Renormalization of the tensor force in effective interaction of nuclear force

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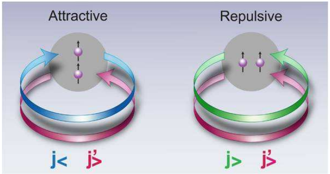
- Change of effective single particle energy can be described as (if $j \neq j'$),
  $\Delta\epsilon_p(j) = \frac{1}{2}\left(V_{jj'}^{T=0} + V_{jj'}^{T=1}\right) n_n(j')$
  n_n : occupation number of neutron in orbit j'
  (using monopole interaction)



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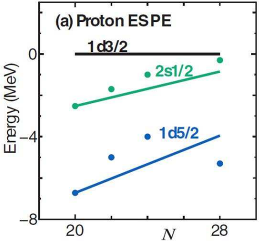

dots: experiment

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Simple modeling: taking tensor force as $\pi + \rho$ meson exchange

Same potential in all nuclei, with no fit.

Question: Can we consider tensor force in medium so simply?
→ to answer this question, examination based on *realistic nuclear force* and *microscopic theory* is needed.

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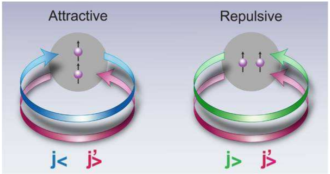

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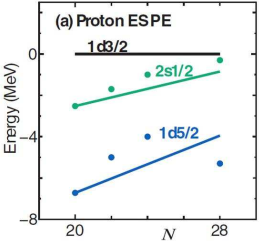

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Original eigenvalue problem and effective interaction

$$H|\Psi_i\rangle = E_i|\Psi_i\rangle \rightarrow P\tilde{H}P|\phi_i\rangle = E_i|\phi_i\rangle, \quad |\phi_i\rangle = P|\Psi_i\rangle.$$

Similarity transformation with decoupling property:

$$\tilde{H} = e^{-\omega}He^{\omega}, \quad Q\omega P = \omega, \quad P\tilde{H}Q = 0$$

Formal solution : $\omega = \sum_{i=1}^{d} Q|\Psi_i\rangle\langle\tilde{\phi}_i|P, \quad |\Psi_i\rangle = |\phi_i\rangle + \omega|\phi_i\rangle$

(Need complete information on true eigenstate $|\Psi_i\rangle$)

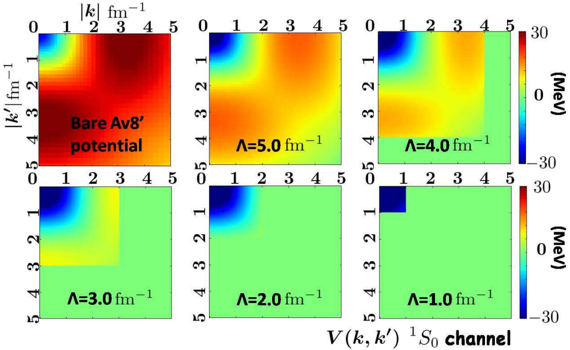
Iterative solution : $V_{\rm eff}^{(n)} = \hat{Q}(E_0) + \sum_m \hat{Q}_m(E_0)\{V_{\rm eff}^{(n-1)}\}^m$

$$\hat{Q}(E_0) \equiv PH_1P + PH_1\frac{1}{E_0 - QHQ}QH_1P, \quad \hat{Q}_m(E_0) \equiv \frac{1}{m!}\frac{d^m\hat{Q}(E_0)}{dE_0^m}.$$

$\hat{Q}(E_0)$: Q-box

(Degenerate unperturbed model space $H_0|\phi_i\rangle = E_0|\phi_i\rangle$)

Solve decoupling equation in momentum space ($\omega = \sum_{i=1}^{d} Q|\Psi_i\rangle\langle\tilde{\phi}_i|P$)



- Coupling between high momentum and low momentum decreases.
- Low momentum attraction increases.

Λ : cutoff parameter

→ boundary between ' low ' momentum and ' high ' momentum.

Conserve all the low-momentum observables and wave functions.

