Application of relativistic density functional theory to charge-exchange neutrino-nucleus reactions

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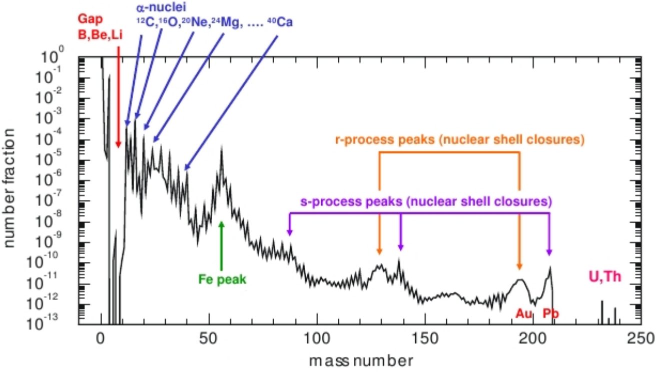
Nucleosynthesis

- H and He created during Big Bang
- Elements up to Fe created during stellar burning
- Foundations of heavy element nucleosynthesis in 1957. (B2FH)



E. M. Burbidge, G. R. Burbidge, W. A. Fowler i F. Hoyle, Rev. Mod. Phys. 29, 547 (1957)

Astrophysical requirements

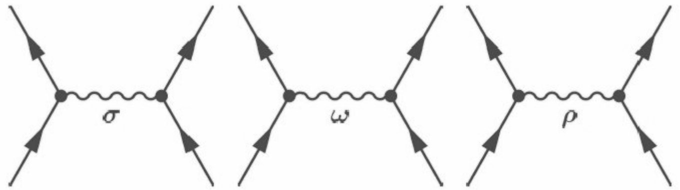
Heavy element nucleosynthesis

- description of very neutron rich nuclei
- delicate balance of weak-interaction processes

Two main approaches

1. Shell model
   - very precise results
   - applicable to a restricted set of nuclei
2. Mean-field models
   - selfconsistent approach
   - applicable to arbitrarily heavy nuclei
   - parameters independent on studied region

Relativistic mean-field theory



- σ scalar, isoscalar effective meson, attractive interaction
- ω vector, isoscalar meson, repulsive interaction
- ρ vector, isovector, isospin dependent interaction

Model is defined by a Lagrangian density

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{int}$$

Variational principle leads to Euler-Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial \varphi} = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \varphi \right)} \right)$$

Dirac equation for the nucleons $\varphi = \bar{\psi}$

$$\left[\gamma^\mu \left(i\partial_\mu + V_\mu \right) + m + S \right] \psi = 0$$

Klein-Gordon equation for mesons $\varphi = \phi_m$

$$-\Delta \phi_m + m_{\phi_m}^2 = \pm \langle \bar{\psi} \Gamma_m \psi \rangle$$

T. Nikšić, D. Vretenar, P. Finelli and P. Ring, Phys. Rev. C 66, 024306 (2002)

Interaction part of the Lagrangian density

$$\mathcal{L}_{int} = -g_\sigma \bar{\psi}\sigma\psi - g_\omega \bar{\psi}\gamma_\mu\omega^\mu\psi - g_\rho \bar{\psi}\gamma_\mu\vec{\rho}^\mu\vec{\tau}\psi - e\bar{\psi}\gamma_\mu A^\mu \frac{1-\tau_3}{2}\psi.$$

Meson-nucleon coupling constants are functions of vector density

$$\rho_v = \sqrt{j_\mu j^\mu}, \qquad j_\mu = \bar{\psi}\gamma_\mu\psi\,,$$

leading to rearrangement terms in the potential

$$\Sigma_\mu^R = \frac{j_\mu}{\rho_v}\left(\frac{\partial g_\omega}{\partial\rho_v}\bar{\psi}\gamma^\nu\psi\omega_\nu + \frac{\partial g_\rho}{\partial\rho_v}\bar{\psi}\gamma^\nu\vec{\tau}\psi\cdot\vec{\rho}_\nu + \frac{\partial g_\sigma}{\partial\rho_v}\bar{\psi}\psi\sigma\right)\,.$$

Interaction parameters are adjusted to infinite nuclear matter and bulk properties of finite nuclei.

