Effect of Tensor Correlations on Spin-Isospin mode

Second EMMI-EFES Workshop on Neutron-Rich Exotic Nuclei RIKEN June 16-18, 2010

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Center for Mathematics and Physics, University of Aizu

- 1.Introduction
- 2. Isotope and Isotone dependence of single particle energies
- 3. Spin and Spin-Isospin excitations and tensor correlations
- 4. Spin Instability of Nuclear matter and Neutron Matter
- 5. Summary

Deformation of deuteron and Tensor Interaction



Rarita-Schwinger, Phys.Rev.59, 436(1941) Blatt-Weisskopf, Theoretical Nuclear Phys.(1952)

Tensor correlations on Spin-Orbit splitting

Skyrme-type tensor interactions

$$\begin{split} V^T &= \frac{T}{2} \{ [(\sigma_1 \cdot \mathbf{k}')(\sigma_2 \cdot \mathbf{k}') - \frac{1}{3} (\sigma_1 \cdot \sigma_2) \, \mathbf{k'^2}] \delta \left(\mathbf{r_1} - \mathbf{r_2}\right) \\ &+ \delta(\mathbf{r_1} - \mathbf{r_2}) \left[(\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) - \frac{1}{3} (\sigma_1 \cdot \sigma_2) \, \mathbf{k^2} \right] \} \\ &+ \frac{U}{2} \{ (\sigma_1 \cdot \mathbf{k}') \, \delta \left(\mathbf{r_1} - \mathbf{r_2}\right) (\sigma_2 \cdot \mathbf{k}) + (\sigma_2 \cdot \mathbf{k}') \, \delta(\mathbf{r_1} - \mathbf{r_2}) (\sigma_1 \cdot \mathbf{k}) \\ &- \frac{2}{3} \left[(\sigma_1 \cdot \sigma_2) \mathbf{k}' \cdot \delta(\mathbf{r_1} - \mathbf{r_2}) \mathbf{k} \right] \} \end{split}$$

T.H.R. Skyrme, Nucl.Phys. 9,615(1959). F.L. Stancu, D. M. Brink and H. Flocard, PLB68,108 (1977).

T.Lesinski, M. Bender, K. Bennaceur, T. Duguet, J. Meyer, Phys. Rev.C76, 014312(2007). G.Colo, H. Sagawa, S. Fracasso, P.F. Bortignon, Phys. Lett. B 646 (2007) 227.

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SD Strength Distributions Wakasa, SIR2010,18-21 Feb.,2010)

- Total strength
 - Asymmetric single bump
 - Extend up to .50 MeV
 - Same as ⁹⁰Zr(p,n)results
 - SIII provides better description
- 0⁻ strength
 - Quenched
 - Seems to be fragmented
- 1⁻ strength
 - Softened compared with theory
 - Peak shift to lower E_x
- 2⁻ strength
 - Hardened compared with theory
 - $Peak shift to higher E_x$

H. Sagawa et al., PRC 76, 024301 (2007).



- No Skyrme int. which reproduces both total and separated strengths
- \triangleleft ΔJ^{π} -dependent correlation ? \rightarrow Require further investigations

Multipole Expansion of Tensor Interactions

$$\begin{split} V^T &= \frac{T}{2} \{ [(\sigma_1 \cdot \mathbf{k}')(\sigma_2 \cdot \mathbf{k}') - \frac{1}{3} \left(\sigma_1 \cdot \sigma_2 \right) \mathbf{k}'^2] \delta \left(\mathbf{r_1} - \mathbf{r_2} \right) \\ &+ \delta(\mathbf{r_1} - \mathbf{r_2}) \left[(\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) - \frac{1}{3} \left(\sigma_1 \cdot \sigma_2 \right) \mathbf{k}^2 \right] \} \\ &+ \frac{U}{2} \{ (\sigma_1 \cdot \mathbf{k}') \, \delta \left(\mathbf{r_1} - \mathbf{r_2} \right) \left(\sigma_2 \cdot \mathbf{k} \right) + \left(\sigma_2 \cdot \mathbf{k}' \right) \delta(\mathbf{r_1} - \mathbf{r_2}) (\sigma_1 \cdot \mathbf{k}) \\ &- \frac{2}{3} \left[(\sigma_1 \cdot \sigma_2) \mathbf{k}' \cdot \delta(\mathbf{r_1} - \mathbf{r_2}) \mathbf{k} \right] \} \end{split}$$

