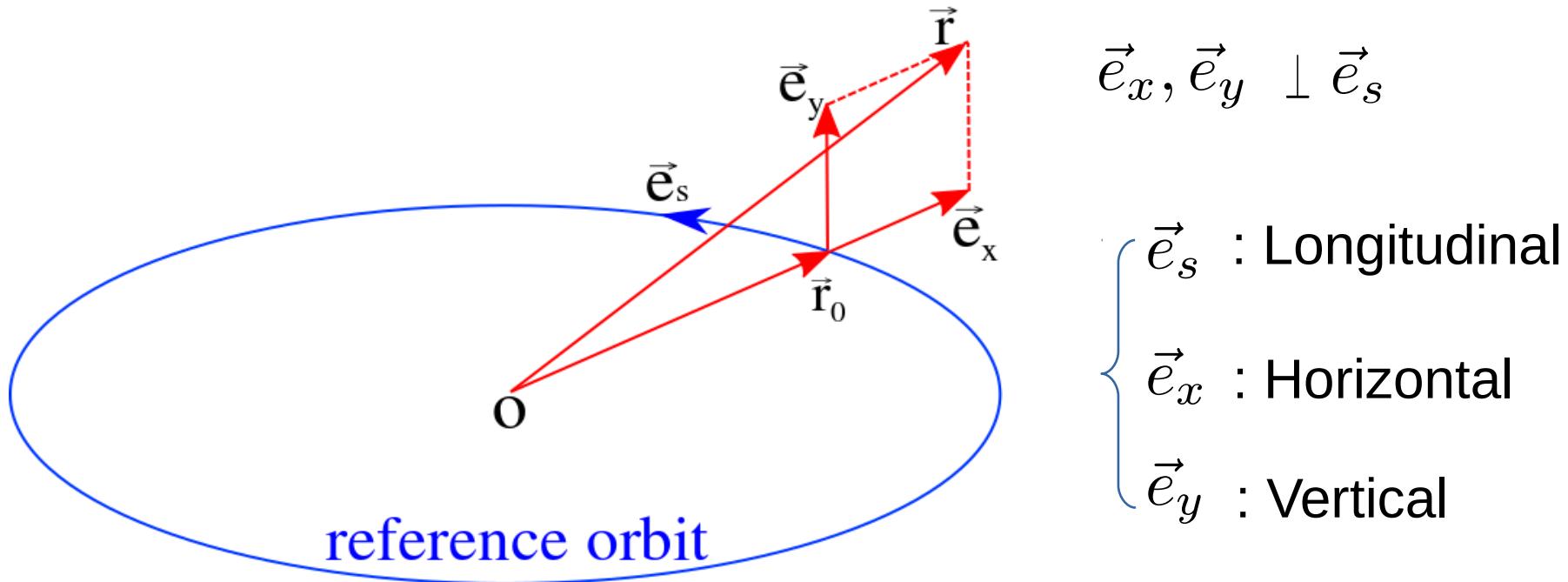
$$\frac{d^2 D_{0,x}}{ds^2} + \kappa_{x0}(s) D_{0,x}(s) = \frac{1}{\rho(s)}$$

- Dispersion is a coupling effect between transverse and longitudinal directions.



(Image source: Internet)

# Coordinate system



A particle's trajectory:  $\vec{r}(s) = \vec{r}_0(s) + x\vec{e}_x + y\vec{e}_y$



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# Intense beam dynamics



- For intense beams, people have to consider the interaction (space charge) between charged particles
- Single particle dynamics + space charge  $\xrightarrow{\text{rms}}$  Intense beam dynamics

Hill's equation

$$\frac{d^2x}{ds^2} + [\kappa_{0,x}(s) - \Delta\kappa_x(s)]x = 0$$

Space-charge defocusing

$\downarrow$  rms

rms Envelope  
equation

$$\sigma''_x + \kappa_{0,x}(s)\sigma_x - \frac{K_{\text{sc}}}{2(\sigma_x + \sigma_y)} - \frac{\tilde{\epsilon}_x^2}{\sigma_x^3} = 0$$

# Transverse rms envelope equations

- As we know, RMS envelope equations can be written as

$$\sigma_x'' + \kappa_{0,x}(s)\sigma_x - \frac{K_{sc}}{2(\sigma_x + \sigma_y)} - \frac{\tilde{\epsilon}_x^2}{\sigma_x^3} = 0$$

(de)focusing of magnets

space charge  $K_{sc} \propto N/\beta^2\gamma^3$

perveance

RMS emittance

RMS beam sizes

(and similar for  $\sigma_y$  in y-direction)

- This equation set characterizes the collective behavior of beams.
- It works for the case of no/neglect energy spread/zero dispersion

# Matched beams

- For a well-designed accelerator, the envelope equations exist an unique periodic solution:

$$\sigma_{x0}(L) = \sigma_{x0}(0) \quad \sigma'_{x0}(L) = \sigma'_{x0}(0)$$

- The matched solutions characterize the maximum beam intensity that can be transported in the accelerator channel

