Stable beam operation of drift tube linac cavities at rf-voltages far below design

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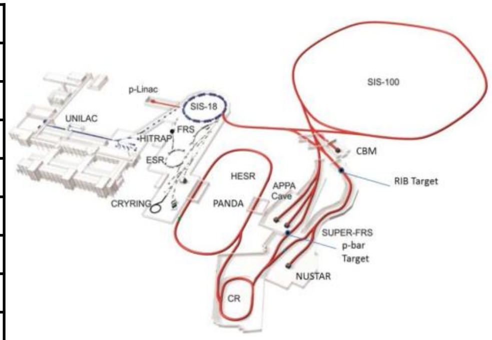
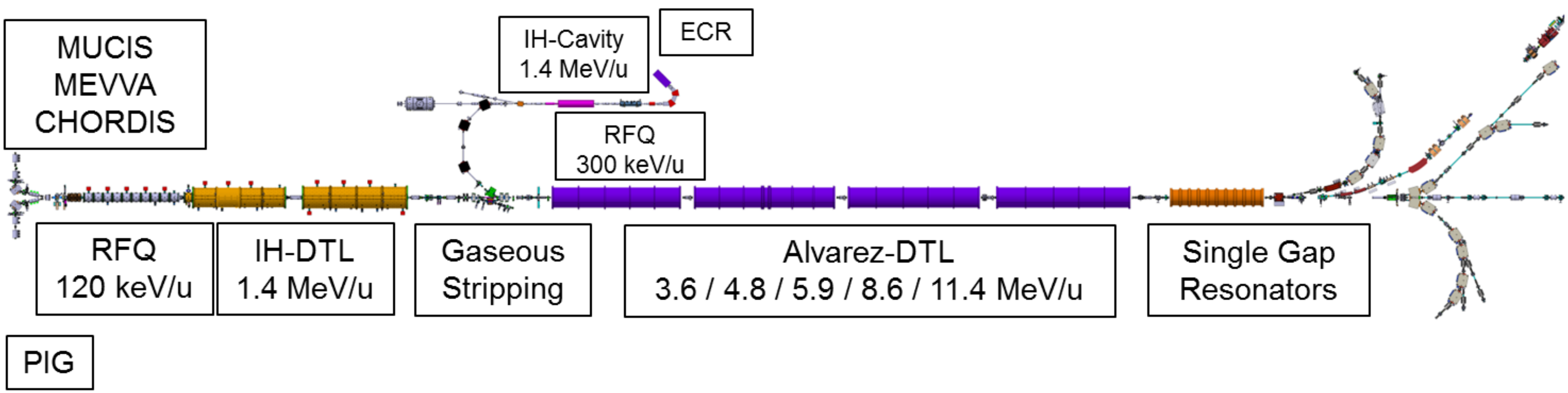
Overview

- Overview
- UNILAC introduction
- Beam dynamics of the Alvarez-type DTL
- Single-particle tracking through the Alvarez-DTL
- Multi-particle tracking through the Alvarez-DTL
- TTF parameter calculation using tracking method
- Conclusion

UNILAC introduction

ion A/q	≤ 8.5, i.e. $^{238}U^{28+}$	
beam current (pulse) * A/q	1.76 (0.5% duty cycle)	emA
input beam energy	2.2	keV/u
output beam energy	3.3 - 11.7	MeV/u
normalized total output emittance, horizontal / vertical	0.8 / 2.5	mm mrad
beam pulse duration	≤ 5000	µs
beam repetition rate	≤ 50	Hz
operating frequency	36.136 / 108.408	MHz
length	≈ 115	m





