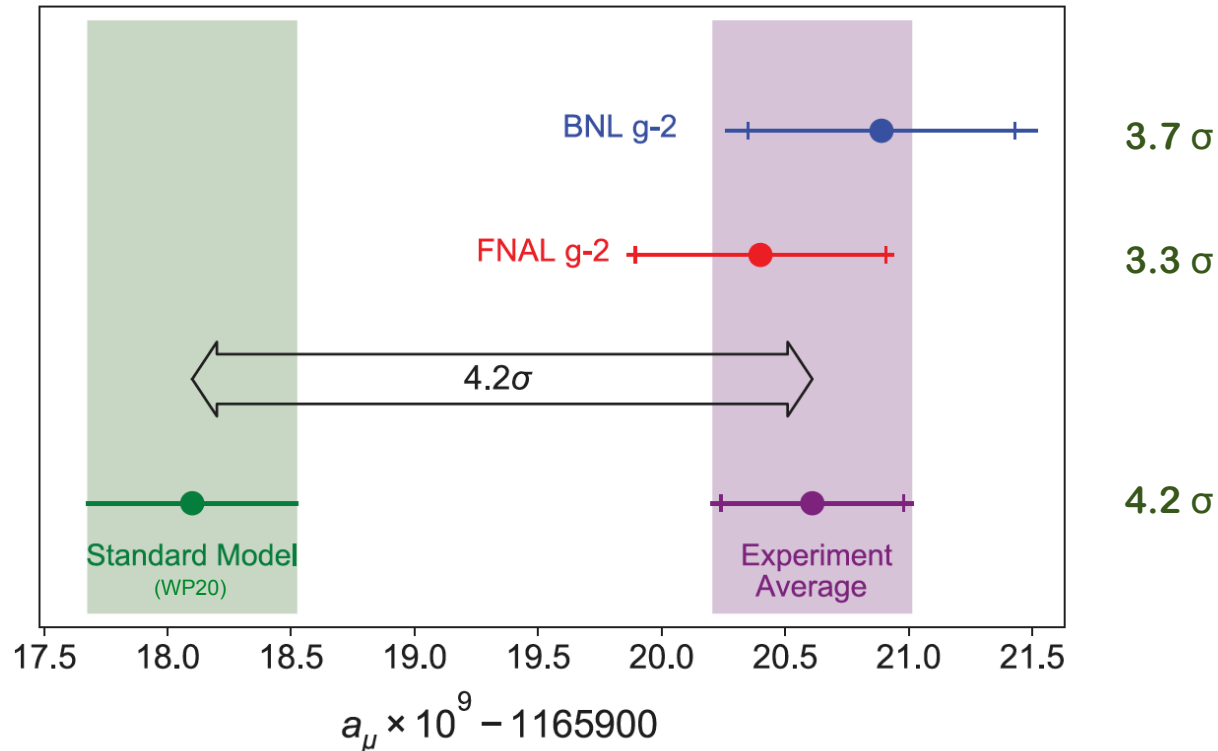


Muon g-2: status of theory & experiment

Paolo Girotti - Pisa University
Massimo Passera - INFN Padova

International Conference on Exotic Atoms and Related Topics
EXA 2021
September 14th 2021



$$a_\mu^{\text{EXP}} = (116592089 \pm 63) \times 10^{-11} [0.54\text{ppm}] \quad \text{BNL E821}$$

$$a_\mu^{\text{EXP}} = (116592040 \pm 54) \times 10^{-11} [0.46\text{ppm}] \quad \text{FNAL E989 Run 1}$$

$$a_\mu^{\text{EXP}} = (116592061 \pm 41) \times 10^{-11} [0.35\text{ppm}] \quad \text{WA}$$

- FNAL aims at 16×10^{-11} . First 4 runs completed, 5th soon.
- Muon g-2 proposal at J-PARC: Phase-1 with $\sim$ BNL precision.

Muon $g-2$: the Standard Model prediction

WP20 = White Paper of the Muon $g-2$ Theory Initiative: [arXiv:2006.04822](https://arxiv.org/abs/2006.04822)

Muon g-2: the QED contribution



$$a_{\mu}^{\text{QED}} = (1/2)(\alpha/\pi) \quad \text{Schwinger 1948}$$

$$+ 0.765857426 (16) (\alpha/\pi)^2$$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$$+ 24.05050988 (28) (\alpha/\pi)^3$$

Remiddi, Laporta, Barbieri ... ; Czarnecki, Skrzypek '99; MP '04;
Friot, Greynat & de Rafael '05, Ananthanarayan, Friot, Ghosh 2020

$$+ 130.8780 (60) (\alpha/\pi)^4$$

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Steinhauser et al. 2013, 2015 & 2016 (all electron & τ loops, analytic);
Laporta, PLB 2017 (mass independent term) **COMPLETED!**

$$+ 750.86 (88) (\alpha/\pi)^5 \quad \text{COMPLETED!}$$

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta, ...
Aoyama, Hayakawa, Kinoshita, Nio 2012, 2015, 2017 & 2019.
Volkov 1909.08015: $A_1^{(10)}$ [no lept loops] at variance, but negligible $\delta a_{\mu} \sim 6 \times 10^{-14}$