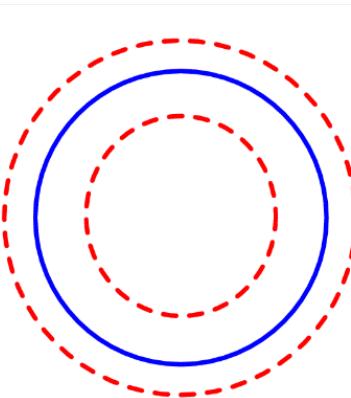
# Envelope oscillation



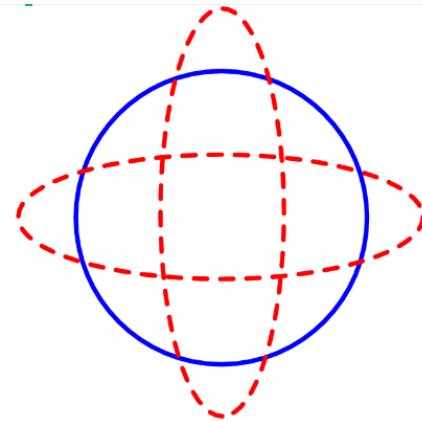
- In reality, because of imperfections/errors, beams perform envelope mismatch oscillations around the matched solutions.

$$\sigma_x = \sigma_{x0} + \xi \quad \sigma_y = \sigma_{y0} + \zeta$$

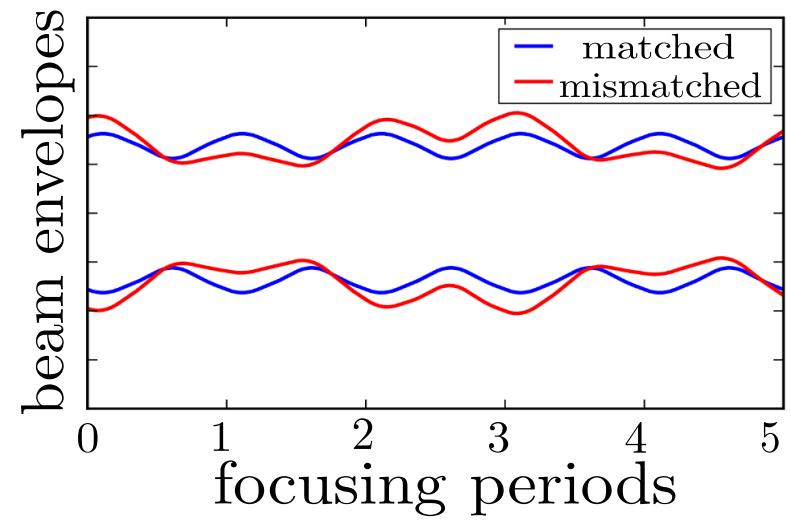
- Two fundamental oscillation “modes”



Breathing mode



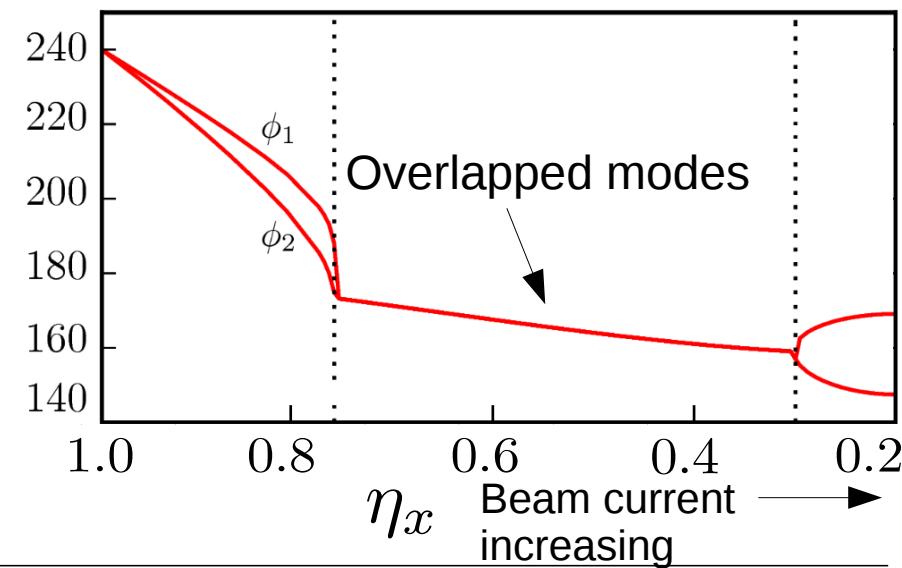
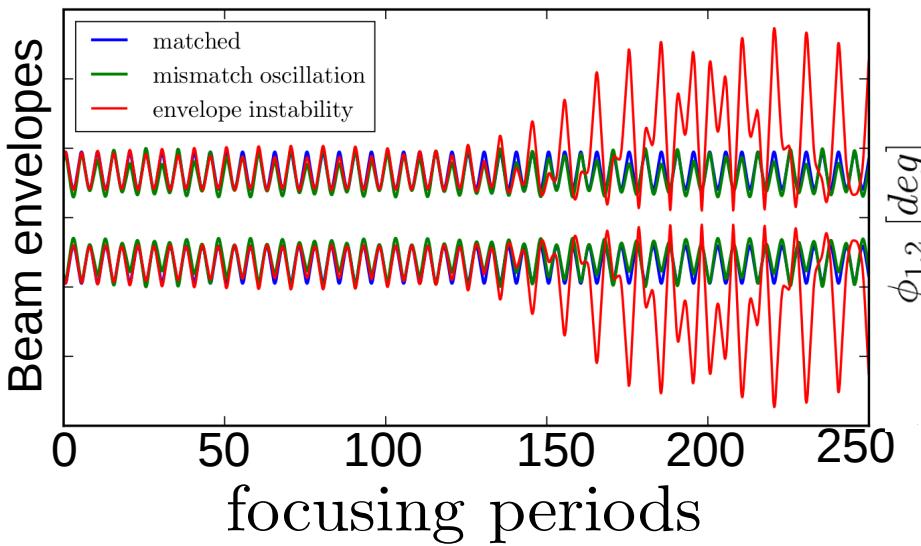
Quadrupolar mode



