Second EMMI-EFES Workshop on Neutron-Rich Exotic Nuclei

RIKEN, June 16 $\sim$ 18, 2010

Gradient method to solve the Hartree-Fock-Bogoliubov equation in the coordinate space

Ying Zhang

June 18, 2010

Peking University
Jie Meng

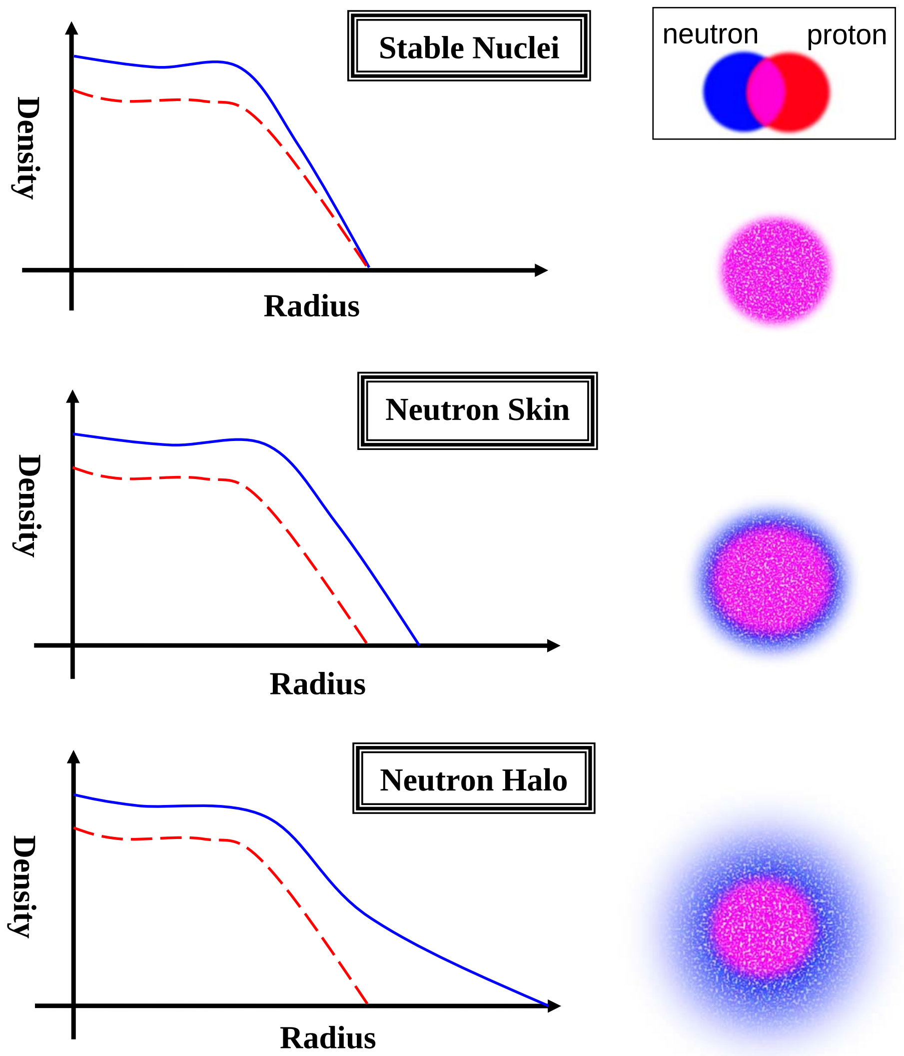
Niigata University
Masayuki Matsuo

Outline

Outline

Introduction — *Frontier in nuclear physics*

★ **Exotic phenomena in nuclear physics**



Tanihata PRL 55(1985)2676,
Hansen ARNPS 45(1995)591,
Jensen RMP 76(2004)215,...

Features:

- ✓ Weakly bound
- ✓ Coupling to the continuum
- ✓ Large spacial density distribution
- ✓ ......

To describe the exotic nuclei

one expects to present the appropriate asymptotic behavior for the w.f. of nucleons in the coordinate space.

Introduction — *Current status of investigation*

★ **Candidate theory for the description of exotic nuclei**

Hartree-Fock-Bogoliubov (HFB) theory: Bogoliubov quasiparticle ⇒ unified description of both the mean field & paring correlation

- Non-relativistic: Skyrme *Dobaczewski NPA(1984)*
- Relativistic: covariant density functional theory
  Vretenar PR(2005), Meng PPNP(2006), Long PRC(2010)

★ **Technique to solve the HFB equation in coordinate space**

✓ Shooting method

✓ Runge-Kutta scheme for coupled channels: *Price PRC (1987)*

✓ Woods-Saxon basis: *Zhou PRC(2003), ISPUN07(2008)*

✓ Green function method: *Oba PRC(2009)*

✓ ...

Introduction — *Current status of investigation*

★ **Candidate theory for the description of exotic nuclei**

Hartree-Fock-Bogoliubov (HFB) theory: Bogoliubov quasiparticle ⇒ unified description of both the mean field & paring correlation

- Non-relativistic: Skyrme *Dobaczewski NPA(1984)*
- Relativistic: covariant density functional theory
  Vretenar PR(2005), Meng PPNP(2006), Long PRC(2010)

★ **Technique to solve the HFB equation in coordinate space**

✓ Shooting method

✓ Runge-Kutta scheme for coupled channels: *Price PRC (1987)*

✓ Woods-Saxon basis: *Zhou PRC(2003), ISPUN07(2008)*

✓ Green function method: *Oba PRC(2009)*

✓ ...

? Gradient step method: *Reinhard NPA(1982)*

- Gradient method: *Mang ZPA(1976)*
- Imaginary time step (ITS) method: *Davies NPA(1980), Gall ZPA(1994)*

Gradient evolution to solve the Hartree-Fock-Bogoliubov equation

★ **Gradient step method**

Starts from an initial state and search for the local minimum on the energy surface

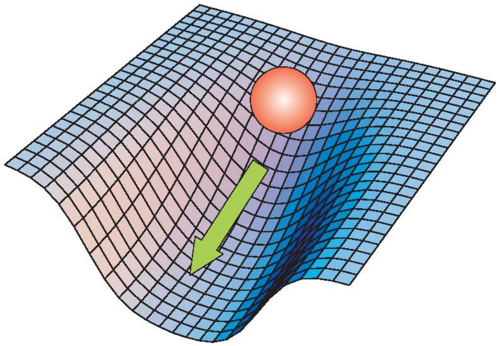
- Imaginary time step (ITS) method
- Gradient method

