

Operator representation of realistic potentials for  
nuclear many-body calculations

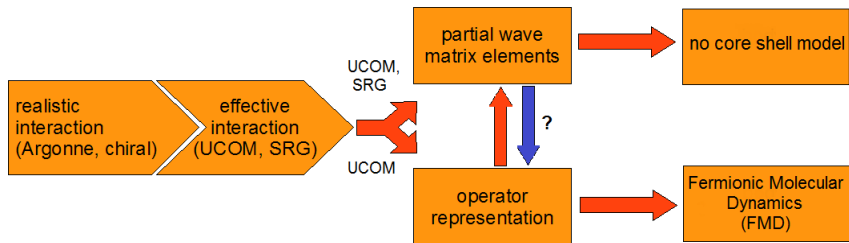
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GSI Helmholtzzentrum für Schwerionenforschung GmbH

17.06.2010

# Motivation

$$\langle k(LS)J; T | \tilde{V}_{NN} | k'(L'S)J; T \rangle$$



$$\begin{aligned} \tilde{V}_{NN} &= \sum_{S,T} V_{ST}^Z(r) \tilde{\Pi}_{ST} \\ &+ \sum_T V_{1T}^{LS}(r) \tilde{L}\tilde{S} \tilde{\Pi}_{1T} \\ &+ \dots \end{aligned}$$

# Argonne potential

## operator representation

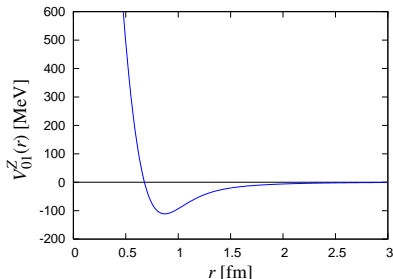


▶ operator representation:

$$\begin{aligned}
 \tilde{V}_{Arg} &= \sum_{S,T} V_{ST}^Z(r) \tilde{\Pi}_{ST} \\
 &+ \sum_{S,T} V_{ST}^{L^2}(r) \tilde{L}^2 \tilde{\Pi}_{ST} \\
 &+ \sum_T V_{1T}^{LS}(r) \tilde{L} \tilde{S} \tilde{\Pi}_{1T} \\
 &+ \sum_T V_{1T}^T(r) \tilde{S}_{12} \tilde{\Pi}_{1T} \\
 &+ \sum_T V_{1T}^{TLL}(r) \tilde{S}_{12}(\vec{L}, \vec{L}) \tilde{\Pi}_{1T}
 \end{aligned}$$

$$\tilde{S}_{12} = \frac{3}{r^2} (\vec{r} \cdot \vec{\sigma}_1) (\vec{r} \cdot \vec{\sigma}_2) - \vec{\sigma}_1 \vec{\sigma}_2$$

$$\tilde{S}_{12}(\vec{a}, \vec{b}) = \frac{3}{2} [(\vec{\sigma}_1 \cdot \vec{a})(\vec{\sigma}_2 \cdot \vec{b}) + (\vec{\sigma}_1 \cdot \vec{b})(\vec{\sigma}_2 \cdot \vec{a})] - \frac{1}{2} (\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a})$$

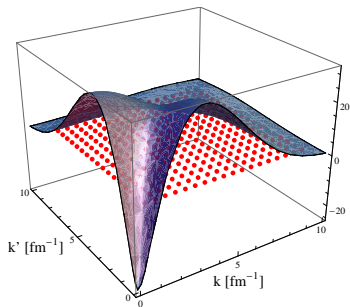


# Argonne potential

partial wave matrix elements

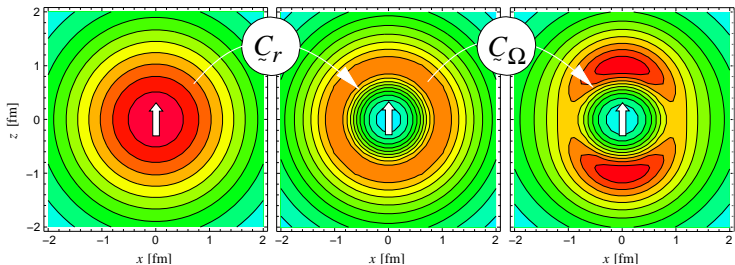
- ▶ for each  $L, L', S, J, T$ :

$$\langle k(LS)J; T | \tilde{V} | k'(L'S)J; T \rangle$$



matrix elements in MeV fm<sup>3</sup> for  $S=0, T=1$  and  $L_1=L_2=0$

# Unitary Correlation Operator Method



taken from T. Neff, H. Feldmeier, Nucl. Phys. A713

- ▶ correlations introduced explicitly by the 'Unitary Correlation Operator Method' UCOM:

- ▶ correlated state

$$|\hat{\Psi}\rangle = \tilde{\mathcal{C}}|\Psi\rangle = \tilde{\mathcal{C}}_{\Omega}\tilde{\mathcal{C}}_r|\Psi\rangle$$