# Envelope instability



- Envelope mismatch oscillation could trigger the **envelope instability** – the resonance between the two modes.
- Criteria: Phase advance  $k_{0,x,y} > 90^\circ$  and, space-charge depressed to  $k_{x,y} < 90^\circ$



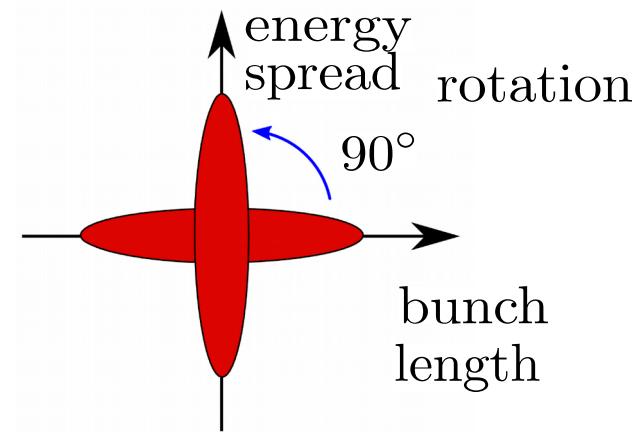
# Coupled envelope equations

- The rms envelope equation is not enough to describe the beam dynamics during bunch compression

- During bunch compression, both of

{ Space charge }  
 { Dispersion }

will be enhanced, and there is a rotation in longi



→ Envelope equations

→ Dispersion function

Generalized  
envelope model

→ longitudinal dynamics

Coupled longitudinal-transverse model

# Generalized envelope model

- Including dispersion, the generalized envelope equations

$$\frac{d^2\sigma_x}{ds^2} + \left[ \kappa_{x0}(s) - \frac{K_{sc}}{2X(X+Y)} \right] \sigma_x - \frac{\varepsilon_{dx}^2}{\sigma_x^3} = 0$$

$$\frac{d^2\sigma_y}{ds^2} + \left[ \kappa_{y0}(s) - \frac{K_{sc}}{2Y(X+Y)} \right] \sigma_y - \frac{\varepsilon_{dy}^2}{\sigma_y^3} = 0$$

$$\frac{d^2D_x}{ds^2} + \left[ \kappa_{x0}(s) - \frac{K_{sc}}{2X(X+Y)} \right] D_x = \frac{1}{\rho(s)}$$