Gradient evolution to solve the Hartree-Fock-Bogoliubov equation

★ **Gradient step method**

Starts from an initial state and search for the local minimum on the energy surface

- Imaginary time step (ITS) method
- Gradient method



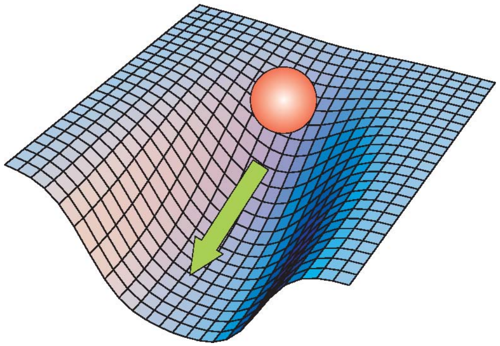
✓ Bound from the bottom

Gradient evolution to solve the Hartree-Fock-Bogoliubov equation

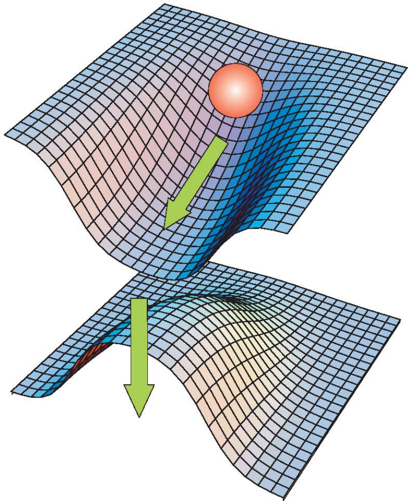
★ **Gradient step method**

Starts from an initial state and search for the local minimum on the energy surface

- Imaginary time step (ITS) method
- Gradient method



✓ Bound from the bottom



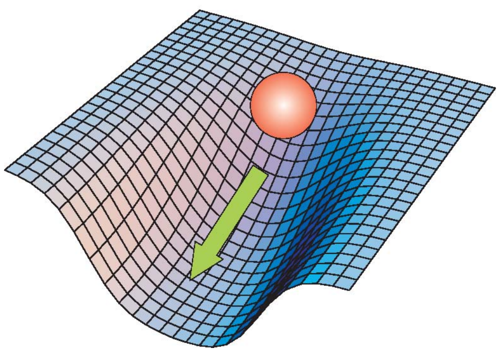
? Not bound from the bottom

Gradient evolution to solve the Hartree-Fock-Bogoliubov equation

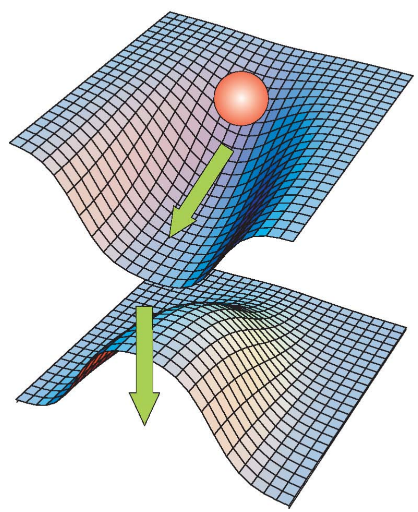
★ **Gradient step method**

Starts from an initial state and search for the local minimum on the energy surface

- Imaginary time step (ITS) method
- Gradient method



✓ Bound from the bottom



? Not bound from the bottom

★ **My adventure**

1. ITS method to solve the Dirac equation
2. ITS method to solve the HFB equation
3. Gradient method to solve the HFB equation

directly in the coordinate space!

Outline

ITS method to solve the Dirac equation

★ **Imaginary time step (ITS) method** *Davies NPA(1980)*

- Evolution of the w. f.

$$|\Phi_j^{(n+1)}\rangle = \left(1 - \eta\hat{h}\right)\left|\phi_j^{(n)}\right\rangle$$

- Dirac equation

$$\begin{pmatrix} V+S & -\frac{d}{dr}+\frac{\kappa_a}{r} \\ +\frac{d}{dr}+\frac{\kappa_a}{r} & V-S-2M \end{pmatrix}\begin{pmatrix} F_a(r) \\ G_a(r) \end{pmatrix} = \varepsilon_a \begin{pmatrix} F_a(r) \\ G_a(r) \end{pmatrix},$$

- Schrödinger-like equation

$$\begin{cases} G_a = \frac{1}{2M_+}\left(\frac{dF_a}{dr}+\frac{\kappa_a}{r}F_a\right), \text{ where } M_+ = M + \frac{S-V+\varepsilon_a}{2}, \\ \hat{h}_F F_a = \varepsilon_a F_a \end{cases}$$

$$\hat{h}_F = -\frac{1}{2M_+}\frac{d^2}{dr^2} + \frac{1}{2M_+^2}\frac{dM_+}{dr}\frac{d}{dr} + \left[(V+S) + \frac{1}{2M_+^2}\frac{dM_+}{dr}\frac{\kappa_a}{r} + \frac{1}{2M_+}\frac{\kappa_a(\kappa_a+1)}{r^2}\right].$$

ITS method to solve the Dirac equation

★ **Imaginary time step (ITS) method** *Davies NPA(1980)*

- Evolution of the w. f.

$$|\Phi_j^{(n+1)}\rangle = \left(1 - \eta\hat{h}\right)\left|\phi_j^{(n)}\right\rangle$$

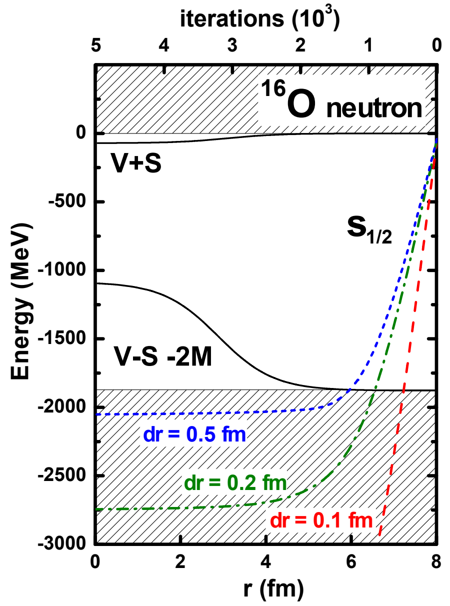
- Dirac equation

$$\begin{pmatrix} V+S & -\frac{d}{dr}+\frac{\kappa_a}{r} \\ +\frac{d}{dr}+\frac{\kappa_a}{r} & V-S-2M \end{pmatrix}\begin{pmatrix} F_a(r) \\ G_a(r)\end{pmatrix} = \varepsilon_a \begin{pmatrix} F_a(r) \\ G_a(r)\end{pmatrix},$$

