



# Shape mixing dynamics in the low-lying states of proton-rich Kr isotopes

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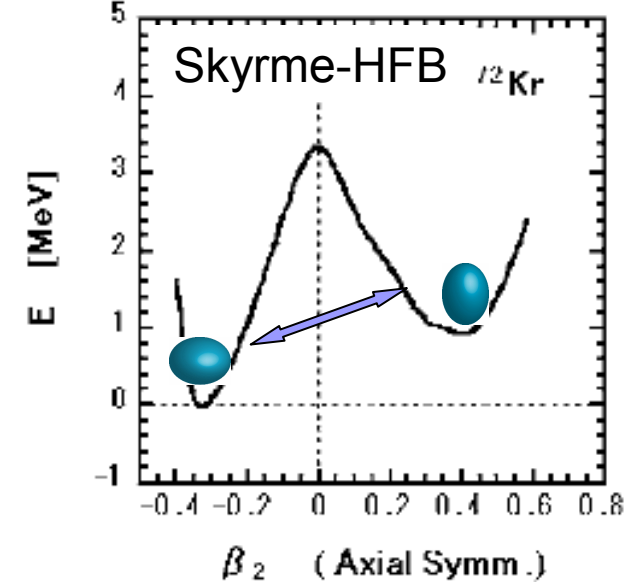
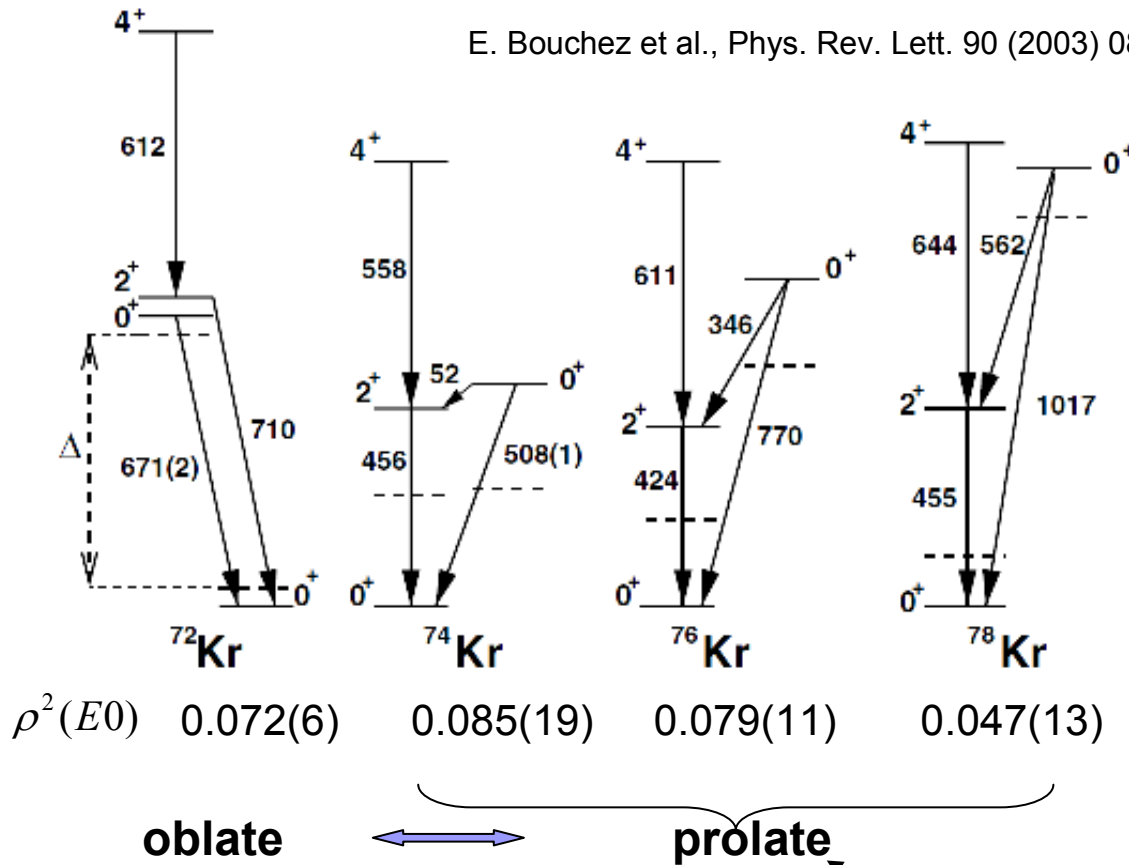
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# Oblate- prolate shape coexistence phenomena in proton-rich Kr isotopes

E. Bouchez et al., Phys. Rev. Lett. 90 (2003) 082502

M. Yamagami et al., NPA 693(2001) 579.



Shape mixing by tunneling between the local minima

- ◆ Low-lying excited  $0^+$  states
- ◆ Irregularities of the excitation spectra
- ◆ Systematics of the excited  $0^+$  energies and  $E0$  transition strengths

◆ Spectroscopic quadrupole moments

Shape transition from the oblate ground state in  $^{72}\text{Kr}$  to the prolate ground state in  $^{74}, ^{76}, ^{78}\text{Kr}$

We study oblate-prolate shape mixing in the low-lying state of  $^{72,74,76}\text{Kr}$  using

