東京大学
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Derivation of IBM Hamiltonian and low-lying states of heavy neutron-rich nuclei

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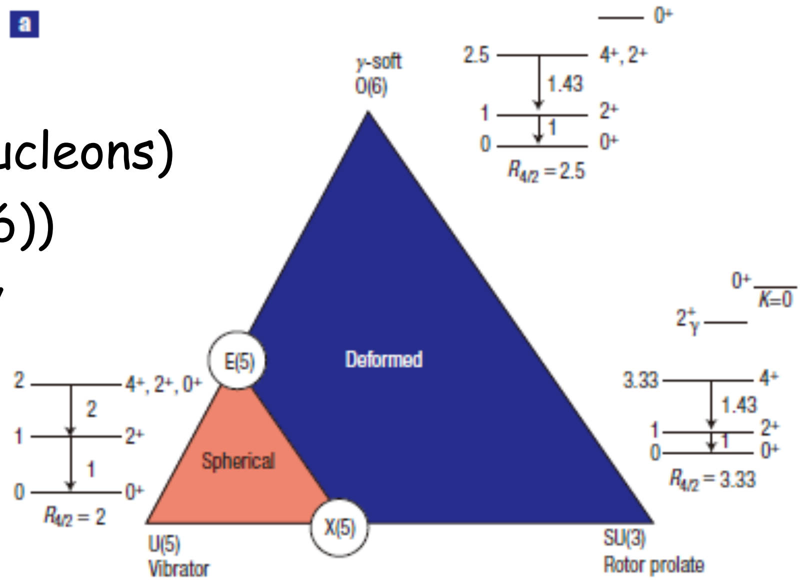
Interacting Boson Model (IBM) and microscopic basis

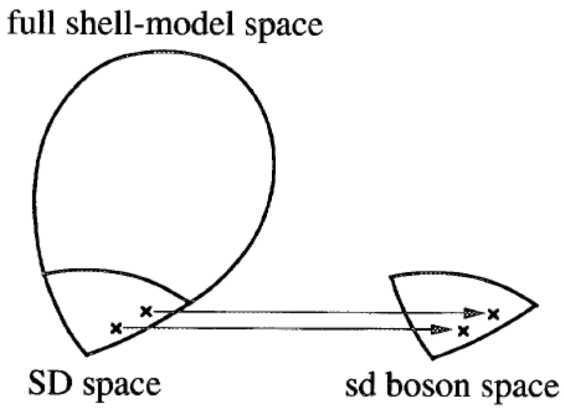
Arima and Iachello (1974)

- *sd* bosons (collective SD pairs of valence nucleons)
- Dynamical symmetries (U(5), SU(3) and O(6)) and their mixtures with phenomenologically adjusted parameters
- Microscopic basis

by shell model: successful for spherical & γ-unstable shapes (OAI mapping)



Casten (2006)



- Otsuka, Arima, Iachello and Talmi (1978)
- Otsuka, Arima and Iachello (1978)
- Gambhir, Ring and Schuck (1982)
- Allaat et al (1986)
- Deleze et al (1993)
- Mizusaki and Otsuka (1997)

General cases → ?

This limitation may be due to the highly complicated shell-model interaction, which becomes unfeasible for strong deformation.

Potential energy surface (PES) by mean-field models (e.g., Skyrme) can be a good starting point for deformed nuclei.

We construct an IBM Hamiltonian starting from the mean-field (Skyrme) model. → low-lying states, shape-phase transitions etc.

Note:

- algebraic features
- boson number counting rule (# of valence nucleons /2 = # of bosons)

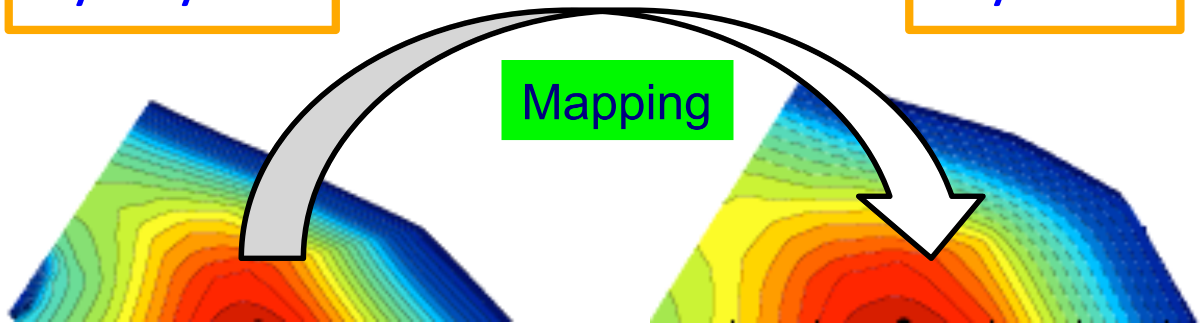
of IBM are kept.

Derivation of IBM interaction strengths

Potential Energy Surface (PES) in $\beta\gamma$ plane

by Skyrme

by IBM



reflects effects of Pauli principle and nuclear forces

simulates basic properties of nucleon system

IBM interaction strengths

Diagonalization of IBM Hamiltonian

Levels and wave functions with good J & N(Z) + predictive power

K.N. et al., PRL101, 142501 (2008)

- HF (Skyrme) PES

density-dependent zero-range pairing force in BCS approximation, mass quadrupole constraint

- Boson Hamiltonian

$$H_{\mathrm{IBM}} = E_0 + \epsilon(n_{d\pi} + n_{d\nu}) + \kappa Q_\pi Q_\nu + \alpha LL$$

kinetic term irrelevant to the PES (discussed later)

IBM PES (coherent state formalism)

Dieperink and Scholten (1980) ; Ginocchiio and Kirson (1980) ; Bohr and Mottelson (1980)

$$\langle H_{\mathrm{IBM}} \rangle = \frac{\langle \Phi | H_{\mathrm{IBM}} | \Phi \rangle}{\langle \Phi | \Phi \rangle}$$