Random phase approximation

If a small amplitude oscillating external field acts on the nucleus

$$\hat{F}(t) = \hat{F}e^{-i\omega t} + \text{h.c.}$$

equation of motion for the density operator becomes

$$i\partial_t \hat{\rho} = \left[\hat{h}(\hat{\rho}) + \hat{f}(t), \hat{\rho}\right] \qquad \hat{\rho}(t) = \hat{\rho}^0 + \delta\hat{\rho}^0$$

leading to the RPA equations

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X^\lambda \\ Y^\lambda \end{pmatrix} = E_\lambda \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X^\lambda \\ Y^\lambda \end{pmatrix}$$

Matrix elements of transition operator $\hat{O}_J$

$$\left\langle J_f \left\| \hat{O}_J \right\| J_i \right\rangle = \sum_{pn} \left\langle p \left\| \hat{O}_J \right\| n \right\rangle \left(X^J_{pn} u_p v_n - Y^J_{pn} v_p u_n \right)$$

N. Paar, D. Vretenar, E. Khan and G. Colò, Rep. Prog. Phys. 70, 691 (2007)

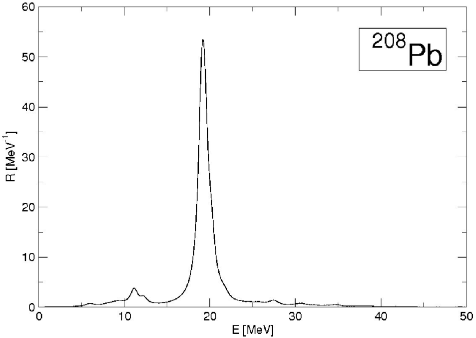
Pion field contributes at the RPA level

$$V_{\delta\pi} = g' \left(\frac{f_\pi}{m_\pi} \right)^2 \vec{\tau}_1 \vec{\tau}_2 \mathbf{\Sigma}_1 \cdot \mathbf{\Sigma}_2 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

$$m_\pi = 138\,\mathrm{MeV}$$

$$\frac{f_\pi^2}{4\pi} = 0.08$$

$$g'_{DD-ME2} = 0.52$$



Formalism

Weak-interaction Hamiltonian

$$\hat{H}_W = -\frac{G}{\sqrt{2}} \int d\boldsymbol{x} \mathcal{J}_\lambda(\boldsymbol{x}) j^\lambda(\boldsymbol{x}),$$

with transition matrix elements being

$$\left\langle f \left| \hat{H}_W \right| i \right\rangle = -\frac{G}{\sqrt{2}} l_\lambda \int d\boldsymbol{x} e^{-i\boldsymbol{qr}} \left\langle f \left| \mathcal{J}_\lambda(\boldsymbol{x}) \right| i \right\rangle .$$

Multipole expansion of the leptonic current matrix element

$$e^{-i\boldsymbol{qr}} = \sum_{J=0}^{\infty} \left[4\pi(2J+1)\right]^{1/2} i^J j_J(\kappa r) Y_{J0}(\Omega_r)$$

