In-medium Similarity Renormalization Group for nuclei

Department of Physics, The University of Tokyo
Koshiroh Tsukiyama

Essential Points

I. Similarity renormalization group (SRG)

II. SRG transformation in many-body medium

- ✤ Decoupling of ground states
- ✤ Non perturbativeness
- ✤ Size-extensivity
- ✤ (Potential applicability to $V_{\mathrm{eff}}/O_{\mathrm{eff}}$)

Nuclei From Scratch

Description of nuclei from nucleonic degrees of freedom (ab initio).

- Binding-energy systematics
- Low-lying excitations and spectroscopy
- Collective excitations with Large- or small amplitude
- Phenomena at the extreme conditions (T, J, N/Z ...)

Nuclei From Scratch

Description of nuclei from nucleonic degrees of freedom (ab initio).

- Binding-energy systematics
- Low-lying excitations and spectroscopy
- Collective excitations with Large- or small amplitude
- Phenomena at the extreme conditions (T, J, N/Z ...)

Tremendous progress has been made over the past decade

- Green's Function Monte Carlo (A=12)
- No-core Shell Model (A=14)
- Coupled-Cluster/ Unitary Model Operator Approach (near closed shell)
- Green's function method (quasi particle structure around closed shell)

Nuclei From Scratch

Description of nuclei from nucleonic degrees of freedom (ab initio).

- Binding-energy systematics
- Low-lying excitations and spectroscopy
- Collective excitations with Large- or small amplitude
- Phenomena at the extreme conditions (T, J, N/Z ...)

Tremendous progress has been made over the past decade

- Green's Function Monte Carlo (A=12)
- No-core Shell Model (A=14)
- Coupled-Cluster/ Unitary Model Operator Approach (near closed shell)
- Green's function method (quasi particle structure around closed shell)

One obstacle to extending such calculations to heavier nuclei

=> Strong coupling between low- and high-momentum states induced by the short-distance details of typical two- and three-nucleon interactions, from low-energy scattering data and deuteron.

Understanding nuclei from RG perspective

Nuclear Hamiltonian is ”resolution” dependent

$$H(\Lambda) = T + V^{(2)}(\Lambda) + V^{(3)}(\Lambda) \cdots$$

Relevant details of high-energy physics ==> Λ-dependent coefficients of operators in a low-energy Hamiltonian.

Decoupling of high momentum d.o.f. can be achieved by lowering the “resolution scale” , or Λ, down to typical nuclear structure momentum scale. ==> Necessary d.o.f. for low-energy observables.

Understanding nuclei from RG perspective

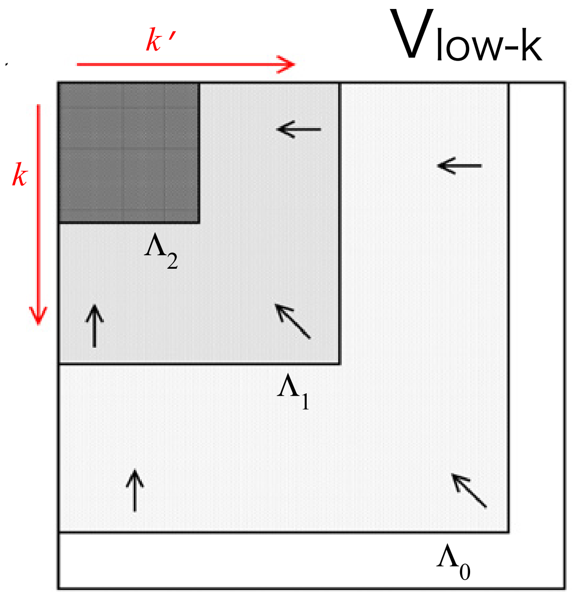
Nuclear Hamiltonian is ”resolution” dependent

$$H(\Lambda) = T + V^{(2)}(\Lambda) + V^{(3)}(\Lambda) \cdots$$

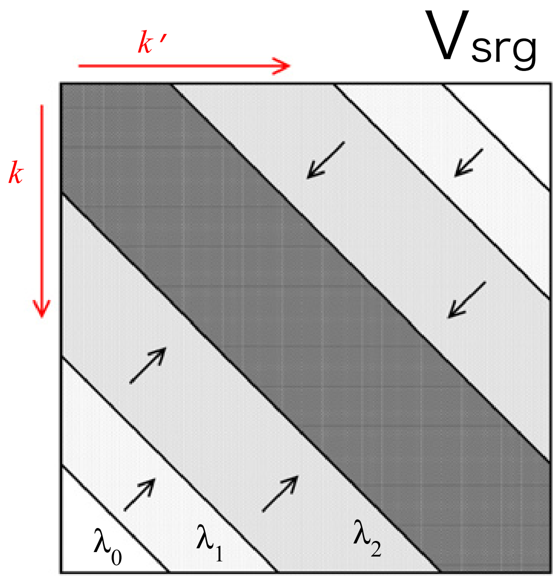
Relevant details of high-energy physics ==> Λ-dependent coefficients of operators in a low-energy Hamiltonian.

Decoupling of high momentum d.o.f. can be achieved by lowering the “resolution scale” , or Λ, down to typical nuclear structure momentum scale. ==> Necessary d.o.f. for low-energy observables.

$V(k,k') \doteq$



or



Understanding nuclei from RG perspective

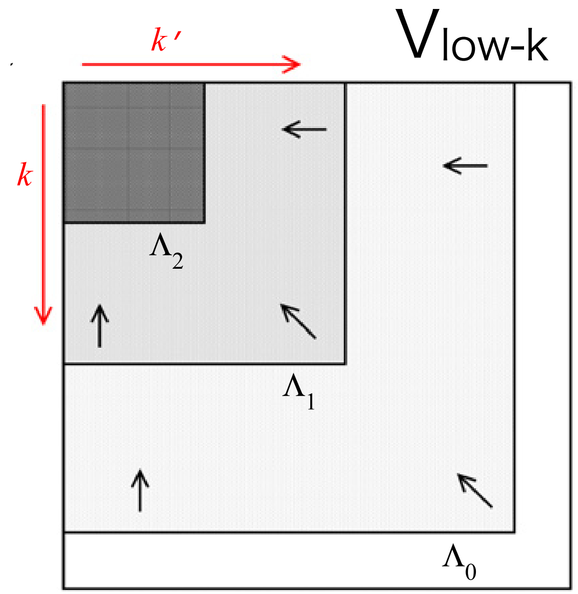
Nuclear Hamiltonian is "resolution" dependent

$$H(\Lambda) = T + V^{(2)}(\Lambda) + V^{(3)}(\Lambda) \cdots$$

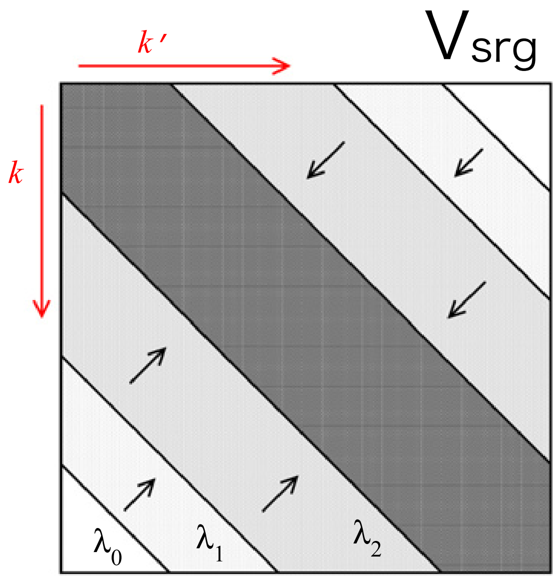
Relevant details of high-energy physics ==> Λ-dependent coefficients of operators in a low-energy Hamiltonian.

Decoupling of high momentum d.o.f. can be achieved by lowering the "resolution scale" , or Λ, down to typical nuclear structure momentum scale. ==> Necessary d.o.f. for low-energy observables.

$V(k,k') \doteq$



or



✦Greatly simplify the nuclear many-body problems, making those calculations converge rapidly.

✦ Varying Λ provides a powerful tool to assess theoretical errors due to truncation in Hamiltonian and many-body approximations.

Similarity Renormalization Group

Glazek and Wilson, Phys. Rev. D**48**, 5863(1993), or Wegner, Ann. Phys. (Leipzig) **3**, 77 (1994)

$$H(s) = U(s)HU^{\dagger}(s) \equiv \underline{H^{d}(s) + H^{od}(s)}$$

can arbitrary be defined

Similarity Renormalization Group

Glazek and Wilson, Phys. Rev. D**48**, 5863(1993), or Wegner, Ann. Phys. (Leipzig) **3**, 77 (1994)

$$H(s) = U(s)HU^{\dagger}(s) \equiv \underbrace{H^{d}(s) + H^{od}(s)}$$

can arbitrary be defined

The evolution of the Hamiltonian with the flow parameter s is given by the infinitesimal representation of the transformation,

$$\frac{dH(s)}{ds} = [\eta(s), H(s)],$$

Similarity Renormalization Group

Glazek and Wilson, Phys. Rev. D**48**, 5863(1993), or Wegner, Ann. Phys. (Leipzig) **3**, 77 (1994)

$$H(s) = U(s)HU^{\dagger}(s) \equiv \underline{H^{d}(s) + H^{od}(s)}$$

can arbitrary be defined

The evolution of the Hamiltonian with the flow parameter s is given by the infinitesimal representation of the transformation,

$$\frac{dH(s)}{ds} = [\eta(s), H(s)],$$

where η (s) is the anti-hermitian generator of the transformation, given by

$$\eta(s) = [H^{d}(s), H(s)]$$

Similarity Renormalization Group

Glazek and Wilson, Phys. Rev. D**48**, 5863(1993), or Wegner, Ann. Phys. (Leipzig) **3**, 77 (1994)

$$H(s) = U(s) H U^{\dagger}(s) \equiv \underline{H^{d}(s) + H^{od}(s)}$$

can arbitrary be defined

The evolution of the Hamiltonian with the flow parameter s is given by the infinitesimal representation of the transformation,

$$\frac{dH(s)}{ds} = [\eta(s), H(s)],$$

where η (s) is the anti-hermitian generator of the transformation, given by

$$\eta(s) = [H^{d}(s), H(s)]$$

Different choice for H^{d}(s) can tailor the SRG evolution for a particular problem.