**5D quadrupole collective Hamiltonian  
(Generalized Bohr-Mottelson Hamiltonian) :**

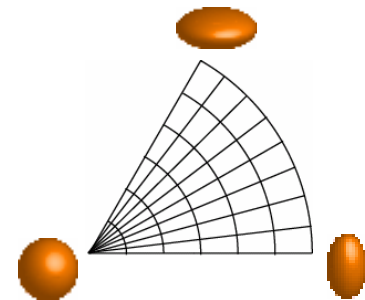
$$H = T_{\text{vib}} + T_{\text{rot}} + \boxed{V(\beta, \gamma)} \text{ collective potential}$$

$$T_{\text{vib}} = \frac{1}{2} \boxed{D_{\beta\beta}(\beta, \gamma)} \dot{\beta}^2 + \boxed{D_{\beta\gamma}(\beta, \gamma)} \dot{\beta} \dot{\gamma} + \frac{1}{2} \boxed{D_{\gamma\gamma}(\beta, \gamma)} \dot{\gamma}^2$$

vibrational inertial masses

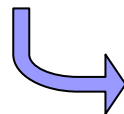
$$T_{\text{rot}} = \sum_{k=1}^3 \frac{1}{2} \boxed{J_k} \omega_k^2$$

rotational moments of inertia



The collective Hamiltonian is derived microscopically by means of the “**Constrained HFB+ Local QRPA**”(CHFB+LQRPA) method, which we recently developed on the basis of the **Adiabatic Self-consistent Collective Coordinate (ASCC)** method

Matsuo, Nakatsukasa, and Matsuyanagi, Prog.Theor. Phys. 103(2000), 959.



Application of 1D ASCC to shape coexistence in Se and Kr

N. Hinohara, et al, Prog. Theor. Phys. 119(2008), 59; PRC 80 (2009),014305.

an approximation of the 2-dimensional ASCC method.

## Constrained HFB + Local QRPA method:

Solve the constrained HFB eq. at each point on the  $(\beta, \gamma)$  plane

$$N, Z, \beta, \gamma \quad \downarrow \quad |\phi(\beta, \gamma)\rangle \quad \boxed{V(\beta, \gamma)} \quad \lambda(\beta, \gamma) \quad \mu(\beta, \gamma)$$

Solve the local QRPA eqs. on top of each CHFB state  $|\phi(\beta, \gamma)\rangle$

$$\omega_\alpha^2(\beta, \gamma) \quad \hat{Q}^{(\alpha)}(\beta, \gamma) \quad \hat{P}^{(\alpha)}(\beta, \gamma)$$

Local Thouless-Valatin eqs.

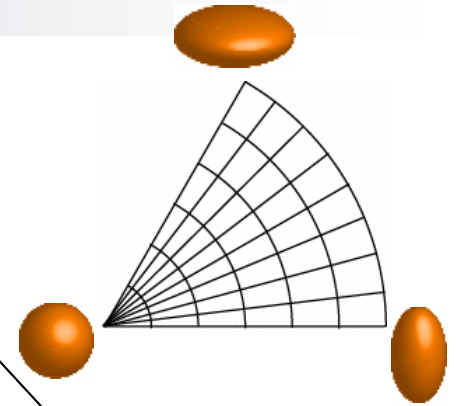
Calculate the vibrational masses

**Local QRPA masses**

$$\boxed{D_{\beta\beta}(\beta, \gamma) \quad D_{\beta\gamma}(\beta, \gamma) \quad D_{\gamma\gamma}(\beta, \gamma) \quad \mathcal{I}_k(\beta, \gamma)}$$

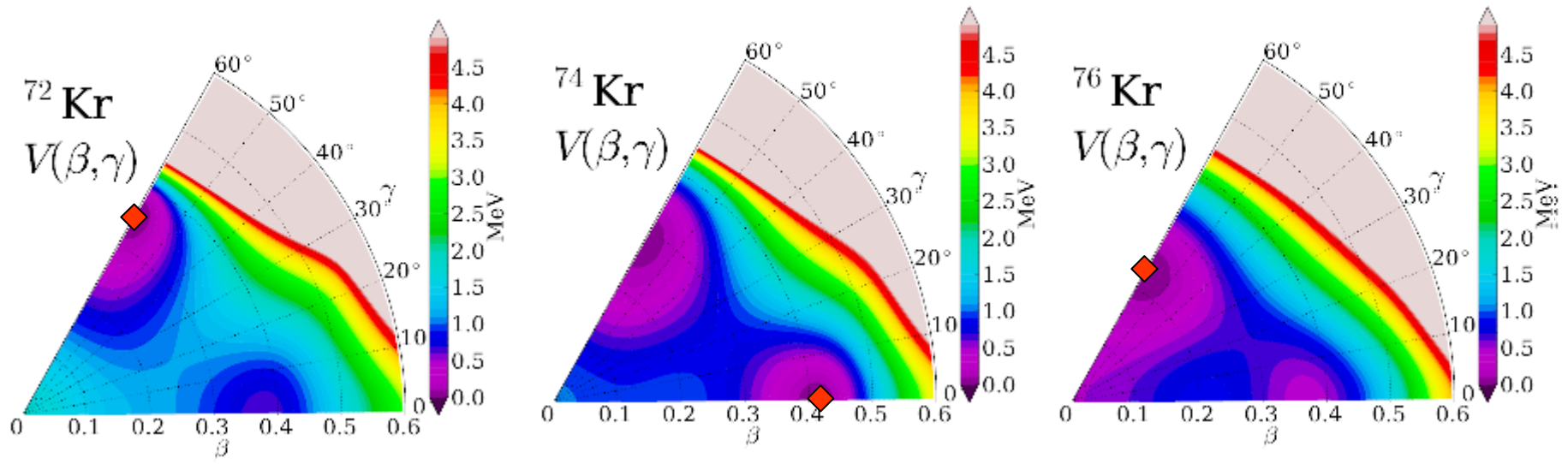
Diagonalize the 5D Quadrupole Hamiltonian to calculate the energy spectrum