- Schrödinger-like equation

$$\begin{cases} G_a = \frac{1}{2M_+}\left(\frac{dF_a}{dr}+\frac{\kappa_a}{r}F_a\right), \text{ where } M_+ = M + \frac{S-V+\varepsilon_a}{2}, \\ \hat{h}_{\mathrm{F}}F_a = \varepsilon_a F_a \end{cases}$$

$$\hat{h}_{\mathrm{F}} = -\frac{1}{2M_+}\frac{d^2}{dr^2} + \frac{1}{2M_+^2}\frac{dM_+}{dr}\frac{d}{dr} + \left[(V+S) + \frac{1}{2M_+^2}\frac{dM_+}{dr}\frac{\kappa_a}{r} + \frac{1}{2M_+}\frac{\kappa_a(\kappa_a+1)}{r^2}\right].$$



ITS method to solve the Dirac equation

★ **Imaginary time step (ITS) method** *Davies NPA(1980)*

- Evolution of the w. f.

$$|\Phi_j^{(n+1)}\rangle = \left(1-\eta\hat{h}\right)\left|\phi_j^{(n)}\right\rangle$$

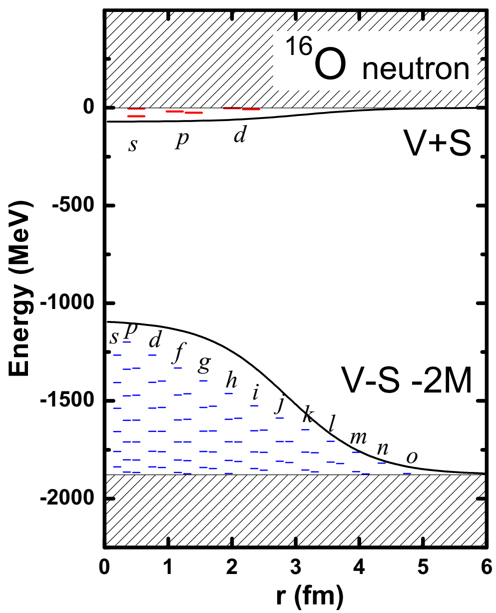
- Dirac equation

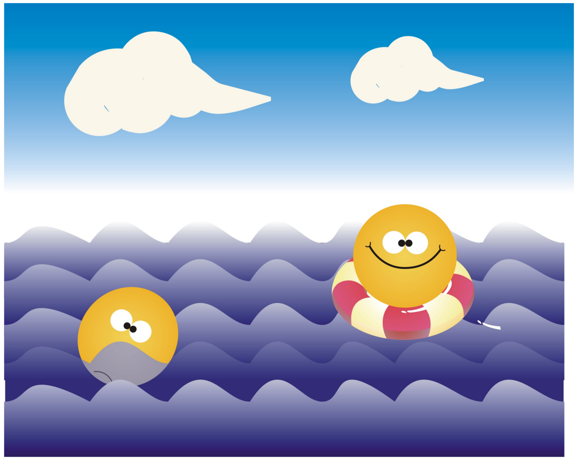
$$\begin{pmatrix} V+S & -\frac{d}{dr}+\frac{\kappa_a}{r} \\ +\frac{d}{dr}+\frac{\kappa_a}{r} & V-S-2M \end{pmatrix}\begin{pmatrix} F_a(r) \\ G_a(r) \end{pmatrix} = \varepsilon_a \begin{pmatrix} F_a(r) \\ G_a(r) \end{pmatrix},$$

- Schrödinger-like equation

$$\begin{cases} G_a = \frac{1}{2M_+}\left(\frac{dF_a}{dr}+\frac{\kappa_a}{r}F_a\right), \text{ where } M_+ = M + \frac{S-V+\varepsilon_a}{2}, \\ \hat{h}_{\mathrm{F}} F_a = \varepsilon_a F_a \end{cases}$$

$$\hat{h}_{\mathrm{F}} = -\frac{1}{2M_+}\frac{d^2}{dr^2}+\frac{1}{2M_+^2}\frac{dM_+}{dr}\frac{d}{dr} + \left[(V+S)+\frac{1}{2M_+^2}\frac{dM_+}{dr}\frac{\kappa_a}{r}+\frac{1}{2M_+}\frac{\kappa_a(\kappa_a+1)}{r^2}\right].$$





Y. Zhang, et al., IJMPE 19(2010)55

ITS method to solve the Hartree-Fock-Bogoliubov equation

★ **Hartree-Fock-Bogoliubov equation**

$$\begin{pmatrix} h-\lambda & \Delta \\ -\Delta^* & -h^*+\lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = \begin{pmatrix} U_k \\ V_k \end{pmatrix} E_k \tag{1}$$

★ **ITS evolution for HFB equation**

$$\begin{pmatrix} U' \\ V' \end{pmatrix} = \left\{ 1 - \eta \begin{pmatrix} h-\lambda & \Delta \\ -\Delta^* & -h^*+\lambda \end{pmatrix} \right\} \begin{pmatrix} U \\ V \end{pmatrix}, \tag{2}$$

ITS method to solve the Hartree-Fock-Bogoliubov equation

★ Hartree-Fock-Bogoliubov equation

$$\sum_{\sigma'}\int d^3r' \begin{pmatrix} h(\mathbf{r},\sigma;\mathbf{r}',\sigma')-\lambda & \Delta(\mathbf{r},\sigma;\mathbf{r}',\sigma') \\ \Delta(\mathbf{r},\sigma;\mathbf{r}',\sigma') & -h(\mathbf{r},\sigma;\mathbf{r}',\sigma')+\lambda \end{pmatrix} \begin{pmatrix} \psi_U^k(\mathbf{r}',\sigma') \\ \psi_V^k(\mathbf{r}',\sigma') \end{pmatrix} = \begin{pmatrix} \psi_U^k(\mathbf{r},\sigma) \\ \psi_V^k(\mathbf{r},\sigma) \end{pmatrix} E_k$$