$$\begin{split} &\delta(\vec{r}_{1}-\vec{r}_{2}) = \sum_{lm} Y_{lm}(\hat{r}_{1})Y_{lm}^{*}(\hat{r}_{2})\frac{\delta(r_{1}-r_{2})}{r_{1}r_{2}} \\ &V^{T} \propto T_{(\lambda,\kappa)} \{ [\sigma_{1} \times [\nabla_{1} \times Y_{l=1}(\hat{r}_{1})]^{(\lambda)} \}^{(\kappa)} [\sigma_{2} \times [\nabla_{2} \times Y_{l=1}(\hat{r}_{2})]^{(\lambda')} \}^{(\kappa)} \}^{(0)} \delta(r_{1}-r_{2}) \\ &1^{+} T_{(\lambda=\lambda=2,\kappa=1)} \Rightarrow repulsive \\ &2^{+} T_{(\lambda=\lambda=2,\kappa=2)} \Rightarrow attractive \end{split}$$

 $3^{+} T_{(\lambda = \lambda = 2, \kappa = 3)} \Rightarrow repulsive$ $1^{+} T_{(\lambda = 2, \lambda = 0, \kappa = 1)} \Rightarrow strong mixing between Gamow - Teller and spin - quadrupole excitations!$

Diagonal p-h matrix element for SD excitations

$$\hat{O}^{\lambda}_{\pm} = \sum_{i} t^{i}_{\pm} r_{i} (Y^{i}_{1} \sigma^{i})_{\lambda}$$

$$\begin{split} V_{TE}^{(\lambda)} &= \frac{5T}{4} \sum_{\ell,k,k'} \frac{(-)^{k+k'+\lambda+\ell+1} \hat{k} \hat{k'}}{2\lambda+1} \begin{cases} k & k' & 2\\ 1 & 1 & \ell \end{cases} \\ &\times \begin{cases} 1 & 1 & 2\\ k' & k & \lambda \end{cases} \langle p || \hat{O}_{k',\lambda} || h \rangle \langle p || \hat{O}_{k,\lambda} || h \rangle^*, \quad (5) \\ V_{TE}^{(\lambda)} &= -\frac{5}{12} T \begin{cases} 1\\ -1/6\\ 1/50 \end{cases} |\langle p || \hat{O}_{1,\lambda} || h \rangle|^2 \quad for \ \lambda = \begin{cases} 0^-\\ 1^-\\ 2^- \end{cases} \end{split}$$

The TO tensor part is also expressed in a similar way as

$$V_{TO}^{(\lambda)} = \frac{5}{12} U \left\{ \begin{array}{c} 1\\ -1/6\\ 1/50 \end{array} \right\} |\langle p||\hat{O}_{1,\lambda}||h\rangle|^2 \quad for \ \lambda = \left\{ \begin{array}{c} 0^-\\ 1^-\\ 2^- \end{array} \right\}$$
(7)

Direct Diagonal Matrix Element

Anti-symmetrized Matrix Element

$$V_T^{(\lambda)} = V_{TE}^{(\lambda)} + V_{TO}^{(\lambda)} \equiv a_\lambda T + b_\lambda U. \qquad \qquad V_{T,AS}^{(\lambda)} = \left[-\frac{1}{2}a_\lambda T + \frac{1}{2}b_\lambda U\right] \langle \tau_1 \cdot \tau_2 \rangle$$