Neutrino capture

$$\nu_e + (Z, N) \rightarrow (Z+1, N-1)^* + e^-,$$

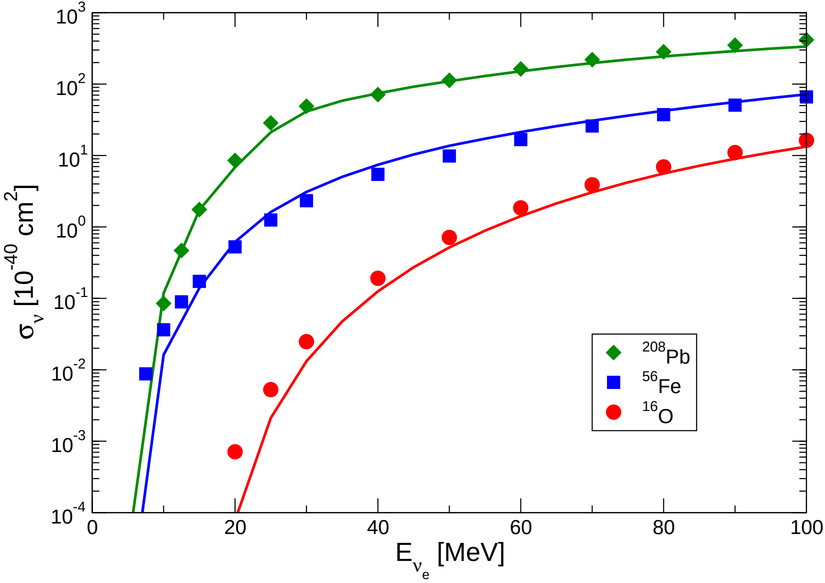
Cross section reads

$$\left(\frac{d\sigma_\nu}{d\Omega}\right) = \frac{V^2 p_l E_l}{(2\pi)^2} \sum_{\text{lepton spins}} \frac{1}{2J_i+1} \sum_{M_i, M_f} \left|\left\langle f \left| \hat{H}_W \right| i \right\rangle\right|^2$$

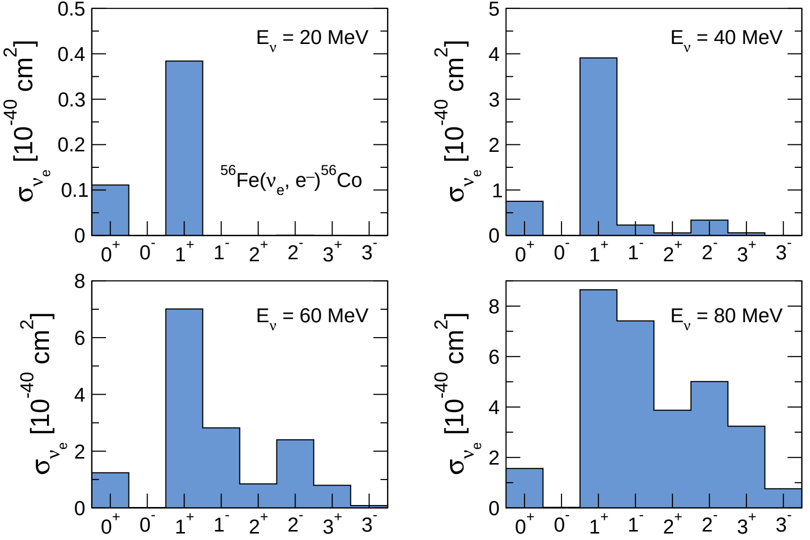
Total cross-section averaged over μ^- decay at rest (Michel) neutrino spectrum

$$f_\nu(E_\nu) = \frac{96 E_\nu^2}{m_\mu^4} (m_\mu - 2E_\nu)$$

$$\begin{aligned} \langle \sigma_{^{56}Fe} \rangle &= 341 \cdot 10^{-42}\,\mathrm{cm}^2 \\ \langle \sigma_{exp.} \rangle &= 256 \pm 108 \pm 43 \cdot 10^{-42}\,\mathrm{cm}^2 \end{aligned}$$

