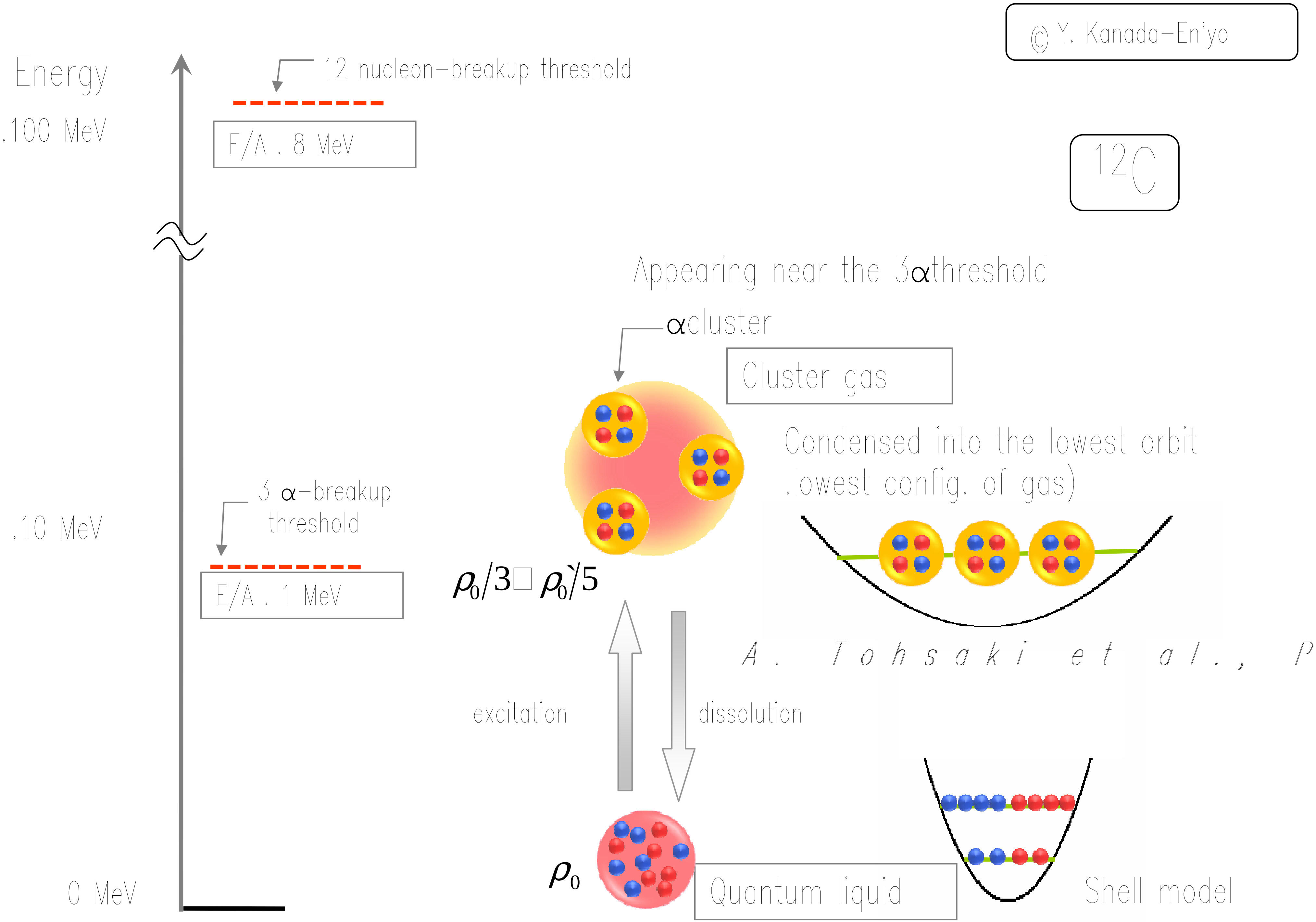
Alpha particle condensation in light nuclei

Yasuro Funaki

University of Tsukuba

EENEN10, RIKEN, , June 16-18 ,

Where is a "gas phase" in finite nuclei?



First example of α condensate state in finite nuclei

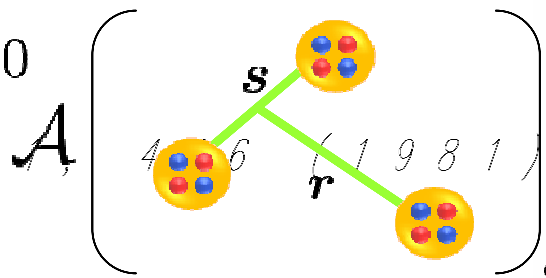
RGM (Full 3α) vs 3αcond. (3αconfined in 0 S orbit)

The Solution of 3αRGM eq. of motion,

$$\langle \phi^3(\alpha) | H - E | \mathcal{A}[\chi(\boldsymbol{s},\boldsymbol{r})\phi^3(\alpha)] \rangle = 0$$

M. Kamimura, NPA 351, 456 (1981).

RGM w.f.



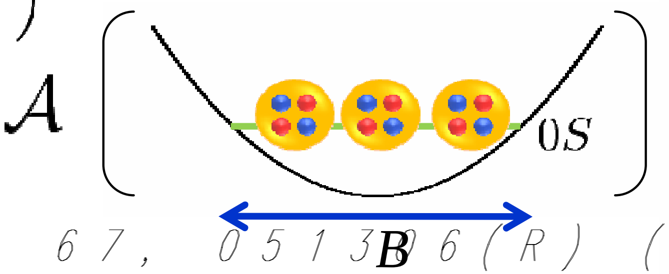
is almost equivalent to the 3αcond. w.f.

$$\chi = \prod_{i=1}^{3} \exp\left(-\frac{2}{B^2}(\boldsymbol{X}_i - \boldsymbol{X}_G)^2\right)$$

X_i : com coordinate of the i-th α

X_G : total com coordinate

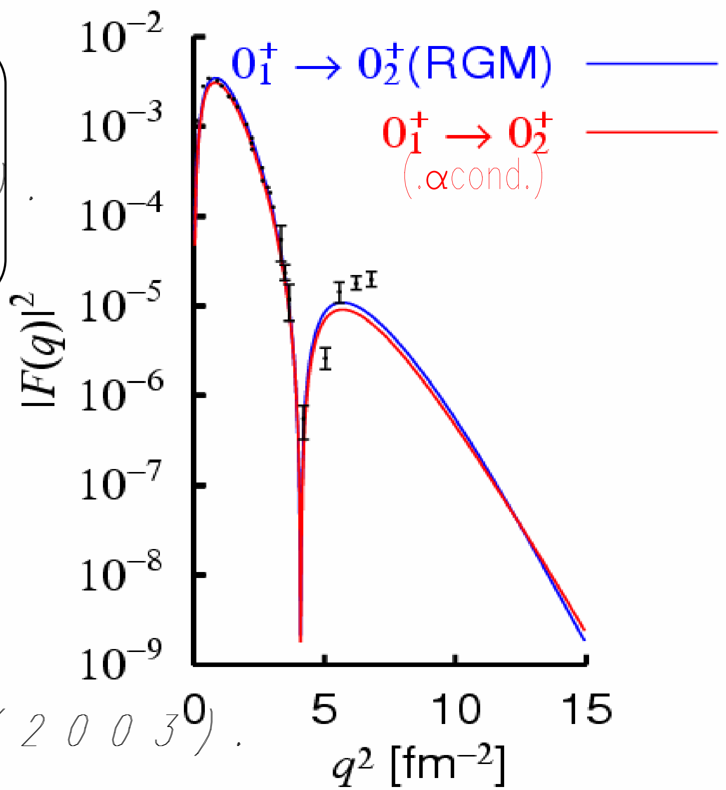
αcond. w.f.



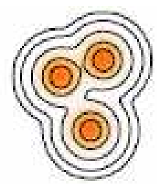
Y. F. et al., PRC 67, 051306(R) (2003).

The full α problem gives the 3αcondensate w.f. as its solution!

Form factor



3αclustering also appears starting without assumption of α's by FMD & AMD



M. Chernykh, T. Neff et al.

Y. Kanada-En'yo, PTP 11, ...

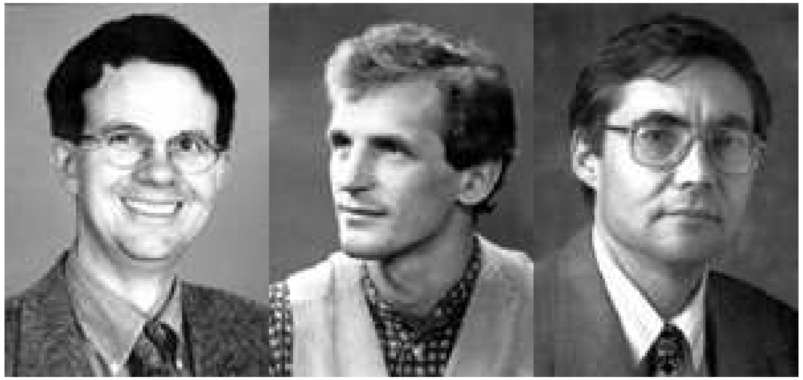
Bose-Einstein Condensation in the atomic world

Year

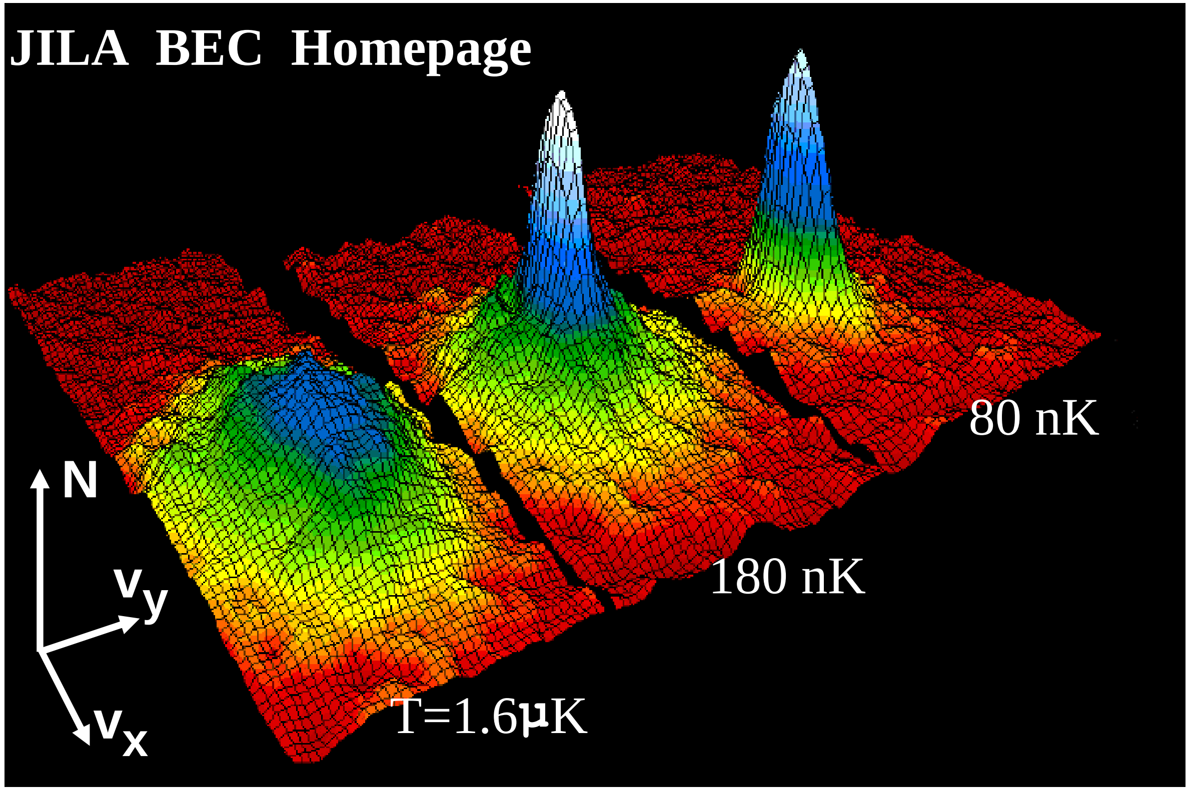
1925 Bose and Einstein predicted BEC

1995 Observation of BEC (^{87}Rb)

2001 Cornell, Ketterle, Wieman won the Nobel Prize in Physics.



Cornell **Ketterle** **Wieman**



Character

Particle Number : $10^3 \sim 10^4$

Interaction : S-wave

Low density : ρa^3 □ 10^{-3}

$a(^{87}Rb)=5.77nm$

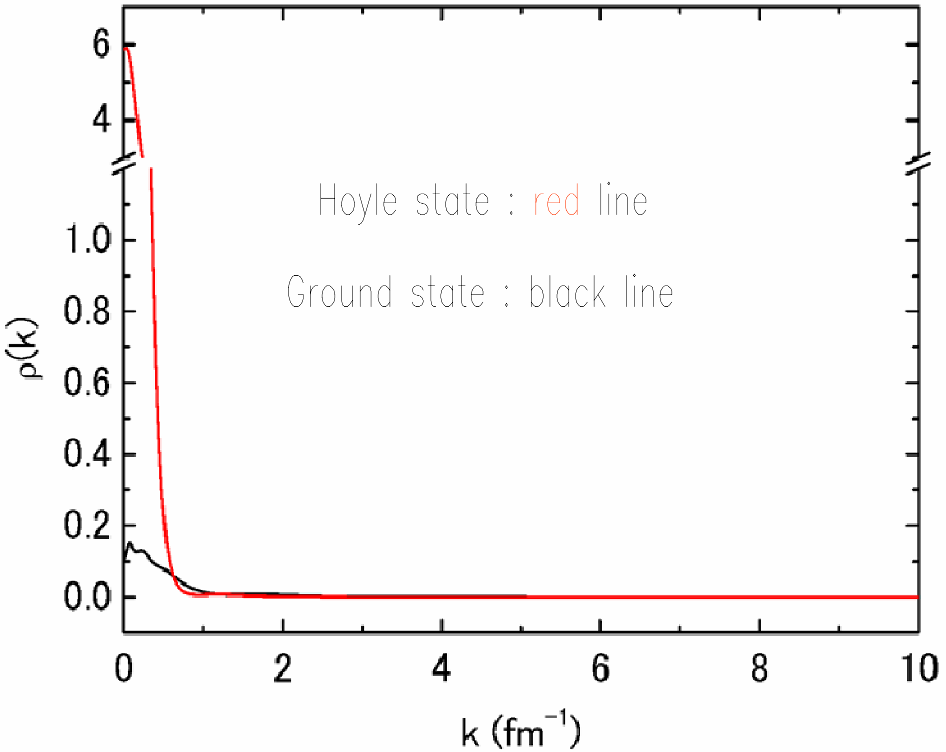
Direct information of alpha condensation

via 3αOCM(Orthogonality Condition Model)

$$\rho(k) = \int d\boldsymbol{r} d\boldsymbol{r}' \frac{e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}}{(2\pi)^{3/2}} \rho(\boldsymbol{r},\boldsymbol{r}') \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}'}}{(2\pi)^{3/2}}$$

$$\int d\mathbf{r}'\, \rho(\mathbf{r},\mathbf{r}')\phi(\mathbf{r}') = \mu\ \phi(\mathbf{r})$$