UNILAC output energies

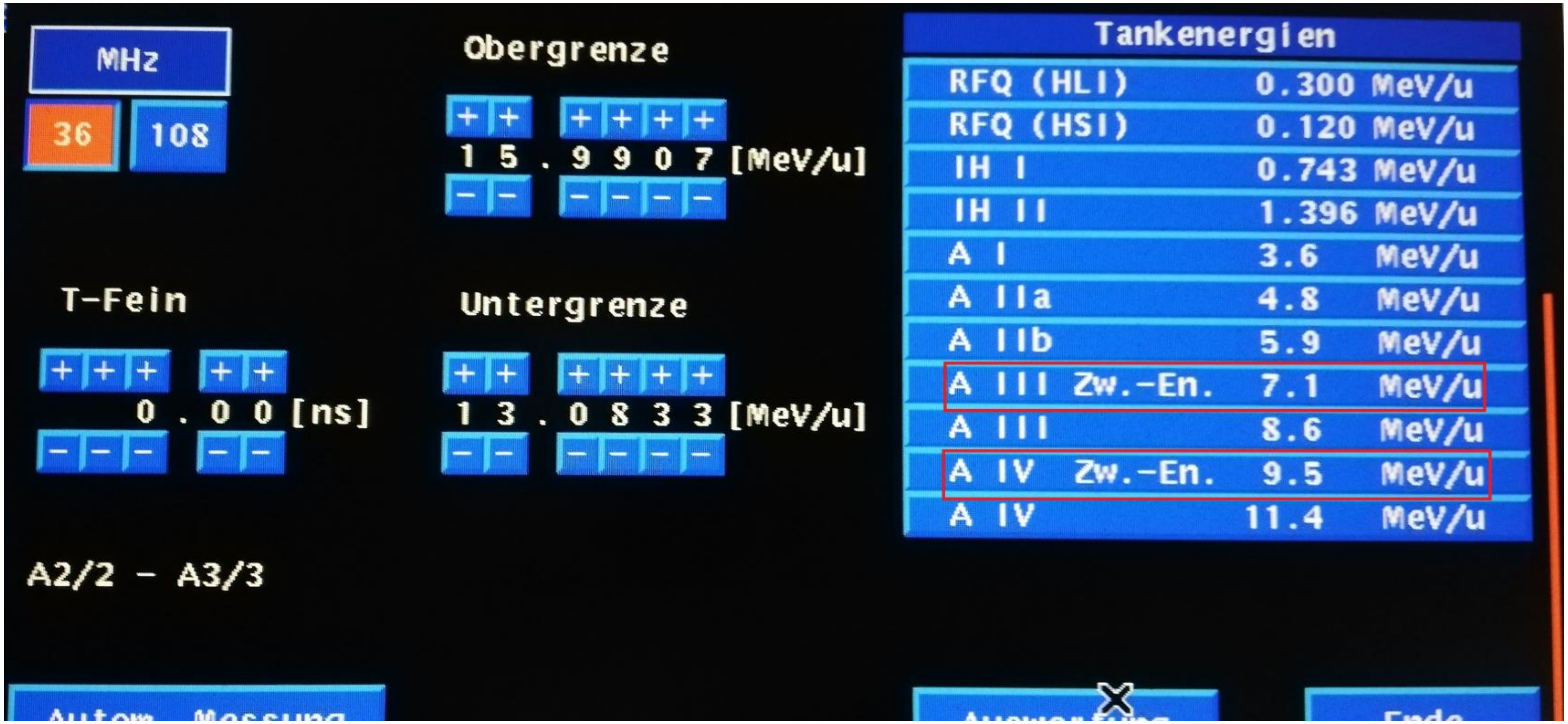
- the Alvarez post-stripper design allows to switch off the rf of cavities starting from the last one
- the focusing quadrupoles are kept powered but rf-power is off along those cavities
- delivered standard output energies are accordingly: 3.6, 4.8, 5.9, 8.6, and 11.4 MeV/u
- these energies are routinely delivered to experiments

Inter-mediate energy phenomenon

during commissioning of cavities #4 & #5 (year 1982) was observed:

- if cavity #5 is rf-powered just with 80% of the design rf-power, the output beam quality is still excellent. The energy is 9.50 MeV/u
- If cavity #4 is rf-powered just with 70% of the design rf-power, the output beam quality is still excellent. The energy is 7.10 MeV/u
- operation at 70% to 80% of the design rf-power is beyond the scheme of perturbation, i.e., cannot be modelled and understood by linearization around the design case. This “inter-mediate“ energy phenomenon has been not understood for about 35 years. However, such beams were (and are) delivered to many experimentalists to their full satisfaction
- the phenomenon was not understood as many codes make too simple assumptions and approximations, namely ignoring the energy dependence of TTF or assuming that phases are equal to the design phases. That means, generally codes work just in the linear regime close to the design case. But the studied cases are far outside the linear regime, i.e., the bucket

Inter-mediate energy phenomenon



recent advances in modeling longitudinal DTL beam dynamics allowed to explain the "inter-mediate" energy phenomenon. See more details in:

<u>Nuclear Inst. and Methods in Physics Research, A 887 (2018) 40–49</u>

Longitudinal beam dynamics of DTL

A Hamiltonian system can be constructed describing single-particle motion in longitudinal phase-space as

$$H = -\frac{\pi w_r^2}{\beta_s^3 \gamma_s^3 \lambda} - \frac{q E_0 T(\beta_r)}{mc^2}(\sin\psi_r - \psi_r \cos\psi_s),$$

since ψ and w are variables canonically dependent on s

$$\frac{d\psi_r}{dz} = -\frac{2\pi w_r}{\beta_s^3 \gamma_s^3 \lambda}, \quad \frac{dw_r}{dz} = \frac{q E_0 T(\beta_r)}{mc^2}(\cos\psi_r - \cos\psi_s).$$

To quantify this longitudinal movement the reference particle vector function is defined as

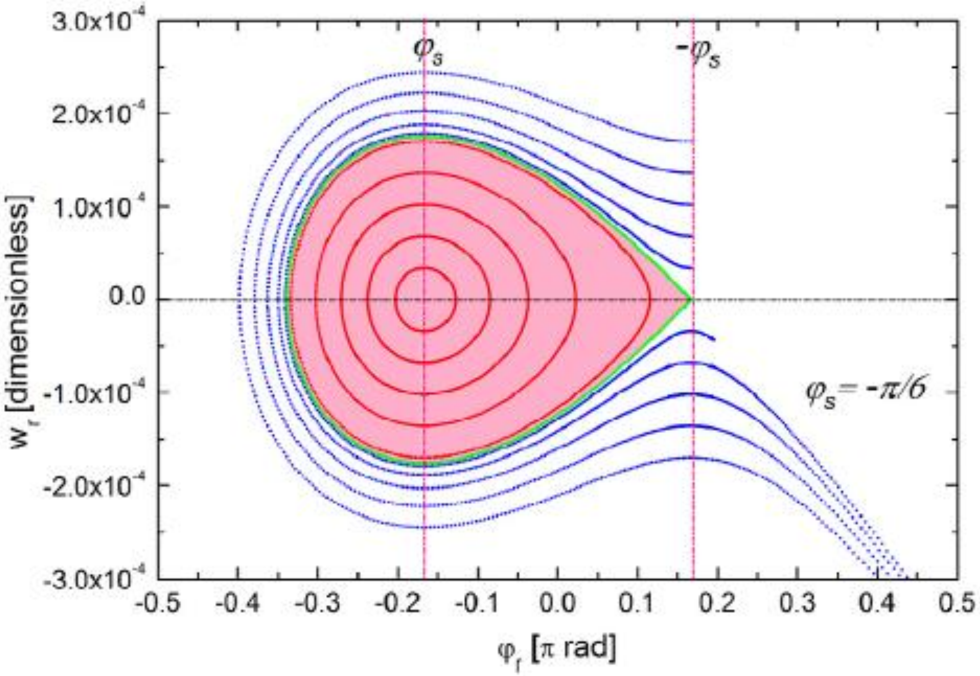
$$\Psi(\psi_r, w_r) := \begin{bmatrix} \psi_r \\ w_r \end{bmatrix}$$

