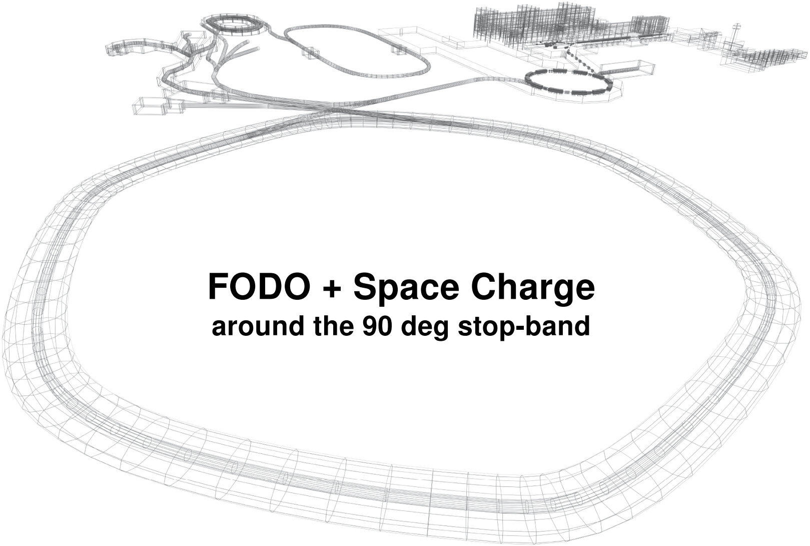
FODO + Space Charge

around the 90 deg stop-band



Simulation Set-up

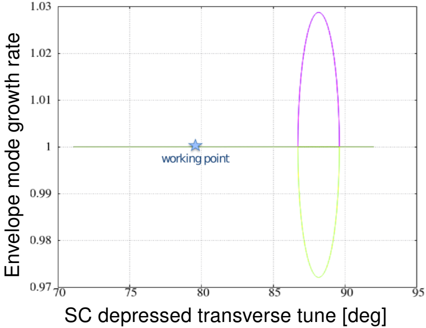
Consider a *proton* beam in a simple 1 m long FODO (actually DOFO) cell with 2 RF cavities (at 1/4 and 3/4 of the length).

parameter	value
intensity	$N = 8.846 \times 10^9$
norm. tr. RMS emittance	$\epsilon_{x,y} = 1\,\text{mm mrad}$
RMS bunch length	$\sigma_z/c = 0.63\,\text{ns}/4 = 2.7\,\text{cm}/c$
betatron tunes	$Q_x \equiv Q_y = 92/360$
synchrotron tune	$Q_s = Q_{x,y}/10 = 9.2/360$
kinetic energy	10 MeV
bunch speed	$\beta = 0.145$
natural chromaticity	$Q'_{x,y} = 0.33$

Space charge (SC) parameters are such that the transverse RMS equivalent tune yields a SC shifted value of $Q_x^{SC} = 79.6/360$.

Ji Qiang's Main IMPACT Results

Ji Qiang simulated the scenario with IMPACT using a 3D particle-in-cell (PIC) open-boundary Poisson solver (FFT + integrated Green's function):



⟶ SC shifted transverse envelope tune sits below 90 deg stop-band

⟹ no coherent (second-order / quadrupolar) resonance

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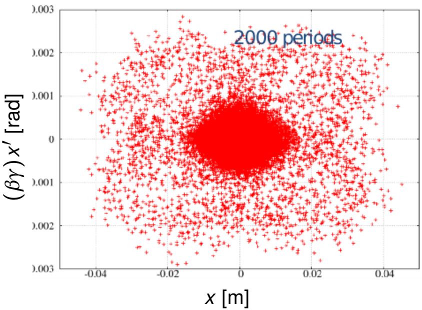
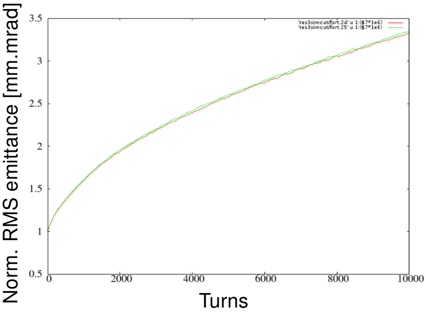