Space charge coupling

→ Total beam size

$$X = \sqrt{\sigma_x^2 + \sigma_\delta^2 D_x^2}$$

Betatron  
beam size

Dispersion  
beam size

→ Generalized emittance  
 $\epsilon_{dx}$  an invariant with  
dispersion

→  $D_x$ : space-charge  
modified dispersion

# Dispersion mode

- Performing perturbations on the generalized equations,

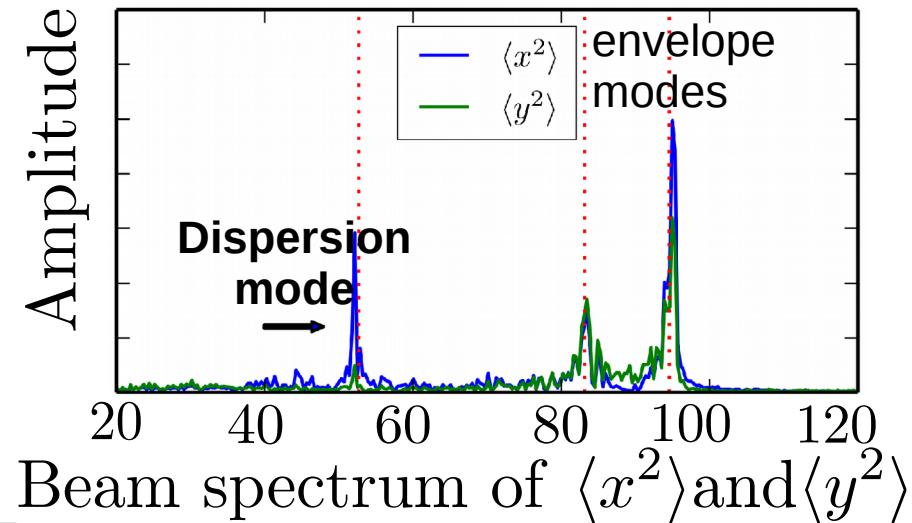
$$\left\{ \begin{array}{l} \sigma_x = \sigma_{x0} + \xi, \\ \sigma_y = \sigma_{y0} + \zeta \\ D_x = D_{x0} + d_x \\ X = X_0 + \frac{\xi \sigma_{x0}}{X_0} + \frac{\sigma_\delta^2 D_{x0} d_x}{X_0} \end{array} \right.$$



oscillation equation set

$$\frac{d^2}{ds^2} \begin{pmatrix} \xi \\ \eta \\ d_x \end{pmatrix} = M \begin{pmatrix} \xi \\ \eta \\ d_x \end{pmatrix}$$

- $M$  is a  $3 \times 3$  matrix, with element of functions of  $\sigma_{x0}, \sigma_{y0}$  and  $D_{x0}$
- Three fundamental mode can be found from  $M$



# Dispersion-induced beam instability



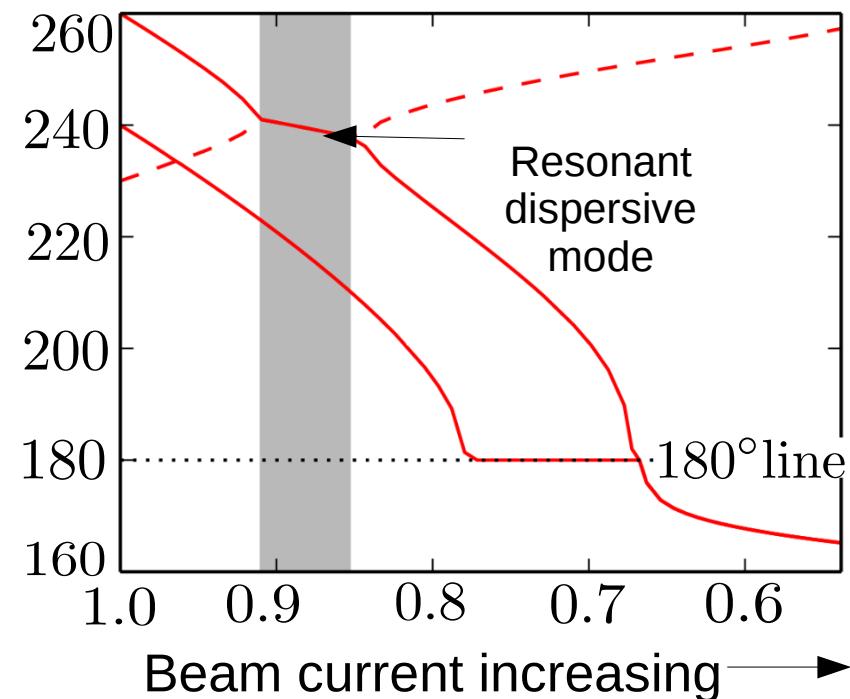
- Dispersion mode becomes unstable.
- The criteria is

$$k_{0,x} > 120^\circ \text{ and } k_x < 120^\circ$$

↑  
Lattice with large phase advance

↑  
Sufficiently high beam current

- It is characterized by the resonance of the dispersion mode and the envelope modes





# Coupled envelope equations

- The combined longitudinal-transverse envelope equations including dispersion are

$$z_m'' + \left( \kappa_{z0} - \frac{K_L}{z_m^3} \right) z_m - \frac{\varepsilon_z^2}{z_m^3} = 0$$

$$\sigma_x'' + \left[ \kappa_{x0} - \frac{K_{sc}}{2X(X+Y)} \right] \sigma_x - \frac{\varepsilon_{dx}^2}{\sigma_x^3} = 0$$

$$\sigma_y'' + \left[ \kappa_{y0} - \frac{K_{sc}}{2Y(X+Y)} \right] \sigma_y - \frac{\varepsilon_{dy}^2}{\sigma_y^3} = 0$$

$$D_x'' + \left[ \kappa_{x0} - \frac{K_{sc}}{2X(X+Y)} \right] D_x = \frac{1}{\rho(s)}$$