==> Simplicity, flexibility

powerful alternative to Lee-Suzuki

Similarity Renormalization Group

Glazek and Wilson, Phys. Rev. D**48**, 5863(1993), or Wegner, Ann. Phys. (Leipzig) **3**, 77 (1994)

$$H(s) = U(s)HU^\dagger(s) \equiv \underline{H^d(s) + H^{od}(s)}$$

can arbitrary be defined

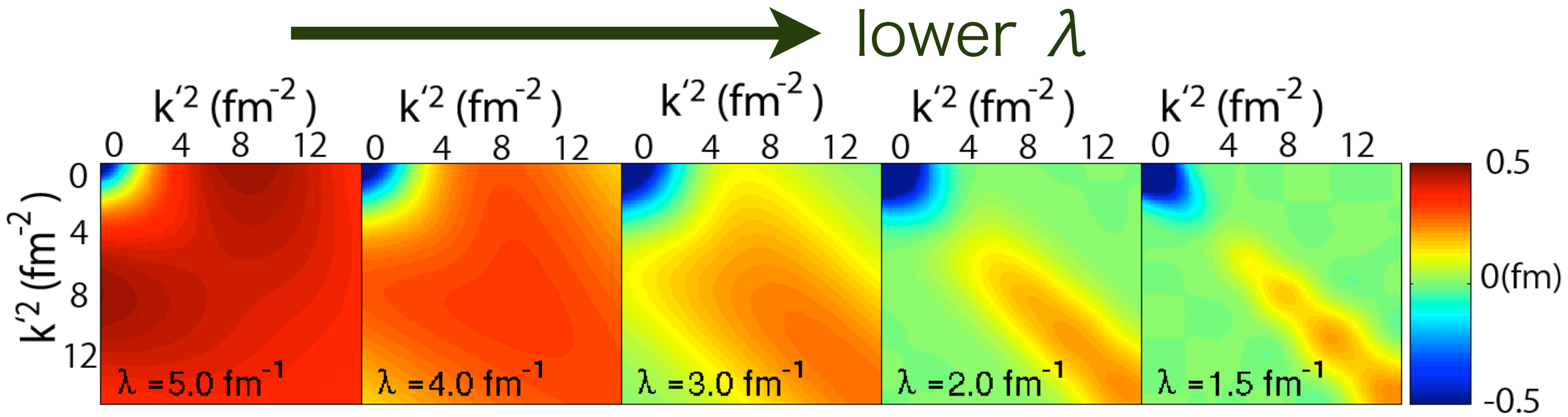
The evolution of the Hamiltonian with the flow parameter s is given by the infinitesimal representation of the transformation,

$$\frac{dH(s)}{ds} = [\eta(s), H(s)],$$

where η (s) is the anti-hermitian generator of the transformation, given by

$$\eta(s) = [H^d(s), H(s)]$$

Different choice for $H^d(s)$ can tailor the SRG evolution for a particular problem.



==> Simplicity, flexibility

powerful alternative to Lee-Suzuki

$$H^d(s) = T$$

Bogner et al, PRC75, 061001(R) (2007)

Similarity Renormalization Group

Glazek and Wilson, Phys. Rev. D**48**, 5863(1993), or Wegner, Ann. Phys. (Leipzig) **3**, 77 (1994)

$$H(s) = U(s)HU^{\dagger}(s) \equiv \underline{H^{d}(s) + H^{od}(s)}$$

can arbitrary be defined

The evolution of the Hamiltonian with the flow parameter s is given by the infinitesimal representation of the transformation,

$$\frac{dH(s)}{ds} = [\eta(s), H(s)],$$

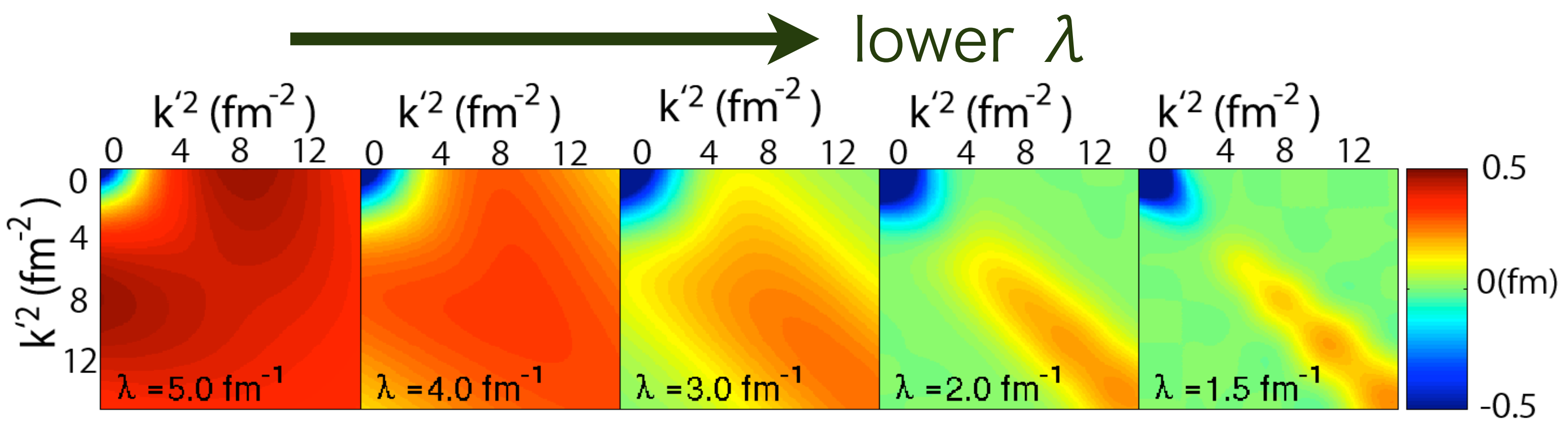
where η (s) is the anti-hermitian generator of the transformation, given by

$$\eta(s) = [H^{d}(s), H(s)]$$

Different choice for H^d(s) can tailor the SRG evolution for a particular problem.

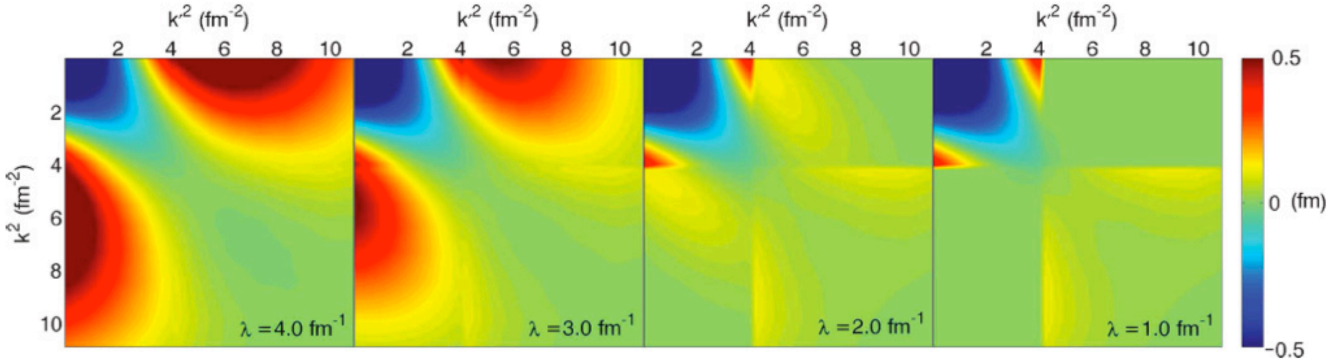
==> Simplicity, flexibility

powerful alternative to Lee-Suzuki



$$H^{d}(s) = T$$

Bogner et al, PRC75, 061001(R) (2007)

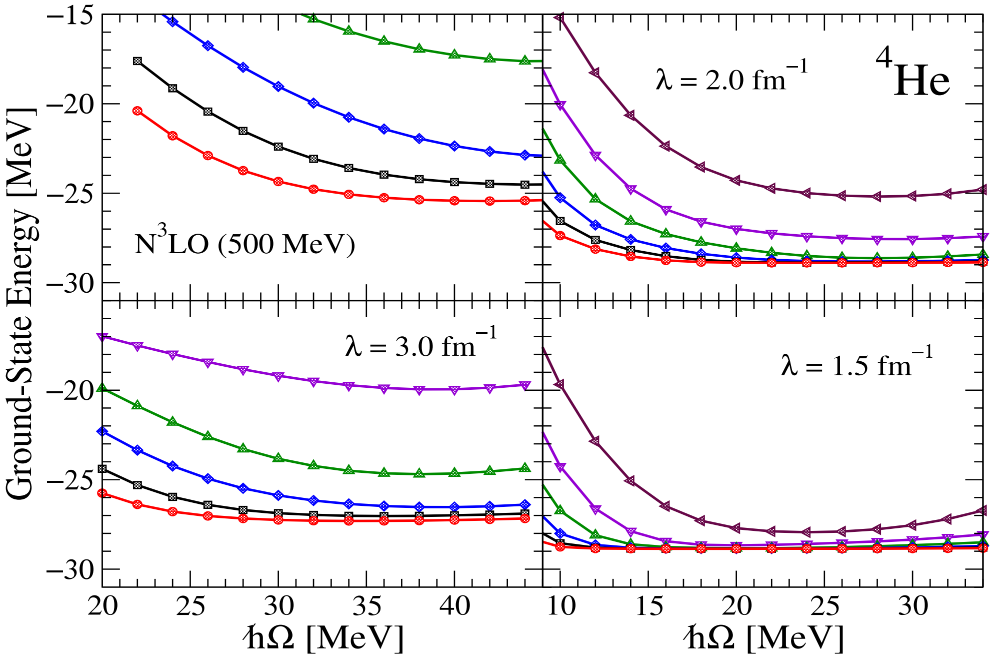


$$H^{d}(s) = \begin{pmatrix} PH(s)P & 0 \\ 0 & QH(s)Q \end{pmatrix}$$

Anderson et al, PRC77, 037001 (2008)

RG and many-body interactions

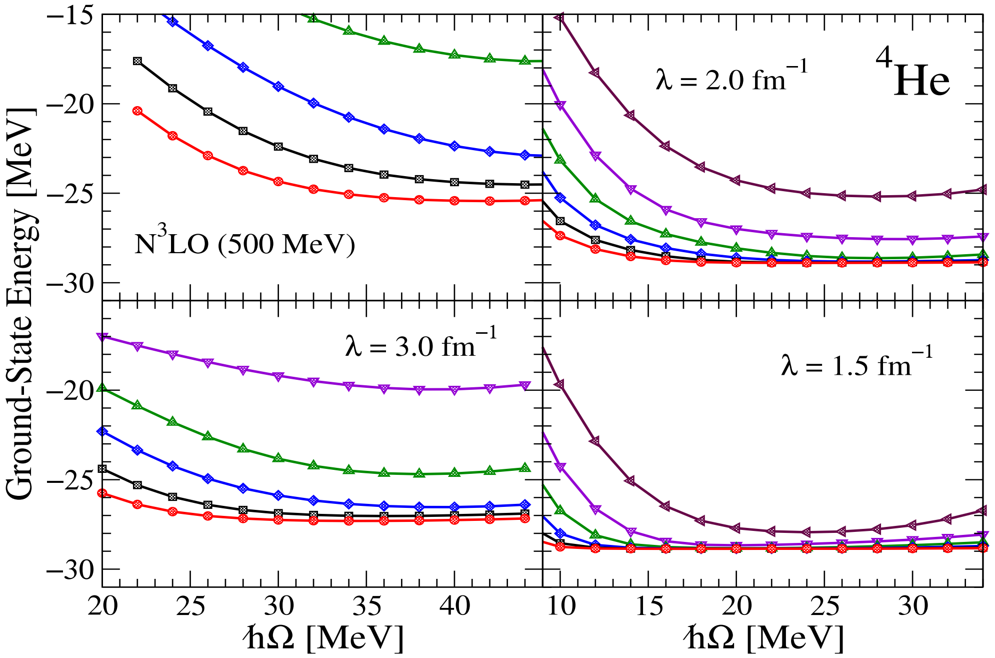
So far, the SRG to NN(N) have been applied in free space to construct NN and/or NNN interactions to be used as input for ab initio calculations (NCSM).