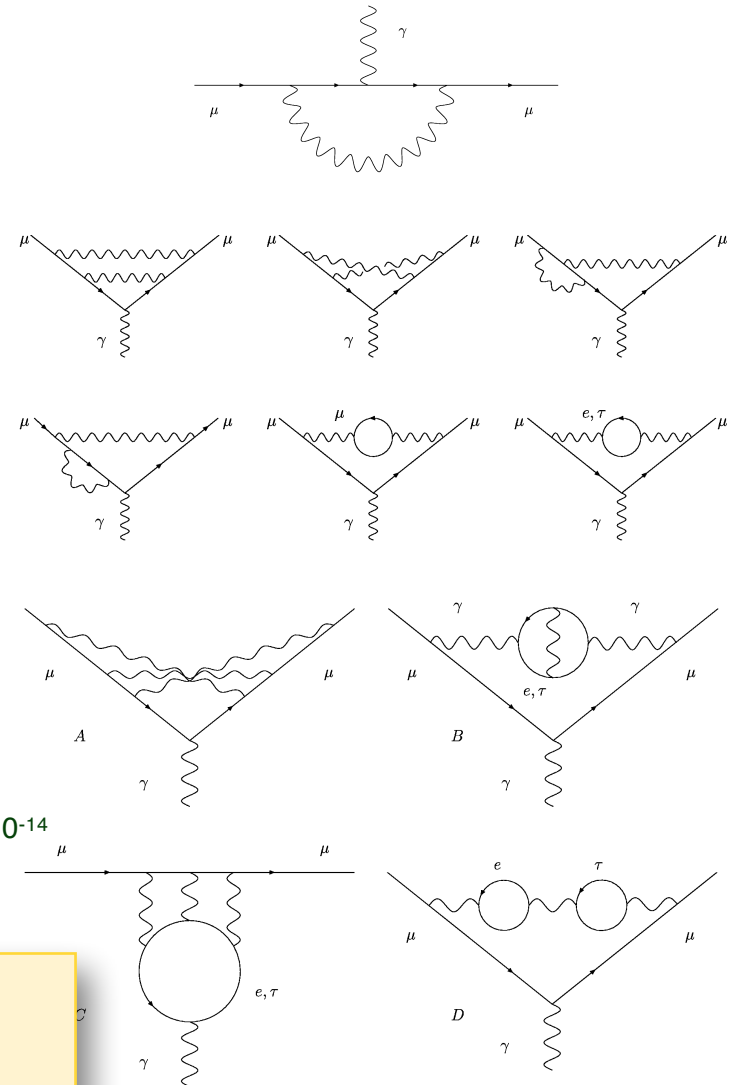
Adding up, we get:

$$a_{\mu}^{\text{QED}} = 116584718.931 (19)(100)(23) \times 10^{-11}$$

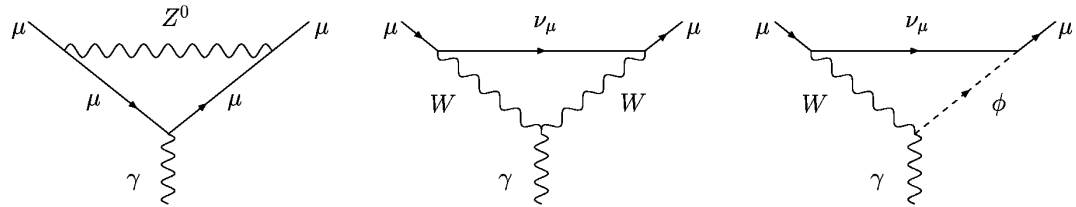
mainly from 4-loop coeff. unc. 6-loop from $\alpha(\text{Cs})$

$\alpha = 1/137.035999046(27)$ [0.2ppb] Parker et al 2018

WP20 value



One-loop term:



$$a_{\mu}^{\text{EW}}(1\text{-loop}) = \frac{5G_{\mu}m_{\mu}^2}{24\sqrt{2}\pi^2} \left[1 + \frac{1}{5} (1 - 4\sin^2\theta_W)^2 + O\left(\frac{m_{\mu}^2}{M_{Z,W,H}^2}\right) \right] \approx 195 \times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

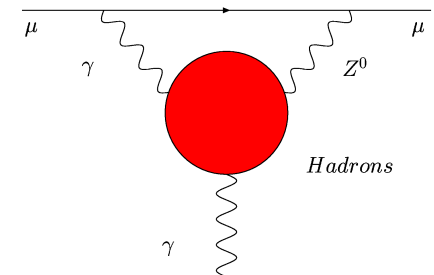
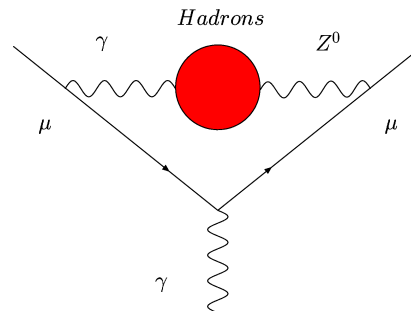
One-loop plus higher-order terms:

$$a_{\mu}^{\text{EW}} = 153.6 (1.0) \times 10^{-11}$$

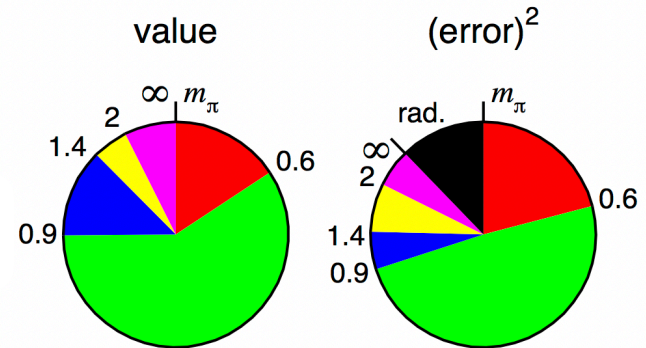
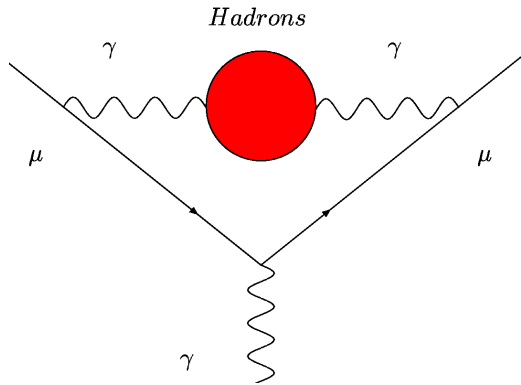
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degraasi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013, Ishikawa, Nakazawa, Yasui, 2019.