Tensor correlations on Spin-Dipole excitations

$$V^{T} \propto T_{(\lambda,\kappa)} \{ [\sigma_{1} \times [\nabla_{1} \times Y_{l=1}(\hat{r}_{1})]^{(\lambda)} \}^{(\kappa)} [\sigma_{2} \times [\nabla_{2} \times Y_{l=1}(\hat{r}_{2})]^{(\lambda)} \}^{(\kappa)} \}^{(0)} \delta(r_{1} - r_{2})$$

 $0^{-} T_{(\lambda = \lambda = 1, \kappa = 2)} \Longrightarrow repulsive$

1⁻
$$T_{(\lambda=\lambda=1,\kappa=1)} \Rightarrow attractive$$

2⁻
$$T_{(\lambda=\lambda=1,\kappa=2)} \Rightarrow replusive$$

2⁻ $T_{(\lambda=3,\lambda=1,\kappa=2)}$ \Rightarrow strong mixing between SD and Spin - octupole

mode	T term (>0)	U term (<0)	
0- IS	attractive	attractive	
IV	repulsive	repulsive	
1- IS	repulsive	repulsive	
IV	attractive	attractive	
2- IS	attractive	attractive	
IV	repulsive	repulsive	

For charge exchange mode,

0- repulsive (hardening)

- 1- attractive (softening)
- 2- repulsive (hardening)



Tensor Force on Multipole Response in Finite Nuclei

Li-Gang Cao, G. Colo, H. Sagawa, P.F.Bortignon and L. Sciacchitano Phys. Rev.C80, 064304(2009)

SLy5 +Tensor [G. Colo et al., PLB646, 227 (2007)]T44[T. Lesinski et al., PRC76, 014312 (2007)]

$$\Delta E_{\rm RPA} \approx \Delta E_{\rm HF} + < V_{\rm tensor} >$$

SLy5+tensor	T=888.0	U=-408.
Т44	T=520.983	U=21.522

²⁰⁸Pb

$\pi h_{11/2} \rightarrow \pi h_{9/2}$ HF 5.85MeV	6.45MeV(+tensor)
$vi_{13/2} \rightarrow vi_{11/2}$ HF 7.49MeV	9.17MeV(+tensor)
[SLy5] RPA 7.39MeV	7.79MeV(+tensor)
[SLy5] RPA 9.14MeV	10.57MeV(+tensor)
$\Delta \overline{E}_{HF} = 1.14 \text{MeV} < \overline{V}_{T}$	>=-0.23MeV
$\pi h_{11/2} \rightarrow \pi h_{9/2}$ HF 6.4MeV	4.6MeV(+tensor)
$vi_{13/2} \rightarrow vi_{11/2}$ HF 8.7MeV	6.9MeV(+tensor)
[T44] RPA 8.0MeV	6.1MeV(+tensor)
$\begin{bmatrix} T44 \end{bmatrix}$ RPA 10 1 MeV	$9.2 M_{\odot} V (+ top cor)$

 $\Delta \overline{E}_{HF} = -1.8 \text{MeV} < \overline{V}_{T} > = -0.05 \text{MeV}$

Exp. T. Shizuma et al., PRC78, 061303(2008) A. Tamii (this symposium) 5.85MeV 7.30MeV Stability condition with tensor interaction

Li-Gang Cao, G. Colo and H.S., PRC (2010)

$$\begin{split} V_{\rm ph} &= \sum_{\ell} (F_{\ell} + F_{\ell}' \tau_1 \cdot \tau_2 + G_{\ell} \sigma_1 \cdot \sigma_2) \\ &+ G_{\ell}' (\tau_1 \cdot \tau_2) (\sigma_1 \cdot \sigma_2)) P_l(\cos \theta) , \\ &+ \frac{q^2}{k_F^2} H(\cos \vartheta) S_{12}(\hat{q}) + \frac{q^2}{k_F^2} H'(\cos \vartheta) S_{12}(\hat{q}) \tau \cdot \tau) \end{split}$$

$$H_{0} = N_{0}k_{F}^{2}\frac{1}{4}\left(\frac{1}{2}T + \frac{3}{2}U\right) \qquad \qquad H_{0}^{'} = N_{0}k_{F}^{2}\frac{1}{4}\left(-\frac{1}{2}T + \frac{1}{2}U\right)$$