- ▶ correlated operator

$$\langle\hat{\Psi}'|\hat{B}|\hat{\Psi}\rangle = \langle\Psi'|\tilde{\mathcal{C}}^{\dagger}\hat{B}\tilde{\mathcal{C}}|\Psi\rangle \equiv \langle\Psi'|\hat{\tilde{B}}|\Psi\rangle$$

# UCOM potential

## operator representation



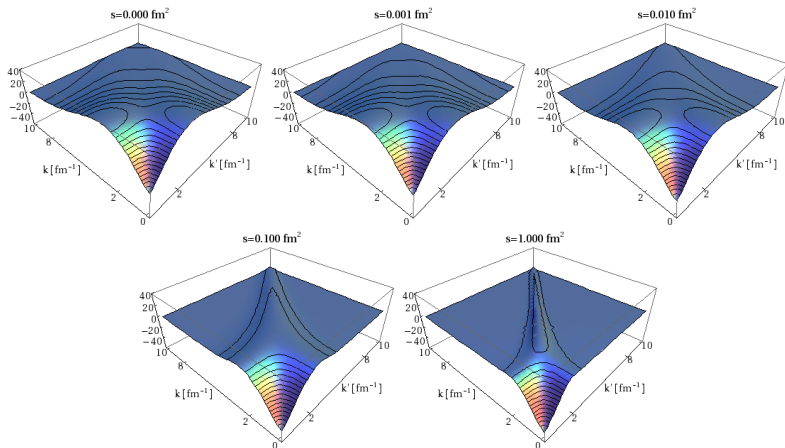
► operator representation:

$$\begin{aligned}
 \mathcal{V}_{UCOM} = \widehat{H}^{[2]} = \widehat{T}^{[2]} + \widehat{V}^{[2]} &= \sum_{S,T} V_{ST}^Z(r) \mathbb{1}_{ST} \\
 &+ \sum_{S,T} V_{ST}^{L2}(r) \widetilde{L}^2 \mathbb{1}_{ST} \\
 &+ \sum_{S,T} \frac{1}{2} (\widetilde{p}^2 V_{ST}^{p2}(r) + V_{ST}^{p2}(r) \widetilde{p}^2) \mathbb{1}_{ST} \\
 &+ \sum_T V_{1T}^{LS}(r) \widetilde{L} \widetilde{S} \mathbb{1}_{1T} \\
 &+ \sum_T V_{1T}^{L2LS}(r) \widetilde{L}^2 \widetilde{L} \widetilde{S} \mathbb{1}_{1T} \\
 &+ \sum_T V_{1T}^T(r) \mathbb{S}_{12} \mathbb{1}_{1T} \\
 &+ \sum_T V_{1T}^{TLL}(r) \mathbb{S}_{12}(\widetilde{L}, \widetilde{L}) \mathbb{1}_{1T} \\
 &+ \sum_T V_{1T}^{Tpp}(r) \widetilde{\mathbb{S}}_{12}(p\Omega, p\Omega) \mathbb{1}_{1T} \\
 &+ \sum_T (p_r V_{1T}^{Trp}(r) + V_{1T}^{Trp}(r) p_r) \widetilde{\mathbb{S}}_{12}(r, p\Omega) \mathbb{1}_{1T} \\
 &+ \dots
 \end{aligned}$$

# Similarity Renormalization Group

## SRG evolution

### ► SRG evolution

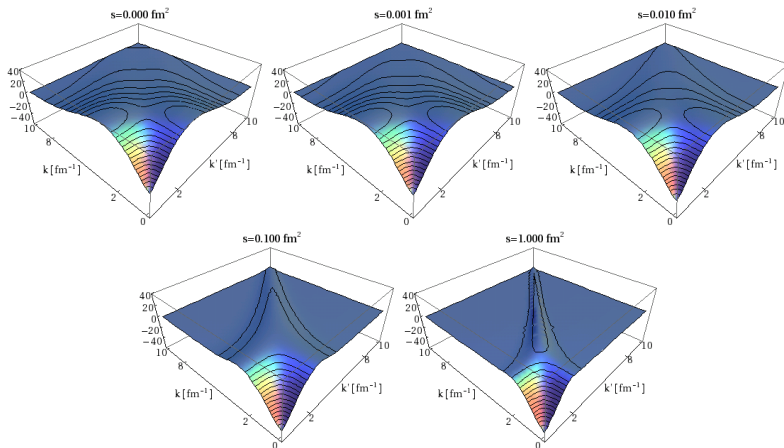


### ► matrix elements, but no operator representation

# Similarity Renormalization Group


## SRG evolution

### ► SRG evolution



### ► matrix elements, **but no operator representation**



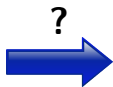
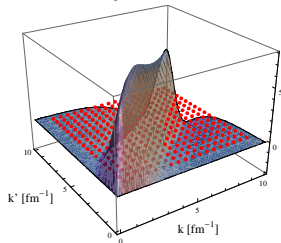
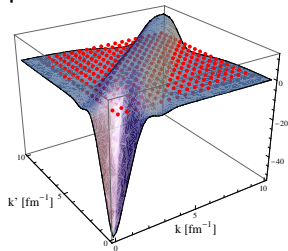


