Nuclear Monopole Interaction and Shell Structure Evolution

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A. Umeya and K. Muto, Phys. Rev. C **69** (2004) 024306; **74** (2006) 034330.
A. Umeya, S. Nagai, G. Kaneko and K. Muto, Phys. Rev. C **77** (2008) 034318.
A. Umeya, G. Kaneko, T. Haneda and K. Muto, Phys. Rev. C **77** (2008) 044301.

Motivation

1. We have understood, since 1970s, a close relation between single-particle energies and monopole:

$$V_{jj'} = \frac{\sum_J (2J+1)\langle jj'|V|jj'\rangle_J}{\sum_J (2J+1)}$$

2. We have found, by experimental studies, new phenomena in neutron-rich nuclei, such as the disappearance of magic numbers (*N*=8, *N*=20).

Discussion of changes of single-particle energies from a nucleus to another → shell structure evolution

Q1: How should single-particle energies be defined, especially in open-shell nuclei?

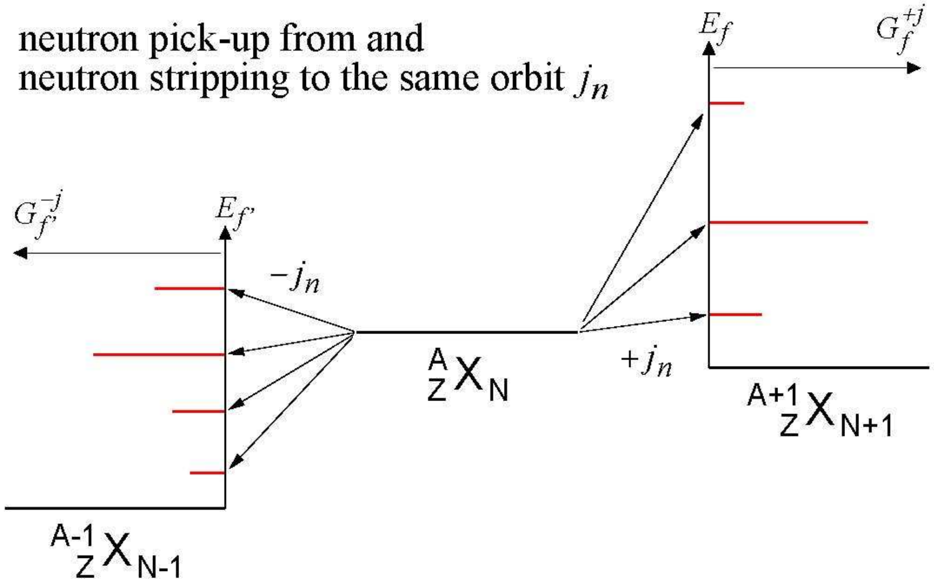
Q2: Roles of C, T, and LS interactions?

Definition of single-particle energies (1)

Prescription by Baranger

Nucl. Phys. A149 (1970) 225

$$\varepsilon_j = \frac{\sum_f G_f^{+j} \mathcal{E}_f^{+j} + \sum_{f'} G_{f'}^{-j} \mathcal{E}_{f'}^{-j}}{\sum_f G_f^{+j} + \sum_{f'} G_{f'}^{-j}}$$



spectroscopic strength
$$\begin{cases} G_f^{+j} & : \text{stripping} \\ G_{f'}^{-j} & : \text{pickup} \end{cases}$$

excitation energy
$$\begin{cases} \mathcal{E}_f^{+j} = E_f - E_i & : \text{stripping} \\ \mathcal{E}_{f'}^{-j} = E_i - E_{f'} & : \text{pickup} \end{cases}$$

Definition of single-particle energies (2)

$$\varepsilon_j = \frac{\sum_f G_f^{+j}\mathcal{E}_f^{+j} + \sum_{f'} G_{f'}^{-j}\mathcal{E}_{f'}^{-j}}{\sum_f G_f^{+j} + \sum_{f'} G_{f'}^{-j}} = \langle\Psi|\frac{1}{2j+1}\sum_m\{[a_{jm}, H], a_{jm}^\dagger\}|\Psi\rangle$$

