Three-Nucleon Forces for Medium-Mass Neutron-Rich Nuclei: A Microscopic Approach

Jason D. Holt

Collaborators: T. Otsuka, A. Schwenk, T. Suzuki

THE UNIVERSITY of TENNESSEE KNOXVILLE

TRIUMF

OAK RIDGE NATIONAL LABORATORY

Second EMMI-EFES Workshop on Neutron-Rich Nuclei
June 17, 2010

Outline

- **Microscopic Theory for Valence Shell Effective NN Interactions**
  - Many-body perturbation theory: methods and interactions
  - Deficiencies: revealed in monopole interaction, oxygen properties
- **3N forces**
  - Chiral Effective Field Theory
  - Inclusion in valence-shell interactions: 1- and 2-body parts
- **Impact on Nuclear Structure**
  - **2-body 3N**: monopole components of valence-shell interaction
  - Evolution of single particle energies (SPE): dripline in oxygen isotopes
  - **1-body 3N**: contribution to microscopic *sd*-shell SPE
    - Parameter-free shell model calculations: impact on spectra

Many-body Problem for Finite Nuclei

- Various methods to solve many-body problem: Coupled Cluster, NCSM, In-medium SRG – we use **many-body perturbation theory (MBPT)**

- Solve the many-body Schrödinger equation for nuclear systems: $H\psi = E\psi$

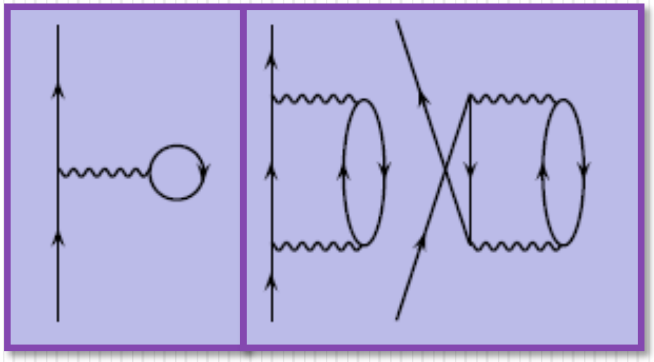
- Impossible to solve in heavy systems in complete Hilbert space

- Consider problem in truncated (model) space defined by projection operator *P*:

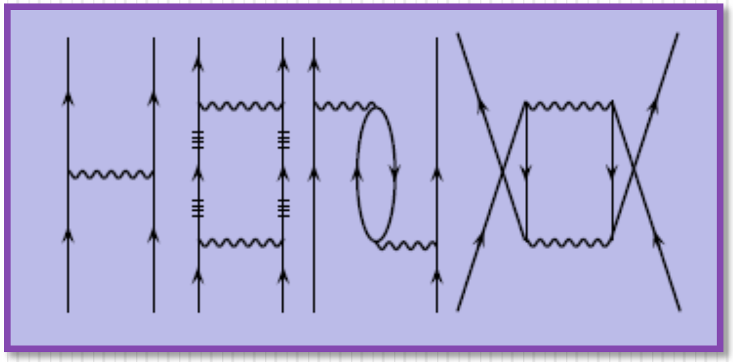
$$PH_{eff}P\psi = EP\psi; \quad H_{eff} = H_0 + V_{eff}$$

and V_{eff} acts in the model space given by *P*

Folded-diagrams: method to construct effective interaction perturbatively



1-body to 2nd order



2-body to 2nd order

Monopole Part of Interaction

- **Microscopic MBPT** typically works for few particles/holes away from closed shell: deteriorates beyond this

- Deficiencies in microscopic interactions can be improved by adjusting a particular set of two-body matrix elements (TBME):

Angular average of interaction

Determines interaction of orbit *a* with *b*: *evolution of orbitals*

$$V_{ab}^{T} = \frac{\sum_J (2J+1)\, V_{abab}^{JT}\, [1-(-1)^{J+T}\delta_{ab}]}{\sum_J (2J+1)[1-(-1)^{J+T}\delta_{ab}]}$$

Phenomenological shell model interactions typically start from MBPT results then exploit importance of monopoles:

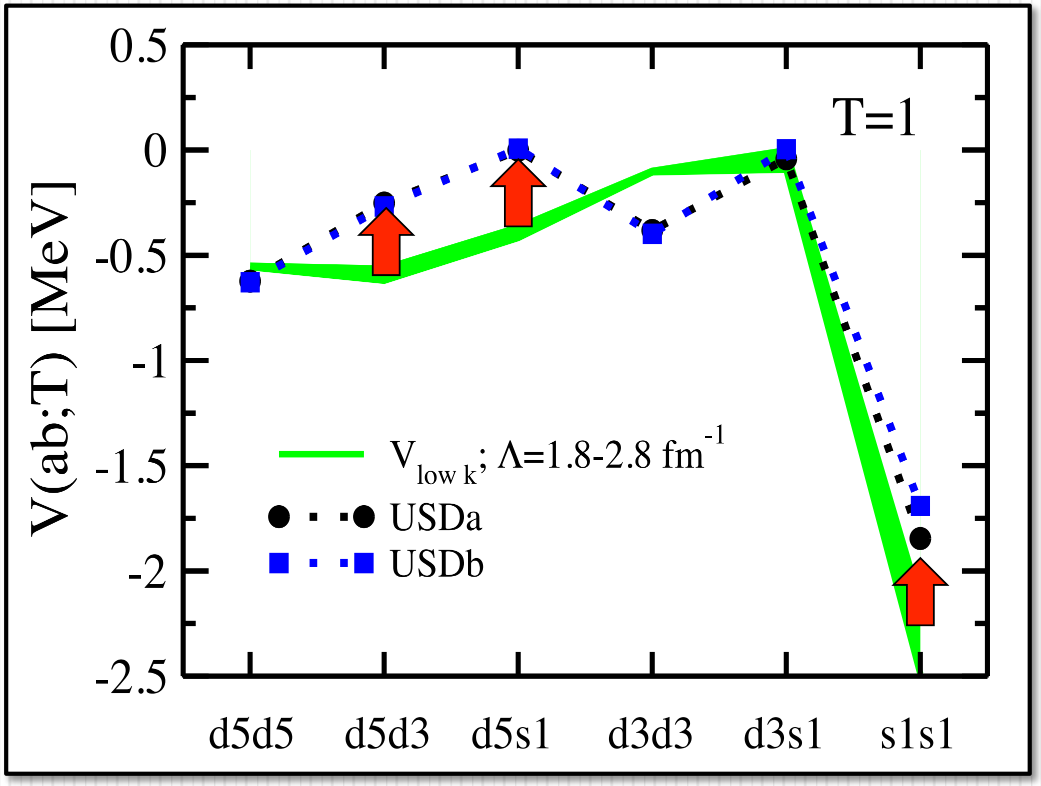
***sd*-shell**: 63 TBME - USD (1984), USDa, USDb (2006)