## Operator representation and matrix elements

# From partial wave matrix elements to operator representation



partial wave matrix elements



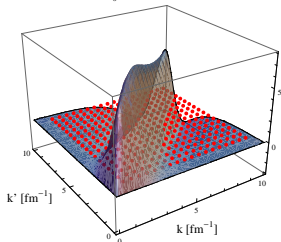
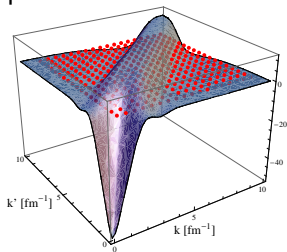
operator representation

$$\begin{aligned}
 \tilde{V}_{NN} &= \sum_{ST,i} V_{ST}^i(r) \tilde{\mathcal{O}}_i \tilde{\Pi}_{ST} \\
 &= \sum_{S,T} V_{ST}^Z(r) \tilde{\Pi}_{ST} \\
 &+ \sum_T V_{1T}^{LS}(r) \tilde{\vec{L}} \tilde{\vec{S}} \tilde{\Pi}_{1T} \\
 &+ \dots
 \end{aligned}$$

# From partial wave matrix elements to operator representation



partial wave matrix elements



Fit  
←  
 $\gamma_{ST,j}^i$

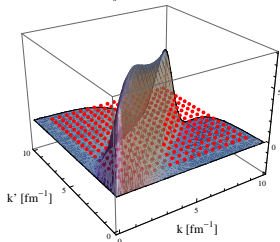
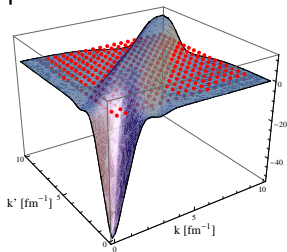
- operator representation
- ▶ choose appropriate set of operators

$$V_{\text{Ansatz}} = \sum_{ST,i} V_{ST}^i(r) \mathcal{O}_i \Pi_{ST}$$

# From partial wave matrix elements to operator representation



partial wave matrix elements



Fit  
←  
 $\gamma_{ST,j}^i$

operator representation

- ▶ choose appropriate set of operators

$$V_{Ansatz} = \sum_{ST,i} V_{ST}^i(r) \mathcal{O}_i \Pi_{ST}$$

- ▶ representation of (unknown!) radial dependencies by a sum of gaussians:

$$V_{ST}^i(r) = \sum_j \gamma_{ST,j}^i \exp\left\{-\frac{r^2}{2\kappa_j}\right\}$$

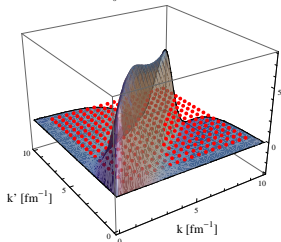
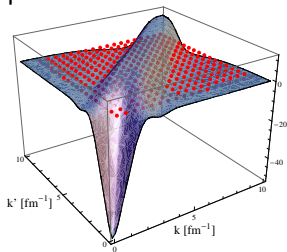
- ▶ choose parameters:  $\kappa_j = \kappa_1 \cdot b^{j-1}$

$$\kappa = \{0.05, 0.05 \cdot \sqrt{2}, 0.1, \dots 6.4\} \text{ fm}^2$$

# From partial wave matrix elements to operator representation



partial wave matrix elements



Fit  
←  
 $\gamma_{ST,j}^i$

operator representation

- ▶ choose appropriate set of operators

$$\mathcal{V}_{Ansatz} = \sum_{ST,i} \mathcal{V}_{ST}^i(r) \mathcal{Q}_i \mathcal{P}_{ST}$$

- ▶ representation of (unknown!) radial dependencies by a sum of gaussians:

$$\mathcal{V}_{ST}^i(r) = \sum_j \gamma_{ST,j}^i \exp\left\{-\frac{r^2}{2\kappa_j}\right\}$$

- ▶ choose parameters:  $\kappa_j = \kappa_1 \cdot b^{j-1}$

$$\kappa = \{0.05, 0.05 \cdot \sqrt{2}, 0.1, \dots, 6.4\} \text{ fm}^2$$

- ▶ calculate **analytically** matrix elements

$$\langle k(LS)JT | \mathcal{V}_{Ansatz} | k'(L'S)JT \rangle$$



# Applications

# Results

UCOM - reduced set of operators

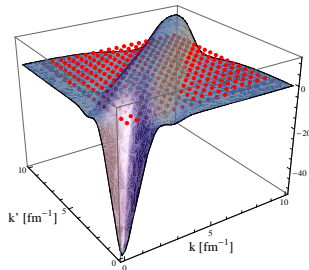
- ▶ Which operators do we **really** need?
- ▶ procedure:
  - ▶ fitting Ansatz with **less** operators to matrix elements containing **full** set of operators
  - ▶ optimized for 'low' angular momenta, contributions from neglected terms 'absorbed' in other terms
  - ▶ reduced set of operators has to describe two-nucleon data correctly