and the derivative $D\Psi$ can be written as

$$D\Psi(z, \Psi) := \frac{d\Psi(\psi_r, w_r)}{dz} = \begin{bmatrix} -\frac{2\pi}{\beta_s^3 \gamma_s^3 \lambda} w_r \\ \frac{q E_0 T(\beta_r)}{mc^2}(\cos\psi_r - \cos\psi_s) \end{bmatrix}$$

In order to solve this differential equation, the Bulirsch–Stoer method in the MATHCAD can be applied

Solutions using Mathcad program



the approximation $T(\beta r) = T(\beta s)$ is made, implying that the difference $\beta r - \beta s$ is negligible

Investigation scenarios of Alvarez-DTL

	cavity#1	cavity#2	cavity#3	cavity#4	cavity#5
A	100%	100%	100%	100%	100%
B	100%	100%	100%	100%	70%
C	100%	100%	100%	80%	off

Mathematic routine of Mathcad

The reference particle vector function is defined as

$$Z(z, \beta c) := \begin{bmatrix} z \\ \beta c \end{bmatrix} = \begin{bmatrix} z \\ \frac{dz}{dt} \end{bmatrix}$$

The starting time is at $t = 0$ at position $z = 0$

$$Z(0) := \begin{bmatrix} 0 \\ \beta_0 c \end{bmatrix}$$ ⇨ initial condition

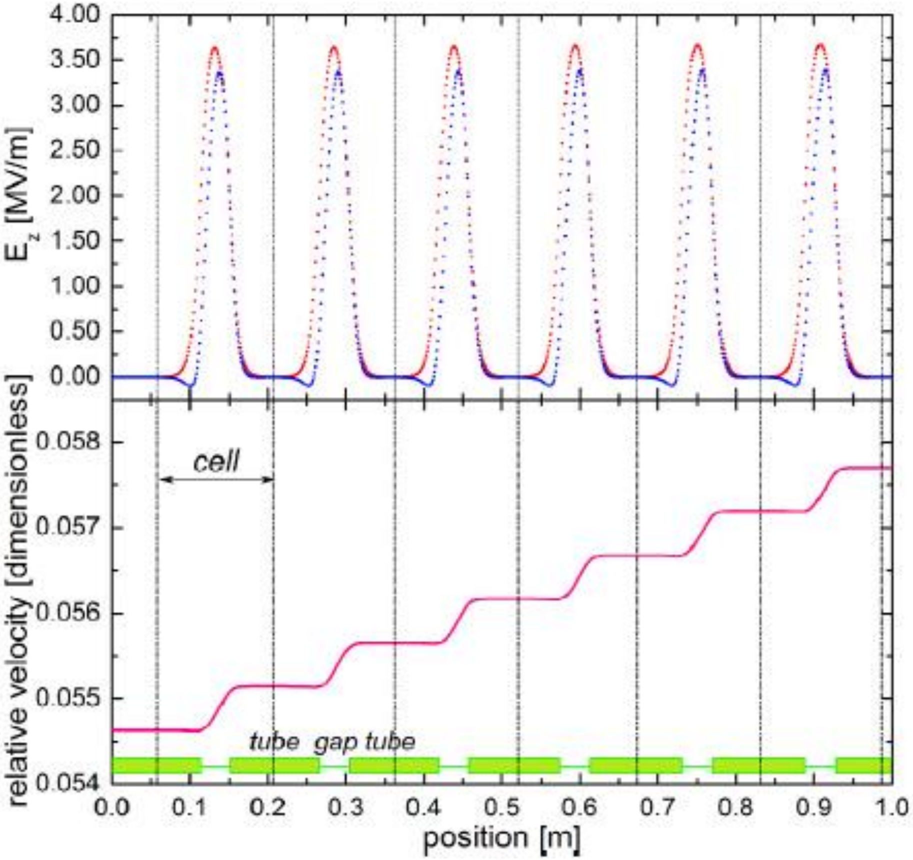
The derivative DZ w.r.t. time t is

$$DZ(t, Z) := \frac{dZ(z, \beta c)}{dt} = \begin{bmatrix} \beta c \\ \frac{\frac{q}{m_0}}{\sqrt{1-\beta^2}} E_z(z) \cos(\omega t + \psi_0) \end{bmatrix}.$$

A dedicated Mathcad program is written to solve the differential equation

The input energy is selected to be 1.3931 MeV/u. This dedicated routine is a time code using time as independent variable

First 1.0 meters of tank #1



Rf-phases at subsequent rf-gap centers are calculated as: −30.00, −30.08, −30.20, −30.34, −30.52, and −30.71 degrees, respectively.