Accelerated convergence, but λ-dependent results

RG and many-body interactions

So far, the SRG to NN(N) have been applied in free space to construct NN and/or NNN interactions to be used as input for ab initio calculations (NCSM).



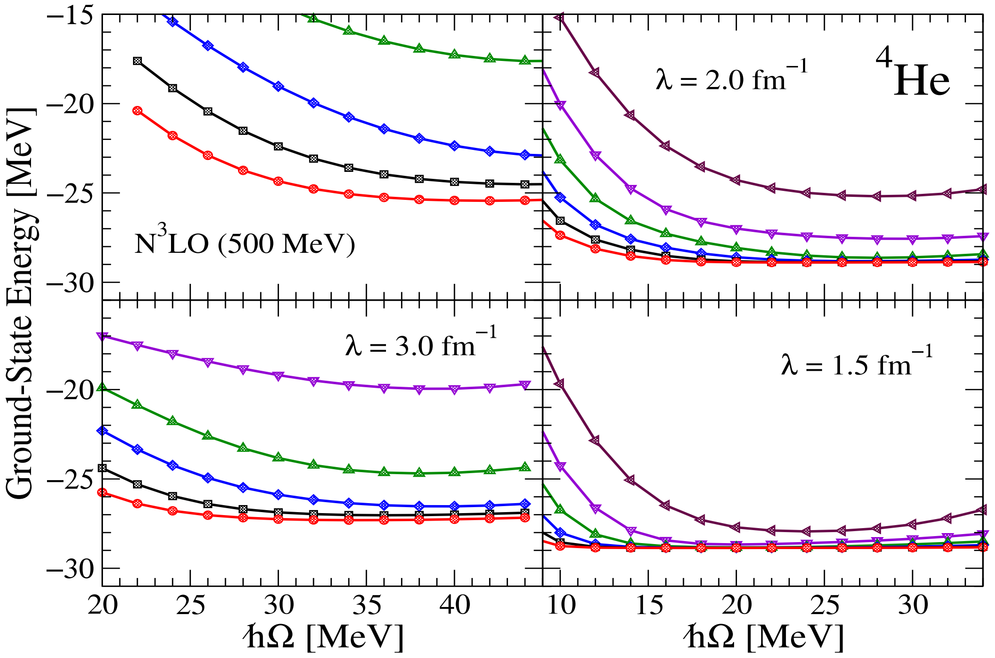
Accelerated convergence, but λ-dependent results

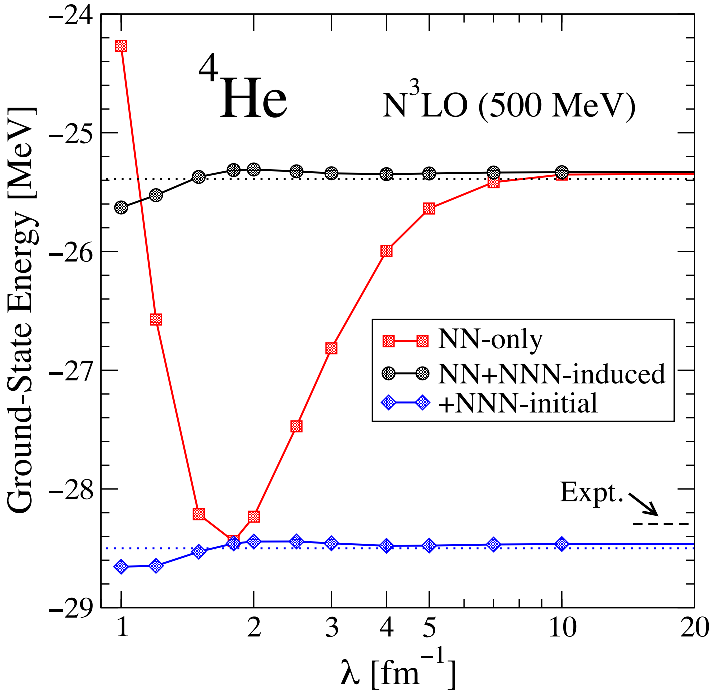
Many-body forces are induced and do affect observables

$$H(s) = U(s)H^{(2)}U^{\dagger}(s) = \tilde{H}^{(2)}(s) + \tilde{H}^{(3)}(s) + \cdots$$

RG and many-body interactions

So far, the SRG to NN(N) have been applied in free space to construct NN and/or NNN interactions to be used as input for ab initio calculations (NCSM).





Accelerated convergence, but λ-dependent results

Jurgeson, Furnstahl and Navratil Phys Rev. Lett. 103, 082501(2009)

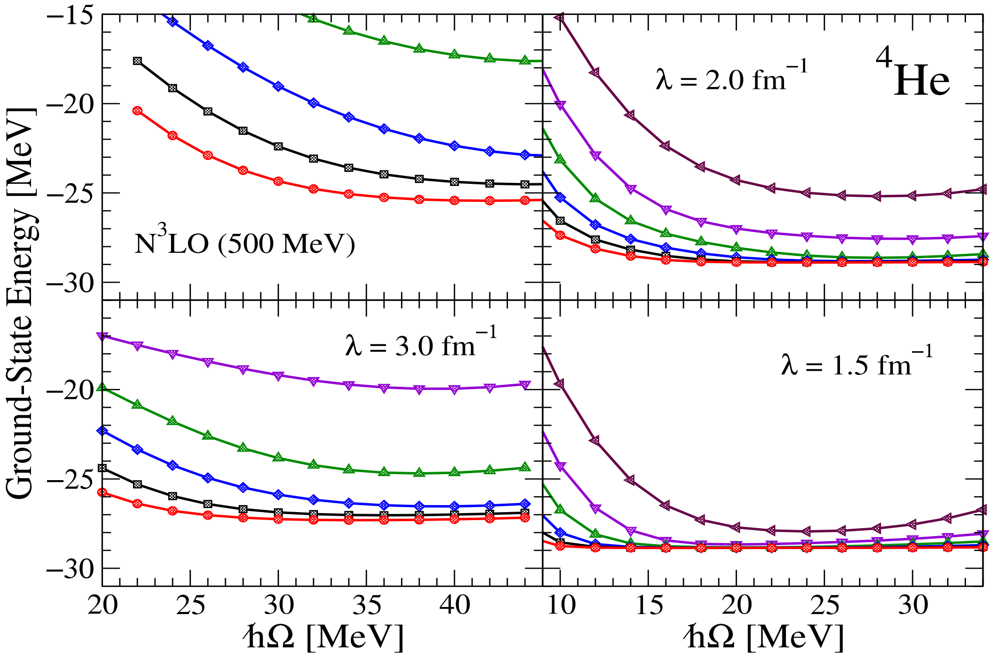
Many-body forces are induced and do affect observables

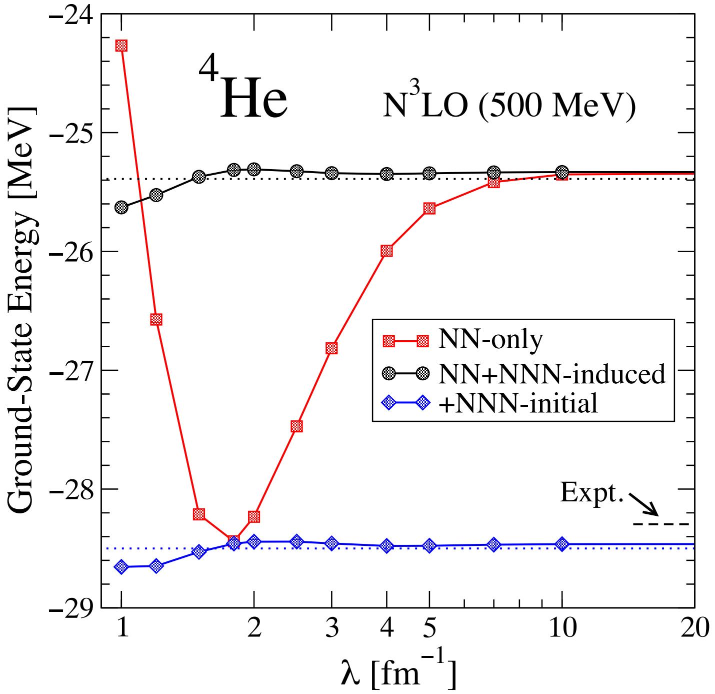
$$H(s) = U(s)H^{(2)}U^{\dagger}(s) = \tilde{H}^{(2)}(s) + \tilde{H}^{(3)}(s) + \cdots$$

Free-space SRG needs the consistent evolution of 3N or higher interactions

RG and many-body interactions

So far, the SRG to NN(N) have been applied in free space to construct NN and/or NNN interactions to be used as input for ab initio calculations (NCSM).





Accelerated convergence, but λ-dependent results

Jurgeson, Furnstahl and Navratil Phys Rev. Lett. 103, 082501(2009)

Many-body forces are induced and do affect observables

$$H(s) = U(s)H^{(2)}U^{\dagger}(s) = \tilde{H}^{(2)}(s) + \tilde{H}^{(3)}(s) + \cdots$$

Free-space SRG needs the consistent evolution of 3N or higher interactions

SRG transformation directly in many-body system

❋Normal-ordering w.r.t. a reference state, and truncate at a finite order.

❋One can approximately evolve 3-body, ..A-body operators within 2b machinery.

K.T. S. Bogner and A. Schwenk, to be submitted

In-medium SRG for Nuclei

Starting with a general Hamiltonian

$$\hat{H} = \sum_{ij} T_{ij} a_i^\dagger a_j + \frac{1}{2!^2} \sum_{ijkl} V_{ijkl}^{(2)} a_i^\dagger a_j^\dagger a_l a_k + \frac{1}{3!^2} \sum_{ijklmn} V_{ijklnm}^{(3)} a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l + \cdots$$

In-medium SRG for Nuclei

Starting with a general Hamiltonian

$$\hat{H} = \sum_{ij} T_{ij} a_i^\dagger a_j + \frac{1}{2!^2} \sum_{ijkl} V_{ijkl}^{(2)} a_i^\dagger a_j^\dagger a_l a_k + \frac{1}{3!^2} \sum_{ijklmn} V_{ijklnm}^{(3)} a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l + \cdots$$

All operators can be normal-ordered wrt a finite-density Fermi vacuum $|\Phi\rangle$

$$\hat{H} = E_0 + \sum_{kk'} g_{kk'} \{a_k^\dagger a_k'\} + \frac{1}{2!^2} \sum_{kpqr} \Gamma_{kpqr} \{a_k^\dagger a_p^\dagger a_r a_q\} + \frac{1}{3!^2} \sum_{kpqrst} W_{kpqrst} \{a_k^\dagger a_p^\dagger a_q^\dagger a_t a_s a_r\},$$

In-medium SRG for Nuclei

Starting with a general Hamiltonian

$$\hat{H} = \sum_{ij} T_{ij} a_i^\dagger a_j + \frac{1}{2!^2} \sum_{ijkl} V^{(2)}_{ijkl} a_i^\dagger a_j^\dagger a_l a_k + \frac{1}{3!^2} \sum_{ijklmn} V^{(3)}_{ijklnm} a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l + \cdots$$