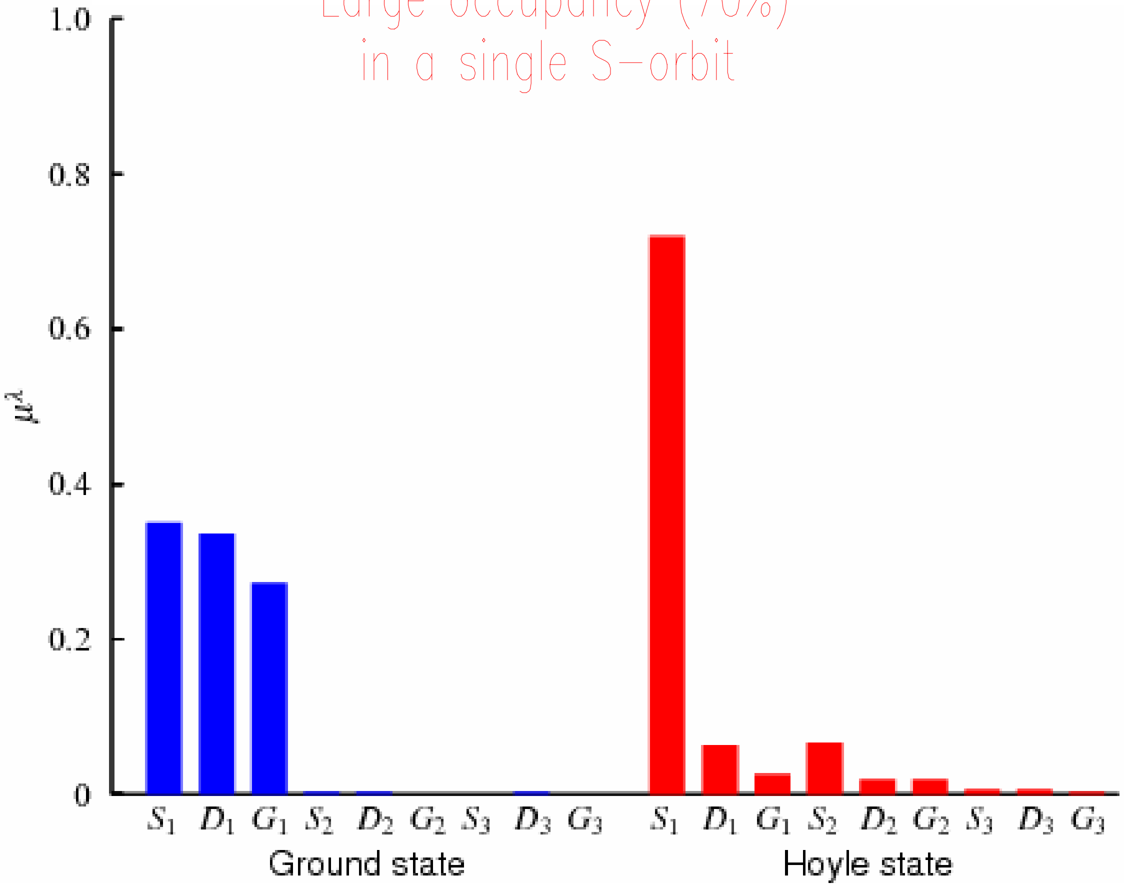
Momentum distribution of α−particle



δ function .like peak
around zero momentum

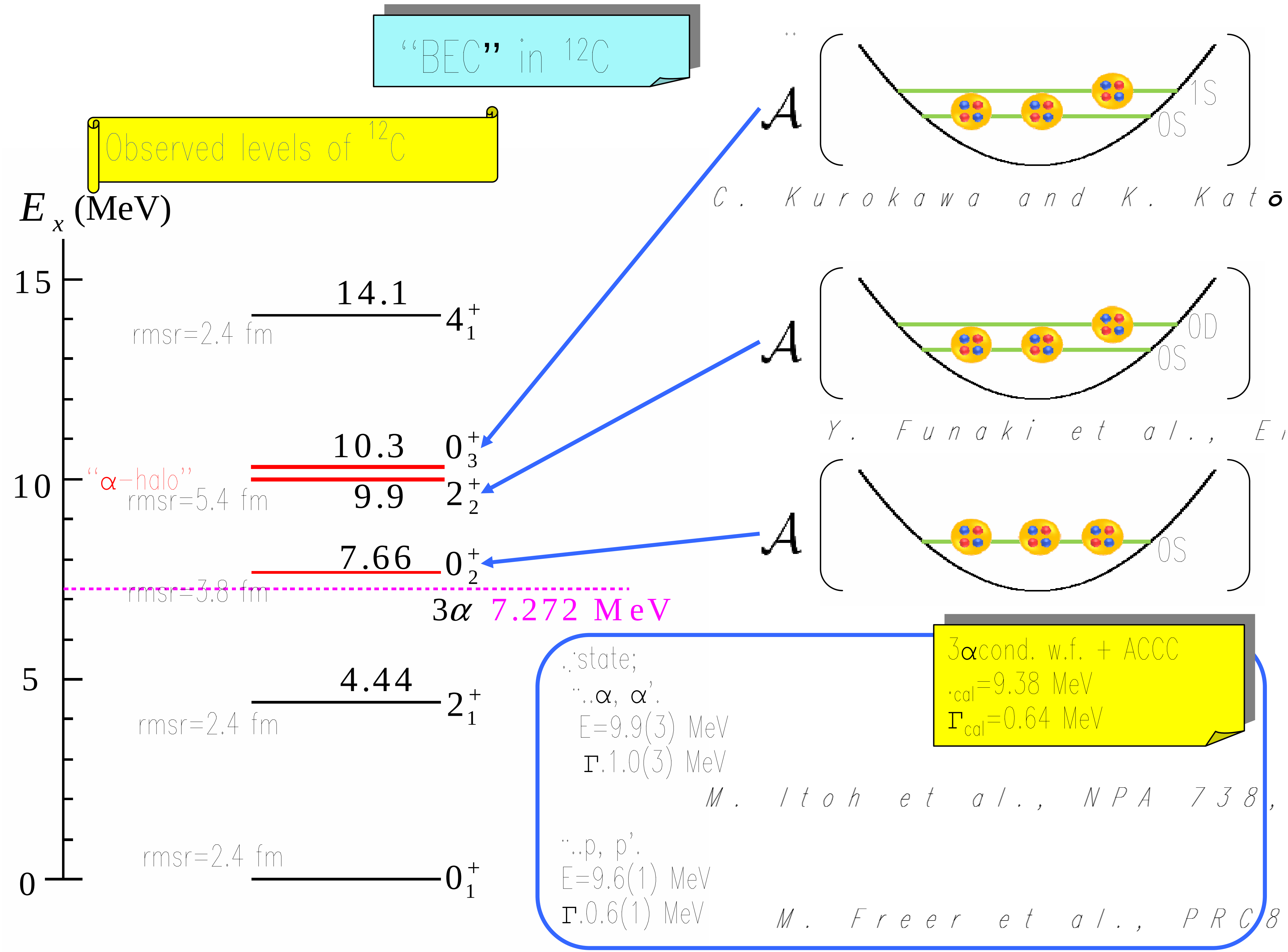
Occupation probability of single α−orbit

Large occupancy (70%)
in a single S−orbit

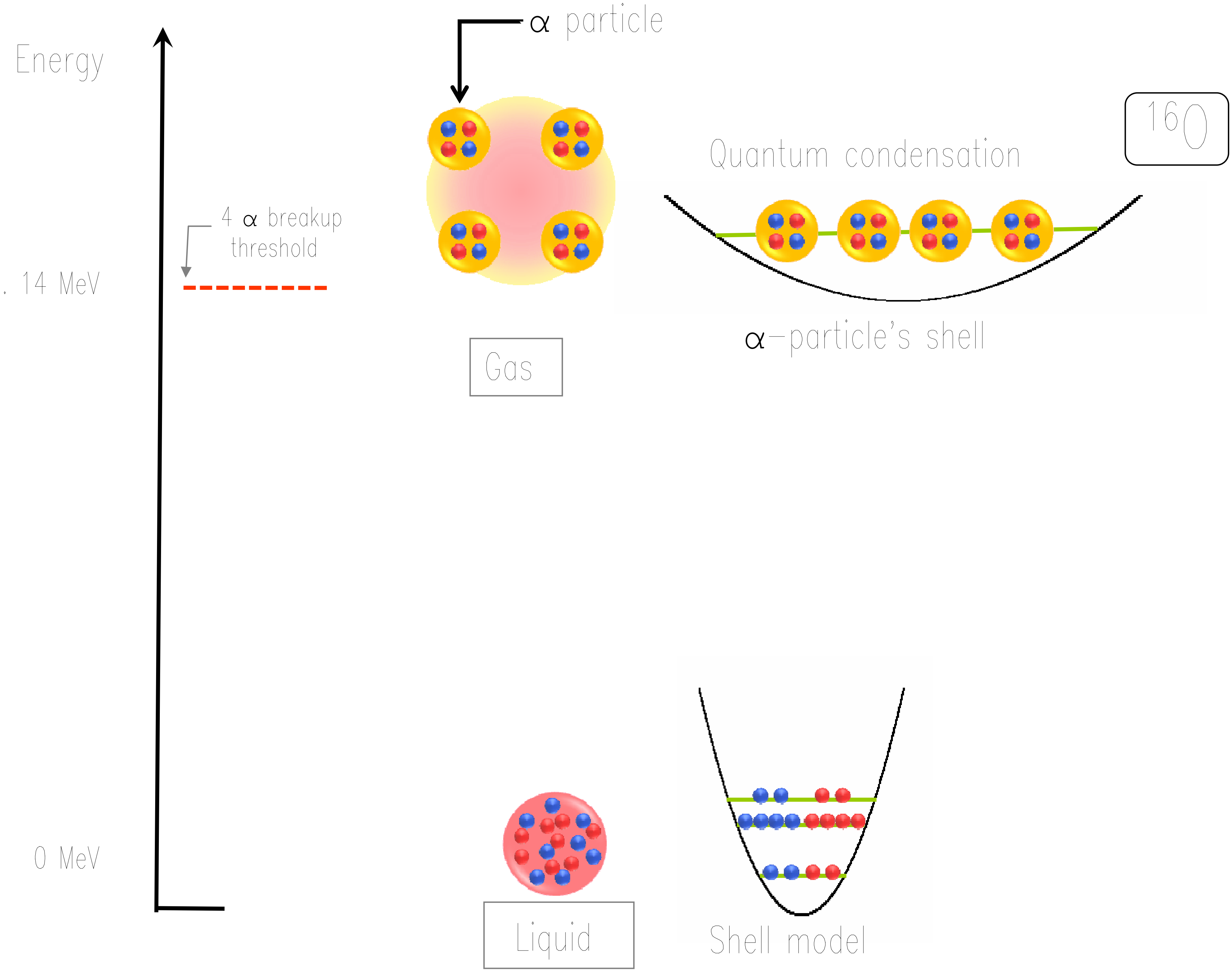


T. Yamada and P. Schuck,
EPJA 26, 185 (2005).

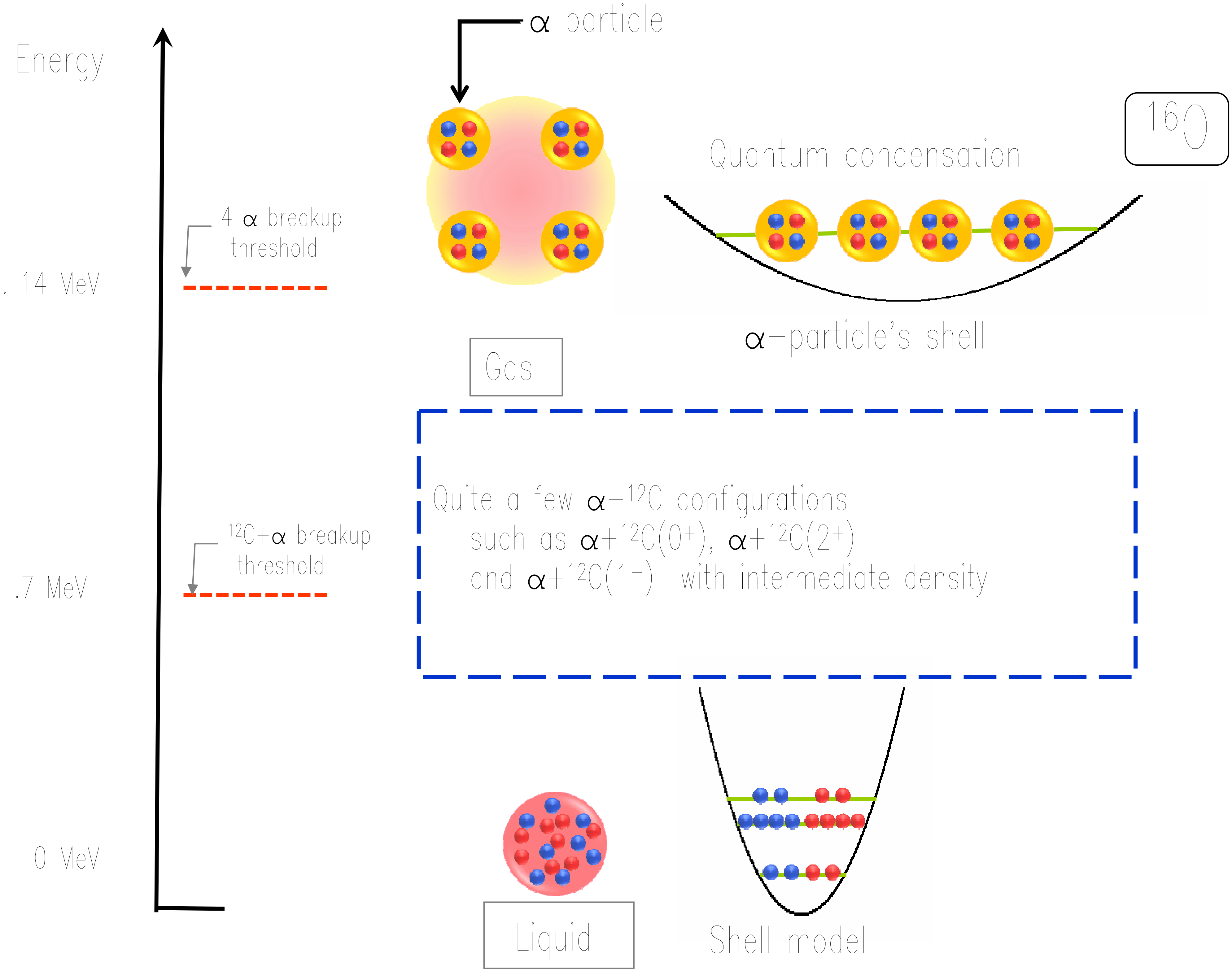
See also H. Matsumura and Y


"BEC" in 12C
Observed levels of 12C
E x (MeV)
15
10
5
0
14.1
4+ 1
rmsr=2.4 fm
10.3
0+ 3
"α−halo"
rmsr=5.4 fm
9.9
2+ 2
7.66
0+ 2
rmsr=3.8 fm
3α 7.272 MeV
4.44
2+ 1
rmsr=2.4 fm
rmsr=2.4 fm
0+ 1
1S
0S
C. Kurokawa and K. Katō
0D
0S
Y. Funaki et al., E
0S
3αcond. w.f. + ACCC
cal=9.38 MeV
Γcal=0.64 MeV
state;
α, α'.
E=9.9(3) MeV
Γ.1.0(3) MeV
M. Itoh et al., NPA 738,
p, p'.
E=9.6(1) MeV
Γ.0.6(1) MeV
M. Freer et al., PRC 8

Analogue to the Hoyle state in ^{16}O?



α particle
Energy
4 α breakup
threshold
14 MeV
0 MeV
16O
Quantum condensation
α−particle's shell
Gas
Liquid
Shell model



Energy
α particle
16O
Quantum condensation
4 α breakup threshold
. 14 MeV
α−particle's shell
Gas
Quite a few α+12C configurations
such as α+12C(0+), α+12C(2+)
and α+12C(1−) with intermediate density
12C+α breakup threshold
.7 MeV
0 MeV
Liquid
Shell model

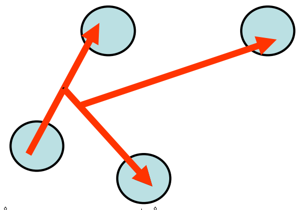
Fully solving 4 α−particles relative motions via Orthogonality Condition Model (OCM)

$$H = T + \sum_{i<j}\left[V_{2\alpha}(r_{ij}) + V_{2\alpha}^{\mathrm{Coul.}}(r_{ij})\right] + V_{3\alpha} + V_{4\alpha} + V_{\mathrm{Pauli}}$$

Pauli blocking operator on α−α motions

$$V_{Pauli} = \lim_{\lambda\to\infty} \lambda \sum_{2n+\ell<4} \sum_{ij} \left|u_{n\ell}(r_{ij})\right\rangle\left\langle u_{n\ell}(r_{ij})\right|$$

Pauli forbidden state: h.o,w.f.