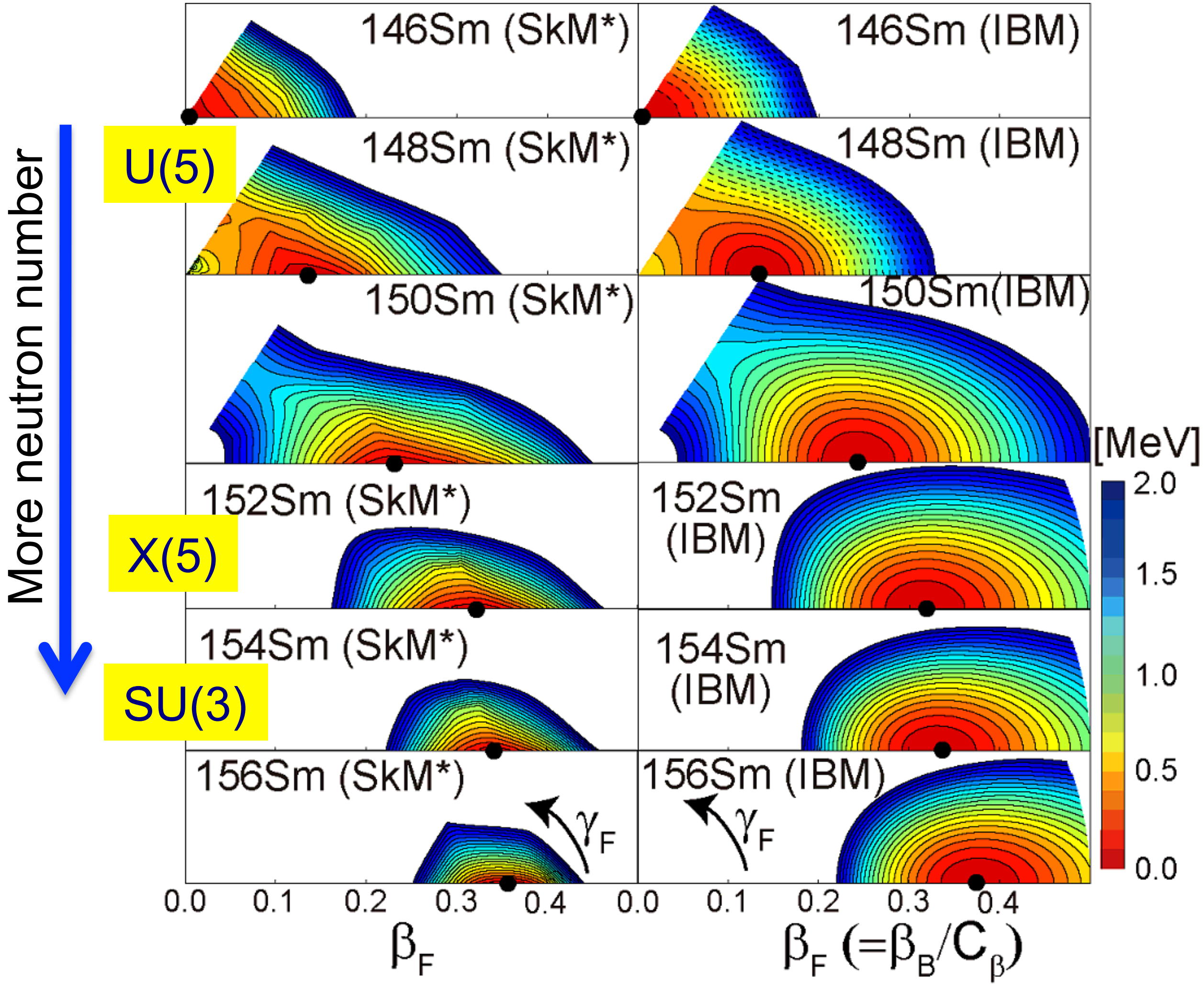
$$|\Phi\rangle \propto \prod_\rho \left[s_\rho^\dagger + \beta_\rho \{ d_{\rho 0}^\dagger \cos\gamma_\rho + \frac{1}{\sqrt{2}} (d_{\rho 2}^\dagger + d_{\rho -2}^\dagger) \sin\gamma_\rho \} \right]^{n_\rho} |0\rangle$$

$$\begin{cases} \beta_\pi = \beta_\nu \equiv \beta_B \\ \gamma_\pi = \gamma_\nu \equiv \gamma_B \end{cases}$$

Some formulas for translation of (β, γ)'s

$\beta_{IBM} = C_\beta \beta_{HF}$, $\gamma_{IBM} = \gamma_{HF}$

Potential energy surfaces in $\beta\gamma$ planes (Sm)



Location of minimum and the overall patter up to several MeV should be reproduced. → By χ^2-fit using wavelet transform.

K.N. et al., PRC81, 044307 (2010)

IBM-2 Hamiltonian

$$H_{\mathrm{IBM}} = \underline{\underline{E_0 + \epsilon(n_{d\pi} + n_{d\nu}) + \kappa Q_\pi^{\chi_\pi} \cdot Q_\nu^{\chi_\nu}}} + \boxed{\alpha LL}$$

determined by the calibration with HF PES

kinetic term irrelevant to the PES

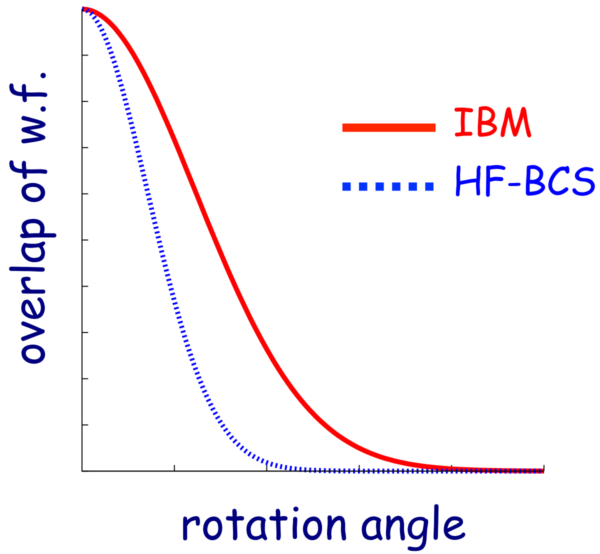
A general boson Hamiltonian contains interaction terms which cannot be determined from PES fits. Then, we propose

mapping

Response of nucleon system → boson system

Rotation of strongly-deformed object is considered.