Include the contribution from the **time-odd component** of the mean field, unlike widely-used cranking masses

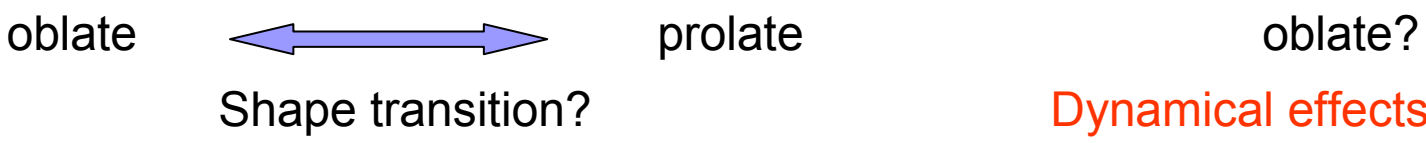


Numerical results

Collective potential



◆: absolute minimum



Dynamical effects beyond the mean field should be taken into account

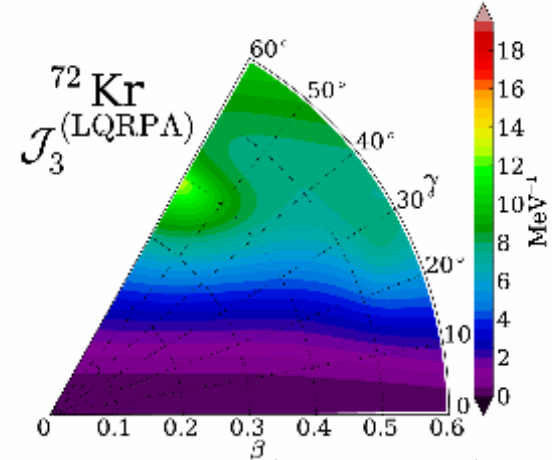
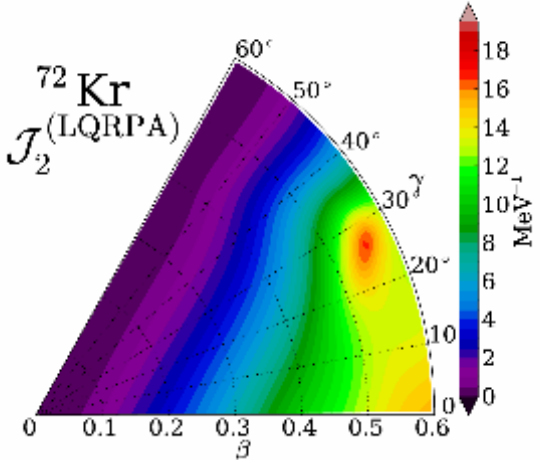
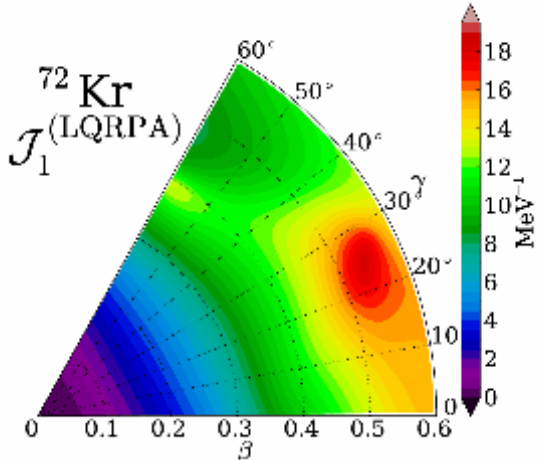
Microscopic Hamiltonian:P+QQ model

parameters are fitted to the pairing gap and the quadrupole deformation obtained with Skyrme-HFB by Yamagami et al.

M. Yamagami et al.,NPA 693(2001) 579.

**LQRPA moments of Inertia**

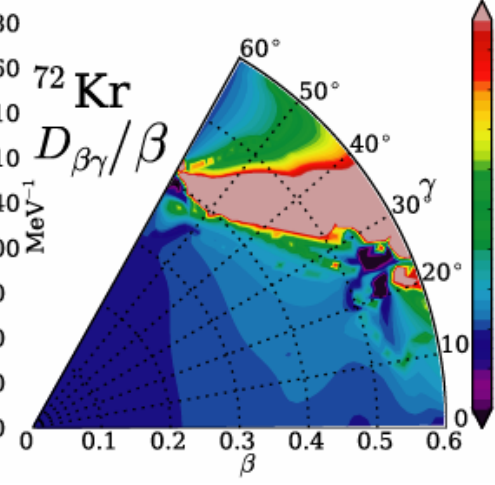
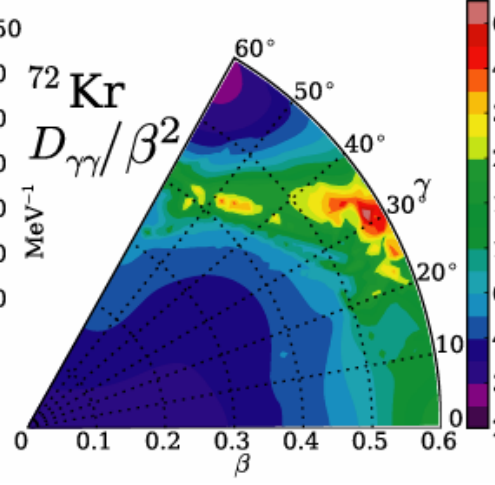
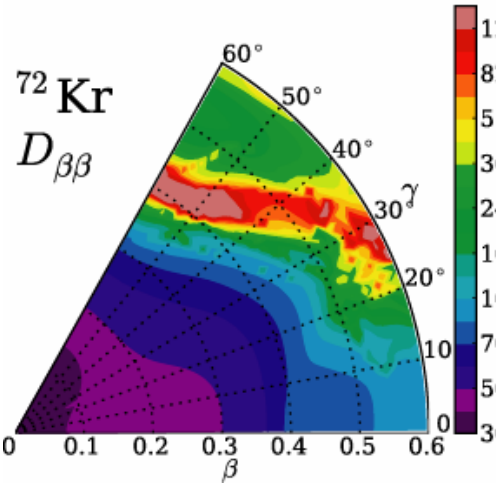
→ Extension of Thouless-Valatin Mol to non-equilibrium HFB pts.