# Results

## UCOM - reduced set of operators

### ► Ansatz

$$\begin{aligned}
 \underline{V}_{\text{Ansatz}} = & \sum_{S,T} V_{ST}^Z(\underline{r}) \underline{\Pi}_{ST} \\
 & + \sum_{S,T} V_{ST}^{L2}(\underline{r}) \underline{\vec{L}}^2 \underline{\Pi}_{ST} \\
 & + \sum_{S,T} \frac{1}{2} (\underline{\vec{p}}^2 V_{ST}^{p2}(\underline{r}) + V_{ST}^{p2}(\underline{r}) \underline{\vec{p}}^2) \underline{\Pi}_{ST} \\
 & + \sum_T V_{1T}^{LS}(\underline{r}) \underline{\vec{L}} \underline{\vec{S}} \underline{\Pi}_{1T} \\
 & + \sum_T V_{1T}^{L2LS}(\underline{r}) \underline{\vec{L}}^2 \underline{\vec{L}} \underline{\vec{S}} \underline{\Pi}_{1T} \\
 & + \sum_T V_{1T}^T(\underline{r}) \underline{S}_{12} \underline{\Pi}_{1T} \\
 & + \sum_T V_{1T}^{TLL}(\underline{r}) \underline{S}_{12}(\underline{\vec{L}}, \underline{\vec{L}}) \underline{\Pi}_{1T} \\
 & + \sum_T V_{1T}^{TPP}(\underline{r}) \underline{\vec{S}}_{12}(\rho_\Omega, \rho_\Omega) \underline{\Pi}_{1T} \\
 & + \sum_T (\underline{p}_r V_{1T}^{Trp}(\underline{r}) + V_{1T}^{Trp}(\underline{r}) \underline{p}_r) \underline{S}_{12}(r, \rho_\Omega) \underline{\Pi}_{1T}
 \end{aligned}$$



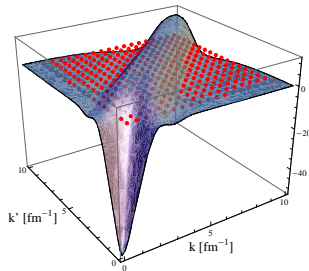


# Results

## UCOM - reduced set of operators

### ► Ansatz

$$\begin{aligned}
 \underline{V}_{\text{Ansatz}} = & \sum_{S,T} V_{ST}^Z(\underline{r}) \underline{\Pi}_{ST} \\
 & + \sum_{S,T} V_{ST}^{L2}(\underline{r}) \underline{\vec{L}}^2 \underline{\Pi}_{ST} \\
 & + \sum_{S,T} \frac{1}{2} (\underline{\vec{p}}^2 V_{ST}^{p2}(\underline{r}) + V_{ST}^{p2}(\underline{r}) \underline{\vec{p}}^2) \underline{\Pi}_{ST} \\
 & + \sum_T V_{1T}^{LS}(\underline{r}) \underline{\vec{L}} \underline{\vec{S}} \underline{\Pi}_{1T} \\
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 \end{aligned}$$

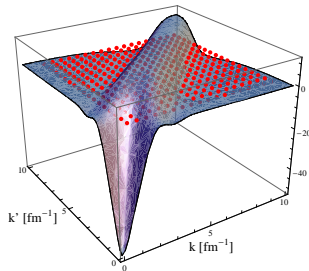


# Results

UCOM - reduced set of operators

## ► Ansatz

$$\begin{aligned}
 \underline{V}_{\text{Ansatz}} = & \sum_{S,T} V_{ST}^Z(\underline{r}) \underline{\Pi}_{ST} \\
 & + \sum_{S,T} V_{ST}^{L2}(\underline{r}) \underline{\vec{L}}^2 \underline{\Pi}_{ST} \\
 & + \sum_{S,T} \frac{1}{2} (\underline{\vec{p}}^2 V_{ST}^{p2}(\underline{r}) + V_{ST}^{p2}(\underline{r}) \underline{\vec{p}}^2) \underline{\Pi}_{ST} \\
 & + \sum_T V_{1T}^{LS}(\underline{r}) \underline{\vec{L}} \underline{\vec{S}} \underline{\Pi}_{1T} \\
 & + \sum_T V_{1T}^{L2LS}(\underline{r}) \underline{\vec{L}}^2 \underline{\vec{L}} \underline{\vec{S}} \underline{\Pi}_{1T} \\
 & + \sum_T V_{1T}^T(\underline{r}) \underline{S}_{12} \underline{\Pi}_{1T} \\
 & + \sum_T V_{1T}^{TLL}(\underline{r}) \underline{S}_{12}(\underline{\vec{L}}, \underline{\vec{L}}) \underline{\Pi}_{1T} \\
 & + \sum_T V_{1T}^{TPP}(\underline{r}) \underline{S}_{12}(\rho_\Omega, \rho_\Omega) \underline{\Pi}_{1T} \\
 & + \sum_T (\underline{p}_r V_{1T}^{Trp}(\underline{r}) + V_{1T}^{Trp}(\underline{r}) \underline{p}_r) \underline{S}_{12}(r, \rho_\Omega) \underline{\Pi}_{1T}
 \end{aligned}$$