Twobody interaction: $V = \sum_p V_p = \sum_p U^p \cdot X^p$

U_p: rank p operator in coordinate space

X_p: rank p operator in spin space

$p = 0$: central
$p = 1$: spin-orbit
$p = 2$: tensor

Spin-tensor decomposition

$$\langle ABLS|V_p|CDL'S'\rangle_{J'T} = (-1)^{J'}\hat{p}\left\{\begin{array}{ccc} L & S & J' \\ S' & L' & p \end{array}\right\} \times \sum_J (-1)^J \hat{J}\left\{\begin{array}{ccc} L & S & J \\ S' & L' & p \end{array}\right\}\langle ABLS|V|CDL'S'\rangle_{JT}$$

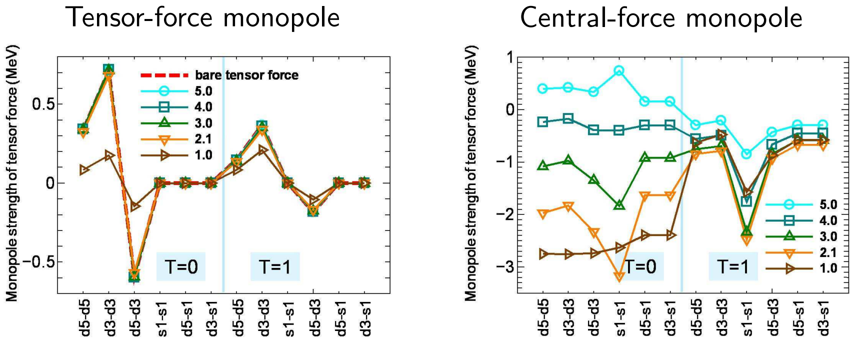
Transformation from jj-coupled matrix elements to LS-coupled matrix elements

$$\langle ABLSJT|V|CDL'S'JT\rangle = [(1+\delta_{AB})(1+\delta_{CD})]^{1/2} \sum_{j_a,j_b,j_c,j_d} \left[\begin{array}{ccc} l_a & \frac{1}{2} & j_a \\ l_b & \frac{1}{2} & j_b \\ L & S & J \end{array}\right]\left[\begin{array}{ccc} l_c & \frac{1}{2} & j_c \\ l_d & \frac{1}{2} & j_d \\ L' & S' & J \end{array}\right] \times [(1+\delta_{ab})(1+\delta_{cd})]^{1/2}\langle abJT|V|cdJT\rangle$$

We can decompose two-body interaction into central, spin-orbit and tensor component uniquely

Cutoff of $V_{\mathrm{low}\,k}$: $\Lambda = 1.0 - 5.0\ \mathrm{fm}^{-1}$

$$V_{ab;T} = \frac{\sum_J (2J+1)\langle ab|V|ab\rangle_{JT}}{\sum_J 2J+1}$$



Tensor force

alomost **no** dependence on cutoff Λ

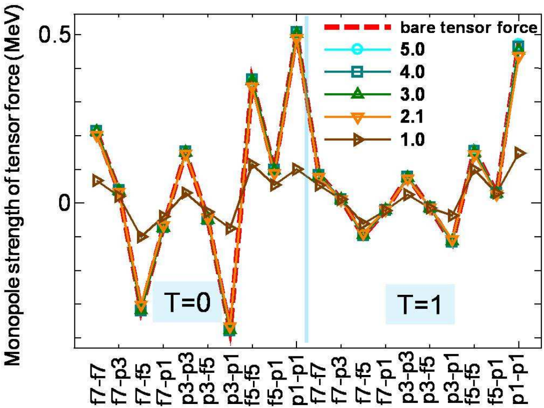
Central force

strongly changed by renormalization

⇒ long-range nature of tensor force

Cutoff of $V_{\mathrm{low}\,k}$: $\Lambda = 1.0 - 5.0\ \mathrm{fm}^{-1}$

Tensor-force monopole



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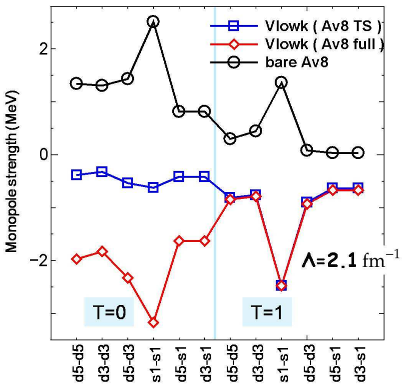
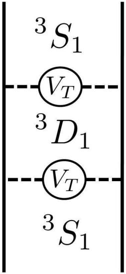
Tensor force survives in *pf*-shell also