All operators can be normal-ordered wrt a finite-density Fermi vacuum $|\Phi\rangle$

$$\hat{H} = E_0 + \sum_{kk'} g_{kk'} \{a_k^\dagger a'_k\} + \frac{1}{2!^2} \sum_{kpqr} \Gamma_{kpqr} \{a_k^\dagger a_p^\dagger a_r a_q\} + \frac{1}{3!^2} \sum_{kpqrst} W_{kpqrst} \{a_k^\dagger a_p^\dagger a_q^\dagger a_t a_s a_r\},$$

where coefficients of normal-ordered operators are given by

$$E_0 = \langle\phi| H |\phi\rangle = \sum_k T_{kk} n_k + \frac{1}{2} \sum_{ij} V^{(2)}_{ijij} n_i n_j + \frac{1}{6} \sum_{ijk} V^{(3)}_{ijkijk} n_i n_j n_k$$

$$g_{ij} = T_{ij} + \sum_k V^{(2)}_{ikjk} n_k + \frac{1}{2} \sum_{kl} V^{(3)}_{ikljkl} n_k n_l$$

$$\Gamma_{pqrs} = V^{(2)}_{pqrs} + \frac{1}{4} \sum_i V^{(3)}_{pqirsi} n_i .$$

$$n_i \equiv \theta(\epsilon_F - \epsilon_i)$$

zero- one- two-body terms include contributions from 3N, 4N.. forces

In-medium SRG for Nuclei

Starting with a general Hamiltonian

$$\hat{H} = \sum_{ij} T_{ij} a_i^\dagger a_j + \frac{1}{2!^2} \sum_{ijkl} V^{(2)}_{ijkl} a_i^\dagger a_j^\dagger a_l a_k + \frac{1}{3!^2} \sum_{ijklmn} V^{(3)}_{ijklnm} a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l + \cdots$$

All operators can be normal-ordered wrt a finite-density Fermi vacuum $|\Phi\rangle$

$$\hat{H} = E_0 + \sum_{kk'} g_{kk'} \{a_k^\dagger a'_k\} + \frac{1}{2!^2} \sum_{kpqr} \Gamma_{kpqr} \{a_k^\dagger a_p^\dagger a_r a_q\} + \frac{1}{3!^2} \sum_{kpqrst} W_{kpqrst} \{a_k^\dagger a_p^\dagger a_q^\dagger a_t a_s a_r\},$$

where coefficients of normal-ordered operators are given by

$$E_0 = \langle\phi|\, H \,|\phi\rangle = \sum_k T_{kk} n_k + \frac{1}{2} \sum_{ij} V^{(2)}_{ijij} n_i n_j + \frac{1}{6} \sum_{ijk} V^{(3)}_{ijkijk} n_i n_j n_k$$

$$g_{ij} = T_{ij} + \sum_k V^{(2)}_{ikjk} n_k + \frac{1}{2} \sum_{kl} V^{(3)}_{ikljkl} n_k n_l$$

$$\Gamma_{pqrs} = V^{(2)}_{pqrs} + \frac{1}{4} \sum_i V^{(3)}_{pqirsi} n_i .$$

$$n_i \equiv \theta(\epsilon_F - \epsilon_i)$$

zero- one- two-body terms include contributions from 3N, 4N.. forces

Truncating the flow equation up to normal-ordered 2-body operators may approximately evolve induced 3-body and higher-body interactions through density- dependent coefficients.

What we drive a Hamiltonian toward

We define what is suppressed in the flow

$$H^{od}(s) = f^{od}(s) + \Gamma^{od}(s)$$

$$\begin{cases} \Gamma^{od}(s) & = \sum_{pp'hh'} \Gamma_{pp'hh'}(s)\{a_p^\dagger a_{p'}^\dagger a_h a_{h'}\} + h.c. \\ f^{od}(s) & = \sum_{ph} f_{ph}(s)\{a_p^\dagger a_h\} + h.c. \end{cases}$$

The coupling between particle and hole is suppressed in the flow.

What we drive a Hamiltonian toward

We define what is suppressed in the flow

$$H^{od}(s) = f^{od}(s) + \Gamma^{od}(s)$$

$$\begin{cases} \Gamma^{od}(s) &= \sum_{pp'hh'} \Gamma_{pp'hh'}(s)\{a_p^\dagger a_{p'}^\dagger a_h a_{h'}\} + h.c. \\ f^{od}(s) &= \sum_{ph} f_{ph}(s)\{a_p^\dagger a_h\} + h.c. \end{cases}$$

The coupling between particle and hole is suppressed in the flow.

Consider two slightly different generators

What we drive a Hamiltonian toward

We define what is suppressed in the flow

$$H^{od}(s) = f^{od}(s) + \Gamma^{od}(s)$$

$$\begin{cases} \Gamma^{od}(s) &= \sum_{pp'hh'} \Gamma_{pp'hh'}(s)\{a_p^\dagger a_{p'}^\dagger a_h a_{h'}\} + h.c. \\ f^{od}(s) &= \sum_{ph} f_{ph}(s)\{a_p^\dagger a_h\} + h.c. \end{cases}$$

The coupling between particle and hole is suppressed in the flow.

Consider two slightly different generators

Wegner's choice

$$\eta^I = [H^d, H] \quad \Longrightarrow \quad \frac{d}{ds}Tr(H^{od}(s))^2 = -2Tr(\eta^\dagger\eta) \le 0$$

What we drive a Hamiltonian toward

We define what is suppressed in the flow

$$H^{od}(s) = f^{od}(s) + \Gamma^{od}(s)$$

$$\begin{cases} \Gamma^{od}(s) & = \sum_{pp'hh'} \Gamma_{pp'hh'}(s)\{a_p^\dagger a_{p'}^\dagger a_h a_{h'}\} + h.c. \\ f^{od}(s) & = \sum_{ph} f_{ph}(s)\{a_p^\dagger a_h\} + h.c. \end{cases}$$

The coupling between particle and hole is suppressed in the flow.

Consider two slightly different generators

Wegner's choice

$$\eta^I = [H^d, H] \Rightarrow \frac{d}{ds}Tr(H^{od}(s))^2 = -2Tr(\eta^\dagger \eta) \leq 0$$

White's choice J. Chem. Phys, 117, 7472 (2002)

$$\eta^{II} = \sum_{ph} \frac{f_{ph}}{f_p - f_h - \underline{\Gamma_{phph}}}\{a_p^\dagger a_h\} - h.c. + \sum_{pp'hh'} \frac{\Gamma_{pp'hh'}}{f_p + f_{p'} - f_h - f_{h'} + \underline{A_{pp'hh'}}}\{a_p^\dagger a_{p'}^\dagger a_{h'} a_h\} - h.c.$$

monopole interaction monopole interaction

$$A_{pp'hh'} = \Gamma_{pp'pp'} + \Gamma_{hh'hh'} - \Gamma_{phph} - \Gamma_{p'h'p'h'} - \Gamma_{ph'ph'} - \Gamma_{p'hp'h}$$

What we drive a Hamiltonian toward

We define what is suppressed in the flow

$$H^{od}(s) = f^{od}(s) + \Gamma^{od}(s)$$

$$\begin{cases} \Gamma^{od}(s) & = \sum_{pp'hh'} \Gamma_{pp'hh'}(s)\{a_p^\dagger a_{p'}^\dagger a_h a_{h'}\} + h.c. \\ f^{od}(s) & = \sum_{ph} f_{ph}(s)\{a_p^\dagger a_h\} + h.c. \end{cases}$$

The coupling between particle and hole is suppressed in the flow.

Consider two slightly different generators

Wegner's choice

$$\eta^I = [H^d, H] \quad \Longrightarrow \quad \frac{d}{ds} Tr(H^{od}(s))^2 = -2Tr(\eta^\dagger \eta) \leq 0$$

White's choice J. Chem. Phys, 117, 7472 (2002)

$$\eta^{II} = \sum_{ph} \frac{f_{ph}}{f_p - f_h - \underline{\Gamma_{phph}}}\{a_p^\dagger a_h\} - h.c. + \sum_{pp'hh'} \frac{\Gamma_{pp'hh'}}{f_p + f_{p'} - f_h - f_{h'} + \underline{A_{pp'hh'}}}\{a_p^\dagger a_{p'}^\dagger a_{h'} a_h\} - h.c.$$

monopole interaction monopole interaction

$$A_{pp'hh'} = \Gamma_{pp'pp'} + \Gamma_{hh'hh'} - \Gamma_{phph} - \Gamma_{p'h'p'h'} - \Gamma_{ph'ph'} - \Gamma_{p'hp'h}$$

- Hamiltonians using η^{I} and η^{II} are unitary equivalent if no truncation is made.
- Any discrepancies in energy eigenvalues provides a measure of the truncation errors from the neglected 3-body or higher-body operators.

Decoupling of Hilbert space

Solving the flow equation for normal-ordered operators

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$

builds up correlations in pp, hh and ph channels to all order in bare couplings

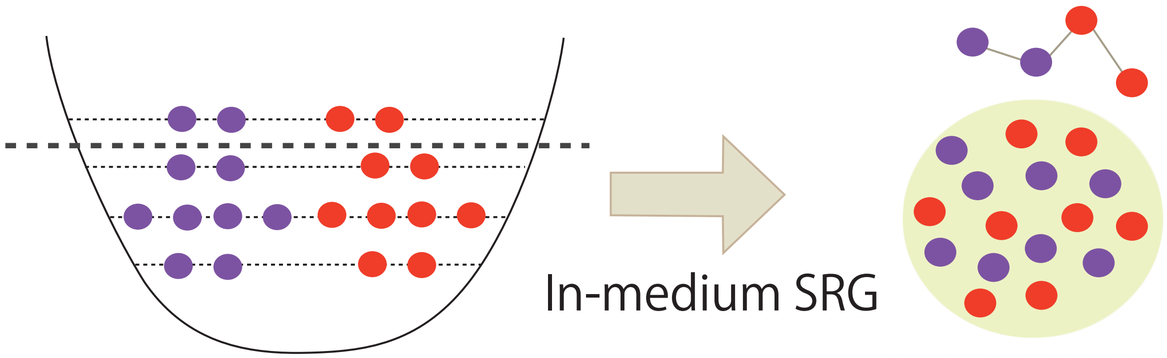
At the end of the flow

$$H(\infty) = E_0(\infty) + f^d(\infty) + \Gamma^d(\infty).$$

- The reference state |Φ⟩ becomes the ground state of H(∞) with $E_0(\infty)$
- The |Φ⟩ is decoupled from the rest of the Hilbert space

$$QH(\infty)P = 0, \quad PH(\infty)Q = 0$$

$$P = |\Phi\rangle\langle\Phi| \text{ and } Q = 1 - |\Phi\rangle\langle\Phi|$$

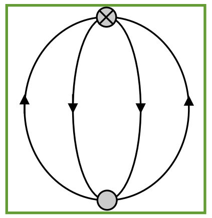


Effective Hamiltonians (and operators) for open-shell systems

Ground-state energy

In-medium SRG(2) Flow Equation

$$\frac{dE_0}{ds} = \sum_{ij} \eta^{(1)}_{ij} f_{ji}(n_i - n_j) + \underline{\frac{1}{2}\sum_{ijkl} \eta^{(2)}_{ijkl}\Gamma_{klij} n_i n_j \bar{n}_k \bar{n}_l} \quad (\bar{n}_i \equiv 1 - n_i),$$