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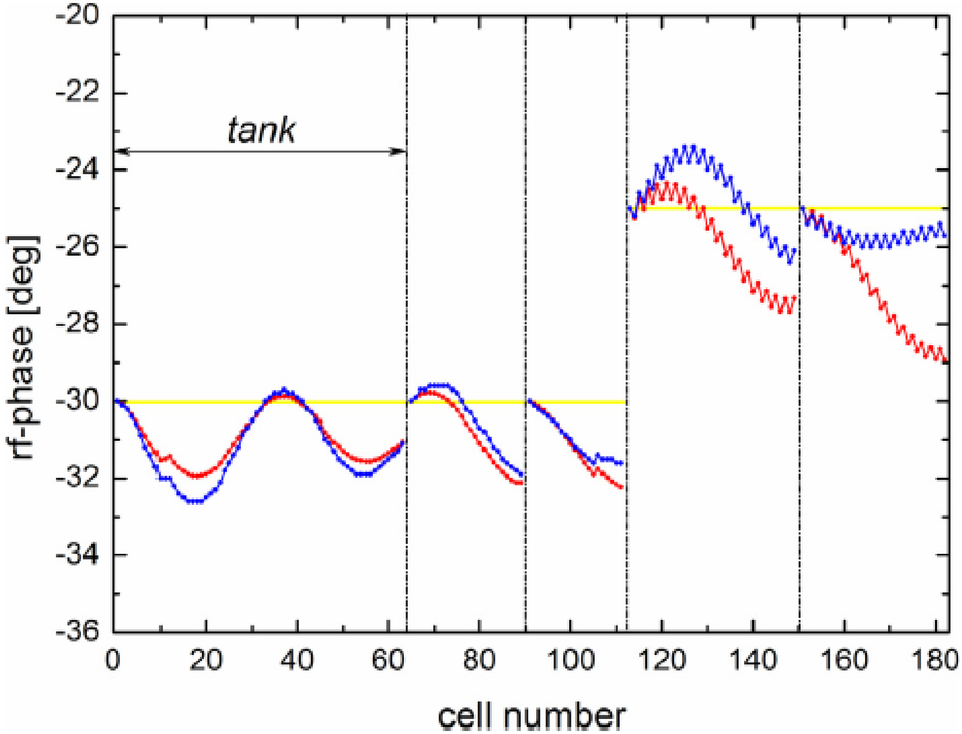
Mathematatic program benchmark

Nominal gap voltages were assumed, scenario-A



Initial cavity phases are determined such that the rf-phases of the reference particle at the center of each entrance gap is equal to the respective design value

For the first application of the numerical routine basing on MATHCAD along the complete Alvarez DTL, the initial cavity phases are determined

$$\psi_0^M = 14.96°, -45.95°, 165.31°, -87.17°, -121.47°$$

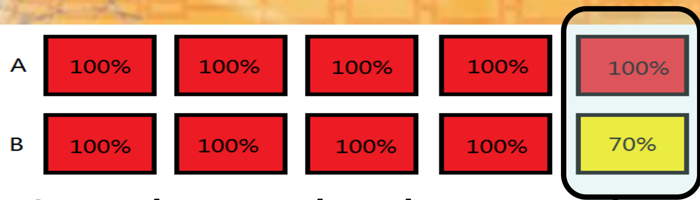
Additionally, BEAMPATH was used to track the reference particle through the complete DTL, the initial cavity phases are determined

$$\psi_0^B = 14.96°, -46.29°, 164.82°, -88.62°, -123.76°$$

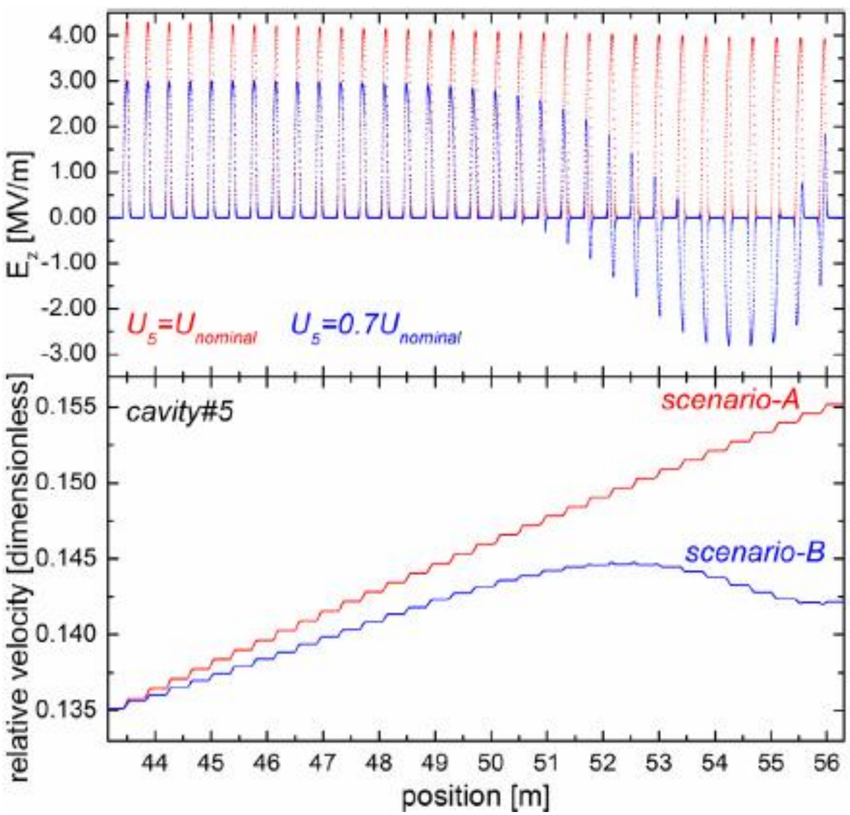
Cavity exit energies of the reference particle obtained from the routine and from BEAMPATH using nominal gap voltages, i.e., scenario-*A*.