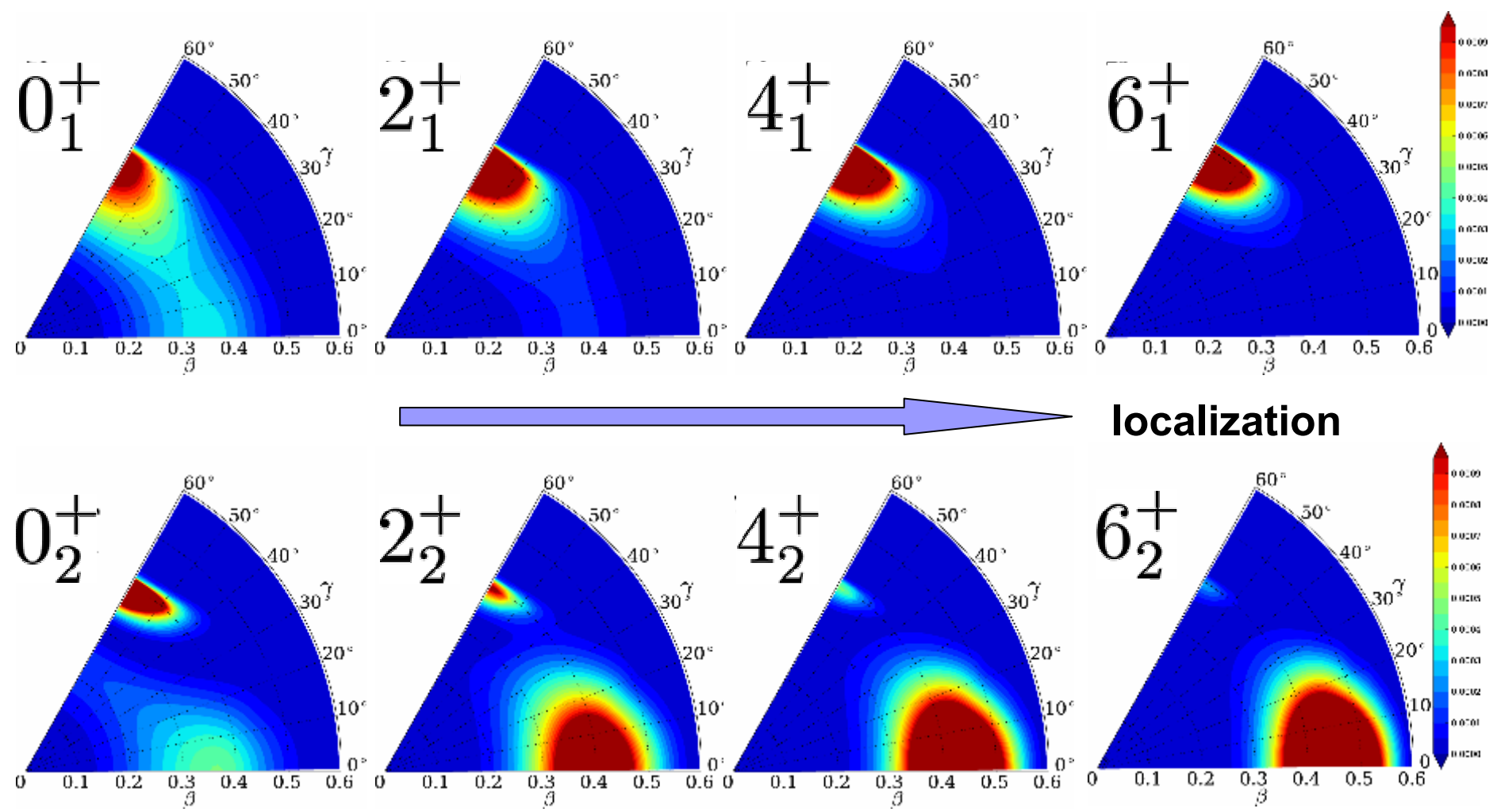
$$\mathcal{J}_k(\beta, \gamma) = 4\beta^2 \underline{D_k(\beta, \gamma)} \sin^2(\gamma - 2\pi k/3)$$

strongly dependent on  $(\beta, \gamma)$

**Local QRPA vibrational masses:**



Collective wave functions squared for <sup>72</sup>Kr  $\beta^4 \sum_K |\Phi_{IKk}(\beta, \gamma)|^2$