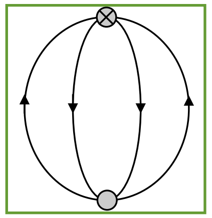


- The commutator form ==> no unlinked diagrams

  ==> size-extensive, with truncation errors scaling linearly with A.
- SRG is intrinsically non-perturbative.
- Modest scaling of computational efforts ==> suitable for medium-mass nuclei

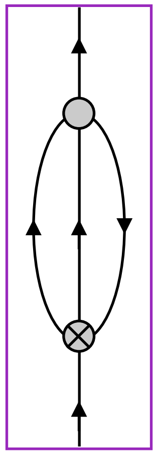
In-medium SRG(2) Flow Equation

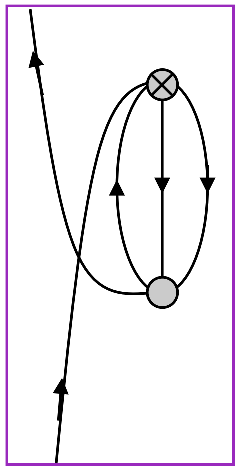
$$\frac{dE_0}{ds} = \sum_{ij} \eta^{(1)}_{ij} f_{ji}(n_i - n_j) + \underline{\frac{1}{2}\sum_{ijkl} \eta^{(2)}_{ijkl}\Gamma_{klij} n_i n_j \bar{n}_k \bar{n}_l} \quad (\bar{n}_i \equiv 1 - n_i),$$



$$\frac{df_{12}}{ds} = \sum_{i} \left\{(\eta^{(1)}_{1i} f_{i2}) + (1 \leftrightarrow 2)\right\} + \sum_{ij}(n_i - n_j)(\eta^{(1)}_{ij}\Gamma_{j1i2} - f_{ij}\eta^{(2)}_{j1i2})$$

$$\underline{+\frac{1}{2}\sum_{ijk}\left\{\eta^{2b}_{k1ij}\Gamma_{ijk2}(n_i n_j \bar{n}_k + \bar{n}_i \bar{n}_j n_k) + (1 \leftrightarrow 2)\right\}},$$



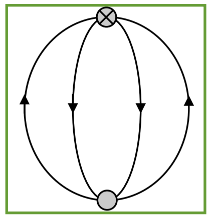


- The commutator form ==> no unlinked diagrams

  ==> size-extensive, with truncation errors scaling linearly with A.
- SRG is intrinsically non-perturbative.
- Modest scaling of computational efforts ==> suitable for medium-mass nuclei

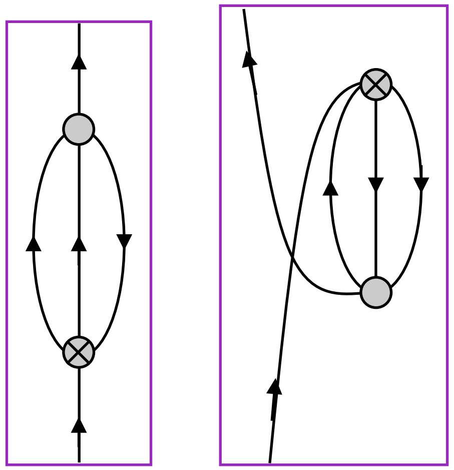
In-medium SRG(2) Flow Equation

$$\frac{dE_0}{ds} = \sum_{ij} \eta^{(1)}_{ij} f_{ji}(n_i - n_j) + \underline{\frac{1}{2}\sum_{ijkl} \eta^{(2)}_{ijkl}\Gamma_{klij} n_i n_j \bar{n}_k \bar{n}_l} \quad (\bar{n}_i \equiv 1 - n_i),$$



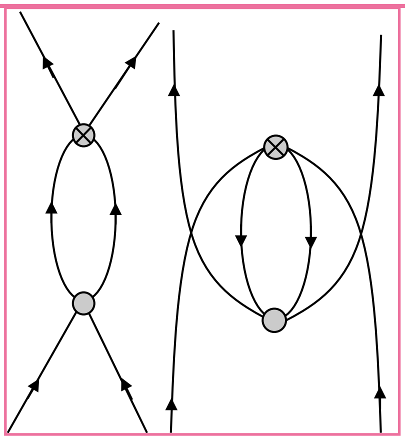
$$\frac{df_{12}}{ds} = \sum_{i}\left\{(\eta^{(1)}_{1i} f_{i2}) + (1 \leftrightarrow 2)\right\} + \sum_{ij}(n_i - n_j)(\eta^{(1)}_{ij}\Gamma_{j1i2} - f_{ij}\eta^{(2)}_{j1i2})$$

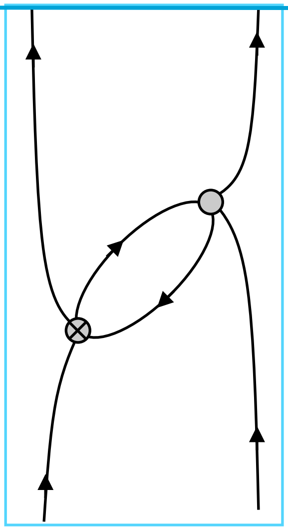
$$\underline{+\frac{1}{2}\sum_{ijk}\{\eta^{2b}_{k1ij}\Gamma_{ijk2}(n_i n_j \bar{n}_k + \bar{n}_i \bar{n}_j n_k) + (1 \leftrightarrow 2)\}},$$



$$\frac{d\Gamma_{1234}}{ds} = \sum_{i}\left\{\left(\eta^{(1)}_{1i}\Gamma_{i234} - f_{1i}\eta^{(2)}_{i234}\right) - (1 \leftrightarrow 2)\right\} - \sum_{i}\left\{\left(\eta^{(1)}_{i3}\Gamma_{12i4} - f_{i3}\eta^{(2)}_{12i4}\right) - (3 \leftrightarrow 4)\right\}$$

$$\underline{+\frac{1}{2}\sum_{ij}\left\{\eta^{(2)}_{12ij}\Gamma_{ij34}(\bar{n}_i\bar{n}_j - n_i n_j) + (1,2 \leftrightarrow 3,4)\right\}} + \underline{\sum_{ij}(\bar{n}_i n_j - n_i \bar{n}_j)\left[\left\{\eta^{(2)}_{j2i4}\Gamma_{i1j3} - \eta^{(2)}_{i1j3}\Gamma_{j2i4}\right\} - (1 \leftrightarrow 2)\right]}$$



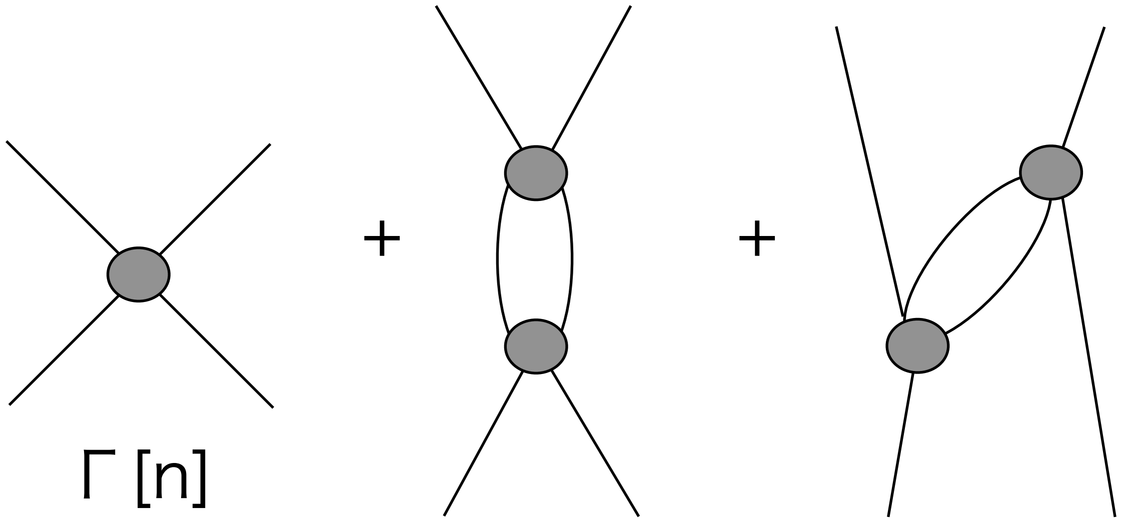


- The commutator form ==> no unlinked diagrams
  ==> size-extensive, with truncation errors scaling linearly with A.
- SRG is intrinsically non-perturbative.
- Modest scaling of computational efforts ==> suitable for medium-mass nuclei

Non-Perturbativeness of IM-SRG: Schematic

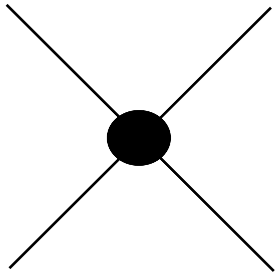
The flow equation can essentially be seen as

Γ [n+1]=



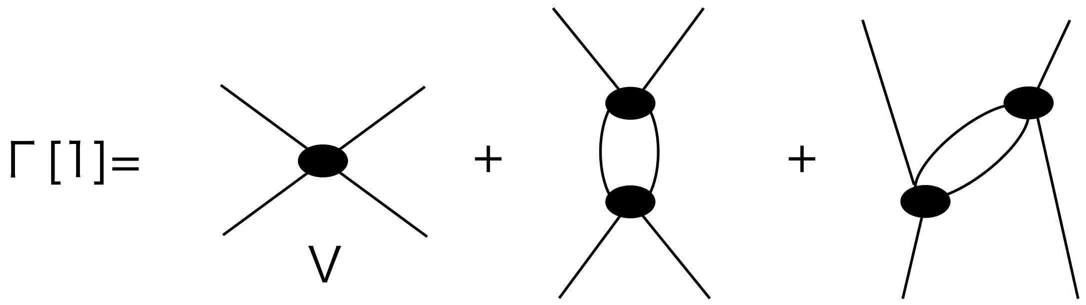
With the initial condition

Γ [0]=



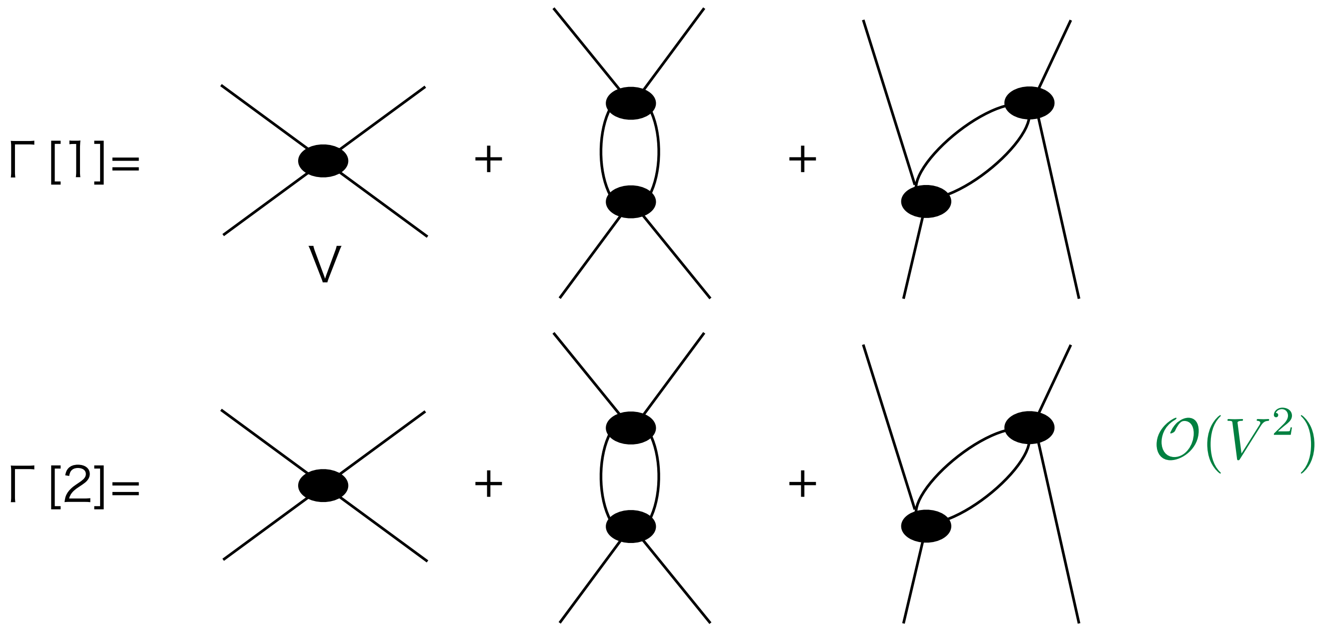
=: V(bare two-body coupling)