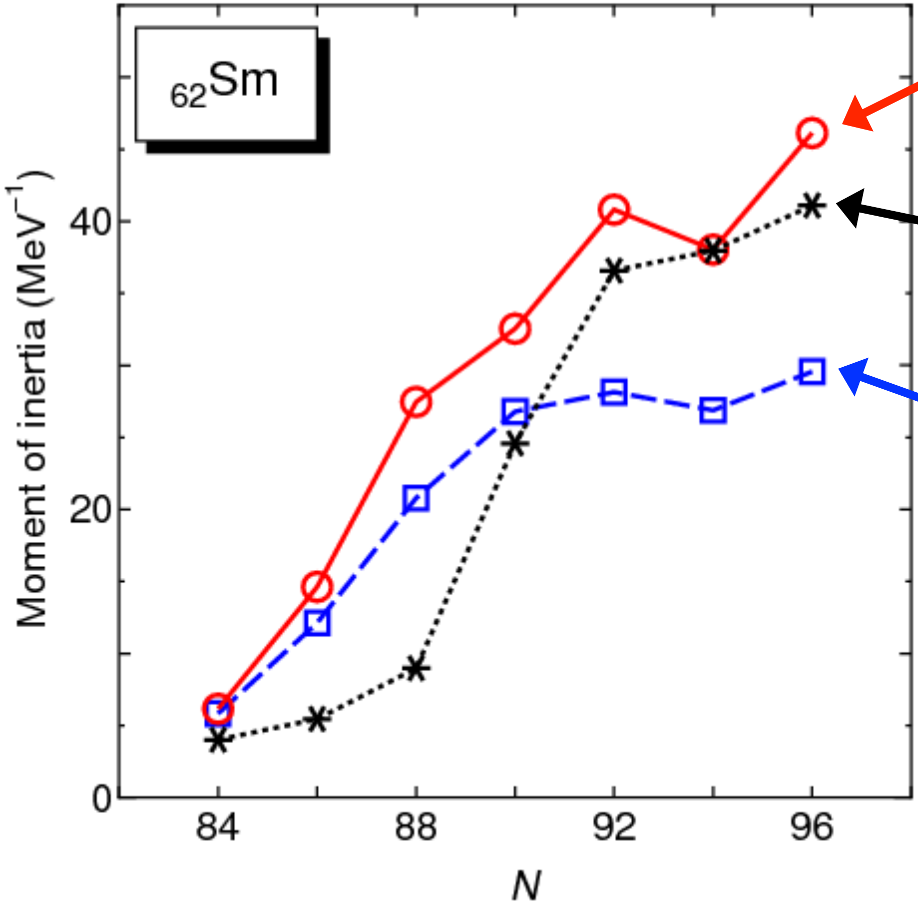
The difference of the overlap requires the rotational "mass" (kinetic) term in the boson Hamiltonian.



This work: comparison of moment of inertia
➔ coefficient of LL

Determination of the coefficient of LL term

Moment of inertia in the intrinsic state



Inglis-Belyaev formula by cranking model in the same Skyrme (SkM*) HF calc.