α−α relative motions:

Many gaussians

Gauss Expansion Method (GEM)

E. Hiyama et al., Prog. P

2−body force (folding MHN force)

$$V_{2\alpha}(r) = \sum_n V_n^{(2)} \exp\left(-\beta_n^{(2)} r^2\right)$$

Coulomb force

$$V_{2\alpha}^{Coul}(r) = \frac{4e^2}{r}\operatorname{erf}(ar)$$

Phenomenological 3−body force (repulsive)

$$V_{3\alpha} = V^{(3)} \sum_{i<j<k} \exp\left[-\beta(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)\right]$$

$$V^{(3)} = 87.5\ \mathrm{MeV},\quad \beta = 0.15\ \mathrm{fm}^{-2}$$

Phenomenological 4−body force (repulsive)

$$V_{4\alpha} = V^{(4)} \exp\left[-\beta(r_{12}^2 + r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2 + r_{34}^2)\right]$$

$$V^{(4)} = 12000\ \mathrm{MeV},\quad \beta = 0.15\ \mathrm{fm}^{-2}$$

Equation of motion

$$\delta\left[\langle\Psi_L(^{16}\mathrm{O})|H - E|\Psi_L(^{16}\mathrm{O})\rangle\right] = 0$$

Energies from 4α threshold

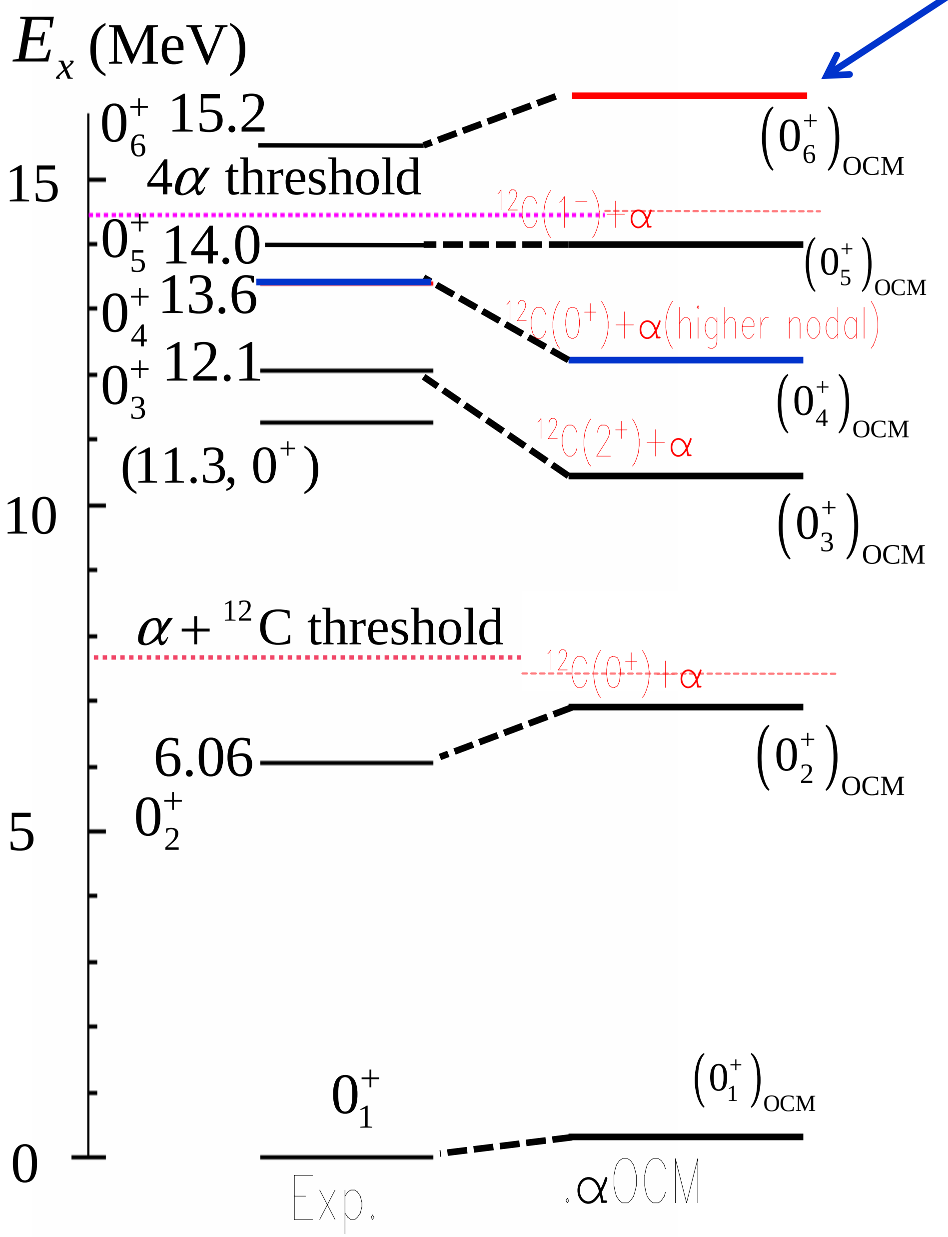
	Cal. (MeV)	Exp. (MeV)
^{12}C(g.s.)	.7.32	.7.28
^{16}O(g.s.)	.14.2	.14.44

$$\left|\langle V_{3\alpha}\rangle\right|, \left|\langle V_{4\alpha}\rangle\right| < \frac{7}{100}\left|\langle V_{2\alpha}\rangle\right|$$

Energy levels, rms radii, monopole matrix elements and density distribution.

4α cond. state

Low lying 0^+ levels of ^{16}O



	R_{rms} (fm)	M(E0)(fm^2)	M(E0)(fm^2) Exp.
$(0_1^+)_{OCM}$	2.7		
$(0_2^+)_{OCM}$	3.0	3.9	0_2^+: 3.55
$(0_3^+)_{OCM}$	3.1	2.4	0_3^+: 4.03
$(0_4^+)_{OCM}$	4.0	2.4	0_4^+: no data
$(0_5^+)_{OCM}$	3.1	2.6	0_5^+: 3.3
$(0_6^+)_{OCM}$	5.6	1.0	0_6^+: no data

Large monopole matrix element can be the evidence of cluster states.

T. Yamada, Y. F. et al

0_4^+ state: T. Wakasa, Y. F. et al., PLB 653, 1

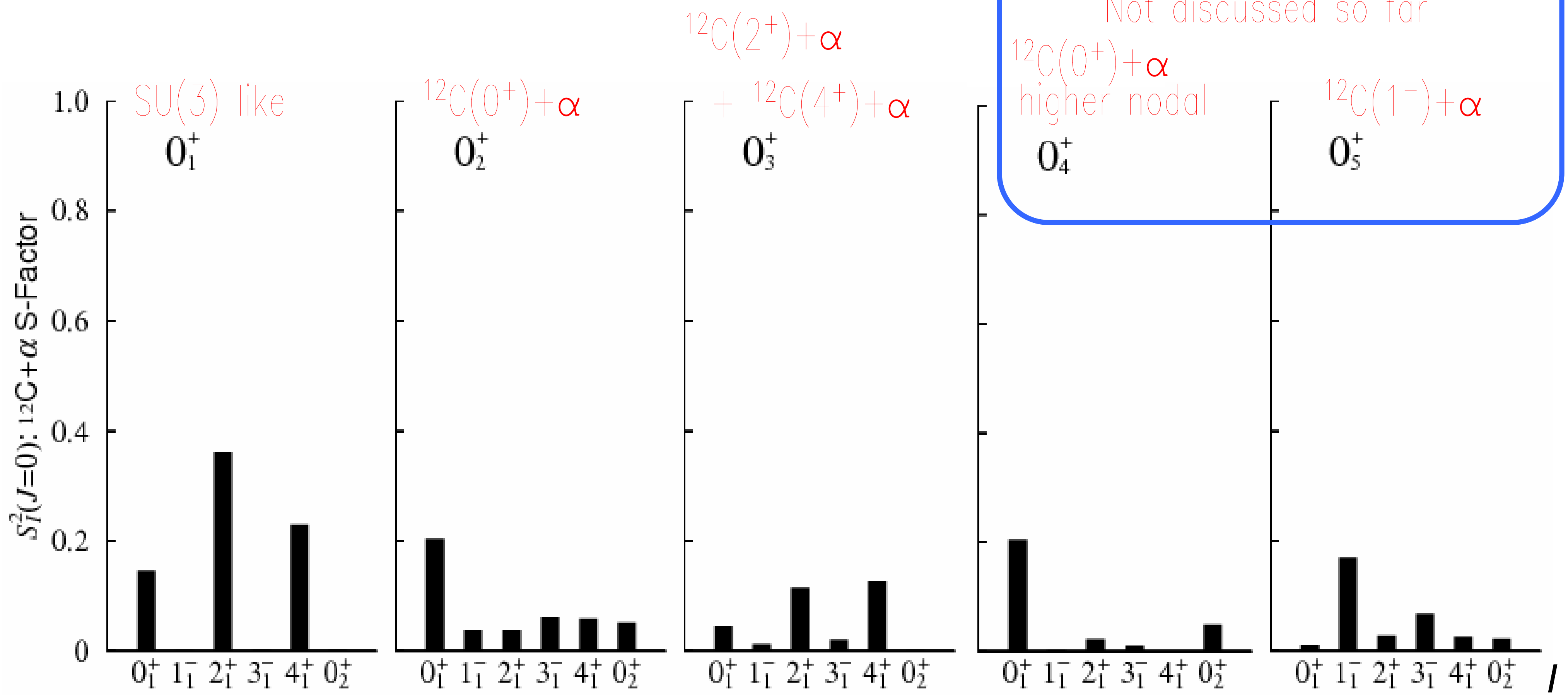
S-factor : $^{12}C(I)+\alpha$ components for the 0_1^+ – 0_6^+ states

$$r\times \mathcal{Y}_{IL,J=0}(r)=r\times\left\langle\left[\frac{\delta(r-r')}{rr'}Y_L(\hat{\mathbf{r}}')\Psi_{\rm OCM}(^{12}{\rm C}(I))\right]_0\middle|\Psi_{\rm OCM}(0_k^+)\right\rangle$$

$$S_{IL}^2(J=0)=\int dr\left(r\times\mathcal{Y}_{IL,J=0}(r)\right)^2$$

New states

Not discussed so far

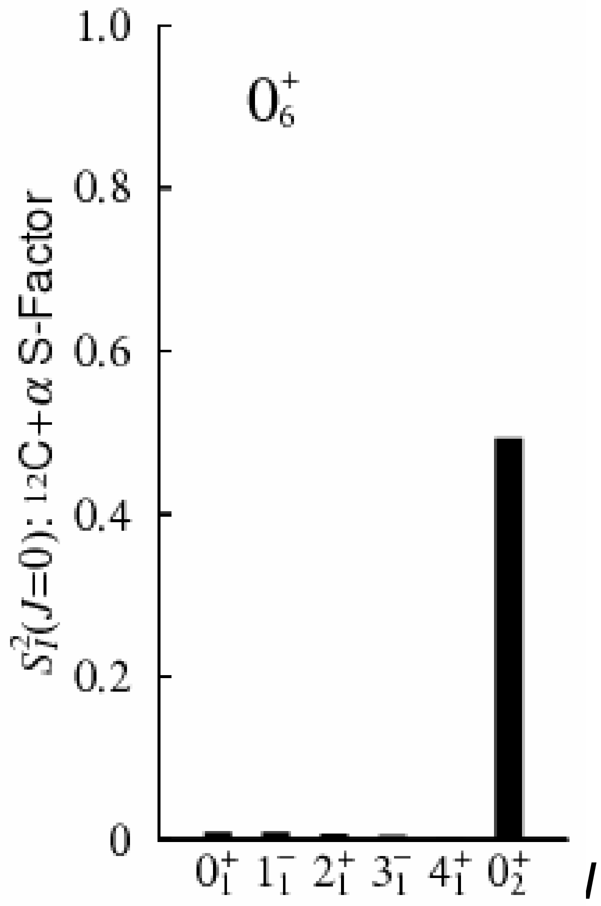


I: Spin of ^{12}C

S-factor : $^{12}C(I)+\alpha$ components for the 0_1^+ – 0_6^+ states

$$r\times \mathcal{Y}_{IL,J=0}(r)=r\times\left\langle\left[\frac{\delta(r-r')}{rr'}Y_L(\hat{\mathbf{r}}')\Psi_{\rm OCM}(^{12}{\rm C}(I))\right]_0\middle|\Psi_{\rm OCM}(0_k^+)\right\rangle$$

$$\mathcal{S}^2_{IL}(J=0)=\int dr\left(r\times\mathcal{Y}_{IL,J=0}(r)\right)^2$$



I: Spin of ^{12}C

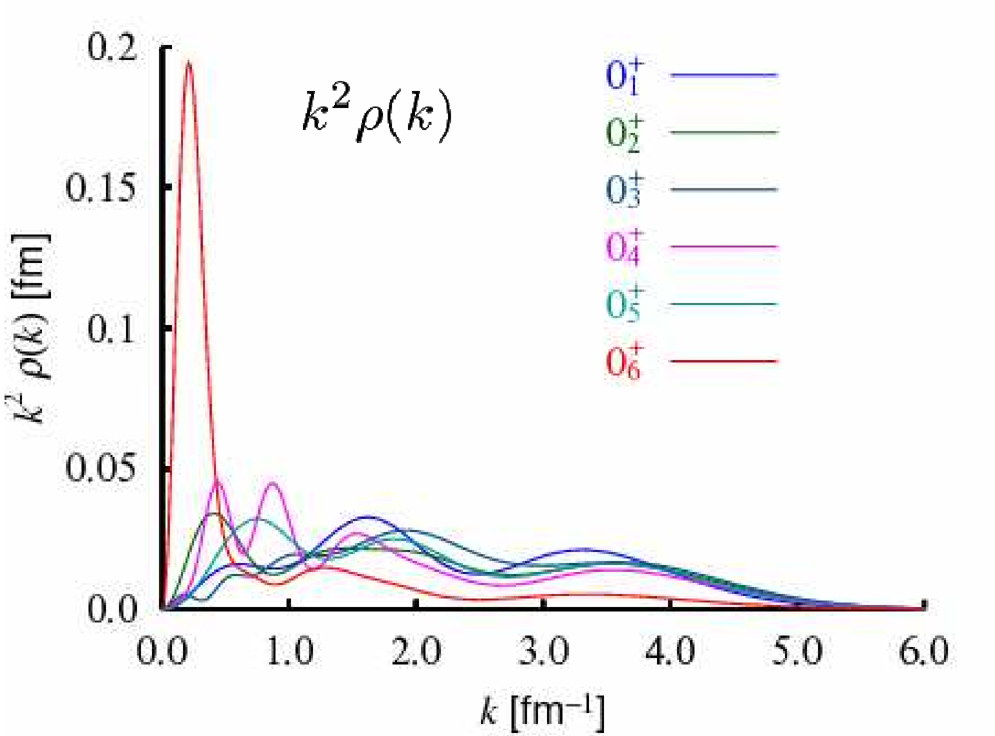
Analogue to the Hoyle state

$\alpha+^{12}C$(Hoyle) configuration is dominant.