Non-Perturbativeness of IM-SRG: Schematic



Solving the flow equation step by step

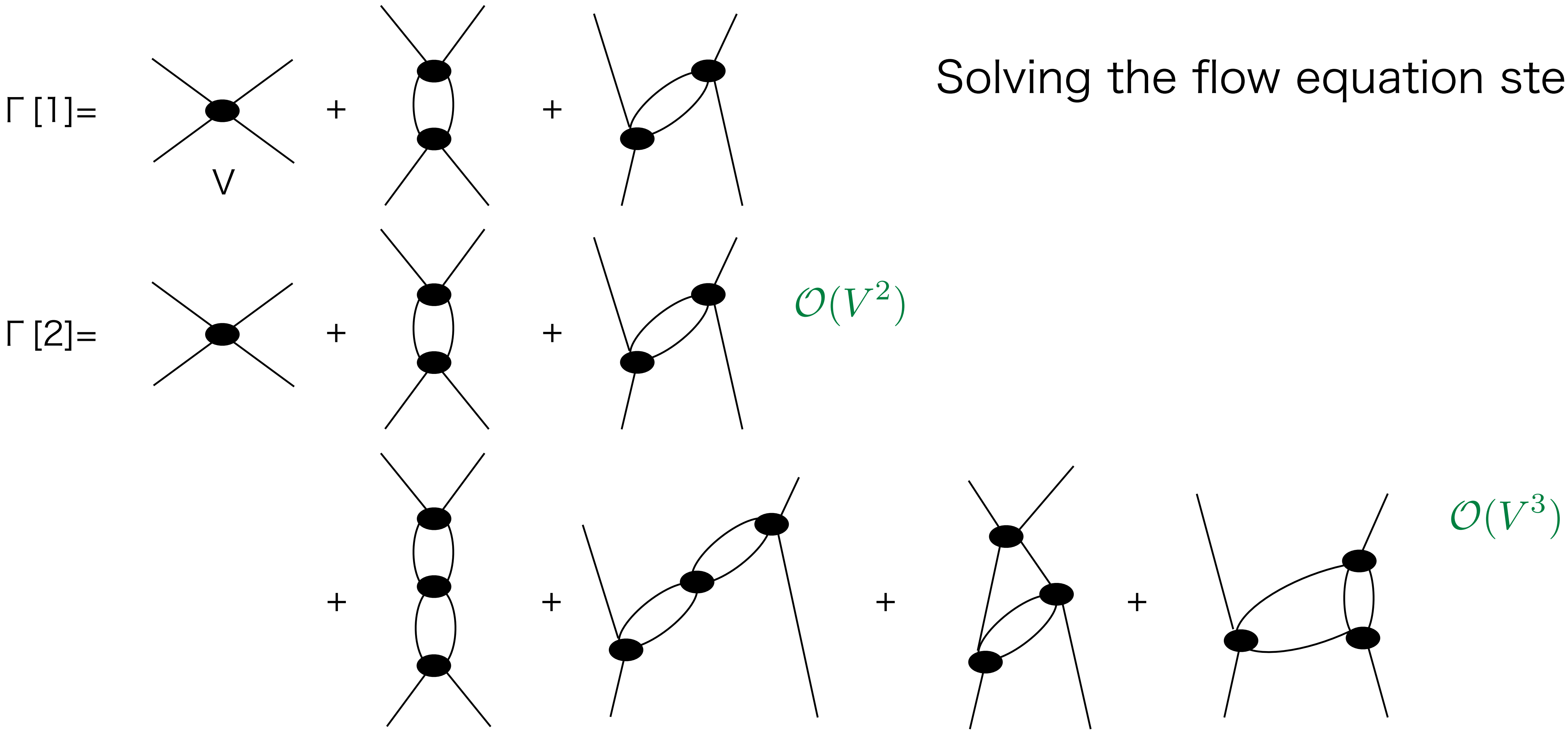
Non-Perturbativeness of IM-SRG: Schematic



Solving the flow equation step by step

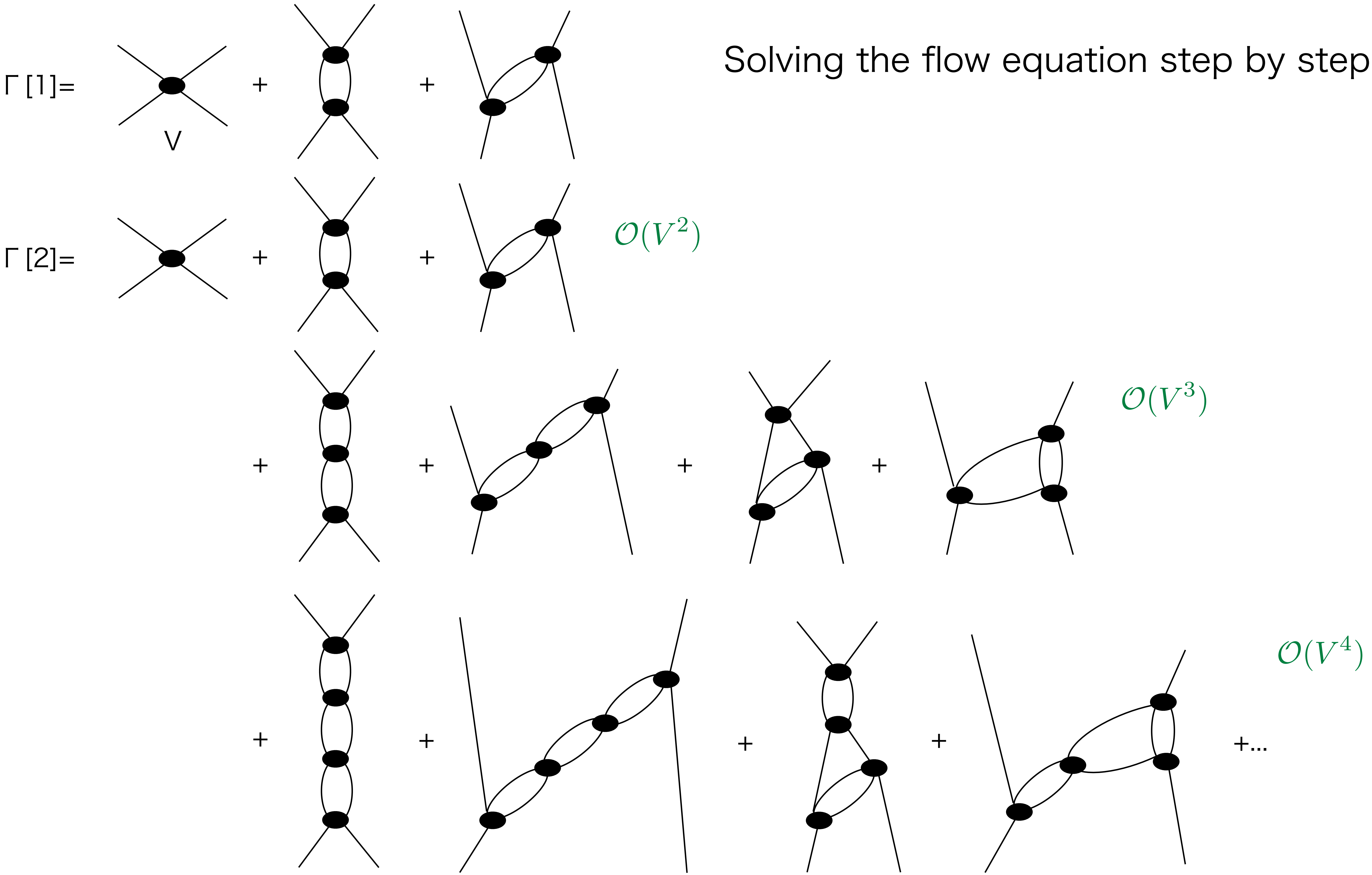
Non-Perturbativeness of IM-SRG: Schematic