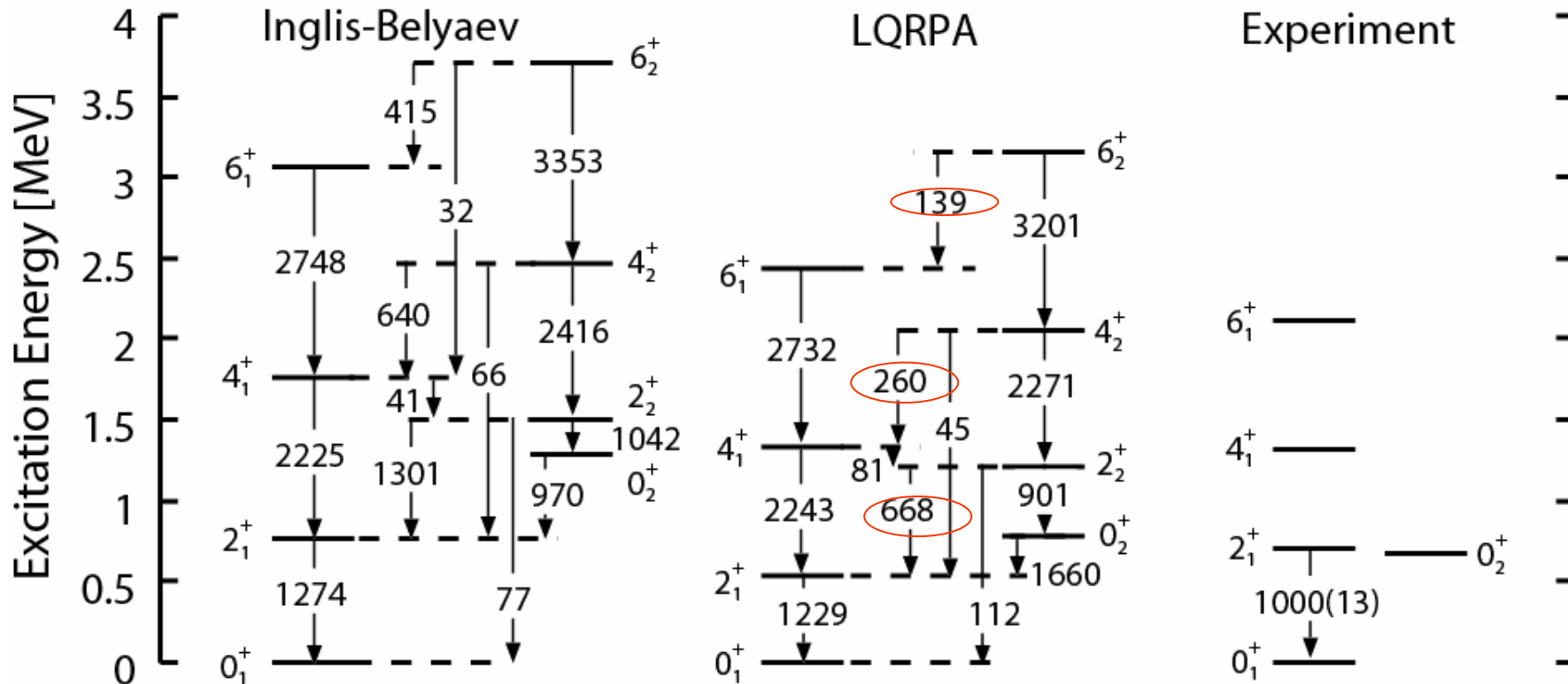


# Excitation Energies and B(E2)

EXP: Fischer et al., Phys.Rev.**C67** (2003) 064318,  
 Bouchez, et al., Phys.Rev.Lett.**90** (2003) 082502.  
 Gade, et al., Phys.Rev.Lett.**95** (2005) 022502, **96** (2006) 189901