Hadronic loop uncertainties (and 3-loop nonleading logs).

WP20 value



The hadronic LO contribution



Keshavarzi, Nomura, Teubner 2018

$$a_{\mu}^{\text{HLO}} = \frac{1}{4\pi^3} \int_{m_{\pi}^2}^{\infty} ds K(s) \sigma_{\text{had}}^{(0)}(s)$$

$$K(s) = \int_0^1 dx \frac{x^2 (1-x)}{x^2 + (1-x)(s/m_{\mu}^2)}$$

$$a_{\mu}^{\text{HLO}} = 6895 (33) \times 10^{-11}$$

F. Jegerlehner, arXiv:1711.06089

$$= 6939 (40) \times 10^{-11}$$

Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921

$$= 6928 (24) \times 10^{-11}$$

Keshavarzi, Nomura, Teubner, arXiv:1911.00367

$$= 6931 (40) \times 10^{-11} (0.6\%)$$

WP20 value



WP20 value obtained merging conservatively DHMZ + KNT + constraints from CHKS

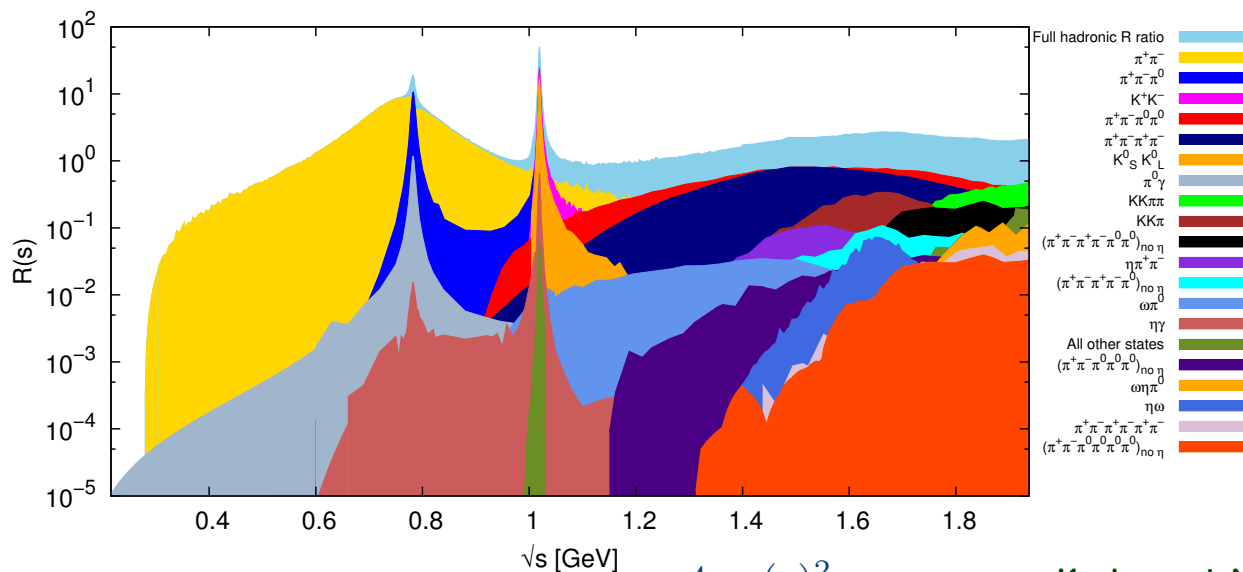
Colangelo, Hoferichter, Hoid, Kubis, Stoffer 2018-19



Radiative Corrections to $\sigma(s)$ are crucial.

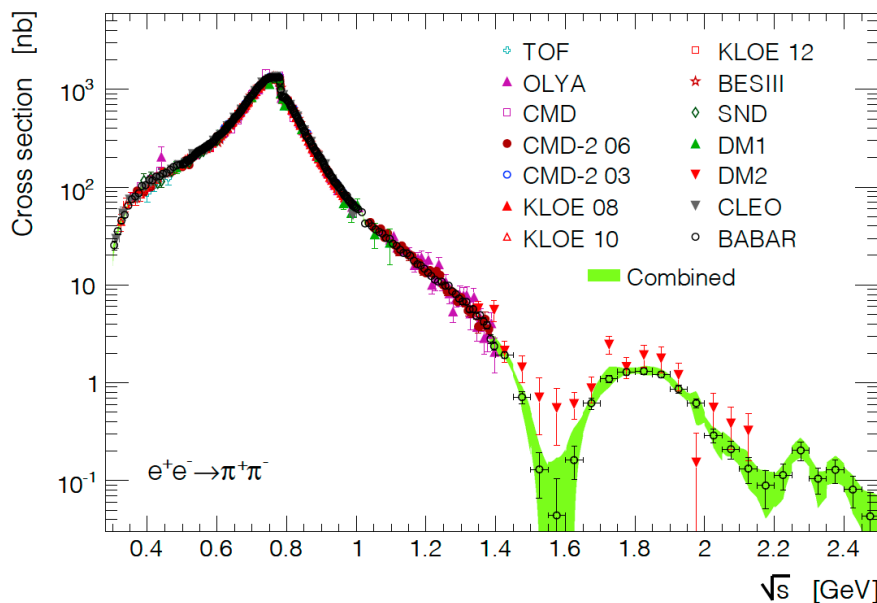
S. Actis et al, Eur. Phys. J. C66 (2010) 585