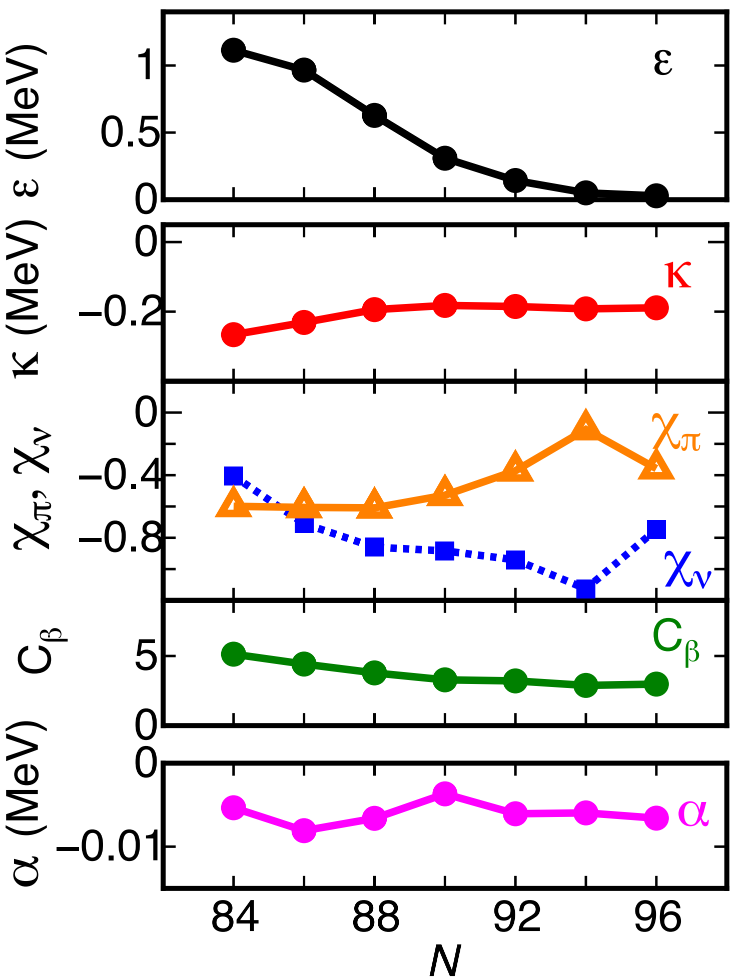
taken from experimental $E(2_1^+)$

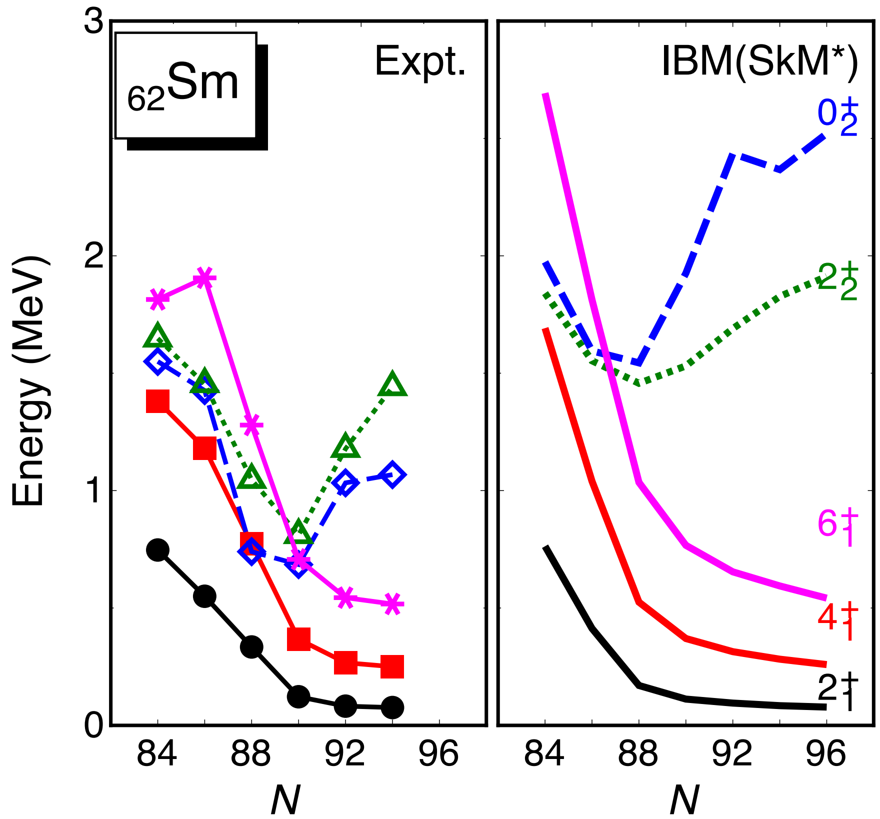
IBM (w/o LL) by the cranking calc.

Coefficient of LL term (α) is determined by adjusting the moment of inertia of boson to that of fermion.

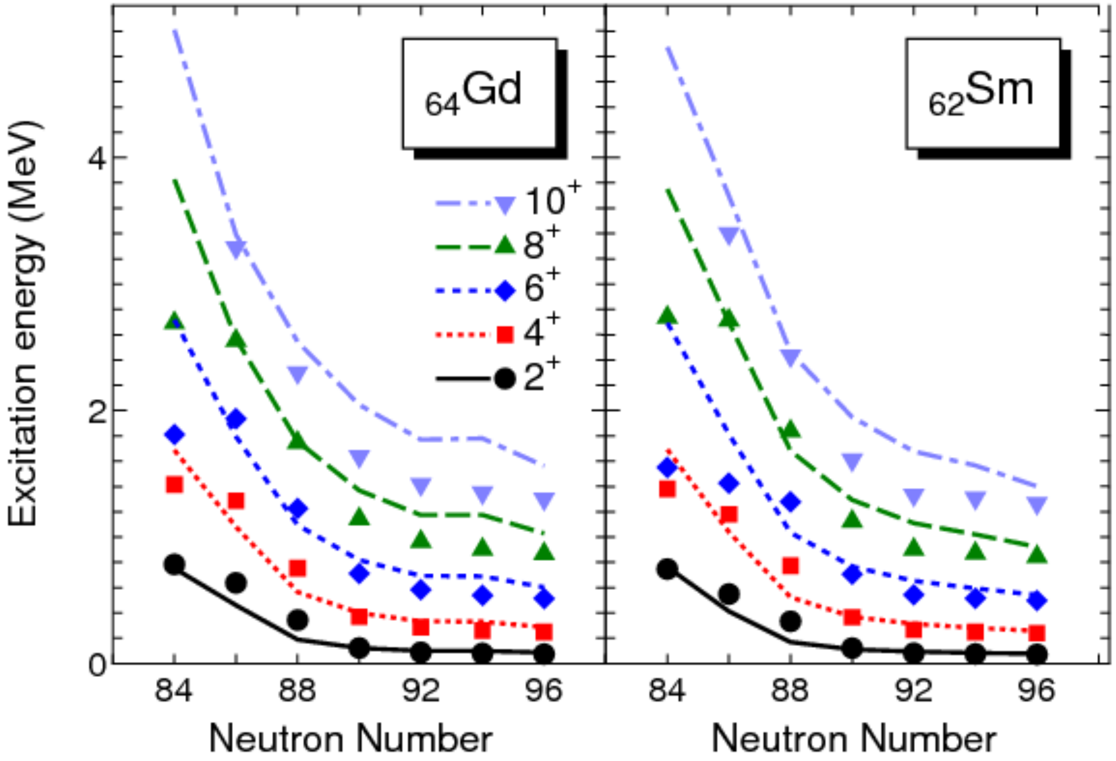
Derived interaction strengths and excitation spectra

Interaction strengths determined microscopically

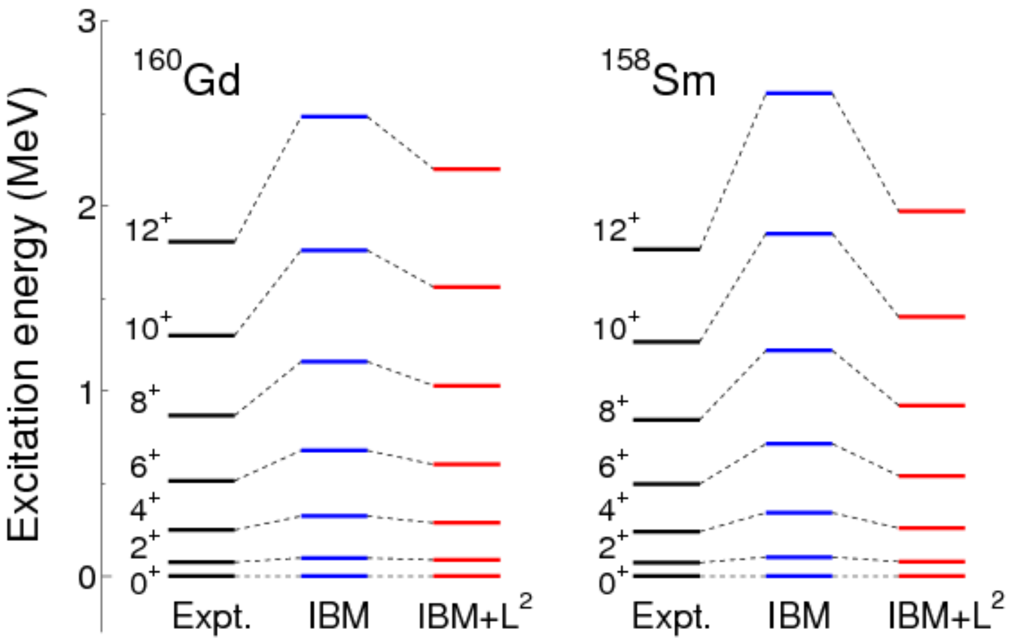




Shape-phase transition occurs between U(5) and SU(3) limits with X(5) critical point



Higher-lying yrast levels



N=96 nuclei

The effect of LL is robust for axially symmetric strong deformation, while it is minor for weakly deformed or γ-soft nuclei.