( ) ...B(E2) e<sup>2</sup> fm<sup>4</sup>

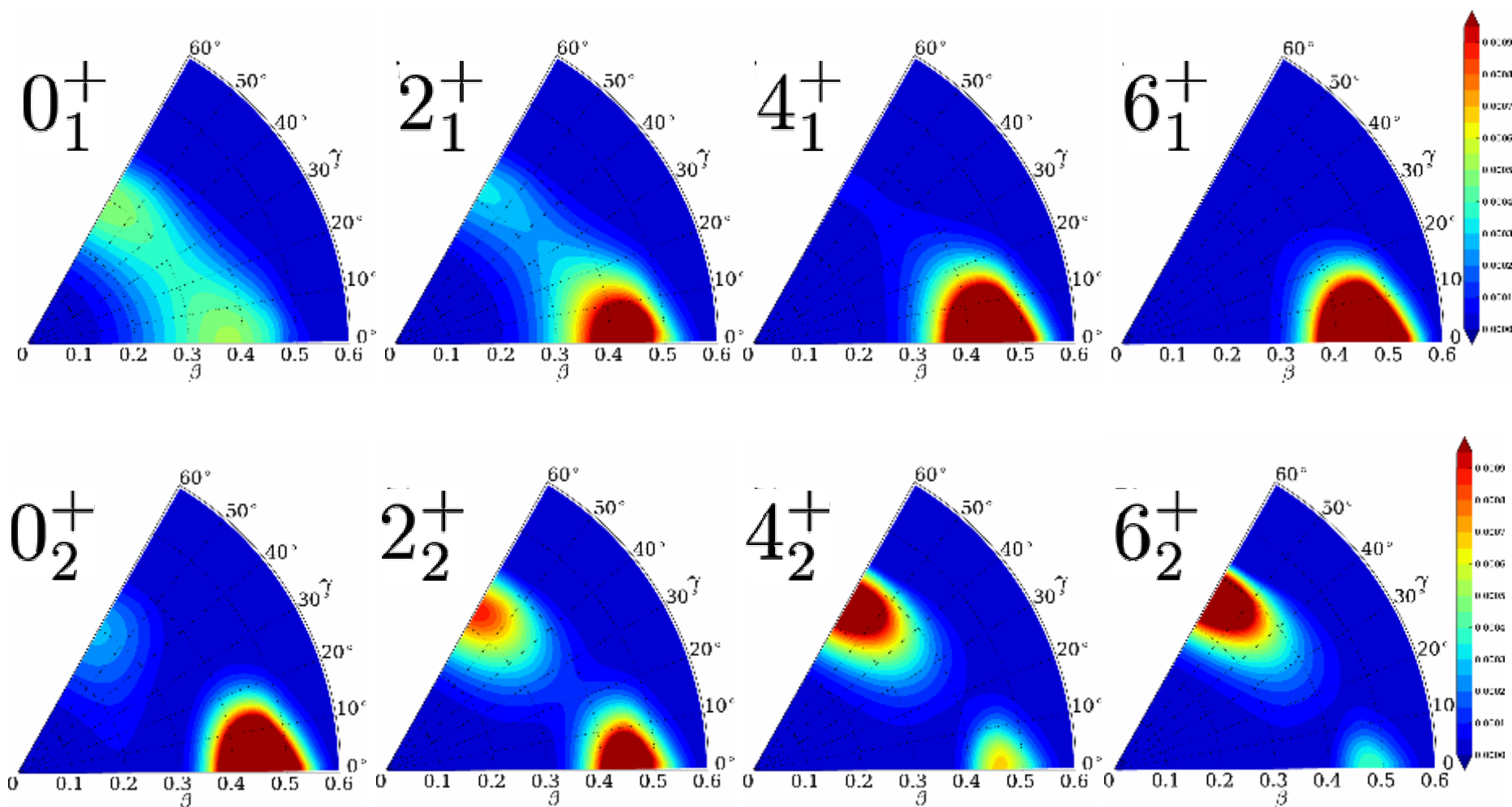
effective charge:  $e_{pol} = 0.834$



- The interband transitions become weaker as angular momentum increases.  
 ↔ development of the localization of w.f.
- The time-odd mean-field lowers the excitation energies.

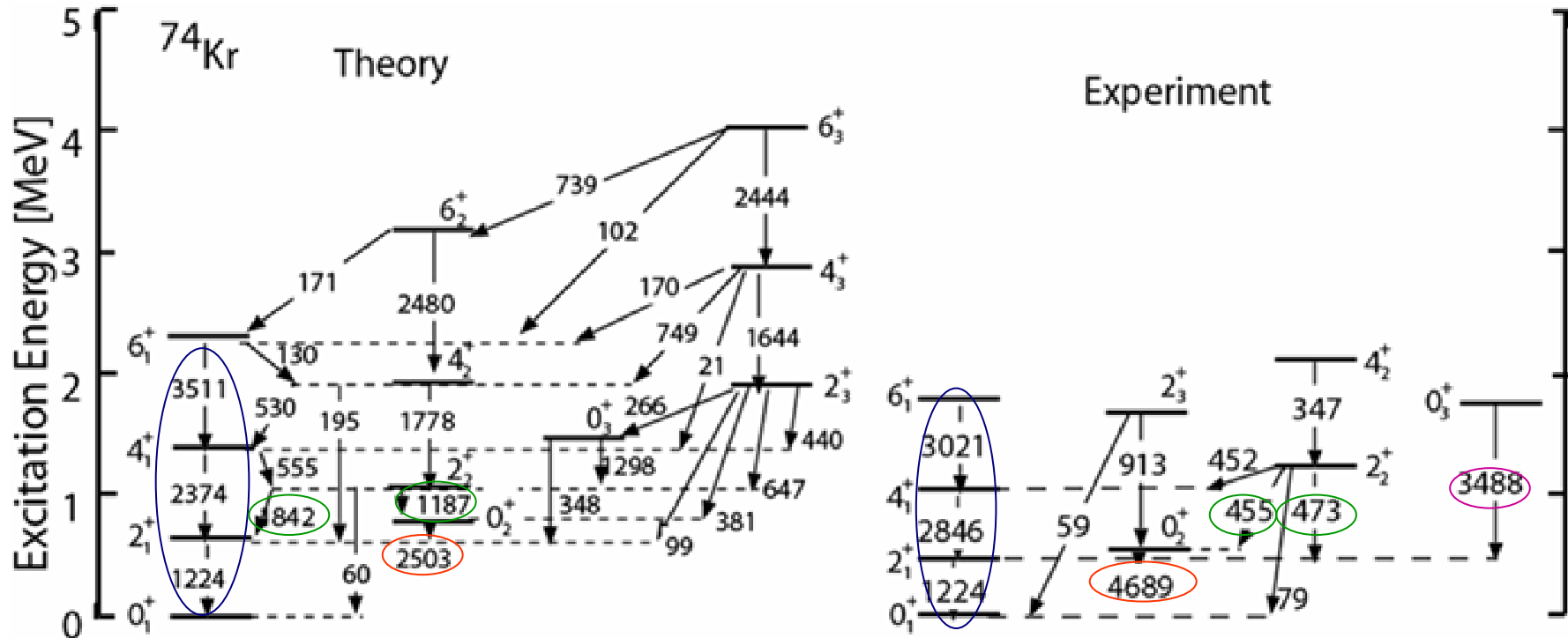


Collective wave functions squared for <sup>74</sup>Kr  $\beta^4 \sum_K |\Phi_{IKk}(\beta, \gamma)|^2$



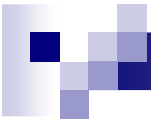
# Excitation Energies and B(E2) for $^{74}\text{Kr}$

EXP: E. Clément et al., PRC 75,054313 (2007).



Main features of the experimental data:

- ☺ Increasing tendency of B(E2) in the ground band (blue oval)
  - ☺ Strong  $0_2 \rightarrow 2_1$  transition (red oval)
  - ☺ Equal strength of the  $2_2 \rightarrow 2_1$  and  $2_2 \rightarrow 0_2$  transitions (green oval)
  - ☹ Strong  $0_3 \rightarrow 2_1$  transition (pink oval) not reproduced
- } well reproduced!