The low-energy hadronic cross section



$$R(s) = \sigma(e^+e^- \rightarrow \text{hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$

Keshavarzi, Nomura Teubner
PRD 2018

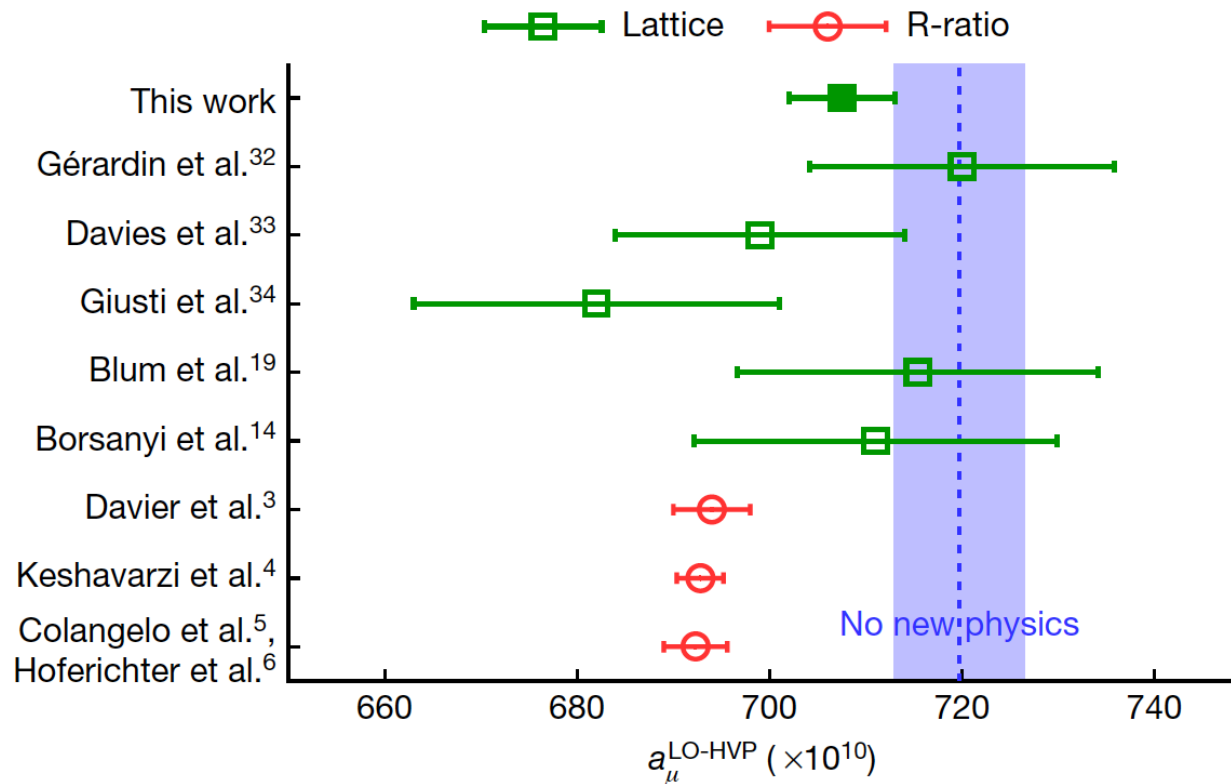


Davier, Hoecker, Malaescu, Zhang
EPJC 2020

- Great progress in lattice QCD results. The BMW collaboration reached 0.8% precision:

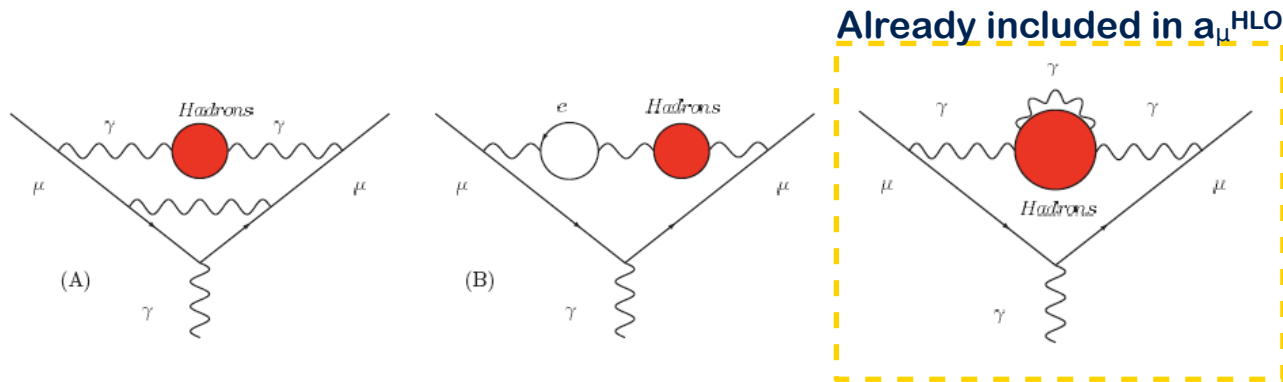
$$a_{\mu}^{\text{HLO}} = 7075(23)_{\text{stat}}(50)_{\text{syst}} [55]_{\text{tot}} \times 10^{-11}$$