- global fit of single particle energies (SPEs) and two-body matrix elements; monopoles most important

***pf*-shell:** 195 TBME

- GXPF1 (2004): quasi-global fit; monopoles most important
- KB3G(2001): modification of monopole part only

Phenomenological vs. Microscopic



Compare monopoles from:

- *Microscopic* **low-momentum** interactions
- *Phenomenological* **USD** interactions

Clear shifts for *low-lying orbitals*:

- T=1 repulsive shift

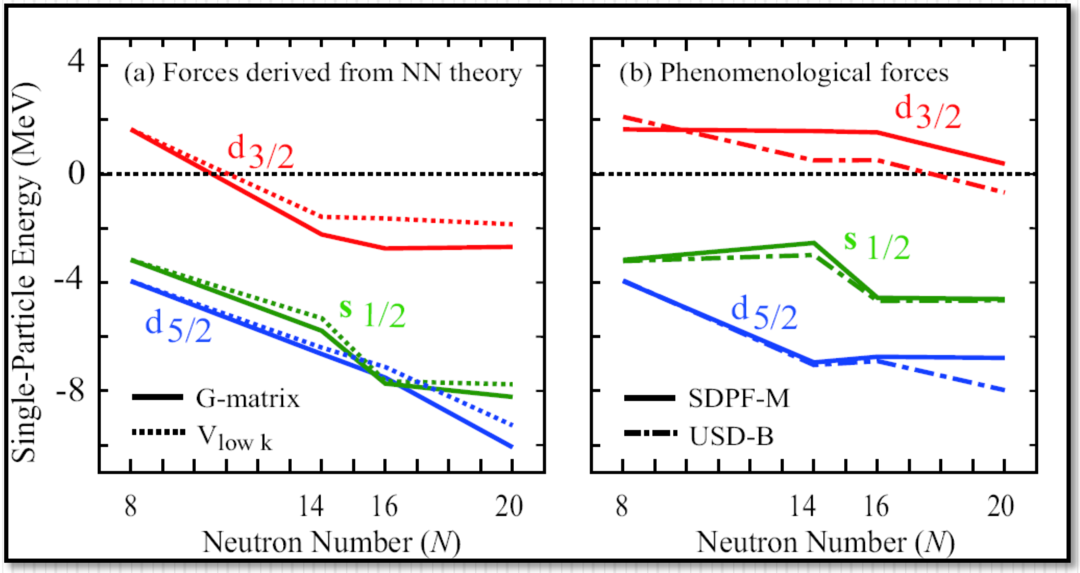
Incorrect hierarchy for $d_{5/2}$-$d_{5/2}$ vs. $d_{5/2}$-$d_{3/2}$