Effect of renormalization of the tensor force

$V_{\text{low}\,k}$ from Av8' Tensor subtracted (Av8' TS)
$V_{\text{low}\,k}$ from Av8' (Av8 full)

Central-force monopole





Short range part of tensor force
⇒ strongly renormalized to central force.

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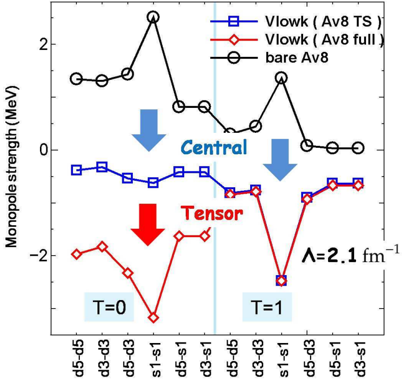

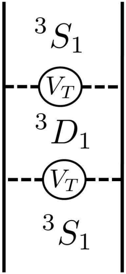

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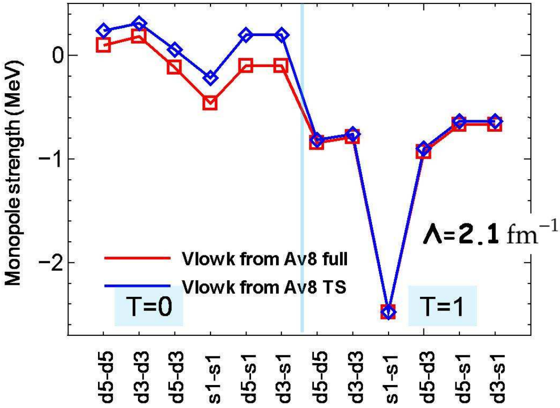
Central-force monopole without ${}^3S_1 - {}^3D_1$ channel



The main contribution comes from ${}^3S_1 - {}^3D_1$ channel

Schrodinger equation for deuteron:

$$-\frac{\hbar^2}{M}\frac{d^2u(r)}{dr^2}+V_C u(r)+\sqrt{8}V_T w(r)=E_d u(r)$$
$$-\frac{\hbar^2}{M}\frac{d^2w(r)}{dr^2}+\left(\frac{6\hbar^2}{Mr^2}+V_C-2V_T-3V_{LS}\right)w(r)+\sqrt{8}V_T u(r)=E_d w(r)$$

Effective central force

$$V_{\rm eff}(r;{}^3S_1)=V_C(r;{}^3S_1)+\Delta V_{\rm eff}(r;{}^3S_1),\qquad \Delta V_{\rm eff}(r;{}^3S_1)\equiv\sqrt{8}V_T(r)\frac{w(r)}{u(r)}.$$

→ This leads additional attraction to the central force

Feshbach's formal solution of the effective interaction

$$PHP|\Psi\rangle+PHQ\frac{1}{E-QHQ}QHP|\Psi\rangle=E|\Psi\rangle,\qquad H_{\rm eff}=PHP+PHQ\frac{1}{E-QHQ}QHP$$

$V_{\mathrm{low}\,k}$

- P-space: low-momentum
- Q-space: high-momentum

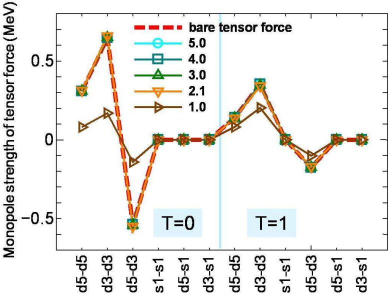
Effective central force

- P-space: S-wave wavefunction
- Q-space: D-wave wavefunction

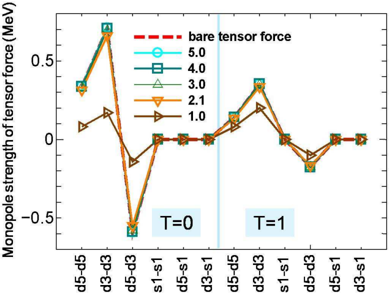
→ can be understood on the same footing

Tensor-force monopole of $V_{\mathrm{low}\,k}$ starting from another realistic nuclear forces