Shell-model Hamiltonian

$$H = \sum_{jm}\varepsilon_j^{\rm core} a_{jm}^\dagger a_{jm} + \sum_{\alpha\alpha' JM}\langle 2\alpha|V|2\alpha'\rangle_J A^\dagger(2\alpha JM)A(2\alpha' JM)$$

$$\varepsilon_j = \varepsilon_j^{\rm core} + \sum_{j'}\Delta\varepsilon_{jj'}\langle\Psi|\widehat{N}_{j'}|\Psi\rangle$$

interaction with the core nucleons

interaction with the valence nucleons

$$\Delta\varepsilon_{jj'} = \frac{\sum_J (1-(-1)^{2j-J}\delta_{jj'})(2J+1)\langle jj'|V|jj'\rangle_J}{(2j+1)(2j'+1)}$$

monopole

Monopole and single-particle energies

$$\varepsilon_j = \varepsilon_j^{\text{core}} + \sum_{j'} \Delta\varepsilon_{jj'} \langle \Psi | \widehat{N}_{j'} | \Psi \rangle \Rightarrow \varepsilon_j^{\text{valence}}$$

$$\Delta\varepsilon_{jj'} = \frac{\sum_J \left(1 - (-1)^{2j-J}\delta_{jj'}\right)(2J+1)\langle jj'|V|jj'\rangle_J}{(2j+1)(2j'+1)}$$

monopole

This formula can be applied to any nucleus (state), and thus allows quantitative discussion of shell structure evolution.

The s.p. energies are determined by the monopole strengths.

Note that $\Delta\varepsilon_{jj'} \neq V_{jj'}$ for $j = j'$

The present definition of s.p. energies is consistent with multipole expansion and binding energies.

Monopole interaction : multipole expansion

The monopole interaction is defined as the lowest-rank term of multipole expansion of two-body *NN* interaction.

Proton-neutron interaction

exchange

$$V_{pn} = \sum_{jJ} \langle j_p j_n | V | j'_p j'_n \rangle_J \, [a^\dagger_{j_p} \otimes a^\dagger_{j_n}]^{(J)} \cdot [\tilde{a}_{j'_p} \otimes \tilde{a}_{j'_n}]^{(J)}$$

$$= \sum_k V_{pn}^{(k)} = V_{pn}^{(0)} + V_{pn}^{(1)} + V_{pn}^{(2)} + \cdots$$

monopole interaction ➡ $V_{pn}^{(0)} = \sum_{j_p j_n} \Delta\varepsilon_{j_p j_n} \widehat{N}_{j_p} \widehat{N}_{j_n} \qquad \widehat{N}_j = \sum_m a^\dagger_{jm} a_{jm}$

Like-nucleon interaction

monopole interaction ➡ $V_{pp/nn}^{(0)} = \sum_{[jj']} \left[\frac{1}{1+\delta_{jj'}} \Delta\varepsilon_{jj'} \widehat{N}_j \widehat{N}_{j'} - \delta_{jj'} \frac{1}{2} \Delta\varepsilon_{jj} \widehat{N}_j \right]$

$\Delta\varepsilon_{jj'}$: exactly the same quantity that appears in s.p. energies.

Roles of C, T and LS interactions (1)

Example for tensor interaction

multipole expansion

Fourier transfrom
of radial potentials

$$V_{\mathrm{TN}} = \int_0^\infty \mathrm{d}q\, q^2 \sum_i v_2^i(q) \left[f^i_{\mathrm{TN}0} + f^i_{\mathrm{TN}\tau} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \right]$$

$$\times \sum_{k_1 k_2 K} g^{(k_1 k_2 K)} \left(\boldsymbol{F}_1^{(1k_1 K)} \cdot \boldsymbol{F}_2^{(1k_2 K)} \right).$$

$$\boldsymbol{F}_m^{(1LJ)} = \left[\boldsymbol{\sigma}_m \times j_L(q r_m) \boldsymbol{C}^{(L)}(\widehat{\boldsymbol{r}}_m) \right]^{(J)}$$

$$g^{(k_1 k_2 K)} = (-1)^{\frac{k_1+k_2}{2}} \sqrt{\frac{2}{3}}\, [k_1]\, [k_2]\, \langle k_1 0 k_2 0 \,|\, 20 \rangle\, W(11 k_1 k_2 ; 2K)$$

Roles of C, T and LS interactions (2)

Direct terms of the monopole vanish for tensor interactions:

$$\Delta\varepsilon_{jj'}^{(\mathrm{dir})}(\mathrm{TN}) \;=\; 0.$$

Sum over spin-orbit partners:

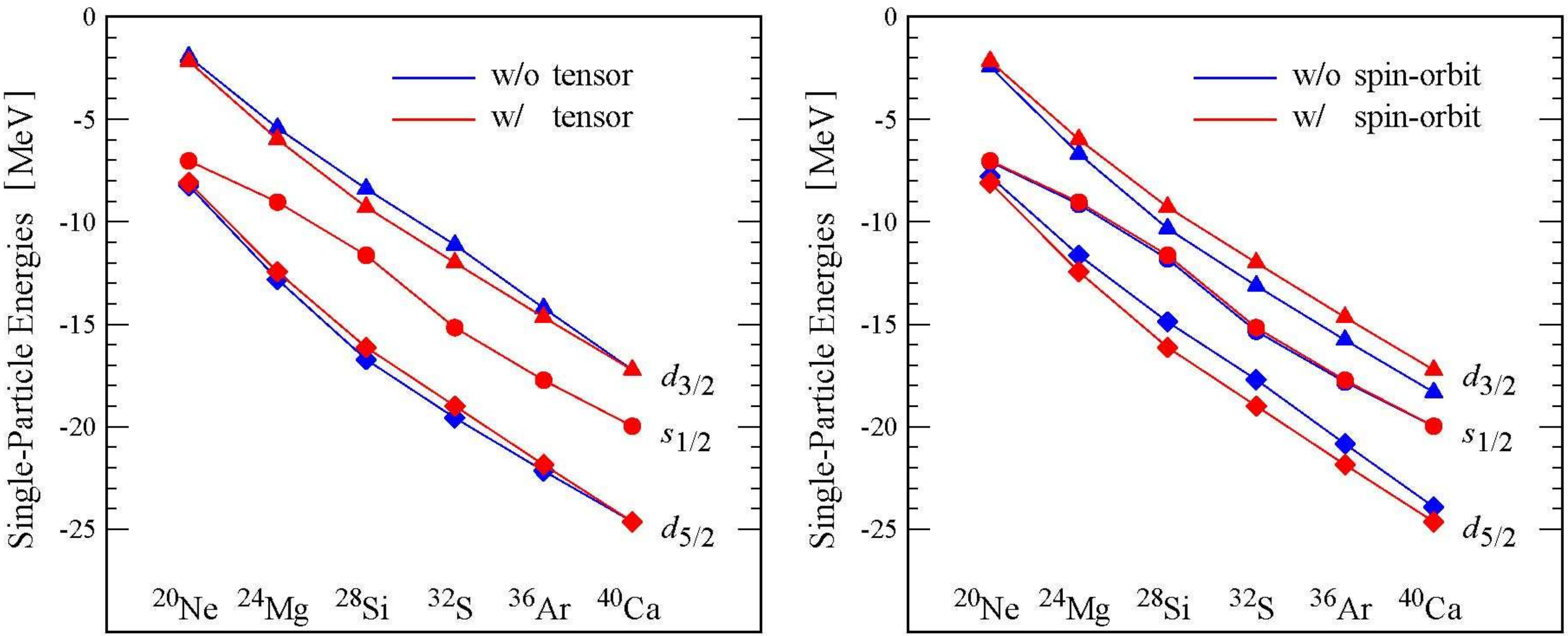
$$\sum_{j=\ell\pm\frac{1}{2}} [j]\, \Delta\varepsilon_{jj'}^{\mathrm{exch}}(V_{\mathrm{TN}}) \;=\; \mathrm{const} \sum_{K} (-1)^K\,[K]\, W(11k_1k_2;2K) \times \sum_{j} [j] \begin{Bmatrix} \frac{1}{2} & \ell & j \\ \frac{1}{2} & \ell' & j' \\ 1 & k_1 & K \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \ell' & j' \\ \frac{1}{2} & \ell & j \\ 1 & k_2 & K \end{Bmatrix} = 0.$$

$$(2j_< + 1)\,\Delta\varepsilon_{j'j_<}(V_{\mathrm{TN}}) + (2j_> + 1)\,\Delta\varepsilon_{j'j_>}(V_{\mathrm{TN}}) \;=\; 0$$