★ ITS evolution for HFB equation

$$\begin{pmatrix} \psi_U^{k'}(\mathbf{r},\sigma) \\ \psi_V^{k'}(\mathbf{r},\sigma) \end{pmatrix} = \begin{pmatrix} \psi_U^k(\mathbf{r},\sigma) \\ \psi_V^k(\mathbf{r},\sigma) \end{pmatrix} - \eta\sum_{\sigma'}\left\{\int d^3r' \begin{pmatrix} h(\mathbf{r},\sigma;\mathbf{r}',\sigma')-\lambda & \Delta(\mathbf{r},\sigma;\mathbf{r}',\sigma') \\ \Delta(\mathbf{r},\sigma;\mathbf{r}',\sigma') & -h(\mathbf{r},\sigma;\mathbf{r}',\sigma')+\lambda \end{pmatrix} \begin{pmatrix} \psi_U^k(\mathbf{r}',\sigma') \\ \psi_V^k(\mathbf{r}',\sigma') \end{pmatrix}\right\}$$

★ Simple test

- single-particle hamiltonian,

$$\hat{h}(\mathbf{r},\sigma;\mathbf{r}',\sigma') = \left[\frac{\hat{p}^2}{2M}+V\right]\delta(\mathbf{r}-\mathbf{r}')\delta_{\sigma\sigma'}$$

- the pairing potential,

$$\Delta(\mathbf{r}\sigma,\mathbf{r}'\sigma') = \sum_{ljm} Y_{ljm}(\hat{\mathbf{r}}\sigma)\frac{\Delta_{lj}(r,r')}{rr'}Y^*_{ljm}(\hat{\mathbf{r}}'\sigma')$$

- single-quasiparticle wave function

$$\psi^i_{U(V)}(\mathbf{r}\sigma) = \frac{\varphi^i_{U(V)}(r)}{r}Y_{ljm}(\hat{\mathbf{r}}\sigma), \quad i=(nljm)$$

ITS method to solve the Hartree-Fock-Bogoliubov equation

★ Details for evolution

- $V(r)$: Harmonic oscillator potential for ^{12}C neutron

$$V(r) \;=\; \frac{1}{2}m\omega^2 r^2 = \frac{1}{2}\frac{mc^2}{\hbar^2 c^2}(\hbar\omega)^2 r^2, \quad \text{where} \quad \hbar\omega = \frac{41}{A^{1/3}} \;\text{MeV}, \tag{1}$$

- Pairing potential:

$$\Delta(r, r') = -V_p \, e^{-\frac{(r-R_0)^2}{a^2}} \, \delta(r - r'), \tag{2}$$

  - $V_p = 0$ MeV
  - $R_0 = r_0 A^{1/3}$, with $r_0 = 1.04$ fm
  - $a = 0.65$ fm

- Fermi level is fixed: $\lambda = -10$ MeV
- Step parameter: $\eta = \Delta t/\hbar, \;\; \Delta t = 10^{-26}$ s
- Box: $R = 20$ fm, $dr = 0.1$ fm
- Initial wave functions: $\psi_V \to$ spherical Bessel function, $\psi_U \to 0$

ITS method to solve the Hartree-Fock-Bogoliubov equation

★ **ITS evolution for $\Delta = 0$**

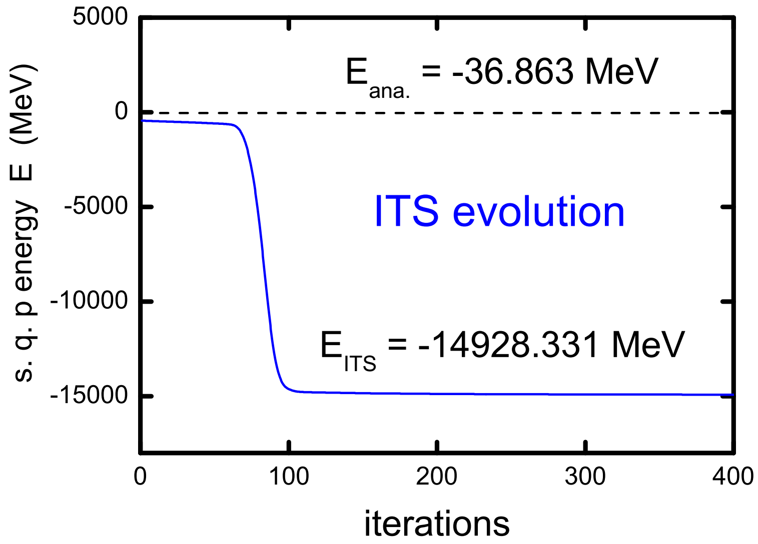


Figure: ITS evolution of the energy for the first $s_{1/2}$ quasi-particle state

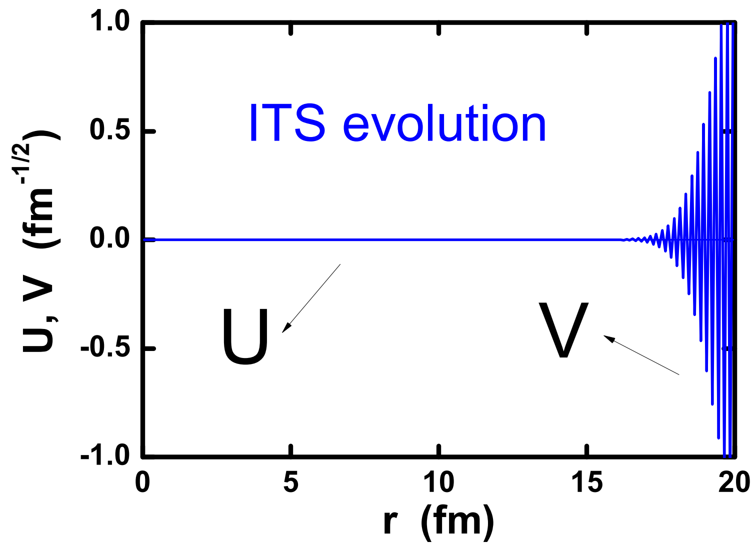


Figure: ITS evolution result of the wave function for the first $s_{1/2}$ quasi-particle state

Gradient method to solve the Hartree-Fock-Bogoliubov equation

★ Gradient method

The energy functional expressed in quasi-particle representation

$$E(Z) = \frac{\langle\Phi'|H|\Phi'\rangle}{\langle\Phi'|\Phi'\rangle} = H^0 + \begin{pmatrix} H^{20*} & H^{20} \end{pmatrix} \begin{pmatrix} Z \\ Z^* \end{pmatrix} + \frac{1}{2} \begin{pmatrix} Z^* & Z \end{pmatrix} \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} Z \\ Z^* \end{pmatrix},$$

the first order derivative of $E(Z)$ is

$$\left.\frac{\partial E(Z)}{\partial Z^*_{\mu\nu}}\right|_{Z=0} = H^{20}_{\mu\nu} \tag{3}$$

Therefore, the energy difference between $|\Phi\rangle$ and $|\Phi'\rangle$ can be expanded as *Mang NPA(1976)*

$$\Delta E = \sum_{\mu\nu} H^{20}_{\mu\nu} Z_{\mu\nu} + O(Z^2_{\mu\nu}) \tag{4}$$

If $Z_{\mu\nu}$ is chosen to be the direction of the steepest energy descent as,

$$Z_{\mu\nu} = -\eta \left.\frac{\partial E}{\partial Z^*_{\mu\nu}}\right|_{Z=0} = -\eta H^{20}_{\mu\nu}, \text{ where } \eta > 0 \tag{5}$$

the energy difference can be written as

$$\Delta E = -\eta (H^{20})^2. \tag{6}$$

The total energy will decrease during this evolution until it finds the state with the lowest energy.