** Origin of the shifts: Can neglected 3N forces explain this?

-- Proposed by **A. Zuker (2003)**

Physical Implications in Oxygen

Monopoles drive *evolution* of single particle energies



Phenomenological Forces

Large gap for ^{22}O

$d_{3/2}$ orbit is unbound

Microscopic NN Theory

No shell gap at ^{22}O

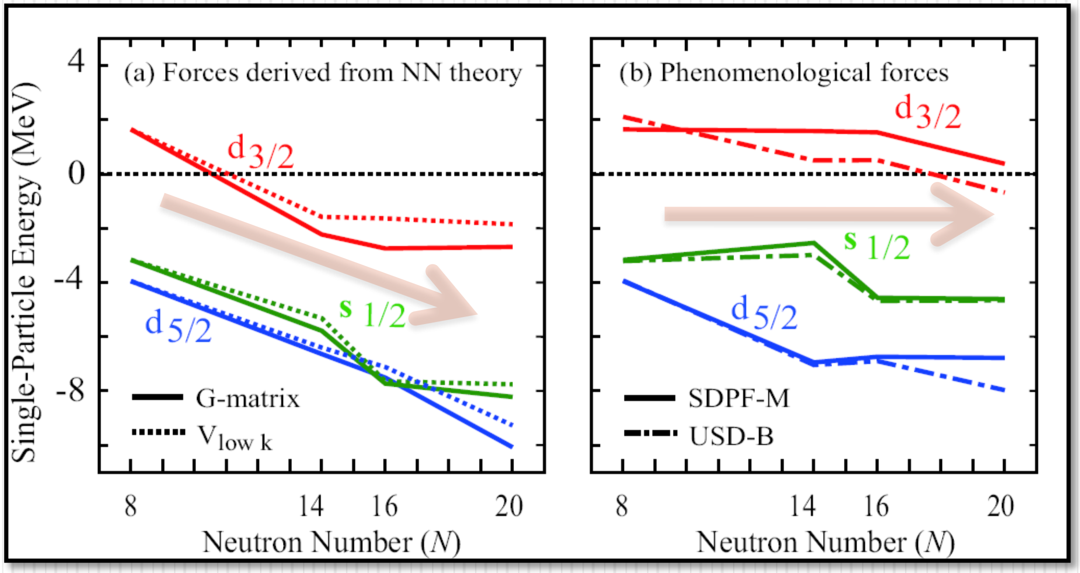
→ Compressed spectra

$d_{3/2}$ orbit is strongly bound

→ Implications for dripline

Physical Implications in Oxygen

Monopoles drive *evolution* of single particle energies



Phenomenological Forces

Large gap for ^{22}O

$d_{3/2}$ orbit is unbound

Microscopic NN Theory

No shell gap at ^{22}O

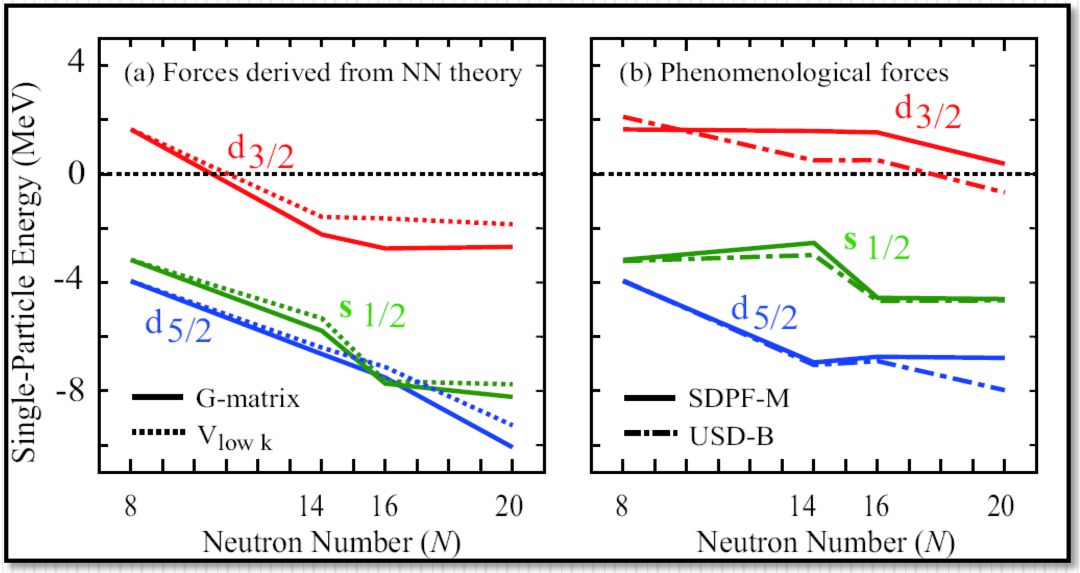
→ Compressed spectra

$d_{3/2}$ orbit is strongly bound

→ Implications for dripline

Physical Implications in Oxygen

Monopoles drive *evolution* of single particle energies



Phenomenological Forces

Large gap for ^{22}O

$d_{3/2}$ orbit is unbound

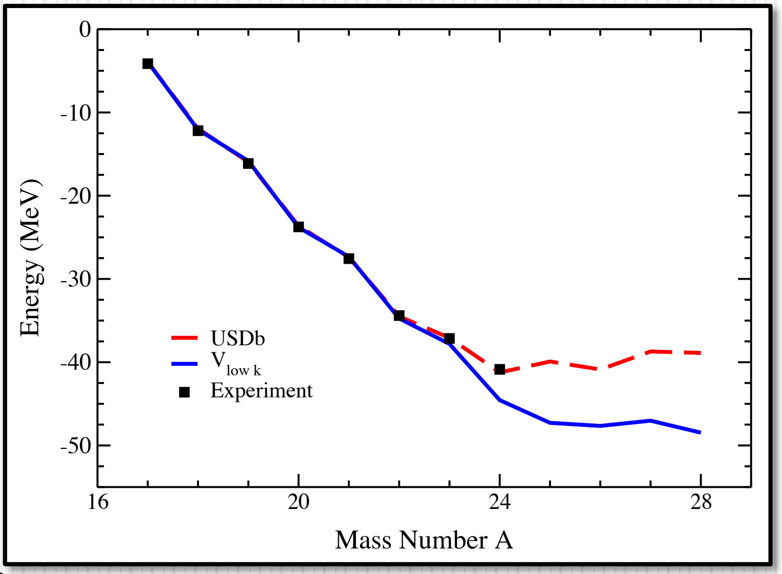
Microscopic NN Theory

No shell gap at ^{22}O

→ Compressed spectra

$d_{3/2}$ orbit is strongly bound

→ Implications for dripline



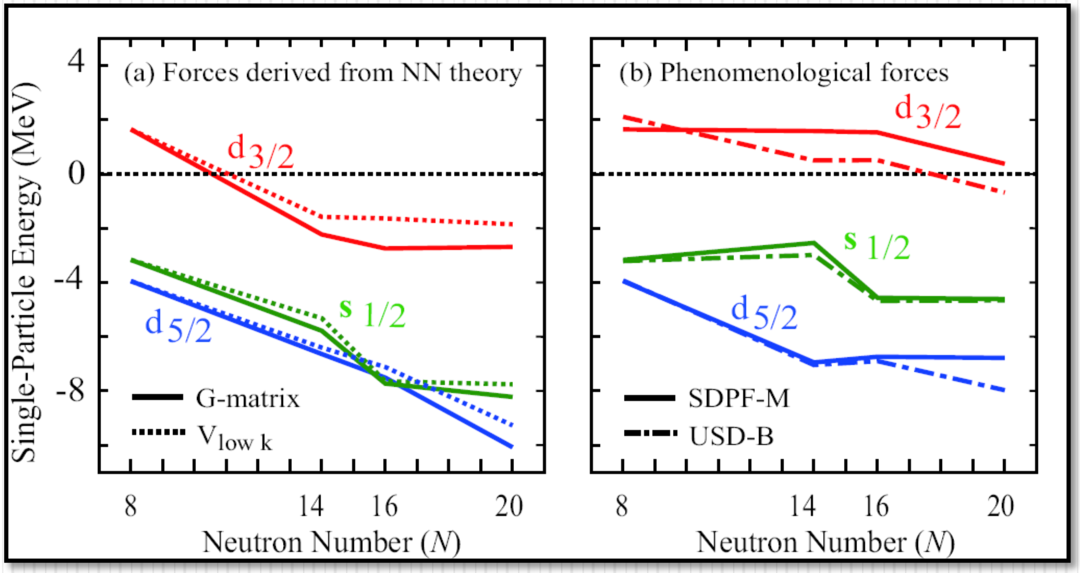
Oxygen: experimentally established dripline at ^{24}O