Stability conditions

no tensor

IS
$$l = 0$$
 $1 + G_0 > 0$
IS $l = 1$ $1 + \frac{1}{3}G_1 > 0$

IS
$$l = 1, J = 0$$
 $1 + \frac{1}{3}G_1 - \frac{10}{3}H_0 > 0$ with tensorIS $l = 1, J = 1$ $1 + \frac{1}{3}G_1 + \frac{5}{3}H_0 > 0$

Summary

- 1. Skyrme Tensor is introduced in HF calculations. Triplet-Even and Triplet-Odd components
- 2. The isotope dependence of energy splitting ($\varepsilon(h11/2) \varepsilon(g7/2)$) of Z=50 isotopes is well reproduced by a parameter set of tensor interactions. The same parameter set gives fairly good description of energy difference $\varepsilon(i13/2) - \varepsilon(h9/2)$ of N=82 isotones.
- 3. HF+RPA calculations are performed for Gamow-Teller and Spin Quadrupole excitations. We found that the sum rule strength of GT transitions are increased, while the main peak energies are slightly shifted to lower energy side. This is due to the coupling between GT and SQR with the tensor interactions.
- 4. 10% of sum rule strength is removed from the main peak to higher energy region of SQR.
- 5. Softening and hardening of Spin-Dipole excitations are found in experimentally and RPA with tensor interactions .

Collaborators

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P.F. Bortignon, University of Milano, Italy

Spin-Isospin mode Diagram



Comparison between DWIA and MDA





Spin-orbit splitting

$$\delta H = \frac{1}{2}\alpha(J_n^2 + J_p^2) + \beta J_n J_p.$$

The contribution of the tensor to the <u>total</u> <u>energy</u> is not very large and does not improve mass systematics.(Thomas Duguet)

however, it may play important role for the **spin-orbit splittings.**

$$\begin{split} U_{s.o.}^{(q)} &= \frac{W_0}{2r} \left(2 \frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr} \right) + \left(\alpha \frac{J_q}{r} + \beta \frac{J_{q'}}{r} \right), \\ q \ (n = 0, p = 1) \qquad q' = 1 - q \\ J_q(r) &= \frac{1}{4\pi r^3} \sum_i v_i^2 (2j_i + 1) \left[j_i (j_i + 1) - l_i (l_i + 1) - \frac{3}{4} \right] R_i^2(r). \\ \textbf{SLy5+T} & \qquad TIJ \text{ family} \\ \alpha_c &= \frac{1}{8} (t_1 - t_2) - \frac{1}{8} (t_1 x_1 + t_2 x_2) = 80.7 \text{MeV.fm}^5 \\ \beta_c &= -\frac{1}{8} (t_1 x_1 + t_2 x_2) = -48.9 \text{MeV.fm}^5 \\ \beta_c &= -\frac{1}{8} (t_1 x_1 + t_2 x_2) = -48.9 \text{MeV.fm}^5 \\ \alpha_r &= \frac{5}{12} U = -170 \text{MeV.fm}^5 \\ \beta_T &= \frac{5}{24} (\text{T} + \text{U}) = 100 \text{MeV.fm}^5 \end{split}$$