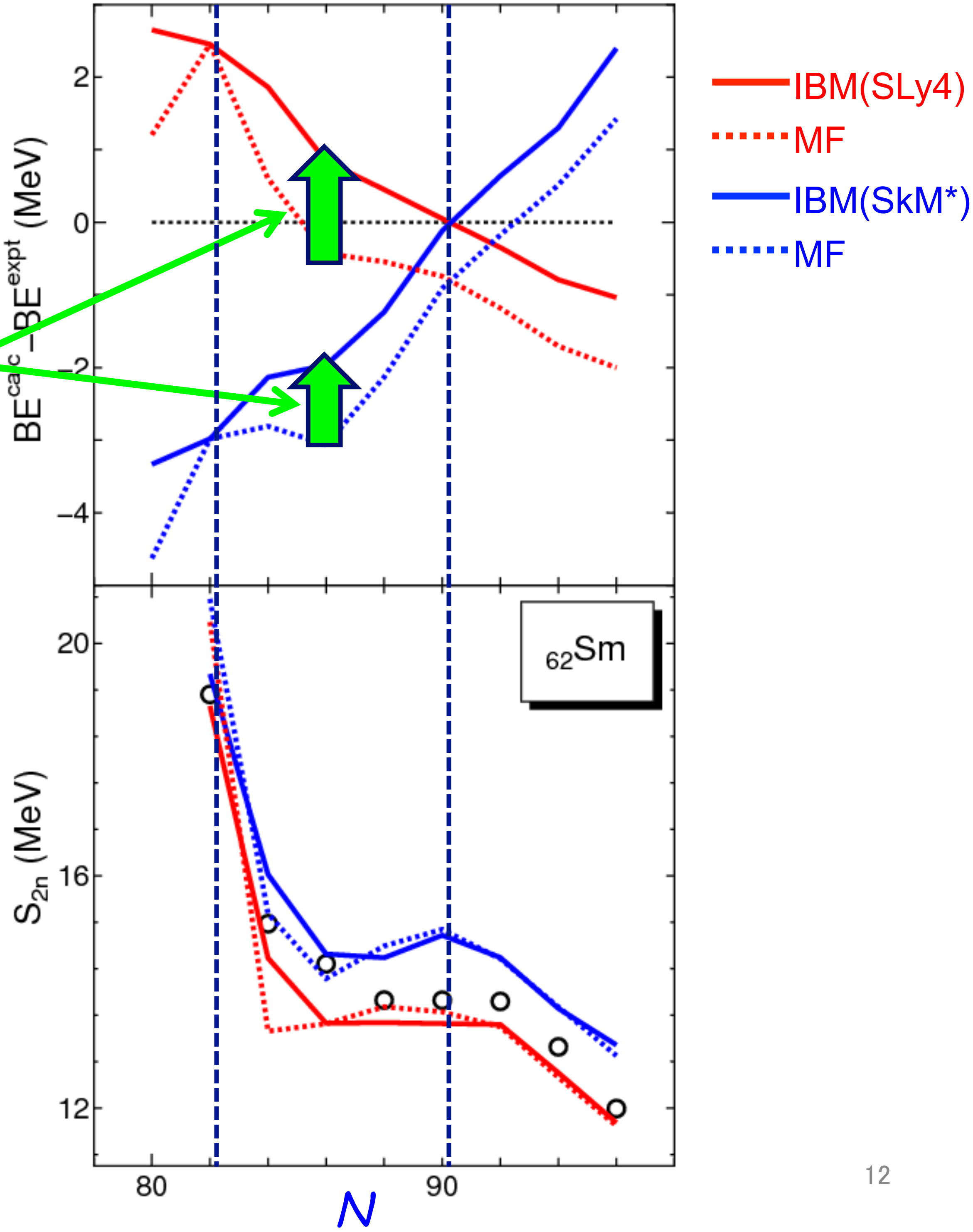
Binding energy

K.N. et al., PRC81, 044307 (2010)

BE^{calc}-BE^{expt}

Correlation effect included by the IBM Hamiltonian

S_{2n}



Similar arguments by
- GCM

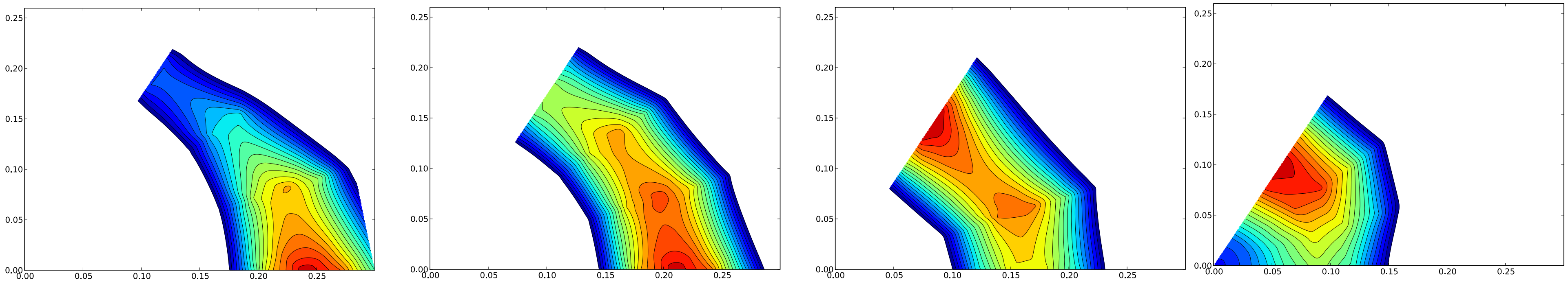
M.Bender et al., PRC73, 034322 (2006)
- IBM phenomenology