^{12}C(Hoyle): 3αcondensate → 4αcondensate

Momentum distributions of the α particle

$$\rho(k) = \int d\boldsymbol{r} d\boldsymbol{r}' \frac{e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}}{(2\pi)^{3/2}} \rho(\boldsymbol{r},\boldsymbol{r}') \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}'}}{(2\pi)^{3/2}}$$



0_6^+ : delta-function-like peak at zero momentum

4αcondensate state character.

de Broglie w.l. $\lambda = \frac{2\pi}{\sqrt{\langle k^2 \rangle}} \geq 20$ fm

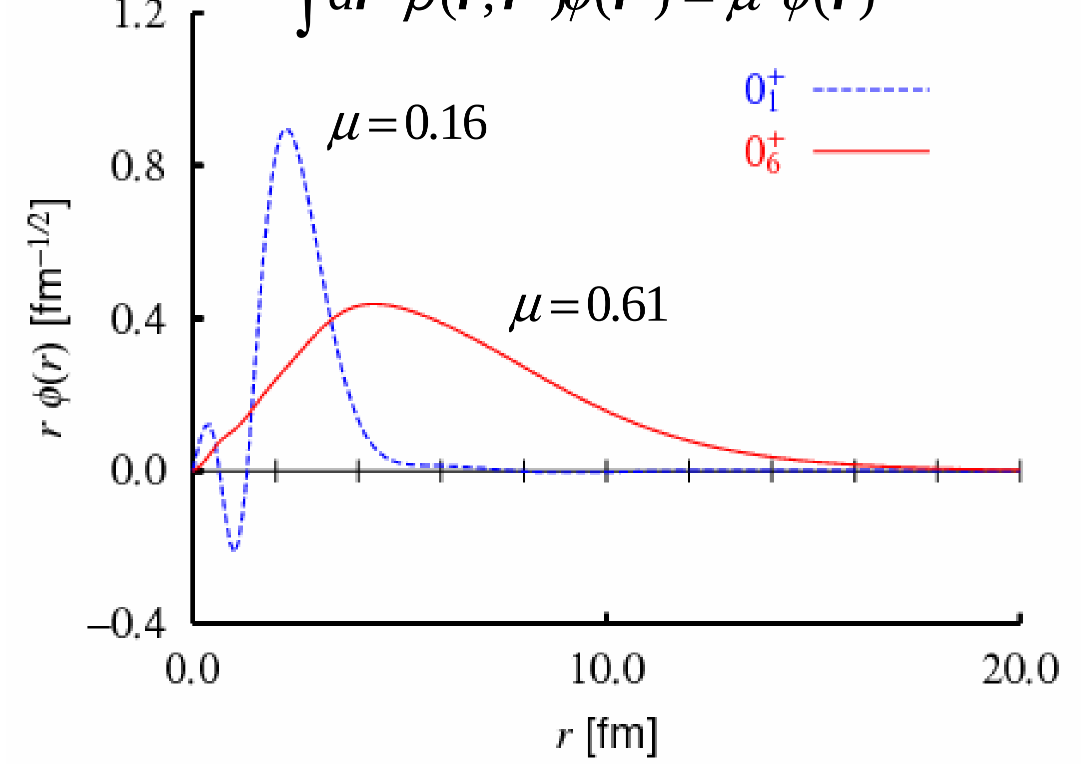
$\boldsymbol{r}_i$: coordinate of the i-th α particle

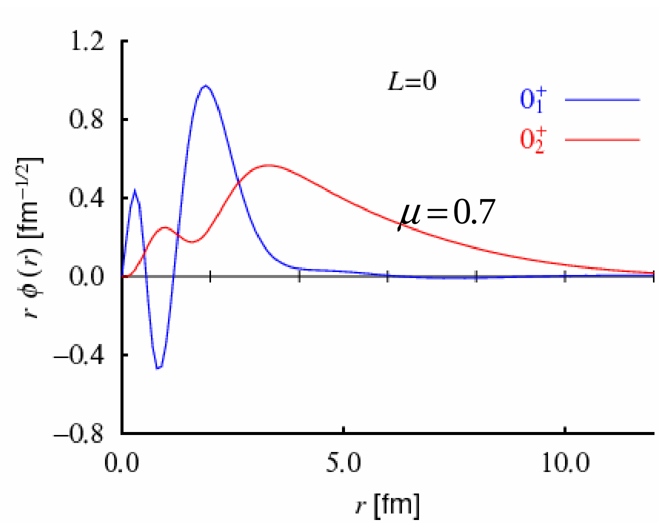
$\boldsymbol{X}_G$: coordinate of total center-of-mass

$$\rho(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{4}\sum_{i=1}^{4} \langle \Psi_{\mathrm{OCM}}(0_k^+) | \delta(\boldsymbol{r}_i - \boldsymbol{X}_G - \boldsymbol{r}') \rangle \langle \delta(\boldsymbol{r}_i - \boldsymbol{X}_G - \boldsymbol{r}) | \Psi_{\mathrm{OCM}}(0_k^+) \rangle$$

Single-α occupancy and single-α orbit for the 0_1^+ and 0_6^+ states
(Only the S orbit (L=0) with the largest occupancy)

$$\int d\mathbf{r}'\rho(\mathbf{r},\mathbf{r}')\phi(\mathbf{r}') = \mu\ \phi(\mathbf{r})$$





Similar to ^{12}C case!

0_6^+ : Large 0S occupancy ! (61%)
Largely extended 0S orbital, large occupancy.
Mean-field-like structure of α particles.

Typical nature of the αcondensate!

0_1^+ : αparticles are dissolved . Reflecting shell structure of nucleons.
SU(3) configuration : 2S nodal behaviour

Y. F. et al., PRL101,

Alpha decay widths

$\Gamma(0_4^+)_{OCM}$. 0.2 MeV

$\Gamma(0_5^+)_{OCM}$ < 0.05 MeV

$\Gamma(0_6^+)_{OCM}$. 0.05 MeV

(calculated based on R-matrix theory)

$$\Gamma_L = P_L \cdot \gamma_L^2(a)$$

$$\gamma_L^2(a) \propto (a\mathcal{Y}_L(a))^2$$

P_L : penetration factor

$\gamma_L^2(a)$: reduced width

a: channel radius

$\Gamma(0_4^+$ at 13.6 MeV$)=$ 0.6 MeV

$\Gamma(0_5^+$ at14.0 MeV$)=$ 0.19 MeV

$\Gamma(0_6^+$ at15.2 MeV$)=$ 0.17 MeV

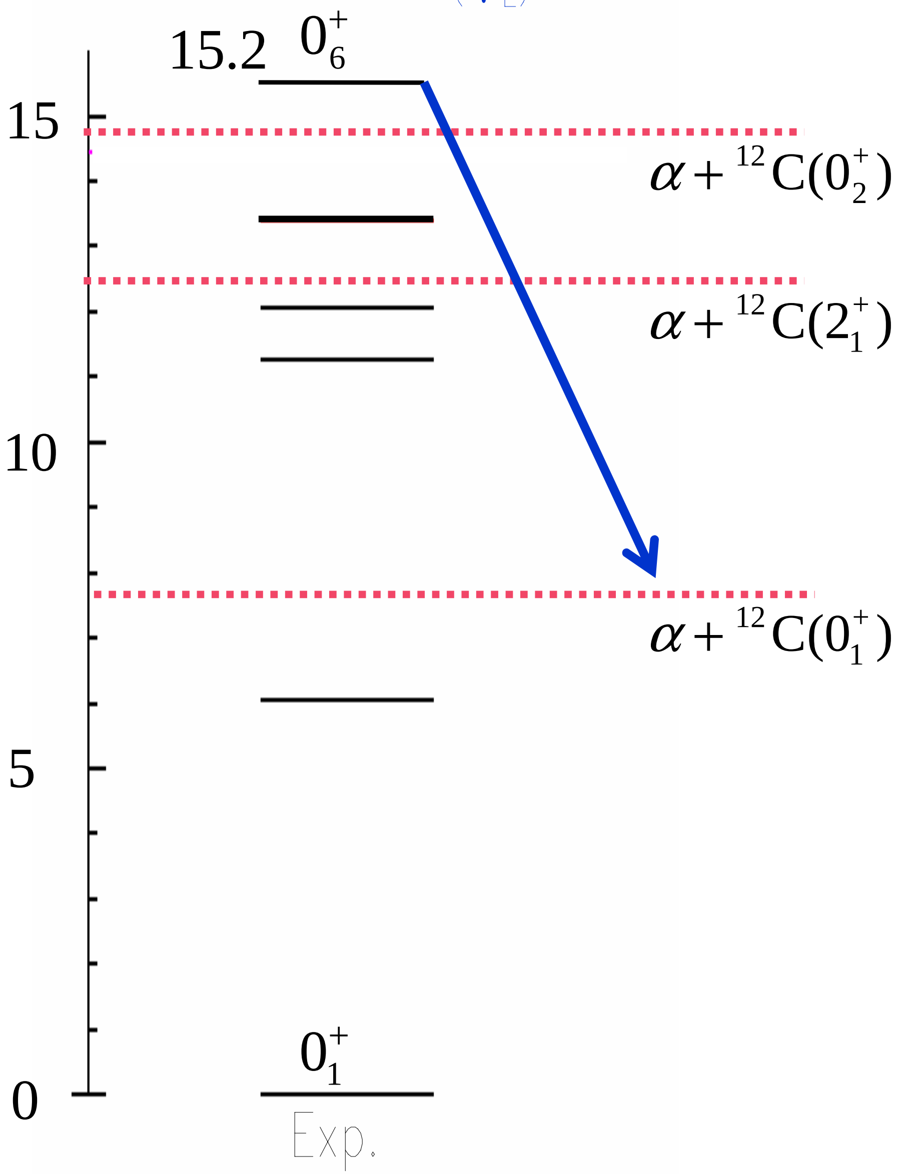
$0_4^+ - 0_6^+$: Consistent with experiment

The reason why 0_6^+ is narrow in spite of the high excitation

$\alpha.{}^{12}C(L=0_1^+)$

P_L : Large

$(\gamma_L)^2$: Small



P_L : depends on decay energy

$(\gamma_L)^2$: $\alpha.{}^{12}C(L)$ components

$\Gamma_L = P_L\,(\gamma_L)^2$: Suppressed

$\Gamma(0_4^+)_{OCM}$. 0.2 MeV

$\Gamma(0_5^+)_{OCM}$ < 0.05 MeV

$\Gamma(0_6^+)_{OCM}$. 0.05 MeV

(calculated based on R−matrix theory)

$$\Gamma_L = P_L \cdot \gamma_L^2(a)$$

$$\gamma_L^2(a) \propto (a\Upsilon_L(a))^2$$

P_L : penetration factor

$\gamma_L^2(a)$: reduced width

a: channel radius

$\Gamma(0_4^+$ at 13.6 MeV$)$= 0.6 MeV

$\Gamma(0_5^+$ at14.0 MeV$)$= 0.19 MeV

$\Gamma(0_6^+$ at15.2 MeV$)$= 0.17 MeV

0_6^+ : Small width, quasi stable

. M. Itoh (CYRIC)

P_L : depends on decay energy

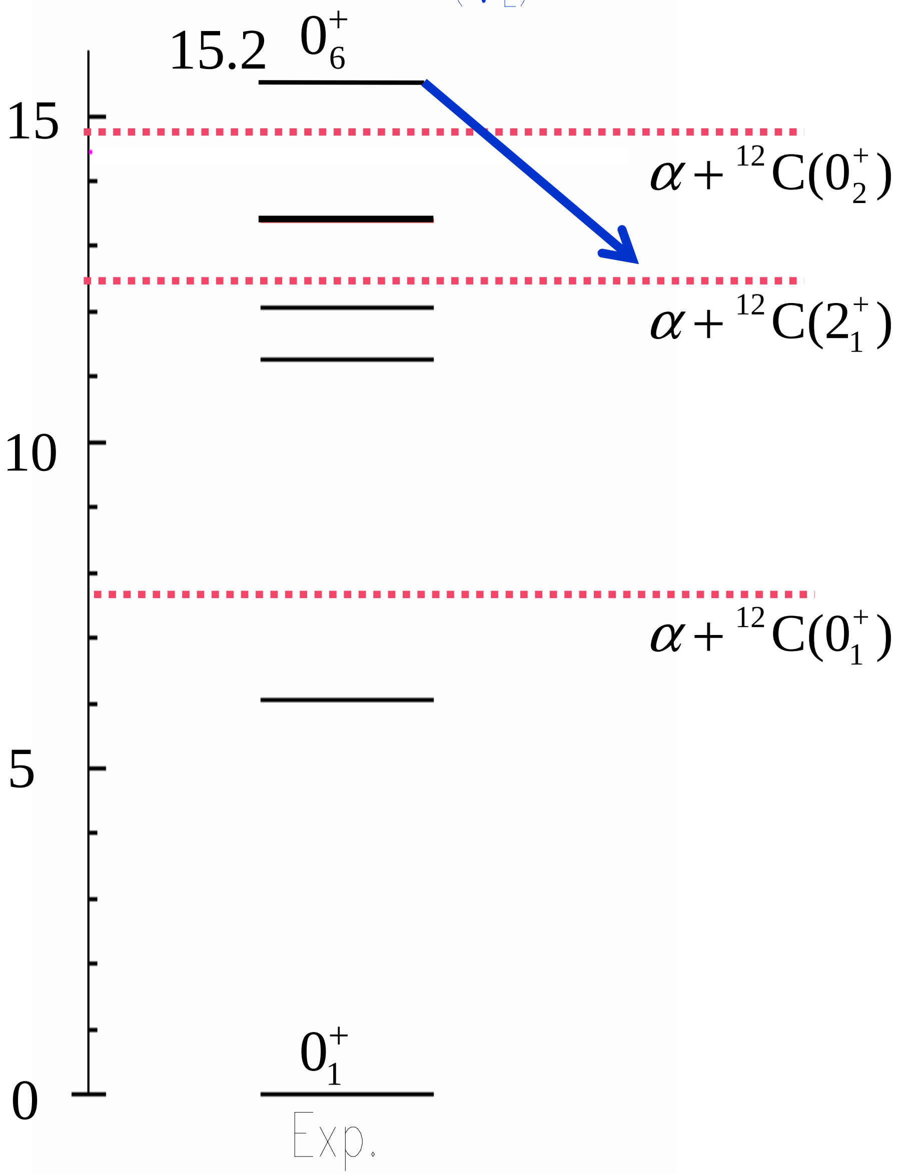
$(\gamma_L)^2$: α.^{12}C(L) components

α.^{12}C($L=2_1^+$)

P_L : Medium

$(\gamma_L)^2$: Small

$\Gamma_L = P_L\,(\gamma_L)^2$: Suppressed



$\Gamma(0_4^+)_{OCM}$. 0.2 MeV

$\Gamma(0_5^+)_{OCM}$ < 0.05 MeV

$\Gamma(0_6^+)_{OCM}$. 0.05 MeV

(calculated based on R–matrix theory)

$$\Gamma_L = P_L \cdot \gamma_L^2(a)$$

$$\gamma_L^2(a) \propto (a\mathcal{Y}_L(a))^2$$

P_L : penetration factor

$\gamma_L^2(a)$: reduced width

a: channel radius

$\Gamma(0_4^+$ at 13.6 MeV$)$= 0.6 MeV

$\Gamma(0_5^+$ at14.0 MeV$)$= 0.19 MeV

$\Gamma(0_6^+$ at15.2 MeV$)$= 0.17 MeV

0_6^+ : Small width, quasi stable

. M. Itoh (CYRIC)

0_6^+ : Decays into all channels are suppressed!