Gradient method to solve the Hartree-Fock-Bogoliubov equation

★ Gradient evolution for HFB equation

$$\begin{cases} U' &= U + V^* Z^*, \\ V' &= V + U^* Z^* \end{cases} \tag{7}$$

where

$$Z = -\eta(H^{20} - \lambda N^{20}). \tag{8}$$

with

$$H^{20} = U^\dagger h V^* - V^\dagger h^* U^* + U^\dagger \Delta U^* - V^\dagger \Delta^* V^*, \quad N^{20} = U^\dagger V^* - V^\dagger U^* \tag{9}$$

Since one should have the relation between U and V as

$$U^\dagger U + V^\dagger V = 1, \quad UU^\dagger + V^* V^T = 1, \tag{10}$$

$$U^T V + V^T U = 0, \quad UV^\dagger + V^* U^T = 0. \tag{11}$$

and the HFB equation they should satisfy, one could get the evolution for U and V can be expressed as

$$\begin{pmatrix} U' \\ V' \end{pmatrix} = \left\{ 1 - \eta \begin{pmatrix} -h + \lambda + E & -\Delta \\ \Delta^* & h^* - \lambda + E \end{pmatrix} \right\} \begin{pmatrix} U \\ V \end{pmatrix} \tag{12}$$

Gradient method to solve the Hartree-Fock-Bogoliubov equation

★ Gradient evolution for HFB equation

$$\begin{cases} U' = U + V^* Z^*, \\ V' = V + U^* Z^* \end{cases} \tag{7}$$

where

$$Z = -\eta(H^{20} - \lambda N^{20}). \tag{8}$$

with

$$H^{20} = U^\dagger h V^* - V^\dagger h^* U^* + U^\dagger \Delta U^* - V^\dagger \Delta^* V^*, \quad N^{20} = U^\dagger V^* - V^\dagger U^* \tag{9}$$

Since one should have the relation between U and V as

$$U^\dagger U + V^\dagger V = 1, \quad UU^\dagger + V^* V^T = 1, \tag{10}$$

$$U^T V + V^T U = 0, \quad UV^\dagger + V^* U^T = 0. \tag{11}$$

and the HFB equation they should satisfy, one could get the evolution for U and V can be expressed as

$$\begin{pmatrix} \psi_U^{k'}(\mathbf{r},\sigma) \\ \psi_V^{k'}(\mathbf{r},\sigma) \end{pmatrix} = \begin{pmatrix} \psi_U^{k}(\mathbf{r},\sigma) \\ \psi_V^{k}(\mathbf{r},\sigma) \end{pmatrix} - \eta \sum_{\sigma'} \left\{ \int d^3r' \begin{pmatrix} -h(\mathbf{r},\sigma;\mathbf{r}',\sigma') + \lambda + E_k & -\Delta(\mathbf{r},\sigma;\mathbf{r}',\sigma') \\ -\Delta(\mathbf{r},\sigma;\mathbf{r}',\sigma') & h(\mathbf{r},\sigma;\mathbf{r}',\sigma') - \lambda + E_k \end{pmatrix} \begin{pmatrix} \psi_U^{k}(\mathbf{r}',\sigma') \\ \psi_V^{k}(\mathbf{r}',\sigma') \end{pmatrix} \right\}$$

Gradient method to solve the Hartree-Fock-Bogoliubov equation

★ **Comparison between the Gradient and ITS evolution**

$$\begin{pmatrix} h-\lambda & \Delta \\ -\Delta^* & -h^*+\lambda \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} U \\ V \end{pmatrix} E, \tag{12}$$

Gradient evolution

$$\begin{pmatrix} U' \\ V' \end{pmatrix} = \left\{ 1 - \eta \begin{pmatrix} -h+\lambda+E & -\Delta \\ \Delta^* & h^*-\lambda+E \end{pmatrix} \right\} \begin{pmatrix} U \\ V \end{pmatrix} \tag{13}$$

ITS evolution

$$\begin{pmatrix} U' \\ V' \end{pmatrix} = \left\{ 1 - \eta \begin{pmatrix} h-\lambda & \Delta \\ -\Delta^* & -h^*+\lambda \end{pmatrix} \right\} \begin{pmatrix} U \\ V \end{pmatrix}, \tag{14}$$

Gradient method to solve the Hartree-Fock-Bogoliubov equation

★ Details for evolution

- $V(r)$: Harmonic oscillator potential for ^{12}C neutron

$$V(r) = \frac{1}{2}m\omega^2 r^2 = \frac{1}{2}\frac{mc^2}{\hbar^2 c^2}(\hbar\omega)^2 r^2, \quad \text{where} \quad \hbar\omega = \frac{41}{A^{1/3}} \text{ MeV}, \tag{15}$$

- Pairing potential:

$$\Delta(r, r') = -V_p\, e^{-\frac{(r-R_0)^2}{a^2}}\, \delta(r - r'), \tag{16}$$

  - $V_p = 0$ MeV
  - $R_0 = r_0 A^{1/3}$, with $r_0 = 1.04$ fm
  - $a = 0.65$ fm
- Fermi level is fixed: $\lambda = -10$ MeV
- Step parameter: $\eta = \Delta t/\hbar, \quad \Delta t = 10^{-26}$ s
- Box: $R = 20$ fm, $dr = 0.1$ fm
- Initial wave functions: $\psi_V \to$ spherical Bessel function, $\psi_U \to 0$