- 2–2.5 σ tension with the dispersive evaluations. BMW collaboration 2021



Borsanyi et al (BMWc), Nature 2021

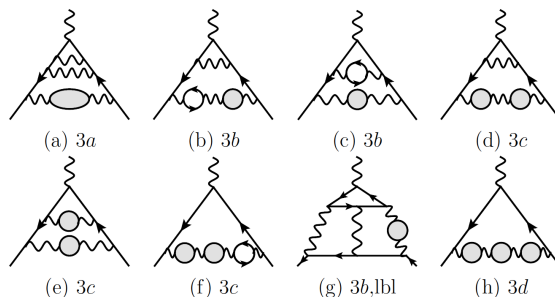
- $O(\alpha^3)$ contributions of diagrams containing HVP insertions:



$$a_\mu^{\text{HNLO}}(\text{vp}) = -98.3 (7) \times 10^{-11}$$

Krause '96; Keshavarzi, Nomura, Teubner 2019; WP20.

- $O(\alpha^4)$ contributions of diagrams containing HVP insertions:

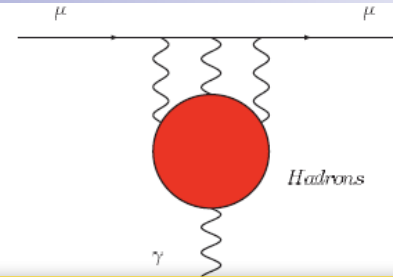


$$a_\mu^{\text{HNNLO}}(\text{vp}) = 12.4 (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

- Hadronic light-by-light at $O(\alpha^3)$

 This term had a troubled life! But nowadays:



$$\begin{aligned}
 a_{\mu}^{\text{HNLO}}(|b|) &= 80 (40) \times 10^{-11} && \text{Knecht \& Nyffeler '02} \\
 &= 136 (25) \times 10^{-11} && \text{Melnikov \& Vainshtein '03} \\
 &= 105 (26) \times 10^{-11} && \text{Prades, de Rafael, Vainshtein '09} \\
 &= 100 (29) \times 10^{-11} && \text{Jegerlehner, arXiv:1705.00263} \\
 &= \mathbf{92 (19) \times 10^{-11}} && \text{WP20 (phenomenology)}
 \end{aligned}$$

 Significant improvements due to data-driven dispersive approach.

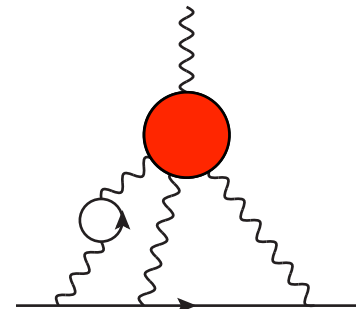
Colangelo, Hoferichter, Procura, Stoffer, 2014–17; Pauk, Vanderhaeghen 2014.

 Lattice: RBC: $\mathbf{82(35) \times 10^{-11}}$ 1911.08123 Mainz: $\mathbf{110(15) \times 10^{-11}}$ 2104.02632

- Hadronic light-by-light at $O(\alpha^4)$

$$a_{\mu}^{\text{HNNLO}}(|b|) = 2 (1) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014; WP20



- Comparing the SM prediction with the measured muon g-2 value:

$$a_{\mu}^{\text{EXP}} = 116592061 (41) \times 10^{-11}$$

BNL+FNAL

$$a_{\mu}^{\text{SM}} = 116591810 (43) \times 10^{-11}$$

WP20

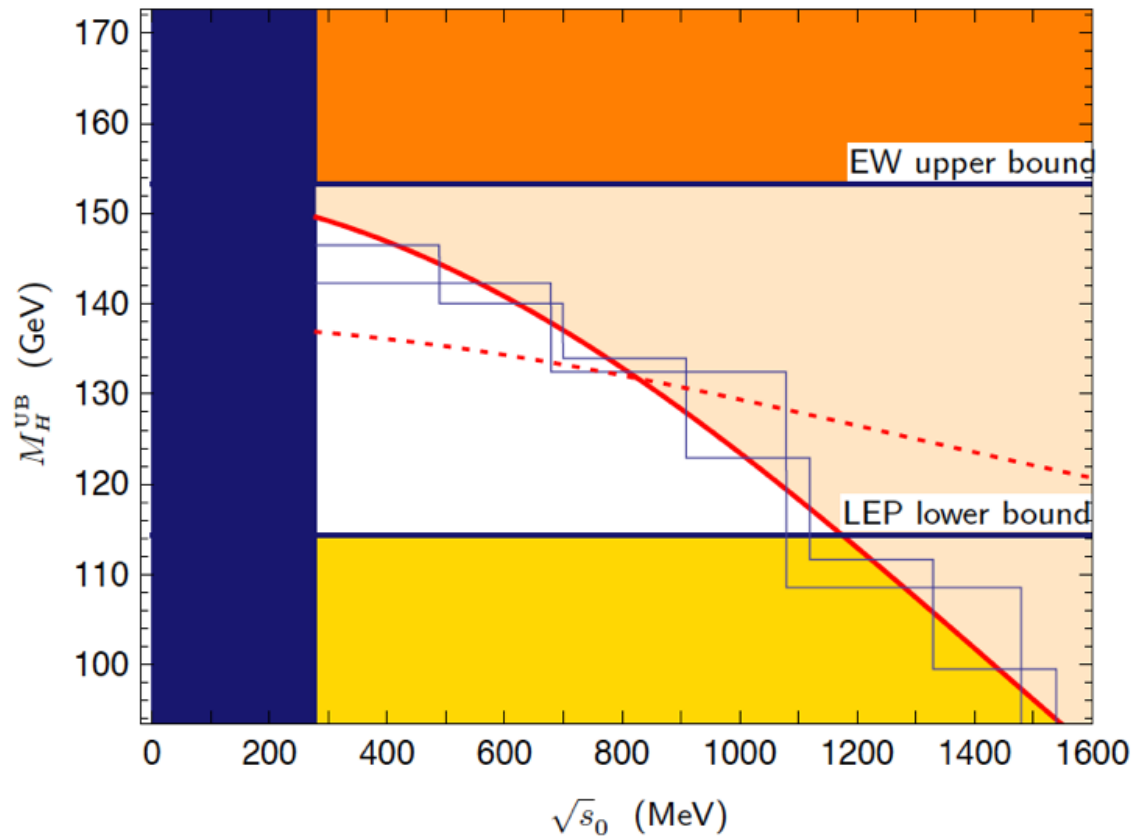
$$\Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} = 251 (59) \times 10^{-11}$$