P_L : depends on decay energy

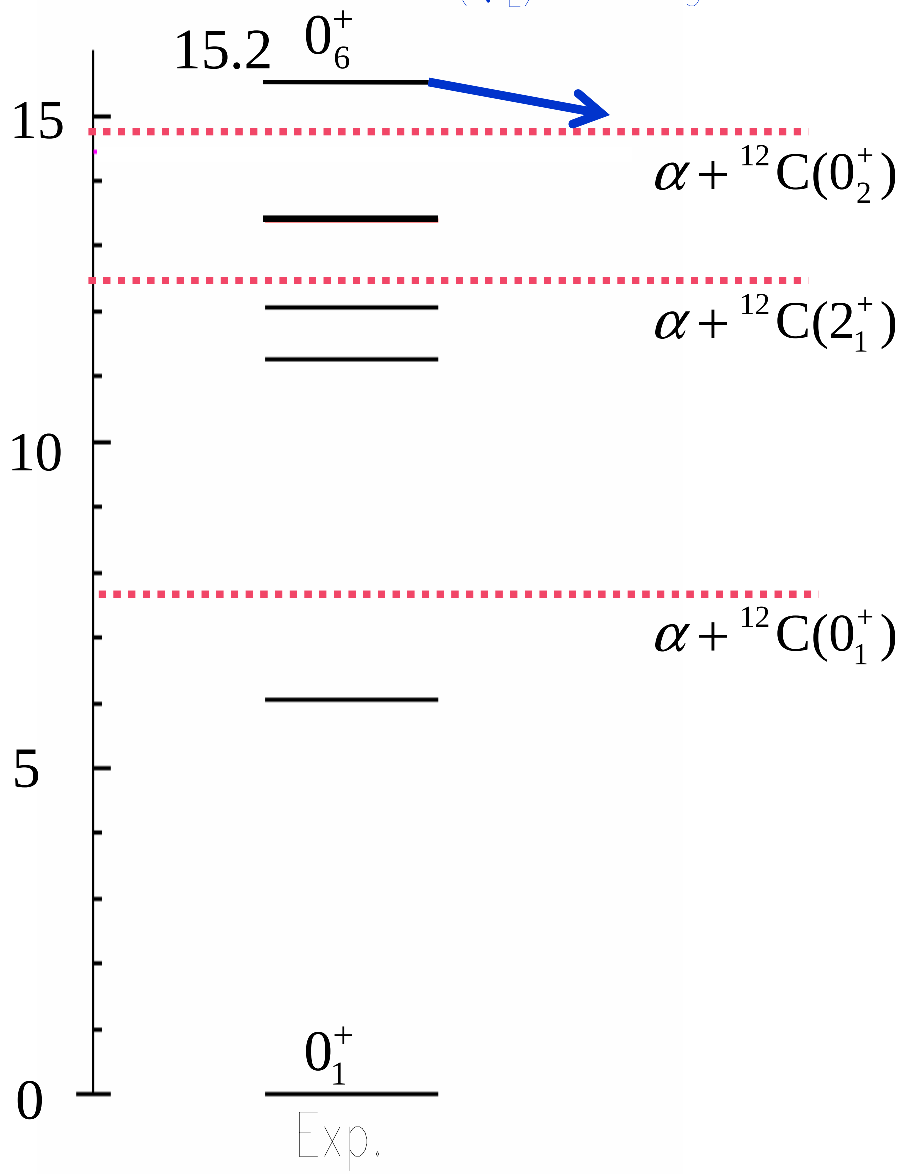
$(\gamma_L)^2$: α-^{12}C(L) components

α-^{12}C(L=0_2^+)

P_L : Small

$(\gamma_L)^2$: Large

$\Gamma_L = P_L\,(\gamma_L)^2$: Suppressed



$\Gamma(0_4^+)_{OCM}$. 0.2 MeV

$\Gamma(0_5^+)_{OCM}$ < 0.05 MeV

$\Gamma(0_6^+)_{OCM}$. 0.05 MeV

(calculated based on R-matrix theory)

$$\Gamma_L = P_L \cdot \gamma_L^2(a)$$

$$\gamma_L^2(a) \propto (a\mathcal{Y}_L(a))^2$$

P_L : penetration factor

$\gamma_L^2(a)$: reduced width

a: channel radius

$\Gamma(0_4^+$ at 13.6 MeV$)$= 0.6 MeV

$\Gamma(0_5^+$ at14.0 MeV$)$= 0.19 MeV

$\Gamma(0_6^+$ at15.2 MeV$)$= 0.17 MeV

In heavier systems, the analogue states may survive stably!

Conclusions

Investigation of loosely bound alpha gas states in finite nuclei

- It is well established that the Hoyle state has not only loosely bound 3α "gas" structure but also the 3αcondensate character.

- More α-particle condensate states very likely to exist.

  Analogue state in ^{16}O to the Hoyle state (found with 4αOCM calc.) as the sixth 0+ state

  Large occupancy into a single-alpha 0S orbit
  Peak around zero momentum
  Stably existing

  Assigned to 15.2 MeV state?
  More experimental information is needed.

To be done: Trying to do 4-alpha CSM (Complex Scaling Method)
Non-zero spin states (excitation of the 4-alpha condensate)
4-alpha linear chain state simultaneously

Thanks

to my Collaborators

Taiichi Yamada (Kanto Gakuin Univ.)

Hisashi Horiuchi (RCNP)

Akihiro Tohsaki (RCNP)

Peter Schuck (IPN, Orsay)

Gerd Röpke (Rostock Univ.)

Masaaki Takashina (RCNP)

Tomotsugu Wakasa (Kyushu Univ.)

Wolfram von Oertzen (HMI, Berlin)

and for your attention.

W. G. Lynch et

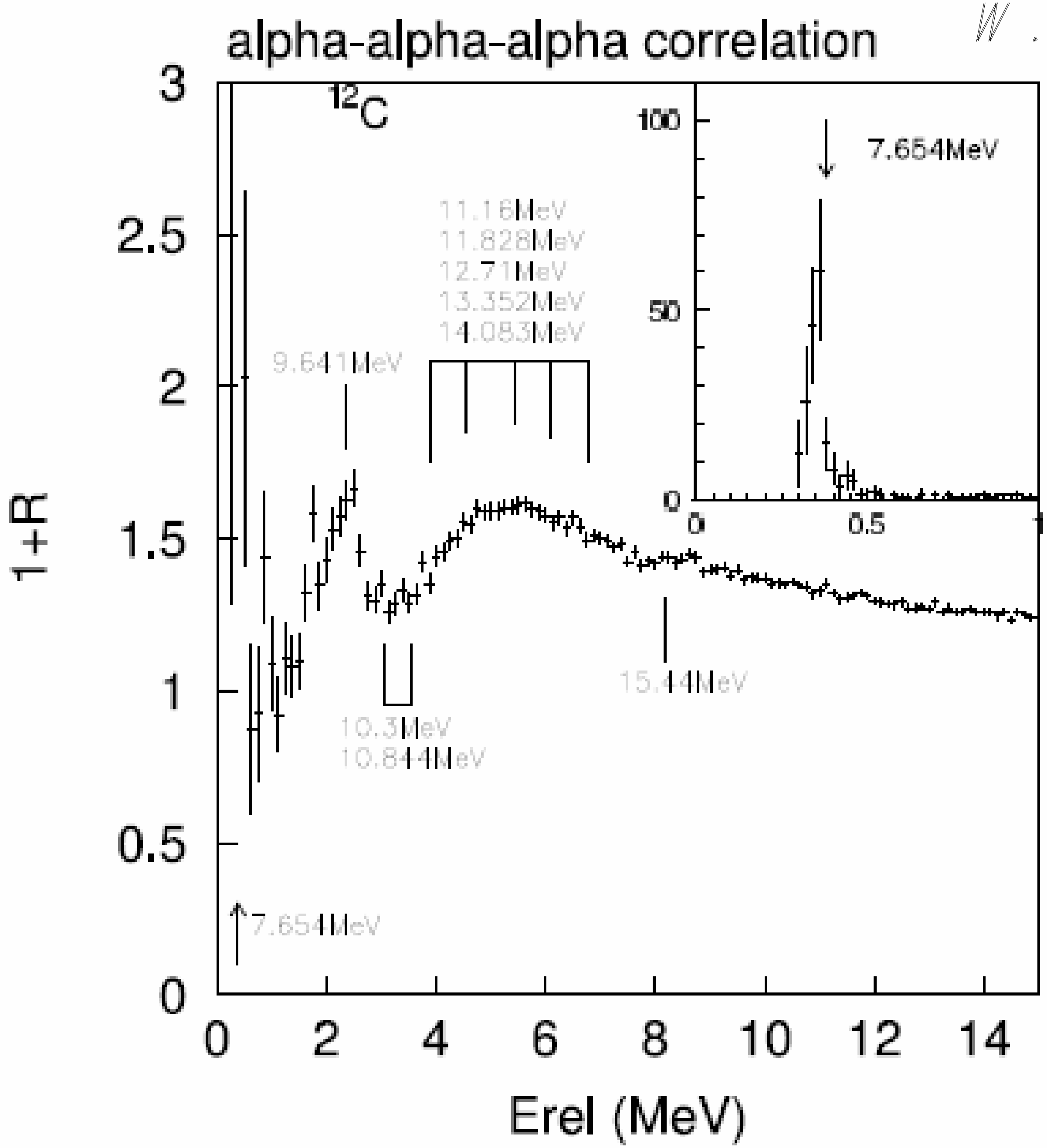
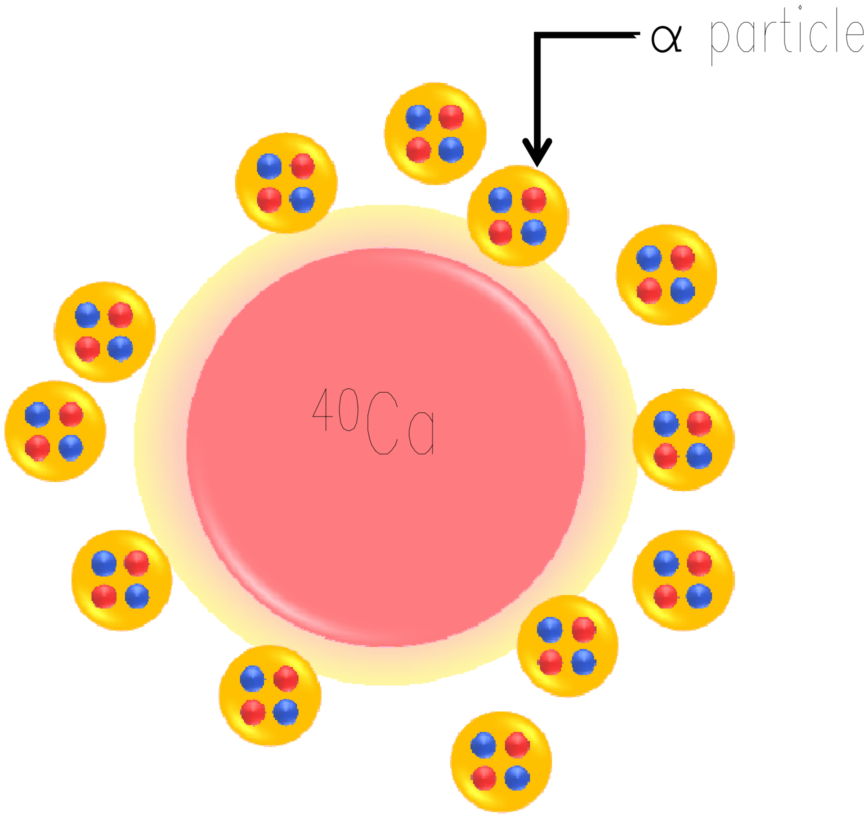


Figure 5.27: The α-α-α correlation function is shown. Resonances from the excited states of ^{12}C are labelled with the first peak seen more clearly in the inner upright panel.

Similar experiment (data ana

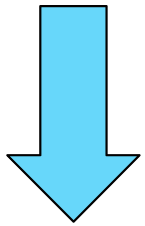
Talk by Verde-s

Compound nuclei with α gas



$^{52}Fe^{*} \rightarrow {}^{40}Ca + 3\alpha$'s

Disintegration into 3α's is enhanced



Sign of α−particle condensate

W. von Oertzen group
Tz. Kokalova et al., EPJA 23, 19(2005); PRL 96, 192502 (2006).

Applied to ^{12}C

The calculated binding energy (MeV)

$E_{3\alpha}^{\text{ths}} = -82.04$ MeV   Volkov No. 2

	Single THSR w. f.	Hill–Wheeler (superposition of THSR)	RGM
.:	−81.55	−81.79	−81.7
2_1˙	−84.65	−86.71	−86.7
.1˙	−87.68	−89.52	−89.4

Slight modification of THSR w.f. where the effect of deformation is taken into account, with $J^{\pi}=0^{+}$ projection.

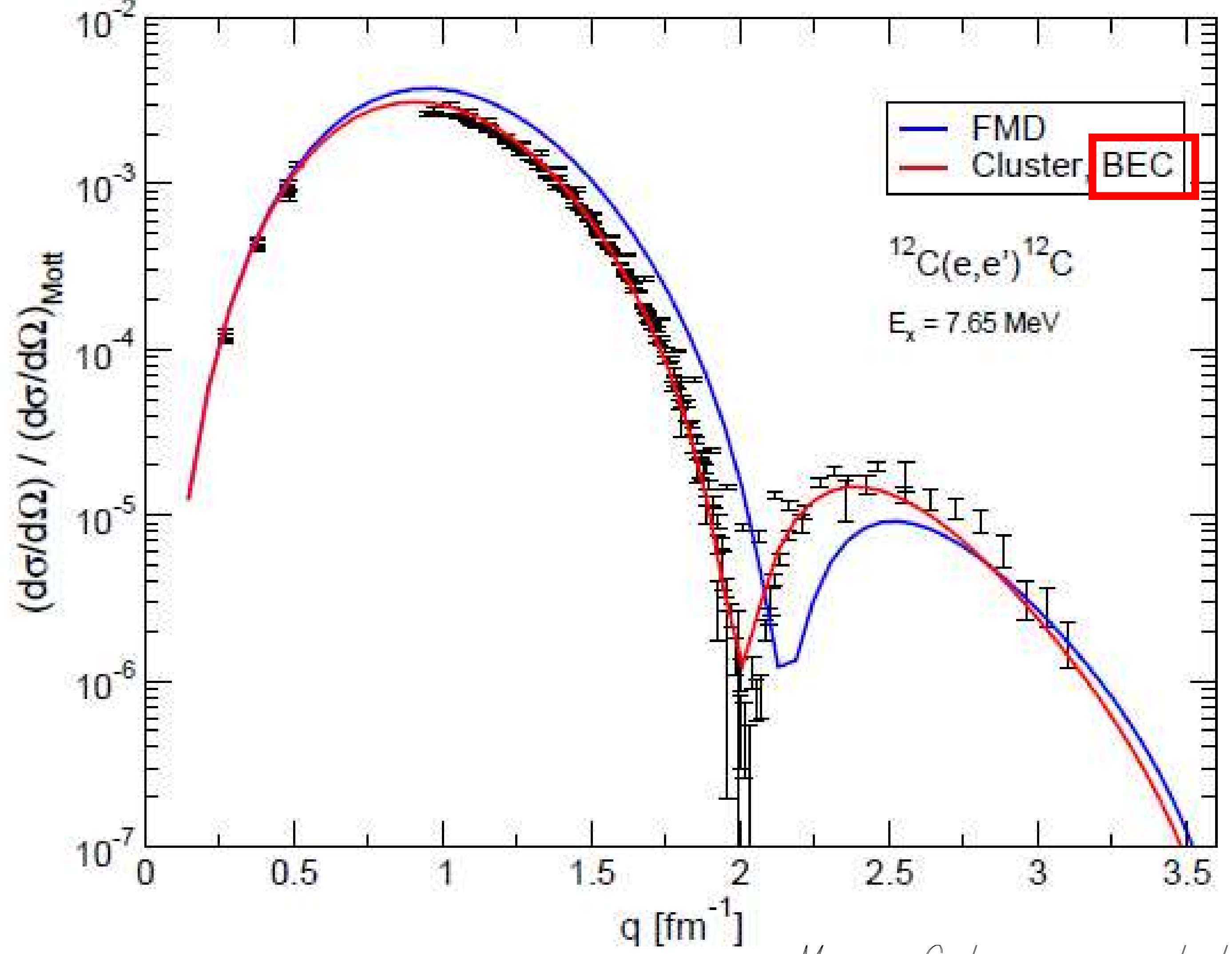
$$\hat{\mathcal{P}}^{\perp}_{\text{g.s.}}\ \hat{\mathcal{P}}_{J=0}\mathcal{A}\left\{\prod_{i=1}^{3}\exp\left(-\frac{2}{B_x^2}\vec{X}_{ix}^{2}-\frac{2}{B_y^2}\vec{X}_{iy}^{2}-\frac{2}{B_z^2}\vec{X}_{iz}^{2}\right)\phi(\alpha_i)\right\}$$

Single THSR w.f.: input of the optimum parameter value,

$(B_x = B_y, B_z) = (8.2 \text{ fm}, 2.3 \text{ fm})$

The RGM w. f. (full three–body) of .: state is almost the same as the single 3αTHSR w. f. ...7...