Gradient evolution to solve the Hartree-Fock-Bogoliubov equation

★ **Gradient evolution for $\Delta = 0$**

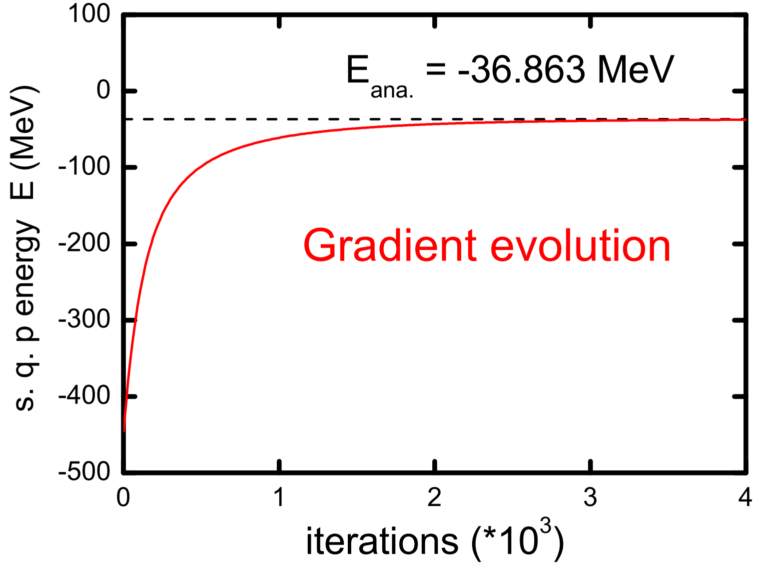


Figure: Gradient evolution of the energy for the first $s_{1/2}$ quasi-particle state

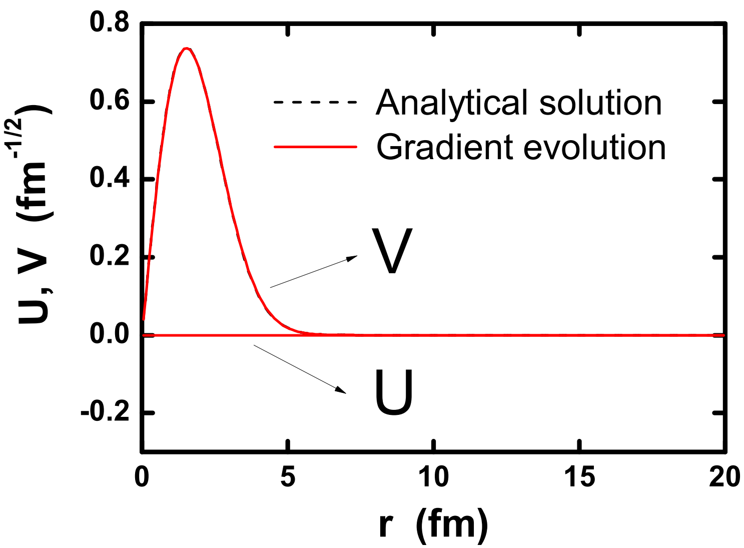


Figure: Gradient evolution result of the wave function for the first $s_{1/2}$ quasi-particle state

Gradient evolution to solve the Hartree-Fock-Bogoliubov equation

★ **Gradient evolution for $\Delta \neq 0$ ($V_p = 10$ MeV)**

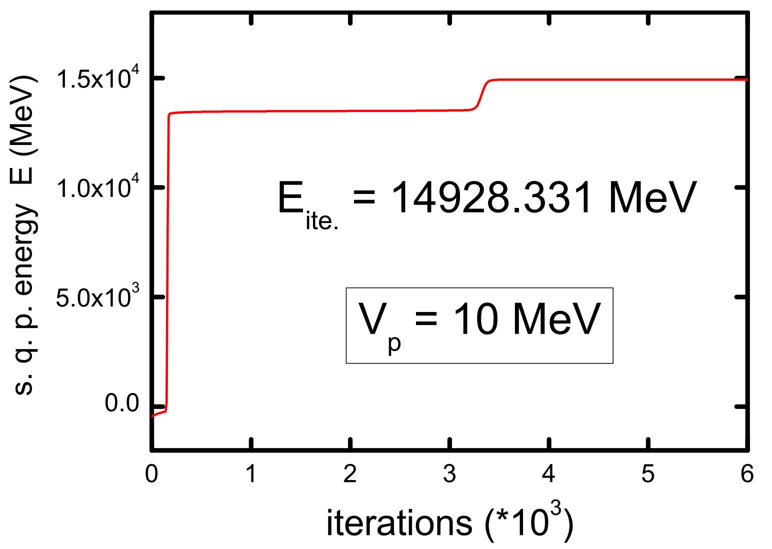


Figure: Gradient evolution of the energy for the first $s_{1/2}$ quasi-particle state

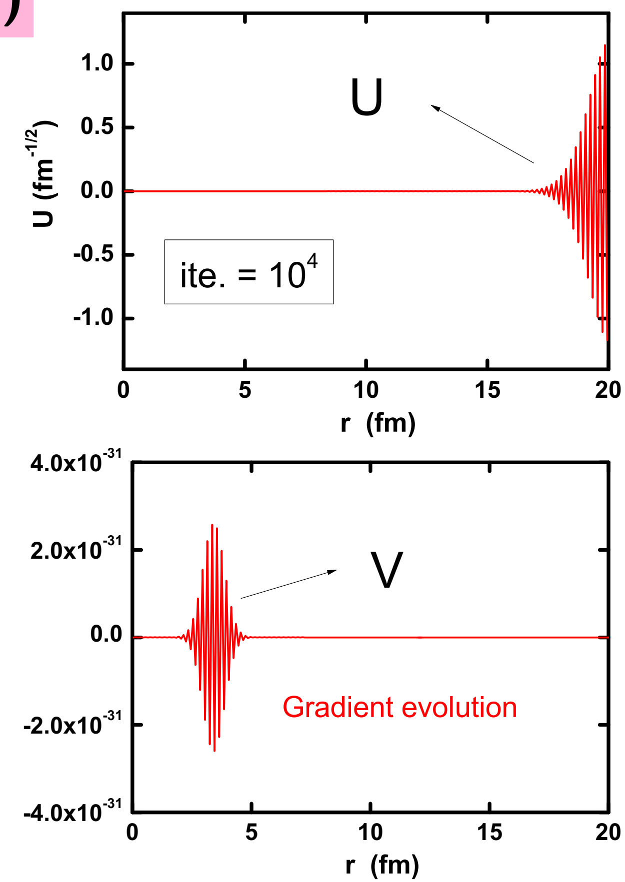


Figure: Gradient evolution result of the wave function for the first $s_{1/2}$ quasi-particle state