- ⟶ space charge field of Gaussian distribution: octupole component
- ⟶ halo particles are resonantly driven to large amplitude for $Q_x = 0.2\overline{5}$
- ⟹ RMS emittance growth of factor 2.5 over 5000 periods

Setting up a thin lattice in MAD-X:

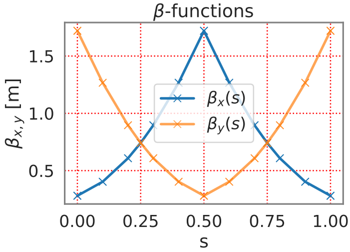
MAD-X set-up

```
kqd := -28.7736 * 0.1;
kqf := 28.7736 * 0.1;
v := 0.041693; ! in MV

qd: multipole, knl := {0, kqd/2.};
qf: multipole, knl := {0, kqf};
rf: rfcavity, volt := v, harmon = 1, lag = 0;

fodo: sequence, l = 1;
qd, at = 0;
rf, at = 1 / 4.;
qf, at = 1 / 2.;
rf, at = 1 * 3 / 4.;
qd, at = 1;
endsequence;
```

Numerical Model

Approach:

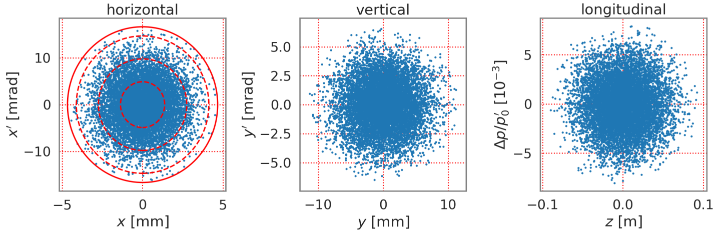
- load MAD-X thin lattice into SixTrackLib (to GPU)
- place 10 PyHEADTAIL SC nodes in regular distance (every 0.1 m)



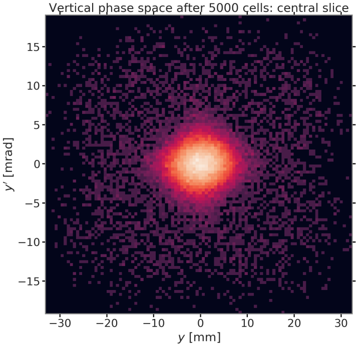
- each SC node runs the same PIC algorithm as in the IMPACT model (but on the GPU): open boundary 3D Poisson solver with FFT and integrated Green's function

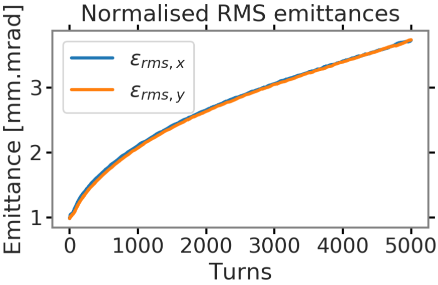
Numerical parameters of 3D PIC:

- 1×10^6 macro-particles, 6D Gaussian distribution
- 256×256 transverse cells spanning a fixed half grid width of 24 maximal RMS amplitudes along the lattice
  $\left(\text{beam size } \sigma_{x,y}(s) = \sqrt{\beta_{x,y}(s)\epsilon_{x,y}/(\beta\gamma)} \text{ oscillates within factor 2}\right)$
  $\longrightarrow$ all particles contained within the grid at all times during simulation
- 64 longitudinal slices spanning a total length of $2 \times 4\sigma_z$
- particle generation is limited by 3.4 RMS action radius (all 3 planes!)