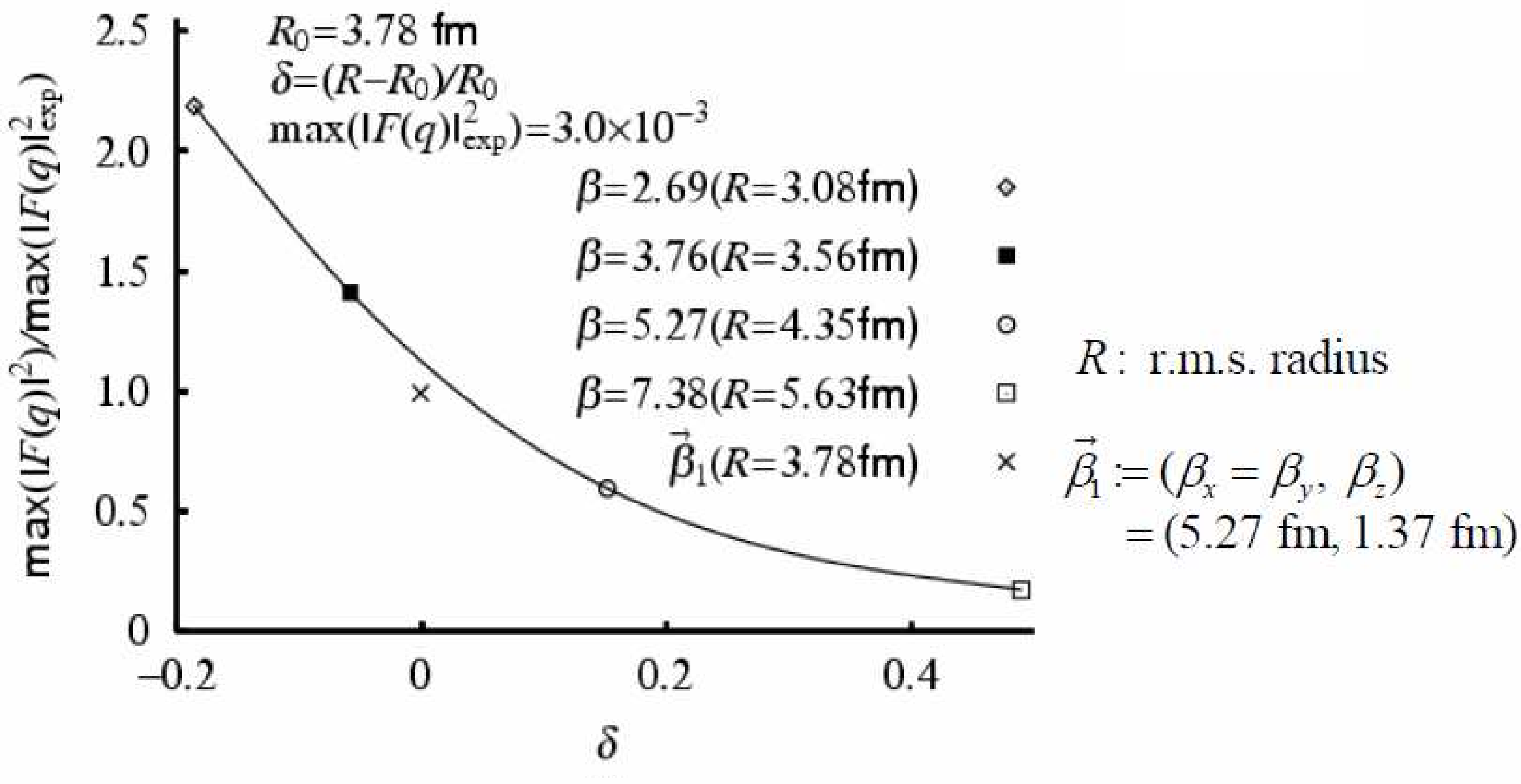
Electron Scattering Data ($0_1^+ \rightarrow 0_2^+$)



M. Chernykh, et al., P
also see M. Chernykh

Very nice reproduction by THSR w.f. (BEC)

Size dependence of the magnitude of the formfactor

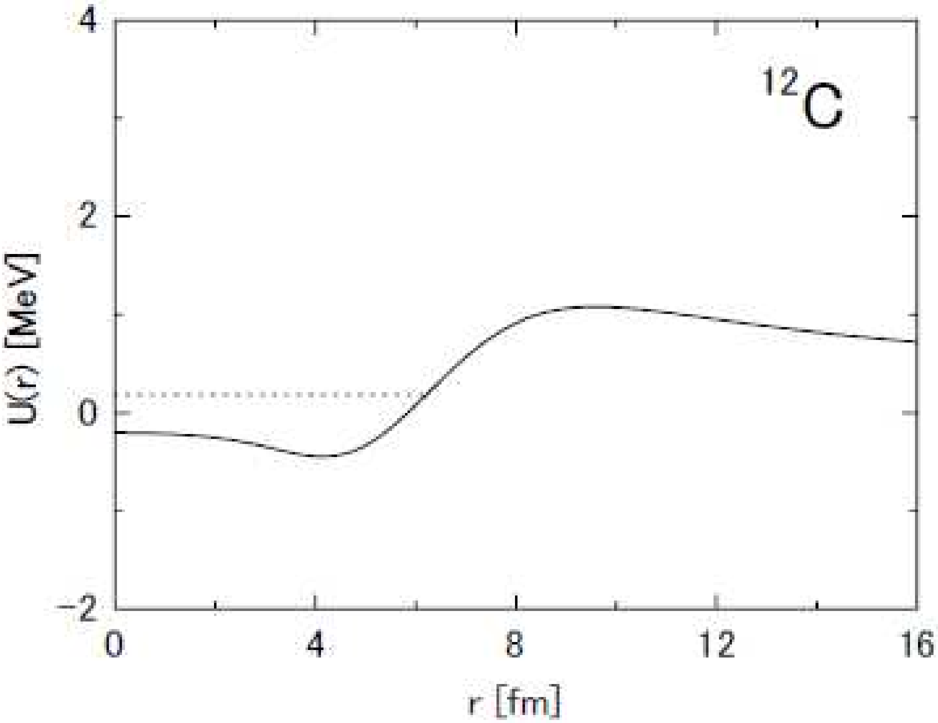


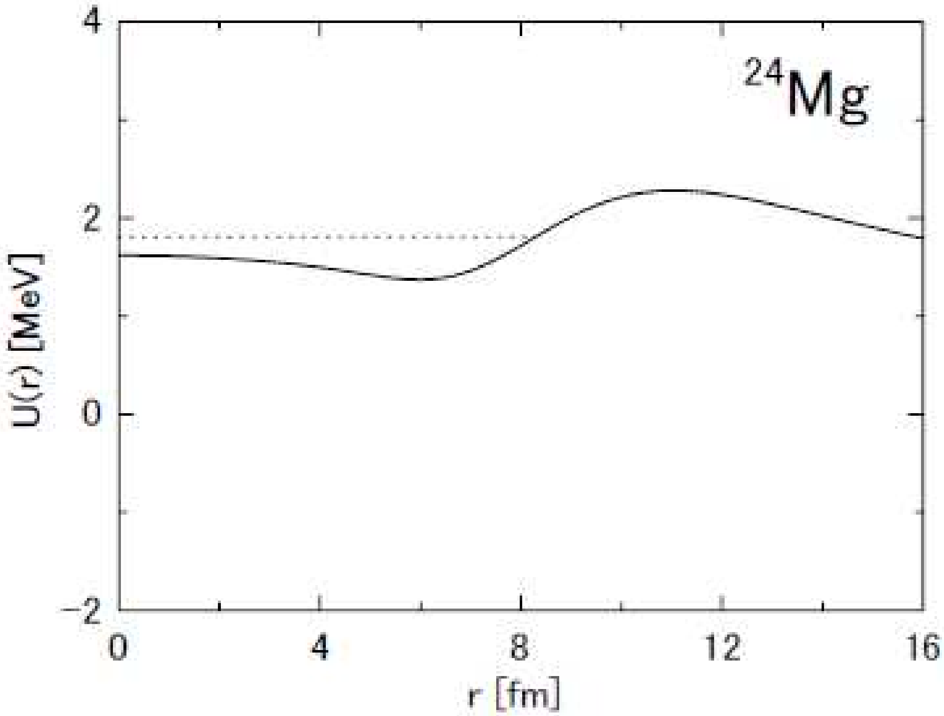
Artificially changing the rms radius of the Hoyle state by varying the parameter in THSR w,f,

The magnitude of form factor sensitively depends on the size of the Hoyle state.
Nevertheless, nicely reproduced!
Indicating that the calculated large rms radius, R_{rms}=3.8 fm is very reliable!
c.f. R_{rms}=2.4 fm for the g.s.

Y. F. et al., EPJ

Single-α potential given via Gross–Pitaevskii approach





.Coulomb barrier → quasi-stable states

.The barrier position is more than 8 fm. Trapped into a loose potential
(interaction range of Ali-Bodmer .4 fm)

→ Loosely bound α gas, α condensate state

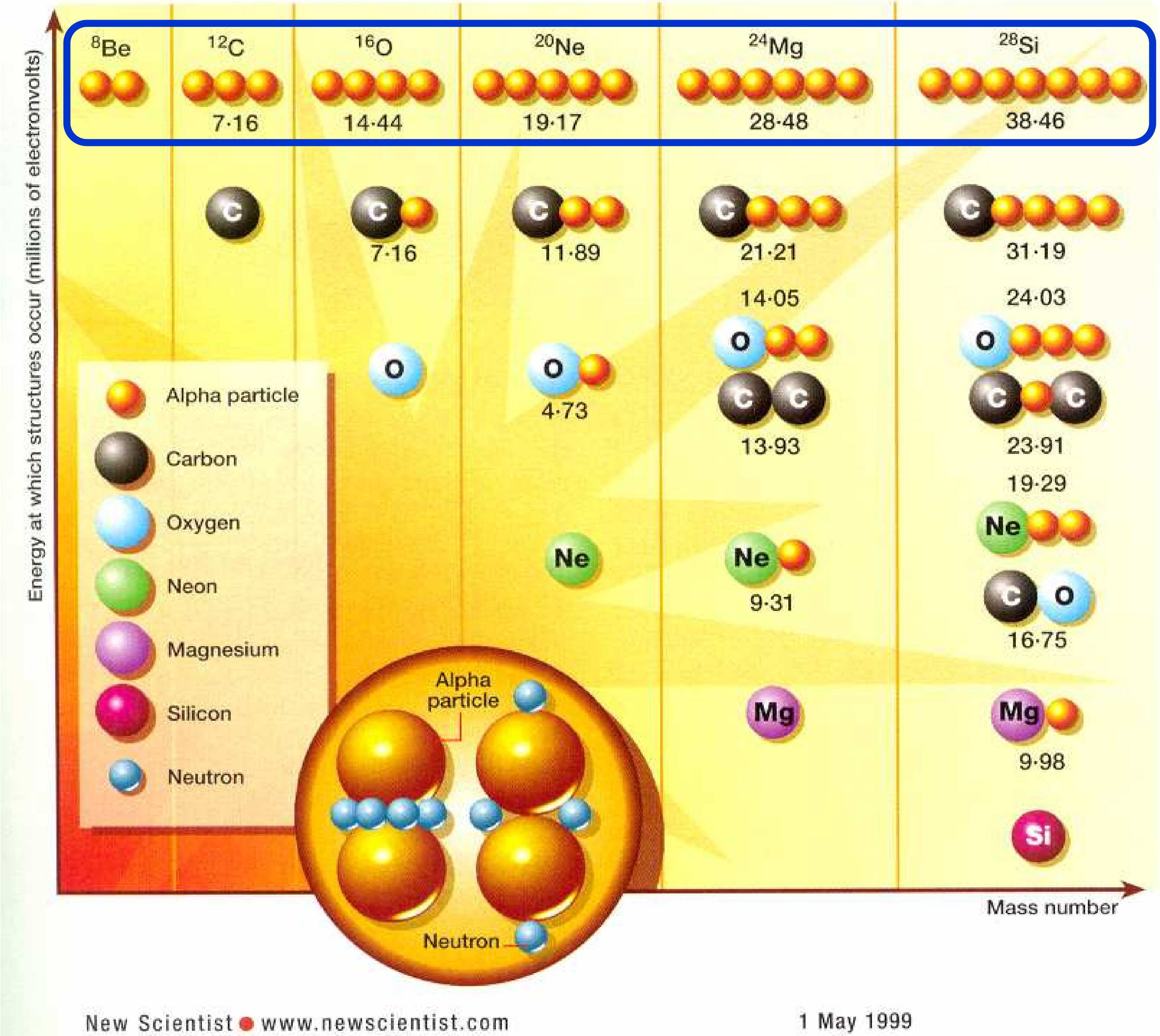
$$\Phi_{n\alpha} = \prod_{i=1}^{n} \phi(\boldsymbol{r}_i) \qquad \Big[-\frac{\hbar^2}{2m_\alpha}(1-\frac{1}{n})\nabla^2 + U(r)\Big]\phi(\boldsymbol{r}) = \varepsilon\phi(\boldsymbol{r})$$

.Pure bosons
.nα states are assumed

$$U(r) = (n-1)\int d\boldsymbol{r}'|\phi(\boldsymbol{r}')|^2 v_2(\boldsymbol{r}',\boldsymbol{r}) + \frac{(n-1)(n-2)}{2}\int d\boldsymbol{r}''d\boldsymbol{r}'|\phi(\boldsymbol{r}'')|^2|\phi(\boldsymbol{r}')|^2 v_3(\boldsymbol{r}'',\boldsymbol{r}',\boldsymbol{r})$$

T. Yamada and P. Schuck

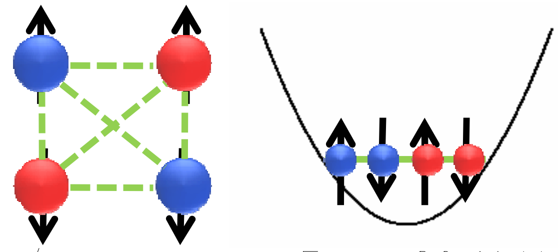
Prediction of cluster states in light nuclei (Ikeda Diagram)



Classified according to the Threshold Rule.

The most tightly bound light cluster

α particle (quartet)

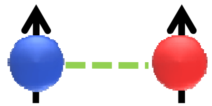


E/ A .7 MeV E .20 MeV

stiff

The most elemental subunit in nuclear cluster structures.

Pair (deuteron) is less bound in nuclear system.