USDb: O unbound beyond ^{24}O

MBPT: bound through ^{28}O

Physical Implications in Oxygen

Monopoles drive *evolution* of single particle energies



Phenomenological Forces

Large gap for ^{22}O

$d_{3/2}$ orbit is unbound

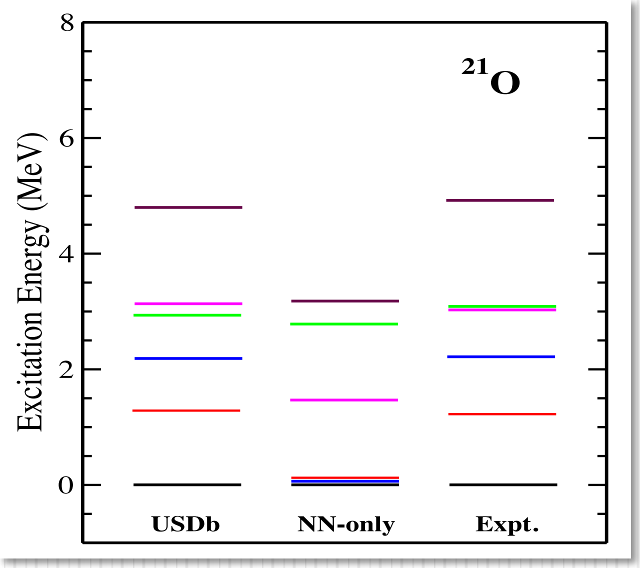
Microscopic NN Theory

No shell gap at ^{22}O

→ Compressed spectra

$d_{3/2}$ orbit is strongly bound

→ Implications for dripline



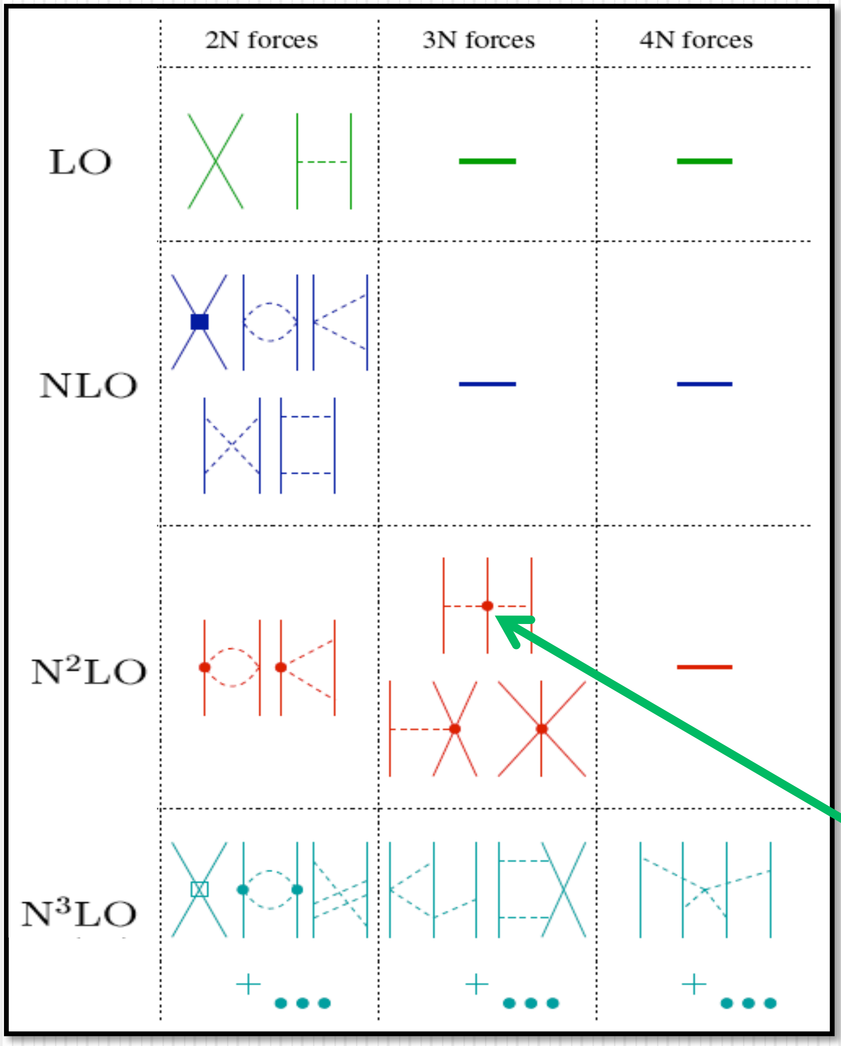
Oxygen: experimentally established dripline at ^{24}O

USDb: O unbound beyond ^{24}O

MBPT: bound through ^{28}O

Spectra much too compressed beyond ^{18}O

Chiral Effective Field Theory



Nucleons interact via π exchange
and contact interactions

Explains hierarchy: $V_{NN} > V_{3N} > \ldots$

Short-range couplings fit to experiment

Systematic way to include 3NF

Only two new couplings at N^{2}LO:
c terms: already constrained by
NN, πN data

$$c_1 = -0.9^{+0.2}_{-0.5},\ c_3 = -4.7^{+1.2}_{-1.0}$$
$$c_4 = 3.5^{+0.5}_{-0.2}$$

No new couplings at N^{3}LO N^{4}LO

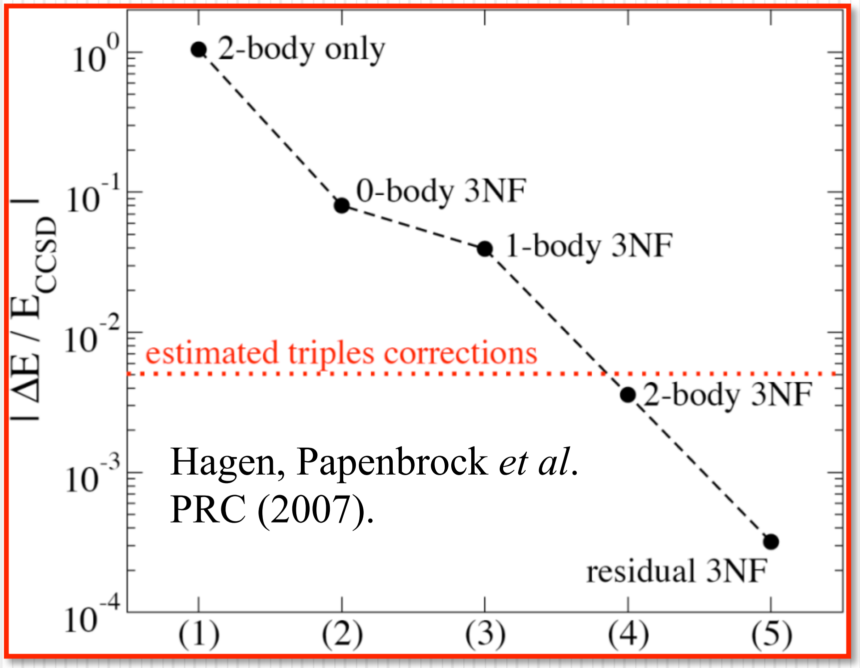
Chiral 2N: large cutoffs not suitable for MBPT – need to renormalize…
Evolve to lower cutoff using **RG methods** (smooth regulator):

3N Forces for Valence-Shell Theories

$V_{\mathrm{low}\,k}(\Lambda)$ + N^{2}LO Chiral $V_{3N}(\Lambda)$

D(Λ), E(Λ) couplings fit in light systems

Approach: inspired by benchmark **Coupled Cluster** results for ^{4}He with 3N



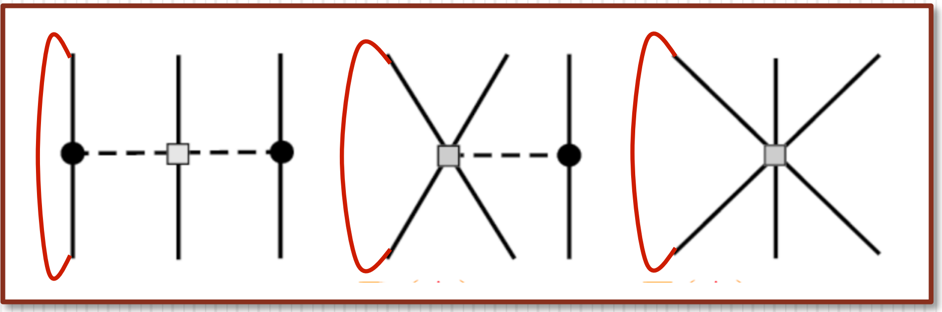
Hagen, Papenbrock *et al.* PRC (2007).