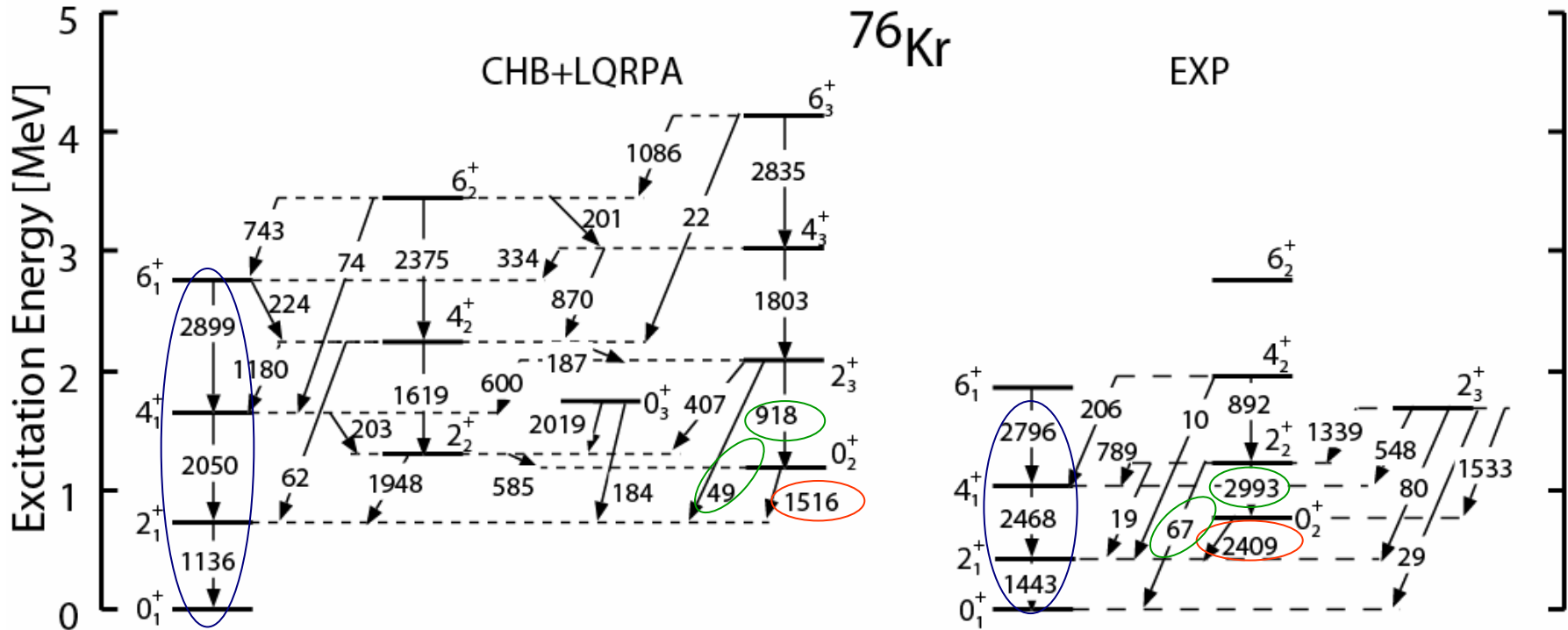


# Excitation Energies and B(E2) for $^{76}\text{Kr}$

( ) ...B(E2)  $e^2 \text{ fm}^4$

effective charge:  $e_{\text{pol}} = 0.834$

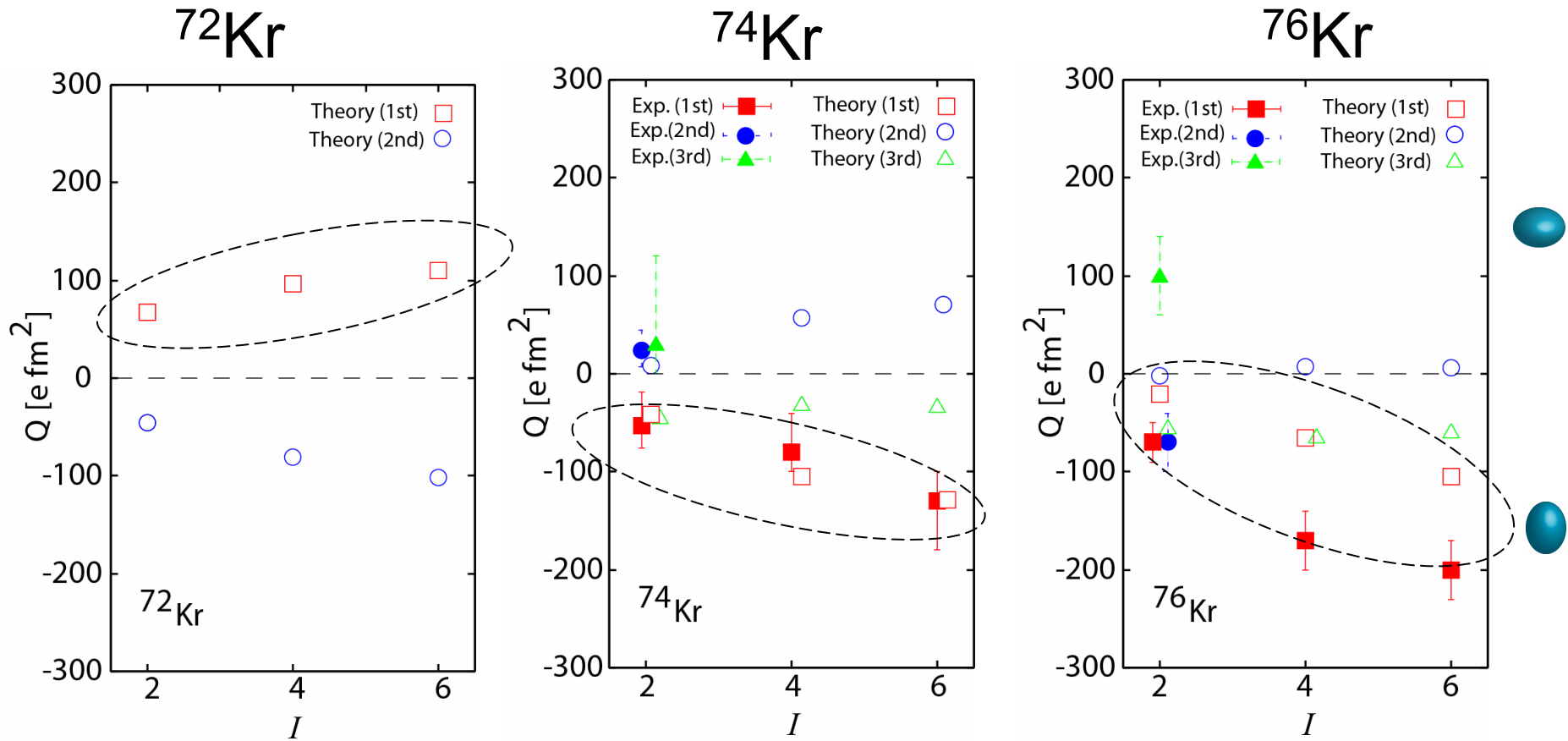
EXP: E. Clément et al., PRC 75,054313 (2007).