→ Coupling terms

Total beam size

$$X = \sqrt{\sigma_x^2 + \boxed{\sigma_\delta^2} D_x^2}$$

RMS momentum spread

$$\sigma_\delta = \sqrt{\left( \frac{5z'_m}{\eta} \right)^2 + \left( \frac{\varepsilon_z}{\eta \sigma_z} \right)^2}$$

( $\eta$  is the slip factor)



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- **Bunch compression in SIS-18**
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# Bunch compression in SIS-18 GSI



- Two approaches

## (1) Envelope solver

Input initial parameters

↓ Iterative method

Solve envelope equations

↓ Matched solutions

Solve perturbation equations

## (2) Multi-particle tracking simulation

→ Macro particles are tracked along accelerator; Space charge calculation with PIC

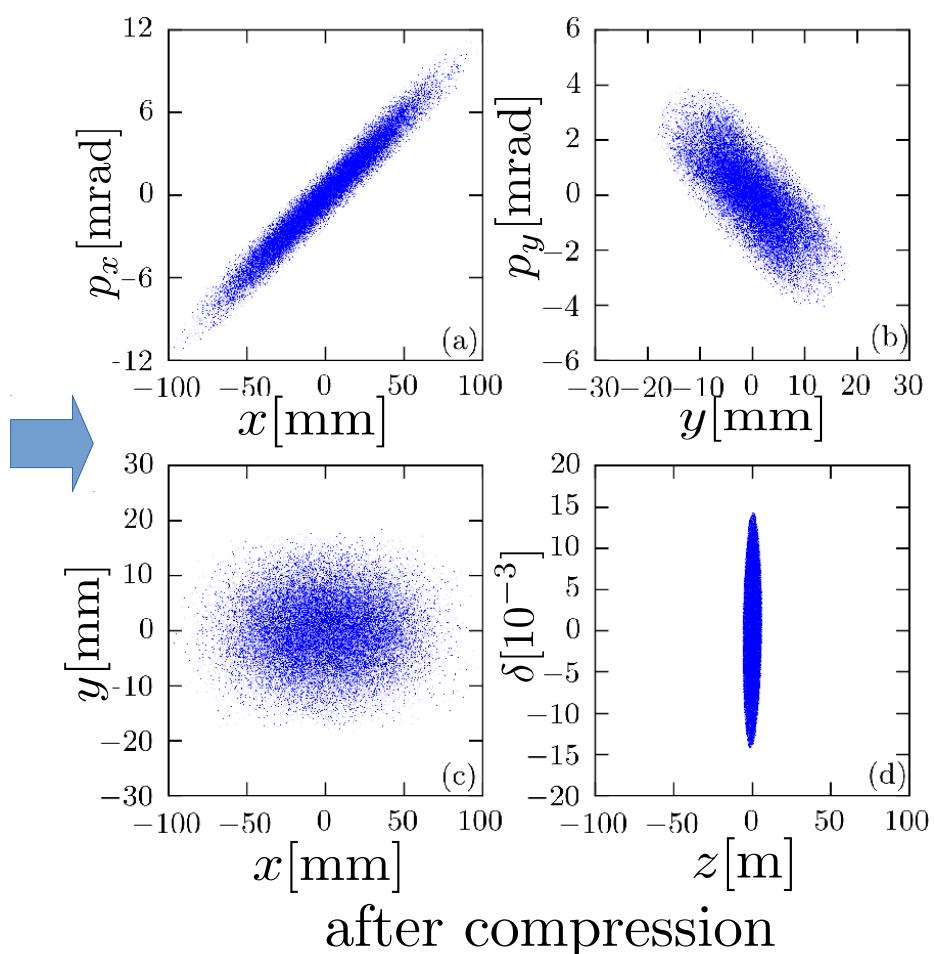
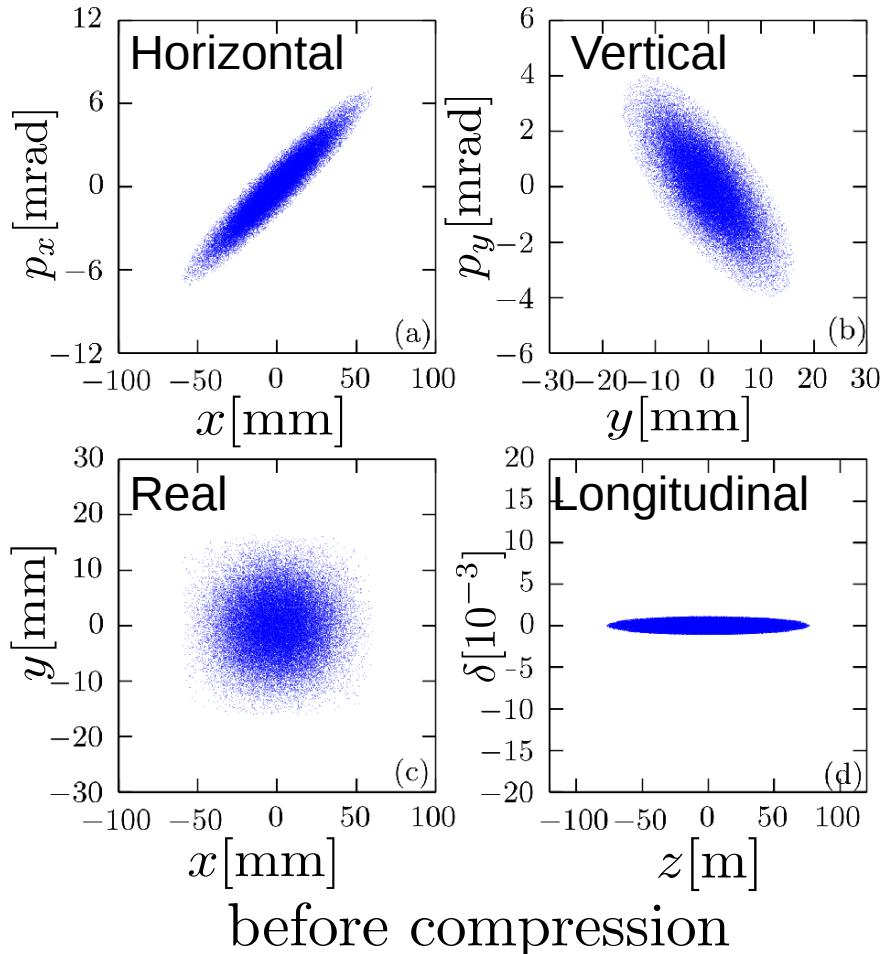
# Bunch compression in SIS-18 GSI