R.B.Cakirli et al., PRL102, 082501 (2009)

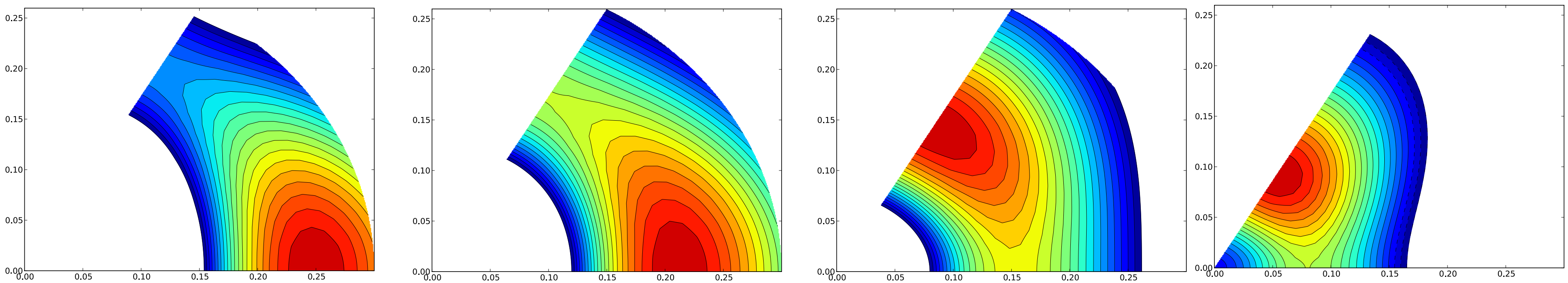
Potential energy surface for W with 82<N<126