SixTrackLib + PyHEADTAIL Simulations
Results





Results:

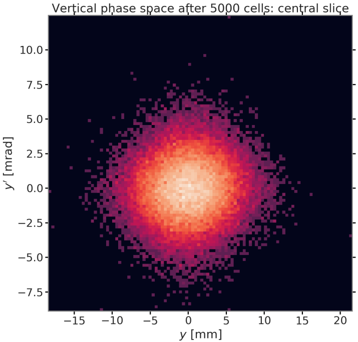
- increased halo population (x and y plane inverted for DOFO here)
- the RMS emittance $\epsilon_{x,y}$ grows by 3.75 over 5000 FODO periods

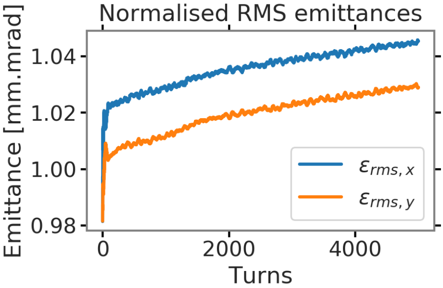
⟶ dynamics confirm IMPACT results (cf. $\epsilon_{x,y}$ growth of only 2.5 tho!)

Cross-checks to investigate results

Cross-check with 90 deg
Non-resonant case

Moving lower to $Q_{x,y} = 90/360 = 0.25$ zero-current tune, the resonant islands move towards infinite amplitude, particles remain stable:





- particles adjust to octupolar deformation inside separatrix (at large but finite amplitude due to finite chromaticity)

$\Longrightarrow$ numerical PIC parameters look fine (no numerical noise issues)

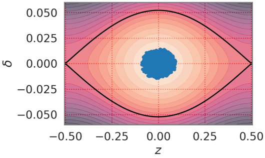
Revisit Models

Model Comparison

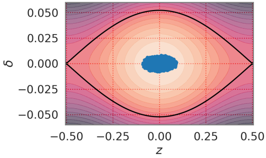
SixTrackLib + PyHEADTAIL	**IMPACT**
Gaussian distrib. matched to zero-current optics functions	Gaussian distrib. based on SC matched RMS envelope figures
cutting at 3.4 RMS action amplitudes in *phase* space	cutting at 3.4 RMS beam sizes in *real* space
non-linear RF	linear RF
thin quadrupole	thick quadrupole
exact drifts	
3D PIC (integrated Green's function)	
same intensity, transverse $\epsilon_{x,y}$, longitudinal σ_z, σ_δ	
1×10^6 macro-particles	600 000 macro-particles
static grid $64 \times 256 \times 256$	dynamic grid $64 \times 64 \times 64$

Optimally we would like to keep longitudinal space charge effects marginal, yet they are always present with 3D PIC.

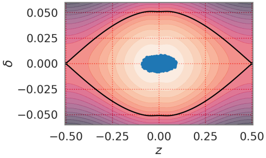
Matching of momentum spread σ_δ to long. SC (fixing σ_z):



(a) only RF matched: $\sigma_\delta = 4.4 \times 10^{-3}$



(b) RF and SC matched: $\sigma_\delta = 2.5 \times 10^{-3}$



(c) potential well distortion

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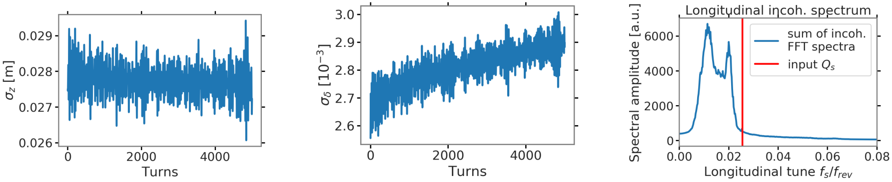
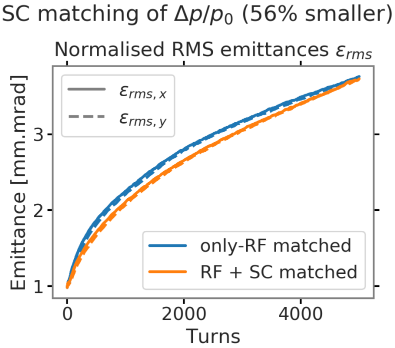