E/ A .1 MeV

K. Ikeda et al., PTP suppl. Extra num., 464 (

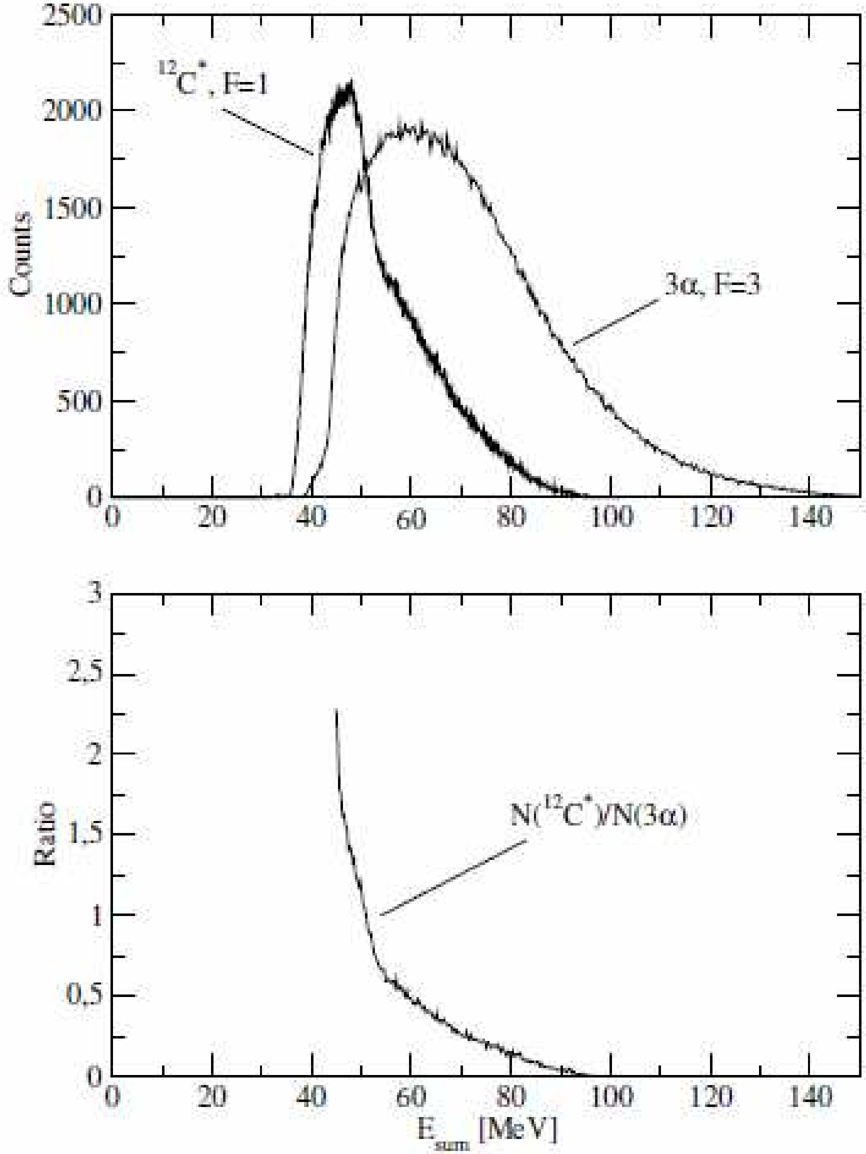
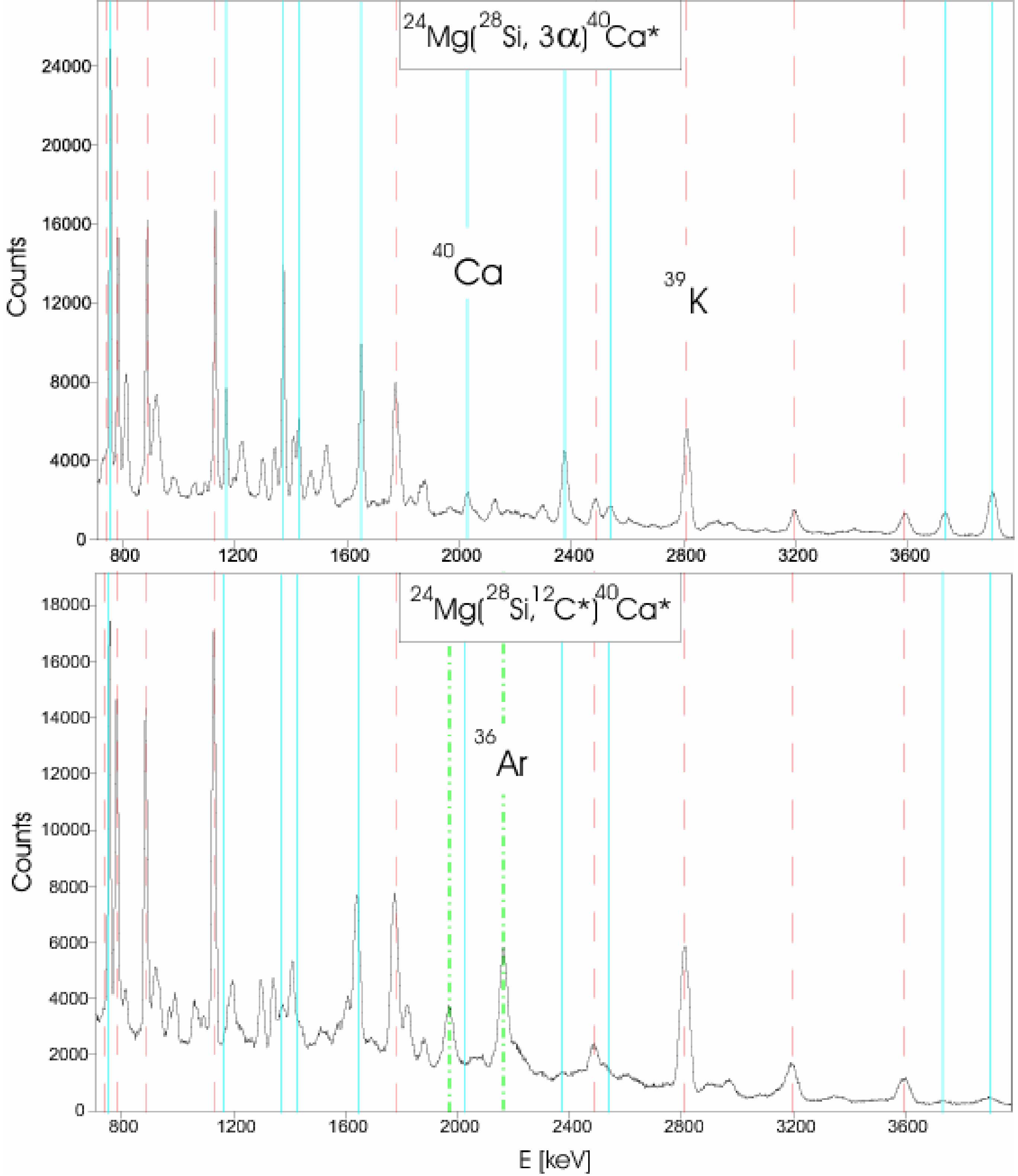


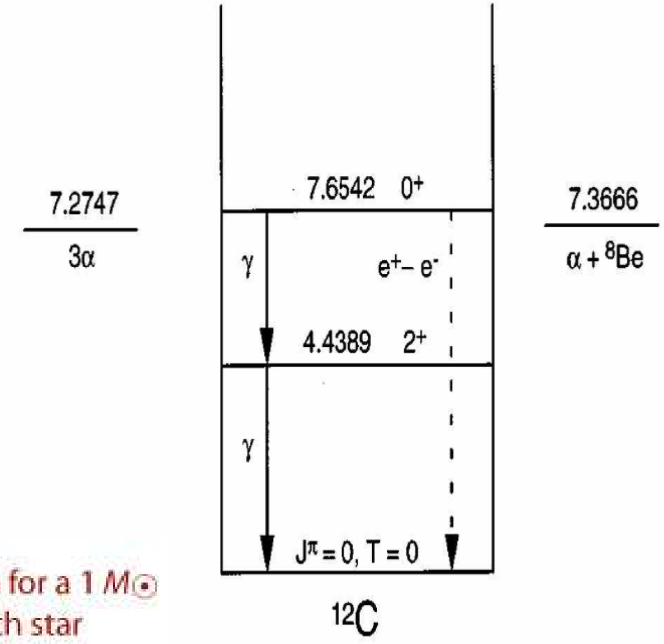
Fig. 9. Upper panel: Total energy spectra ($\Delta E+E$ signals) as observed with the ISIS charged-particle detector system for the emission of three single α's ($F=3$), and of $^{12}C^*(0_2^+)$ in the reaction $^{28}Si + {}^{24}Mg$. Note that the three-α curve is constructed from events with $F=3$, but with the energy scale multiplied by a factor of three to enable a comparison between the $^{12}C^*(0_2^+)$ and $F=3$ distributions. The vertical scale has been adjusted to show qualitatively the differences. Lower panel: the ratio between these two curves.



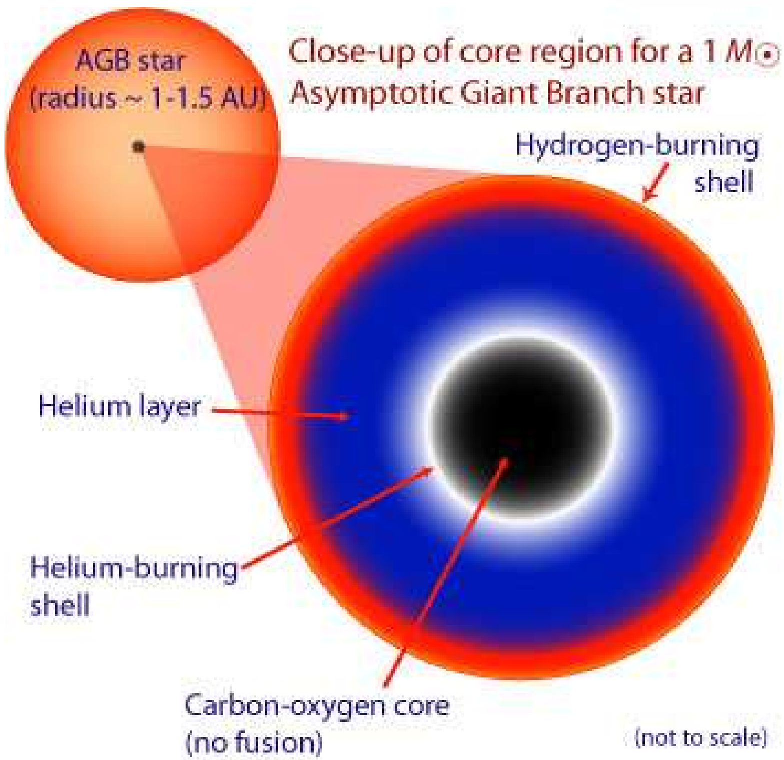
$^{24}Mg(^{28}Si, 3\alpha)^{40}Ca^*$
^{40}Ca
^{39}K
Counts
0
4000
8000
12000
16000
20000
24000
800
1200
1600
2000
2400
2800
3200
3600
$^{24}Mg(^{28}Si,^{12}C^*)^{40}Ca^*$
^{36}Ar
Counts
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
E [keV]

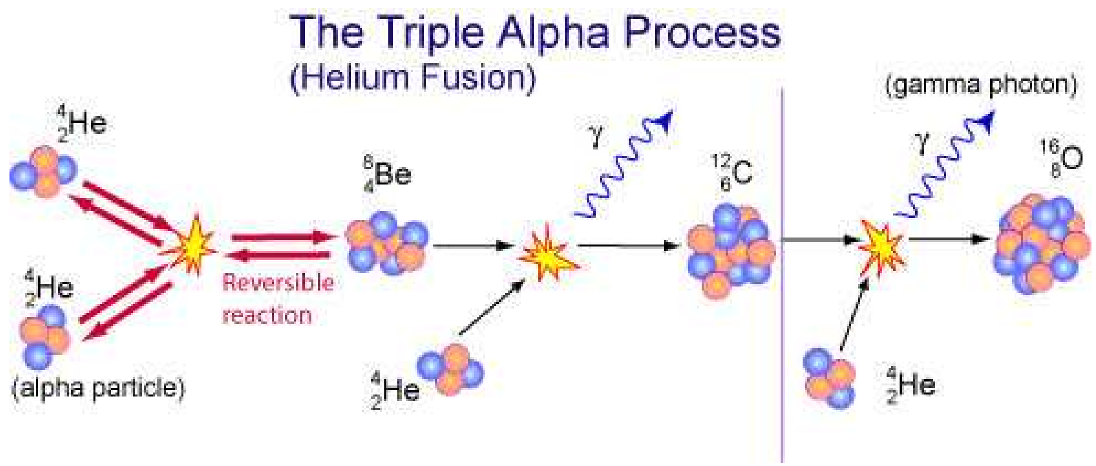
Hoyle state

.0_2^+ state of ^{12}C.



Fred Hoyle

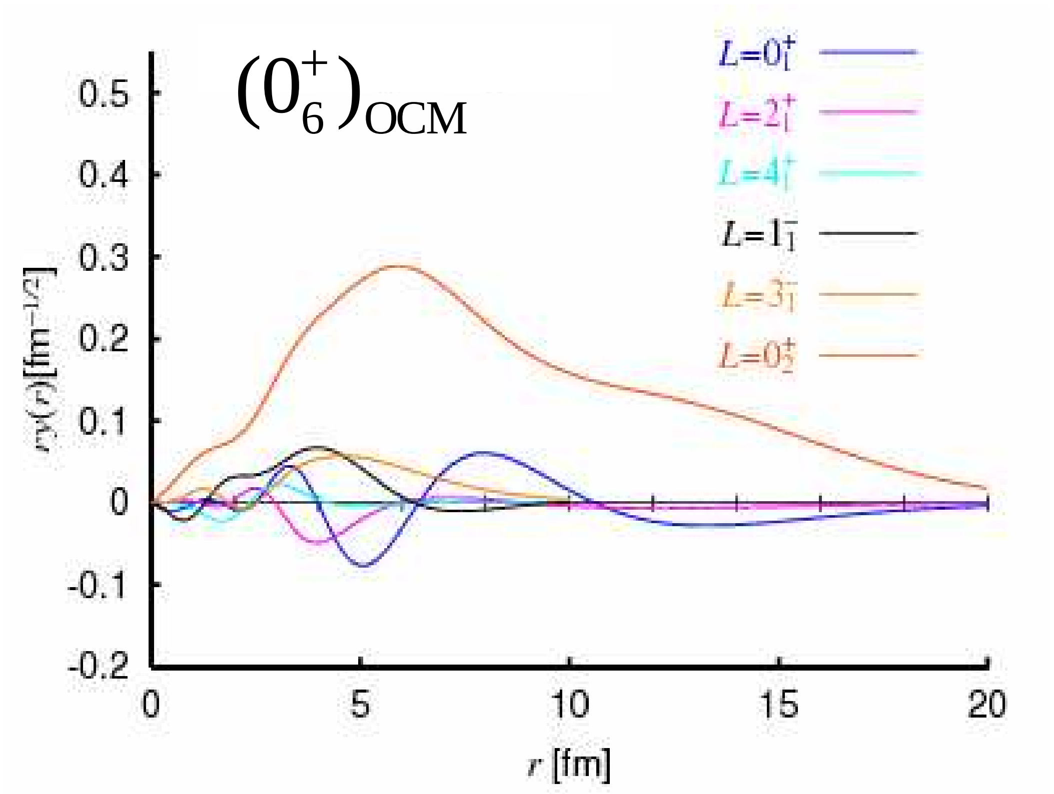




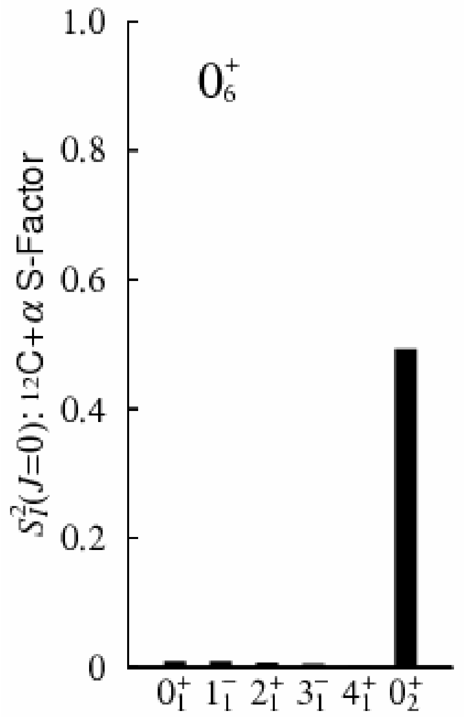
http://outreach.atnf.csiro.au/education/senior/astrophysics/stellarevolution_postmain.html

Reduced width amplitudes of 0_6^+ state obtained with 4αOCM

Defined as
$$r\times\mathcal{Y}_{IL,J=0}(r)=r\times\left\langle\left[\frac{\delta(r-r')}{rr'}Y_L(\hat{\mathbf{r}}')\Psi_{\rm OCM}(^{12}{\rm C}(I))\right]_0\middle|\Psi_{\rm OCM}(0_k^+)\right\rangle$$



$$S_{IL}^2(J=0)=\int dr\left(r\times\mathcal{Y}_{IL,J=0}(r)\right)^2$$



I: Spin of ^{12}C

α+^{12}C(Hoyle) configuration is dominant.
^{12}C(Hoyle): 3αcondensate

→ 4αcondensate

For ^{8}Be Full α RGM solution, which is given by superposing many Brink w.f.s is completely equivalent to a single THSR w.f. **99.99**..