- The coupled envelope approach is used to investigate the bunch compression in GSI SIS-18

Parameters [unit]	Value
Circumference [m]	216
Kinetic energy [MeV/u]	295
Initial half bunch length [m]	78
Final half bunch length [m]	6.0
Required turns	77
Periodic phase advance	128,104

# Bunch compression in SIS-18 GSI

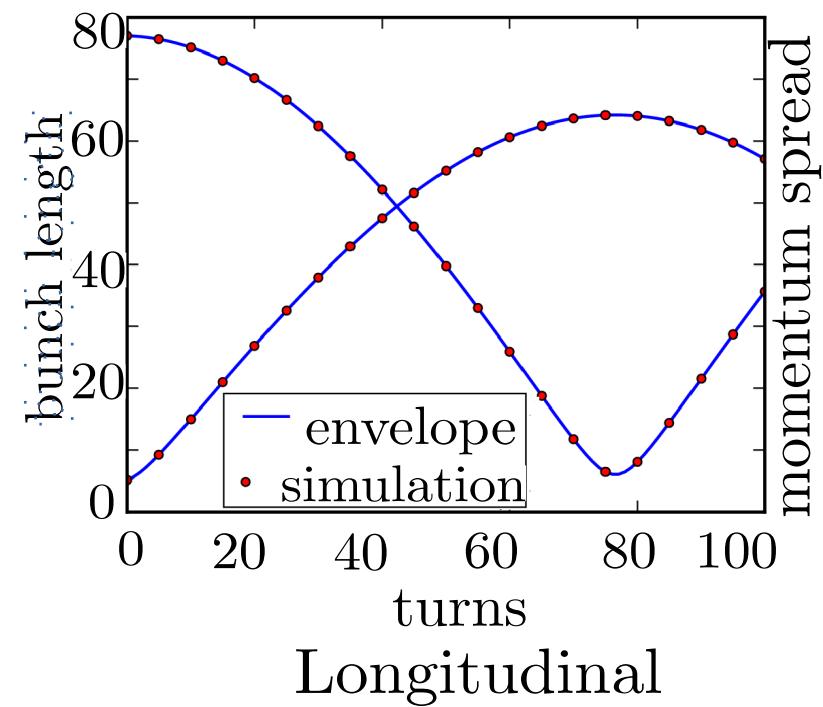
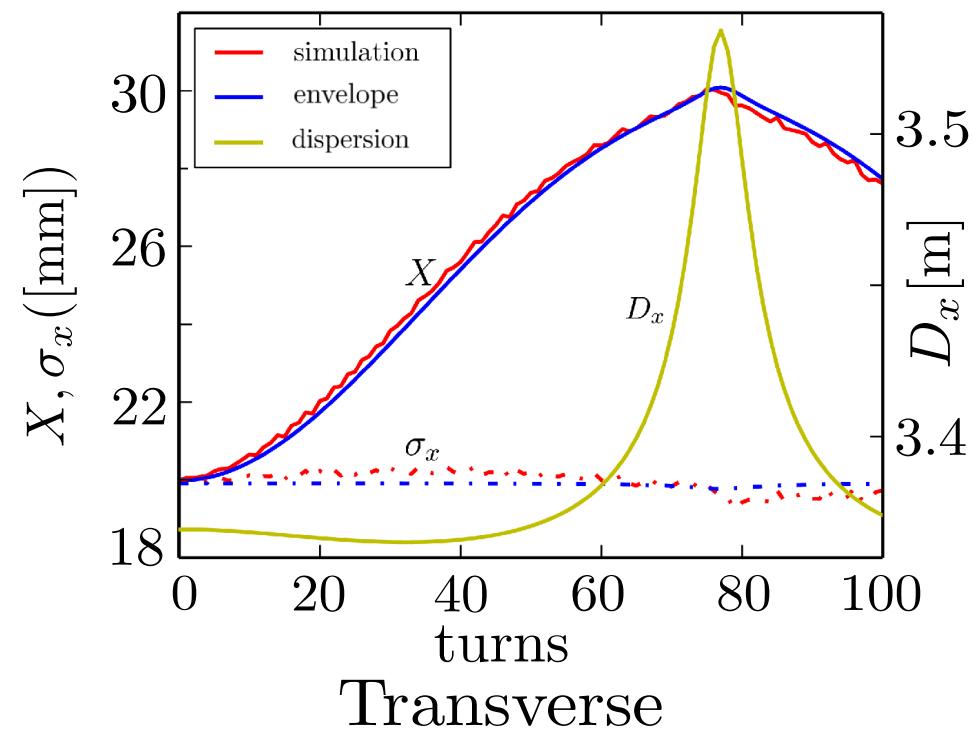