Position	Routine [MeV/u]	BEAMPATH [MeV/u]
cavity#1 exit	3.6006	3.5996
cavity#2 exit	4.7696	4.7703
cavity#3 exit	5.8878	5.8861
cavity#4 exit	8.6251	8.6284
cavity#5 exit	11.4232	11.4228

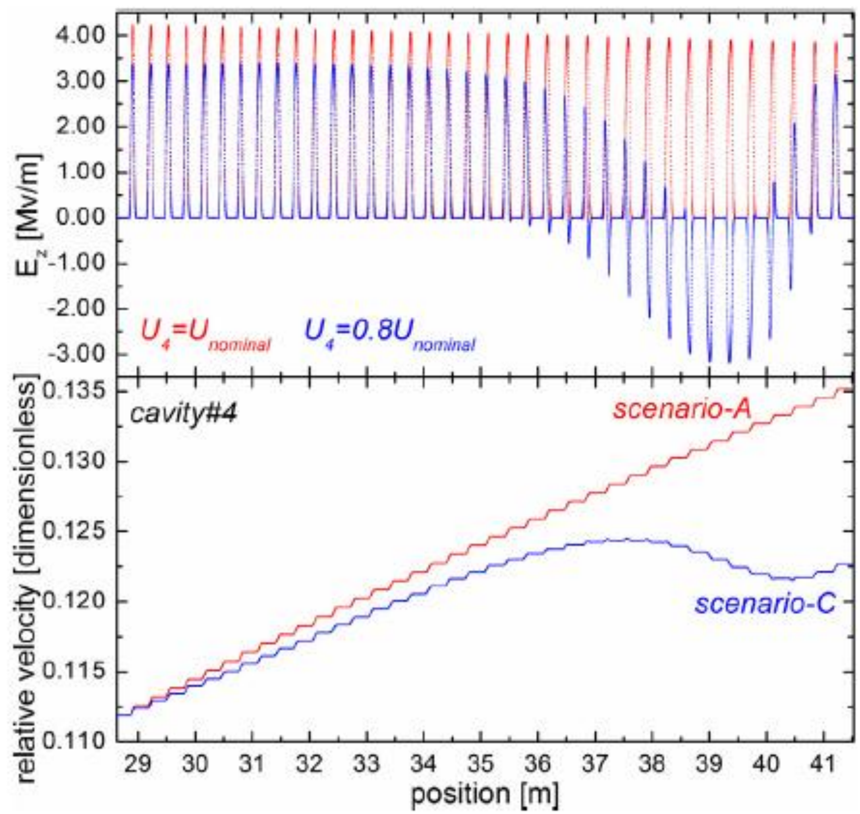
Single particle tracking using Mathcad



Scenario-A,B using given gap-voltages



Scenario-A,C using given gap-voltages



Using the advanced modeling of the routine, the solution for any off-design particle can always be obtained by direct numerical integration through known longitudinal fields