Tensor part of the Skyrme energy density functional: Spherical nuclei

T. Lesinski,^{1,*} M. Bender,^{2,3,†} K. Bennaceur,^{1,2} T. Duguet,⁴ and J. Meyer¹



Effect of tensor interaction on spin-orbit splitting



SLy5+T

$$\alpha = \alpha_C + \alpha_T = -89.3 \text{MeV.fm}^5$$
$$\beta = \beta_C + \beta_T = 51.1 \text{MeV.fm}^5$$

T44

 $\alpha = \alpha_C + \alpha_T = 120 \text{MeV.fm}^5$ $\beta = \beta_C + \beta_T = 120 \text{MeV.fm}^5$



G.Colo, H. Sagawa, S. Fracasso, P.F. Bortignon, Phys. Lett. B 646 (2007) 227.



Tensor effect of pion and rho meson exchange potentials on Spin-orbit interaction



T. Otsuka et al., PRL95,232502 (2005)

Tensor correlations on Spin-Isospin mode

Effect of Tensor Correlations on Gamow-Teller States in ⁹⁰Zr and ²⁰⁸Pb

C.L. Bai^{1,2)}, H. Sagawa³⁾, H.Q. Zhang^{1,2)}, X.Z. Zhang²⁾, G. Colò⁴⁾ and F.R. Xu¹⁾

$$V^{T} = \frac{T}{2} \{ [(\sigma_{1} \cdot \mathbf{k}')(\sigma_{2} \cdot \mathbf{k}') - \frac{1}{3} (\sigma_{1} \cdot \sigma_{2}) \mathbf{k}'^{2}] \delta (\mathbf{r_{1}} - \mathbf{r_{2}}) \\ + \delta(\mathbf{r_{1}} - \mathbf{r_{2}}) [(\sigma_{1} \cdot \mathbf{k})(\sigma_{2} \cdot \mathbf{k}) - \frac{1}{3} (\sigma_{1} \cdot \sigma_{2}) \mathbf{k}^{2}] \} \\ + \frac{U}{2} \{ (\sigma_{1} \cdot \mathbf{k}') \delta (\mathbf{r_{1}} - \mathbf{r_{2}}) (\sigma_{2} \cdot \mathbf{k}) + (\sigma_{2} \cdot \mathbf{k}') \delta (\mathbf{r_{1}} - \mathbf{r_{2}}) (\sigma_{1} \cdot \mathbf{k}) \\ - \frac{2}{3} [(\sigma_{1} \cdot \sigma_{2}) \mathbf{k}' \cdot \delta (\mathbf{r_{1}} - \mathbf{r_{2}}) \mathbf{k}] \}$$

$$m_{-}(0) - m_{+}(0) = \sum_{\nu} \left(|\langle \nu | O_{-} | 0 \rangle|^{2} - |\langle \nu | O_{+} | 0 \rangle|^{2} \right)$$
$$= \langle 0 | [O_{-}, O_{+}] | 0 \rangle,$$

 $m_{-}(1) + m_{+}(1) = \sum_{\nu} \left(|\langle \nu | O_{-} | 0 \rangle| + |\langle \nu | O_{+} | 0 \rangle|^{2} \right) E_{\nu}$ = $\langle 0 | [O_{+}, [H, O_{-}]] | 0 \rangle,$

$$\begin{aligned} \Delta E_{GT} &= \frac{m_{-}(1)}{m_{-}(0)} \\ &\sim \frac{m_{-}(1) + m_{+}(1)}{m_{-}(0) - m_{+}(0)} \\ &= \frac{4\pi}{3(N-Z)} \int dr r^{2} \left[-\left(\frac{5}{2}U + \frac{5}{2}T\right) J_{n} J_{p} - \frac{5}{3}U(J_{n}^{2} + J_{p}^{2}) \right] \end{aligned}$$

S3T

 $m_{-}(1; \text{no tensor}) m_{-}(1; \text{with tensor}) \delta E_{RPA} \delta E_{DC}$

	MeV	MeV	MeV	MeV	
90 Zr	271.45	338.68	2.241	2.276	
208 Pb	1854.12	2000.76	1.111	1.118	

The tensor force in RPA



Gamow-Teller



The main peak is moved downward by the tensor force but the centroid is moved upwards !