Y. F. et al., PTP 108, 297 (2002); Y. F. et al., submitted to PRC

2αRGM eq.

$$(H - EN)\chi = 0 \Longleftrightarrow \left(\frac{1}{\sqrt{N}} H \frac{1}{\sqrt{N}} - E\right)\Psi_{2\alpha} = 0$$

$$\langle \chi | N | \chi \rangle = 1 \qquad \langle \Psi_{2\alpha} | \Psi_{2\alpha} \rangle = 1$$

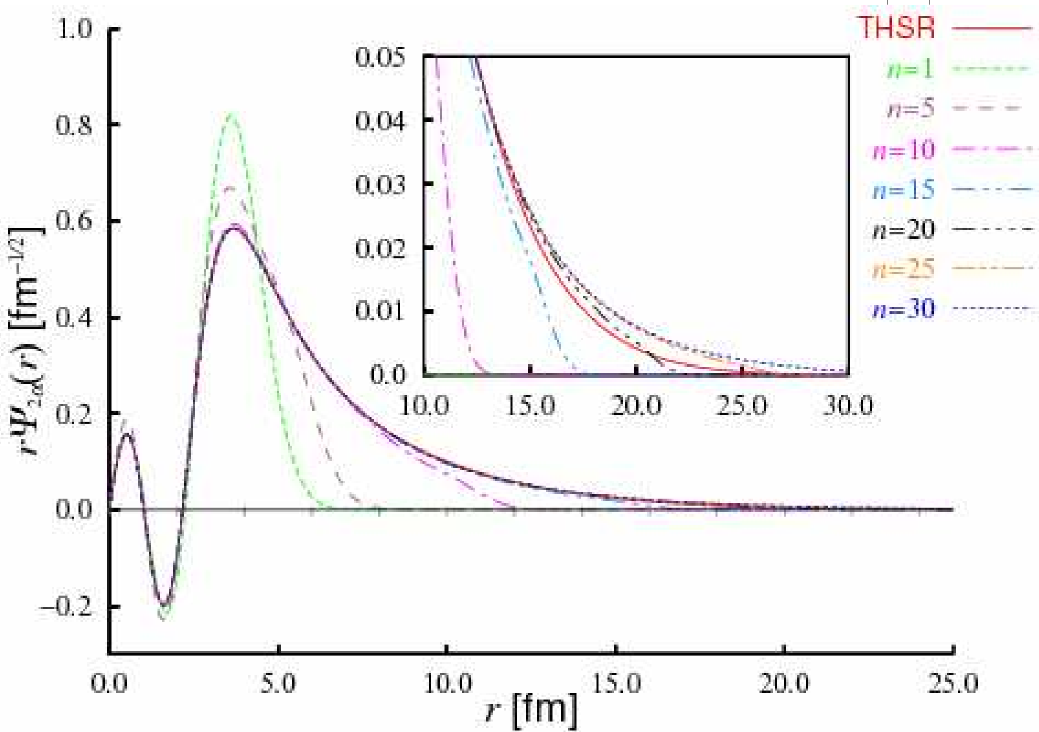
$$\left|\left\langle \Psi_{2\alpha}^{\text{Brink}} \middle| \Psi_{2\alpha}^{\text{THSR}} \right\rangle\right|^2 = 0.9999$$

$$\Psi_{2\alpha} = \sqrt{N}\chi = \int d^3\boldsymbol{b} \sqrt{N(\boldsymbol{a},\boldsymbol{b})}\chi(\boldsymbol{b})$$

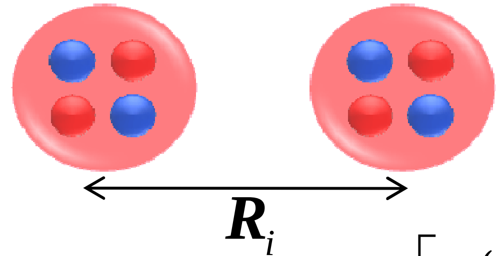
$$N(\boldsymbol{a},\boldsymbol{b}) = \langle \delta(\boldsymbol{r}-\boldsymbol{a})\phi(\alpha_1)\phi(\alpha_2) | \mathcal{A}[\delta(\boldsymbol{r}-\boldsymbol{b})\phi(\alpha_1)\phi(\alpha_2)] \rangle$$

$$H(\boldsymbol{a},\boldsymbol{b}) = \langle \delta(\boldsymbol{r}-\boldsymbol{a})\phi(\alpha_1)\phi(\alpha_2) | H | \mathcal{A}[\delta(\boldsymbol{r}-\boldsymbol{b})\phi(\alpha_1)\phi(\alpha_2)] \rangle$$

n: num. of superposition



Superposition of dumbells



$$\chi^{\text{Brink}}(\boldsymbol{r}) = \sum_{i=1}^{n} f(\boldsymbol{R}_i)\hat{P}_{J=0}\exp\left[-\frac{(\boldsymbol{r}-\boldsymbol{R}_i)^2}{b^2}\right]$$

$$\Psi_{2\alpha} = \sqrt{N}\chi^{\text{Brink}}$$

VS

THSR

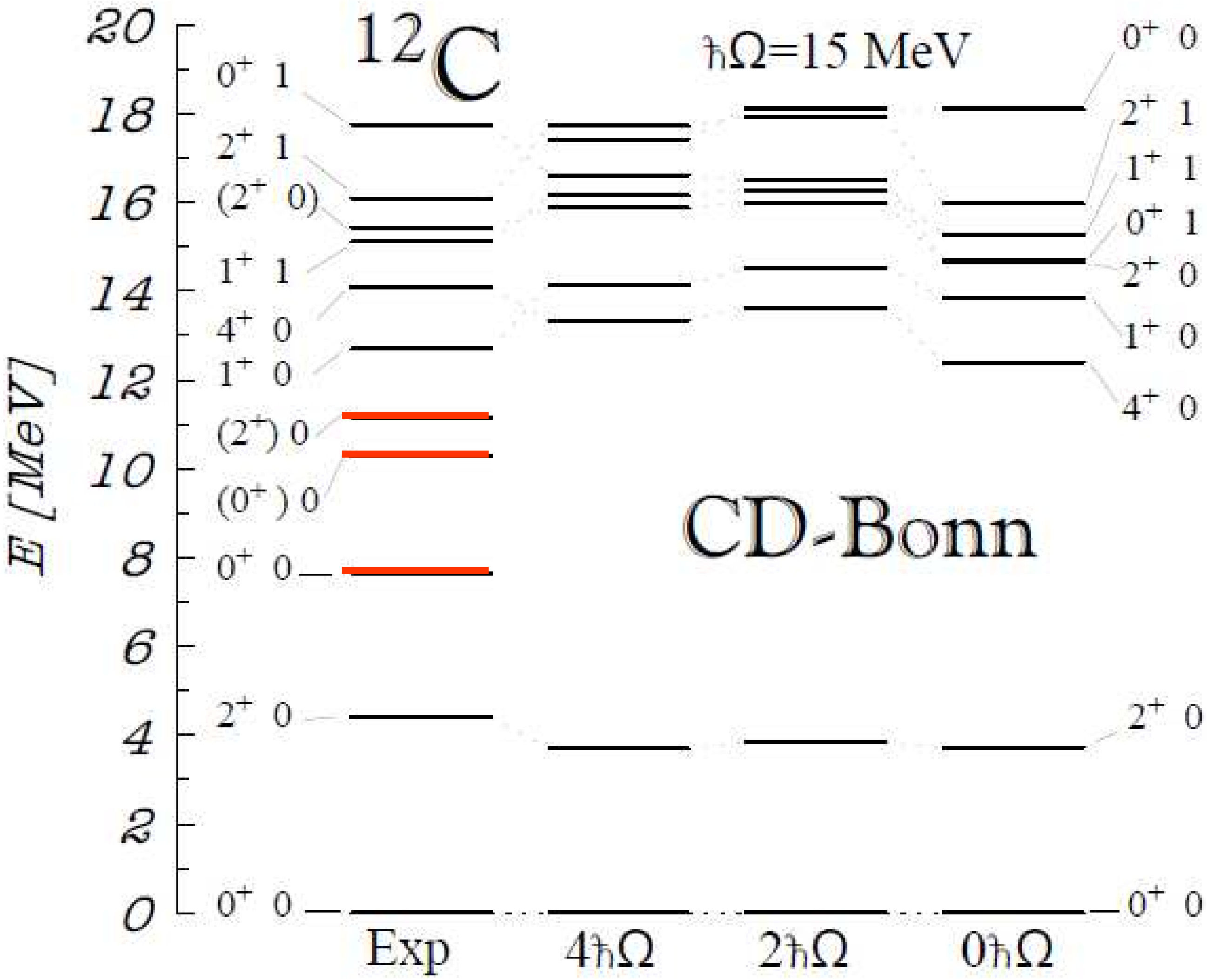
$$\chi^{\text{THSR}}(\boldsymbol{r}) = \hat{P}_{J=0}\exp\left[-\frac{r_x^2 + r_y^2}{B_\perp^2} - \frac{r_z^2}{B_z^2}\right]$$

$$\Psi_{2\alpha} = \sqrt{N}\chi^{\text{THSR}}$$

Typical mysterious 0^+ states in nuclear structure problem

0_2^+ state of ^{12}C (Hoyle state) indispensable to ^{12}C production in stars

Ab initio non-core shell model calculation



0_2^+ , 2_2^+ , 0_3^+states : missing

0_2^+ state: excitation energy is not lower than 20 MeV

The typical excited states which resist a shell model description

P. Navratil et al., PRL 84, 5728 (2001)

Hoyle state:
×shell structure
×3α linear chain structure
(by Morinaga)

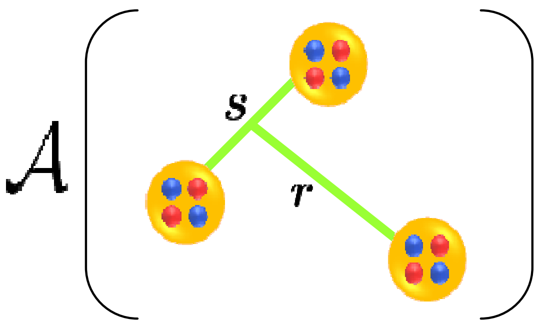
○ "Gas-like" 3α structure
coupled with relative S-waves
(3α OCM by Horiuchi)

	Exp.	Theor.
Energy (MeV)	7.65	7.74
α decay width (eV)	8.7±2.7	7.7
$M(0_2^+ \rightarrow 0_1^+)$ (fm^2)	5.4±0.2	6.7
$B(\ldots;\ 0_2^+ \rightarrow 2_1^+)$ ($e^2\ fm^4$)	13±4	5.6

.Many experimental data exist.
.Three-body (3α) problem was fully solved thirty years ago.
.The cluster model w. f. well reproduces almost all experimental data.
.Kamimura et al. (RGM), Uegaki et al. (GCM).

Resonating Group Method.....

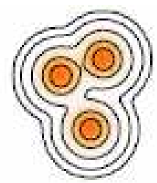
$$\langle \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)|H - E|\mathcal{A}[\chi(\boldsymbol{s},\boldsymbol{r})\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)]\rangle = 0$$



Fully solved without any model assumption
w.r.t. inter-α motions

3α clustering also appears starting without
assumption of α's by FMD & AMD

M. Chernykh, T. Neff et
Y. Kanada-En'yo, PTP 11



Condensate model

Particle number projected BCS w.f.

$$\langle \boldsymbol{r}_1, \cdots, \boldsymbol{r}_{2n} | \mathrm{BCS} \rangle = \mathcal{A}\Big\{ \Phi(\boldsymbol{r}_1, \boldsymbol{r}_2)\Phi(\boldsymbol{r}_3, \boldsymbol{r}_4)\cdots\Phi(\boldsymbol{r}_{2n-1}, \boldsymbol{r}_{2n}) \Big\}$$

n α condensate w.f.

$$\langle \boldsymbol{r}_1, \cdots, \boldsymbol{r}_{4n} | \Phi_{n\alpha} \rangle = \mathcal{A}\Big\{ \Phi(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \boldsymbol{r}_4)\Phi(\boldsymbol{r}_5, \boldsymbol{r}_6, \boldsymbol{r}_7, \boldsymbol{r}_8)\cdots\Phi(\boldsymbol{r}_{4n-3}, \boldsymbol{r}_{4n-2}, \boldsymbol{r}_{4n-1}, \boldsymbol{r}_{4n}) \Big\}$$

Variational ansatz (two parameters B and b)

(THSR ansatz) A. Tohsaki, H. Horiuchi, P. Schuck and G. Röpke et al., PRL 87, 192501 (2001).

$$\Phi(\boldsymbol{r}_{4i-3}, \cdots, \boldsymbol{r}_{4i}) = e^{-\frac{2}{B^2}(\boldsymbol{X}_i - \boldsymbol{X}_G)^2} \phi_\alpha(\boldsymbol{r}_{4i-3}, \cdots, \boldsymbol{r}_{4i})$$

$$\phi_\alpha \propto e^{-\frac{1}{8b^2}\sum_{k<l}(\boldsymbol{r}_k - \boldsymbol{r}_l)^2}$$

c.o.m. of i-th α particle

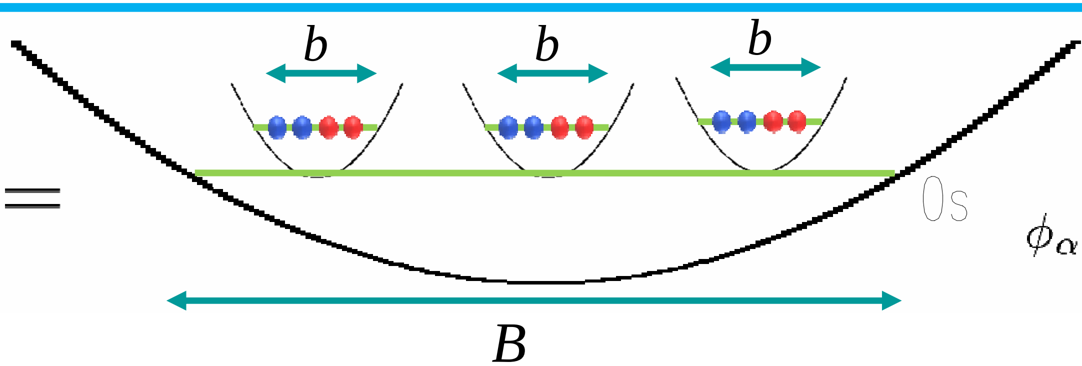
$$\boldsymbol{X}_i = \frac{\boldsymbol{r}_{4i-3} + \cdots + \boldsymbol{r}_{4i}}{4}$$

Total c.o.m.

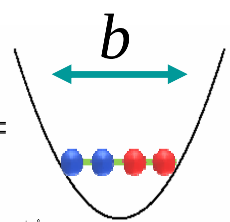
$$\boldsymbol{X}_G = \frac{\boldsymbol{r}_1 + \cdots + \boldsymbol{r}_{4n}}{4n}$$

n=3 case

$$\Phi_{3\alpha}(B, b) =$$



$$\phi_\alpha(\boldsymbol{r}_1, \cdots, \boldsymbol{r}_4) \doteq$$



$0s^4$ configuration

Two limits

B =b. Slater determinant

B >>b. Gas of independent α-particles