$$\text{for any } j', \quad \text{including } j' = j_<,\ j' = j_>$$

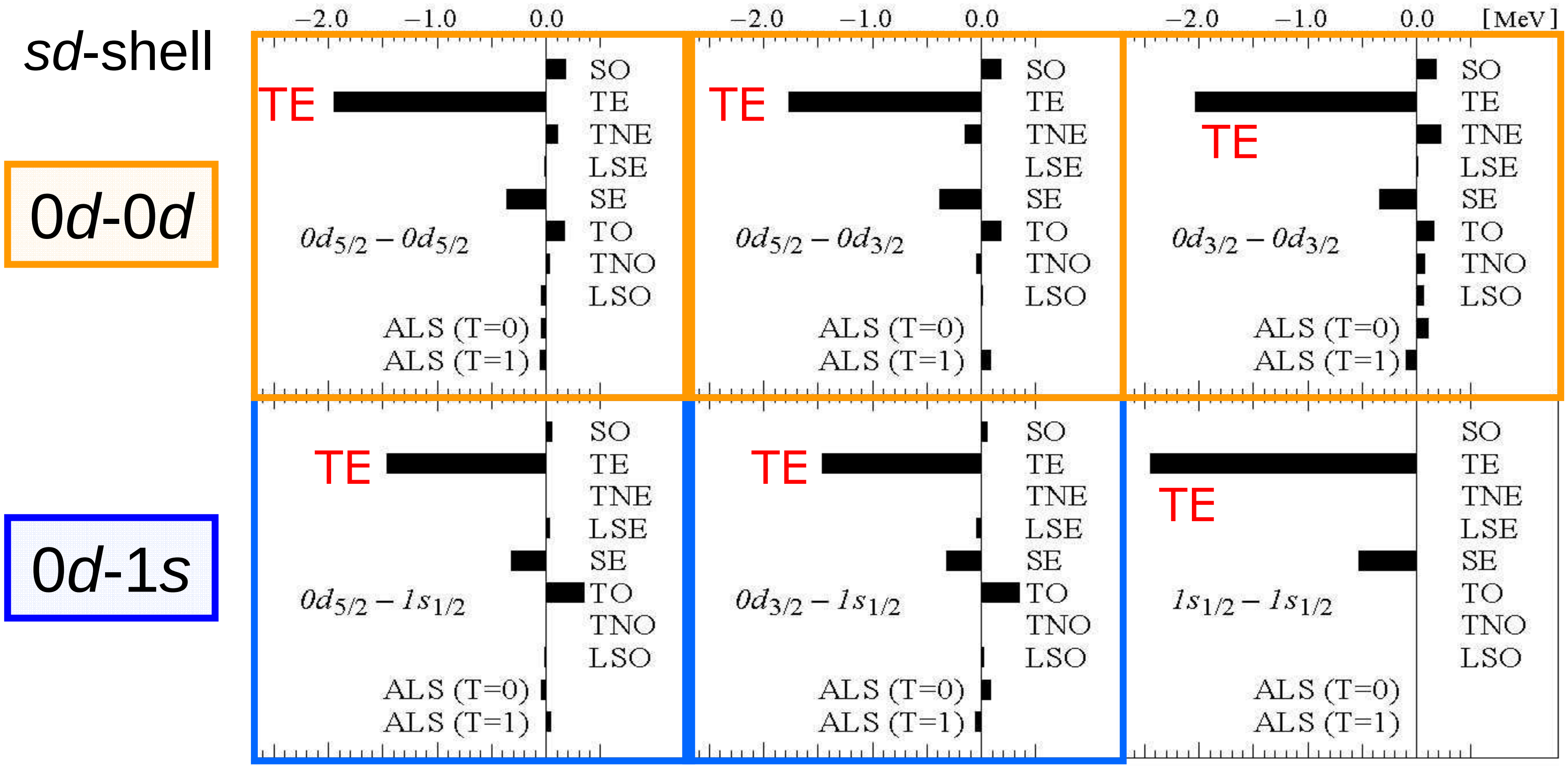
Example-1 **: s.p. energies in *sd*-shell nuclei**

Shell-model calculation in the $(sd)^n$ configuration on ^{16}O core
USD interaction (B. H. Wildenthal, Prog. Part. Nucl. Phys. **11**, 5 (1984))