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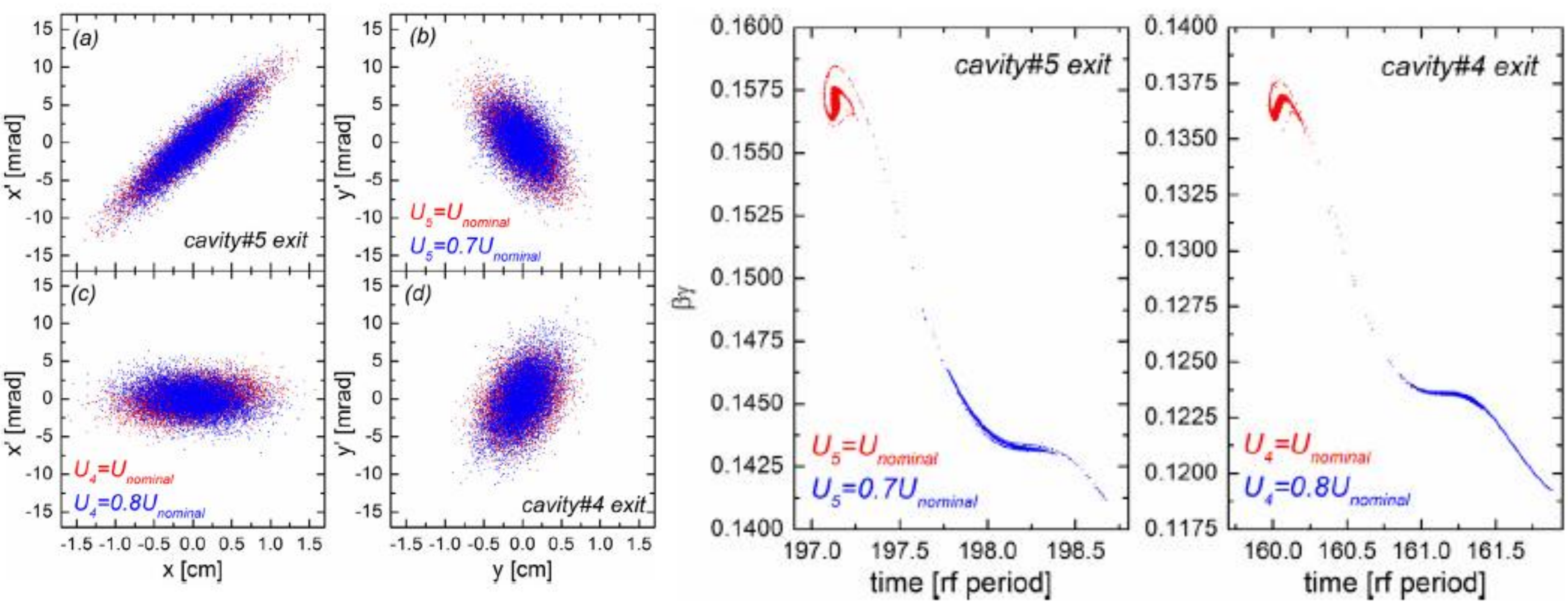
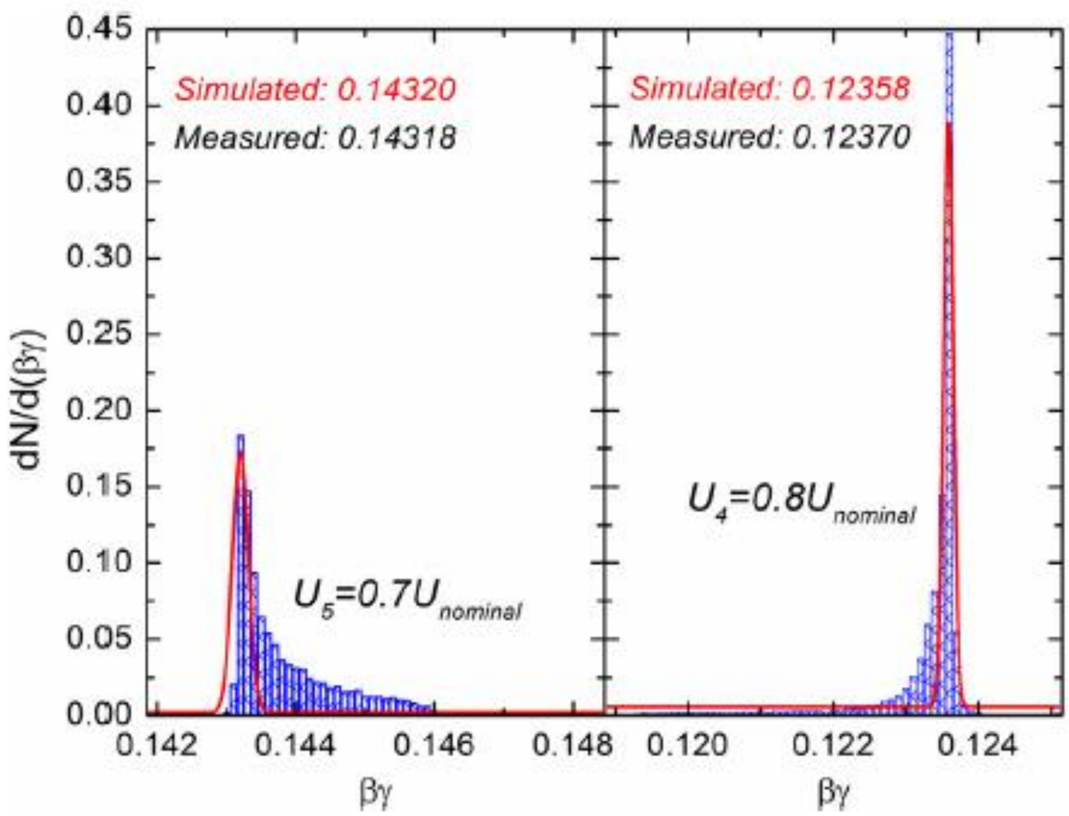
Multi-particle tracking using BEAMPATH

Transverse phase-space *Longitudinal phase-space*



The relative momentum spread is within the requirements of the experiments, as just about 0.4(3.1)% of the particles are out of the total relative momentum spread range of ±2% for scenario-B(C)

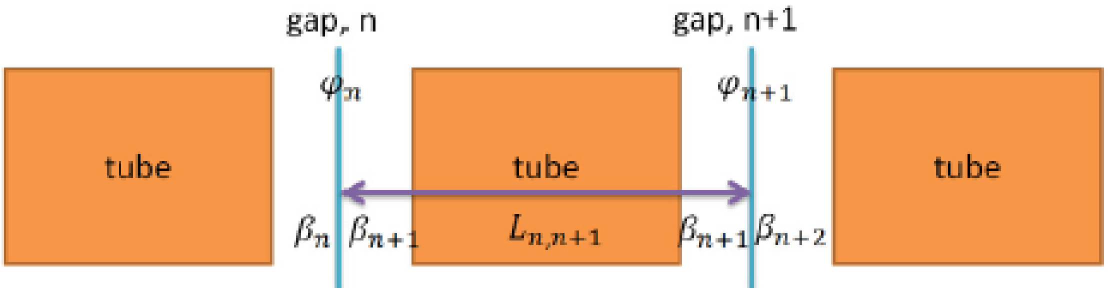
Final energy and energy spread analysis



Left: at exit of the fifth DTL cavity from scenario-B. Energy of maximum abundance are calculated (measured) as 9.5021 MeV/u (9.5 MeV/u).