4.2 σ

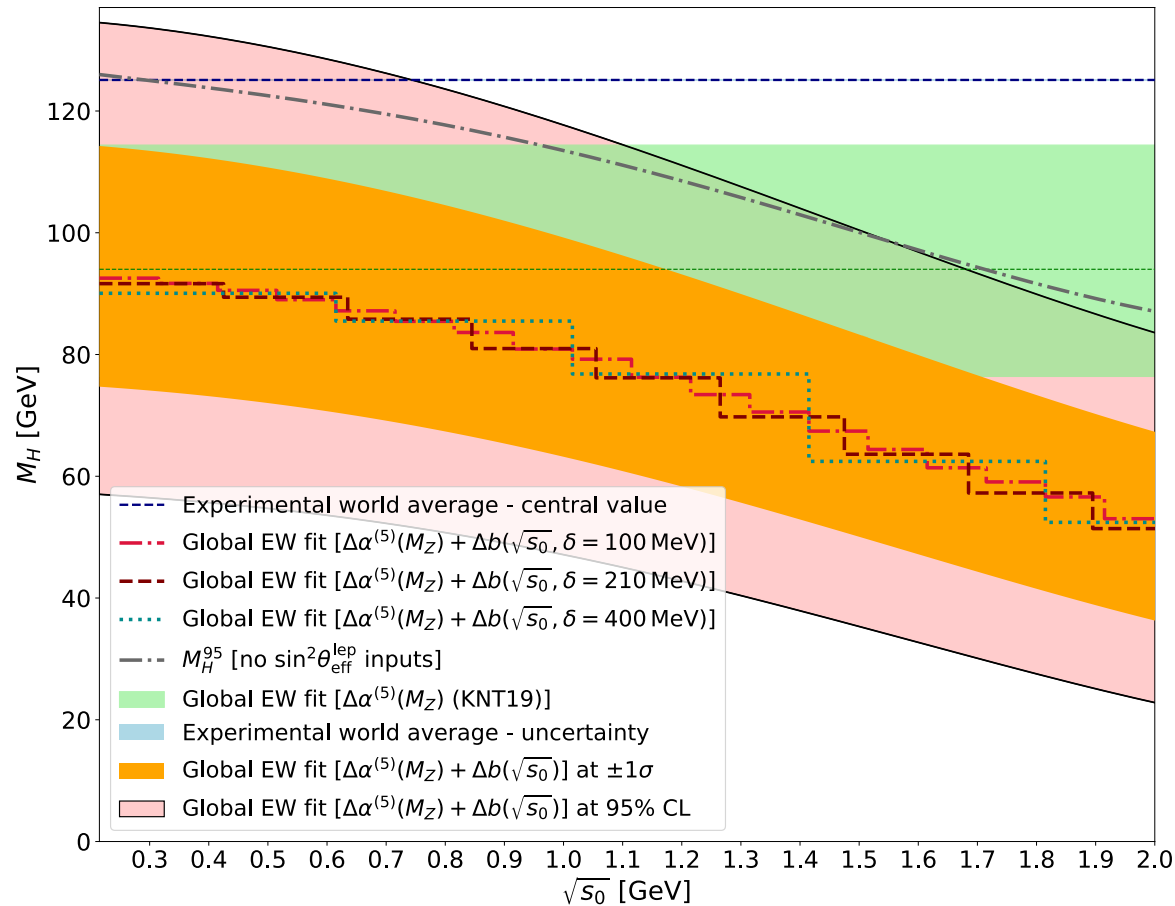
If BMW 2021 HLO instead of WP20, EXP & SM differ only by **1.6 σ**

- Is Δa_{μ} due to **new physics** beyond the SM? Could be due to:
 - NP at the weak scale and weakly coupled to SM particles
 - NP very heavy and strongly coupled to SM particles
 - NP very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles

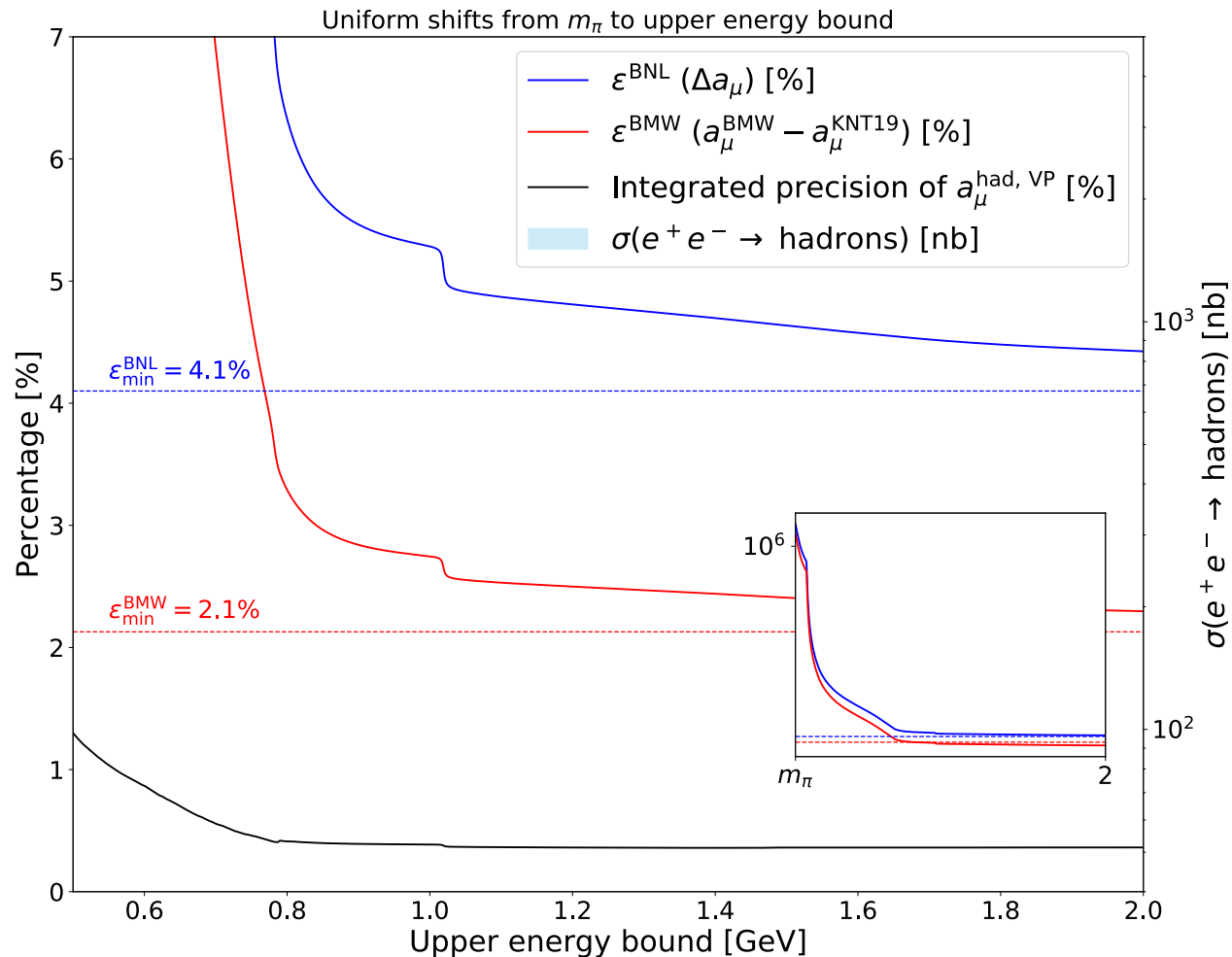
How much does the M_H upper bound from the EW fit change when we shift up $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$] to fix Δa_μ ?



Marciano, MP, Sirlin, PRD 2008



Shifts $\Delta\sigma(s)$ to fix Δa_μ are possible,
but conflict with the EW fit if they occur above ~ 1 GeV



Shifts below ~ 1 GeV conflict with the quoted exp. precision of $\sigma(s)$

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (updated 2021)

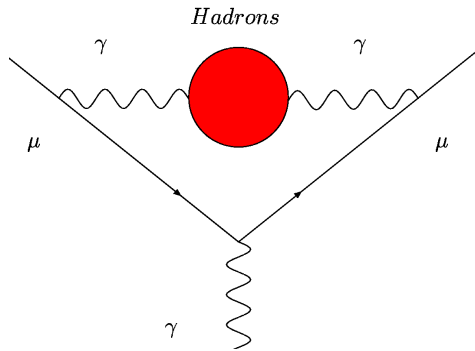
- Crivellin, Hoferichter, Manzari and Montull, “Hadronic vacuum polarization: $(g-2)_\mu$ versus global electroweak fits,” arXiv:2003.04886.
- Eduardo de Rafael, “On Constraints Between $\Delta\alpha_{\text{had}}(M_Z^2)$ and $(g_\mu-2)_{\text{HVP}}$,” arXiv:2006.13880.
- Malaescu and Schott, “Impact of correlations between a_μ and α_{QED} on the EW fit,” arXiv:2008.08107.
- Colangelo, Hoferichter and Stoffer, “Constraints on the two-pion contribution to hadronic vacuum polarization,” arXiv:2010.07943.

The MUonE project



The spacelike method for a_μ^{HLO}

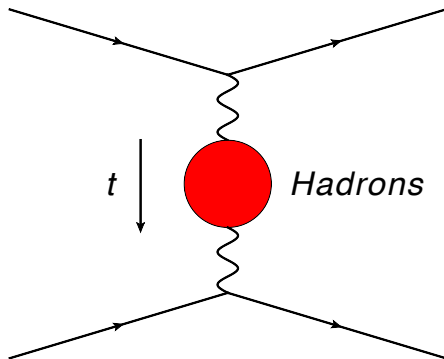
- Leading hadronic contribution computed via the usual dispersive (timelike) formula:



$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{m_\pi^2}^{\infty} ds K(s) \sigma_{\text{had}}^{(0)}(s)$$

$$K(s) = \int_0^1 dx \frac{x^2 (1-x)}{x^2 + (1-x) (s/m_\mu^2)}$$

- Alternatively, simply exchanging the x and s integrations:



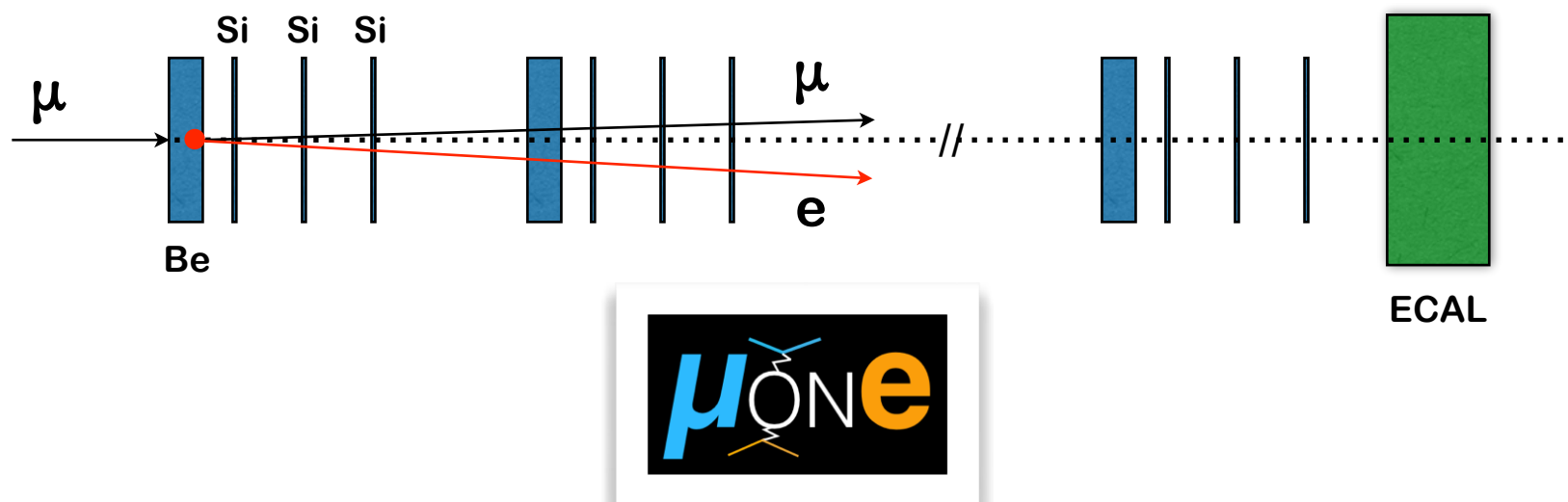
$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

$\Delta\alpha_{\text{had}}(t)$ is the hadronic contribution to the running of α in the spacelike region: **measure a_μ^{HLO} via scattering data!**

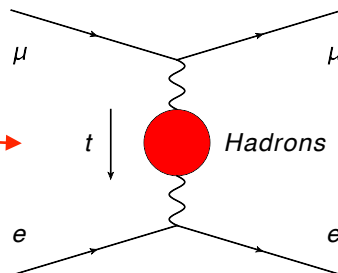
- $\Delta\alpha_{\text{had}}(t)$ can be measured via the **elastic scattering** $\mu e \rightarrow \mu e$.
- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.



Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna,
Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987

- **Statistics:** With CERN's 150 GeV muon beam M2 ($1.3 \times 10^7 \mu/s$), incident on 40 15mm Be targets (total Be thickness: 60cm), 2-3 years of data taking (2×10^7 s/yr) $\rightarrow \mathcal{L}_{\text{int}} \sim 1.5 \times 10^7 \text{ nb}^{-1}$.
- With this $\mathcal{L}_{\text{int}}$ we estimate that measuring the shape of $d\sigma/dt$ we can reach a statistical sensitivity of **$\sim 0.3\%$ on a_μ^{HLO}** , ie $\sim 20 \times 10^{-11}$.
- **Systematic** effects must be known at $\leq 10\text{ppm}$!
- Test beams performed at CERN in 2017 & 2018 [arXiv:1905.11677, 2102.11111](#)
- Lol submitted to CERN SPSC in 2019: **Test run approved for 2021, delayed to 2022.**
- Full-statistics run hopefully in 2023–24.

- To extract $\Delta\alpha_{\text{had}}(t)$ from MUonE's measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at $\leq 10\text{ppm}$!



- Fully differential fixed-order MC @ NLO ready Pavia and PSI 2018-19
- NNLO QED: Master Integrals for 2-loop box diagrams computed. Full 2-loop amplitude completed! ($m_e=0$) Padova 2017 - present
- Two MC built including partial subsets of the NNLO QED corrections due to electron and muon radiation Pavia and PSI 2020
- NNLO hadronic effects computed Padova and KIT 2019
- Extraction of the leading electron mass effects from the massless muon-electron scattering amplitudes PSI 2019-present
- New Physics extracting $\Delta\alpha_{\text{had}}(t)$ at MUonE? Padova and Heidelberg 2020
- ...

Theory for muon-electron scattering @ 10 ppm:
A report of the MUonE theory initiative. arXiv:2004.13663

Conclusions

- Fermilab's Muon $g-2$ experiment confirms BNL's result: the discrepancy between experiment and SM increases to 4.2σ .
- The BMWc lattice QCD result weakens the exp-SM discrepancy. It must be confirmed or refuted by other lattice calculations.
- Is Δa_μ due to missed contributions in the hadronic cross section? Shifts above 1 GeV to fix Δa_μ conflict with the electroweak fit.
- Leading hadronic contribution to a_μ : dispersive vs lattice. MUonE will provide a new independent & alternative determination.