0- 1- and 2-body parts (summed over occupied states) of 3NF dominate:

Neglect residual 3NF

Generate **effective two-body** force from 3N by similar sum (as in nuclear matter)

$$\langle ab | V_{3N,\mathrm{eff}} | a'b' \rangle = \sum_{\alpha = core} \langle ab\alpha | V_{3N} | a'b'\alpha \rangle$$

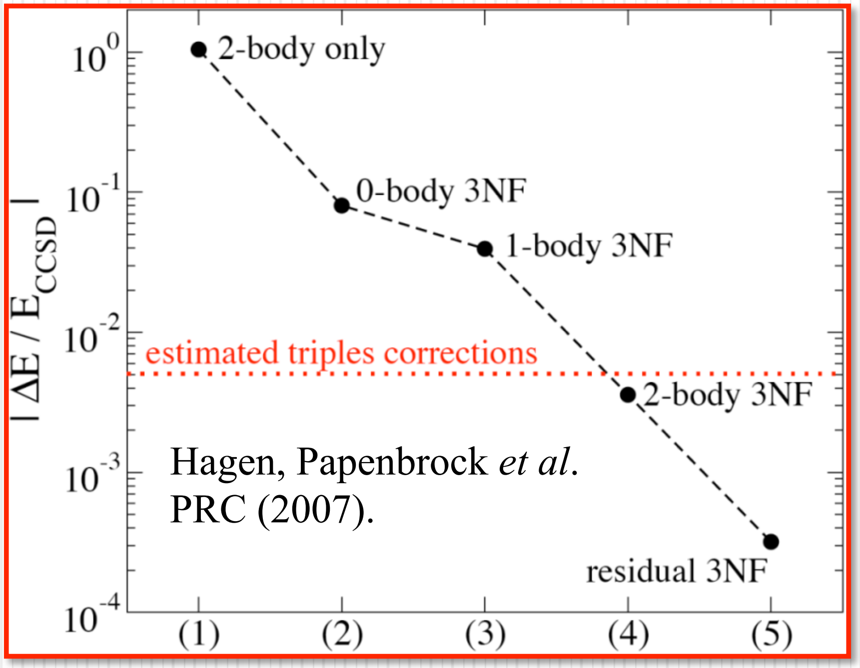


3N Forces for Valence-Shell Theories

$V_{\text{low }k}(\Lambda)$ + N^{2}LO Chiral $V_{3N}(\Lambda)$

$D(\Lambda)$, $E(\Lambda)$ couplings fit in light systems

Approach: inspired by benchmark **Coupled Cluster** results for ^{4}He with 3N

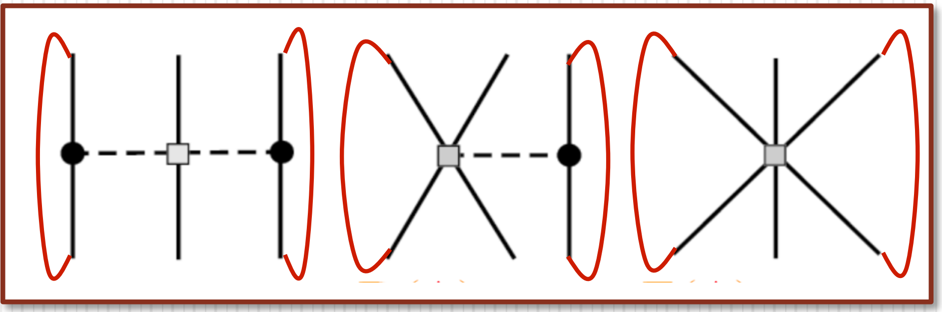


0- 1- and 2-body parts (summed over occupied states) of 3NF dominate:

Neglect residual 3NF

Generate **effective two-body** force from 3N by similar sum (as in nuclear matter)

$$\langle ab | V_{3N,\text{eff}} | a'b' \rangle = \sum_{\alpha = core} \langle ab\alpha | V_{3N} | a'b'\alpha \rangle$$



Effective one-body contribution:

$$\langle a | V_{3N,\text{eff}} | a' \rangle = \frac{1}{2} \sum_{\alpha\beta = core} \langle a\alpha\beta | V_{3N} | a'\alpha\beta \rangle$$

- 3N forces tractable in shell model

Calculation Details

Focus on $T=1$ monopoles and systems in the following details:

① NN matrix elements derived from:

- Chiral N^{3}LO (Machleidt, 500 MeV) using smooth-regulator $V_{\text{low }k}$ with range of cutoffs
- 3$^{\text{rd}}$-order in MBPT
- 20$\hbar\omega$ intermediate state configurations (converged)

② 3N forces: calculate monopole components from:

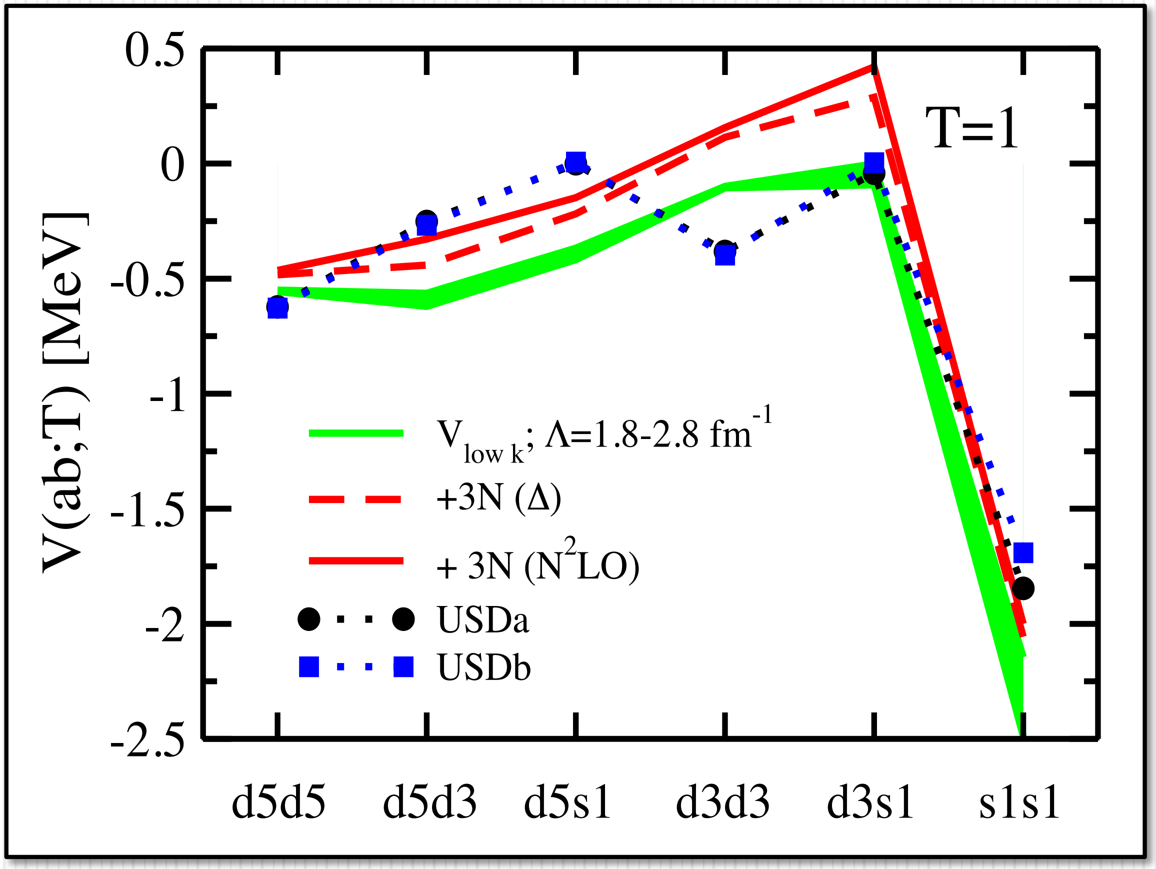
A) Chiral N^{2}LO fit to above $V_{low\ k}$ with $\Lambda = 2.0$ fm^{-1}

B) One-Delta excitation from N^{2}LO: specific choice of ***c*-terms**

Converged in 3NF partial waves up to $J \leq \frac{7}{2}$

Included to **first order** in perturbation theory

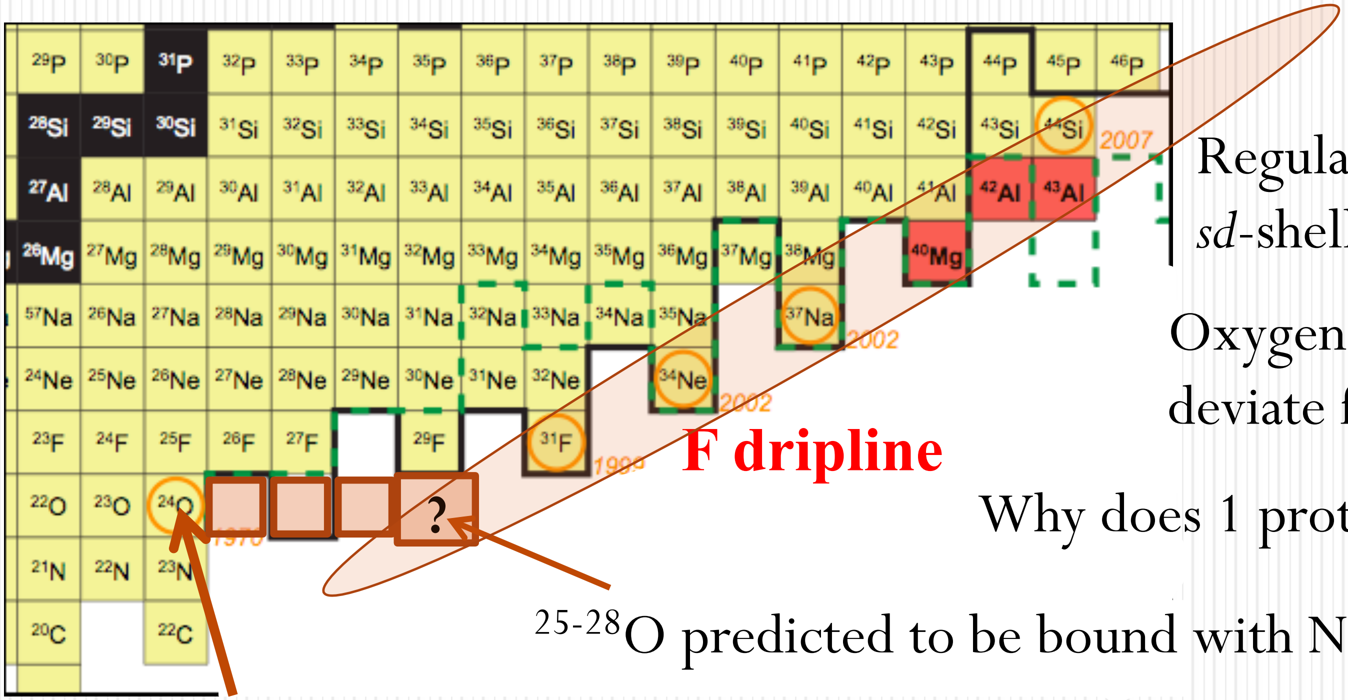
Two-body 3N: Monopoles in *sd*-shell



- Dominant effect from **one-Δ** – as expected from cutoff variation
- 3N forces produce clear repulsive shift in monopoles

- First calculations to show missing monopole strength due to neglected 3N
- Restores monopole hierarchy $d_{5/2}$-$d_{5/2}$ vs. $d_{5/2}$-$d_{3/2}$
- **Future**: Improved treatment of high-lying orbits – treat as holes in ^{40}Ca core

Oxygen-Flourine Anomaly



F dripline

Regular trend for dripline of *sd*-shell nuclei

Oxygen dripline observed to deviate from this trend

Why does 1 proton change so much?