(a) SC matched σ_z (b) SC matched σ_δ (c) incoherent spectrum sum

Compare Longitudinal SC Matching

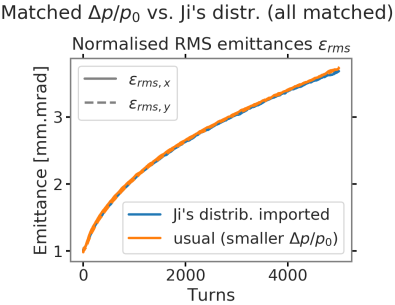
Effect on RMS emittance growth:



Observations:

- weaker initial RMS emittance growth, after 5000 periods identical

Import initial IMPACT distribution by Ji Qiang into SixTrackLib + PyHEADTAIL, compare to previous smaller SC-matched σ_δ simulation:



Observations:

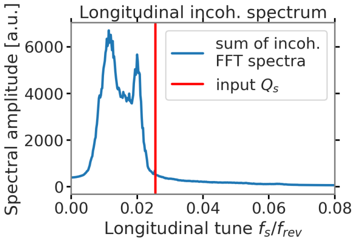
- no discrepancy between distributions generated by either code!

$\Longrightarrow$ different final $\epsilon_{x,y}$ must originate from different modelling (lattice/SC)

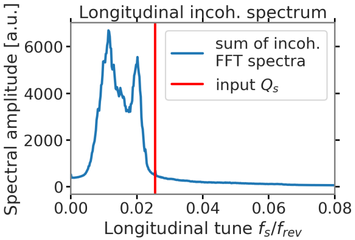
Non-linear vs. Linear Synchrotron Motion

Removing RF cavities from SixTrackLib model, undoing the longitudinal drift and inserting a linear synchrotron map from PyHEADTAIL:

non-linear RF case



linear RF case



- incoherent synchrotron tune spread remains the same

  ⟶ longitudinal space charge dominates anyway

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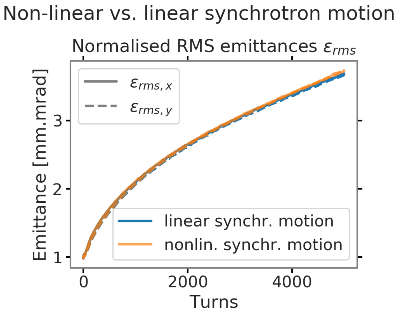
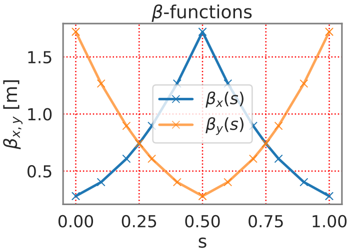


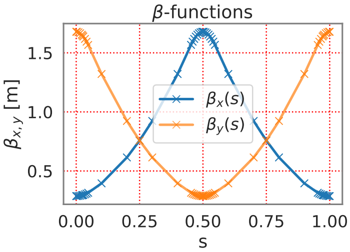
- incoherent synchrotron tune spread remains the same
  - ⟶ longitudinal space charge dominates anyway
- no impact on RMS $\epsilon_{x,y}$ growth

Cross-check Quadrupole Magnet Model

Replacing the single thin lens quadrupole by a thick quadrupole and using the TEAPOT algorithm in MAD-X to slice the magnets into 16 thin lenses:



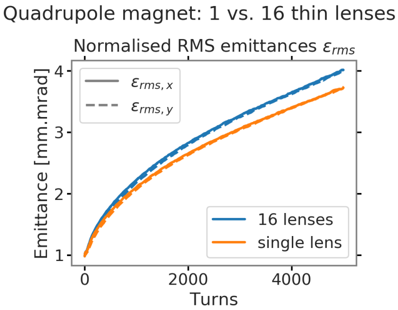
$\rightsquigarrow$



- matching thick quadrupoles of length 0.1 m gives $\kappa_{x,y} = 3.09217\,\mathrm{m}^{-1}$
  $\longrightarrow$ identical to IMPACT (while 1 single thin lens gave $\kappa_{x,y} = 2.87736\,\mathrm{m}^{-1}$)
- 16 slices are essentially converged, as
  $Q_x = 0.255555556 \rightsquigarrow 0.2555524323$ after MAD-X `makethin`