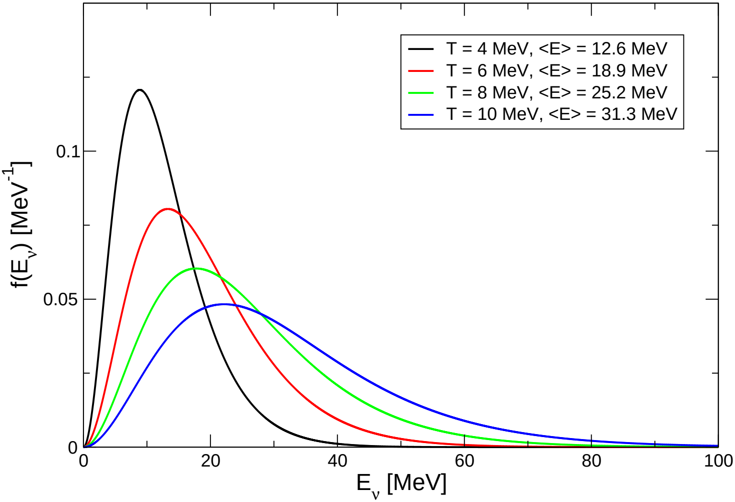


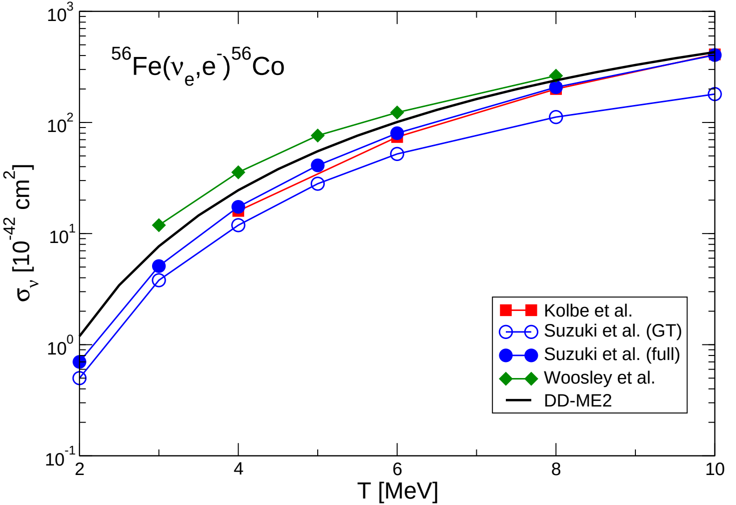
R. Lazauskas and C. Volpe, arxiv:nucl-th/0704.2724



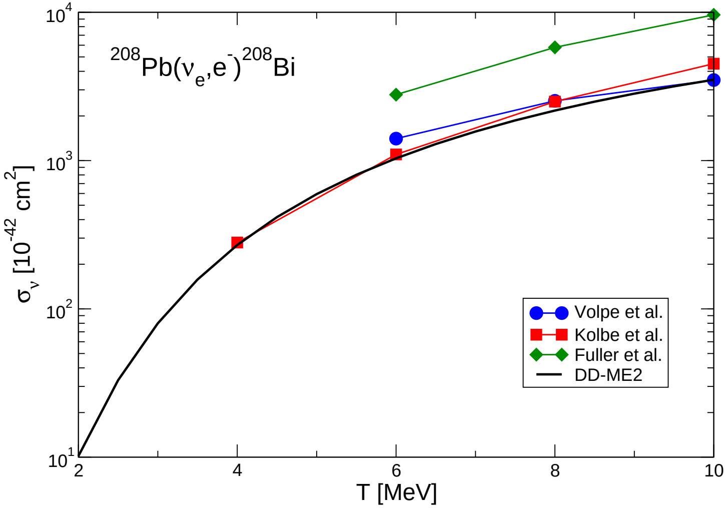
E_ν = 20 MeV
$^{56}Fe(\nu_e, e^-)^{56}Co$
σ_{ν_e} [10^{-40} cm^2]
0.5
0.4
0.3
0.2
0.1
0
0^+ 0^- 1^+ 1^- 2^+ 2^- 3^+ 3^-
E_ν = 40 MeV
5
4
3
2
1
0
E_ν = 60 MeV
8
6
4
2
E_ν = 80 MeV

$$f_{SN}(E_\nu) = \frac{1}{T^3} \frac{E_\nu^2}{e^{E_\nu/T-\alpha} + 1}$$





E. Kolbe and K. Langanke, Phys. Rev. C 63, 025802 (2001)
T. Suzuki et al., Phys. Rev. C 79, 061603(R) (2009)
S. E. Woosley, D. H. Hartmann, R. D. Hoffman and W. C. Haxton, Astrophys. J. 356, 272 (1990)



C. Volpe, N. Auerbach, G. Colò and N. Van Giai, Phys. Rev. C 65, 044603 (2002)
E. Kolbe and K. Langanke, Phys. Rev. C 63, 025802 (2001)
G. M. Fuller, W. C. Haxton and G. C. McLaughlin, Phys. Rev. D 59, 085005 (1999)