Right: at exit of the fourth DTL cavity from scenario-C. Energy of maximum abundance is calculated (measured) as 7.0858 MeV/u (7.1 MeV/u)

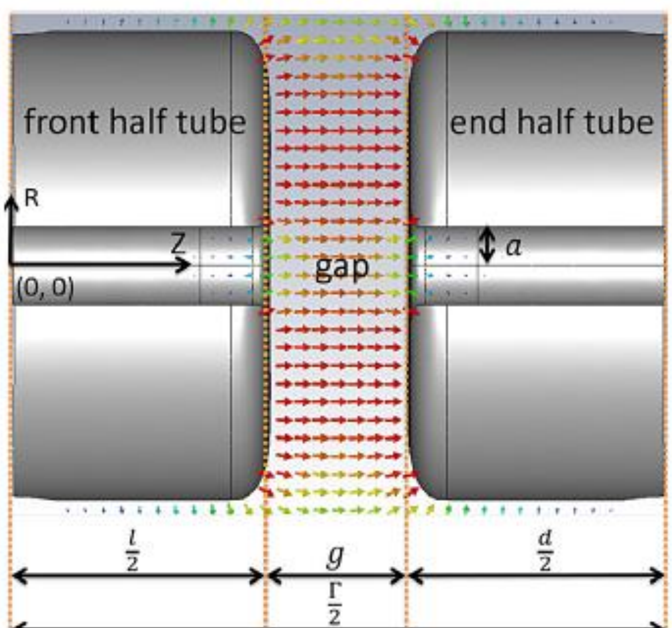
Simulation of particle tracking



Standard method using potential

$$W_{r,n+1} = W_{r,n} + qU_n T_n(\beta_n)\cos\psi_{r,n}$$

$$\psi_{r,n+1} = \psi_{r,n} + \frac{2\pi L_{n,n+1}}{\beta_{r,n+1}\lambda} - 2\pi\,,$$



Fourier–Bessel expansion using three-dimensional field-map

$$E_z(z,r,t) = -\cos(\omega t + \psi_0)\sum_{m=1}^{M} E_m I_0(\mu_m r)\sin\left(\frac{2\pi m z}{\Gamma}\right),$$

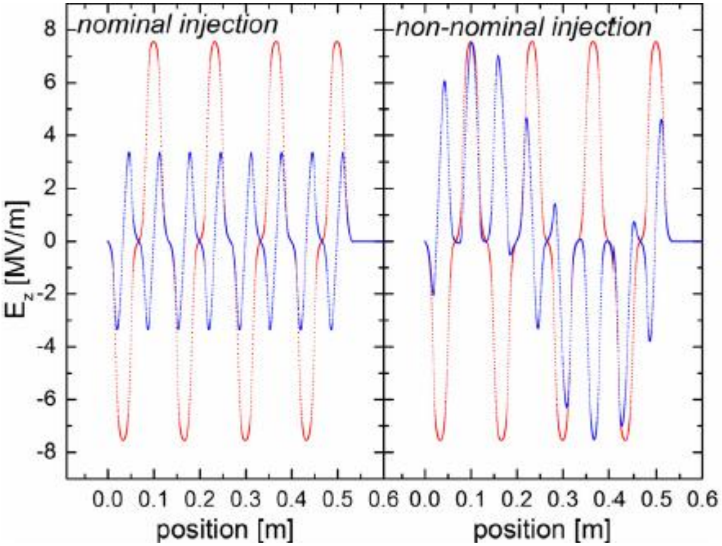
$$E_r(z,r,t) = \cos(\omega t + \psi_0)\sum_{m=1}^{M} \frac{2\pi m E_m}{\mu_m \Gamma} I_1(\mu_m r)\cos\left(\frac{2\pi m z}{\Gamma}\right),$$

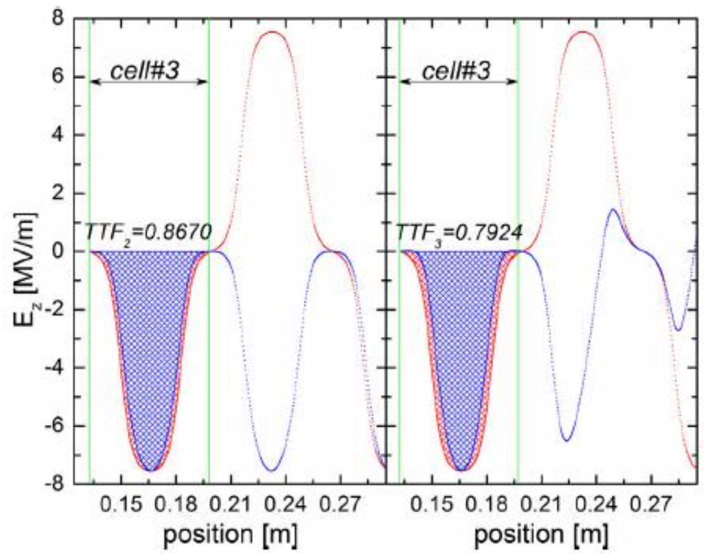
$$B_\theta(z,r,t) = \sin(\omega t + \psi_0)\sum_{m=1}^{M} \frac{2\pi E_m}{\mu_m \lambda c} I_1(\mu_m r)\sin\left(\frac{2\pi m z}{\Gamma}\right),$$