Solving the flow equation step by step

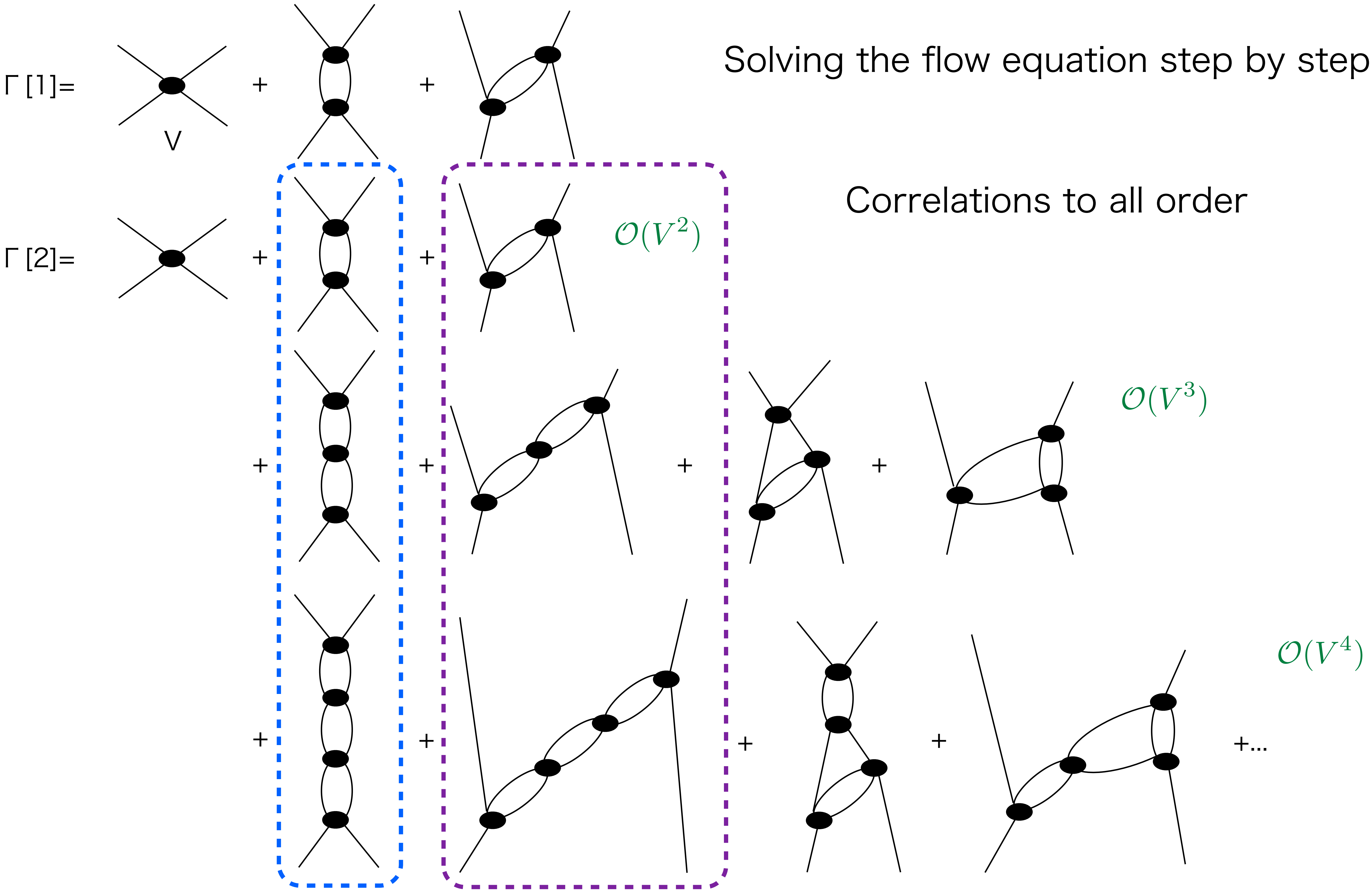


Non-Perturbativeness of IM-SRG: Schematic

Solving the flow equation step by step



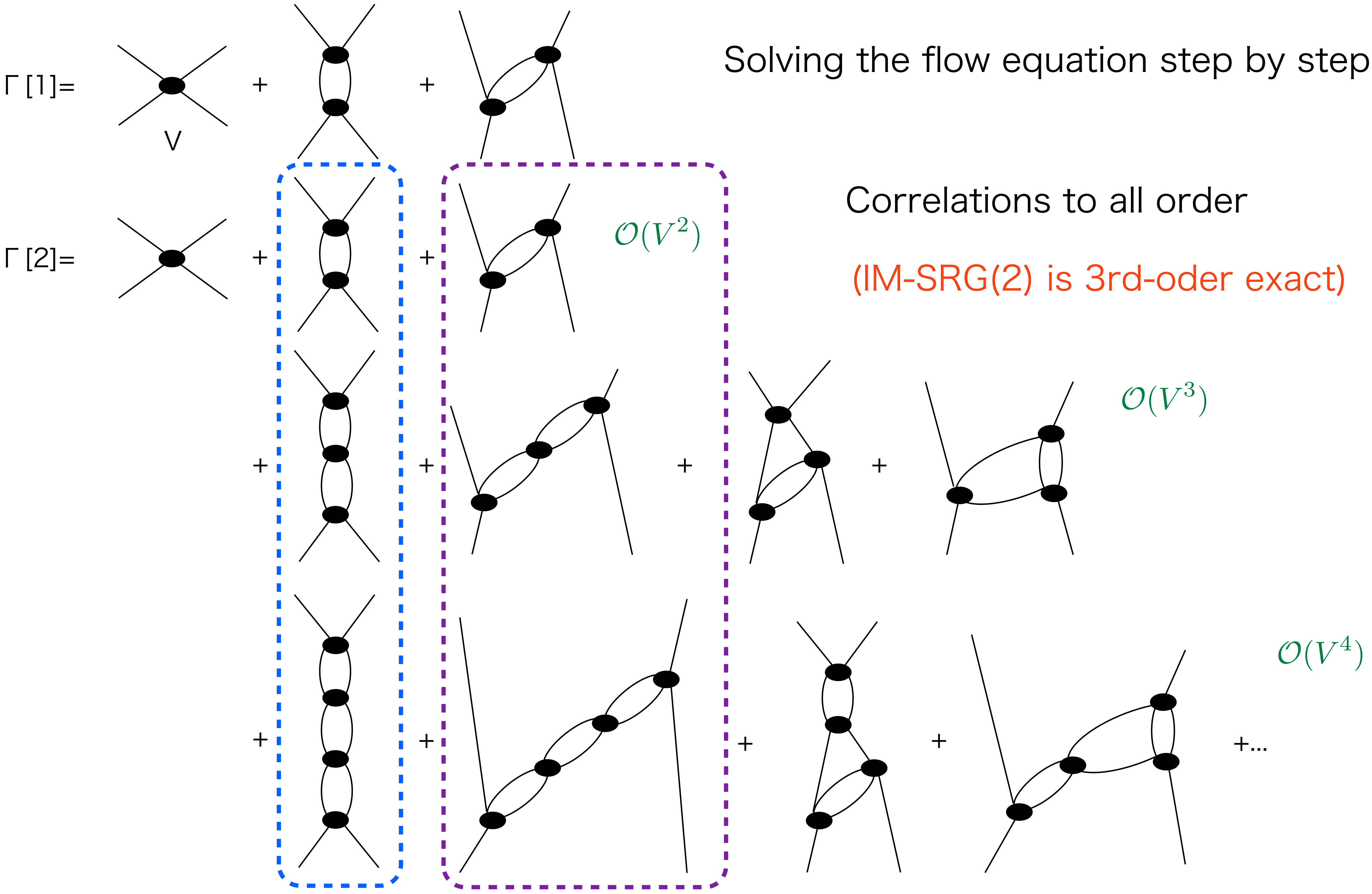
Non-Perturbativeness of IM-SRG: Schematic



Solving the flow equation step by step

Correlations to all order

Non-Perturbativeness of IM-SRG: Schematic



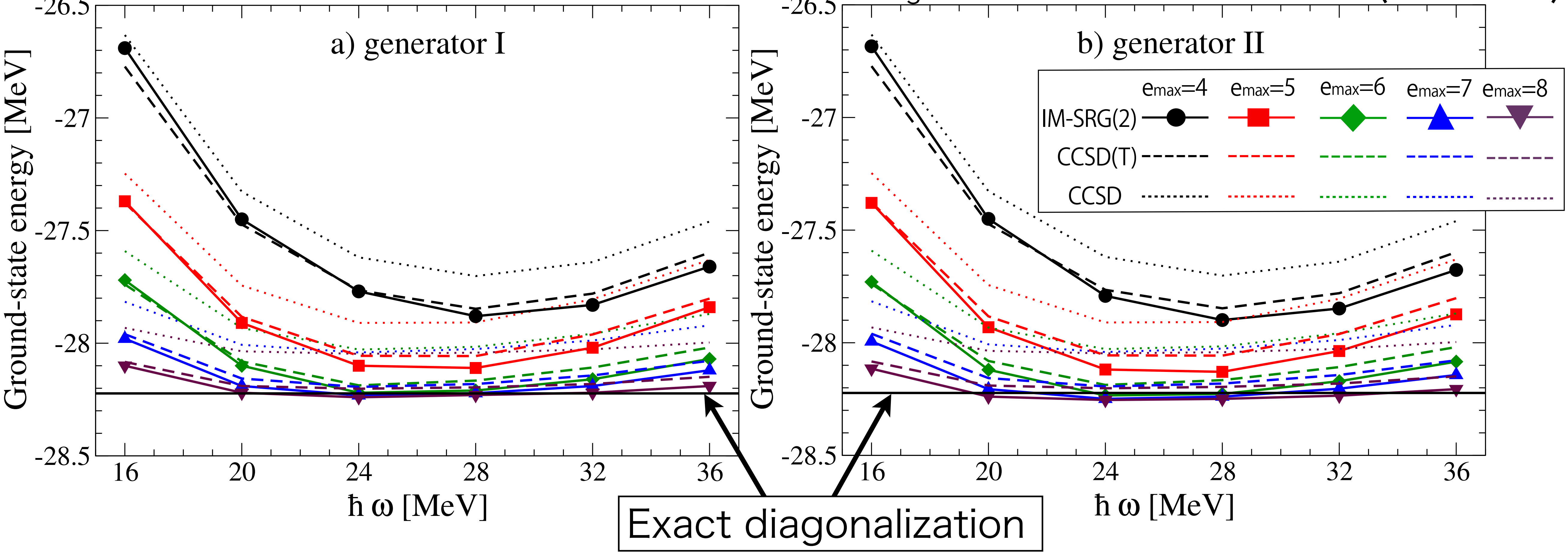
Solving the flow equation step by step

Correlations to all order

(IM-SRG(2) is 3rd-oder exact)

Ground-State Energy of ^{4}He

V_{srg} λ=2.0 fm^{-1} from N3LO(500MeV)



Exact diagonalization

✤The IM-SRG(2) for both generators are in good agreement with the CCSD(T)

✤The η^{I} and η^{II} results agree to within 20 keV of each other.

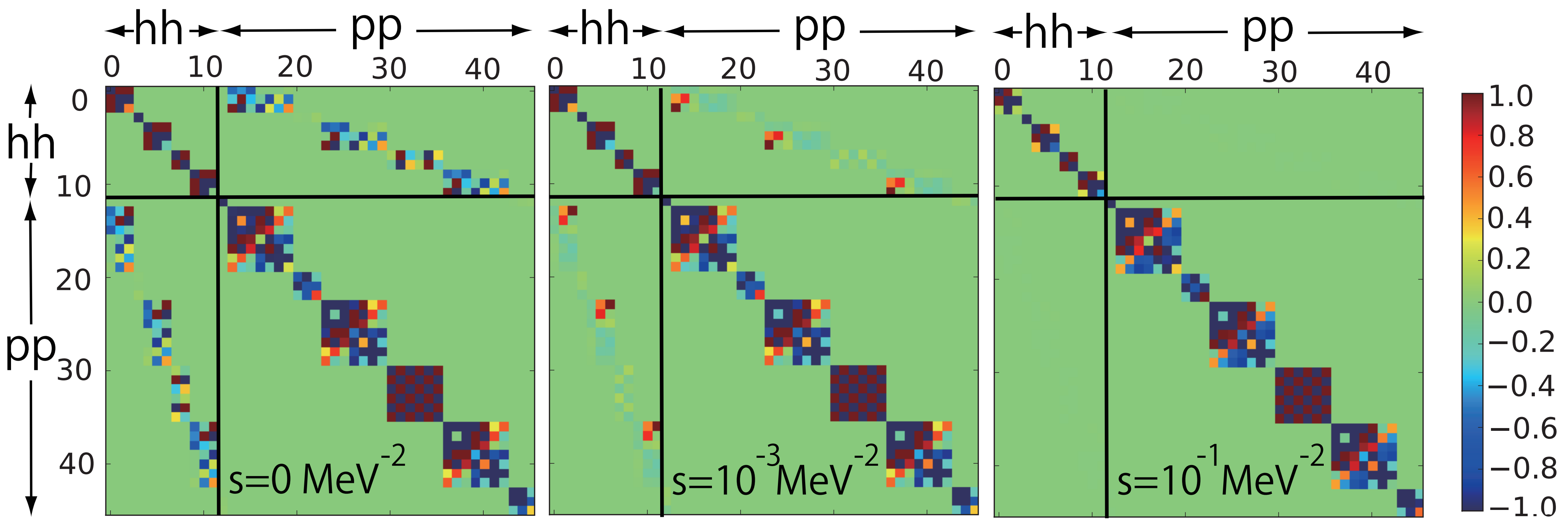
✤Essentially converged e_{max}=8 results falls within 20 keV of the exact NCSM.