HF-SkM*

N=110 | N=114 | N=118 | N=122

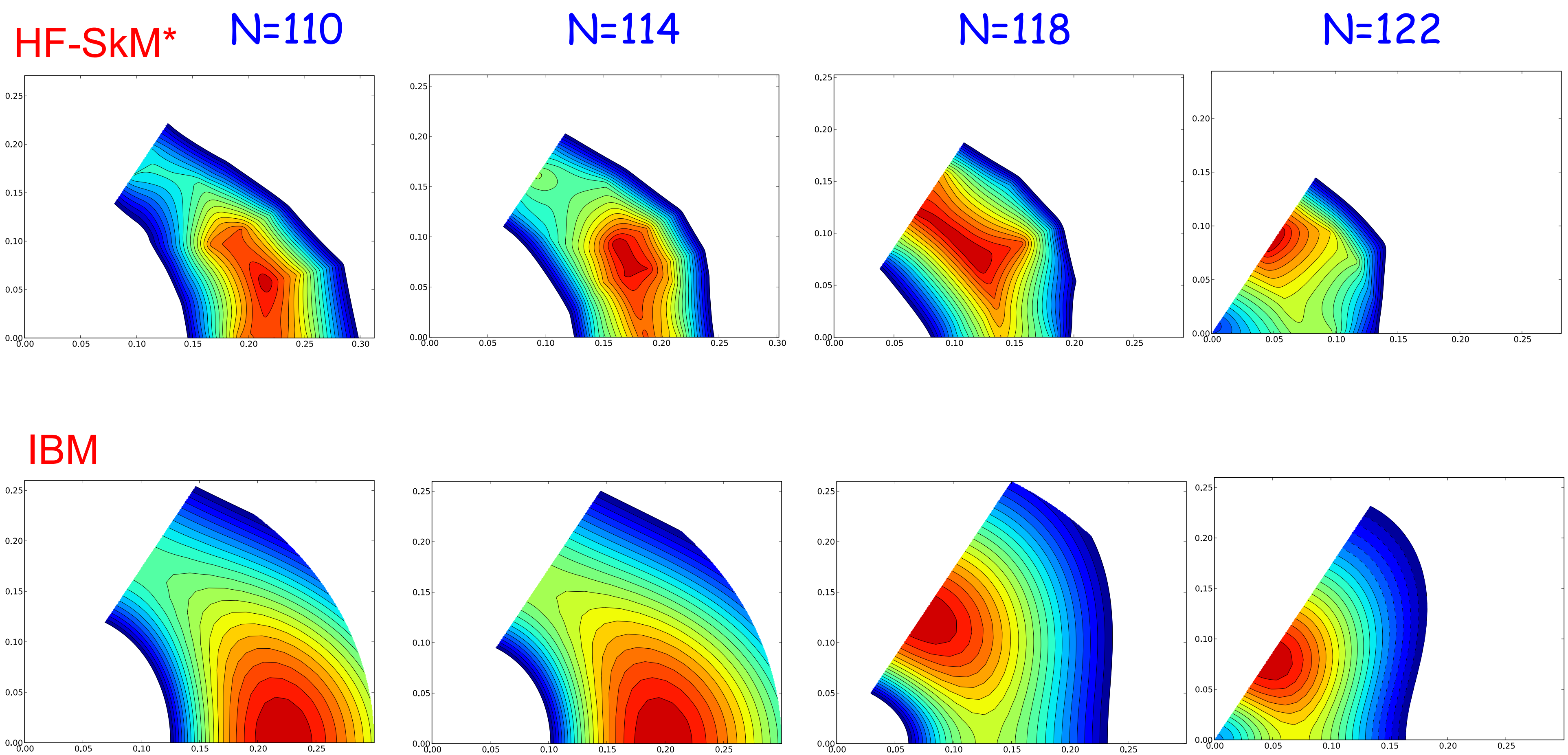


IBM



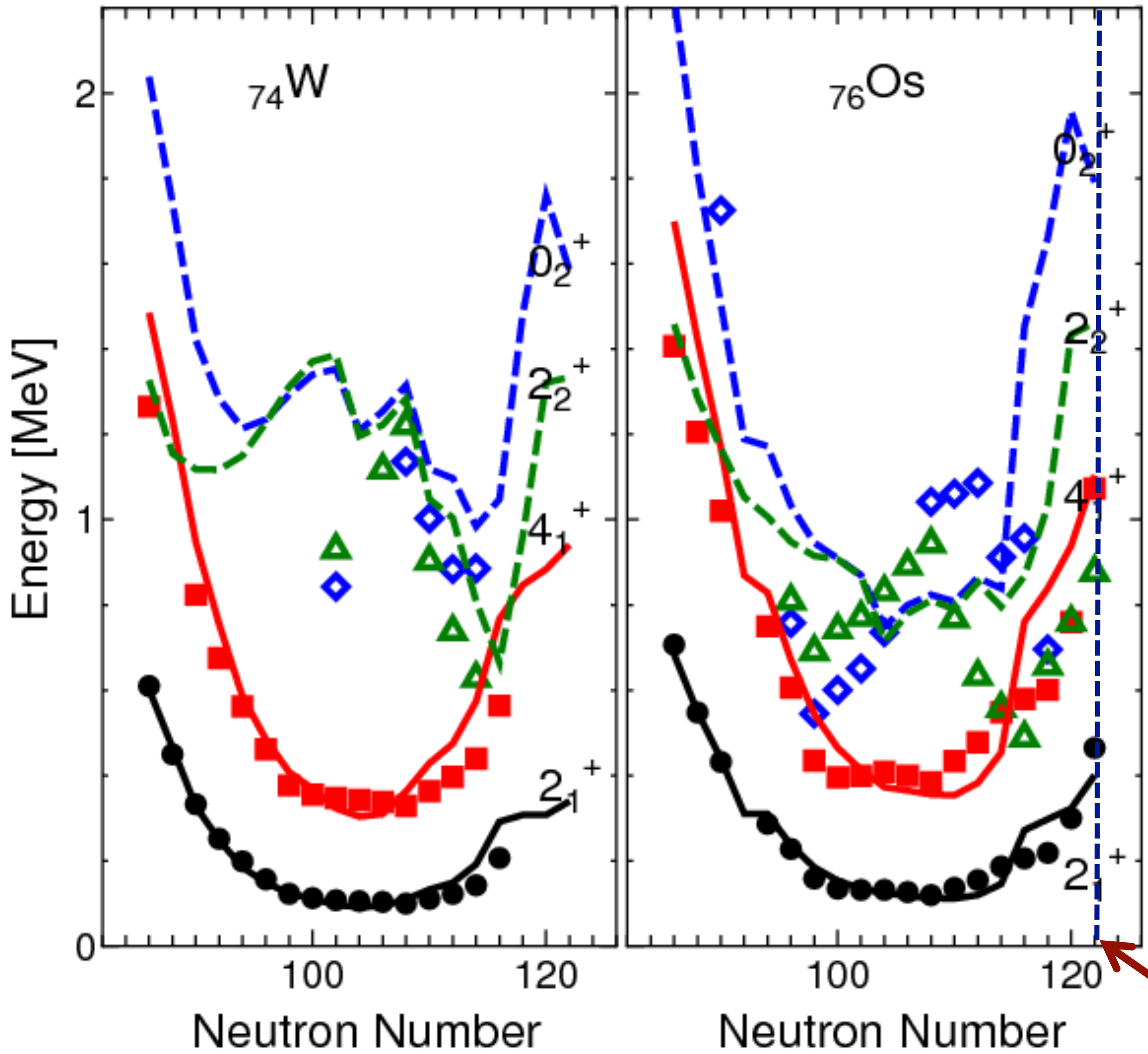
More neutron holes ←

Potential energy surface for Os with 82<N<126



More neutron holes

Evolution of low-lying spectra for 82<N<126



For ^{198}Os, E(2_1^+), E(4^+_1) are consistent with the recent experiment at GSI.

Zs.Podolyak et al., PRC79, 031305(R) (2009)

N=122

Summary and Outlook

- Determination of IBM Hamiltonian by mean-field

  • spectra and wave function with good J& N
  • dynamical and critical-point symmetries
    (prediction for exotic nuclei)
  • quantum fluctuation effect on binding energy

- Some response of the nucleonic system can be formulated, which corresponds to microscopic origin of kinetic *LL* term.

- Use of other interactions, e.g., Gogny (in progress), a more realistic force

References:
• K.N., N. Shimizu, and T. Otsuka, Phys. Rev. Lett. 101, 142501 (2008)
• K.N., N. Shimizu, and T. Otsuka, Phys. Rev. C 81, 044307 (2010)
• K.N., T. Otsuka, N. Shimizu, and L. Guo, in preparation (2010)

For strong deformation, the difference between the overlap of the nucleon wave function and that of the corresponding boson wave function is supposed to become larger.

This leads to the introduction of the rotational mass (kinetic) term in the boson Hamiltonian

We formulate the response of the rotating nucleon system in terms of bosons, in order to determine the coefficient of kinetic LL term of IBM.

Fit by using wavelet transform (WT), which is suitable for analyzing localized signal.

WT of the PES with axial sym.

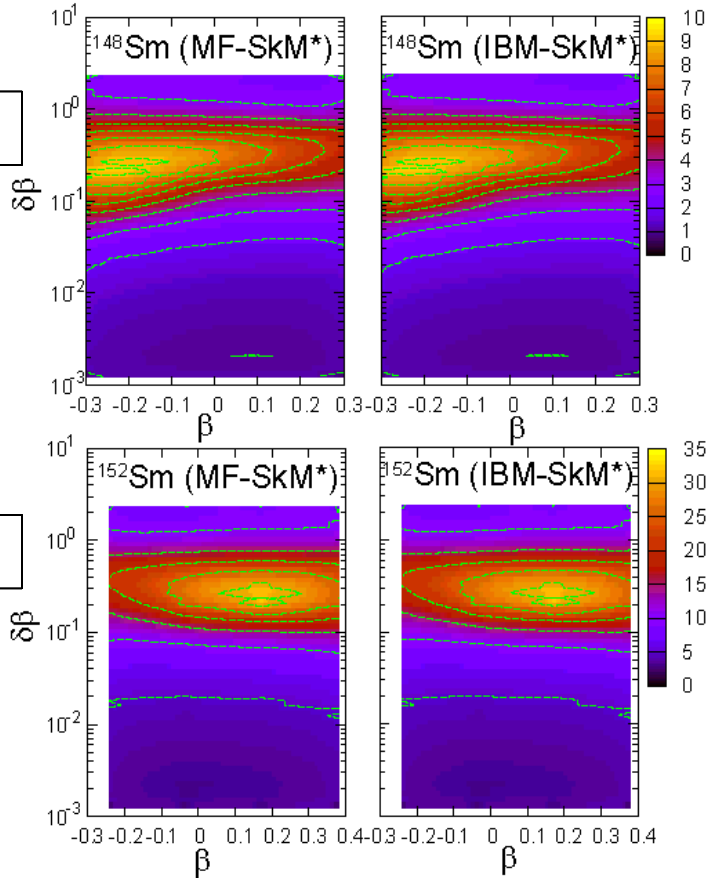
$$\tilde{E}(\delta\beta, \beta) = \frac{1}{\sqrt{\delta\beta}} \int E(\beta')\Phi^{\star}\left(\frac{\beta - \beta'}{\delta\beta}\right) d\beta'$$