# Results

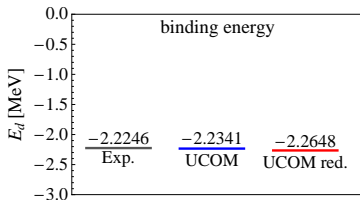
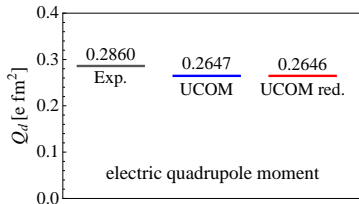
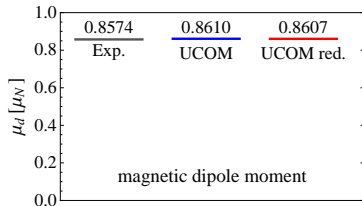
UCOM - reduced set of operators

- ▶ Does the reduced set of operators keep the features of the interaction?
  - ▶ deuteron
  - ▶ phase-shifts
  - ▶ few-nucleon systems

# Results

UCOM - reduced set of operators

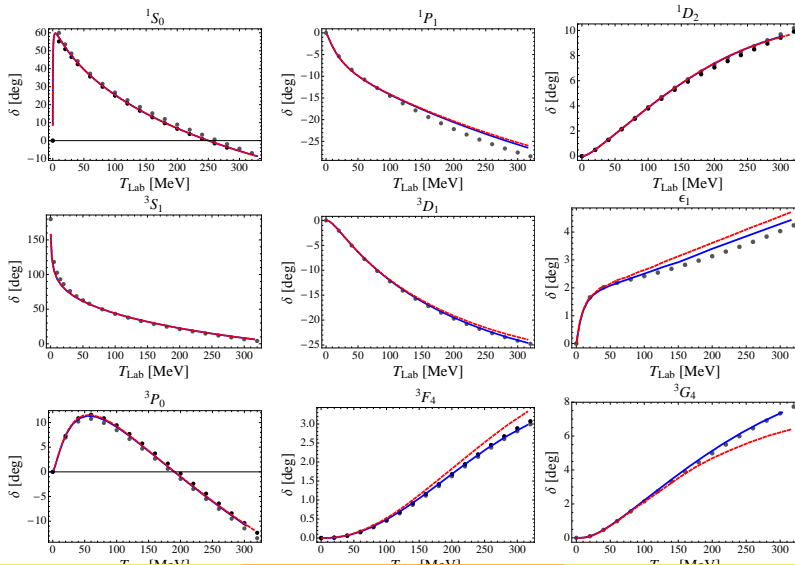
► deuteron properties:



$P_d$  and  $Q_d$  calculated with uncorrelated operators **GSI**

# Results

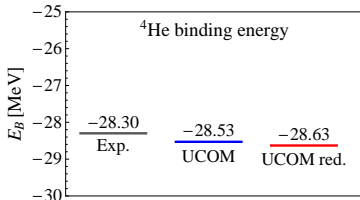
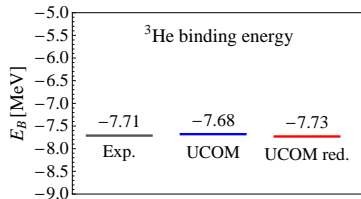
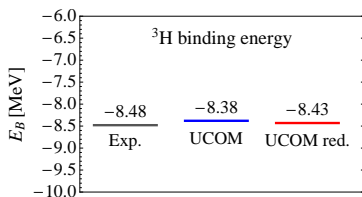
## UCOM - reduced set of operators



# Results

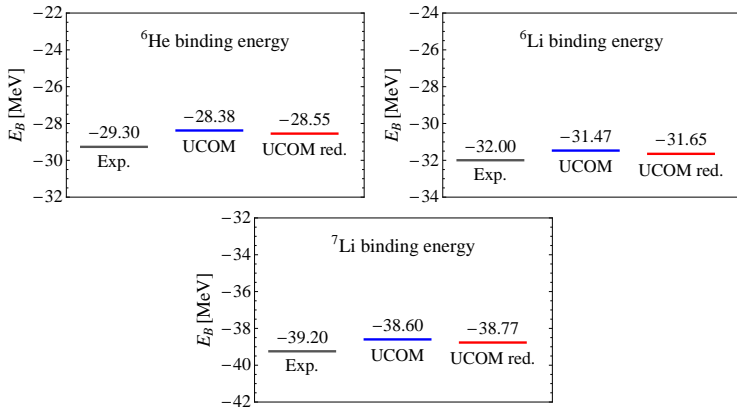
UCOM - reduced set of operators

- 'no core shell model' calculations for light nuclei



# Results

## UCOM - reduced set of operators



- ▶ results for full UCOM and reduced operator form in good agreement
- ▶ neglecting more terms changes few-nucleon properties