Gradient method to solve the Hartree-Fock-Bogoliubov equation

★ **Gradient evolution with constraint of N**

The Gradient method is extremely useful in cases where we must fulfill a subsidiary condition – for instance, the particle number condition $\langle \hat{N} \rangle = N$. Starting from $|\Phi_0\rangle$ with arbitrary particle number N_0, we do not proceed in the direction of the gradient H^{20} alone, but we admix the gradient of the particle number N^{20}

$$Z = -\eta(H^{20} - \lambda N^{20}). \tag{17}$$

The parameter λ is determined in such a way that $\Phi(Z_1)$ has the right particle number N up to linear order in Z. This gives

$$N - N_0 = \sum_{k<k'} Z^*_{kk'} N^{20}_{kk'} + c.c. = Z \cdot N^{20} = -\eta(H^{20} - \lambda N^{20}) \cdot N^{20}. \tag{18}$$

Therefore, one could have

$$\lambda = \frac{H^{20} \cdot N^{20}}{N^{20} \cdot N^{20}} + \frac{N - N_0}{\eta N^{20} \cdot N^{20}}, \tag{19}$$

where $Z \cdot N$ is the scalar product of the vectors (Z, Z^*) and (N, N^*).

Gradient evolution to solve the Hartree-Fock-Bogoliubov equation

★ **Gradient evolution for $\Delta \neq 0$ ($V_p = 10$ MeV)**

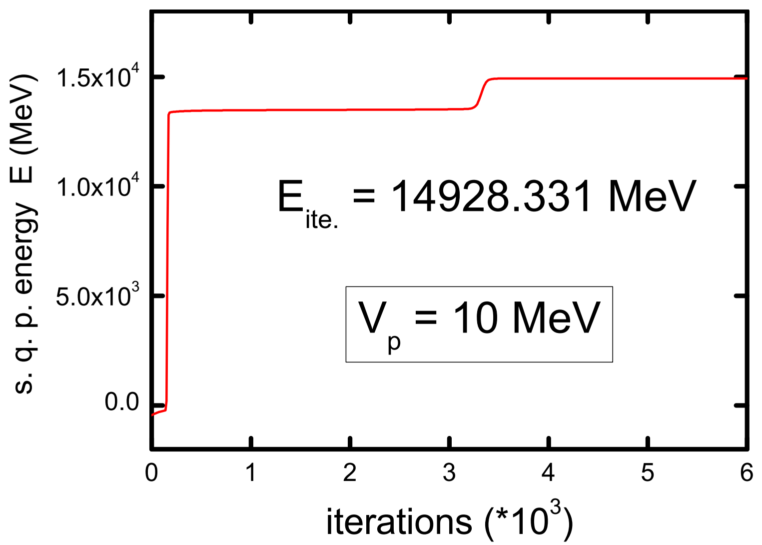


Figure: Gradient evolution of the energy for the first $s_{1/2}$ quasi-particle state

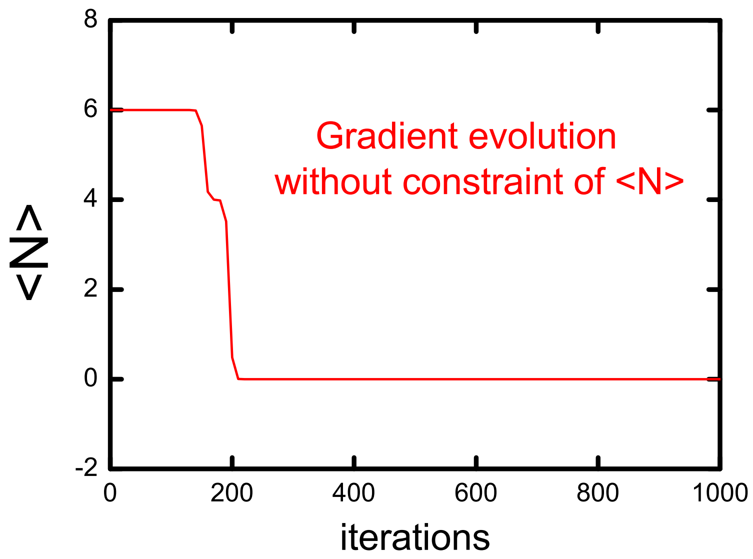


Figure: Gradient evolution of the particle number expectation

Gradient evolution to solve the Hartree-Fock-Bogoliubov equation

★ **Gradient evolution for $\Delta \neq 0$ ($V_p = 10$ MeV)**

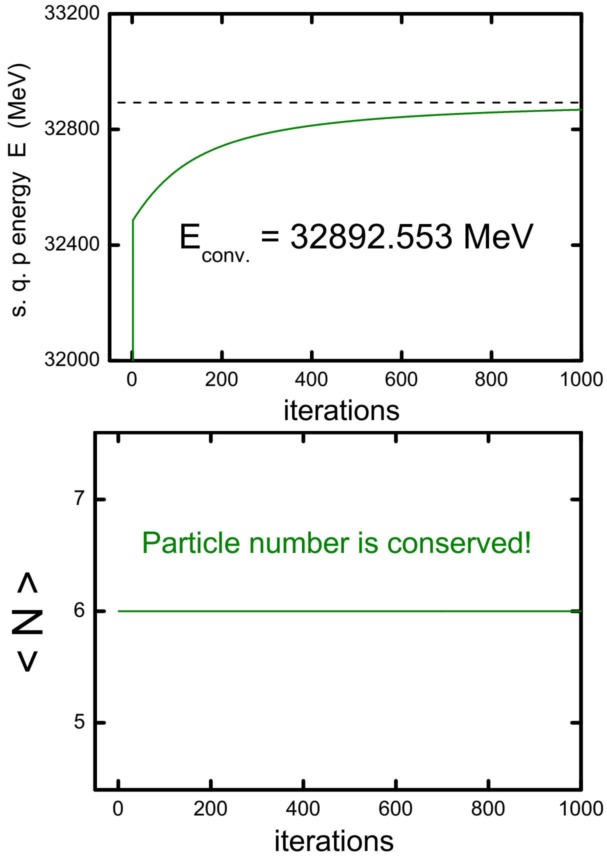


Figure: Gradient evolution of the energy and the Fermi level for the first $s_{1/2}$ quasi-particle state