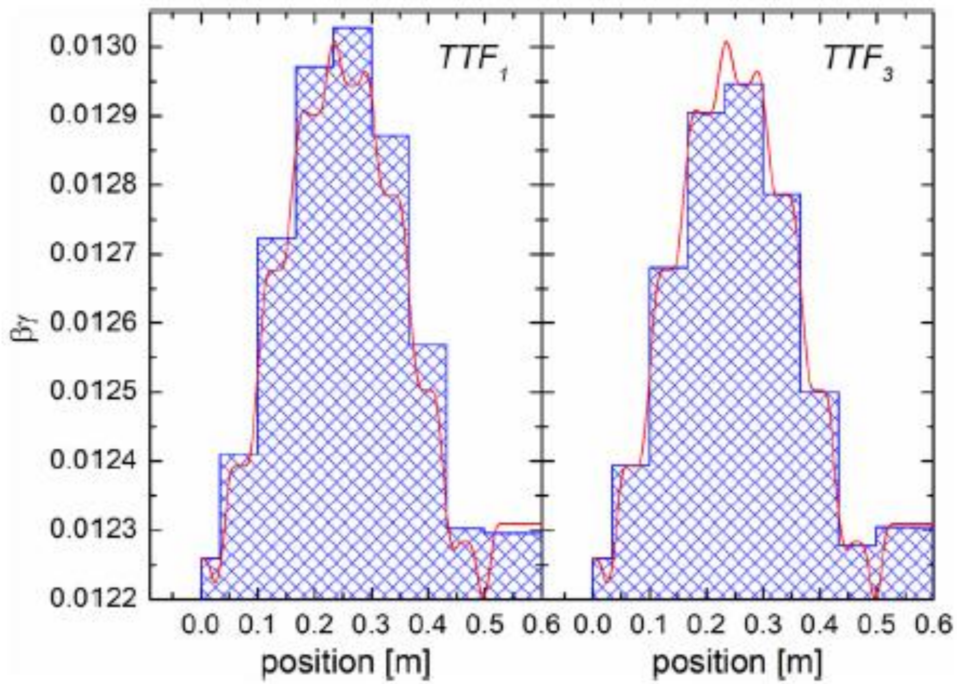
and

$$\mu_m = \frac{2\pi}{\lambda}\sqrt{\left(\frac{m\lambda}{\Gamma}\right)^2 - 1},\quad \Gamma = l + 2g + d,\quad E_0 = \frac{U}{g},$$

Transit time factor calculation





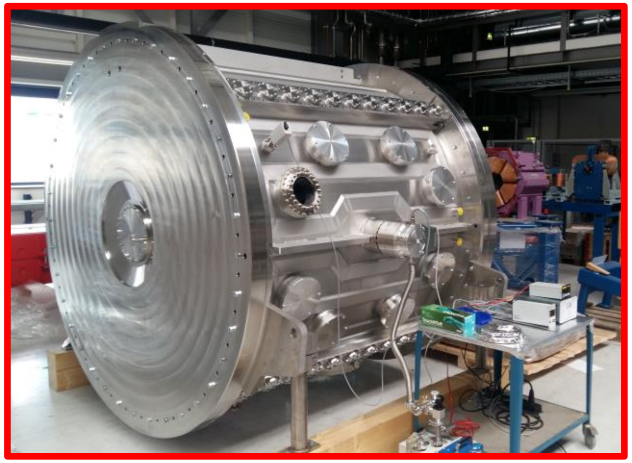


New post-stripper inter-mediate energies

from the design of the new post-stripper, eight inter-mediate energies are expected

Normalized DTL cavity voltages and resulting energy at the DTL exit for the new post-stripper DTL of the GSI UNILAC.

U_1/U_{d1}	U_2/U_{d2}	U_3/U_{d3}	U_4/U_{d4}	U_5/U_{d5}	E_{fin} [MeV/u]
1.0	1.0	1.0	1.0	1.0	11.4
1.0	1.0	1.0	1.0	0.42	9.5
1.0	1.0	1.0	1.0	0.20	9.3
1.0	1.0	1.0	1.0	0	9.1
1.0	1.0	1.0	0.72	0	7.5
1.0	1.0	1.0	0.42	0	6.9
1.0	1.0	1.0	0	0	6.7
1.0	1.0	0.80	0	0	5.5
1.0	1.0	0.70	0	0	5.0
1.0	1.0	0.52	0	0	4.8
1.0	1.0	0.0	0	0	4.5
1.0	0.45	0	0	0	3.5
1.0	0.0	0	0	0	3.3



Conclusion

- *The inter-mediate energy phenomenon of the last two Alvarez-type cavities at GSI's UNILAC was finally modelled 35 years after it has been observed for the first time*
- *This modeling bases on identifying and dropping of approximations being not applicable to these complex scenarios*
- *Applying this advanced model, the inter-mediate energies were exactly reproduced in single-particle and multi-particle simulations with MATHCAD and BEAMPATH*
- *More information given in:*
  - *Nuclear Inst. and Methods in Physics Research, A 887 (2018) 40–49*
  - *Nuclear Inst. and Methods in Physics Research, A 928 (2019) 70–78*

Thank you for your attention!