Simulation results

# Bunch compression in SIS-18 GSI



- Comparison between the envelope calculation and simulation results



# Bunch compression in SIS-18 GSI

- Numerical results agrees well with the PIC simulations.
- In the given parameters, the bunch compression works well.
- The horizontal beam size growth is due to dispersion effect, and no beam instabilities are observed.



# Intensity limitations due to space charge



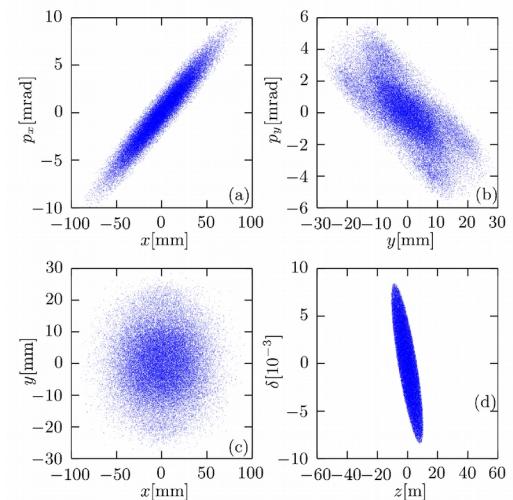
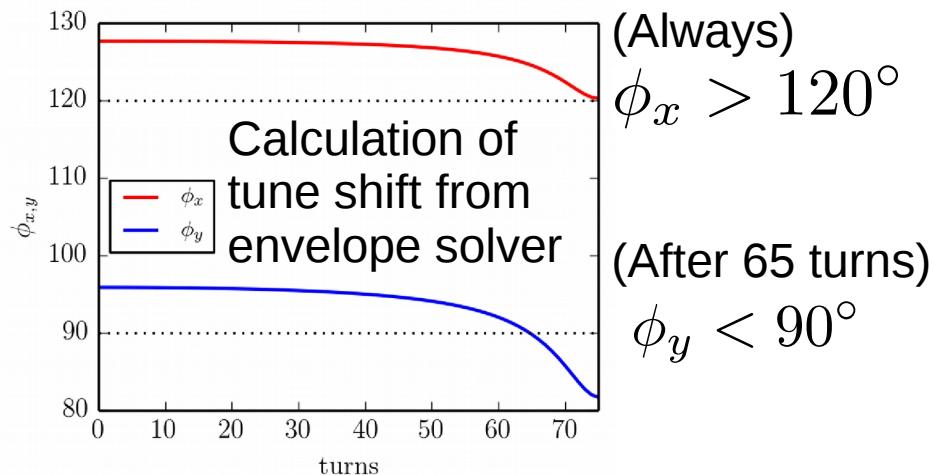
- During bunch compression, the bunch length is being compressed, and transverse space charge will be enhanced
  - Both of { Transverse space charge | Energy spread (dispersion) } will be enhanced, and have a combined effect on beam dynamics
- One of the major beam intensity limitations would be space-charge driven beam instabilities
  - { (1) 90°-related limitation
  - { (2) 120° related limitation

# Intensity limitations due to space charge



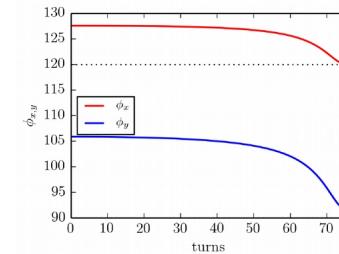
## (1) 90°-related limitation

- In order to investigate the 90° instability, we **double the beam intensity**, resulting in the phase advance shift lower than 90°