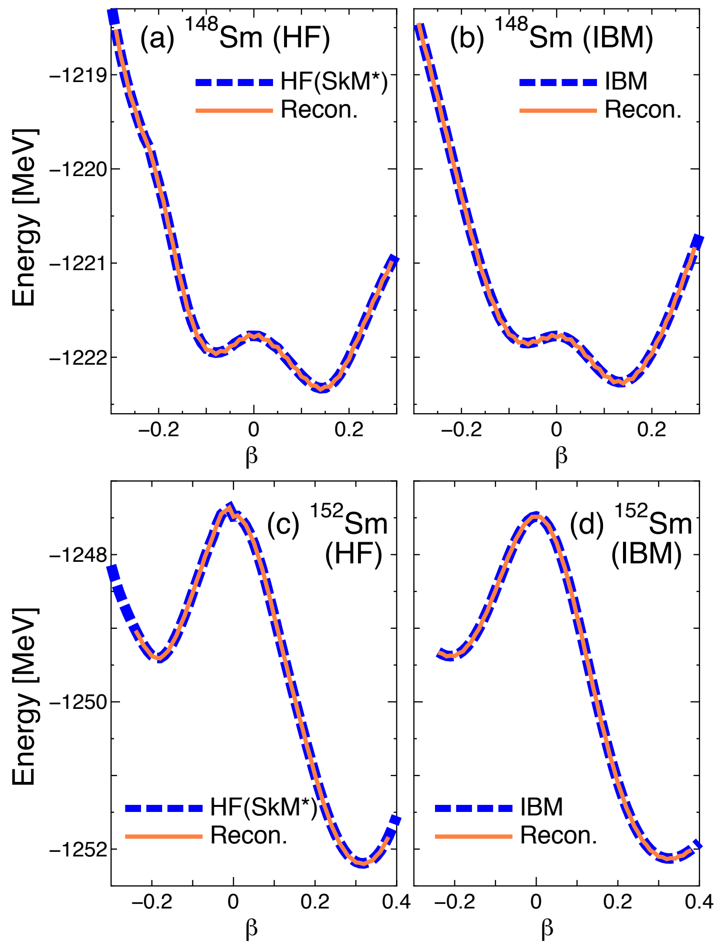
scale (frequency) — position — PES (HF/IBM) — basis (wavelet)

χ^2 fits of $|E|^2$

^{148}Sm

^{152}Sm





other application to physical system, e.g.,

Shevchenko et al, PRC77, 024302 (2008)

Derivation of kinetic LL interaction strength

Cranking calculation

- MOI of IBM (only one parameter α):

$$|\Phi\rangle = \prod_{\rho}(s_\rho^\dagger + \sum_\mu a_{\rho\mu} d_{\rho\mu}^\dagger)^{n_\rho}|0\rangle, \quad a_{\pm 1} \neq 0$$

$$\delta\langle\Phi|H_{\mathrm{IBM}} - \omega L_x|\Phi\rangle = 0 \Longrightarrow \mathcal{I} = \frac{\langle \hat{L}_x\rangle}{\omega}\bigg|_{a_{\pi 1}=a_{\nu 1}\to 0}$$

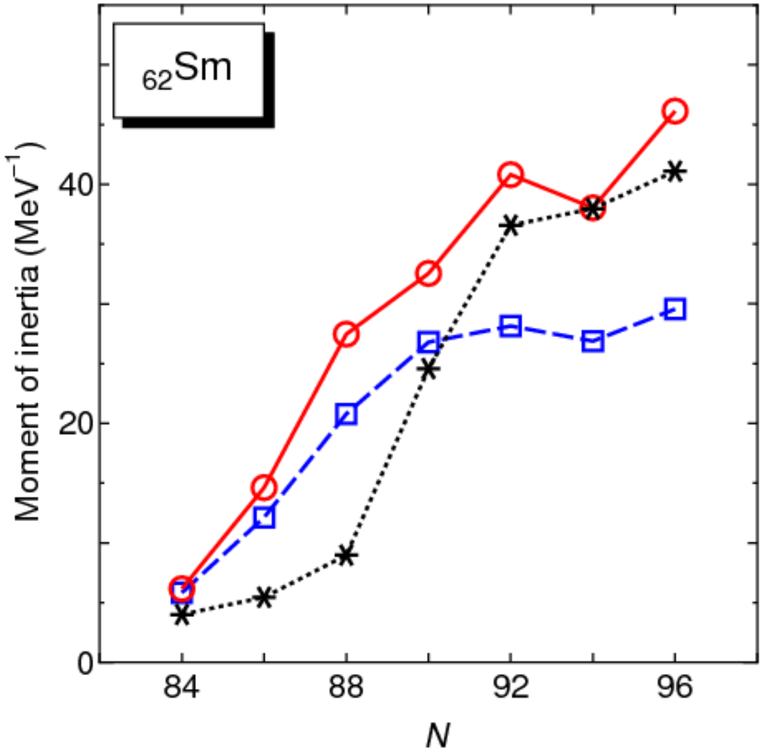
Schaaser and Brink (1984)

- Microscopic input (Inglis-Belyaev MOI):

$$\mathcal{I} \equiv \mathcal{I}_{\mathrm{IB}} = 2 \cdot \sum_{k,k'>0} \frac{|\langle k|\hat{L}_x|k'\rangle|^2}{E_k + E_{k'}}(u_k v_{k'} - u_{k'} v_k)^2$$

Strength of LL term is determined by adjusting the moment of inertia (MOI) of IBM to that of HF.

Moment of inertia



Inglis-Belyaev
Experimental E(2_1^+)
IBM w/o LL