- ▶ UCOM fit: local potential, parameterized by gaussians

$$V(\underline{r}) \propto \sum_k \gamma_k \exp \left\{ -\frac{\underline{\tilde{r}}^2}{2\kappa_k} \right\}$$

(+ quadratic momentum dependence)

- ▶ SRG momentum dependence more complicated: fit with nonlocal potential

$$V(\underline{\tilde{r}}^2, \underline{\tilde{p}}^2) \propto \sum_{k,l} \gamma_{kl} \exp \left\{ -\frac{\lambda_l}{4} \underline{\tilde{p}}^2 \right\} \exp \left\{ -\frac{\underline{\tilde{r}}^2}{2(\kappa_k - \lambda_l/4)} \right\} \exp \left\{ -\frac{\lambda_l}{4} \underline{\tilde{p}}^2 \right\}$$

- ▶  $\lambda$  shows how 'local' the potential is:
  - ▶  $\lambda = 0$ : local
  - ▶  $\lambda k_f^2 \ll 1$ : quadratic momentum dependence

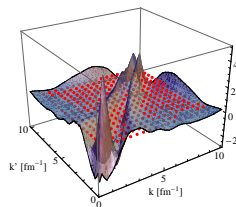
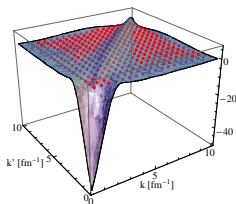
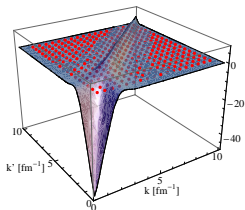
$$V(\underline{\tilde{r}}^2, \underline{\tilde{p}}^2) \rightarrow V^Z(\underline{r}) + \frac{1}{2} (\underline{\tilde{p}}^2 V^{p2}(\underline{r}) + V^{p2}(\underline{r}) \underline{\tilde{p}}^2)$$



# Results

## SRG - partial wave matrix elements

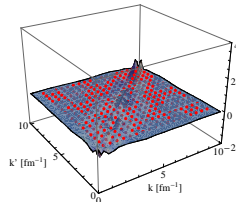
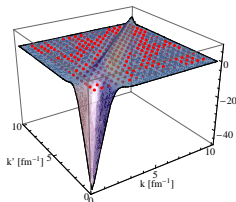
- ▶ SRG matrix element fit with local ( $+p^2$ )



- ▶ and nonlocal Ansatz

$$s = 1.0 \text{ fm}^2$$

matrix elements in  
 $\text{MeV fm}^3$



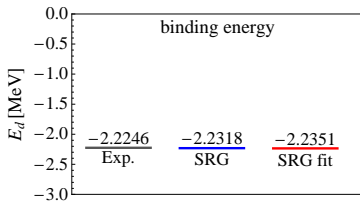
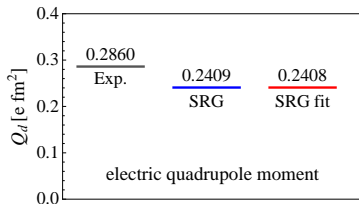
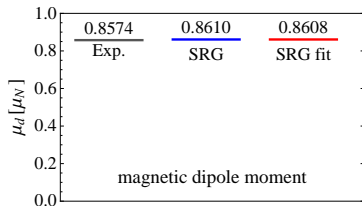
- ▶ operators of Argonne potential, but  $V_{ST}^i(r) \rightarrow V_{ST}^i(\vec{r}^2, \vec{p}^2)$

$$\begin{aligned}
 \tilde{V}_{Ansatz} &= \sum_{S,T} V_{ST}^Z(\vec{r}^2, \vec{p}^2) \tilde{\Pi}_{ST} \\
 &+ \sum_{S,T} V_{ST}^{L2}(\vec{r}^2, \vec{p}^2) \vec{L}^2 \tilde{\Pi}_{ST} \\
 &+ \sum_T V_{1T}^{LS}(\vec{r}^2, \vec{p}^2) \vec{L} \vec{S} \tilde{\Pi}_{1T} \\
 &+ \sum_T V_{1T}^T(\vec{r}^2, \vec{p}^2) \mathcal{S}_{12} \tilde{\Pi}_{1T} \\
 &+ \sum_T V_{1T}^{TLL}(\vec{r}^2, \vec{p}^2) \mathcal{S}_{12}(\vec{L}, \vec{L}) \tilde{\Pi}_{1T} \\
 &+ \dots \mathcal{S}_{12}(r, p_\Omega)?
 \end{aligned}$$