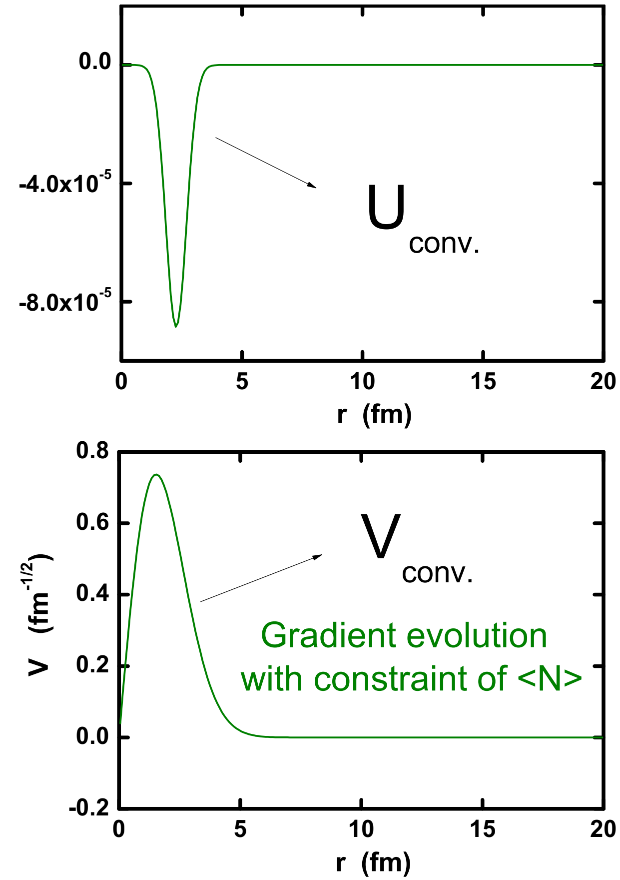


Figure: Gradient evolution result of the wave functions for the first $s_{1/2}$ quasi-particle state at convergence.

Gradient evolution to solve the Hartree-Fock-Bogoliubov equation

★ **Gradient evolution for $\Delta \neq 0$ ($V_p = 10$ MeV)**

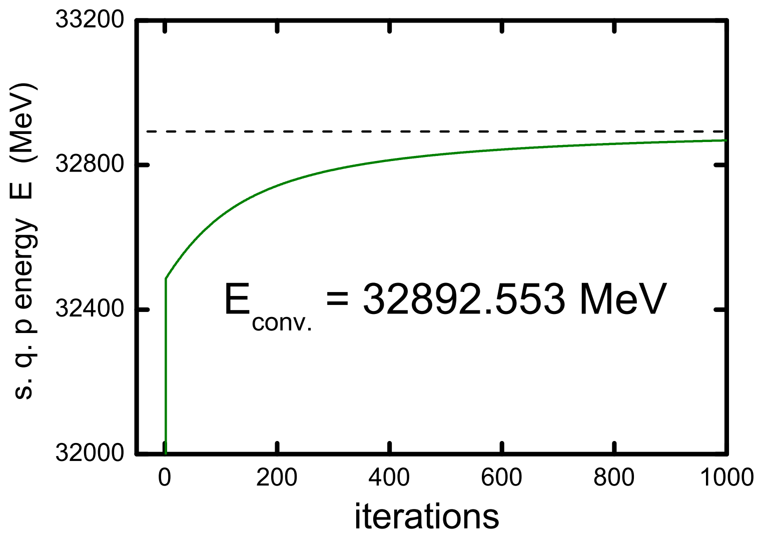


Figure: Gradient evolution of the energy for the first $s_{1/2}$ quasi-particle state

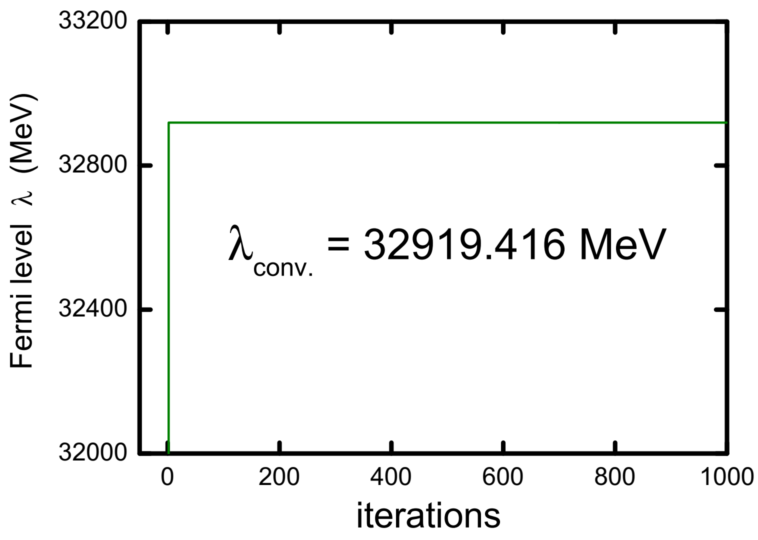


Figure: Gradient evolution of the Fermi level for the first $s_{1/2}$ quasi-particle state

Outline

1. Introduction
2. My adventure
   - ITS method to solve the Dirac equation
   - ITS method to solve the Hartree-Fock-Bogoliubov equation
   - Gradient method to solve the HFB equation
3. Summary and Perspectives

Summary and Perspectives

- *Recipe in coordinate space for the description of exotic nuclei*
  ⟶ Gradient step method
- *Dirac equation*
  ⟶ Disaster for the direct ITS evolution for Dirac equation
  ⟶ ITS method for the Schrödinger-like equation
- *HFB equation*
  ⟶ ITS method failed
  ⟶ Gradient method should be promising, but still some problems left...
- *To be continued...*

Summary and Perspectives

- *Recipe in coordinate space for the description of exotic nuclei*
  $\longrightarrow$ Gradient step method
- *Dirac equation*
  $\longrightarrow$ Disaster for the direct ITS evolution for Dirac equation
  $\longrightarrow$ ITS method for the Schrödinger-like equation
- *HFB equation*
  $\longrightarrow$ ITS method failed
  $\longrightarrow$ Gradient method should be promising, but still some problems left...
- *To be continued...*

Thanks: Jie Meng, Masayuki Matsuo, Peter Ring, Hiroyuki Sagawa, ...

Summary and Perspectives

- *Recipe in coordinate space for the description of exotic nuclei*
  $\longrightarrow$ Gradient step method
- *Dirac equation*
  $\longrightarrow$ Disaster for the direct ITS evolution for Dirac equation
  $\longrightarrow$ ITS method for the Schrödinger-like equation
- *HFB equation*
  $\longrightarrow$ ITS method failed
  $\longrightarrow$ Gradient method should be promising, but still some problems left...
- *To be continued...*

Thanks: Jie Meng, Masayuki Matsuo, Peter Ring, Hiroyuki Sagawa, ...