Compare Quadrupole Magnet Models

Effect on RMS emittance growth:

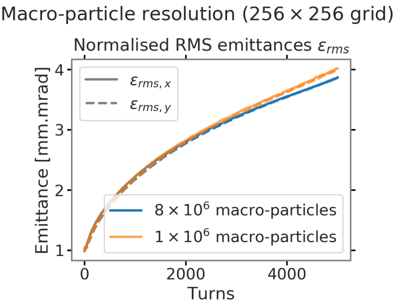


Observation:

- more resolved model even yields higher emittance growth from start
  - ⟶ not the explanation for smaller emittance growth in IMPACT

Cross-check Macro-particle Number

Increasing macro-particle number from 1×10^6 to 8×10^6 macro-particles:

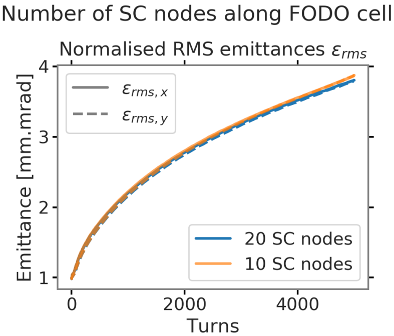


Observation:

- no impact, only slightly suppresses numerical noise in late part of simulation (where resonance dynamics already happened)

Cross-check Amount of SC Nodes

Increasing from 10 SC nodes to 20 SC nodes along the 1 m FODO cell:

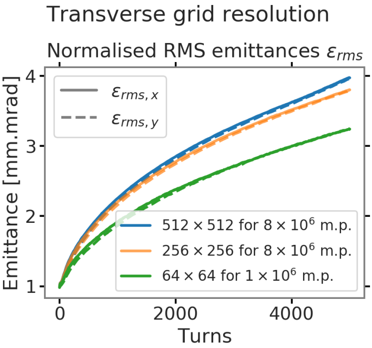


Observation:

- no impact, time scale of space charge integration is small enough

Cross-check PIC Grid Resolution

Varying the number of transverse grid cells in 3D PIC:



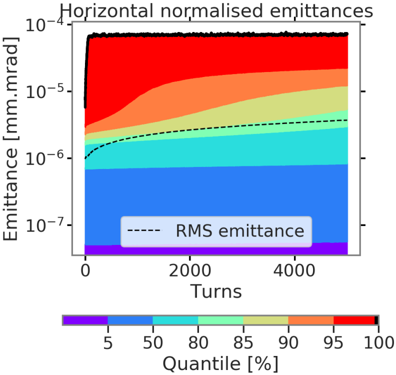
Observations:

- 256 × 256 cells almost converged (512 × 512 changes very little)
- 64 × 64 case significantly suppresses initial resonance dynamics

  ⟹ could more grid cells in IMPACT possibly give larger $\epsilon_{x,y}$ growth, too?

Overview

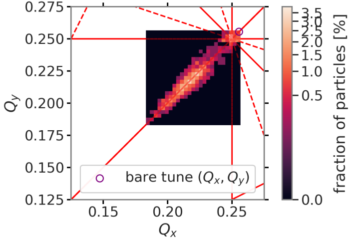
Below coherent (second-order / quadrupolar) 90 deg envelope stop-band exists an incoherent-like space charge driven octupolar resonance, into which halo particles (at action amplitudes of 80% and higher) are drawn:

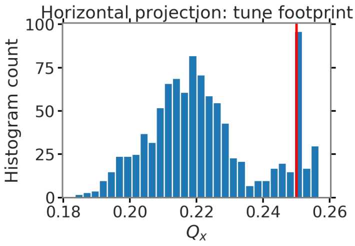


⟶ outermost particle rapidly (< 100 turns) saturates at $12.2\times$ action

Final Incoherent Tune Footprint

While most core particles remain in place and their space charge depressed tunes do not change, the halo particles are drawn into the 90 deg resonance condition:



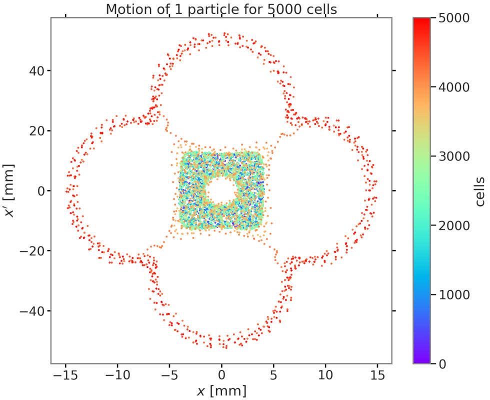


(Tune footprint of 1000 particles based on `PyNAFF` harmonic fitting during final 128 turns, i.e. ≈ 3 synchrotron periods.)

Overview Resonance Dynamics

Particle Phase Space

The Poincaré section of a high amplitude particle shows the octupolar resonance driven by the space charge field of the beam core:



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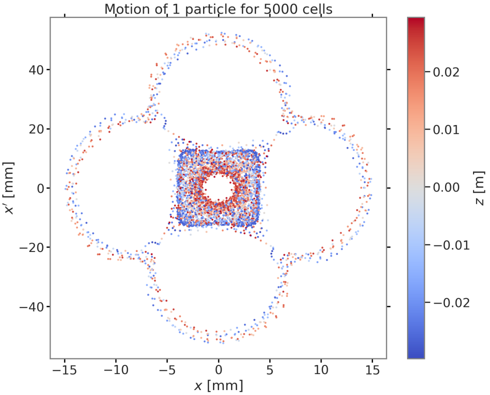

→ here, the energy increase happens during 1 synchrotron period and predominantly around $z = 0$

Modelling

Both IMPACT and SixTrackLib+PyHEADTAIL codes can simulate this case with self-consistent space charge:

- resulting beam dynamics are equivalent
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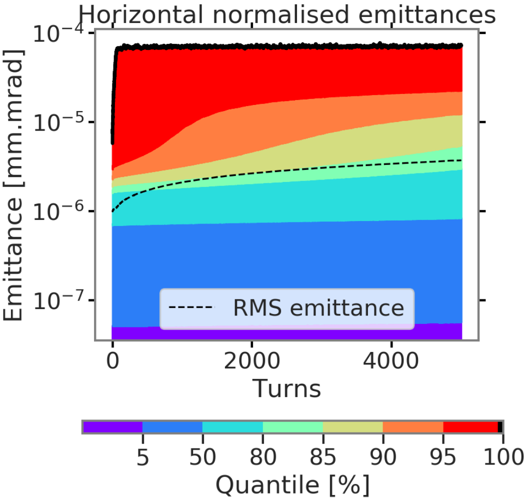
Physics

3D case with synchrotron motion is **much more severe** than 2D coasting beam case (Ji's presentation): $> 250\%$ vs. 10% RMS emittance growth

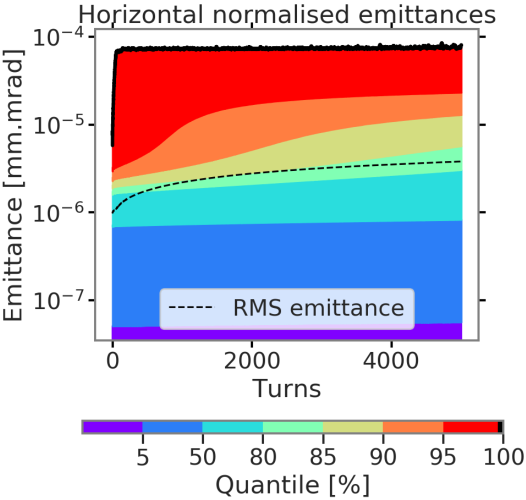
Thanks!

Appendix

Standard Case 011



Hi Resolution Case 015



Xtra Hi Resolution Case 016