# Results

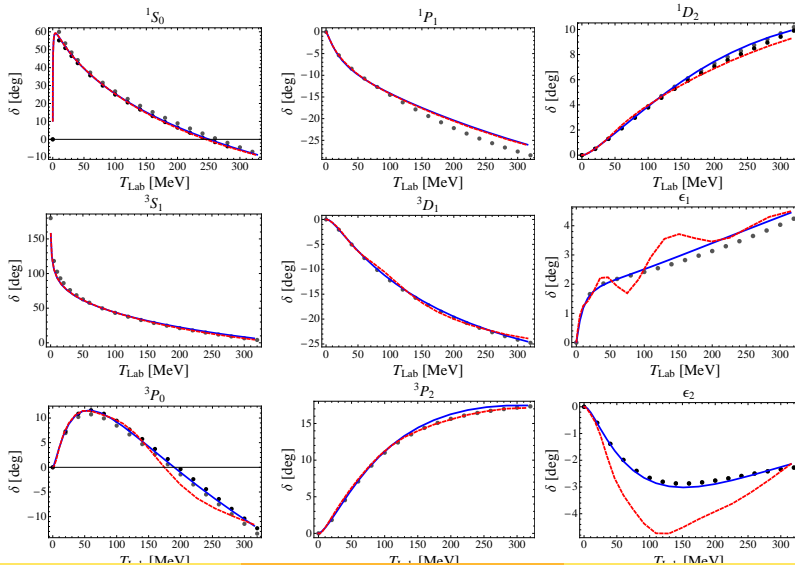
## SRG fit - deuteron

- ▶ deuteron properties:




# Results

## SRG fit - phase-shifts



# Summary

- ▶ operator representation of potentials needed for FMD, AMD
- ▶ operator representation starting from partial wave matrix elements
- ▶ UCOM interaction with reduced set of operators
  - ▶ less complicated structure, but same features
- ▶ operator representation for SRG potentials
  - ▶ nonlocal Ansatz
  - ▶ improve fit of nonlocal matrix elements



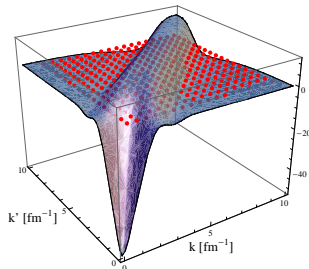
Thanks to my collaborators  
H.Feldmeier  
T.Neff

# Results

## UCOM - reduced set of operators 2

- ▶ which operators do we **really** need?
- ▶ Ansatz

$$\begin{aligned}
 \tilde{V}_{Ansatz} = & \sum_{S,T} V_{ST}^Z(\underline{r}) \tilde{\Pi}_{ST} \\
 & + \sum_{S,T} V_{ST}^{L2}(\underline{r}) \tilde{L}^2 \tilde{\Pi}_{ST} \\
 & + \sum_{S,T} \frac{1}{2} (\tilde{p}^2 V_{ST}^{p2}(\underline{r}) + V_{ST}^{p2}(\underline{r}) \tilde{p}^2) \tilde{\Pi}_{ST} \\
 & + \sum_T V_{1T}^{LS}(\underline{r}) \tilde{L} \tilde{S} \tilde{\Pi}_{1T} \\
 & + \sum_T V_{1T}^{L2LS}(\underline{r}) \tilde{L}^2 \tilde{L} \tilde{S} \tilde{\Pi}_{1T} \\
 & + \sum_T V_{1T}^T(\underline{r}) \tilde{S}_{12} \tilde{\Pi}_{1T} \\
 & + \sum_T V_{1T}^{TLL}(\underline{r}) \tilde{S}_{12}(\tilde{L}, \tilde{L}) \tilde{\Pi}_{1T} \\
 & + \sum_T V_{1T}^{TPP}(\underline{r}) \tilde{S}_{12}(\rho_\Omega, \rho_\Omega) \tilde{\Pi}_{1T} \\
 & + \sum_T (\tilde{p}_r V_{1T}^{Trp}(\underline{r}) + V_{1T}^{Trp}(\underline{r}) \tilde{p}_r) \tilde{S}_{12}(r, \rho_\Omega) \tilde{\Pi}_{1T}
 \end{aligned}$$