Truncation to normal-ordered two-body interaction is a controlled approximation.

Suppression of $H^{od}(s)$

η^{I}-evolution of the matrix elements $\Gamma^{JTz}_{abcd}(s)$ at three different steps in s.

^{16}O, $V_{low\text{-}k}$ $\Lambda = 1.8\mathrm{fm}^{-1}$ from N3LO(500MeV)



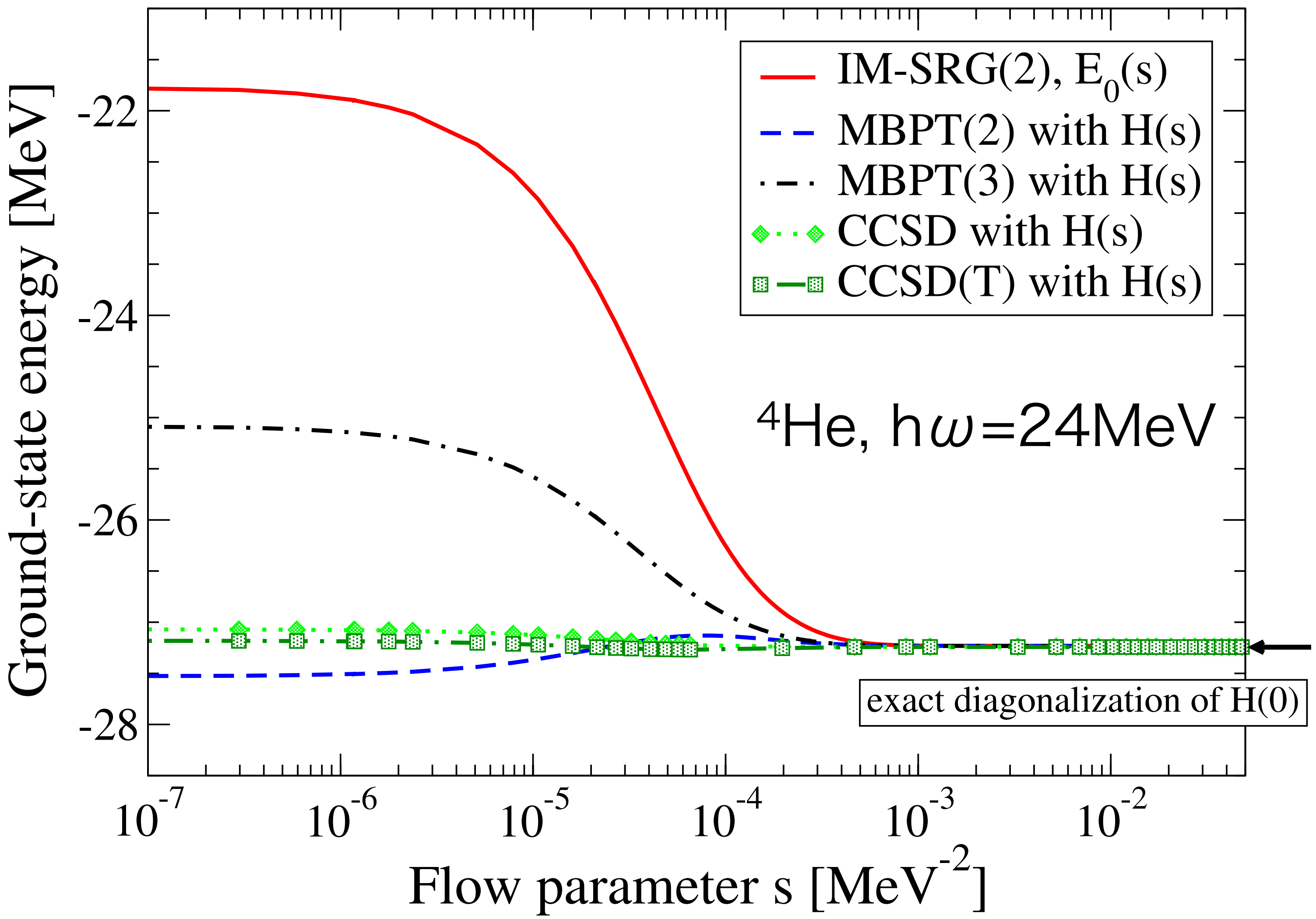
The off-diagonal couplings in the initial Hamiltonian are rapidly driven to zero

Perturbative many-body approximations become more effective in SRG evolution well before the the complete decoupling.

Non-perturbativeness

Many-body methods with the flowing Hamiltonian H(s)

$H(0)=T+V-T_{CM}$, $V=V_{srg}$ $\lambda=2.0$ fm^{-1} from N3LO(500MeV)



✤With the initial Hamiltonian, MBPT breaks down.

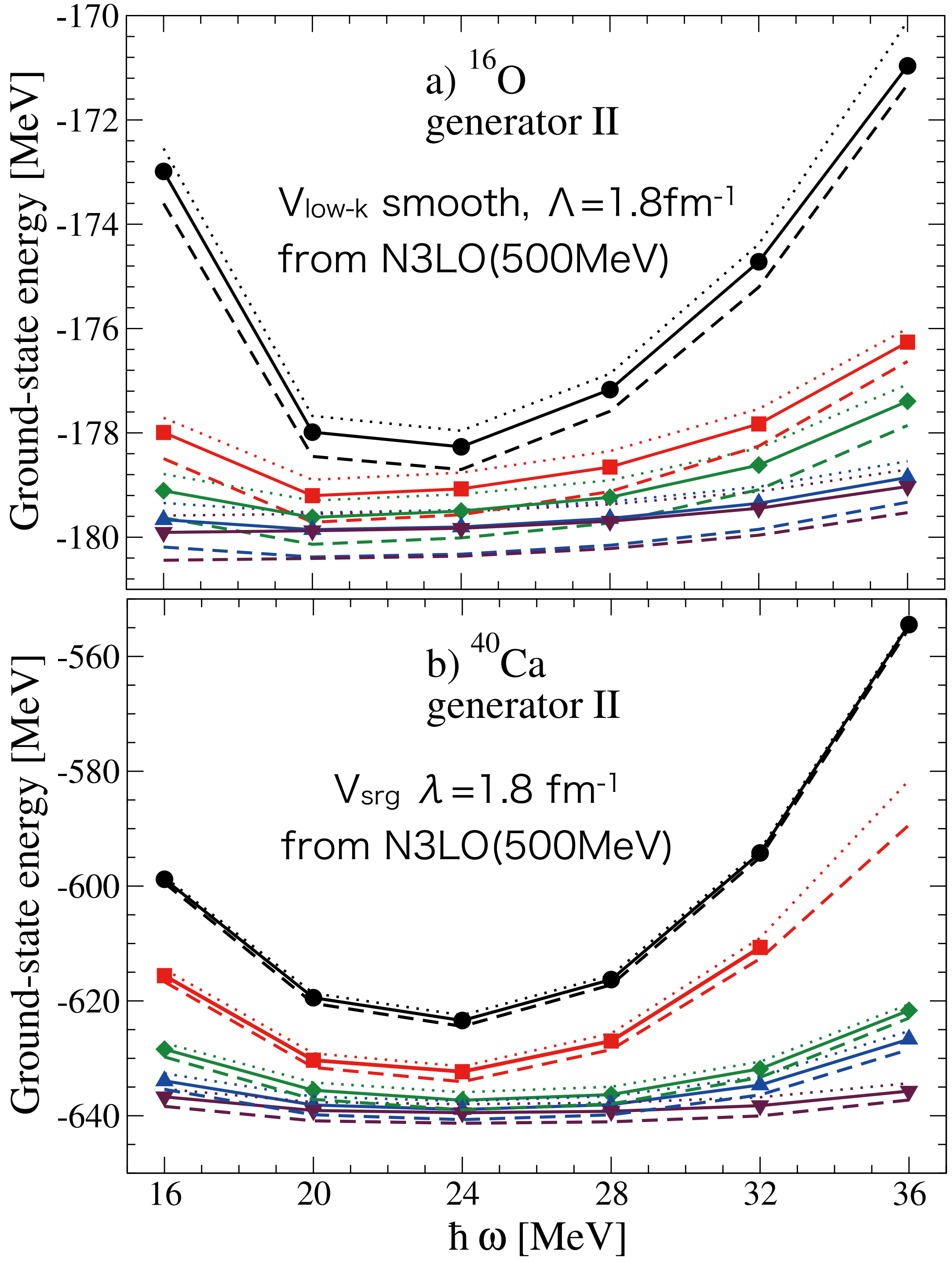
✤With increasing s, MBPT become small.

The 2nd order happens to be nice
BUT the 3rd order goes bad.

✤All many-body methods approach the exact results, where the transformed Hamiltonian can be diagonalized by the simplest state.

✤Almost s-independent CCSD(T) ==> IM-SRG(2) is controllable approximation.

Size-extensibity: Numerically manifested



- ✤ The calculations for ^{16}O, ^{40}Ca are well-converged and very good quality,
- ✤ Falling between CCSD and CCSD(T)
- ✤ IM-SRG can be used for medium-mass nuclei, probably heavier ones as well.

Summary and Outlook

Summary

- ☑ We introduced SRG evolution of Hamiltonian in many-body medium (IM-SRG).
- ☑ The flow equation is derived for finite system in M- and J-scheme representation.
- ☑ We numerically demonstrated the features of in-medium SRG
  - ☑ Decoupling of the ground state.
  - ☑ Size-extensivity
  - ☑ Non-perturbative

Summary and Outlook

Summary

- ☑ We introduced SRG evolution of Hamiltonian in many-body medium (IM-SRG).
- ☑ The flow equation is derived for finite system in M- and J-scheme representation.
- ☑ We numerically demonstrated the features of in-medium SRG
  - ☑ Decoupling of the ground state.
  - ☑ Size-extensivity
  - ☑ Non-perturbative

Work in Progress

- ☐ Application to heavier system ==> frontier of ab-initio method
- ☐ Derivation of effective operator/Hamiltonian for open-shell systems.
  - ☐ effective interaction for valence shell nucleons. <= ready for p-, sd-shell!
  - ☐ effective charge ==> B(E2) for C, Ca, Ni and Sn.
  - ☐ quenching factor for GT transition,
  - ☐ charge/matter radii

$$\boxed{\frac{d\mathcal{O}(s)}{ds} = [\eta(s), \mathcal{O}(s)]}$$

- ☐ An Initial 3NF ==> Impact of 3NF in medium, neutron-rich nuclei.

<= start with normal-ordered 3NF

Thanks to Collaborators

In-medium SRG

Achim Schwenk, TU Darmstadt, EMMI/GSI
Scott Bogner, NSCL/MSU

Applications to the nuclear shell-model

Taka Otsuka (Supervisor), U. Tokyo
Noritaka Shimizu, U. Tokyo

Danke schön!