- ☺ Increasing tendency in the ground band
- ☺ Strong  $0_2 \rightarrow 2_1$  transition
- ☺  $B(E2; 2_2 \rightarrow 0_2) \gg B(E2; 2_2 \rightarrow 2_1)$

} well reproduced!

# Spectroscopic quadrupole moments $Q$

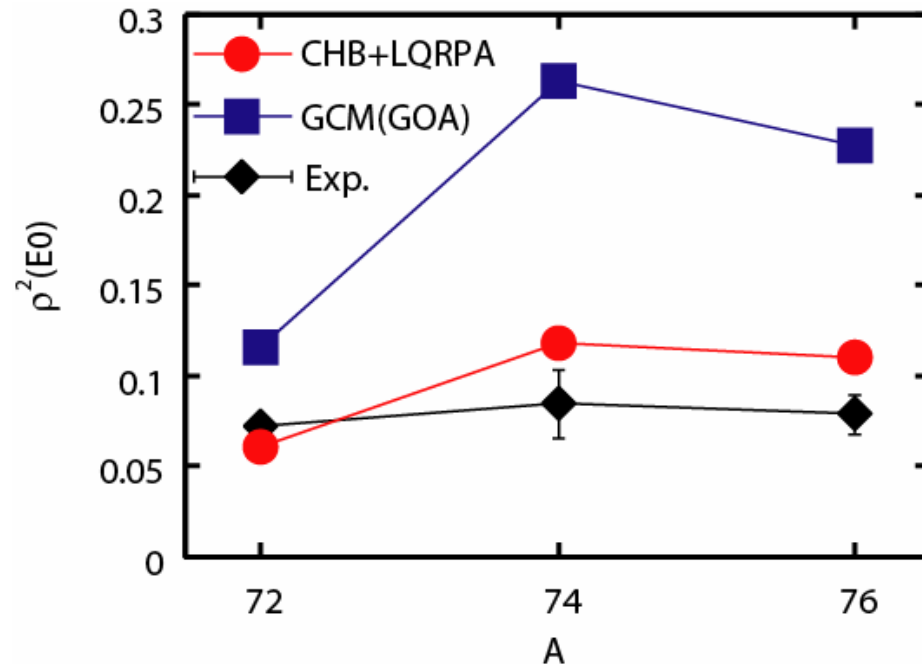


Exp. (1st) :  $\blacksquare$  Theory(1st):  $\square$   
 Exp.(2nd) :  $\bullet$  Theory(2nd):  $\circ$   
 Exp. (3rd) :  $\blacktriangle$  Theory(3rd):  $\triangle$

Shape transition from oblate in  $^{72}\text{Kr}$  to prolate in  $^{74}, ^{76}\text{Kr}$  was reproduced.



## E0 transition strengths $\rho^2(E0; 0_2^+ \rightarrow 0_1^+)$



GCM(GOA):

HFB-based configuration  
mixing calculation using the  
Gogny D1S interaction

M. Girod et al., PLB 676 (2009) 39.

Exp: C. Chandler et al. PRC 56 (1997) 2924.  
E. Bouchex et al., PRL 90 (2003) 082502.

The calculated result reproduces well the experimental data both qualitatively and quantitatively!

The  $\rho(E0)$  takes a maximal value at  $A=74$ , which reflects the shape transition.



## Summary

- We have studied the shape coexistence/mixing in 72,74,76Kr using 5D quadrupole Hamiltonian derived by means of the CHFB+LQRPA method.
- Our results indicate a shape transition from the oblate ground state in 72Kr to the prolate one in 74,76Kr, which is consistent with the experiment.
- The basic features of the low-lying states in these nuclei are determined by the interplay of the large-amplitude shape fluctuation in the triaxial shape degree of freedom, the  $\beta$ -vibrational excitations and the rotational motions.
- The rotational motion plays an important role for the growth of the localization of the vibrational wave functions in the  $(\beta, \gamma)$  plane.

## Outlook

More realistic interaction

Full 2D ASCC Method