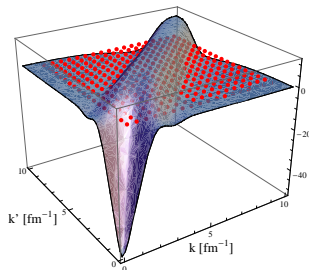


# Results

## UCOM - reduced set of operators 2

- ▶ which operators do we **really** need?
- ▶ Ansatz

$$\begin{aligned}
 \tilde{V}_{\text{Ansatz}} = & \sum_{S,T} V_{ST}^Z(\underline{r}) \tilde{\Pi}_{ST} \\
 & + \sum_{S,T} V_{ST}^{L2}(\underline{r}) \tilde{L}^2 \tilde{\Pi}_{ST} \\
 & + \sum_{S,T} \frac{1}{2} (\tilde{p}^2 V_{ST}^{P2}(\underline{r}) + V_{ST}^{P2}(\underline{r}) \tilde{p}^2) \tilde{\Pi}_{ST} \\
 & + \sum_T V_{1T}^{LS}(\underline{r}) \tilde{L} \tilde{S} \tilde{\Pi}_{1T} \\
 & + \sum_T V_{1T}^{L2LS}(\underline{r}) \tilde{L}^2 \tilde{L} \tilde{S} \tilde{\Pi}_{1T} \\
 & + \sum_T V_{1T}^T(\underline{r}) \tilde{S}_{12} \tilde{\Pi}_{1T} \\
 & + \sum_T V_{1T}^{TLL}(\underline{r}) \tilde{S}_{12}(\tilde{L}, \tilde{L}) \tilde{\Pi}_{1T} \\
 & + \sum_T V_{1T}^{TPP}(\underline{r}) \tilde{S}_{12}(\rho_\Omega, \rho_\Omega) \tilde{\Pi}_{1T} \\
 & + \sum_T (\tilde{p}_r V_{1T}^{Trp}(\underline{r}) + V_{1T}^{Trp}(\underline{r}) \tilde{p}_r) \tilde{S}_{12}(r, \rho_\Omega) \tilde{\Pi}_{1T}
 \end{aligned}$$

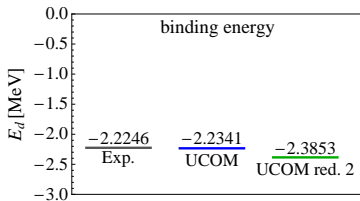
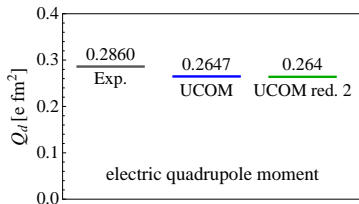
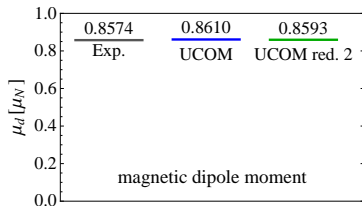




# Results

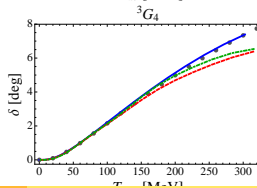
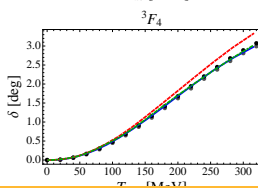
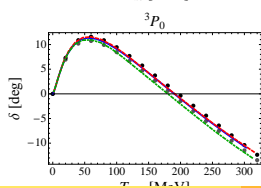
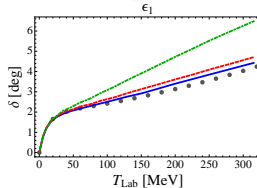
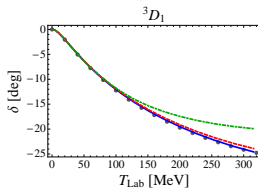
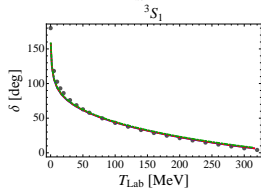
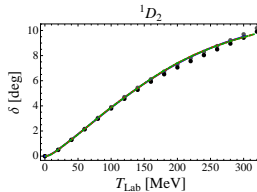
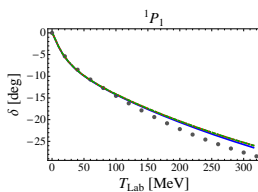
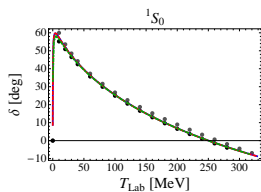
## UCOM - reduced set of operators 2

- ▶ deuteron properties:



# Results

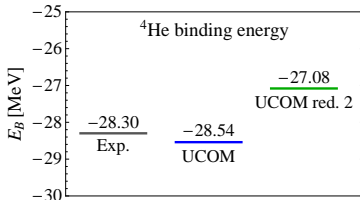
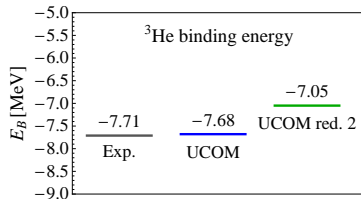
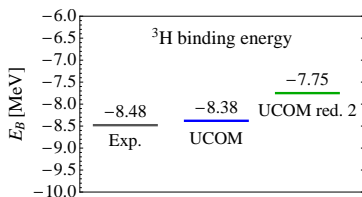
## UCOM - reduced set of operators 2



# Results

## UCOM - reduced set of operators 2

- 'no core shell model' calculations for light nuclei



# Results

## UCOM - reduced set of operators 2

▶ A=6,7 nuclei