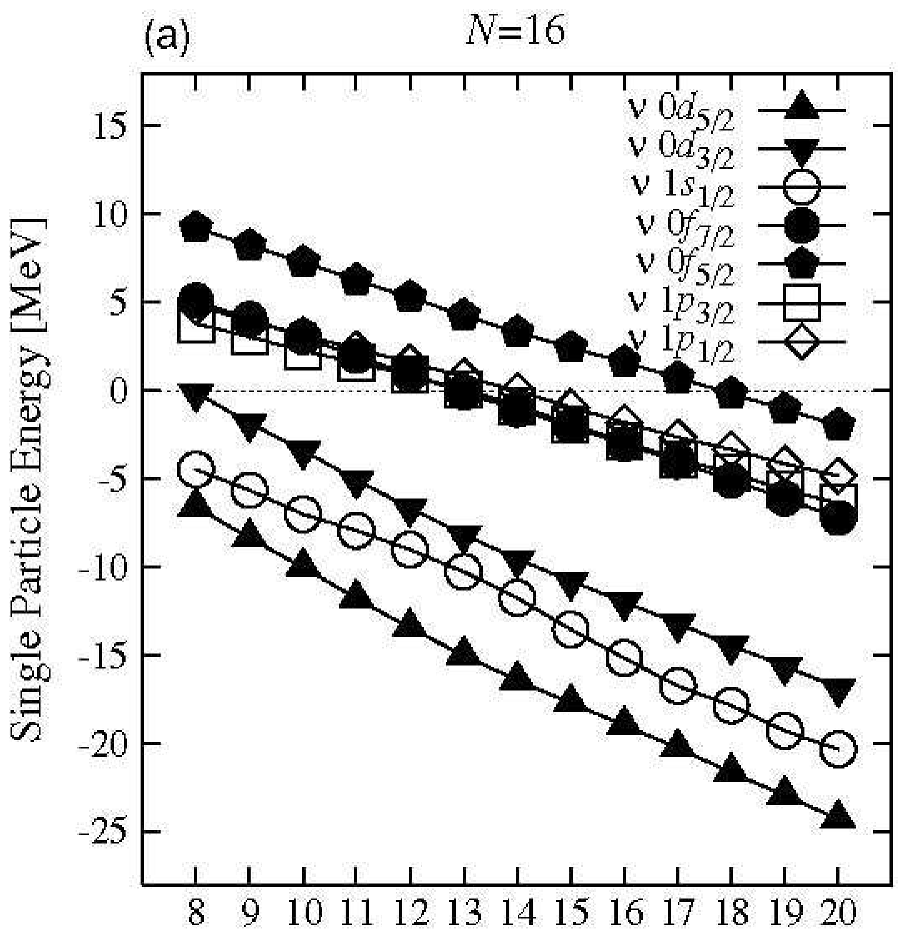


The monopole due to Triplet-Even attraction

The spin-tensor decomposition clarifies that the monopole strengths are dominated by triplet-even attraction of central interaction, which includes the second order tensor effects.



Example-2 : s.p. energies in *N* = 16 nuclei



Neutron s.p. energies

USD + Millener Kurath

A. Umeya and K. Muto
Phys. Rev. C74 (2006) 034330

In the neutron-rich region, $1p_{3/2}$ is lower than $0f_{7/2}$, due to different slopes between 1*p*- and 0*f*-orbits, since 1*p-0d* interaction is less attractive than 0*f-0d*.

p-wave neutron halo in ^{31}Ne

T. Nakamura *et al*.
Phys. Rev. Lett. 103, 262501 (2009)

Conclusions - 1

Single-particle energies, defined according to the prescription by Baranger, are

1. expressed as

$$\varepsilon_j = \varepsilon_j^{\text{core}} + \sum_{j'} \Delta\varepsilon_{jj'} \langle \Psi | \widehat{N}_{j'} | \Psi \rangle \Rightarrow \varepsilon_j^{\text{valence}}$$

2. applied to any nucleus

3. determined by the monopole strengths

$$\Delta\varepsilon_{jj'} = \frac{\sum_J \left(1 - (-1)^{2j-J}\delta_{jj'}\right)(2J+1)\langle jj'|V|jj'\rangle_J}{(2j+1)(2j'+1)}$$

which are consistent with the multipole expansion of *NN* interaction (and binding energies).

Conclusions - 2

Roles of **central**, **tensor** and **spin-orbit** interactions are studied by multipole expansion and evaluating their contributions to the monopole strengths.

When both spin-orbit partners are completely filled, the interaction with these nucleons gives, for single-particle energies of a pair of spin-orbit partners,

1. **the same large gain** by the **central** interaction, due to the strong triplet-even channel attraction,
2. **no shift** by the **tensor** interaction,
3. **expansion of the splitting** by the **spin-orbit** interaction.