χ N3LO (*sd*-shell)



CD-bonn (*sd*-shell)



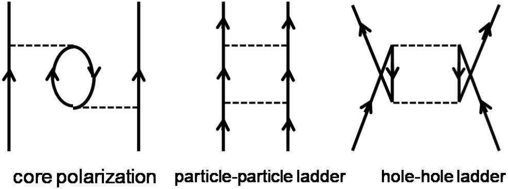
Similar results are obtained in case of N3LO and CD-bonn
$\rightarrow$ irrelevant to spcific model of the nuclear force
(Same conculusion also in *pf*-shell)

Effective interaction for shell model

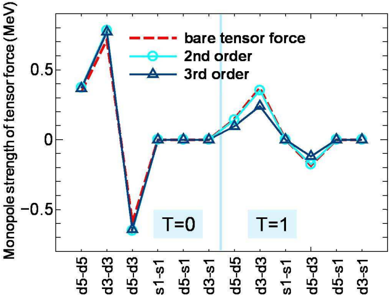
$$V_{\text{eff}}^{(n)} = \hat{Q}(E_0) + \sum_m \hat{Q}_m(E_0)\{V_{\text{eff}}^{(n-1)}\}^m$$

- Starting from $V_{\text{low}\,k}$
- Q-box and its folded diagrams is taken into account
- Effective interaction for *sd*-shell and *pf*-shell
- Harmonic oscillator basis

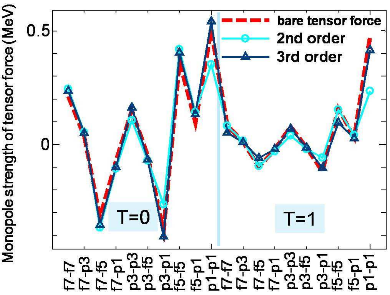
Diagrams included in 2nd order Q-box



Tensor-force monopole (*sd*-shell)



Tensor-force monopole (*pf*-shell)



- Tensor force survives even in the effective interaction for shell model, in both *sd*-shell and *pf*-shell
- Complicated and specific structure of the tensor force

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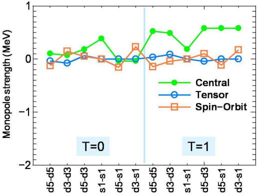
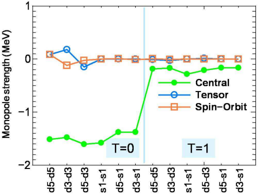
Evaluation of the Q-box

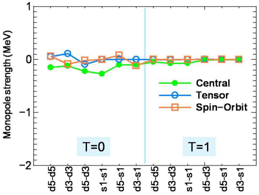
core-polarization

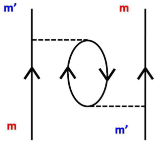


particle-particle ladder



hole-hole ladder





- Predominantly the central force
- Tensor component is not coherent

Only exchange diagram contribute to Tensor-force monopole
→ always has the same sign
→ not predominantly the tensor force
(tensor force has strong state dependence)

- Tensor force is not affected much by renormalizations, at least in its monopole part.
  - Tensor-force monopole in $V_{\mathrm{low}k}$ is quite similar to that of original potential.
    → long-range nature of tensor force.
  - Tensor-force monopole in the effective interaction for the shell model calculated by the Q-box expansion is also similar to that of original potential.
    → because tensor force has complicated and specific structure, tensor force is hardly induced by renormalization through second or third order in interaction.
  - These results do not rely on the spcific model of the original nuclear force. (Av8', CD-bonn, χN3LO ...)

→ Tensor force survives both renormalization of short-range correlations and in-medium effects.

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