C.L.Bai, HS, H.Q.Zhang, X.Z.Zhang, G.Colo and F.R.Xu, P.L.B675,28 (2009). C.L.Bai, H.Q. Zhang, X.Z.Zhang, F,R,Xu, HS and G.Colo, PRC79, 041301(R) (2009).

	type of	$m_{-}(0)$	$m_{-}(0)$	$m_{-}(1)$	$m_{-}(1)$	$m_{-}(1)$	$m_{+}(1)$
	calculation	$0-30 \mathrm{MeV}$	$30-60 \mathrm{MeV}$	$0-30 { m MeV}$	$30-60 { m MeV}$	total	total
⁹⁰ Zr	00	29.16	0.71	395	26.2	421.8	10.1
	10	29.16	0.79	444	22	466	11.1
	11	27.00	2.89	366.9	122	493.2	10.3
²⁰⁸ Pb	00	127.54	3.43	2080	124.5	2212.8	18.8
	10	127.38	3.68	2176	93	2269	21
	11	114.10	16.58	1658	694	2370	19.3

About 10% of strength is moved by the tensor correlations to the energy region above 30 MeV.

Relevance for the GT quenching problem.













 $f_{7/2} \rightarrow f_{5/2}$ HF 8.90MeV 8.60MeV(+tensor) [T44] RPA 10.9MeV 10.47MeV(+tensor) ΔE_{HF} = 0.30MeV < V_T >= −0.13MeV Exp.

(p. 10.23MeV





Stability under extreme conditions

Large asymmetries, high densities, finite T, ...

RPA framework: probe the fluctuations around the ground-state \rightarrow local stability criterium

Validity check of residual interaction \rightarrow Landau parameters

$$V_{\rm ph} = \sum_{\ell} (F_{\ell} + F'_{\ell} \tau_1 \cdot \tau_2 + G_{\ell} \sigma_1 \cdot \sigma_2 + G'_{\ell} (\tau_1 \cdot \tau_2) (\sigma_1 \cdot \sigma_2)) P_l(\cos\theta) ,$$

Matter is stable if

 $F_l > -2l - 1$











Ferromagnetic phase diagram: G & G'









IS l = 0 $1 + G_0 > 0$ (without tensor)



IS
$$J = 1^+$$
 $(2 + G_0) \pm \sqrt{G_0^2 + 8H_0^2} > 0$





IV l = 0 $1 + G_0 > 0$ (with











Summary and Future perspectives II

6. Spin and Spin-Isospin instability are strongly affected by the tensor inetractions.

7. Spin and Spin-isospin Landau parameters are improved by introducing spin and spin-isopsin density dependent interactions.

8. Good mass systematics is preserved within the rms deviation of less than 600keV by introducing new terms (S. Goriery).

Mean Field Theories

Non-relativistic model

HF: Skyrme Interaction

T.H.R Skyrme(1956,1959)

D. Vautherin and D. Brink(1972)

F. Stancu, D. Brink and H. Flocard +tensor (1977)

Gogny Interaction (1975)

J. Decharge and D. Gogny (1980)

Relativistic model

RMF(Hartree)

J. D. Walecka (1974)

B. D. Serot and J. D. Walecka (1986)

RHF

A. Bouyssy, J.F.Mathiot, N.V.Giai and S. Marcos(1987)

TDA and RPA theory

Linear Response theory (coordinate space RPA)

G.F.Bertsch and S.F.Tsai (1975)

K.F.Liu and Nguyen Van Giai (1976) + continuum

M. Matsuo (2001) +pairing

RRPA

D. Vretenar, P. Ring ----, TDRMF(1994)

Z.Y. Ma, N.V.Giai, H. Toki ----(1997) RRPA linear response