$^{25\text{-}28}$O predicted to be bound with NN-only

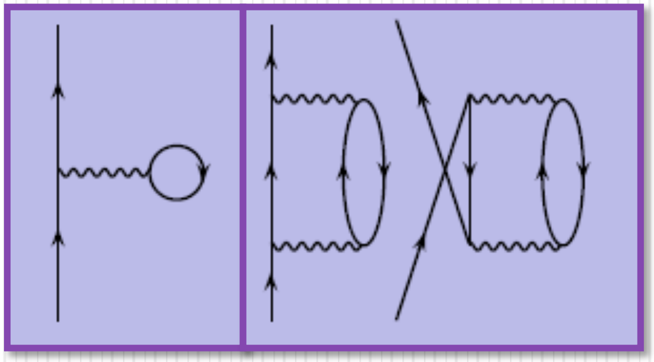
Experimental Oxygen dripline observed at ^{24}O

Monopole changes multiplied by neutron number – small changes will impact neutron-rich regions

Use 3N forces to investigate this anomaly – probe limits of nuclear existence with microscopic theory

One-body 3N: Calculation of SPEs

So far phenomenological SPEs: NN-only microscopic SPE yield “poor” results

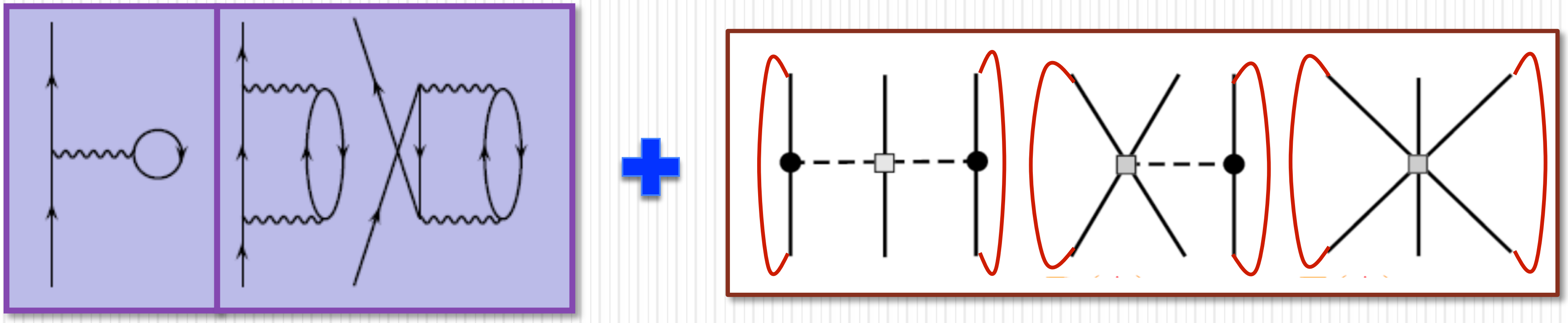


- *sd*-shell: Self-consistent calculation in MBPT $20\hbar\omega$ (converged) to 3rd-order
- NN-only insufficient: consistent with similar studies (Coraggio, *et. al*, 2007)

Orbit	“Exp”	USDb	NN
$d_{5/2}$	-4.14	-3.93	-5.43
$s_{1/2}$	-3.27	-3.21	-5.32
$d_{3/2}$	0.944	2.11	-0.97

One-body 3N: Calculation of SPEs

So far phenomenological SPEs: NN-only microscopic SPE yield “poor” results



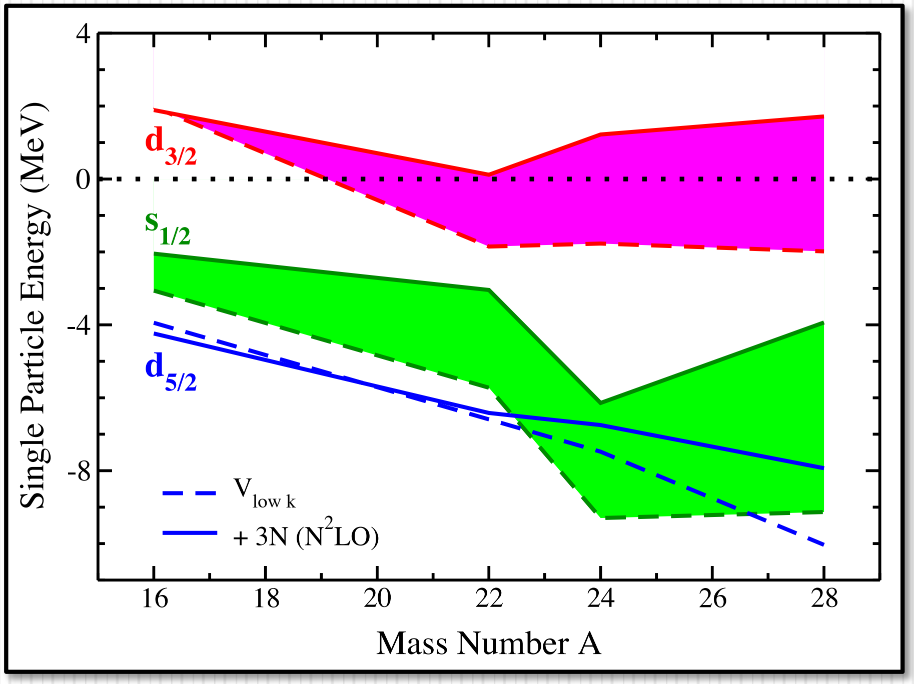
- *sd*-shell: Self-consistent calculation in MBPT $20\hbar\omega$ (converged) to 3rd-order
- NN-only insufficient: consistent with similar studies (Coraggio, *et. al*, 2007)

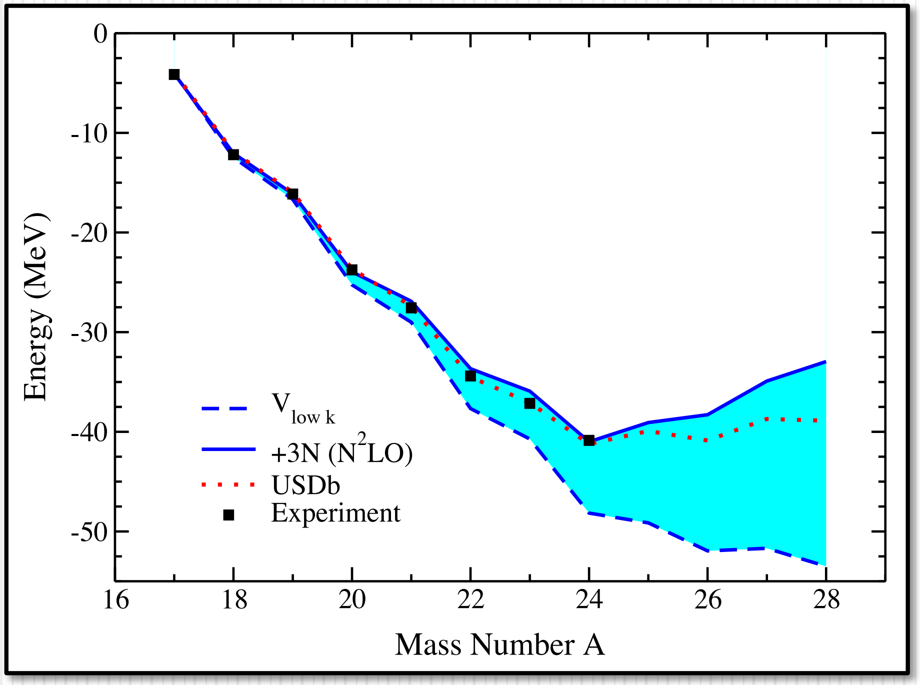
Orbit	**“Exp”**	**USDb**	**NN**	**3N**	**NN+3N**
$d_{5/2}$	-4.14	-3.93	-5.43	1.36	-4.07
$s_{1/2}$	-3.27	-3.21	-5.32	3.39	-1.93
$d_{3/2}$	0.944	2.11	-0.97	3.06	2.09