- To avoid the beam instability, we adjust the lattice

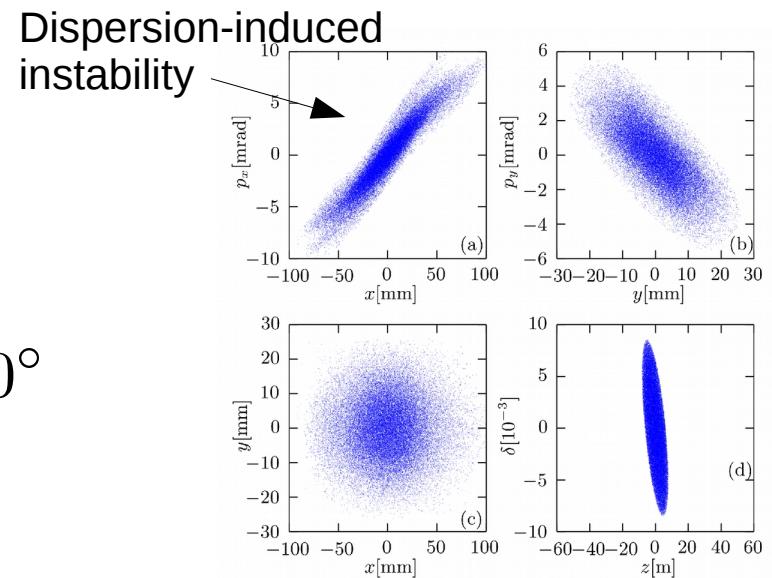
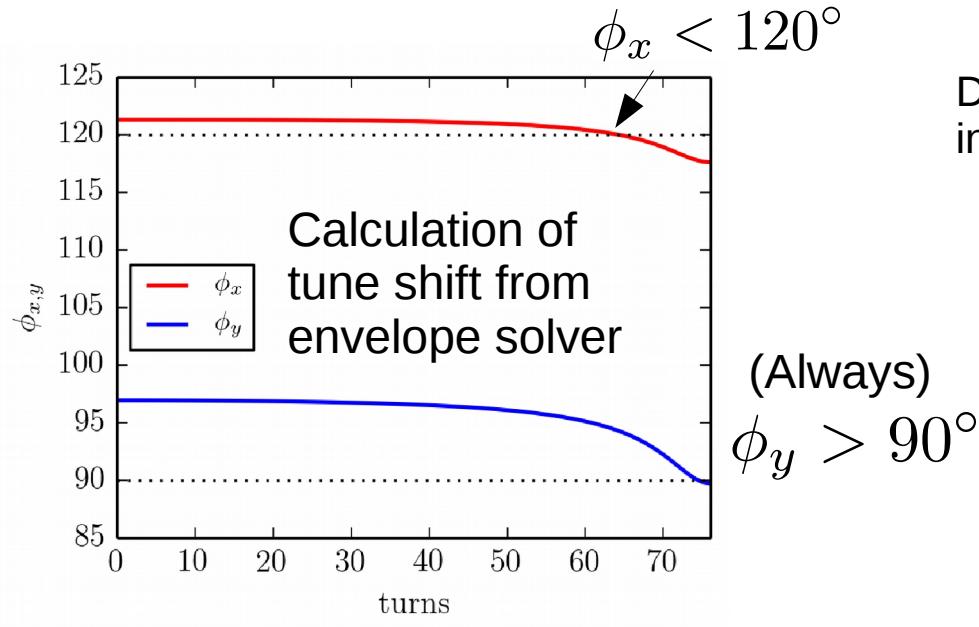
$$\text{from } \begin{cases} \phi_x = 129^\circ \\ \phi_y = 98^\circ \end{cases} \text{ to } \begin{cases} \phi_x = 129^\circ \\ \phi_y = 108^\circ \end{cases}$$





- Another instability is 120° dispersion-induced instability
- In order to investigate it, the phase advance from

$$\left\{ \begin{array}{l} \phi_{x0} = 129^\circ \\ \phi_{y0} = 98^\circ \end{array} \right. \text{to} \left\{ \begin{array}{l} 122^\circ \\ 98^\circ \end{array} \right.$$



- To avoid the beam instability, the phase advance should be sufficiently far away from 120°



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# Summary

- The coupled transverse-longitudinal envelope set is established to investigate the bunch compression in SIS-18.
- During bunch compression, two major intensity limitation due to space charge are discussed:  $90^\circ$  and  $120^\circ$ -related stop band, caused by a unstable dispersion mode.
- Methods of how to avoid the stop band (increase the intensity limitation) are shown.

# Outlook

- An experiment are proposed to measure the dispersion mode in SIS-18



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# Thank you for your attention!