Charged lepton (μ^-) capture

Bound μ^- capture process

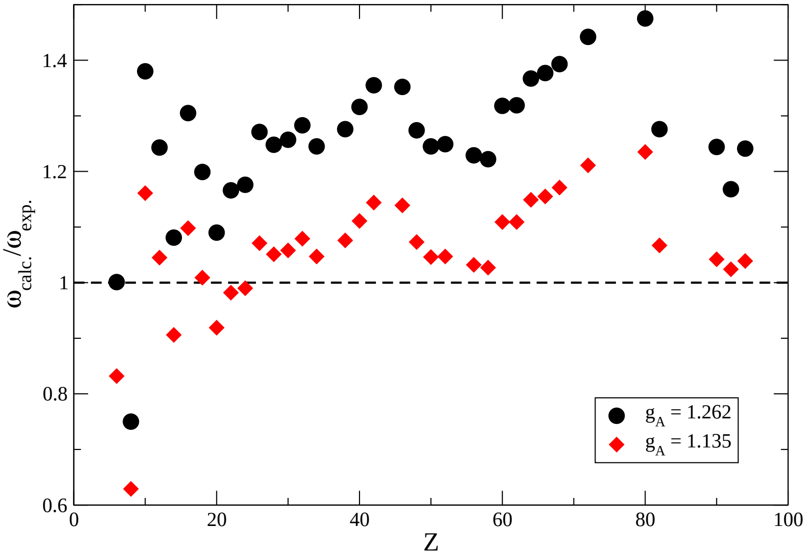
$$\mu^- + (Z, N) \rightarrow (Z-1, N+1)^* + \nu_\mu$$

The capture rate follows from the Fermi Golden Rule

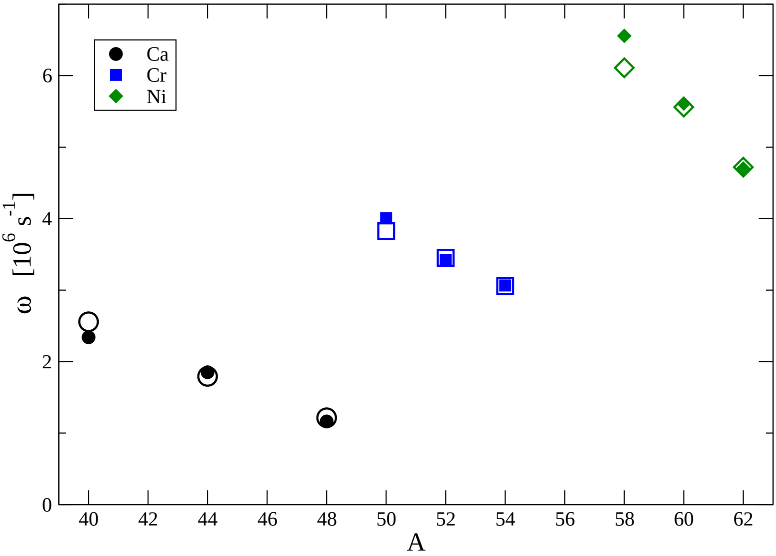
$$\omega_{fi} = \frac{V^2\nu^2}{2\pi} \sum_{\text{lepton spins}} \frac{1}{2J_i+1} \sum_{M_i, M_f} \left| \left\langle f \left| \hat{H}_W \right| i \right\rangle \right|^2$$

and reads

$$\omega_{fi} = \frac{2G^2\nu^2}{1+\nu/M_T} \frac{1}{2J_i+1} \left\{ \sum_{J=0}^{\infty} \left| \left\langle J_f \left\| \phi_{1s} \left(\hat{\mathcal{M}}_J - \hat{\mathcal{L}}_J \right) \right\| J_i \right\rangle \right|^2 + \sum_{J=1}^{\infty} \left| \left\langle J_f \left\| \phi_{1s} \left(\hat{\mathcal{T}}_J^{el} - \hat{\mathcal{T}}_J^{mag} \right) \right\| J_i \right\rangle \right|^2 \right\}$$



1.4
1.2
1
0.8
0.6
0
20
40
60
80
100
Z
$\omega_{calc.}/\omega_{exp.}$
g_A = 1.262
g_A = 1.135


Ca
Cr
Ni
ω [10^6 s^-1]
A
0
2
4
6
40
42
44
46
48
50
52
54
56
58
60
62

Conclusion

- astrophysical models require precise knowledge of weak-interaction processes on many exotic nuclei
- self-consistent models can be reliably used on experimentally unreachable nuclei
- relativistic mean-field model has been successfully applied to calculations of astrophysically relevant processes

- model will be expanded to include effects of temperature and applied to other processes