- Consistent with CC hierarchy: 1-body 3N > 2-body 3N ~ order of magnitude
- Microscopic SPEs: Reasonable agreement with USD, experimental

Fully-Microscopic Calculations

- Does the same conclusions hold when using microscopic SPEs?
- Use microscopic NN+3N monopole matrix elements and NN+3N SPEs
- Compare with NN-only MBPT, USD SPEs

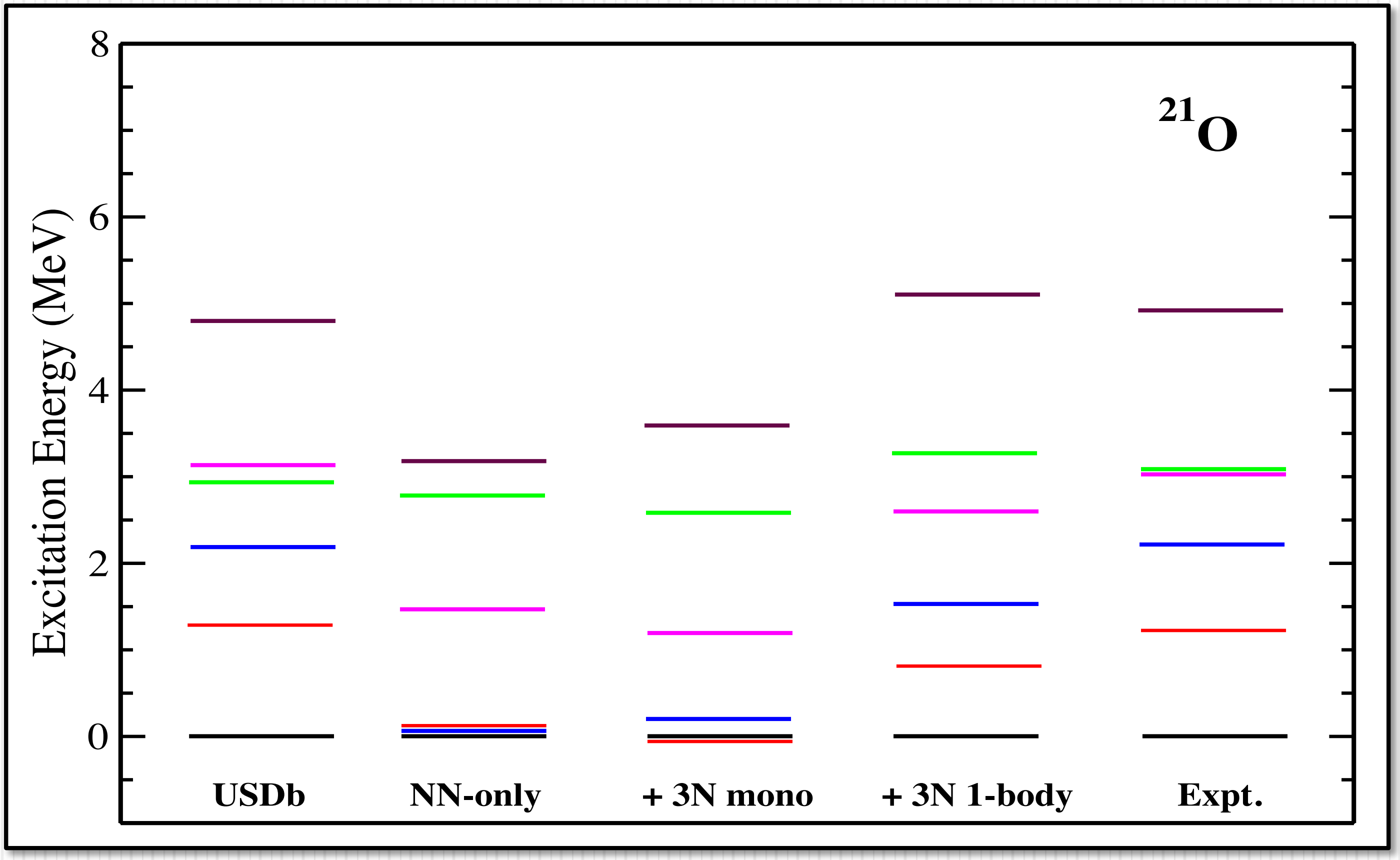




- ***Fully-microscopic*** calculation still predicts dripline at ^{24}O

Impact on Spectra

- ***Fully-microscopic*** calculation of spectra in oxygen isotopes



- NN-only: poor agreement with experiment
- 3N monopoles offer some help: spectrum still too compressed
- Using microscopic 3N monopoles, NN+3N SPEs: beneficial for spectrum
  - Correct ordering, improved spacing – need to include full 3N multipoles

Outlook

- Exploring frontiers of nuclear structure of medium-mass nuclei with 3N forces
- **2-body 3NF**: contribution to TBME – monopoles
  - **Repulsive shift** seen in T=1 monopoles due to 3N forces in *sd*- and *pf*-shells
- First shell model results in *sd*-, *pf*-shells using chiral 3N forces:
  - Leads to correct predicted binding energies and evolution of shell structure
  - Cures NN-only failings: Dripline, spectra in oxygen, shell gap in ^{48}Ca
- **1-body 3NF**: Microscopic SPEs in *sd*-shell – parameter-free shell model calculations
- **Near Future**:
  - T=0: need NN-3N to **2nd order**
  - Ni, Sn Isotopes
  - Continuum effects near driplines with K. Tsukiyama (Tokyo)

Thanks to Collaborators: T. Otsuka (Tokyo) , A. Schwenk (Darmstadt), T. Suzuki (Nihon U.)

Travel support from JUSTIPEN